Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/46-1.2.3.2-d-x-^m-a+b-x^n+c-x^-2-n-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [664]. This is test number [46].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (664)	0.00 (0)
Mathematica	99.70 (662)	0.30 (2)
Fricas	80.57 (535)	19.43 (129)
Maple	74.70 (496)	25.30 (168)
Giac	65.51 (435)	34.49 (229)
Mupad	54.22 (360)	45.78 (304)
Maxima	45.63 (303)	54.37 (361)
Sympy	41.27 (274)	58.73 (390)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

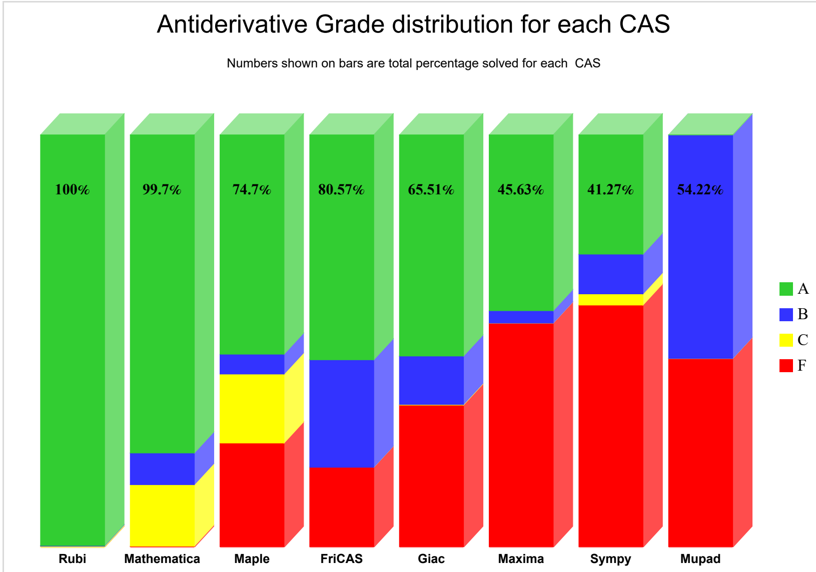
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

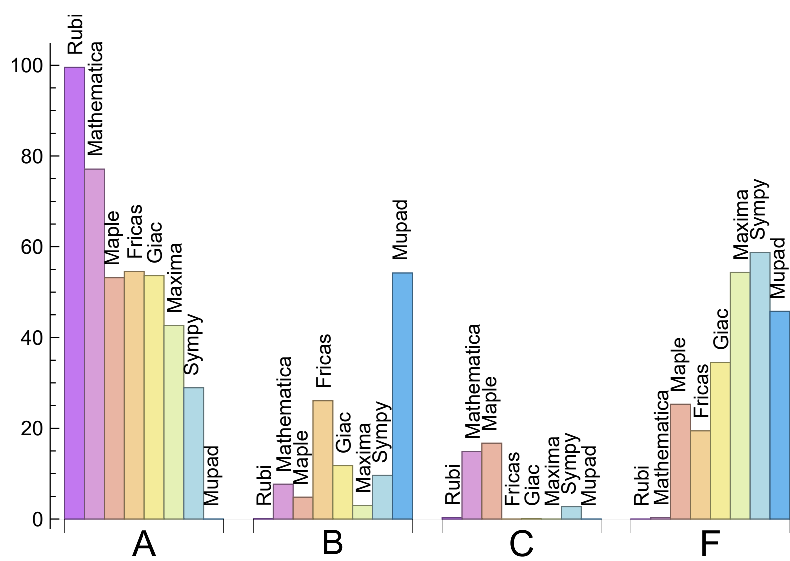
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.55	0.15	0.30	0.00
Mathematica	77.11	7.68	14.91	0.30
Fricas	54.52	26.05	0.00	19.43
Giac	53.61	11.75	0.15	34.49
Maple	53.16	4.82	16.72	25.30
Maxima	42.62	3.01	0.00	54.37
Sympy	28.92	9.64	2.71	58.73
Mupad	N/A	54.22	0.00	45.78

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	168	100.00 %	0.00 %	0.00 %
Fricas	129	66.67 %	10.85 %	22.48 %
Giac	229	96.51 %	1.31 %	2.18 %
Maxima	361	77.56 %	0.00 %	22.44 %
Sympy	390	73.85 %	22.56 %	3.59 %
Mupad	304	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

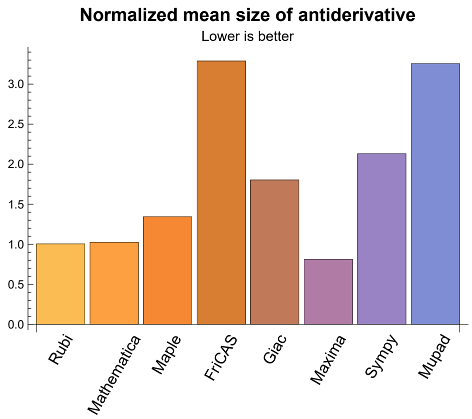
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

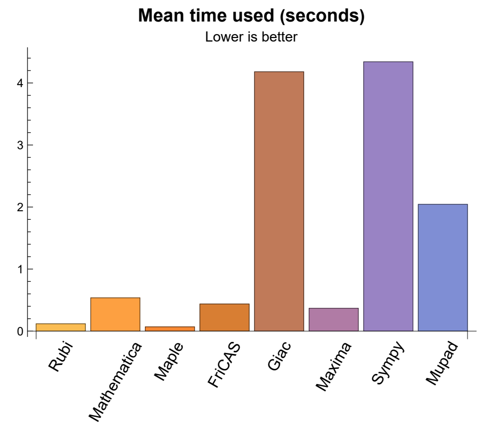
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	157.95	1.00	137.00	1.00
Mathematica	0.54	145.73	1.02	83.00	0.93
Maple	0.07	185.47	1.34	75.00	0.79
Maxima	0.37	82.42	0.81	55.00	0.76
Fricas	0.44	768.25	3.29	142.00	1.48
Sympy	4.34	221.72	2.13	58.00	0.95
Giac	4.18	310.25	1.80	96.00	0.84
Mupad	2.04	573.54	3.25	119.00	0.99

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {132, 229, 230, 231, 250, 256, 257, 258, 260, 261, 262, 263, 265, 266, 268, 269, 309, 327, 347, 367, 385, 602, 605}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

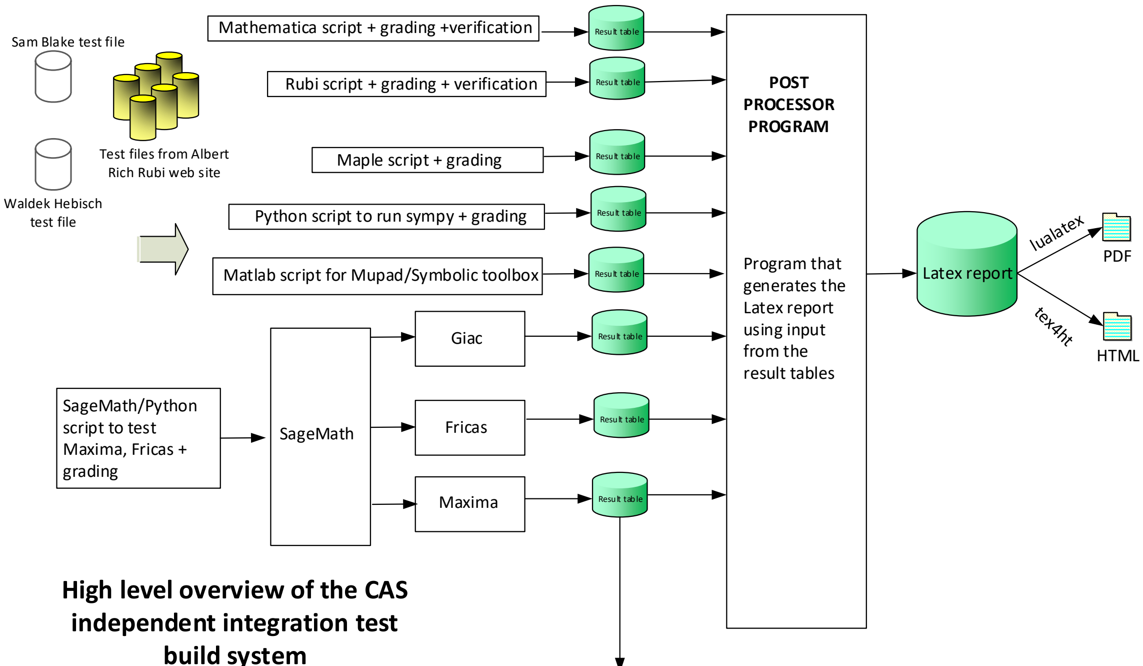
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664 }

B grade: { 154 }

C grade: { 176, 478 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 139, 140, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 241, 242, 248, 249, 253, 254, 255, 256, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 328, 330, 332, 339, 342, 345, 348, 349, 350, 352, 355, 368, 369, 370, 371, 372, 373, 374, 376, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 548, 549, 550, 551, 552, 553, 554, 555, 562, 563, 565, 566, 567, 572, 579, 582, 583, 584, 585, 586, 587, 588, 593, 596, 597, 598, 599, 604, 606, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663 }

B grade: { 61, 154, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 232, 233, 243, 244, 245, 246, 247, 252, 564, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 589, 590, 591, 592, 594, 595, 600, 601, 602, 603, 605, 607, 608, 609, 610, 611, 612, 664 }

C grade: { 132, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 184, 250, 251, 257, 258, 309, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 331, 333, 334, 335, 336, 337, 338, 340, 341, 343, 344, 346, 347, 351, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 409, 410, 411, 457, 458, 478, 500, 501, 502, 503, 504, 505, 546, 547, 556, 557, 558, 559, 560, 561 }

F grade: { 661, 662 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 111, 113, 114, 117, 118, 119, 120, 125, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 237, 238, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 328, 329, 330, 331, 332, 333, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 451, 452, 453, 454, 455, 456, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 502, 503, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 523, 524, 525, 526, 527, 528, 533, 540, 544, 572, 579 }

B grade: { 82, 107, 109, 110, 112, 115, 116, 154, 248, 334, 336, 433, 434, 438, 449, 450, 549, 550, 551, 552, 553, 554, 555, 565, 607, 608, 609, 610, 611, 612, 663, 664 }

C grade: { 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 319, 320, 321, 322, 323, 324, 325, 326, 358, 359, 360, 361, 362, 363, 364, 365, 366, 378, 379, 380, 381, 382, 383, 384, 409, 410, 411, 457, 458, 500, 501, 504, 505, 515, 522, 548, 556, 557, 558, 559, 560, 561, 596, 597, 598, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660 }

F grade: { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 472, 473, 474, 475, 476, 477, 478, 506, 507, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 543, 545, 546, 547, 562, 563, 564, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 83, 84, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 130, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 248, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 348, 350, 352, 354, 356, 368, 370, 372, 374, 376, 386, 387, 388, 390, 392, 393, 394, 395, 405, 406, 407, 408, 409, 410, 440, 441, 442, 443, 444, 445, 446, 447, 448, 455, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 473, 474, 475, 479, 480, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 506, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 533, 540, 548, 596, 597, 598 }

B grade: { 9, 21, 42, 45, 48, 79, 82, 85, 88, 154, 389, 391, 607, 608, 609, 610, 611, 612, 663, 664 }

C grade: { }

F grade: { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 351, 353, 355, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 371, 373, 375, 377, 378, 379, 380, 381, 382, 383, 384, 385, 396, 397, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 472, 476, 477, 478, 481, 500, 501, 502, 503, 504, 505, 507, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 543, 544, 545, 546, 547, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 234, 235, 236, 237, 238, 240, 241, 242, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 294, 296, 298, 310, 312, 314, 316, 318, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 345, 348, 349, 350, 352, 354, 355, 356, 359, 362, 365, 368, 370, 374, 376, 386, 388, 392, 394, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 455, 460, 462, 463, 464, 465, 466, 467, 468, 469, 473, 474, 475, 478, 479, 491, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 533, 540, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 565, 572, 579, 586, 616, 617, 639, 641, 642 }

B grade: { 82, 143, 144, 145, 146, 147, 148, 149, 150, 154, 170, 172, 173, 175, 176, 178, 179, 181, 182, 184, 239, 248, 293, 295, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 313, 315, 317, 319, 320, 321, 322, 323, 324, 325, 326, 343, 344, 346, 351, 353, 357, 358, 360, 361, 363, 364, 366, 369, 371, 372, 373, 375, 377, 378, 379, 380, 381, 382, 383, 384, 387, 389, 390, 391, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 454, 456, 457, 458, 470, 471, 552, 556, 557, 558, 559, 560, 561, 593, 596, 597, 598, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 640, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663, 664 }

C grade: { }

F grade: { 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 196, 197, 198, 199, 200, 214, 215, 216, 217, 218, 229, 230, 231, 232, 233, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 459, 461, 472, 476, 477, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 492, 529, 530, 531, 532, 534, 535, 536, 537, 538, 539, 541, 542, 562, 563, 564, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

2.1.6 Sympy

A grade: { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 89, 90, 91, 92, 93, 94, 95, 96, 97, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 311, 313, 315, 317, 322, 323, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 342, 345, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 388, 390, 392, 394, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 440, 441, 442, 443, 444, 445, 446, 447, 448, 456, 457, 460, 493, 494, 498, 499, 613, 615, 618, 620, 638, 640, 643, 645 }

B grade: { 138, 139, 140, 141, 248, 249, 310, 312, 314, 316, 387, 389, 391, 393, 395, 412, 413, 414, 415, 416, 417, 418, 422, 423, 424, 425, 426, 427, 431, 432, 433, 434, 435, 436, 437, 495, 496, 565, 597, 598, 607, 608, 609, 610, 611, 612, 614, 616, 617, 621, 622, 624, 631, 633, 639, 641, 642, 646, 647, 649, 655, 657, 663, 664 }

C grade: { 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 338, 340, 341, 343, 344, 346 }

F grade: { 1, 2, 3, 4, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 142, 150, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 318, 319, 320, 321, 324, 325, 326, 327, 347, 367, 385, 419, 420, 421, 428, 429, 430, 438, 439, 449, 450, 451, 452, 453, 454, 455, 458, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 497, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 599, 600, 601, 602, 603, 604, 605, 606, 619, 623, 625, 626, 627, 628, 629, 630, 632, 634, 635, 636, 637, 644, 648, 650, 651, 652, 653, 654, 656, 658, 659, 660, 661, 662 }

2.1.7 Giac

A grade: { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 120, 126, 127, 130, 138, 139, 140, 141, 142, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 171, 174, 177, 180, 183, 188, 189, 204, 205, 222, 237, 238, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 314, 316, 318, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 453, 455, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 475, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 496, 500, 501, 502, 515, 516, 517, 518, 523, 524, 525, 552, 614, 616, 619, 622, 624, 628, 637, 641, 663, 664 }

B grade: { 45, 61, 82, 118, 119, 125, 154, 170, 172, 173, 175, 178, 179, 181, 223, 248, 249, 311, 313, 315, 317, 349, 355, 389, 391, 454, 456, 473, 474, 522, 558, 596, 597, 598, 607, 608, 609, 610, 611, 612, 613, 615, 617, 620, 621, 623, 625, 626, 627, 629, 630, 631, 632, 633, 634, 635, 636, 638, 639, 640, 642, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660 }

C grade: { 176 }

F grade: { 1, 2, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 143, 144, 145, 146, 147, 148, 149, 150, 182, 184, 185, 186, 187, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 319, 320, 321, 322, 323, 324, 325, 326, 327, 347, 367, 385, 449, 450, 451, 452, 457, 458, 459, 472, 476, 477, 478, 479, 493, 494, 495, 497, 498, 499, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 519, 520, 521, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 618, 643, 661, 662 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 9, 12, 15, 16, 17, 18, 19, 20, 21, 22, 27, 30, 43, 44, 45, 46, 47, 48, 49, 61, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 94, 97, 100, 108, 111, 125, 126, 127, 130, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 204, 205, 222, 223, 224, 225, 236, 237, 238, 248, 249, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 451, 452, 455, 456, 457, 458, 460, 465, 468, 469, 470, 471, 473, 474, 475, 478, 485, 496, 552, 565, 596, 597, 598, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663, 664 }

C grade: { }

F grade: { 7, 8, 10, 11, 13, 14, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 89, 90, 92, 93, 95, 96, 98, 99, 101, 102, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 128, 129, 131, 132, 133, 134, 135, 136, 137, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 290, 309, 327, 347, 367, 385, 449, 450, 453, 454, 459, 461, 462, 463, 464, 466, 467, 472, 476, 477, 479, 480, 481, 482, 483, 484, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 599, 600, 601, 602, 603, 604, 605, 606, 661, 662 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	46	53	0	0	40
N.S.	1	1.00	0.67	0.75	0.88	1.02	0.00	0.00	0.77
time (sec)	N/A	0.034	0.030	0.128	0.289	0.395	0.000	0.000	1.226

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	0	29
N.S.	1	1.00	1.00	1.16	0.56	1.12	0.00	0.00	1.16
time (sec)	N/A	0.004	0.015	0.129	0.278	0.401	0.000	0.000	1.152

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	17	21	0	14	21
N.S.	1	1.00	1.00	0.96	0.74	0.91	0.00	0.61	0.91
time (sec)	N/A	0.004	0.088	0.128	0.275	0.381	0.000	4.130	1.151

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	46	46	38	54	0	52	51
N.S.	1	1.00	0.60	0.60	0.49	0.70	0.00	0.68	0.66
time (sec)	N/A	0.039	0.184	0.143	0.284	0.460	0.000	5.013	1.282

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	38	37	46	48	38	51
N.S.	1	1.00	1.00	0.79	0.77	0.96	1.00	0.79	1.06
time (sec)	N/A	0.016	0.011	0.157	0.481	0.457	0.055	3.659	0.095

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	83	13	12	23	59
N.S.	1	1.00	0.49	0.46	1.05	0.16	0.15	0.29	0.75
time (sec)	N/A	0.017	0.009	0.050	0.272	0.410	0.010	2.886	1.259

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	12	29	-1
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01
time (sec)	N/A	0.015	0.005	0.054	0.283	0.359	0.010	3.128	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	12	29	-1
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01
time (sec)	N/A	0.015	0.005	0.054	0.286	0.348	0.010	4.122	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	24	52	13	12	22	33
N.S.	1	1.00	1.06	0.67	1.44	0.36	0.33	0.61	0.92
time (sec)	N/A	0.018	0.006	0.049	0.280	0.346	0.011	5.113	1.229

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	13	13	12	29	-1
N.S.	1	1.00	0.49	0.46	0.16	0.16	0.15	0.37	-0.01
time (sec)	N/A	0.013	0.006	0.017	0.269	0.342	0.010	5.553	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	36	33	10	10	8	20	-1
N.S.	1	1.00	0.49	0.45	0.14	0.14	0.11	0.27	-0.01
time (sec)	N/A	0.009	0.004	0.016	0.279	0.339	0.013	6.192	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	37	34	96	11	10	28	109
N.S.	1	1.00	0.49	0.45	1.28	0.15	0.13	0.37	1.45
time (sec)	N/A	0.013	0.006	0.023	0.273	0.348	0.023	6.205	1.377

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	38	36	14	14	8	29	-1
N.S.	1	1.00	0.49	0.47	0.18	0.18	0.10	0.38	-0.01
time (sec)	N/A	0.015	0.007	0.020	0.268	0.331	0.020	6.410	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	37	34	15	15	8	26	-1
N.S.	1	1.00	0.50	0.46	0.20	0.20	0.11	0.35	-0.01
time (sec)	N/A	0.015	0.005	0.018	0.277	0.328	0.025	5.773	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	39	38	99	17	10	43	112
N.S.	1	1.00	0.52	0.51	1.32	0.23	0.13	0.57	1.49
time (sec)	N/A	0.016	0.006	0.019	0.278	0.363	0.039	5.051	1.384

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	37	34	13	13	14	30	33
N.S.	1	1.00	0.48	0.44	0.17	0.17	0.18	0.39	0.43
time (sec)	N/A	0.015	0.005	0.020	0.273	0.338	0.042	4.951	1.208

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.016	0.005	0.020	0.271	0.375	0.045	5.757	1.176

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	37	34	86	13	14	30	33
N.S.	1	1.00	0.47	0.43	1.09	0.16	0.18	0.38	0.42
time (sec)	N/A	0.015	0.006	0.019	0.280	0.344	0.051	5.665	1.184

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.015	0.006	0.019	0.276	0.376	0.055	4.486	1.267

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.014	0.006	0.022	0.275	0.342	0.055	4.189	1.164

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	117	15	15	31	35
N.S.	1	1.00	0.49	0.46	1.48	0.19	0.19	0.39	0.44
time (sec)	N/A	0.016	0.006	0.020	0.282	0.344	0.060	2.559	1.151

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	15	15	15	31	35
N.S.	1	1.00	0.49	0.46	0.19	0.19	0.19	0.39	0.44
time (sec)	N/A	0.016	0.006	0.020	0.276	0.348	0.064	4.996	1.162

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.030	0.012	0.056	0.289	0.327	0.000	4.892	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	167	61	58	114	35	0	67	-1
N.S.	1	1.40	0.51	0.49	0.96	0.29	0.00	0.56	-0.01
time (sec)	N/A	0.036	0.011	0.060	0.280	0.336	0.000	5.584	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.028	0.011	0.057	0.301	0.347	0.000	4.780	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.028	0.010	0.054	0.271	0.379	0.000	4.178	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	61	58	83	35	0	45	46
N.S.	1	1.00	0.78	0.74	1.06	0.45	0.00	0.58	0.59
time (sec)	N/A	0.034	0.010	0.045	0.271	0.358	0.000	3.205	1.249

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.027	0.010	0.064	0.284	0.383	0.000	3.705	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.028	0.010	0.056	0.273	0.356	0.000	3.390	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	60	24	52	35	0	44	36
N.S.	1	1.00	1.67	0.67	1.44	0.97	0.00	1.22	1.00
time (sec)	N/A	0.019	0.010	0.046	0.276	0.341	0.000	3.384	1.220

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	35	35	0	67	-1
N.S.	1	1.00	0.37	0.35	0.21	0.21	0.00	0.40	-0.01
time (sec)	N/A	0.024	0.012	0.055	0.292	0.356	0.000	3.260	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	59	56	32	32	0	64	-1
N.S.	1	1.00	0.36	0.35	0.20	0.20	0.00	0.40	-0.01
time (sec)	N/A	0.022	0.010	0.018	0.292	0.363	0.000	4.485	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	60	57	152	32	0	65	-1
N.S.	1	1.00	0.38	0.36	0.95	0.20	0.00	0.41	-0.01
time (sec)	N/A	0.031	0.014	0.039	0.282	0.373	0.000	5.041	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	37	37	0	67	-1
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	-0.01
time (sec)	N/A	0.028	0.011	0.023	0.280	0.363	0.000	4.673	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	37	37	0	65	-1
N.S.	1	1.00	0.37	0.36	0.23	0.23	0.00	0.40	-0.01
time (sec)	N/A	0.028	0.013	0.024	0.278	0.338	0.000	5.497	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	62	59	156	38	0	85	-1
N.S.	1	1.00	0.39	0.37	0.97	0.24	0.00	0.53	-0.01
time (sec)	N/A	0.033	0.013	0.040	0.280	0.384	0.000	3.989	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	37	37	0	69	-1
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	-0.01
time (sec)	N/A	0.028	0.010	0.023	0.280	0.384	0.000	3.876	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	37	37	0	68	-1
N.S.	1	1.00	0.37	0.36	0.23	0.23	0.00	0.42	-0.01
time (sec)	N/A	0.027	0.012	0.025	0.268	0.354	0.000	3.904	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	61	60	220	39	0	86	-1
N.S.	1	1.00	0.38	0.37	1.36	0.24	0.00	0.53	-0.01
time (sec)	N/A	0.031	0.010	0.028	0.285	0.352	0.000	3.519	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	37	37	0	70	-1
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	-0.01
time (sec)	N/A	0.028	0.009	0.026	0.272	0.363	0.000	3.387	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	61	58	37	37	0	67	-1
N.S.	1	1.00	0.38	0.36	0.23	0.23	0.00	0.41	-0.01
time (sec)	N/A	0.027	0.010	0.023	0.272	0.368	0.000	3.781	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	63	60	253	39	0	85	-1
N.S.	1	1.00	0.39	0.37	1.57	0.24	0.00	0.53	-0.01
time (sec)	N/A	0.033	0.014	0.023	0.288	0.355	0.000	3.760	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	61	58	37	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.42	0.92
time (sec)	N/A	0.028	0.009	0.023	0.281	0.363	0.000	3.691	1.214

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	37	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.029	0.011	0.023	0.276	0.364	0.000	4.201	1.224

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	59	56	148	35	0	68	151
N.S.	1	1.00	1.44	1.37	3.61	0.85	0.00	1.66	3.68
time (sec)	N/A	0.013	0.010	0.024	0.275	0.342	0.000	4.701	1.208

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	37	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.028	0.009	0.024	0.276	0.383	0.000	3.904	1.191

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	37	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.029	0.011	0.026	0.278	0.357	0.000	3.741	1.207

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	61	58	179	37	0	69	151
N.S.	1	1.00	0.73	0.69	2.13	0.44	0.00	0.82	1.80
time (sec)	N/A	0.028	0.009	0.026	0.289	0.347	0.000	3.800	1.209

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	37	37	0	69	151
N.S.	1	1.00	0.37	0.35	0.22	0.22	0.00	0.41	0.90
time (sec)	N/A	0.027	0.010	0.024	0.335	0.376	0.000	4.004	1.215

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.043	0.017	0.063	0.272	0.380	0.000	3.338	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.040	0.013	0.057	0.274	0.373	0.000	3.761	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	83	80	145	57	0	105	-1
N.S.	1	1.00	0.52	0.50	0.91	0.36	0.00	0.66	-0.01
time (sec)	N/A	0.078	0.014	0.056	0.273	0.349	0.000	4.391	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.038	0.013	0.063	0.292	0.389	0.000	4.062	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.038	0.013	0.059	0.288	0.362	0.000	4.860	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	83	80	114	57	0	105	-1
N.S.	1	1.00	0.70	0.67	0.96	0.48	0.00	0.88	-0.01
time (sec)	N/A	0.060	0.015	0.055	0.277	0.372	0.000	4.779	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.038	0.014	0.058	0.279	0.353	0.000	4.441	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.039	0.013	0.055	0.276	0.376	0.000	5.345	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	83	80	83	57	0	67	-1
N.S.	1	1.00	1.06	1.03	1.06	0.73	0.00	0.86	-0.01
time (sec)	N/A	0.036	0.014	0.048	0.275	0.339	0.000	4.880	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	57	57	0	105	-1
N.S.	1	1.00	0.33	0.31	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.037	0.014	0.055	0.287	0.370	0.000	4.548	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	56	56	0	104	-1
N.S.	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.037	0.013	0.059	0.286	0.378	0.000	8.079	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	82	24	52	57	0	66	36
N.S.	1	1.00	2.28	0.67	1.44	1.58	0.00	1.83	1.00
time (sec)	N/A	0.019	0.014	0.051	0.281	0.358	0.000	4.193	1.244

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	56	56	0	104	-1
N.S.	1	1.00	0.33	0.32	0.22	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.035	0.013	0.063	0.275	0.351	0.000	4.443	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	81	78	53	53	0	101	-1
N.S.	1	1.00	0.33	0.32	0.21	0.21	0.00	0.41	-0.00
time (sec)	N/A	0.033	0.013	0.021	0.311	0.432	0.000	3.928	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	82	79	206	55	0	104	-1
N.S.	1	1.00	0.33	0.31	0.82	0.22	0.00	0.41	-0.00
time (sec)	N/A	0.045	0.016	0.024	0.292	0.378	0.000	3.348	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	59	59	0	105	-1
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	-0.00
time (sec)	N/A	0.039	0.014	0.045	0.276	0.335	0.000	4.151	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	59	59	0	103	-1
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.41	-0.00
time (sec)	N/A	0.039	0.015	0.036	0.299	0.348	0.000	3.511	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	214	61	0	124	-1
N.S.	1	1.00	0.34	0.33	0.85	0.24	0.00	0.49	-0.00
time (sec)	N/A	0.049	0.024	0.027	0.318	0.376	0.000	3.799	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	83	80	59	59	0	107	-1
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.041	0.016	0.027	0.272	0.350	0.000	3.038	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	59	59	0	106	-1
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	-0.00
time (sec)	N/A	0.039	0.014	0.026	0.280	0.364	0.000	3.935	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	282	61	0	126	-1
N.S.	1	1.00	0.34	0.33	1.12	0.24	0.00	0.50	-0.00
time (sec)	N/A	0.047	0.016	0.026	0.290	0.402	0.000	3.961	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	83	80	59	59	0	107	-1
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.039	0.014	0.026	0.276	0.346	0.000	3.135	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	59	59	0	105	-1
N.S.	1	1.00	0.34	0.32	0.24	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.038	0.014	0.029	0.277	0.356	0.000	2.901	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	313	61	0	127	-1
N.S.	1	1.00	0.34	0.33	1.24	0.24	0.00	0.50	-0.00
time (sec)	N/A	0.048	0.019	0.058	0.285	0.423	0.000	3.876	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	59	59	0	108	-1
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.43	-0.00
time (sec)	N/A	0.039	0.012	0.027	0.277	0.361	0.000	4.254	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	59	59	0	106	-1
N.S.	1	1.00	0.34	0.32	0.24	0.24	0.00	0.43	-0.00
time (sec)	N/A	0.038	0.011	0.027	0.278	0.368	0.000	4.566	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	85	82	342	61	0	125	-1
N.S.	1	1.00	0.34	0.33	1.36	0.24	0.00	0.50	-0.00
time (sec)	N/A	0.045	0.013	0.028	0.287	0.365	0.000	5.016	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	59	59	0	108	-1
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.43	-0.00
time (sec)	N/A	0.039	0.012	0.027	0.275	0.347	0.000	4.323	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	83	80	59	59	0	105	-1
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.42	-0.00
time (sec)	N/A	0.039	0.013	0.025	0.280	0.362	0.000	3.495	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	85	82	374	61	0	123	-1
N.S.	1	1.00	0.34	0.33	1.49	0.24	0.00	0.49	-0.00
time (sec)	N/A	0.046	0.019	0.026	0.296	0.428	0.000	4.548	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	59	59	0	107	231
N.S.	1	1.00	0.33	0.32	0.24	0.24	0.00	0.43	0.92
time (sec)	N/A	0.039	0.012	0.024	0.279	0.360	0.000	4.304	1.260

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	59	59	0	107	231
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.040	0.012	0.025	0.275	0.380	0.000	4.374	1.323

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	81	78	210	57	0	106	231
N.S.	1	1.00	1.98	1.90	5.12	1.39	0.00	2.59	5.63
time (sec)	N/A	0.014	0.012	0.026	0.285	0.364	0.000	3.464	1.222

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	59	59	0	107	231
N.S.	1	1.00	0.33	0.32	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.039	0.013	0.029	0.275	0.385	0.000	4.079	1.305

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	59	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.039	0.012	0.026	0.271	0.391	0.000	5.558	1.241

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	80	241	59	0	107	231
N.S.	1	1.00	0.99	0.95	2.87	0.70	0.00	1.27	2.75
time (sec)	N/A	0.028	0.012	0.027	0.286	0.365	0.000	4.286	1.217

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	59	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.040	0.012	0.026	0.275	0.362	0.000	3.849	1.220

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	59	59	0	107	231
N.S.	1	1.00	0.33	0.31	0.23	0.23	0.00	0.42	0.91
time (sec)	N/A	0.039	0.012	0.026	0.275	0.348	0.000	5.662	1.230

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	83	80	272	59	0	107	231
N.S.	1	1.00	0.65	0.62	2.12	0.46	0.00	0.84	1.80
time (sec)	N/A	0.040	0.013	0.028	0.304	0.367	0.000	3.256	1.222

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	131	113	109	123	32	146	-1
N.S.	1	1.00	0.55	0.47	0.45	0.51	0.13	0.61	-0.00
time (sec)	N/A	0.079	0.036	0.139	0.493	0.409	0.067	3.378	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	128	110	106	106	22	143	-1
N.S.	1	1.00	0.54	0.47	0.45	0.45	0.09	0.61	-0.00
time (sec)	N/A	0.076	0.021	0.131	0.531	0.372	0.063	2.794	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	35	32	15	13	10	22	33
N.S.	1	1.00	0.80	0.73	0.34	0.30	0.23	0.50	0.75
time (sec)	N/A	0.024	0.006	0.123	0.278	0.372	0.048	3.696	1.392

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	109	97	98	304	24	124	-1
N.S.	1	1.00	0.54	0.48	0.49	1.50	0.12	0.61	-0.00
time (sec)	N/A	0.056	0.017	0.141	0.493	0.379	0.049	4.319	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	109	97	98	299	20	122	-1
N.S.	1	1.00	0.54	0.48	0.49	1.48	0.10	0.60	-0.00
time (sec)	N/A	0.076	0.014	0.128	0.490	0.416	0.057	4.908	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	42	37	43	18	15	32	48
N.S.	1	1.00	0.52	0.46	0.54	0.22	0.19	0.40	0.60
time (sec)	N/A	0.023	0.008	0.148	0.274	0.370	0.103	4.630	1.387

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	133	111	106	103	29	131	-1
N.S.	1	1.00	0.56	0.47	0.45	0.43	0.12	0.55	-0.00
time (sec)	N/A	0.069	0.021	0.135	0.516	0.381	0.076	4.523	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	240	140	118	106	143	32	125	-1
N.S.	1	0.99	0.58	0.49	0.44	0.59	0.13	0.51	-0.00
time (sec)	N/A	0.070	0.023	0.131	0.495	0.389	0.092	4.141	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	122	54	52	73	33	31	50	75
N.S.	1	0.98	0.43	0.42	0.58	0.26	0.25	0.40	0.60
time (sec)	N/A	0.032	0.011	0.161	0.287	0.360	0.149	4.115	1.399

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	235	301	149	512	0	185	-1
N.S.	1	1.00	0.84	1.08	0.53	1.83	0.00	0.66	-0.00
time (sec)	N/A	0.090	0.049	0.034	0.491	0.357	0.000	4.099	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	235	297	146	503	0	176	-1
N.S.	1	1.00	0.85	1.08	0.53	1.82	0.00	0.64	-0.00
time (sec)	N/A	0.089	0.043	0.030	0.507	0.384	0.000	3.832	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	16	26	0	24	34
N.S.	1	1.00	0.71	0.63	0.42	0.68	0.00	0.63	0.89
time (sec)	N/A	0.019	0.008	0.020	0.274	0.361	0.000	3.567	1.193

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	237	301	147	514	0	173	-1
N.S.	1	1.00	0.86	1.09	0.53	1.86	0.00	0.62	-0.00
time (sec)	N/A	0.088	0.047	0.030	0.497	0.382	0.000	4.064	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	235	299	145	499	0	177	-1
N.S.	1	1.00	0.82	1.05	0.51	1.74	0.00	0.62	-0.00
time (sec)	N/A	0.094	0.046	0.030	0.491	0.376	0.000	2.992	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	74	107	88	90	0	87	-1
N.S.	1	1.00	0.50	0.73	0.60	0.61	0.00	0.59	-0.01
time (sec)	N/A	0.055	0.020	0.048	0.279	0.367	0.000	3.391	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	260	316	148	201	0	201	-1
N.S.	1	1.00	0.82	1.00	0.47	0.64	0.00	0.64	-0.00
time (sec)	N/A	0.103	0.055	0.057	0.510	0.348	0.000	4.009	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	266	322	150	242	0	184	-1
N.S.	1	1.00	0.84	1.02	0.47	0.77	0.00	0.58	-0.00
time (sec)	N/A	0.107	0.058	0.040	0.495	0.402	0.000	4.190	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	97	133	117	119	0	120	-1
N.S.	1	1.00	0.52	0.71	0.62	0.63	0.00	0.64	-0.01
time (sec)	N/A	0.066	0.022	0.036	0.279	0.369	0.000	5.016	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	218	519	195	723	0	205	-1
N.S.	1	1.00	0.61	1.45	0.54	2.01	0.00	0.57	-0.00
time (sec)	N/A	0.126	0.086	0.036	0.515	0.373	0.000	4.766	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	39	32	43	58	0	32	42
N.S.	1	1.00	0.50	0.41	0.55	0.74	0.00	0.41	0.54
time (sec)	N/A	0.034	0.012	0.030	0.282	0.341	0.000	4.313	1.284

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	229	521	195	734	0	207	-1
N.S.	1	1.00	0.62	1.42	0.53	1.99	0.00	0.56	-0.00
time (sec)	N/A	0.129	0.090	0.036	0.497	0.360	0.000	4.469	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	221	519	193	723	0	199	-1
N.S.	1	1.00	0.61	1.44	0.54	2.01	0.00	0.55	-0.00
time (sec)	N/A	0.125	0.088	0.038	0.518	0.374	0.000	4.938	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	16	48	0	24	34
N.S.	1	1.00	0.71	0.63	0.42	1.26	0.00	0.63	0.89
time (sec)	N/A	0.020	0.007	0.026	0.273	0.330	0.000	5.912	1.259

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	219	521	191	734	0	195	-1
N.S.	1	1.00	0.61	1.45	0.53	2.04	0.00	0.54	-0.00
time (sec)	N/A	0.129	0.077	0.031	0.506	0.376	0.000	5.615	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	211	519	189	719	0	199	-1
N.S.	1	1.00	0.58	1.43	0.52	1.98	0.00	0.55	-0.00
time (sec)	N/A	0.134	0.074	0.032	0.513	0.368	0.000	3.233	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	96	193	132	178	0	109	-1
N.S.	1	1.00	0.43	0.87	0.59	0.80	0.00	0.49	-0.00
time (sec)	N/A	0.083	0.029	0.038	0.282	0.359	0.000	3.228	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	242	536	192	311	0	223	-1
N.S.	1	1.00	0.61	1.35	0.48	0.78	0.00	0.56	-0.00
time (sec)	N/A	0.144	0.084	0.047	0.507	0.401	0.000	2.941	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	234	542	194	352	0	215	-1
N.S.	1	1.00	0.59	1.36	0.49	0.88	0.00	0.54	-0.00
time (sec)	N/A	0.150	0.090	0.045	0.518	0.370	0.000	3.580	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	119	219	163	207	0	143	-1
N.S.	1	1.00	0.44	0.81	0.61	0.77	0.00	0.53	-0.00
time (sec)	N/A	0.101	0.033	0.040	0.285	0.347	0.000	4.358	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	111	453	243	369	0	900	-1
N.S.	1	1.00	0.35	1.45	0.78	1.18	0.00	2.88	-0.00
time (sec)	N/A	0.095	0.188	0.021	0.282	0.383	0.000	4.224	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	131	199	119	159	0	384	-1
N.S.	1	1.00	0.64	0.97	0.58	0.78	0.00	1.87	-0.00
time (sec)	N/A	0.056	0.074	0.033	0.290	0.351	0.000	3.017	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	53	56	35	35	0	83	-1
N.S.	1	1.00	0.55	0.58	0.36	0.36	0.00	0.86	-0.01
time (sec)	N/A	0.025	0.027	0.012	0.277	0.368	0.000	2.414	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.029	0.011	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.034	0.006	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.037	0.006	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	66	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.028	0.031	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	110	150	115	163	0	375	207
N.S.	1	1.00	0.64	0.87	0.67	0.95	0.00	2.18	1.20
time (sec)	N/A	0.077	0.069	0.020	0.273	0.370	0.000	3.495	1.310

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	77	96	79	108	0	235	137
N.S.	1	1.00	0.59	0.74	0.61	0.83	0.00	1.81	1.05
time (sec)	N/A	0.057	0.049	0.018	0.281	0.347	0.000	3.637	1.219

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	51	58	54	70	0	132	85
N.S.	1	1.00	0.61	0.69	0.64	0.83	0.00	1.57	1.01
time (sec)	N/A	0.039	0.038	0.027	0.280	0.354	0.000	3.032	1.193

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.047	0.018	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.040	0.013	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	32	31	30	37	0	58	46
N.S.	1	1.00	0.78	0.76	0.73	0.90	0.00	1.41	1.12
time (sec)	N/A	0.019	0.040	0.018	0.280	0.452	0.000	3.589	1.155

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	60	51	0	0	0	0	0	-1
N.S.	1	1.03	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.010	0.040	0.007	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	55	204	0	0	0	0	0	-1
N.S.	1	1.04	3.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.119	0.007	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.033	0.005	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.055	0.010	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.013	0.057	0.015	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.033	0.016	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.057	0.018	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	83	0	254	316	75	1758
N.S.	1	1.00	0.96	1.02	0.00	3.14	3.90	0.93	21.70
time (sec)	N/A	0.054	0.036	0.050	0.000	0.390	1.683	3.395	1.981

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	197	223	59	1199
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	19.03
time (sec)	N/A	0.040	0.016	0.040	0.000	0.374	0.820	5.276	1.800

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	129	131	36	174
N.S.	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	4.58
time (sec)	N/A	0.024	0.007	0.023	0.000	0.371	0.346	5.208	1.232

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	65	0	223	253	66	1362
N.S.	1	1.00	0.96	0.94	0.00	3.23	3.67	0.96	19.74
time (sec)	N/A	0.046	0.016	0.033	0.000	0.458	16.284	4.168	1.922

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	92	85	0	293	0	93	2500
N.S.	1	1.00	1.03	0.96	0.00	3.29	0.00	1.04	28.09
time (sec)	N/A	0.088	0.021	0.042	0.000	0.425	0.000	4.384	2.026

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	636	636	70	61	0	6271	279	0	2500
N.S.	1	1.00	0.11	0.10	0.00	9.86	0.44	0.00	3.93
time (sec)	N/A	0.782	0.021	0.224	0.000	2.181	140.557	0.000	12.146

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	70	59	0	5841	196	0	2280
N.S.	1	1.00	0.11	0.09	0.00	9.26	0.31	0.00	3.61
time (sec)	N/A	0.648	0.020	0.023	0.000	1.717	50.674	0.000	3.397

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	44	43	0	4107	175	0	2695
N.S.	1	1.00	0.08	0.08	0.00	7.36	0.31	0.00	4.83
time (sec)	N/A	0.321	0.012	0.033	0.000	0.625	1.442	0.000	8.111

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	42	43	0	2571	122	0	2129
N.S.	1	1.00	0.08	0.08	0.00	4.61	0.22	0.00	3.82
time (sec)	N/A	0.362	0.011	0.023	0.000	0.418	0.995	0.000	7.712

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	43	41	0	2980	158	0	1543
N.S.	1	1.00	0.08	0.07	0.00	5.34	0.28	0.00	2.77
time (sec)	N/A	0.272	0.012	0.026	0.000	0.434	0.813	0.000	5.388

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	558	45	40	0	4279	155	0	2597
N.S.	1	1.00	0.08	0.07	0.00	7.67	0.28	0.00	4.65
time (sec)	N/A	0.349	0.011	0.021	0.000	0.600	3.686	0.000	8.495

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	71	61	0	5924	252	0	2978
N.S.	1	1.00	0.12	0.10	0.00	9.71	0.41	0.00	4.88
time (sec)	N/A	0.473	0.021	0.042	0.000	1.434	2.264	0.000	6.889

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	612	612	75	62	0	6396	0	0	2500
N.S.	1	1.00	0.12	0.10	0.00	10.45	0.00	0.00	4.08
time (sec)	N/A	0.578	0.021	0.039	0.000	1.512	0.000	0.000	10.654

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	27	27	29	29	27
N.S.	1	1.00	1.00	0.80	0.77	0.77	0.83	0.83	0.77
time (sec)	N/A	0.018	0.005	0.033	0.275	0.342	0.041	3.224	1.253

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	22	22	24	22
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.79	0.86	0.79
time (sec)	N/A	0.013	0.004	0.022	0.313	0.352	0.041	3.232	0.046

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	19	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.90	0.81
time (sec)	N/A	0.010	0.003	0.031	0.283	0.408	0.037	4.808	0.055

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	21	21	18	17	17	15	19	16
N.S.	1	2.10	2.10	1.80	1.70	1.70	1.50	1.90	1.60
time (sec)	N/A	0.009	0.003	0.018	0.308	0.341	0.034	4.272	0.383

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	31	23	21	20	24	21
N.S.	1	1.00	1.00	1.15	0.85	0.78	0.74	0.89	0.78
time (sec)	N/A	0.014	0.004	0.023	0.291	0.350	0.049	3.555	1.261

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	36	28	35	29	36	26
N.S.	1	1.00	1.00	1.06	0.82	1.03	0.85	1.06	0.76
time (sec)	N/A	0.022	0.004	0.027	0.286	0.391	0.062	4.193	1.233

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	41	35	40	34	41	32
N.S.	1	1.00	1.00	1.00	0.85	0.98	0.83	1.00	0.78
time (sec)	N/A	0.025	0.004	0.027	0.284	0.353	0.075	4.405	0.043

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	118	94	94	102	144	96	124
N.S.	1	1.00	0.95	0.76	0.76	0.82	1.16	0.77	1.00
time (sec)	N/A	0.081	0.036	0.032	0.508	0.371	0.305	5.465	0.244

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	114	92	92	90	129	94	119
N.S.	1	1.00	0.93	0.75	0.75	0.74	1.06	0.77	0.98
time (sec)	N/A	0.067	0.019	0.031	0.507	0.379	0.308	4.304	1.420

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	89	89	99	134	91	118
N.S.	1	1.00	0.93	0.75	0.75	0.83	1.13	0.76	0.99
time (sec)	N/A	0.054	0.018	0.034	0.500	0.360	0.306	3.657	0.186

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	111	85	85	88	126	87	104
N.S.	1	1.00	0.98	0.75	0.75	0.78	1.12	0.77	0.92
time (sec)	N/A	0.052	0.017	0.026	0.496	0.374	0.306	3.168	0.160

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	107	84	84	106	134	86	114
N.S.	1	1.00	0.96	0.75	0.75	0.95	1.20	0.77	1.02
time (sec)	N/A	0.047	0.016	0.026	0.498	0.372	0.299	7.426	1.371

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	106	84	84	102	110	86	113
N.S.	1	1.00	0.95	0.75	0.75	0.91	0.98	0.77	1.01
time (sec)	N/A	0.045	0.016	0.027	0.503	0.359	0.295	3.176	1.363

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	108	84	84	84	119	86	113
N.S.	1	1.00	0.96	0.75	0.75	0.75	1.06	0.77	1.01
time (sec)	N/A	0.045	0.016	0.025	0.504	0.395	1.128	3.846	1.364

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	107	84	84	124	124	86	110
N.S.	1	1.00	0.96	0.75	0.75	1.11	1.11	0.77	0.98
time (sec)	N/A	0.043	0.016	0.026	0.515	0.370	1.115	3.765	0.225

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	89	89	117	139	91	119
N.S.	1	1.00	0.99	0.75	0.75	0.98	1.17	0.76	1.00
time (sec)	N/A	0.056	0.027	0.035	0.521	0.356	1.094	3.548	1.380

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	113	89	89	126	128	91	118
N.S.	1	1.00	0.95	0.75	0.75	1.06	1.08	0.76	0.99
time (sec)	N/A	0.050	0.034	0.049	0.647	0.366	1.016	4.285	1.362

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	118	94	96	112	141	98	124
N.S.	1	1.00	0.94	0.75	0.76	0.89	1.12	0.78	0.98
time (sec)	N/A	0.067	0.031	0.035	0.572	0.388	1.177	4.253	0.189

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	118	94	96	153	136	98	121
N.S.	1	1.00	0.94	0.75	0.76	1.21	1.08	0.78	0.96
time (sec)	N/A	0.069	0.038	0.039	0.504	0.382	1.005	5.596	1.399

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	59	44	0	768	26	641	320
N.S.	1	1.00	0.14	0.11	0.00	1.86	0.06	1.56	0.78
time (sec)	N/A	0.279	0.009	0.017	0.000	0.419	0.068	4.863	1.821

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.025	0.007	0.020	0.595	0.347	0.043	4.316	1.214

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	41	40	0	994	26	827	304
N.S.	1	1.00	0.10	0.10	0.00	2.42	0.06	2.01	0.74
time (sec)	N/A	0.187	0.007	0.019	0.000	0.450	0.066	3.765	1.717

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	39	40	0	765	24	640	327
N.S.	1	1.00	0.09	0.10	0.00	1.86	0.06	1.56	0.80
time (sec)	N/A	0.181	0.006	0.020	0.000	0.423	0.067	3.669	1.840

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	20
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87
time (sec)	N/A	0.016	0.005	0.015	0.554	0.364	0.037	3.592	1.217

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	40	38	0	994	26	815	304
N.S.	1	1.00	0.11	0.10	0.00	2.65	0.07	2.17	0.81
time (sec)	N/A	0.156	0.006	0.020	0.000	0.447	0.066	4.179	0.451

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	375	42	37	0	767	20	632	327
N.S.	1	2.02	0.23	0.20	0.00	4.12	0.11	3.40	1.76
time (sec)	N/A	0.151	0.006	0.014	0.000	0.427	0.070	3.950	1.788

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	35	38	34	41	35	36
N.S.	1	1.00	1.34	0.85	0.93	0.83	1.00	0.85	0.88
time (sec)	N/A	0.026	0.008	0.021	0.524	0.391	0.053	3.787	1.231

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	61	50	0	1007	24	829	286
N.S.	1	1.00	0.15	0.12	0.00	2.42	0.06	1.99	0.69
time (sec)	N/A	0.176	0.009	0.024	0.000	0.482	0.076	4.202	1.656

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	65	50	0	800	31	645	324
N.S.	1	1.00	0.16	0.12	0.00	1.91	0.07	1.54	0.78
time (sec)	N/A	0.207	0.009	0.023	0.000	0.409	0.080	4.339	1.717

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	40	43	51	48	45	41
N.S.	1	1.00	1.06	0.83	0.90	1.06	1.00	0.94	0.85
time (sec)	N/A	0.035	0.009	0.024	0.528	0.349	0.063	4.377	0.064

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	54	51	0	1034	39	839	318
N.S.	1	1.00	0.13	0.12	0.00	2.44	0.09	1.98	0.75
time (sec)	N/A	0.245	0.009	0.024	0.000	0.437	0.083	4.282	1.592

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	38	33	0	1491	24	0	513
N.S.	1	1.00	0.10	0.09	0.00	3.91	0.06	0.00	1.35
time (sec)	N/A	0.260	0.006	0.015	0.000	0.797	0.053	0.000	2.615

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	20
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87
time (sec)	N/A	0.017	0.006	0.029	0.535	0.359	0.035	3.367	0.046

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	37	36	0	1048	24	0	351
N.S.	1	1.00	0.09	0.09	0.00	2.63	0.06	0.00	0.88
time (sec)	N/A	0.190	0.006	0.016	0.000	0.431	0.052	0.000	2.614

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	206	0	0	451	0	0	543
N.S.	1	1.00	0.89	0.00	0.00	1.95	0.00	0.00	2.35
time (sec)	N/A	0.200	0.419	0.006	0.000	0.394	0.000	0.000	2.939

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	166	0	0	367	0	0	315
N.S.	1	1.00	0.97	0.00	0.00	2.15	0.00	0.00	1.84
time (sec)	N/A	0.106	0.292	0.004	0.000	0.381	0.000	0.000	1.866

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	132	0	0	303	0	0	193
N.S.	1	1.00	0.86	0.00	0.00	1.98	0.00	0.00	1.26
time (sec)	N/A	0.088	0.220	0.006	0.000	0.368	0.000	0.000	1.591

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	101	0	0	237	0	98	87
N.S.	1	1.00	0.94	0.00	0.00	2.19	0.00	0.91	0.81
time (sec)	N/A	0.055	0.203	0.004	0.000	0.375	0.000	3.925	1.390

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	85	0	0	197	0	76	72
N.S.	1	1.00	1.02	0.00	0.00	2.37	0.00	0.92	0.87
time (sec)	N/A	0.040	0.137	0.004	0.000	0.381	0.000	4.528	1.390

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	105	0	0	566	0	0	88
N.S.	1	1.00	0.96	0.00	0.00	5.19	0.00	0.00	0.81
time (sec)	N/A	0.068	0.173	0.007	0.000	0.409	0.000	0.000	1.362

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	107	0	0	601	0	0	91
N.S.	1	1.00	0.96	0.00	0.00	5.37	0.00	0.00	0.81
time (sec)	N/A	0.071	0.167	0.004	0.000	0.397	0.000	0.000	1.553

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	91	0	0	215	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	2.44	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.228	0.005	0.000	0.395	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	108	0	0	259	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	2.23	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.407	0.004	0.000	0.405	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	141	0	0	325	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.502	0.004	0.000	0.417	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	176	0	0	389	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.671	0.006	0.000	0.469	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	358	0	0	0	0	0	-1
N.S.	1	1.00	2.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	9.190	0.004	0.000	0.000	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	337	0	0	0	0	0	-1
N.S.	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	10.285	0.002	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	335	0	0	0	0	0	-1
N.S.	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	10.239	0.007	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	340	0	0	0	0	0	-1
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	10.252	0.004	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	340	0	0	0	0	0	-1
N.S.	1	1.00	2.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	10.209	0.003	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	289	0	0	641	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.255	0.851	0.005	0.000	0.411	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	220	0	0	535	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	2.40	0.00	0.00	-0.00
time (sec)	N/A	0.140	0.638	0.006	0.000	0.433	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	194	0	0	451	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	2.21	0.00	0.00	-0.00
time (sec)	N/A	0.124	0.527	0.005	0.000	0.394	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	140	0	0	361	0	172	223
N.S.	1	1.00	0.93	0.00	0.00	2.41	0.00	1.15	1.49
time (sec)	N/A	0.080	0.379	0.005	0.000	0.405	0.000	3.653	1.575

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	110	0	0	297	0	135	115
N.S.	1	1.00	0.89	0.00	0.00	2.40	0.00	1.09	0.93
time (sec)	N/A	0.058	0.308	0.004	0.000	0.392	0.000	3.461	1.444

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	143	0	0	727	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	4.69	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.486	0.007	0.000	0.479	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	131	0	0	713	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	4.75	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.488	0.004	0.000	0.452	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	131	0	0	713	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	4.72	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.511	0.004	0.000	0.443	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	148	0	0	771	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	4.73	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.701	0.003	0.000	0.487	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	118	0	0	319	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	2.40	0.00	0.00	-0.01
time (sec)	N/A	0.075	0.640	0.004	0.000	0.425	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	160	0	0	383	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.950	0.004	0.000	0.465	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	201	0	0	473	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.138	1.322	0.005	0.000	0.549	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	244	0	0	557	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	2.18	0.00	0.00	-0.00
time (sec)	N/A	0.207	1.539	0.006	0.000	0.651	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	453	0	0	0	0	0	-1
N.S.	1	1.00	3.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	10.562	0.004	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	410	0	0	0	0	0	-1
N.S.	1	1.00	2.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	10.462	0.003	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	408	0	0	0	0	0	-1
N.S.	1	1.00	3.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	10.432	0.005	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	379	0	0	0	0	0	-1
N.S.	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	10.328	0.005	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	379	0	0	0	0	0	-1
N.S.	1	1.00	2.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	10.326	0.005	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	138	0	0	303	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	1.77	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.255	0.009	0.000	0.385	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	101	0	0	241	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.216	0.008	0.000	0.404	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	0	0	203	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.179	0.006	0.000	0.396	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	70	0	0	161	0	61	55
N.S.	1	1.00	1.03	0.00	0.00	2.37	0.00	0.90	0.81
time (sec)	N/A	0.035	0.115	0.006	0.000	0.428	0.000	3.866	1.492

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	0	0	118	0	76	34
N.S.	1	1.00	0.95	0.00	0.00	2.74	0.00	1.77	0.79
time (sec)	N/A	0.024	0.076	0.007	0.000	0.363	0.000	3.662	1.575

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	124	0	0	36
N.S.	1	1.00	1.00	0.00	0.00	2.82	0.00	0.00	0.82
time (sec)	N/A	0.025	0.073	0.007	0.000	0.362	0.000	0.000	1.571

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	179	0	0	56
N.S.	1	1.00	1.00	0.00	0.00	2.49	0.00	0.00	0.78
time (sec)	N/A	0.040	0.135	0.005	0.000	0.389	0.000	0.000	1.560

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	91	0	0	221	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	2.05	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.222	0.008	0.000	0.382	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	110	0	0	263	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.346	0.007	0.000	0.408	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	141	0	0	327	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.470	0.007	0.000	0.484	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	168	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	10.068	0.008	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	168	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	10.072	0.005	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	163	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.051	10.049	0.006	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	343	0	0	0	0	0	-1
N.S.	1	1.00	2.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	10.228	0.005	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	342	0	0	0	0	0	-1
N.S.	1	1.00	2.44	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	10.203	0.005	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	170	0	0	591	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	3.03	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.672	0.006	0.000	0.423	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	0	0	459	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	3.35	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.450	0.005	0.000	0.462	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	97	0	0	387	0	0	84
N.S.	1	1.00	0.81	0.00	0.00	3.22	0.00	0.00	0.70
time (sec)	N/A	0.066	0.376	0.007	0.000	0.391	0.000	0.000	1.658

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	0	68	0	45	38
N.S.	1	1.00	1.00	0.97	0.00	1.74	0.00	1.15	0.97
time (sec)	N/A	0.022	0.251	0.026	0.000	0.360	0.000	2.978	1.429

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	37	0	67	0	45	37
N.S.	1	1.00	1.00	0.97	0.00	1.76	0.00	1.18	0.97
time (sec)	N/A	0.017	0.252	0.026	0.000	0.367	0.000	2.958	1.370

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	131	0	0	389	0	0	-1
N.S.	1	1.00	1.42	0.00	0.00	4.23	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.416	0.007	0.000	0.428	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	125	0	0	485	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	3.42	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.534	0.004	0.000	0.397	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	166	0	0	615	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.145	0.720	0.006	0.000	0.513	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	210	0	0	705	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	2.75	0.00	0.00	-0.00
time (sec)	N/A	0.199	0.951	0.006	0.000	0.547	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	340	0	0	0	0	0	-1
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	10.209	0.007	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	362	0	0	0	0	0	-1
N.S.	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	10.338	0.006	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	359	0	0	0	0	0	-1
N.S.	1	1.00	2.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	10.323	0.004	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	407	0	0	0	0	0	-1
N.S.	1	1.00	2.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	10.498	0.006	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	405	0	0	0	0	0	-1
N.S.	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	10.444	0.005	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	301	110	241	1459	449	260
N.S.	1	1.00	0.69	2.98	1.09	2.39	14.45	4.45	2.57
time (sec)	N/A	0.040	0.544	0.014	0.341	0.411	0.807	4.338	1.524

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	51	50	71	299	119	89
N.S.	1	1.00	0.67	0.98	0.96	1.37	5.75	2.29	1.71
time (sec)	N/A	0.013	0.045	0.018	0.292	0.358	0.312	3.255	1.358

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	84	0	0	0	0	0	-1
N.S.	1	1.00	0.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.097	0.020	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	78	0	0	0	0	0	-1
N.S.	1	1.00	0.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.477	0.151	0.007	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	357	0	0	0	0	0	-1
N.S.	1	1.00	2.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.778	0.006	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.502	0.006	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	181	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.706	0.008	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	221	0	0	0	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	5.784	0.006	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	179	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.185	0.027	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	162	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.325	0.007	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	162	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.252	0.019	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	138	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.150	0.012	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.383	0.014	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.403	0.013	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.401	0.009	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	161	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.114	0.007	0.000	0.000	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	157	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.209	0.006	0.000	0.000	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	164	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.388	0.011	0.000	0.000	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.404	0.013	0.000	0.000	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	162	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.256	0.014	0.000	0.000	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.414	0.017	0.000	0.000	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	166	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.398	0.019	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	164	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.287	0.020	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	34	0	0	0	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.006	0.024	0.006	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	24	25	24	31	22	24	25
N.S.	1	1.00	0.80	0.83	0.80	1.03	0.73	0.80	0.83
time (sec)	N/A	0.009	0.011	0.020	0.520	0.333	0.041	3.174	0.047

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	18	23	15	18	18
N.S.	1	1.00	0.82	0.86	0.82	1.05	0.68	0.82	0.82
time (sec)	N/A	0.007	0.005	0.016	0.289	0.330	0.033	3.737	1.315

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	19	24	15	19	21
N.S.	1	1.00	1.00	0.87	0.83	1.04	0.65	0.83	0.91
time (sec)	N/A	0.006	0.007	0.017	0.507	0.341	0.040	4.722	1.365

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	9	8	9	11
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	1.00
time (sec)	N/A	0.002	0.002	0.013	0.282	0.345	0.027	4.345	0.018

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	19	23	15	19	20
N.S.	1	1.00	0.87	0.87	0.83	1.00	0.65	0.83	0.87
time (sec)	N/A	0.005	0.004	0.017	0.504	0.327	0.038	3.600	0.026

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	24	32	19	29	20
N.S.	1	1.00	1.00	0.88	1.00	1.33	0.79	1.21	0.83
time (sec)	N/A	0.009	0.007	0.037	0.306	0.332	0.050	3.690	0.044

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	25	31	26	25	25
N.S.	1	1.00	1.00	0.83	0.83	1.03	0.87	0.83	0.83
time (sec)	N/A	0.009	0.008	0.023	0.555	0.333	0.052	3.837	0.040

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	33	44	31	33	31
N.S.	1	1.00	1.00	0.85	1.00	1.33	0.94	1.00	0.94
time (sec)	N/A	0.011	0.008	0.023	0.333	0.337	0.060	3.248	0.054

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	28	30	36	29	31	30
N.S.	1	1.00	0.89	0.76	0.81	0.97	0.78	0.84	0.81
time (sec)	N/A	0.010	0.008	0.028	0.518	0.331	0.062	4.481	0.046

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	94	64	83	137	90	83	45
N.S.	1	1.00	0.90	0.62	0.80	1.32	0.87	0.80	0.43
time (sec)	N/A	0.036	0.046	0.037	0.512	0.353	0.069	5.039	1.369

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	93	65	84	134	90	84	47
N.S.	1	1.00	0.94	0.66	0.85	1.35	0.91	0.85	0.47
time (sec)	N/A	0.034	0.040	0.017	0.507	0.351	0.067	6.492	1.326

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	63	82	131	82	82	45
N.S.	1	1.00	0.93	0.65	0.85	1.35	0.85	0.85	0.46
time (sec)	N/A	0.035	0.043	0.018	0.502	0.368	0.061	6.860	0.082

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	92	65	84	133	83	84	46
N.S.	1	1.00	0.93	0.66	0.85	1.34	0.84	0.85	0.46
time (sec)	N/A	0.037	0.033	0.014	0.509	0.387	0.063	4.338	0.045

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	91	63	82	132	88	82	44
N.S.	1	1.00	0.94	0.65	0.85	1.36	0.91	0.85	0.45
time (sec)	N/A	0.035	0.032	0.015	0.505	0.349	0.065	4.099	1.305

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	98	70	88	135	97	88	49
N.S.	1	1.00	0.92	0.66	0.83	1.27	0.92	0.83	0.46
time (sec)	N/A	0.037	0.047	0.023	0.544	0.346	0.076	2.757	1.314

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	96	68	90	145	99	87	51
N.S.	1	1.00	0.91	0.64	0.85	1.37	0.93	0.82	0.48
time (sec)	N/A	0.037	0.050	0.023	0.526	0.349	0.080	3.307	1.365

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	103	75	95	150	102	96	55
N.S.	1	1.00	0.91	0.66	0.84	1.33	0.90	0.85	0.49
time (sec)	N/A	0.038	0.052	0.024	0.514	0.343	0.086	4.309	0.093

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	101	73	95	150	102	94	55
N.S.	1	1.00	0.89	0.65	0.84	1.33	0.90	0.83	0.49
time (sec)	N/A	0.040	0.051	0.032	0.500	0.358	0.112	5.635	0.104

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	32	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.004	0.105	0.006	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	39	41	34	46	34	35	26
N.S.	1	1.00	1.22	1.28	1.06	1.44	1.06	1.09	0.81
time (sec)	N/A	0.010	0.016	0.040	0.289	0.339	0.041	3.797	0.047

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	22	19	18	23	15	19	20
N.S.	1	1.00	0.85	0.73	0.69	0.88	0.58	0.73	0.77
time (sec)	N/A	0.009	0.006	0.016	0.280	0.348	0.033	3.209	0.048

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	36	29	40	26	30	21
N.S.	1	1.00	1.32	1.44	1.16	1.60	1.04	1.20	0.84
time (sec)	N/A	0.007	0.008	0.020	0.284	0.347	0.039	3.115	1.269

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	9	8	9	11
N.S.	1	1.00	0.85	0.77	0.69	0.69	0.62	0.69	0.85
time (sec)	N/A	0.002	0.002	0.014	0.321	0.326	0.037	4.121	0.022

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	36	29	40	26	30	21
N.S.	1	1.00	1.32	1.44	1.16	1.60	1.04	1.20	0.84
time (sec)	N/A	0.006	0.006	0.020	0.283	0.335	0.039	4.706	0.035

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	47	24	32	19	30	22
N.S.	1	1.00	0.93	1.68	0.86	1.14	0.68	1.07	0.79
time (sec)	N/A	0.010	0.007	0.029	0.310	0.337	0.045	5.773	0.058

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	50	37	54	36	38	26
N.S.	1	1.00	1.28	1.56	1.16	1.69	1.12	1.19	0.81
time (sec)	N/A	0.009	0.012	0.034	0.286	0.341	0.054	5.854	0.044

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	54	35	50	29	36	32
N.S.	1	1.00	0.95	1.46	0.95	1.35	0.78	0.97	0.86
time (sec)	N/A	0.013	0.009	0.031	0.297	0.362	0.057	5.203	0.052

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	49	55	42	59	41	42	32
N.S.	1	1.00	1.26	1.41	1.08	1.51	1.05	1.08	0.82
time (sec)	N/A	0.011	0.010	0.037	0.335	0.336	0.067	4.970	0.051

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	43	28	49	32	30	26
N.S.	1	1.00	1.12	1.26	0.82	1.44	0.94	0.88	0.76
time (sec)	N/A	0.006	0.011	0.029	0.515	0.332	0.058	3.649	1.285

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	35	42	29	46	32	31	23
N.S.	1	1.00	1.21	1.45	1.00	1.59	1.10	1.07	0.79
time (sec)	N/A	0.006	0.010	0.029	0.530	0.333	0.058	3.024	0.034

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	31	42	27	43	26	29	21
N.S.	1	1.00	1.15	1.56	1.00	1.59	0.96	1.07	0.78
time (sec)	N/A	0.005	0.010	0.027	0.538	0.345	0.056	3.098	0.034

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	33	42	29	45	27	31	23
N.S.	1	1.00	1.14	1.45	1.00	1.55	0.93	1.07	0.79
time (sec)	N/A	0.005	0.008	0.027	0.554	0.358	0.056	4.178	0.032

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	33	42	27	44	31	29	21
N.S.	1	1.00	1.22	1.56	1.00	1.63	1.15	1.07	0.78
time (sec)	N/A	0.004	0.007	0.026	0.565	0.378	0.060	5.017	0.029

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	40	47	35	55	37	37	26
N.S.	1	1.00	1.11	1.31	0.97	1.53	1.03	1.03	0.72
time (sec)	N/A	0.007	0.012	0.032	0.514	0.348	0.069	4.513	0.042

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	47	37	63	39	34	28
N.S.	1	1.00	1.06	1.31	1.03	1.75	1.08	0.94	0.78
time (sec)	N/A	0.007	0.013	0.034	0.519	0.320	0.091	3.283	1.292

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	51	52	42	68	44	43	34
N.S.	1	1.00	1.19	1.21	0.98	1.58	1.02	1.00	0.79
time (sec)	N/A	0.008	0.015	0.035	0.503	0.340	0.081	3.407	0.044

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	52	42	68	44	41	34
N.S.	1	1.00	1.00	1.21	0.98	1.58	1.02	0.95	0.79
time (sec)	N/A	0.008	0.014	0.035	0.543	0.355	0.098	2.937	0.047

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	82	0	0	0	0	0	-1
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.241	0.006	0.000	0.000	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	83	0	254	316	75	2500
N.S.	1	1.00	0.96	1.02	0.00	3.14	3.90	0.93	30.86
time (sec)	N/A	0.059	0.035	0.043	0.000	0.390	2.755	6.999	2.687

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	210	176	0	1071	134	2041	2500
N.S.	1	1.00	1.09	0.92	0.00	5.58	0.70	10.63	13.02
time (sec)	N/A	0.227	0.079	0.050	0.000	0.361	2.877	6.303	3.071

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	197	223	59	2654
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	42.13
time (sec)	N/A	0.041	0.016	0.032	0.000	0.358	1.323	8.358	2.608

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	171	153	0	567	76	1034	1220
N.S.	1	1.00	1.08	0.96	0.00	3.57	0.48	6.50	7.67
time (sec)	N/A	0.104	0.056	0.045	0.000	0.374	1.500	5.985	2.810

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	37	0	129	131	36	260
N.S.	1	1.00	1.11	0.97	0.00	3.39	3.45	0.95	6.84
time (sec)	N/A	0.024	0.007	0.021	0.000	0.367	0.446	7.812	1.370

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	133	121	0	619	88	1030	1105
N.S.	1	1.00	0.86	0.79	0.00	4.02	0.57	6.69	7.18
time (sec)	N/A	0.076	0.051	0.045	0.000	0.413	2.505	7.421	2.271

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	66	0	223	253	68	1690
N.S.	1	1.00	0.96	0.96	0.00	3.23	3.67	0.99	24.49
time (sec)	N/A	0.047	0.016	0.037	0.000	0.389	98.459	7.260	2.185

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	75	159	0	1134	153	2055	2500
N.S.	1	1.00	0.41	0.86	0.00	6.16	0.83	11.17	13.59
time (sec)	N/A	0.166	0.021	0.051	0.000	0.414	146.825	5.641	2.416

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	92	84	0	293	0	94	2500
N.S.	1	1.00	1.03	0.94	0.00	3.29	0.00	1.06	28.09
time (sec)	N/A	0.089	0.021	0.049	0.000	0.471	0.000	8.910	2.788

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	70	63	0	6646	0	0	2500
N.S.	1	1.00	0.18	0.17	0.00	17.44	0.00	0.00	6.56
time (sec)	N/A	0.419	0.027	0.065	0.000	2.560	0.000	0.000	3.491

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	70	59	0	5310	0	0	2500
N.S.	1	1.00	0.19	0.16	0.00	14.12	0.00	0.00	6.65
time (sec)	N/A	0.400	0.025	0.025	0.000	0.992	0.000	0.000	3.969

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	44	43	0	4054	0	0	2500
N.S.	1	1.00	0.14	0.13	0.00	12.47	0.00	0.00	7.69
time (sec)	N/A	0.215	0.017	0.023	0.000	0.561	0.000	0.000	3.511

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	42	43	0	2479	126	0	2500
N.S.	1	1.00	0.13	0.13	0.00	7.63	0.39	0.00	7.69
time (sec)	N/A	0.201	0.015	0.025	0.000	0.417	3.159	0.000	3.633

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	43	43	0	2762	172	0	2500
N.S.	1	1.00	0.14	0.14	0.00	8.77	0.55	0.00	7.94
time (sec)	N/A	0.202	0.016	0.023	0.000	0.425	3.078	0.000	2.336

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	45	40	0	4041	0	0	2500
N.S.	1	1.00	0.14	0.13	0.00	12.83	0.00	0.00	7.94
time (sec)	N/A	0.207	0.017	0.021	0.000	0.531	0.000	0.000	3.416

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	71	63	0	5375	0	0	2500
N.S.	1	1.00	0.20	0.17	0.00	14.81	0.00	0.00	6.89
time (sec)	N/A	0.288	0.025	0.037	0.000	1.089	0.000	0.000	2.809

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	75	62	0	6664	0	0	2500
N.S.	1	1.00	0.21	0.17	0.00	18.26	0.00	0.00	6.85
time (sec)	N/A	0.265	0.028	0.041	0.000	1.834	0.000	0.000	5.568

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	488	0	0	0	0	0	-1
N.S.	1	1.00	3.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.943	0.008	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	35	35	42	35	37
N.S.	1	1.00	1.00	0.82	0.80	0.80	0.95	0.80	0.84
time (sec)	N/A	0.025	0.010	0.022	0.498	0.353	0.050	5.224	0.046

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	98	43	42	40	51	42	43
N.S.	1	1.00	1.81	0.80	0.78	0.74	0.94	0.78	0.80
time (sec)	N/A	0.037	0.110	0.028	0.497	0.337	0.047	6.340	0.042

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	37	30	32
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86
time (sec)	N/A	0.021	0.006	0.019	0.544	0.339	0.045	5.356	0.040

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	94	62	61	61	76	61	51
N.S.	1	1.00	1.25	0.83	0.81	0.81	1.01	0.81	0.68
time (sec)	N/A	0.050	0.073	0.023	0.494	0.350	0.083	4.732	0.093

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	26	18	17
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	0.74
time (sec)	N/A	0.015	0.005	0.017	0.508	0.338	0.048	3.019	1.302

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	62	61	61	76	61	51
N.S.	1	1.00	1.05	0.83	0.81	0.81	1.01	0.81	0.68
time (sec)	N/A	0.044	0.030	0.021	0.481	0.349	0.090	3.319	1.275

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	138	87	36	32	41	36	34
N.S.	1	1.00	3.54	2.23	0.92	0.82	1.05	0.92	0.87
time (sec)	N/A	0.023	0.056	0.029	0.523	0.366	0.064	2.943	1.295

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	100	57	42	45	53	42	43
N.S.	1	1.00	1.85	1.06	0.78	0.83	0.98	0.78	0.80
time (sec)	N/A	0.035	0.033	0.029	0.506	0.349	0.059	3.051	0.036

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	141	94	41	49	48	46	41
N.S.	1	1.00	2.94	1.96	0.85	1.02	1.00	0.96	0.85
time (sec)	N/A	0.035	0.069	0.032	0.497	0.373	0.070	3.098	0.063

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	142	95	73	84	88	73	62
N.S.	1	1.00	1.60	1.07	0.82	0.94	0.99	0.82	0.70
time (sec)	N/A	0.066	0.074	0.038	0.485	0.350	0.104	3.571	0.039

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	139	110	0	216	192	109	100
N.S.	1	1.00	0.99	0.78	0.00	1.53	1.36	0.77	0.71
time (sec)	N/A	0.070	0.170	0.042	0.000	0.357	0.359	3.507	0.103

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	67	0	70	82	66	38
N.S.	1	1.00	0.77	0.76	0.00	0.80	0.93	0.75	0.43
time (sec)	N/A	0.039	0.014	0.033	0.000	0.336	0.072	2.863	1.310

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	135	121	0	215	197	108	99
N.S.	1	1.00	0.96	0.86	0.00	1.54	1.41	0.77	0.71
time (sec)	N/A	0.071	0.101	0.035	0.000	0.358	0.384	2.935	0.067

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	135	109	0	215	214	108	97
N.S.	1	1.00	0.96	0.78	0.00	1.54	1.53	0.77	0.69
time (sec)	N/A	0.060	0.094	0.043	0.000	0.398	0.372	3.754	1.311

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	68	67	0	70	82	66	40
N.S.	1	1.00	0.77	0.76	0.00	0.80	0.93	0.75	0.45
time (sec)	N/A	0.037	0.011	0.028	0.000	0.376	0.071	4.426	0.037

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	140	126	0	228	218	113	102
N.S.	1	1.00	0.97	0.87	0.00	1.57	1.50	0.78	0.70
time (sec)	N/A	0.078	0.128	0.038	0.000	0.373	0.369	4.276	0.048

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	148	114	0	244	197	113	104
N.S.	1	1.00	1.01	0.78	0.00	1.66	1.34	0.77	0.71
time (sec)	N/A	0.071	0.172	0.040	0.000	0.377	0.420	4.127	0.030

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	95	75	0	90	94	100	52
N.S.	1	1.00	0.97	0.77	0.00	0.92	0.96	1.02	0.53
time (sec)	N/A	0.056	0.022	0.040	0.000	0.351	0.085	4.013	0.038

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	171	131	0	250	209	120	110
N.S.	1	1.00	1.11	0.85	0.00	1.62	1.36	0.78	0.71
time (sec)	N/A	0.101	0.212	0.046	0.000	0.356	0.430	3.947	0.032

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	79	0	0	0	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.076	0.007	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	38	37	37	42	37	39
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.91	0.80	0.85
time (sec)	N/A	0.026	0.008	0.019	0.495	0.358	0.058	3.966	0.047

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	44	0	47	48	99	29
N.S.	1	1.00	0.96	0.77	0.00	0.82	0.84	1.74	0.51
time (sec)	N/A	0.032	0.010	0.023	0.000	0.346	0.041	3.930	1.305

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.023	0.007	0.020	0.524	0.344	0.046	3.310	1.281

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	98	77	0	172	70	76	53
N.S.	1	1.00	1.20	0.94	0.00	2.10	0.85	0.93	0.65
time (sec)	N/A	0.050	0.080	0.026	0.000	0.364	0.089	3.453	0.050

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	26	18	17
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.13	0.78	0.74
time (sec)	N/A	0.015	0.005	0.014	0.505	0.378	0.054	3.752	1.285

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	65	0	172	70	64	53
N.S.	1	1.00	1.01	0.79	0.00	2.10	0.85	0.78	0.65
time (sec)	N/A	0.043	0.031	0.024	0.000	0.370	0.082	3.471	0.045

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	35	38	34	41	38	36
N.S.	1	1.00	1.34	0.85	0.93	0.83	1.00	0.93	0.88
time (sec)	N/A	0.026	0.009	0.022	0.556	0.364	0.057	3.187	1.292

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	44	0	50	49	99	29
N.S.	1	1.00	0.96	0.77	0.00	0.88	0.86	1.74	0.51
time (sec)	N/A	0.030	0.010	0.028	0.000	0.348	0.078	3.346	1.273

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	51	40	43	51	48	48	41
N.S.	1	1.00	1.06	0.83	0.90	1.06	1.00	1.00	0.85
time (sec)	N/A	0.036	0.010	0.026	0.513	0.348	0.073	3.479	0.066

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	56	87	0	194	83	56	63
N.S.	1	1.00	0.58	0.91	0.00	2.02	0.86	0.58	0.66
time (sec)	N/A	0.064	0.012	0.033	0.000	0.363	0.105	3.111	0.056

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	59	44	0	720	26	254	209
N.S.	1	1.00	0.17	0.12	0.00	2.02	0.07	0.71	0.59
time (sec)	N/A	0.231	0.011	0.030	0.000	0.364	1.590	2.895	0.157

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	41	32	0	220	165	205	53
N.S.	1	1.00	0.15	0.12	0.00	0.80	0.60	0.75	0.19
time (sec)	N/A	0.167	0.008	0.021	0.000	0.361	0.126	3.097	0.096

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	39	40	0	581	24	253	474
N.S.	1	1.00	0.11	0.12	0.00	1.67	0.07	0.73	1.37
time (sec)	N/A	0.134	0.007	0.024	0.000	0.376	1.525	2.789	1.328

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	40	40	0	573	26	253	286
N.S.	1	1.00	0.11	0.11	0.00	1.61	0.07	0.71	0.81
time (sec)	N/A	0.137	0.008	0.036	0.000	0.395	1.560	3.595	0.079

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	42	30	0	220	165	205	53
N.S.	1	1.00	0.15	0.11	0.00	0.80	0.60	0.75	0.19
time (sec)	N/A	0.142	0.007	0.018	0.000	0.348	0.107	5.918	0.037

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	61	52	0	736	29	258	253
N.S.	1	1.00	0.17	0.14	0.00	2.04	0.08	0.72	0.70
time (sec)	N/A	0.161	0.010	0.037	0.000	0.400	1.485	4.717	1.292

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	65	50	0	760	31	258	213
N.S.	1	1.00	0.18	0.14	0.00	2.05	0.08	0.70	0.58
time (sec)	N/A	0.158	0.009	0.034	0.000	0.453	1.513	4.162	1.288

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	54	43	0	243	182	217	63
N.S.	1	1.00	0.19	0.15	0.00	0.85	0.63	0.76	0.22
time (sec)	N/A	0.165	0.011	0.031	0.000	0.344	0.123	3.624	1.296

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	54	51	0	628	37	265	486
N.S.	1	1.00	0.14	0.14	0.00	1.67	0.10	0.70	1.29
time (sec)	N/A	0.190	0.011	0.034	0.000	0.379	1.548	3.293	0.064

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	79	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.074	0.007	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	38	50	62	60	50	64
N.S.	1	1.00	0.92	0.61	0.81	1.00	0.97	0.81	1.03
time (sec)	N/A	0.039	0.022	0.035	0.534	0.361	0.052	3.611	0.134

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	97	79	0	142	54	66	130
N.S.	1	1.00	1.08	0.88	0.00	1.58	0.60	0.73	1.44
time (sec)	N/A	0.094	0.096	0.063	0.000	0.435	0.121	3.213	1.343

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	33	45	56	53	45	59
N.S.	1	1.00	0.96	0.60	0.82	1.02	0.96	0.82	1.07
time (sec)	N/A	0.022	0.015	0.020	0.513	0.341	0.047	3.318	1.358

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	75	70	0	149	49	47	117
N.S.	1	1.00	0.93	0.86	0.00	1.84	0.60	0.58	1.44
time (sec)	N/A	0.056	0.028	0.040	0.000	0.367	0.086	2.895	0.119

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	38	19	31	43	42	31	30
N.S.	1	1.00	1.65	0.83	1.35	1.87	1.83	1.35	1.30
time (sec)	N/A	0.017	0.007	0.017	0.563	0.355	0.040	2.828	1.333

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	60	0	128	49	41	125
N.S.	1	1.00	0.99	0.80	0.00	1.71	0.65	0.55	1.67
time (sec)	N/A	0.036	0.025	0.033	0.000	0.374	0.105	3.218	0.054

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	35	51	58	58	51	42
N.S.	1	1.00	0.96	0.61	0.89	1.02	1.02	0.89	0.74
time (sec)	N/A	0.024	0.020	0.024	0.501	0.349	0.060	4.011	1.412

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	65	75	0	144	56	68	130
N.S.	1	1.00	0.73	0.84	0.00	1.62	0.63	0.76	1.46
time (sec)	N/A	0.050	0.012	0.049	0.000	0.351	0.102	4.323	1.304

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	42	56	76	65	63	49
N.S.	1	1.00	0.91	0.64	0.85	1.15	0.98	0.95	0.74
time (sec)	N/A	0.044	0.022	0.027	0.643	0.402	0.074	4.005	1.358

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	73	84	0	172	65	77	136
N.S.	1	1.00	0.75	0.87	0.00	1.77	0.67	0.79	1.40
time (sec)	N/A	0.093	0.013	0.043	0.000	0.365	0.150	4.016	0.124

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	440	58	46	0	1026	29	240	216
N.S.	1	0.96	0.13	0.10	0.00	2.23	0.06	0.52	0.47
time (sec)	N/A	0.287	0.010	0.024	0.000	0.380	1.028	3.834	1.438

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	41	40	0	761	26	239	149
N.S.	1	1.00	0.10	0.09	0.00	1.77	0.06	0.55	0.35
time (sec)	N/A	0.213	0.008	0.023	0.000	0.401	1.073	3.950	1.465

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	451	451	39	40	0	859	24	239	454
N.S.	1	1.00	0.09	0.09	0.00	1.90	0.05	0.53	1.01
time (sec)	N/A	0.201	0.008	0.021	0.000	0.397	0.954	3.403	0.196

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	431	40	40	0	963	26	239	275
N.S.	1	1.01	0.09	0.09	0.00	2.26	0.06	0.56	0.64
time (sec)	N/A	0.179	0.007	0.023	0.000	0.405	0.949	3.553	0.086

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	42	37	0	769	26	239	403
N.S.	1	1.00	0.10	0.09	0.00	1.86	0.06	0.58	0.97
time (sec)	N/A	0.192	0.007	0.026	0.000	0.391	1.197	4.060	0.083

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	61	52	0	1025	32	244	292
N.S.	1	1.00	0.15	0.12	0.00	2.46	0.08	0.59	0.70
time (sec)	N/A	0.208	0.011	0.034	0.000	0.389	1.140	3.682	1.291

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	466	65	50	0	1071	34	244	492
N.S.	1	1.00	0.14	0.11	0.00	2.30	0.07	0.52	1.06
time (sec)	N/A	0.257	0.011	0.039	0.000	0.402	0.955	4.645	0.184

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	575	0	0	0	0	0	-1
N.S.	1	1.00	4.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.502	0.007	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	38	50	62	58	53	64
N.S.	1	1.00	0.90	0.61	0.81	1.00	0.94	0.85	1.03
time (sec)	N/A	0.032	0.024	0.025	0.503	0.341	0.061	4.030	0.120

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	103	67	92	114	170	97	90
N.S.	1	1.00	1.14	0.74	1.02	1.27	1.89	1.08	1.00
time (sec)	N/A	0.049	0.035	0.036	0.951	0.347	0.250	3.330	1.328

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	33	45	57	53	48	59
N.S.	1	1.00	0.96	0.60	0.82	1.04	0.96	0.87	1.07
time (sec)	N/A	0.020	0.013	0.019	0.507	0.379	0.049	3.878	0.102

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	91	62	87	109	165	92	77
N.S.	1	1.00	1.12	0.77	1.07	1.35	2.04	1.14	0.95
time (sec)	N/A	0.040	0.022	0.031	0.508	0.388	0.174	3.591	1.381

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	38	19	31	43	42	33	30
N.S.	1	1.00	1.65	0.83	1.35	1.87	1.83	1.43	1.30
time (sec)	N/A	0.019	0.008	0.017	0.608	0.340	0.039	3.550	1.570

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	91	62	87	107	165	92	83
N.S.	1	1.00	1.21	0.83	1.16	1.43	2.20	1.23	1.11
time (sec)	N/A	0.026	0.018	0.023	0.504	0.339	0.175	3.128	1.302

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	64	51	59	58	54	42
N.S.	1	1.00	0.96	1.12	0.89	1.04	1.02	0.95	0.74
time (sec)	N/A	0.021	0.020	0.026	0.520	0.351	0.060	2.989	0.429

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	103	67	92	125	172	97	88
N.S.	1	1.00	1.16	0.75	1.03	1.40	1.93	1.09	0.99
time (sec)	N/A	0.038	0.033	0.034	0.507	0.391	0.193	3.344	0.064

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	61	71	56	76	66	66	49
N.S.	1	1.00	0.92	1.08	0.85	1.15	1.00	1.00	0.74
time (sec)	N/A	0.043	0.023	0.051	0.507	0.382	0.077	3.997	1.348

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	111	72	99	130	199	104	95
N.S.	1	1.00	1.14	0.74	1.02	1.34	2.05	1.07	0.98
time (sec)	N/A	0.066	0.045	0.039	0.550	0.341	0.243	5.590	1.385

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	160	131	0	312	58	148	246
N.S.	1	1.00	0.94	0.77	0.00	1.84	0.34	0.87	1.45
time (sec)	N/A	0.079	0.172	0.077	0.000	0.364	0.762	4.184	1.445

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	160	130	0	243	53	147	147
N.S.	1	1.00	0.96	0.78	0.00	1.46	0.32	0.88	0.88
time (sec)	N/A	0.062	0.100	0.074	0.000	0.389	0.725	4.103	0.194

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	132	130	0	261	49	147	269
N.S.	1	1.00	0.76	0.75	0.00	1.51	0.28	0.85	1.55
time (sec)	N/A	0.053	0.116	0.056	0.000	0.381	0.714	3.877	1.472

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	166	131	110	0	255	53	147	269
N.S.	1	1.14	0.90	0.76	0.00	1.76	0.37	1.01	1.86
time (sec)	N/A	0.041	0.029	0.044	0.000	0.385	0.709	3.929	0.081

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	160	130	0	241	53	147	245
N.S.	1	1.00	0.95	0.77	0.00	1.43	0.31	0.87	1.45
time (sec)	N/A	0.038	0.090	0.041	0.000	0.365	0.799	3.024	0.079

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	174	135	0	321	63	152	250
N.S.	1	1.00	1.01	0.78	0.00	1.87	0.37	0.88	1.45
time (sec)	N/A	0.061	0.157	0.046	0.000	0.373	0.813	3.541	1.339

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	166	135	0	335	63	152	268
N.S.	1	1.00	0.91	0.74	0.00	1.84	0.35	0.84	1.47
time (sec)	N/A	0.085	0.159	0.051	0.000	0.370	0.767	3.420	0.204

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	189	148	0	292	73	159	257
N.S.	1	1.00	1.09	0.86	0.00	1.69	0.42	0.92	1.49
time (sec)	N/A	0.098	0.162	0.059	0.000	0.378	0.748	3.956	1.487

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	189	148	0	324	70	159	291
N.S.	1	1.00	1.00	0.78	0.00	1.71	0.37	0.84	1.54
time (sec)	N/A	0.116	0.168	0.060	0.000	0.353	0.860	4.036	0.210

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	15	17	16
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.71	0.81	0.76
time (sec)	N/A	0.008	0.004	0.018	0.292	0.329	0.036	3.927	0.063

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	22	19	22	22
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.73	0.85	0.85
time (sec)	N/A	0.012	0.004	0.019	0.285	0.361	0.044	3.944	1.315

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	37	30	32
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86
time (sec)	N/A	0.022	0.009	0.023	0.507	0.341	0.051	4.906	1.346

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	18	18	27	18	20
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.17	0.78	0.87
time (sec)	N/A	0.015	0.005	0.016	0.498	0.348	0.063	5.206	1.333

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	197	131	36	32	41	33	34
N.S.	1	1.00	5.05	3.36	0.92	0.82	1.05	0.85	0.87
time (sec)	N/A	0.024	0.025	0.034	0.533	0.344	0.063	6.491	0.060

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	208	138	41	49	48	45	41
N.S.	1	1.00	4.33	2.88	0.85	1.02	1.00	0.94	0.85
time (sec)	N/A	0.033	0.028	0.042	0.515	0.364	0.080	3.317	1.369

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	197	131	0	32	41	33	34
N.S.	1	1.00	5.05	3.36	0.00	0.82	1.05	0.85	0.87
time (sec)	N/A	0.025	0.014	0.031	0.000	0.352	0.080	3.651	0.033

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	140	164	0	466	605	145	183
N.S.	1	1.00	0.95	1.12	0.00	3.17	4.12	0.99	1.24
time (sec)	N/A	0.095	0.082	0.056	0.000	0.359	0.701	3.969	0.159

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	112	128	0	383	498	113	151
N.S.	1	1.00	0.95	1.08	0.00	3.25	4.22	0.96	1.28
time (sec)	N/A	0.072	0.056	0.049	0.000	0.374	0.653	3.354	1.396

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	98	0	297	381	86	112
N.S.	1	1.00	0.94	1.10	0.00	3.34	4.28	0.97	1.26
time (sec)	N/A	0.060	0.068	0.036	0.000	0.356	0.453	3.562	0.131

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	75	0	235	306	67	172
N.S.	1	1.00	1.04	1.07	0.00	3.36	4.37	0.96	2.46
time (sec)	N/A	0.036	0.042	0.027	0.000	0.354	0.316	3.735	1.418

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	185	216	55	112
N.S.	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	2.00
time (sec)	N/A	0.023	0.021	0.022	0.000	0.336	0.157	3.830	0.168

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	120	124	34	46
N.S.	1	1.00	1.06	0.97	0.00	3.33	3.44	0.94	1.28
time (sec)	N/A	0.023	0.005	0.019	0.000	0.335	0.094	4.101	0.046

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	61	0	211	564	62	213
N.S.	1	1.00	0.98	0.98	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.032	0.044	0.033	0.000	0.351	4.605	4.111	1.715

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	81	0	269	0	79	339
N.S.	1	1.00	0.95	1.00	0.00	3.32	0.00	0.98	4.19
time (sec)	N/A	0.070	0.053	0.042	0.000	0.408	0.000	4.147	1.814

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	128	0	358	0	105	447
N.S.	1	1.00	0.98	1.23	0.00	3.44	0.00	1.01	4.30
time (sec)	N/A	0.100	0.090	0.056	0.000	0.429	0.000	3.397	1.866

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	157	0	445	0	136	524
N.S.	1	1.00	0.96	1.15	0.00	3.25	0.00	0.99	3.82
time (sec)	N/A	0.134	0.067	0.059	0.000	0.390	0.000	3.586	1.923

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	163	238	0	1029	1012	188	382
N.S.	1	1.00	0.83	1.21	0.00	5.25	5.16	0.96	1.95
time (sec)	N/A	0.147	0.146	0.064	0.000	0.350	1.484	3.275	1.822

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	132	198	0	837	842	161	261
N.S.	1	1.00	0.88	1.32	0.00	5.58	5.61	1.07	1.74
time (sec)	N/A	0.105	0.117	0.059	0.000	0.396	1.009	2.893	1.798

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	109	169	0	635	729	125	279
N.S.	1	1.00	0.96	1.48	0.00	5.57	6.39	1.10	2.45
time (sec)	N/A	0.068	0.095	0.046	0.000	0.342	0.758	2.783	1.859

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	81	97	0	387	280	88	135
N.S.	1	1.00	1.14	1.37	0.00	5.45	3.94	1.24	1.90
time (sec)	N/A	0.030	0.059	0.035	0.000	0.367	0.351	2.926	1.370

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	338	253	76	110
N.S.	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	1.67
time (sec)	N/A	0.023	0.043	0.031	0.000	0.346	0.292	3.427	1.369

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	341	265	76	119
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	1.80
time (sec)	N/A	0.021	0.051	0.029	0.000	0.371	0.308	6.501	0.083

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	177	0	781	0	126	620
N.S.	1	1.00	0.99	1.64	0.00	7.23	0.00	1.17	5.74
time (sec)	N/A	0.097	0.123	0.059	0.000	0.443	0.000	5.708	2.096

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	131	205	0	975	0	171	775
N.S.	1	1.00	0.89	1.39	0.00	6.59	0.00	1.16	5.24
time (sec)	N/A	0.130	0.179	0.059	0.000	0.483	0.000	5.748	2.134

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	175	255	0	1226	0	229	914
N.S.	1	1.00	0.87	1.26	0.00	6.07	0.00	1.13	4.52
time (sec)	N/A	0.169	0.229	0.060	0.000	0.532	0.000	7.054	2.298

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	260	401	0	1926	1714	282	705
N.S.	1	1.00	1.09	1.68	0.00	8.09	7.20	1.18	2.96
time (sec)	N/A	0.202	0.242	0.081	0.000	0.385	3.125	5.905	2.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	221	357	0	1603	1510	245	620
N.S.	1	1.00	1.16	1.88	0.00	8.44	7.95	1.29	3.26
time (sec)	N/A	0.181	0.201	0.068	0.000	0.386	2.044	5.389	2.203

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	174	260	0	953	547	202	343
N.S.	1	1.00	1.57	2.34	0.00	8.59	4.93	1.82	3.09
time (sec)	N/A	0.046	0.115	0.054	0.000	0.370	0.801	3.682	0.195

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	126	223	0	872	513	163	271
N.S.	1	1.00	1.18	2.08	0.00	8.15	4.79	1.52	2.53
time (sec)	N/A	0.035	0.130	0.044	0.000	0.386	0.663	3.355	1.429

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	131	210	0	887	570	154	313
N.S.	1	1.00	1.14	1.83	0.00	7.71	4.96	1.34	2.72
time (sec)	N/A	0.045	0.091	0.049	0.000	0.367	0.739	3.235	1.500

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	102	118	0	788	481	135	253
N.S.	1	1.00	0.99	1.15	0.00	7.65	4.67	1.31	2.46
time (sec)	N/A	0.029	0.064	0.049	0.000	0.352	0.623	4.048	1.430

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	101	97	116	0	785	474	136	285
N.S.	1	0.98	0.94	1.13	0.00	7.62	4.60	1.32	2.77
time (sec)	N/A	0.029	0.064	0.046	0.000	0.371	0.722	3.317	1.424

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	178	352	0	1985	0	239	1089
N.S.	1	1.00	0.96	1.90	0.00	10.73	0.00	1.29	5.89
time (sec)	N/A	0.157	0.229	0.061	0.000	0.610	0.000	2.939	2.456

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	221	404	0	2280	0	309	1255
N.S.	1	1.00	0.92	1.69	0.00	9.54	0.00	1.29	5.25
time (sec)	N/A	0.186	0.282	0.062	0.000	0.765	0.000	3.154	2.554

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	31	30	30	34	32	26
N.S.	1	1.00	1.00	0.78	0.75	0.75	0.85	0.80	0.65
time (sec)	N/A	0.016	0.004	0.017	0.316	0.387	0.048	2.842	0.046

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	25	27	27	21
N.S.	1	1.00	1.00	0.79	0.76	0.76	0.82	0.82	0.64
time (sec)	N/A	0.015	0.003	0.018	0.306	0.361	0.044	4.191	1.310

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	20	22	16
N.S.	1	1.00	1.00	0.81	0.77	0.77	0.77	0.85	0.62
time (sec)	N/A	0.010	0.004	0.016	0.298	0.364	0.039	4.821	0.077

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	17	19	13
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.81	0.90	0.62
time (sec)	N/A	0.007	0.002	0.013	0.306	0.342	0.033	4.165	0.066

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	18	17	17	15	19	8
N.S.	1	1.00	0.91	0.78	0.74	0.74	0.65	0.83	0.35
time (sec)	N/A	0.010	0.002	0.031	0.292	0.355	0.031	3.772	1.373

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	21	21	24	24	17
N.S.	1	1.00	1.00	0.81	0.78	0.78	0.89	0.89	0.63
time (sec)	N/A	0.012	0.003	0.017	0.297	0.392	0.049	3.832	1.385

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	27	26	30	31	29	22
N.S.	1	1.00	1.00	0.79	0.76	0.88	0.91	0.85	0.65
time (sec)	N/A	0.022	0.003	0.021	0.299	0.346	0.059	4.211	0.044

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	31	39	36	34	26
N.S.	1	1.00	1.00	0.78	0.76	0.95	0.88	0.83	0.63
time (sec)	N/A	0.024	0.004	0.021	0.297	0.334	0.061	3.246	1.312

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	36	44	41	39	32
N.S.	1	1.00	1.00	0.77	0.75	0.92	0.85	0.81	0.67
time (sec)	N/A	0.030	0.004	0.023	0.299	0.347	0.068	3.147	0.047

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	208	701	0	959	0	0	-1
N.S.	1	1.00	1.02	3.44	0.00	4.70	0.00	0.00	-0.00
time (sec)	N/A	0.158	1.235	0.061	0.000	0.478	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	158	334	0	709	0	0	-1
N.S.	1	1.00	1.09	2.30	0.00	4.89	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.543	0.041	0.000	0.422	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	131	121	0	590	0	0	100
N.S.	1	1.00	1.25	1.15	0.00	5.62	0.00	0.00	0.95
time (sec)	N/A	0.056	0.178	0.021	0.000	0.396	0.000	0.000	0.126

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	88	88	0	171	0	0	53
N.S.	1	1.00	1.31	1.31	0.00	2.55	0.00	0.00	0.79
time (sec)	N/A	0.028	0.129	0.049	0.000	0.349	0.000	0.000	1.452

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	140	197	0	465	0	237	-1
N.S.	1	1.00	1.05	1.48	0.00	3.50	0.00	1.78	-0.01
time (sec)	N/A	0.068	0.437	0.056	0.000	0.398	0.000	5.561	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	246	376	0	1081	0	507	-1
N.S.	1	1.00	1.12	1.71	0.00	4.91	0.00	2.30	-0.00
time (sec)	N/A	0.134	1.105	0.102	0.000	0.536	0.000	5.422	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	32	40	8	8	0	29	134
N.S.	1	1.00	0.44	0.55	0.11	0.11	0.00	0.40	1.84
time (sec)	N/A	0.024	0.013	0.041	0.298	0.424	0.000	4.385	0.112

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	202	169	0	1059	129	2109	3026
N.S.	1	1.00	1.13	0.94	0.00	5.92	0.72	11.78	16.91
time (sec)	N/A	0.186	0.077	0.036	0.000	0.383	1.375	4.268	2.079

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	70	59	0	5841	196	0	2280
N.S.	1	1.00	0.11	0.09	0.00	9.26	0.31	0.00	3.61
time (sec)	N/A	0.792	0.025	0.205	0.000	1.671	52.834	0.000	4.540

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	70	59	0	5310	0	0	2500
N.S.	1	1.00	0.19	0.16	0.00	14.12	0.00	0.00	6.65
time (sec)	N/A	0.441	0.029	0.056	0.000	1.027	0.000	0.000	3.776

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	105	84	0	0	0	0	-1
N.S.	1	1.00	0.99	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.153	0.017	0.000	0.000	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	55	52	54	53	51	49	44
N.S.	1	1.00	1.38	1.30	1.35	1.32	1.28	1.22	1.10
time (sec)	N/A	0.014	0.015	0.056	0.283	0.407	0.094	3.815	0.038

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	50	23	0	0	45	-1
N.S.	1	1.00	0.67	0.67	0.31	0.00	0.00	0.60	-0.01
time (sec)	N/A	0.027	0.025	0.056	0.340	0.000	0.000	4.728	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	117	109	114	84	0	140	-1
N.S.	1	1.00	0.85	0.80	0.83	0.61	0.00	1.02	-0.01
time (sec)	N/A	0.054	0.035	0.089	0.296	0.398	0.000	4.659	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	93	87	114	61	0	102	-1
N.S.	1	1.00	0.68	0.64	0.83	0.45	0.00	0.74	-0.01
time (sec)	N/A	0.047	0.025	0.056	0.288	0.329	0.000	5.119	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	67	65	114	32	0	64	-1
N.S.	1	1.00	0.49	0.47	0.83	0.23	0.00	0.47	-0.01
time (sec)	N/A	0.040	0.020	0.051	0.300	0.336	0.000	4.748	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	43	43	114	10	0	26	71
N.S.	1	1.00	0.49	0.49	1.30	0.11	0.00	0.30	0.81
time (sec)	N/A	0.027	0.007	0.089	0.424	0.322	0.000	4.423	1.561

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	65	103	36	33	0	61	-1
N.S.	1	1.00	0.44	0.70	0.24	0.22	0.00	0.41	-0.01
time (sec)	N/A	0.048	0.029	0.059	0.316	0.357	0.000	3.889	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	72	92	55	113	0	64	-1
N.S.	1	1.00	0.55	0.71	0.42	0.87	0.00	0.49	-0.01
time (sec)	N/A	0.051	0.047	0.044	0.409	0.362	0.000	4.075	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	56	54	53	136	0	43	53
N.S.	1	1.00	0.41	0.40	0.39	1.01	0.00	0.32	0.39
time (sec)	N/A	0.052	0.040	0.058	0.284	0.384	0.000	2.598	2.804

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	56	54	53	209	0	43	53
N.S.	1	1.00	0.41	0.39	0.39	1.53	0.00	0.31	0.39
time (sec)	N/A	0.053	0.041	0.051	0.415	0.403	0.000	3.525	3.228

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	56	54	53	275	0	43	53
N.S.	1	1.00	0.41	0.39	0.39	2.01	0.00	0.31	0.39
time (sec)	N/A	0.051	0.040	0.042	0.313	0.388	0.000	4.847	3.654

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	56	54	53	343	0	43	53
N.S.	1	1.00	0.41	0.39	0.39	2.50	0.00	0.31	0.39
time (sec)	N/A	0.053	0.041	0.052	0.267	0.393	0.000	5.214	4.344

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	68	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.067	0.006	0.000	0.000	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	207	0	362	579	0	1564	777
N.S.	1	1.00	0.44	0.00	0.77	1.24	0.00	3.34	1.66
time (sec)	N/A	0.158	0.199	0.005	0.302	0.482	0.000	3.997	3.515

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	143	0	198	297	0	745	390
N.S.	1	1.00	0.45	0.00	0.63	0.94	0.00	2.37	1.24
time (sec)	N/A	0.094	0.161	0.004	0.305	0.445	0.000	4.501	2.166

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	83	0	77	110	0	229	138
N.S.	1	1.00	0.58	0.00	0.54	0.77	0.00	1.61	0.97
time (sec)	N/A	0.046	0.075	0.005	0.280	0.389	0.000	3.009	1.542

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.063	0.004	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	61	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.069	0.004	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	162	101	0	0	82	0	0	69
N.S.	1	1.11	0.69	0.00	0.00	0.56	0.00	0.00	0.47
time (sec)	N/A	0.066	0.246	0.007	0.000	0.557	0.000	0.000	1.647

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	93	114	114	147	0	0	-1
N.S.	1	1.00	0.53	0.65	0.65	0.84	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.074	0.076	0.292	2.123	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	121	174	119	0	0	105	-1
N.S.	1	1.00	0.45	0.65	0.44	0.00	0.00	0.39	-0.00
time (sec)	N/A	0.099	0.107	0.099	0.309	0.000	0.000	5.404	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	66	68	0	0	0	80	-1
N.S.	1	1.00	0.37	0.38	0.00	0.00	0.00	0.45	-0.01
time (sec)	N/A	0.062	0.034	0.063	0.000	0.000	0.000	4.149	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	125	115	79	0	0	173	-1
N.S.	1	1.00	0.32	0.29	0.20	0.00	0.00	0.44	-0.00
time (sec)	N/A	0.126	0.055	0.072	0.332	0.000	0.000	4.532	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	99	91	57	0	0	128	-1
N.S.	1	1.00	0.34	0.31	0.20	0.00	0.00	0.44	-0.00
time (sec)	N/A	0.093	0.048	0.040	0.338	0.000	0.000	3.150	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	75	69	30	0	0	79	-1
N.S.	1	1.00	0.40	0.37	0.16	0.00	0.00	0.42	-0.01
time (sec)	N/A	0.066	0.032	0.038	0.293	0.000	0.000	3.503	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	49	50	10	0	0	34	39
N.S.	1	1.00	0.56	0.57	0.11	0.00	0.00	0.39	0.44
time (sec)	N/A	0.036	0.014	0.050	0.285	0.000	0.000	3.180	1.426

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	86	78	44	0	0	77	-1
N.S.	1	1.00	0.45	0.41	0.23	0.00	0.00	0.41	-0.01
time (sec)	N/A	0.081	0.030	0.038	0.281	0.000	0.000	3.477	0.000

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	126	141	97	0	0	127	-1
N.S.	1	1.00	0.42	0.47	0.32	0.00	0.00	0.42	-0.00
time (sec)	N/A	0.129	0.080	0.042	0.303	0.000	0.000	4.274	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	152	199	139	0	0	147	-1
N.S.	1	1.00	0.37	0.49	0.34	0.00	0.00	0.36	-0.00
time (sec)	N/A	0.175	0.080	0.043	0.292	0.000	0.000	4.702	0.000

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	98	94	57	0	0	126	-1
N.S.	1	1.00	0.34	0.33	0.20	0.00	0.00	0.44	-0.00
time (sec)	N/A	0.094	0.045	0.069	0.286	0.000	0.000	5.208	0.000

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	101	91	52	0	0	125	-1
N.S.	1	1.00	0.35	0.31	0.18	0.00	0.00	0.43	-0.00
time (sec)	N/A	0.091	0.046	0.065	0.287	0.000	0.000	6.526	0.000

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	98	152	99	302	0	84	-1
N.S.	1	1.00	0.44	0.68	0.45	1.36	0.00	0.38	-0.00
time (sec)	N/A	0.084	0.073	0.054	0.285	0.448	0.000	6.019	0.000

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	124	116	79	0	0	172	-1
N.S.	1	1.00	0.32	0.30	0.20	0.00	0.00	0.44	-0.00
time (sec)	N/A	0.118	0.051	0.073	0.296	0.000	0.000	4.486	0.000

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	47	45	38	42	0	-1
N.S.	1	1.00	0.83	1.02	0.98	0.83	0.91	0.00	-0.02
time (sec)	N/A	0.027	0.033	0.225	0.281	0.446	7.247	0.000	0.000

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	31	32	24	26	0	-1
N.S.	1	1.00	0.96	1.11	1.14	0.86	0.93	0.00	-0.04
time (sec)	N/A	0.018	0.027	0.217	0.284	0.454	6.819	0.000	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	18	18	19	15	73	0	-1
N.S.	1	1.00	1.20	1.20	1.27	1.00	4.87	0.00	-0.07
time (sec)	N/A	0.008	0.019	0.201	0.282	0.423	3.019	0.000	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	26	27	22	42	25	20
N.S.	1	1.00	1.09	1.13	1.17	0.96	1.83	1.09	0.87
time (sec)	N/A	0.013	0.027	0.199	0.280	0.374	2.759	5.829	1.369

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	58	58	59	0	0	-1
N.S.	1	1.00	0.84	1.02	1.02	1.04	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.061	0.196	0.295	0.409	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	63	75	71	72	73	0	-1
N.S.	1	1.00	0.83	0.99	0.93	0.95	0.96	0.00	-0.01
time (sec)	N/A	0.031	0.072	0.202	0.293	0.370	21.499	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	76	90	84	85	88	0	-1
N.S.	1	1.00	0.82	0.97	0.90	0.91	0.95	0.00	-0.01
time (sec)	N/A	0.037	0.088	0.205	0.277	0.378	40.507	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	34	54	0	272	0	203	-1
N.S.	1	1.00	0.14	0.23	0.00	1.15	0.00	0.86	-0.00
time (sec)	N/A	0.147	0.030	0.253	0.000	0.397	0.000	5.042	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	34	54	0	212	0	136	-1
N.S.	1	1.00	0.21	0.34	0.00	1.32	0.00	0.85	-0.01
time (sec)	N/A	0.089	0.028	0.201	0.000	0.372	0.000	6.079	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	32	79	0	151	0	38	-1
N.S.	1	1.00	0.64	1.58	0.00	3.02	0.00	0.76	-0.02
time (sec)	N/A	0.023	0.027	0.191	0.000	0.490	0.000	6.361	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	34	97	0	161	0	0	-1
N.S.	1	1.00	0.50	1.43	0.00	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.028	0.204	0.000	0.379	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	34	73	0	171	0	0	-1
N.S.	1	1.00	0.19	0.41	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.028	0.207	0.000	0.364	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	34	73	0	259	0	0	-1
N.S.	1	1.00	0.13	0.29	0.00	1.03	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.028	0.215	0.000	0.362	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	38	0	43	59	0	0	-1
N.S.	1	1.00	1.03	0.00	1.16	1.59	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.038	0.047	0.482	0.397	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	43	0	0	59	0	0	-1
N.S.	1	1.00	1.13	0.00	0.00	1.55	0.00	0.00	-0.03
time (sec)	N/A	0.027	0.066	0.048	0.000	0.358	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	96	208	74	74	0	0	-1
N.S.	1	1.00	0.86	1.86	0.66	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.050	0.040	0.320	0.349	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	70	135	48	48	0	0	-1
N.S.	1	1.00	0.62	1.21	0.43	0.43	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.037	0.044	0.473	0.360	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	44	64	22	22	0	0	-1
N.S.	1	1.00	0.44	0.65	0.22	0.22	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.024	0.025	0.457	0.419	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	47	71	32	24	0	0	-1
N.S.	1	1.00	0.52	0.79	0.36	0.27	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.039	0.029	0.538	0.366	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	40	37	41	41	0	0	-1
N.S.	1	1.00	0.83	0.77	0.85	0.85	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.040	0.026	0.414	0.338	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	40	37	69	69	0	0	-1
N.S.	1	1.00	0.45	0.42	0.78	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.047	0.034	0.529	0.343	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	40	37	97	97	0	0	-1
N.S.	1	1.00	0.45	0.42	1.10	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.048	0.032	0.405	0.369	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	55	132	47	57	0	173	-1
N.S.	1	1.00	0.51	1.22	0.44	0.53	0.00	1.60	-0.01
time (sec)	N/A	0.031	0.035	0.048	0.315	0.364	0.000	3.826	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	46	61	25	28	0	53	-1
N.S.	1	1.00	0.49	0.66	0.27	0.30	0.00	0.57	-0.01
time (sec)	N/A	0.020	0.027	0.036	0.334	0.369	0.000	5.889	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	46	61	25	28	0	53	-1
N.S.	1	1.00	0.49	0.66	0.27	0.30	0.00	0.57	-0.01
time (sec)	N/A	0.017	0.023	0.020	0.303	0.348	0.000	3.895	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	39	56	19	20	0	25	-1
N.S.	1	1.00	0.44	0.64	0.22	0.23	0.00	0.28	-0.01
time (sec)	N/A	0.012	0.008	0.018	0.295	0.367	0.000	5.674	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	38	54	13	15	0	0	-1
N.S.	1	1.00	0.45	0.64	0.15	0.18	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.017	0.034	0.330	0.361	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	42	61	22	23	0	0	-1
N.S.	1	1.00	0.45	0.65	0.23	0.24	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.026	0.023	0.271	0.385	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	47	61	22	23	0	0	-1
N.S.	1	1.00	0.49	0.64	0.23	0.24	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.028	0.024	0.275	0.420	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	90	532	276	390	0	2719	-1
N.S.	1	1.00	0.38	2.24	1.16	1.64	0.00	11.42	-0.00
time (sec)	N/A	0.070	0.091	0.049	0.297	0.392	0.000	5.529	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	123	146	108	144	0	292	-1
N.S.	1	1.00	0.58	0.69	0.51	0.68	0.00	1.38	-0.00
time (sec)	N/A	0.045	0.070	0.036	0.311	0.351	0.000	4.617	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	124	145	109	145	0	292	-1
N.S.	1	1.00	0.59	0.69	0.52	0.69	0.00	1.38	-0.00
time (sec)	N/A	0.041	0.058	0.023	0.284	0.357	0.000	2.966	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	122	138	101	130	0	263	-1
N.S.	1	1.00	0.59	0.67	0.49	0.63	0.00	1.28	-0.00
time (sec)	N/A	0.035	0.072	0.021	0.316	0.406	0.000	7.472	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	67	127	43	44	0	0	-1
N.S.	1	1.00	0.34	0.65	0.22	0.22	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.040	0.021	0.286	0.367	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	124	147	101	131	0	0	-1
N.S.	1	1.00	0.58	0.69	0.48	0.62	0.00	0.00	-0.00
time (sec)	N/A	0.048	0.079	0.023	0.323	0.370	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	124	145	101	134	0	0	-1
N.S.	1	1.00	0.57	0.67	0.46	0.61	0.00	0.00	-0.00
time (sec)	N/A	0.050	0.082	0.025	0.298	0.376	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	62	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.035	0.011	0.000	0.000	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.029	0.007	0.000	0.000	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	53	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.019	0.009	0.000	0.000	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	44	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.017	0.008	0.000	0.000	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	45	66	27	22	0	0	-1
N.S.	1	1.00	0.53	0.78	0.32	0.26	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.031	0.021	0.280	0.400	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.027	0.007	0.000	0.000	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.026	0.008	0.000	0.000	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	61	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.035	0.009	0.000	0.000	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.034	0.004	0.000	0.000	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.023	0.004	0.000	0.000	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.019	0.003	0.000	0.000	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	79	104	70	106	0	0	-1
N.S.	1	1.00	0.50	0.65	0.44	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.075	0.023	0.286	0.344	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.030	0.004	0.000	0.000	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	55	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.030	0.005	0.000	0.000	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	0	0	79	0	0	-1
N.S.	1	1.00	1.12	0.00	0.00	1.52	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.048	0.030	0.000	0.367	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	32	51	0	45	0	0	-1
N.S.	1	1.00	0.74	1.19	0.00	1.05	0.00	0.00	-0.02
time (sec)	N/A	0.009	0.054	0.074	0.000	0.441	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	80	0	0	103	0	0	-1
N.S.	1	1.00	0.62	0.00	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.119	0.030	0.000	0.375	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	59	0	0	82	0	0	-1
N.S.	1	1.00	0.58	0.00	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.051	0.028	0.000	0.371	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	124	75	0	0	165	0	0	-1
N.S.	1	1.06	0.64	0.00	0.00	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.074	0.026	0.000	0.406	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	54	148	59	78	0	0	-1
N.S.	1	1.00	0.52	1.44	0.57	0.76	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.048	0.083	0.303	0.350	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	97	973	0	353	0	0	-1
N.S.	1	1.00	0.87	8.77	0.00	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.216	0.111	0.000	0.386	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	82	664	0	285	0	0	-1
N.S.	1	1.00	0.94	7.63	0.00	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.147	0.086	0.000	0.388	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	402	0	231	0	0	-1
N.S.	1	1.00	0.97	5.91	0.00	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.091	0.070	0.000	0.362	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	43	113	0	159	0	39	39
N.S.	1	1.00	1.10	2.90	0.00	4.08	0.00	1.00	1.00
time (sec)	N/A	0.023	0.055	0.060	0.000	0.353	0.000	4.308	1.467

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	90	658	0	333	0	0	-1
N.S.	1	1.00	0.92	6.71	0.00	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.196	0.143	0.000	0.377	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	115	958	0	429	0	0	-1
N.S.	1	1.00	0.91	7.60	0.00	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.345	0.119	0.000	0.393	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	147	1300	0	522	0	0	-1
N.S.	1	1.00	0.90	7.93	0.00	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.286	0.135	0.000	0.360	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	62	280	0	4426	0	0	-1
N.S.	1	1.00	0.18	0.79	0.00	12.54	0.00	0.00	-0.00
time (sec)	N/A	0.345	0.134	0.488	0.000	0.625	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	62	260	0	4699	0	0	-1
N.S.	1	1.00	0.10	0.43	0.00	7.70	0.00	0.00	-0.00
time (sec)	N/A	0.712	0.115	0.311	0.000	0.607	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	60	114	0	801	0	1035	-1
N.S.	1	1.00	0.36	0.67	0.00	4.74	0.00	6.12	-0.01
time (sec)	N/A	0.127	0.111	0.148	0.000	0.363	0.000	6.500	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	105	268	0	1229	0	0	-1
N.S.	1	1.00	0.51	1.31	0.00	6.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	0.134	0.233	0.000	0.412	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	699	699	107	534	0	6279	0	0	-1
N.S.	1	1.00	0.15	0.76	0.00	8.98	0.00	0.00	-0.00
time (sec)	N/A	0.928	0.141	0.495	0.000	1.740	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	105	630	0	5712	0	0	-1
N.S.	1	1.00	0.25	1.52	0.00	13.80	0.00	0.00	-0.00
time (sec)	N/A	0.518	0.141	0.773	0.000	0.978	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	265	0	0	0	0	0	-1
N.S.	1	1.00	1.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.496	0.019	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	263	0	0	0	0	0	-1
N.S.	1	1.00	1.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.425	0.016	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	261	0	0	0	0	0	-1
N.S.	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.392	0.013	0.000	0.000	0.000	0.000	0.000

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	72	397	0	259	362	0	224
N.S.	1	1.00	0.97	5.36	0.00	3.50	4.89	0.00	3.03
time (sec)	N/A	0.044	0.131	0.066	0.000	0.378	28.262	0.000	1.608

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	240	0	0	0	0	0	-1
N.S.	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.314	0.017	0.000	0.000	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	258	0	0	0	0	0	-1
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.342	0.021	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	365	0	0	0	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.558	0.005	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	366	0	0	0	0	0	-1
N.S.	1	1.00	2.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.519	0.003	0.000	0.000	0.000	0.000	0.000

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	364	0	0	0	0	0	-1
N.S.	1	1.00	2.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.467	0.003	0.000	0.000	0.000	0.000	0.000

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	351	0	0	0	0	0	-1
N.S.	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.459	0.003	0.000	0.000	0.000	0.000	0.000

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	113	125	0	658	0	0	-1
N.S.	1	1.00	0.95	1.05	0.00	5.53	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.260	0.099	0.000	0.412	0.000	0.000	0.000

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	365	0	0	0	0	0	-1
N.S.	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.476	0.003	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	365	0	0	0	0	0	-1
N.S.	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.444	0.003	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	518	0	0	0	0	0	-1
N.S.	1	1.00	3.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	1.019	0.004	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	524	0	0	0	0	0	-1
N.S.	1	1.00	3.52	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	1.026	0.005	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	520	0	0	0	0	0	-1
N.S.	1	1.00	3.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.093	1.020	0.004	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	513	0	0	0	0	0	-1
N.S.	1	1.00	3.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	1.053	0.003	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	159	209	0	827	0	0	-1
N.S.	1	1.00	0.92	1.21	0.00	4.78	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.626	0.104	0.000	0.455	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	526	0	0	0	0	0	-1
N.S.	1	1.00	3.51	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	1.006	0.003	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	520	0	0	0	0	0	-1
N.S.	1	1.00	3.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	1.028	0.003	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.156	0.007	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.136	0.007	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	175	0	0	0	0	0	-1
N.S.	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.142	0.008	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	166	0	0	0	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.103	0.007	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	46	0	0	148	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	3.15	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.121	0.008	0.000	0.355	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	173	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.150	0.011	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	175	0	0	0	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.147	0.007	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	-1
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.676	0.007	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	-1
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.633	0.008	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	398	0	0	0	0	0	-1
N.S.	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.615	0.006	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	384	0	0	0	0	0	-1
N.S.	1	1.00	2.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.685	0.007	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	0	0	449	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	4.58	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.630	0.009	0.000	0.456	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	395	0	0	0	0	0	-1
N.S.	1	1.00	2.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.594	0.009	0.000	0.000	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	399	0	0	0	0	0	-1
N.S.	1	1.00	2.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.602	0.007	0.000	0.000	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	137	3798	273	2303	0	25656	1734
N.S.	1	1.00	0.75	20.87	1.50	12.65	0.00	140.97	9.53
time (sec)	N/A	0.111	0.559	0.081	0.297	0.501	0.000	3.747	2.156

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	86	1065	152	706	12124	5454	543
N.S.	1	1.00	0.74	9.10	1.30	6.03	103.62	46.62	4.64
time (sec)	N/A	0.051	0.162	0.050	0.300	0.374	142.979	3.701	1.617

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	205	65	142	1091	557	83
N.S.	1	1.00	0.71	3.53	1.12	2.45	18.81	9.60	1.43
time (sec)	N/A	0.018	0.078	0.051	0.285	0.358	18.635	3.738	1.406

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	307	0	0	0	0	0	-1
N.S.	1	1.00	1.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.640	0.011	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	1890	0	0	0	0	0	-1
N.S.	1	1.00	5.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.680	4.361	0.012	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	12289	0	0	0	0	0	-1
N.S.	1	1.00	19.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	7.040	6.895	0.013	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	618	0	0	0	0	0	-1
N.S.	1	1.00	3.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	2.216	0.007	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	388	0	0	0	0	0	-1
N.S.	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.518	0.007	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	183	0	0	0	0	0	-1
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.149	0.011	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	428	0	0	0	0	0	-1
N.S.	1	1.00	2.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	1.021	0.011	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	181	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.307	0.025	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	150	298	155	137	178	169	141
N.S.	1	1.00	3.26	6.48	3.37	2.98	3.87	3.67	3.07
time (sec)	N/A	0.040	0.030	0.239	0.293	0.324	0.026	5.397	0.076

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	401	1314	464	394	559	493	383
N.S.	1	1.00	4.51	14.76	5.21	4.43	6.28	5.54	4.30
time (sec)	N/A	0.127	0.080	0.231	0.494	0.329	0.065	4.412	1.479

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	797	7550	1019	859	1314	1109	777
N.S.	1	1.00	5.78	54.71	7.38	6.22	9.52	8.04	5.63
time (sec)	N/A	0.250	0.190	0.245	0.303	0.339	0.135	4.047	1.657

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	154	349	179	161	240	213	164
N.S.	1	1.00	2.80	6.35	3.25	2.93	4.36	3.87	2.98
time (sec)	N/A	0.039	0.008	0.197	0.286	0.396	0.029	3.757	0.077

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	405	1413	500	430	722	615	419
N.S.	1	1.00	3.89	13.59	4.81	4.13	6.94	5.91	4.03
time (sec)	N/A	0.112	0.055	0.240	0.530	0.340	0.070	3.236	1.472

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	801	7697	1067	907	1654	1360	825
N.S.	1	1.00	5.04	48.41	6.71	5.70	10.40	8.55	5.19
time (sec)	N/A	0.210	0.030	0.236	0.284	0.364	0.148	5.029	1.650

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	219	158	0	1167	178	1194	2500
N.S.	1	1.00	1.13	0.82	0.00	6.05	0.92	6.19	12.95
time (sec)	N/A	0.294	0.096	0.153	0.000	0.377	1.732	4.142	2.324

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	151	0	424	280	130	278
N.S.	1	1.00	0.95	1.86	0.00	5.23	3.46	1.60	3.43
time (sec)	N/A	0.089	0.031	0.182	0.000	0.363	0.923	3.944	1.764

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	175	140	0	599	104	1285	590
N.S.	1	1.00	1.07	0.85	0.00	3.65	0.63	7.84	3.60
time (sec)	N/A	0.120	0.066	0.157	0.000	0.448	0.741	4.273	1.737

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	129	0	266	168	53	61
N.S.	1	1.00	1.07	3.00	0.00	6.19	3.91	1.23	1.42
time (sec)	N/A	0.042	0.012	0.045	0.000	0.338	0.560	4.500	0.094

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	128	184	0	460	320	274	2173
N.S.	1	1.00	1.36	1.96	0.00	4.89	3.40	2.91	23.12
time (sec)	N/A	0.090	0.056	0.177	0.000	0.374	13.548	4.828	2.504

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	206	168	0	1275	211	0	2500
N.S.	1	1.00	1.06	0.86	0.00	6.54	1.08	0.00	12.82
time (sec)	N/A	0.208	0.240	0.193	0.000	0.444	3.012	0.000	2.389

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	154	213	0	802	0	102	2500
N.S.	1	1.00	1.27	1.76	0.00	6.63	0.00	0.84	20.66
time (sec)	N/A	0.134	0.094	0.239	0.000	0.413	0.000	3.830	5.855

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	235	188	0	1970	347	1243	2500
N.S.	1	1.00	1.05	0.84	0.00	8.79	1.55	5.55	11.16
time (sec)	N/A	0.346	0.141	0.230	0.000	0.415	101.087	3.551	2.833

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	263	323	0	2336	573	1304	2500
N.S.	1	1.00	0.97	1.20	0.00	8.65	2.12	4.83	9.26
time (sec)	N/A	0.411	0.315	0.175	0.000	0.407	11.684	3.630	4.755

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	100	276	0	1007	495	171	427
N.S.	1	1.00	1.03	2.85	0.00	10.38	5.10	1.76	4.40
time (sec)	N/A	0.093	0.091	0.193	0.000	0.406	2.770	4.754	1.770

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	247	319	0	2358	0	1312	2500
N.S.	1	1.00	0.97	1.26	0.00	9.28	0.00	5.17	9.84
time (sec)	N/A	0.277	0.642	0.174	0.000	0.427	0.000	4.138	3.953

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	96	98	270	0	1028	495	172	417
N.S.	1	0.98	1.00	2.76	0.00	10.49	5.05	1.76	4.26
time (sec)	N/A	0.082	0.088	0.082	0.000	0.357	2.678	3.972	1.720

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	271	364	0	3148	0	1357	2500
N.S.	1	1.00	0.91	1.22	0.00	10.53	0.00	4.54	8.36
time (sec)	N/A	0.491	0.580	0.072	0.000	0.426	0.000	4.481	4.846

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	235	399	0	2452	0	454	2500
N.S.	1	1.00	1.45	2.46	0.00	15.14	0.00	2.80	15.43
time (sec)	N/A	0.198	0.330	0.206	0.000	0.503	0.000	3.648	11.354

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	339	441	0	4244	0	847	2500
N.S.	1	1.00	0.97	1.27	0.00	12.20	0.00	2.43	7.18
time (sec)	N/A	1.105	1.056	0.211	0.000	0.626	0.000	4.253	6.384

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	284	462	0	4518	0	224	2500
N.S.	1	1.00	1.33	2.17	0.00	21.21	0.00	1.05	11.74
time (sec)	N/A	0.245	0.343	0.229	0.000	0.630	0.000	4.340	12.317

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	384	489	0	5636	0	1987	2500
N.S.	1	1.00	0.94	1.20	0.00	13.81	0.00	4.87	6.13
time (sec)	N/A	2.463	1.931	0.214	0.000	0.731	0.000	3.366	8.725

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	328	704	0	6493	0	1688	2500
N.S.	1	1.00	0.96	2.06	0.00	19.04	0.00	4.95	7.33
time (sec)	N/A	0.640	2.963	0.239	0.000	0.498	0.000	3.540	7.019

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	146	544	0	3701	1671	365	1182
N.S.	1	1.00	0.97	3.63	0.00	24.67	11.14	2.43	7.88
time (sec)	N/A	0.130	0.149	0.212	0.000	0.414	7.813	4.015	3.855

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	382	885	0	7597	0	2295	2500
N.S.	1	1.00	1.05	2.44	0.00	20.93	0.00	6.32	6.89
time (sec)	N/A	0.714	3.214	0.253	0.000	0.636	0.000	3.383	7.429

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	150	147	541	0	3670	1646	365	1157
N.S.	1	0.99	0.97	3.56	0.00	24.14	10.83	2.40	7.61
time (sec)	N/A	0.132	0.124	0.176	0.000	0.442	7.483	3.602	3.802

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	424	1010	0	8450	0	2487	2500
N.S.	1	1.00	0.97	2.31	0.00	19.34	0.00	5.69	5.72
time (sec)	N/A	3.651	4.825	0.159	0.000	0.797	0.000	3.652	7.800

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	391	966	0	9844	0	1012	2500
N.S.	1	1.00	1.53	3.79	0.00	38.60	0.00	3.97	9.80
time (sec)	N/A	0.343	2.637	0.255	0.000	1.009	0.000	4.038	17.982

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	560	1197	0	10150	0	1412	2500
N.S.	1	1.00	1.16	2.47	0.00	20.97	0.00	2.92	5.17
time (sec)	N/A	0.827	6.162	0.285	0.000	1.155	0.000	3.635	14.376

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	491	1141	0	15081	0	377	2500
N.S.	1	1.00	1.51	3.51	0.00	46.40	0.00	1.16	7.69
time (sec)	N/A	0.388	6.128	0.335	0.000	1.594	0.000	4.731	22.450

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	222	162	0	1282	219	1245	2500
N.S.	1	1.00	1.10	0.80	0.00	6.35	1.08	6.16	12.38
time (sec)	N/A	0.266	0.090	0.199	0.000	0.388	2.087	4.786	1.341

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	80	154	0	436	332	162	287
N.S.	1	1.00	0.92	1.77	0.00	5.01	3.82	1.86	3.30
time (sec)	N/A	0.089	0.028	0.268	0.000	0.361	1.028	4.131	0.443

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	178	143	0	731	124	1325	683
N.S.	1	1.00	1.05	0.84	0.00	4.30	0.73	7.79	4.02
time (sec)	N/A	0.121	0.066	0.234	0.000	0.365	0.857	4.021	1.792

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	47	130	0	268	189	62	477
N.S.	1	1.00	1.07	2.95	0.00	6.09	4.30	1.41	10.84
time (sec)	N/A	0.046	0.012	0.043	0.000	0.346	0.615	3.814	1.622

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	131	188	0	466	348	285	2520
N.S.	1	1.00	1.27	1.83	0.00	4.52	3.38	2.77	24.47
time (sec)	N/A	0.097	0.051	0.232	0.000	0.372	17.430	3.038	3.462

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	209	172	0	1413	258	0	2500
N.S.	1	1.00	1.02	0.84	0.00	6.93	1.26	0.00	12.25
time (sec)	N/A	0.206	0.231	0.198	0.000	0.367	3.254	0.000	3.872

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	157	217	0	820	0	348	2500
N.S.	1	1.00	1.18	1.63	0.00	6.17	0.00	2.62	18.80
time (sec)	N/A	0.134	0.045	0.240	0.000	0.487	0.000	3.881	6.981

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	238	192	0	2138	411	1249	2500
N.S.	1	1.00	1.01	0.81	0.00	9.06	1.74	5.29	10.59
time (sec)	N/A	0.335	0.140	0.200	0.000	0.406	103.092	4.330	3.173

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	266	327	0	2498	641	1370	2500
N.S.	1	1.00	0.95	1.17	0.00	8.95	2.30	4.91	8.96
time (sec)	N/A	0.411	0.304	0.172	0.000	0.389	12.011	3.698	5.092

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	103	280	0	1063	556	211	460
N.S.	1	1.00	1.00	2.72	0.00	10.32	5.40	2.05	4.47
time (sec)	N/A	0.101	0.090	0.231	0.000	0.393	2.944	3.671	1.903

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	250	323	0	2520	0	1378	2500
N.S.	1	1.00	0.95	1.23	0.00	9.58	0.00	5.24	9.51
time (sec)	N/A	0.274	0.630	0.221	0.000	0.402	0.000	3.762	4.398

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	99	272	0	1052	525	211	442
N.S.	1	1.00	1.01	2.78	0.00	10.73	5.36	2.15	4.51
time (sec)	N/A	0.088	0.084	0.082	0.000	0.389	2.716	3.782	1.905

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	238	403	0	2462	0	476	2500
N.S.	1	1.00	1.37	2.32	0.00	14.15	0.00	2.74	14.37
time (sec)	N/A	0.207	0.299	0.288	0.000	0.605	0.000	3.982	11.689

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	342	445	0	4442	0	999	2500
N.S.	1	1.00	0.95	1.24	0.00	12.34	0.00	2.78	6.94
time (sec)	N/A	1.126	1.031	0.212	0.000	0.536	0.000	4.022	7.289

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	287	466	0	4560	0	687	2500
N.S.	1	1.00	1.26	2.04	0.00	20.00	0.00	3.01	10.96
time (sec)	N/A	0.253	0.338	0.253	0.000	1.337	0.000	3.058	13.520

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	387	493	0	5856	0	2002	2500
N.S.	1	1.00	0.91	1.17	0.00	13.84	0.00	4.73	5.91
time (sec)	N/A	2.376	1.909	0.240	0.000	0.717	0.000	3.222	10.447

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	331	708	0	6666	0	1844	2500
N.S.	1	1.00	0.94	2.01	0.00	18.88	0.00	5.22	7.08
time (sec)	N/A	0.593	2.746	0.236	0.000	0.559	0.000	3.257	7.520

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	149	548	0	3805	1794	447	1267
N.S.	1	1.00	0.94	3.45	0.00	23.93	11.28	2.81	7.97
time (sec)	N/A	0.136	0.144	0.263	0.000	0.436	7.930	3.902	4.019

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	385	889	0	7734	0	2527	2500
N.S.	1	1.00	1.03	2.37	0.00	20.62	0.00	6.74	6.67
time (sec)	N/A	0.673	3.263	0.204	0.000	0.613	0.000	3.694	7.944

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	148	543	0	3710	1707	445	1199
N.S.	1	1.00	0.97	3.55	0.00	24.25	11.16	2.91	7.84
time (sec)	N/A	0.134	0.124	0.194	0.000	0.477	7.704	3.097	3.994

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	394	970	0	9862	0	1044	2500
N.S.	1	1.00	1.46	3.59	0.00	36.53	0.00	3.87	9.26
time (sec)	N/A	0.327	2.683	0.275	0.000	1.714	0.000	4.400	18.492

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	499	575	1201	0	10408	0	1658	2500
N.S.	1	1.00	1.15	2.41	0.00	20.86	0.00	3.32	5.01
time (sec)	N/A	0.743	6.148	0.289	0.000	1.168	0.000	4.005	15.398

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	509	1145	0	15147	0	1735	2500
N.S.	1	1.00	1.48	3.34	0.00	44.16	0.00	5.06	7.29
time (sec)	N/A	0.384	6.102	0.335	0.000	4.192	0.000	4.828	24.912

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	10.797	0.012	0.000	0.000	0.000	0.000	0.000

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.466	10.332	0.027	0.000	0.000	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	105	104	104	107	28	29
N.S.	1	1.00	1.00	3.09	3.06	3.06	3.15	0.82	0.85
time (sec)	N/A	0.024	0.014	0.210	0.278	0.363	0.029	3.126	1.584

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	188	175	174	174	187	46	46
N.S.	1	1.00	3.36	3.12	3.11	3.11	3.34	0.82	0.82
time (sec)	N/A	0.066	0.010	0.235	0.270	0.405	0.053	3.305	1.602

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [478] had the largest ratio of [77]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	15	0.133
2	A	1	1	1.00	15	0.067
3	A	1	1	1.00	15	0.067
4	A	3	3	1.00	15	0.200
5	A	8	8	1.00	11	0.727
6	A	3	2	1.00	26	0.077
7	A	3	2	1.00	26	0.077
8	A	3	2	1.00	26	0.077
9	A	2	2	1.00	26	0.077
10	A	3	2	1.00	24	0.083
11	A	2	1	1.00	22	0.045
12	A	3	2	1.00	26	0.077
13	A	3	2	1.00	26	0.077
14	A	3	2	1.00	26	0.077
15	A	3	2	1.00	26	0.077
16	A	3	2	1.00	26	0.077
17	A	3	2	1.00	26	0.077
18	A	3	2	1.00	26	0.077
19	A	3	2	1.00	26	0.077
20	A	3	2	1.00	26	0.077
21	A	3	2	1.00	26	0.077
22	A	3	2	1.00	26	0.077
23	A	3	2	1.00	26	0.077
24	A	4	3	1.40	26	0.115
25	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	2	1.00	26	0.077
27	A	4	3	1.00	26	0.115
28	A	3	2	1.00	26	0.077
29	A	3	2	1.00	26	0.077
30	A	2	2	1.00	26	0.077
31	A	3	2	1.00	24	0.083
32	A	3	2	1.00	22	0.091
33	A	4	3	1.00	26	0.115
34	A	3	2	1.00	26	0.077
35	A	3	2	1.00	26	0.077
36	A	4	3	1.00	26	0.115
37	A	3	2	1.00	26	0.077
38	A	3	2	1.00	26	0.077
39	A	4	3	1.00	26	0.115
40	A	3	2	1.00	26	0.077
41	A	3	2	1.00	26	0.077
42	A	4	3	1.00	26	0.115
43	A	3	2	1.00	26	0.077
44	A	3	2	1.00	26	0.077
45	A	2	2	1.00	26	0.077
46	A	3	2	1.00	26	0.077
47	A	3	2	1.00	26	0.077
48	A	4	4	1.00	26	0.154
49	A	3	2	1.00	26	0.077
50	A	3	2	1.00	26	0.077
51	A	3	2	1.00	26	0.077
52	A	4	3	1.00	26	0.115
53	A	3	2	1.00	26	0.077
54	A	3	2	1.00	26	0.077
55	A	4	3	1.00	26	0.115
56	A	3	2	1.00	26	0.077
57	A	3	2	1.00	26	0.077
58	A	4	3	1.00	26	0.115
59	A	3	2	1.00	26	0.077
60	A	3	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.00	26	0.077
62	A	3	2	1.00	24	0.083
63	A	3	2	1.00	22	0.091
64	A	4	3	1.00	26	0.115
65	A	3	2	1.00	26	0.077
66	A	3	2	1.00	26	0.077
67	A	4	3	1.00	26	0.115
68	A	3	2	1.00	26	0.077
69	A	3	2	1.00	26	0.077
70	A	4	3	1.00	26	0.115
71	A	3	2	1.00	26	0.077
72	A	3	2	1.00	26	0.077
73	A	4	3	1.00	26	0.115
74	A	3	2	1.00	26	0.077
75	A	3	2	1.00	26	0.077
76	A	4	3	1.00	26	0.115
77	A	3	2	1.00	26	0.077
78	A	3	2	1.00	26	0.077
79	A	4	3	1.00	26	0.115
80	A	3	2	1.00	26	0.077
81	A	3	2	1.00	26	0.077
82	A	2	2	1.00	26	0.077
83	A	3	2	1.00	26	0.077
84	A	3	2	1.00	26	0.077
85	A	4	4	1.00	26	0.154
86	A	3	2	1.00	26	0.077
87	A	3	2	1.00	26	0.077
88	A	5	4	1.00	26	0.154
89	A	8	8	1.00	26	0.308
90	A	8	8	1.00	26	0.308
91	A	3	3	1.00	26	0.115
92	A	7	7	1.00	24	0.292
93	A	7	7	1.00	22	0.318
94	A	5	5	1.00	26	0.192
95	A	8	8	1.00	26	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	8	8	0.99	26	0.308
97	A	4	3	0.98	26	0.115
98	A	9	9	1.00	26	0.346
99	A	9	9	1.00	26	0.346
100	A	2	2	1.00	26	0.077
101	A	9	8	1.00	24	0.333
102	A	9	8	1.00	22	0.364
103	A	4	3	1.00	26	0.115
104	A	10	9	1.00	26	0.346
105	A	10	9	1.00	26	0.346
106	A	4	3	1.00	26	0.115
107	A	11	9	1.00	26	0.346
108	A	4	3	1.00	26	0.115
109	A	11	9	1.00	26	0.346
110	A	11	9	1.00	26	0.346
111	A	2	2	1.00	26	0.077
112	A	11	8	1.00	24	0.333
113	A	11	8	1.00	22	0.364
114	A	4	3	1.00	26	0.115
115	A	12	9	1.00	26	0.346
116	A	12	9	1.00	26	0.346
117	A	4	3	1.00	26	0.115
118	A	3	2	1.00	28	0.071
119	A	3	2	1.00	28	0.071
120	A	3	2	1.00	28	0.071
121	A	2	2	1.00	28	0.071
122	A	2	2	1.00	28	0.071
123	A	2	2	1.00	28	0.071
124	A	2	2	1.00	26	0.077
125	A	4	3	1.00	24	0.125
126	A	4	3	1.00	24	0.125
127	A	4	3	1.00	24	0.125
128	A	2	2	1.00	24	0.083
129	A	2	2	1.00	24	0.083
130	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	2	1.03	22	0.091
132	A	3	3	1.04	20	0.150
133	A	3	3	1.00	24	0.125
134	A	2	2	1.00	24	0.083
135	A	2	2	1.00	24	0.083
136	A	3	3	1.00	24	0.125
137	A	2	2	1.00	24	0.083
138	A	6	6	1.00	18	0.333
139	A	5	5	1.00	18	0.278
140	A	3	3	1.00	18	0.167
141	A	7	7	1.00	18	0.389
142	A	8	7	1.00	18	0.389
143	A	14	8	1.00	18	0.444
144	A	14	8	1.00	18	0.444
145	A	13	7	1.00	18	0.389
146	A	13	7	1.00	18	0.389
147	A	13	7	1.00	16	0.438
148	A	13	7	1.00	14	0.500
149	A	14	8	1.00	18	0.444
150	A	14	8	1.00	18	0.444
151	A	6	4	1.00	16	0.250
152	A	5	4	1.00	16	0.250
153	A	4	3	1.00	16	0.188
154	B	4	3	2.10	16	0.188
155	A	6	5	1.00	16	0.312
156	A	4	3	1.00	16	0.188
157	A	4	3	1.00	16	0.188
158	A	15	10	1.00	16	0.625
159	A	15	10	1.00	16	0.625
160	A	14	9	1.00	16	0.562
161	A	14	9	1.00	16	0.562
162	A	13	8	1.00	16	0.500
163	A	13	8	1.00	16	0.500
164	A	13	8	1.00	14	0.571
165	A	13	8	1.00	12	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	14	9	1.00	16	0.562
167	A	14	9	1.00	16	0.562
168	A	15	10	1.00	16	0.625
169	A	15	10	1.00	16	0.625
170	A	14	8	1.00	16	0.500
171	A	5	5	1.00	16	0.312
172	A	13	7	1.00	16	0.438
173	A	13	7	1.00	16	0.438
174	A	3	3	1.00	16	0.188
175	A	13	7	1.00	14	0.500
176	C	13	7	2.02	12	0.583
177	A	7	7	1.00	16	0.438
178	A	14	8	1.00	16	0.500
179	A	14	8	1.00	16	0.500
180	A	8	7	1.00	16	0.438
181	A	16	10	1.00	16	0.625
182	A	13	7	1.00	10	0.700
183	A	3	3	1.00	14	0.214
184	A	13	7	1.00	14	0.500
185	A	7	7	1.00	20	0.350
186	A	6	6	1.00	20	0.300
187	A	6	6	1.00	20	0.300
188	A	5	5	1.00	20	0.250
189	A	4	4	1.00	20	0.200
190	A	7	6	1.00	20	0.300
191	A	7	6	1.00	20	0.300
192	A	4	4	1.00	20	0.200
193	A	5	5	1.00	20	0.250
194	A	6	6	1.00	20	0.300
195	A	7	7	1.00	20	0.350
196	A	2	2	1.00	20	0.100
197	A	2	2	1.00	18	0.111
198	A	2	2	1.00	16	0.125
199	A	2	2	1.00	20	0.100
200	A	2	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	8	7	1.00	20	0.350
202	A	7	6	1.00	20	0.300
203	A	7	6	1.00	20	0.300
204	A	6	5	1.00	20	0.250
205	A	5	4	1.00	20	0.200
206	A	8	7	1.00	20	0.350
207	A	8	7	1.00	20	0.350
208	A	8	7	1.00	20	0.350
209	A	8	7	1.00	20	0.350
210	A	5	4	1.00	20	0.200
211	A	6	5	1.00	20	0.250
212	A	7	6	1.00	20	0.300
213	A	8	7	1.00	20	0.350
214	A	2	2	1.00	20	0.100
215	A	2	2	1.00	18	0.111
216	A	2	2	1.00	16	0.125
217	A	2	2	1.00	20	0.100
218	A	2	2	1.00	20	0.100
219	A	6	6	1.00	20	0.300
220	A	5	5	1.00	20	0.250
221	A	5	5	1.00	20	0.250
222	A	4	4	1.00	20	0.200
223	A	3	3	1.00	20	0.150
224	A	3	3	1.00	20	0.150
225	A	4	4	1.00	20	0.200
226	A	5	5	1.00	20	0.250
227	A	6	6	1.00	20	0.300
228	A	7	6	1.00	20	0.300
229	A	2	2	1.00	20	0.100
230	A	2	2	1.00	18	0.111
231	A	2	2	1.00	16	0.125
232	A	2	2	1.00	20	0.100
233	A	2	2	1.00	20	0.100
234	A	6	6	1.00	20	0.300
235	A	5	5	1.00	20	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	5	5	1.00	20	0.250
237	A	2	2	1.00	20	0.100
238	A	2	2	1.00	20	0.100
239	A	5	5	1.00	20	0.250
240	A	5	5	1.00	20	0.250
241	A	6	6	1.00	20	0.300
242	A	7	6	1.00	20	0.300
243	A	2	2	1.00	20	0.100
244	A	2	2	1.00	18	0.111
245	A	2	2	1.00	16	0.125
246	A	2	2	1.00	20	0.100
247	A	2	2	1.00	20	0.100
248	A	2	1	1.00	20	0.050
249	A	2	1	1.00	18	0.056
250	A	3	2	1.00	20	0.100
251	A	4	3	1.00	20	0.150
252	A	2	2	1.00	22	0.091
253	A	2	2	1.00	22	0.091
254	A	2	2	1.00	22	0.091
255	A	2	2	1.00	22	0.091
256	A	2	2	1.00	20	0.100
257	A	4	4	1.00	18	0.222
258	A	3	3	1.00	18	0.167
259	A	2	2	1.00	18	0.111
260	A	2	2	1.00	18	0.111
261	A	2	2	1.00	18	0.111
262	A	2	2	1.00	16	0.125
263	A	2	2	1.00	14	0.143
264	A	3	3	1.00	18	0.167
265	A	2	2	1.00	18	0.111
266	A	2	2	1.00	18	0.111
267	A	3	3	1.00	18	0.167
268	A	2	2	1.00	18	0.111
269	A	2	2	1.00	18	0.111
270	A	3	3	1.00	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	2	2	1.00	16	0.125
272	A	5	5	1.00	16	0.312
273	A	4	3	1.00	16	0.188
274	A	4	4	1.00	16	0.250
275	A	2	2	1.00	16	0.125
276	A	4	4	1.00	14	0.286
277	A	4	3	1.00	16	0.188
278	A	5	5	1.00	16	0.312
279	A	4	3	1.00	16	0.188
280	A	6	5	1.00	16	0.312
281	A	12	9	1.00	16	0.562
282	A	11	8	1.00	16	0.500
283	A	11	8	1.00	16	0.500
284	A	11	8	1.00	16	0.500
285	A	11	8	1.00	12	0.667
286	A	12	9	1.00	16	0.562
287	A	12	9	1.00	16	0.562
288	A	13	9	1.00	16	0.562
289	A	13	9	1.00	16	0.562
290	A	2	2	1.00	16	0.125
291	A	5	5	1.00	16	0.312
292	A	4	3	1.00	16	0.188
293	A	4	4	1.00	16	0.250
294	A	2	2	1.00	16	0.125
295	A	4	4	1.00	14	0.286
296	A	4	3	1.00	16	0.188
297	A	5	5	1.00	16	0.312
298	A	4	3	1.00	16	0.188
299	A	6	5	1.00	16	0.312
300	A	6	6	1.00	16	0.375
301	A	5	5	1.00	16	0.312
302	A	5	5	1.00	16	0.312
303	A	5	5	1.00	16	0.312
304	A	5	5	1.00	12	0.417
305	A	6	6	1.00	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	6	6	1.00	16	0.375
307	A	7	6	1.00	16	0.375
308	A	7	6	1.00	16	0.375
309	A	3	2	1.00	18	0.111
310	A	6	6	1.00	18	0.333
311	A	5	4	1.00	18	0.222
312	A	5	5	1.00	18	0.278
313	A	4	3	1.00	18	0.167
314	A	3	3	1.00	18	0.167
315	A	4	3	1.00	16	0.188
316	A	7	7	1.00	18	0.389
317	A	5	4	1.00	18	0.222
318	A	8	7	1.00	18	0.389
319	A	8	5	1.00	18	0.278
320	A	8	5	1.00	18	0.278
321	A	7	4	1.00	18	0.222
322	A	7	4	1.00	18	0.222
323	A	7	4	1.00	18	0.222
324	A	7	4	1.00	14	0.286
325	A	8	5	1.00	18	0.278
326	A	8	5	1.00	18	0.278
327	A	3	2	1.00	14	0.143
328	A	6	6	1.00	14	0.429
329	A	7	5	1.00	14	0.357
330	A	5	5	1.00	14	0.357
331	A	10	7	1.00	14	0.500
332	A	3	3	1.00	14	0.214
333	A	10	6	1.00	12	0.500
334	A	7	7	1.00	14	0.500
335	A	7	5	1.00	14	0.357
336	A	8	7	1.00	14	0.500
337	A	13	10	1.00	14	0.714
338	A	20	7	1.00	14	0.500
339	A	9	6	1.00	14	0.429
340	A	19	7	1.00	14	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	19	6	1.00	14	0.429
342	A	9	6	1.00	10	0.600
343	A	20	8	1.00	14	0.571
344	A	20	7	1.00	14	0.500
345	A	12	9	1.00	14	0.643
346	A	22	10	1.00	14	0.714
347	A	3	2	1.00	16	0.125
348	A	6	6	1.00	16	0.375
349	A	5	4	1.00	16	0.250
350	A	5	5	1.00	16	0.312
351	A	10	7	1.00	16	0.438
352	A	3	3	1.00	16	0.188
353	A	10	6	1.00	14	0.429
354	A	7	7	1.00	16	0.438
355	A	5	4	1.00	16	0.250
356	A	8	7	1.00	16	0.438
357	A	13	10	1.00	16	0.625
358	A	20	7	1.00	16	0.438
359	A	19	6	1.00	16	0.375
360	A	19	7	1.00	16	0.438
361	A	19	6	1.00	16	0.375
362	A	19	6	1.00	12	0.500
363	A	22	8	1.00	16	0.500
364	A	20	7	1.00	16	0.438
365	A	22	9	1.00	16	0.562
366	A	22	10	1.00	16	0.625
367	A	3	2	1.00	16	0.125
368	A	5	4	1.00	16	0.250
369	A	5	4	1.00	16	0.250
370	A	4	3	1.00	16	0.188
371	A	4	3	1.00	16	0.188
372	A	3	3	1.00	16	0.188
373	A	4	3	1.00	14	0.214
374	A	6	5	1.00	16	0.312
375	A	5	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	7	5	1.00	16	0.312
377	A	6	5	1.00	16	0.312
378	A	20	8	0.96	16	0.500
379	A	19	7	1.00	16	0.438
380	A	19	7	1.00	16	0.438
381	A	19	7	1.01	16	0.438
382	A	19	7	1.00	12	0.583
383	A	20	8	1.00	16	0.500
384	A	20	8	1.00	16	0.500
385	A	3	2	1.00	16	0.125
386	A	5	4	1.00	16	0.250
387	A	5	4	1.00	16	0.250
388	A	4	3	1.00	16	0.188
389	A	4	3	1.00	16	0.188
390	A	3	3	1.00	16	0.188
391	A	4	3	1.00	14	0.214
392	A	6	5	1.00	16	0.312
393	A	5	4	1.00	16	0.250
394	A	7	5	1.00	16	0.312
395	A	6	5	1.00	16	0.312
396	A	8	5	1.00	16	0.312
397	A	7	4	1.00	16	0.250
398	A	7	4	1.00	16	0.250
399	A	7	4	1.14	16	0.250
400	A	7	4	1.00	12	0.333
401	A	8	5	1.00	16	0.312
402	A	8	5	1.00	16	0.312
403	A	9	6	1.00	16	0.375
404	A	9	6	1.00	16	0.375
405	A	4	3	1.00	16	0.188
406	A	5	4	1.00	16	0.250
407	A	5	5	1.00	14	0.357
408	A	3	3	1.00	14	0.214
409	A	7	7	1.00	14	0.500
410	A	8	7	1.00	14	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	8	8	1.00	10	0.800
412	A	7	6	1.00	18	0.333
413	A	7	6	1.00	18	0.333
414	A	7	6	1.00	16	0.375
415	A	6	6	1.00	14	0.429
416	A	5	5	1.00	18	0.278
417	A	3	3	1.00	18	0.167
418	A	7	7	1.00	18	0.389
419	A	8	7	1.00	18	0.389
420	A	8	7	1.00	18	0.389
421	A	8	7	1.00	18	0.389
422	A	8	7	1.00	16	0.438
423	A	8	7	1.00	14	0.500
424	A	7	7	1.00	18	0.389
425	A	4	4	1.00	18	0.222
426	A	4	4	1.00	18	0.222
427	A	4	4	1.00	18	0.222
428	A	8	7	1.00	18	0.389
429	A	8	7	1.00	18	0.389
430	A	8	7	1.00	18	0.389
431	A	9	8	1.00	14	0.571
432	A	8	8	1.00	18	0.444
433	A	5	4	1.00	18	0.222
434	A	5	5	1.00	18	0.278
435	A	5	5	1.00	18	0.278
436	A	5	5	1.00	18	0.278
437	A	5	4	0.98	18	0.222
438	A	9	8	1.00	18	0.444
439	A	9	8	1.00	18	0.444
440	A	6	4	1.00	18	0.222
441	A	6	4	1.00	16	0.250
442	A	5	4	1.00	14	0.286
443	A	4	3	1.00	18	0.167
444	A	4	3	1.00	18	0.167
445	A	6	5	1.00	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	4	3	1.00	18	0.167
447	A	4	3	1.00	18	0.167
448	A	4	3	1.00	18	0.167
449	A	9	7	1.00	16	0.438
450	A	8	7	1.00	16	0.438
451	A	7	6	1.00	16	0.375
452	A	4	4	1.00	16	0.250
453	A	5	5	1.00	16	0.312
454	A	6	6	1.00	16	0.375
455	A	4	3	1.00	22	0.136
456	A	5	4	1.00	14	0.286
457	A	15	9	1.00	14	0.643
458	A	9	6	1.00	14	0.429
459	A	7	6	1.00	20	0.300
460	A	4	3	1.00	23	0.130
461	A	4	4	1.00	22	0.182
462	A	4	3	1.00	26	0.115
463	A	4	3	1.00	26	0.115
464	A	3	2	1.00	26	0.077
465	A	4	3	1.00	26	0.115
466	A	4	3	1.00	26	0.115
467	A	4	3	1.00	26	0.115
468	A	4	3	1.00	26	0.115
469	A	4	3	1.00	26	0.115
470	A	4	3	1.00	26	0.115
471	A	4	3	1.00	26	0.115
472	A	4	4	1.00	30	0.133
473	A	4	3	1.00	28	0.107
474	A	4	3	1.00	26	0.115
475	A	4	3	1.00	24	0.125
476	A	3	3	1.00	28	0.107
477	A	3	3	1.00	28	0.107
478	C	7	3	1.11	77	0.039
479	A	4	3	1.00	26	0.115
480	A	4	3	1.00	26	0.115

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
481	A	5	4	1.00	24	0.167
482	A	5	4	1.00	26	0.154
483	A	5	4	1.00	26	0.154
484	A	5	4	1.00	26	0.154
485	A	4	3	1.00	26	0.115
486	A	5	4	1.00	26	0.154
487	A	5	4	1.00	26	0.154
488	A	5	4	1.00	26	0.154
489	A	5	4	1.00	26	0.154
490	A	5	4	1.00	26	0.154
491	A	4	3	1.00	26	0.115
492	A	5	4	1.00	26	0.154
493	A	4	3	1.00	23	0.130
494	A	4	3	1.00	23	0.130
495	A	2	2	1.00	23	0.087
496	A	5	5	1.00	21	0.238
497	A	4	3	1.00	23	0.130
498	A	4	3	1.00	23	0.130
499	A	4	3	1.00	23	0.130
500	A	12	9	1.00	25	0.360
501	A	9	9	1.00	25	0.360
502	A	5	5	1.00	25	0.200
503	A	6	6	1.00	25	0.240
504	A	11	11	1.00	25	0.440
505	A	14	11	1.00	25	0.440
506	A	1	1	1.00	26	0.038
507	A	1	1	1.00	28	0.036
508	A	4	3	1.00	32	0.094
509	A	4	3	1.00	32	0.094
510	A	3	2	1.00	32	0.062
511	A	4	3	1.00	32	0.094
512	A	2	2	1.00	32	0.062
513	A	4	3	1.00	32	0.094
514	A	4	3	1.00	32	0.094
515	A	5	4	1.00	30	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	3	2	1.00	28	0.071
517	A	3	2	1.00	26	0.077
518	A	2	1	1.00	24	0.042
519	A	3	2	1.00	28	0.071
520	A	3	2	1.00	28	0.071
521	A	3	2	1.00	28	0.071
522	A	9	4	1.00	30	0.133
523	A	3	2	1.00	28	0.071
524	A	3	2	1.00	26	0.077
525	A	3	2	1.00	24	0.083
526	A	4	3	1.00	28	0.107
527	A	3	2	1.00	28	0.071
528	A	3	2	1.00	28	0.071
529	A	2	2	1.00	30	0.067
530	A	2	2	1.00	28	0.071
531	A	2	2	1.00	26	0.077
532	A	2	2	1.00	24	0.083
533	A	5	5	1.00	28	0.179
534	A	2	2	1.00	28	0.071
535	A	2	2	1.00	28	0.071
536	A	2	2	1.00	30	0.067
537	A	2	2	1.00	28	0.071
538	A	2	2	1.00	26	0.077
539	A	2	2	1.00	24	0.083
540	A	4	3	1.00	28	0.107
541	A	2	2	1.00	28	0.071
542	A	2	2	1.00	28	0.071
543	A	2	2	1.00	36	0.056
544	A	2	2	1.00	33	0.061
545	A	3	3	1.00	34	0.088
546	A	3	3	1.00	33	0.091
547	A	3	3	1.06	35	0.086
548	A	4	3	1.00	30	0.100
549	A	7	6	1.00	24	0.250
550	A	6	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	5	5	1.00	24	0.208
552	A	3	3	1.00	22	0.136
553	A	8	7	1.00	24	0.292
554	A	8	7	1.00	24	0.292
555	A	8	7	1.00	24	0.292
556	A	8	5	1.00	26	0.192
557	A	14	8	1.00	26	0.308
558	A	4	3	1.00	26	0.115
559	A	6	5	1.00	26	0.192
560	A	16	10	1.00	26	0.385
561	A	10	7	1.00	26	0.269
562	A	3	2	1.00	20	0.100
563	A	3	2	1.00	18	0.111
564	A	3	2	1.00	16	0.125
565	A	7	7	1.00	20	0.350
566	A	3	2	1.00	20	0.100
567	A	3	2	1.00	20	0.100
568	A	2	2	1.00	22	0.091
569	A	2	2	1.00	22	0.091
570	A	2	2	1.00	20	0.100
571	A	2	2	1.00	18	0.111
572	A	7	6	1.00	22	0.273
573	A	2	2	1.00	22	0.091
574	A	2	2	1.00	22	0.091
575	A	2	2	1.00	22	0.091
576	A	2	2	1.00	22	0.091
577	A	2	2	1.00	20	0.100
578	A	2	2	1.00	18	0.111
579	A	8	7	1.00	22	0.318
580	A	2	2	1.00	22	0.091
581	A	2	2	1.00	22	0.091
582	A	2	2	1.00	22	0.091
583	A	2	2	1.00	22	0.091
584	A	2	2	1.00	20	0.100
585	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	3	3	1.00	22	0.136
587	A	2	2	1.00	22	0.091
588	A	2	2	1.00	22	0.091
589	A	2	2	1.00	22	0.091
590	A	2	2	1.00	22	0.091
591	A	2	2	1.00	20	0.100
592	A	2	2	1.00	18	0.111
593	A	5	5	1.00	22	0.227
594	A	2	2	1.00	22	0.091
595	A	2	2	1.00	22	0.091
596	A	14	3	1.00	22	0.136
597	A	10	3	1.00	22	0.136
598	A	6	3	1.00	20	0.150
599	A	3	2	1.00	22	0.091
600	A	5	3	1.00	22	0.136
601	A	6	4	1.00	22	0.182
602	A	2	2	1.00	24	0.083
603	A	2	2	1.00	24	0.083
604	A	2	2	1.00	24	0.083
605	A	2	2	1.00	24	0.083
606	A	2	2	1.00	22	0.091
607	A	3	2	1.00	28	0.071
608	A	4	3	1.00	30	0.100
609	A	4	3	1.00	30	0.100
610	A	3	2	1.00	31	0.065
611	A	4	3	1.00	33	0.091
612	A	4	3	1.00	33	0.091
613	A	5	4	1.00	30	0.133
614	A	6	6	1.00	30	0.200
615	A	4	3	1.00	30	0.100
616	A	4	4	1.00	28	0.143
617	A	8	8	1.00	30	0.267
618	A	5	4	1.00	30	0.133
619	A	9	8	1.00	30	0.267
620	A	6	5	1.00	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	5	4	1.00	30	0.133
622	A	5	5	1.00	30	0.167
623	A	5	4	1.00	30	0.133
624	A	5	5	0.98	28	0.179
625	A	5	4	1.00	22	0.182
626	A	9	8	1.00	30	0.267
627	A	6	5	1.00	30	0.167
628	A	9	8	1.00	30	0.267
629	A	7	5	1.00	30	0.167
630	A	6	5	1.00	30	0.167
631	A	6	6	1.00	30	0.200
632	A	6	5	1.00	30	0.167
633	A	6	5	0.99	28	0.179
634	A	6	5	1.00	22	0.227
635	A	10	9	1.00	30	0.300
636	A	7	6	1.00	30	0.200
637	A	10	9	1.00	30	0.300
638	A	5	4	1.00	33	0.121
639	A	6	6	1.00	33	0.182
640	A	4	3	1.00	33	0.091
641	A	4	4	1.00	31	0.129
642	A	8	8	1.00	33	0.242
643	A	5	4	1.00	33	0.121
644	A	9	8	1.00	33	0.242
645	A	6	5	1.00	33	0.152
646	A	5	4	1.00	33	0.121
647	A	5	5	1.00	33	0.152
648	A	5	4	1.00	33	0.121
649	A	5	5	1.00	31	0.161
650	A	9	8	1.00	33	0.242
651	A	6	5	1.00	33	0.152
652	A	9	8	1.00	33	0.242
653	A	7	5	1.00	33	0.152
654	A	6	5	1.00	33	0.152
655	A	6	6	1.00	33	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
656	A	6	5	1.00	33	0.152
657	A	6	5	1.00	31	0.161
658	A	10	9	1.00	33	0.273
659	A	7	6	1.00	33	0.182
660	A	10	9	1.00	33	0.273
661	A	7	6	1.00	26	0.231
662	A	10	9	1.00	28	0.321
663	A	3	2	1.00	24	0.083
664	A	4	3	1.00	26	0.115

Chapter 3

Listing of integrals

Local contents

3.1 $\int \left(ax^3+bx^6\right)^{5/3}\,dx$

Optimal. Leaf size=52

$$-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}+\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}$$

```
[Out] -3/88*a*(b*x^6+a*x^3)^(8/3)/b^2/x^8+1/11*(b*x^6+a*x^3)^(8/3)/b/x^5
```

Rubi [A]
time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2027, 2039}

$$\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(5/3),x]
```

```
[Out] (-3*a*(a*x^3 + b*x^6)^(8/3))/(88*b^2*x^8) + (a*x^3 + b*x^6)^(8/3)/(11*b*x^5
)
```

Rule 2027

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[b*((n*p + n - j + 1)/(a*(
j*p + 1))), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \left(ax^3+bx^6\right)^{5/3}\,dx&=\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}-\frac{(3a)\int \frac{\left(ax^3+bx^6\right)^{5/3}}{x^3}\,dx}{11b}\\&=-\frac{3a\left(ax^3+bx^6\right)^{8/3}}{88b^2x^8}+\frac{\left(ax^3+bx^6\right)^{8/3}}{11bx^5}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 35, normalized size = 0.67

$$\frac{(x^3(a+bx^3))^{8/3}(-3a+8bx^3)}{88b^2x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(5/3),x]
[Out] ((x^3*(a + b*x^3))^(8/3)*(-3*a + 8*b*x^3))/(88*b^2*x^8)
```

Maple [A]
time = 0.13, size = 39, normalized size = 0.75

method	result	size
gosper	$-\frac{(b\,x^3+a)\,(-8b\,x^3+3a)\,(b\,x^6+a\,x^3)^{\frac{5}{3}}}{88b^2x^5}$	39
trager	$-\frac{(-8b^3x^9-13a\,b^2x^6-2a^2b\,x^3+3a^3)\,(b\,x^6+a\,x^3)^{\frac{2}{3}}}{88b^2x^2}$	54
risch	$-\frac{(x^3\,(b\,x^3+a))^{\frac{2}{3}}\,(-8b^3x^9-13a\,b^2x^6-2a^2b\,x^3+3a^3)}{88x^2b^2}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^6+a*x^3)^(5/3),x,method=_RETURNVERBOSE)
[Out] -1/88*(b*x^3+a)*(-8*b*x^3+3*a)*(b*x^6+a*x^3)^(5/3)/b^2/x^5
```

Maxima [A]
time = 0.29, size = 46, normalized size = 0.88

$$\frac{(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3)(bx^3+a)^{\frac{2}{3}}}{88\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^3 + a)^(2/3)/b^2
```

Fricas [A]
time = 0.40, size = 53, normalized size = 1.02

$$\frac{(8\,b^3x^9+13\,ab^2x^6+2\,a^2bx^3-3\,a^3)(bx^6+ax^3)^{\frac{2}{3}}}{88\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
```

```
[Out] 1/88*(8*b^3*x^9 + 13*a*b^2*x^6 + 2*a^2*b*x^3 - 3*a^3)*(b*x^6 + a*x^3)^(2/3)
/(b^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^3 + bx^6\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**6+a*x**3)**(5/3),x)
[Out] Integral((a*x**3 + b*x**6)**(5/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(5/3),x, algorithm="giac")
[Out] integrate((b*x^6 + a*x^3)^(5/3), x)
```

Mupad [B]
time = 1.23, size = 40, normalized size = 0.77

$$-\frac{\left(b\,x^3+a\right)^2\,\left(b\,x^6+a\,x^3\right)^{2/3}\,\left(3\,a-8\,b\,x^3\right)}{88\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 + b*x^6)^(5/3),x)
[Out] -((a + b*x^3)^2*(a*x^3 + b*x^6)^(2/3)*(3*a - 8*b*x^3))/(88*b^2*x^2)
```

3.2 $\int \left(ax^3+bx^6\right)^{2/3} dx$

Optimal. Leaf size=25

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

```
[Out] 1/5*(b*x^6+a*x^3)^(5/3)/b/x^5
```

Rubi [A]

time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2025}

$$\frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(2/3),x]
```

```
[Out] (a*x^3 + b*x^6)^(5/3)/(5*b*x^5)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \left(ax^3+bx^6\right)^{2/3} dx = \frac{\left(ax^3+bx^6\right)^{5/3}}{5bx^5}$$

Mathematica [A]

time = 0.02, size = 25, normalized size = 1.00

$$\frac{\left(x^3\left(a+bx^3\right)\right)^{5/3}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(2/3),x]
```

```
[Out] (x^3*(a + b*x^3))^(5/3)/(5*b*x^5)
```

Maple [A]
time = 0.13, size = 29, normalized size = 1.16

method	result	size
gosper	$\frac{\left(b\,x^{3}+a\right)\left(b\,x^{6}+a\,x^{3}\right)^{\frac{2}{3}}}{5b\,x^{2}}$	29
trager	$\frac{\left(b\,x^{3}+a\right)\left(b\,x^{6}+a\,x^{3}\right)^{\frac{2}{3}}}{5b\,x^{2}}$	29
risch	$\frac{\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{2}{3}}\left(b\,x^{3}+a\right)}{5x^{2}b}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^6+a*x^3)^(2/3),x,method=_RETURNVERBOSE)
[Out] 1/5*(b*x^3+a)/b/x^2*(b*x^6+a*x^3)^(2/3)
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.56

$$\frac{\left(bx^{3}+a\right)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
[Out] 1/5*(b*x^3 + a)^(5/3)/b
```

Fricas [A]
time = 0.40, size = 28, normalized size = 1.12

$$\frac{\left(bx^{6}+ax^{3}\right)^{\frac{2}{3}}\left(bx^{3}+a\right)}{5\,bx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
[Out] 1/5*(b*x^6 + a*x^3)^(2/3)*(b*x^3 + a)/(b*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax^{3}+bx^{6}\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**6+a*x**3)**(2/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^6+a*x^3)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^6 + a*x^3)^(2/3), x)
```

Mupad [**B**]
time = 1.15, size = 29, normalized size = 1.16

$$\frac{\left(\frac{a}{5\,b}+\frac{x^3}{5}\right)\,{\left(b\,x^6+a\,x^3\right)}^{2/3}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 + b*x^6)^(2/3),x)
```

```
[Out] ((a/(5*b) + x^3/5)*(a*x^3 + b*x^6)^(2/3))/x^2
```

3.3 $\int \frac{1}{\left(ax^3+bx^6\right)^{2/3}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

```
[Out] -(b*x^6+a*x^3)^(1/3)/a/x^2
```

Rubi [A]
time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2025}

$$-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(-2/3),x]
[Out] -((a*x^3 + b*x^6)^(1/3)/(a*x^2))
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(ax^3+bx^6\right)^{2/3}}\,dx=-\frac{\sqrt[3]{ax^3+bx^6}}{ax^2}$$

Mathematica [A]
time = 0.09, size = 23, normalized size = 1.00

$$-\frac{\sqrt[3]{x^3\left(a+bx^3\right)}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(-2/3),x]
[Out] -((x^3*(a + b*x^3))^(1/3)/(a*x^2))
```

Maple [A]
time = 0.13, size = 22, normalized size = 0.96

method	result	size
trager	$-\frac{\left(b\,x^{6}+a\,x^{3}\right)^{\frac{1}{3}}}{a\,x^{2}}$	22
gosper	$-\frac{x\left(b\,x^{3}+a\right)}{a\left(b\,x^{6}+a\,x^{3}\right)^{\frac{2}{3}}}$	27
risch	$-\frac{x\left(b\,x^{3}+a\right)}{\left(x^{3}\left(b\,x^{3}+a\right)\right)^{\frac{2}{3}}a}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a*x^3)^(2/3),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^6+a*x^3)^(1/3)/a/x^2
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.74

$$-\frac{(bx^3+a)^{\frac{1}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -(b*x^3 + a)^(1/3)/(a*x)
```

Fricas [A]
time = 0.38, size = 21, normalized size = 0.91

$$-\frac{(bx^6+ax^3)^{\frac{1}{3}}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -(b*x^6 + a*x^3)^(1/3)/(a*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^3+bx^6)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a*x**3)**(2/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(-2/3), x)
```

Giac [A]
time = 4.13, size = 14, normalized size = 0.61

$$-\frac{\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(2/3),x, algorithm="giac")
```

```
[Out] -(b + a/x^3)^(1/3)/a
```

Mupad [B]
time = 1.15, size = 21, normalized size = 0.91

$$-\frac{\left(b\,x^6+a\,x^3\right)^{1/3}}{a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b*x^6)^(2/3),x)
```

```
[Out] -(a*x^3 + b*x^6)^(1/3)/(a*x^2)
```

3.4 $\int \frac{1}{\left(ax^3+bx^6\right)^{5/3}}\,dx$

Optimal. Leaf size=77

$$\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}$$

```
[Out] 1/2/a/x^2/(b*x^6+a*x^3)^(2/3)-3/4*(b*x^6+a*x^3)^(1/3)/a^2/x^5+9/4*b*(b*x^6+
a*x^3)^(1/3)/a^3/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2026, 2041, 2025}

$$\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{1}{2ax^2\left(ax^3+bx^6\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^6)^(-5/3),x]
```

```
[Out] 1/(2*a*x^2*(a*x^3 + b*x^6)^(2/3)) - (3*(a*x^3 + b*x^6)^(1/3))/(4*a^2*x^5) +
 (9*b*(a*x^3 + b*x^6)^(1/3))/(4*a^3*x^2)
```

Rule 2025

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2026

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
, j, n}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1
)/(n - j)], 0] && LtQ[p, -1]
```

Rule 2041

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Dist[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1))), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax^3+bx^6)^{5/3}}\,dx &= \frac{1}{2ax^2(ax^3+bx^6)^{2/3}}+\frac{3\int \frac{1}{x^3(ax^3+bx^6)^{2/3}}\,dx}{a}\\ &= \frac{1}{2ax^2(ax^3+bx^6)^{2/3}}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}-\frac{(9b)\int \frac{1}{(ax^3+bx^6)^{2/3}}\,dx}{4a^2}\\ &= \frac{1}{2ax^2(ax^3+bx^6)^{2/3}}-\frac{3\sqrt[3]{ax^3+bx^6}}{4a^2x^5}+\frac{9b\sqrt[3]{ax^3+bx^6}}{4a^3x^2}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 46, normalized size = 0.60

$$\frac{-a^2+6abx^3+9b^2x^6}{4a^3x^2\left(x^3\left(a+bx^3\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^6)^(-5/3),x]
[Out] (-a^2 + 6*a*b*x^3 + 9*b^2*x^6)/(4*a^3*x^2*(x^3*(a + b*x^3))^(2/3))
```

Maple [A]
time = 0.14, size = 46, normalized size = 0.60

method	result	size
gosper	$-\frac{x\left(b\,x^3+a\right)\left(-9b^2x^6-6ab\,x^3+a^2\right)}{4a^3\left(b\,x^6+a\,x^3\right)^{\frac{5}{3}}}$	46
trager	$-\frac{\left(-9b^2x^6-6ab\,x^3+a^2\right)\left(b\,x^6+a\,x^3\right)^{\frac{1}{3}}}{4\left(b\,x^3+a\right)a^3x^5}$	50
risch	$-\frac{\left(b\,x^3+a\right)\left(-7b\,x^3+a\right)}{4a^3x^2\left(x^3\left(b\,x^3+a\right)\right)^{\frac{2}{3}}}+\frac{b^2x^4}{2a^3\left(x^3\left(b\,x^3+a\right)\right)^{\frac{2}{3}}}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a*x^3)^(5/3),x,method=_RETURNVERBOSE)
[Out] -1/4*x*(b*x^3+a)*(-9*b^2*x^6-6*a*b*x^3+a^2)/a^3/(b*x^6+a*x^3)^(5/3)
```

Maxima [A]
time = 0.28, size = 38, normalized size = 0.49

$$\frac{9\,b^2x^6+6\,abx^3-a^2}{4\left(bx^3+a\right)^{\frac{2}{3}}a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="maxima")
```

```
[Out] 1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)/((b*x^3 + a)^(2/3)*a^3*x^4)
```

Fricas [A]
time = 0.46, size = 54, normalized size = 0.70

$$\frac{(9\,b^2x^6+6\,abx^3-a^2)(bx^6+ax^3)^{\frac{1}{3}}}{4\,(a^3bx^8+a^4x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="fricas")
```

```
[Out] 1/4*(9*b^2*x^6 + 6*a*b*x^3 - a^2)*(b*x^6 + a*x^3)^(1/3)/(a^3*b*x^8 + a^4*x^
5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ax^3+bx^6)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a*x**3)**(5/3),x)
```

```
[Out] Integral((a*x**3 + b*x**6)**(-5/3), x)
```

Giac [A]
time = 5.01, size = 52, normalized size = 0.68

$$\frac{b^2}{2\,a^3\left(b+\frac{a}{x^3}\right)^{\frac{2}{3}}}-\frac{a^9\left(b+\frac{a}{x^3}\right)^{\frac{4}{3}}-8\,a^9\left(b+\frac{a}{x^3}\right)^{\frac{1}{3}}b}{4\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a*x^3)^(5/3),x, algorithm="giac")
```

```
[Out] 1/2*b^2/(a^3*(b + a/x^3)^(2/3)) - 1/4*(a^9*(b + a/x^3)^(4/3) - 8*a^9*(b + a
/x^3)^(1/3)*b)/a^12
```

Mupad [B]
time = 1.28, size = 51, normalized size = 0.66

$$\frac{{\left(b\,x^6+a\,x^3\right)}^{1/3}\,\left(-a^2+6\,a\,b\,x^3+9\,b^2\,x^6\right)}{4\,a^3\,x^5\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b*x^6)^(5/3),x)
```

```
[Out] ((a*x^3 + b*x^6)^(1/3)*(9*b^2*x^6 - a^2 + 6*a*b*x^3))/(4*a^3*x^5*(a + b*x^3
))
```

3.5 $\int \frac{1}{-x^3+x^6}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)$$

```
[Out] 1/2/x^2+1/3*ln(1-x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {1607, 331, 206, 31, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-x^3+x^6}\,dx &= \int \frac{1}{x^3\left(-1+x^3\right)}\,dx\\
&= \frac{1}{2x^2}+\int \frac{1}{-1+x^3}\,dx\\
&= \frac{1}{2x^2}+\frac{1}{3}\int \frac{1}{-1+x}\,dx+\frac{1}{3}\int \frac{-2-x}{1+x+x^2}\,dx\\
&= \frac{1}{2x^2}+\frac{1}{3}\log(1-x)-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\\
&= \frac{1}{2x^2}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\
&= \frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.00

$$\frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Maple [A]
time = 0.16, size = 38, normalized size = 0.79

method	result	size
default	$-\frac{\ln(x^2+x+1)}{6}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{1}{2x^2}+\frac{\ln(-1+x)}{3}$	38
risch	$\frac{1}{2x^2}-\frac{\ln(4x^2+4x+4)}{6}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{3}+\frac{\ln(-1+x)}{3}$	42
meijerg	$-\frac{(-1)^{\frac{2}{3}}\left(\frac{3(-1)^{\frac{1}{3}}}{2x^2}+\frac{x(-1)^{\frac{1}{3}}\left(\ln\left(1-(x^3)^{\frac{1}{3}}\right)-\frac{\ln\left(1+(x^3)^{\frac{1}{3}}+(x^3)^{\frac{2}{3}}\right)}{2}-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x^3)^{\frac{1}{3}}}{2+(x^3)^{\frac{1}{3}}}\right)\right)}{(x^3)^{\frac{1}{3}}}\right)}{3}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-x^3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*ln(x^2+x+1)-1/3*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/2/x^2+1/3*ln(-1+
x)
```

Maxima [A]
time = 0.48, size = 37, normalized size = 0.77

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(x - 1)
```

Fricas [A]
time = 0.46, size = 46, normalized size = 0.96

$$-\frac{2\sqrt{3}\,x^2\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+x^2\log\left(x^2+x+1\right)-2\,x^2\log\left(x-1\right)-3}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) + x^2*log(x^2 + x + 1) -
2*x^2*log(x - 1) - 3)/x^2
```

Sympy [A]
time = 0.05, size = 48, normalized size = 1.00

$$\frac{\log\left(x-1\right)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-x**3),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3 + 1/(2*x**2)
```

Giac [A]
time = 3.66, size = 38, normalized size = 0.79

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{2\,x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(abs(x - 1))
```

Mupad [B]
time = 0.09, size = 51, normalized size = 1.06

$$\frac{\ln\left(x-1\right)}{3}+\ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^3 - x^6),x)
```

```
[Out] log(x - 1)/3 + log(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 - 1/6) - log(x
 + (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*1i)/6 + 1/6) + 1/(2*x^2)
```

3.6 $\int x^5 \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\,(a+bx^3)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\,(a+bx^3)}$$

```
[Out] 1/6*a*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/9*b*x^9*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 14}

$$\frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\,(a+bx^3)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*(a + b*x^3)) + (b*x^9*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^5(ab+b^2x^3)\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (abx^5+b^2x^8)\,dx}{ab+b^2x^3}\\ &= \frac{ax^6\sqrt{a^2+2abx^3+b^2x^6}}{6\,(a+bx^3)}+\frac{bx^9\sqrt{a^2+2abx^3+b^2x^6}}{9\,(a+bx^3)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a+bx^3)^2}\,(3ax^6+2bx^9)}{18\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(3*a*x^6 + 2*b*x^9))/(18*(a + b*x^3))
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.46

method	result	size
gosper	$\frac{x^{6}\left(2 b \,x^{3}+3 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{18 b \,x^{3}+18 a}$	36
default	$\frac{x^{6}\left(2 b \,x^{3}+3 a \right) \sqrt{\left(b \,x^{3}+a \right)^{2}}}{18 b \,x^{3}+18 a}$	36
risch	$\frac{a \,x^{6} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{6 b \,x^{3}+6 a}+\frac{b \,x^{9} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{9 b \,x^{3}+9 a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*x^6*(2*b*x^3+3*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A]
time = 0.27, size = 83, normalized size = 1.05

$$-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,ax^3}{6\,b}-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,a^2}{6\,b^2}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*x^3/b - 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
 + a^2)*a^2/b^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/b^2
```

Fricas [A]
time = 0.41, size = 13, normalized size = 0.16

$$\frac{1}{9}\,bx^9+\frac{1}{6}\,ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*b*x^9 + 1/6*a*x^6
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.15

$$\frac{ax^6}{6} + \frac{bx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**6/6 + b*x**9/9
```

Giac [A]
time = 2.89, size = 23, normalized size = 0.29

$$\frac{1}{18}\left(2\,bx^9 + 3\,ax^6\right)\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/18*(2*b*x^9 + 3*a*x^6)*sgn(b*x^3 + a)
```

Mupad [B]
time = 1.26, size = 59, normalized size = 0.75

$$\frac{\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}\,\left(8\,b^2\,\left(a^2 + b^2\,x^6\right) - 12\,a^2\,b^2 + 4\,a\,b^3\,x^3\right)}{72\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)*(8*b^2*(a^2 + b^2*x^6) - 12*a^2*b^2 + 4*
a*b^3*x^3))/(72*b^4)
```

3.7 $\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}$$

```
[Out] 1/5*a*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/8*b*x^8*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 14}

$$\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b*x^8*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^4(ab+b^2x^3)\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (abx^4+b^2x^7)\,dx}{ab+b^2x^3}\\ &= \frac{ax^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a+bx^3)^2}\,(8ax^5+5bx^8)}{40\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(8*a*x^5 + 5*b*x^8))/(40*(a + b*x^3))
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.46

method	result	size
gosper	$\frac{x^5(5b\,x^3+8a)\sqrt{(b\,x^3+a)^2}}{40b\,x^3+40a}$	36
default	$\frac{x^5(5b\,x^3+8a)\sqrt{(b\,x^3+a)^2}}{40b\,x^3+40a}$	36
risch	$\frac{a\,x^5\sqrt{(b\,x^3+a)^2}}{5b\,x^3+5a}+\frac{b\,x^8\sqrt{(b\,x^3+a)^2}}{8b\,x^3+8a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/40*x^5*(5*b*x^3+8*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.16

$$\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*b*x^8 + 1/5*a*x^5
```

Fricas [A]
time = 0.36, size = 13, normalized size = 0.16

$$\frac{1}{8}\,bx^8+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*b*x^8 + 1/5*a*x^5
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.15

$$\frac{ax^5}{5} + \frac{bx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**5/5 + b*x**8/8
```

Giac [A]
time = 3.13, size = 29, normalized size = 0.37

$$\frac{1}{8}\,bx^8\mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{5}\,ax^5\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*b*x^8*sgn(b*x^3 + a) + 1/5*a*x^5*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\sqrt{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x^4*((a + b*x^3)^2)^(1/2), x)
```

3.8 $\int x^3 \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

```
[Out] 1/4*a*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/7*b*x^7*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1369, 14\}$

$$\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b*x^7*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(ab+b^2x^3\right)\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(abx^3+b^2x^6\right)\,dx}{ab+b^2x^3}\\
&= \frac{ax^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a+bx^3)^2}\,(7ax^4+4bx^7)}{28\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(7*a*x^4 + 4*b*x^7))/(28*(a + b*x^3))
```

Maple [A]
time = 0.05, size = 36, normalized size = 0.46

method	result	size
gosper	$\frac{x^{4}\left(4b\,x^{3}+7a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{28b\,x^{3}+28a}$	36
default	$\frac{x^{4}\left(4b\,x^{3}+7a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{28b\,x^{3}+28a}$	36
risch	$\frac{a\,x^{4}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{4b\,x^{3}+4a}+\frac{b\,x^{7}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{7b\,x^{3}+7a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*x^4*(4*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.16

$$\frac{1}{7}\,bx^7+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

Fricas [A]
time = 0.35, size = 13, normalized size = 0.16

$$\frac{1}{7}\,bx^7+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/7*b*x^7 + 1/4*a*x^4
```

Sympy [**A**]
time = 0.01, size = 12, normalized size = 0.15

$$\frac{ax^4}{4} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**4/4 + b*x**7/7
```

Giac [**A**]
time = 4.12, size = 29, normalized size = 0.37

$$\frac{1}{7}\,bx^7\mathrm{sgn}\left(bx^3 + a\right) + \frac{1}{4}\,ax^4\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7*b*x^7*sgn(b*x^3 + a) + 1/4*a*x^4*sgn(b*x^3 + a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\sqrt{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x^3*((a + b*x^3)^2)^(1/2), x)
```

3.9 $\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

```
[Out] 1/6*(b*x^3+a)*((b*x^3+a)^2)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1366, 623}

$$\frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*b)
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^3\right)\\ &= \frac{(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}}{6b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.06

$$\frac{\sqrt{(a+bx^3)^2}\,(2ax^3+bx^6)}{6\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(2*a*x^3 + b*x^6))/(6*(a + b*x^3))
```

Maple [A]
time = 0.05, size = 24, normalized size = 0.67

method	result	size
default	$\frac{(b\,x^{3}+a)\,\sqrt{(b\,x^{3}+a)^{2}}}{6b}$	24
risch	$\frac{(b\,x^{3}+a)\,\sqrt{(b\,x^{3}+a)^{2}}}{6b}$	24
gosper	$\frac{x^{3}\,(b\,x^{3}+2a)\,\sqrt{(b\,x^{3}+a)^{2}}}{6b\,x^{3}+6a}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(b*x^3+a)*((b*x^3+a)^2)^(1/2)/b
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(23) = 46$.
time = 0.28, size = 52, normalized size = 1.44

$$\frac{1}{6}\,\sqrt{b^2x^6+2\,abx^3+a^2}\,x^3+\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,a}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*x^3 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^
2)*a/b
```

Fricas [A]
time = 0.35, size = 13, normalized size = 0.36

$$\frac{1}{6}\,bx^6+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/3*a*x^3
```

Sympy [A]
time = 0.01, size = 12, normalized size = 0.33

$$\frac{ax^3}{3} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**3/3 + b*x**6/6
```

Giac [A]
time = 5.11, size = 22, normalized size = 0.61

$$\frac{1}{6}\left(bx^6 + 2\,ax^3\right)\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(b*x^6 + 2*a*x^3)*sgn(b*x^3 + a)
```

Mupad [B]
time = 1.23, size = 33, normalized size = 0.92

$$\left(\frac{a}{6\,b} + \frac{x^3}{6}\right)\,\sqrt{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] (a/(6*b) + x^3/6)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
```

3.10 $\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=79

$$\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

```
[Out] 1/2*a*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5*b*x^5*((b*x^3+a)^2)^(1/2)/(b*x^
3+a)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = $\{1369, 14\}$

$$\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b*x^5*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(ab+b^2x^3\right)\,dx}{ab+b^2x^3}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(abx+b^2x^4\right)\,dx}{ab+b^2x^3}\\ &= \frac{ax^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a+bx^3)^2}\,(5ax^2+2bx^5)}{10\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(5*a*x^2 + 2*b*x^5))/(10*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.46

method	result	size
gosper	$\frac{x^2(2b\,x^3+5a)\sqrt{(b\,x^3+a)^2}}{10b\,x^3+10a}$	36
default	$\frac{x^2(2b\,x^3+5a)\sqrt{(b\,x^3+a)^2}}{10b\,x^3+10a}$	36
risch	$\frac{a\,x^2\sqrt{(b\,x^3+a)^2}}{2b\,x^3+2a}+\frac{b\,x^5\sqrt{(b\,x^3+a)^2}}{5b\,x^3+5a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x^2*(2*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.16

$$\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/2*a*x^2
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.16

$$\frac{1}{5}\,bx^5+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*b*x^5 + 1/2*a*x^2
```

Sympy [**A**]
time = 0.01, size = 12, normalized size = 0.15

$$\frac{ax^2}{2} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x**2/2 + b*x**5/5
```

Giac [**A**]
time = 5.55, size = 29, normalized size = 0.37

$$\frac{1}{5}\,bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,ax^2\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5*sgn(b*x^3 + a) + 1/2*a*x^2*sgn(b*x^3 + a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x*((a + b*x^3)^2)^(1/2), x)
```

3.11 $\int \sqrt{a^2+2abx^3+b^2x^6}\,dx$

Optimal. Leaf size=74

$$\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}$$

```
[Out] a*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*b*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1357}

$$\frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b*x^4*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2abx^3+b^2x^6}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(2ab+2b^2x^3\right)\,dx}{2ab+2b^2x^3}\\ &= \frac{ax\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{bx^4\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 36, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\,\left(4ax+bx^4\right)}{4\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(4*a*x + b*x^4))/(4*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 33, normalized size = 0.45

method	result	size
gosper	$\frac{x\left(b\,x^{3}+4a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{4b\,x^{3}+4a}$	33
default	$\frac{x\left(b\,x^{3}+4a\right)\sqrt{\left(b\,x^{3}+a\right)^{2}}}{4b\,x^{3}+4a}$	33
risch	$\frac{ax\sqrt{\left(b\,x^{3}+a\right)^{2}}}{b\,x^{3}+a}+\frac{b\,x^{4}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{4b\,x^{3}+4a}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x*(b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Maxima [A]
time = 0.28, size = 10, normalized size = 0.14

$$\frac{1}{4}\,bx^4 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + a*x
```

Fricas [A]
time = 0.34, size = 10, normalized size = 0.14

$$\frac{1}{4}\,bx^4 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*b*x^4 + a*x
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.11

$$ax + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2),x)
```

```
[Out] a*x + b*x**4/4
```

Giac [A]
time = 6.19, size = 20, normalized size = 0.27

$$\frac{1}{4}\left(bx^4 + 4\,ax\right)\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(b*x^4 + 4*a*x)*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{{\left(b\,x^3+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(((a + b*x^3)^2)^(1/2), x)
```

3.12 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx$

Optimal. Leaf size=75

$$\frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\,(a+bx^3)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\,\log(x)}{a+bx^3}$$

```
[Out] 1/3*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+
a)
```

Rubi [A]
time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{1369, 14\}$

$$\frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\,(a+bx^3)}+\frac{a\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
```

```
[Out] (b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (a*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x}\,dx}{ab+b^2x^3}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x}+b^2x^2\right)\,dx}{ab+b^2x^3}$$
$$= \frac{bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)} + \frac{a\sqrt{a^2+2abx^3+b^2x^6}\,\log(x)}{a+bx^3}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^3\right)^2}\left(bx^3+3a\log(x)\right)}{3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x,x]
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3 + 3*a*Log[x]))/(3*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 34, normalized size = 0.45

method	result	size
default	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(b\,x^3+3a\ln(x)\right)}{3b\,x^3+3a}$	34
risch	$\frac{b\,x^3\sqrt{\left(b\,x^3+a\right)^2}}{3b\,x^3+3a}+\frac{a\ln(x)\sqrt{\left(b\,x^3+a\right)^2}}{b\,x^3+a}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/3*((b*x^3+a)^2)^(1/2)*(b*x^3+3*a*ln(x))/(b*x^3+a)
```

Maxima [A]
time = 0.27, size = 96, normalized size = 1.28

$$\frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a\log\left(2\,b^2x^3+2\,ab\right)-\frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{1}{3}\,\sqrt{b^2x^6+2\,abx^3+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*a*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3
 + 2*a^2)*a*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/3*sqrt(b^2*x^6 + 2
*a*b*x^3 + a^2)
```

Fricas [A]
time = 0.35, size = 11, normalized size = 0.15

$$\frac{1}{3}\,bx^3 + a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + a*log(x)
```

Sympy [A]
time = 0.02, size = 10, normalized size = 0.13

$$a\log\left(x\right) + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x,x)
```

```
[Out] a*log(x) + b*x**3/3
```

Giac [A]
time = 6.20, size = 28, normalized size = 0.37

$$\frac{1}{3}\,bx^3\mathrm{sgn}\left(bx^3 + a\right) + a\log\left(|x|\right)\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*sgn(b*x^3 + a) + a*log(abs(x))*sgn(b*x^3 + a)
```

Mupad [B]
time = 1.38, size = 109, normalized size = 1.45

$$\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3}-\frac{\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)\,\sqrt{a^2}}{3}+\frac{a\,b\,\ln\left(a\,b+\sqrt{{\left(b\,x^3+a\right)}^2}\,\sqrt{b^2}+b^2\,x^3\right)}{3\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/3 - (log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a^
2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)*(a^2)^(1/2))/3 + (a*b*log(a*b + ((a +
b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3))/(3*(b^2)^(1/2))
```

3.13 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx$

Optimal. Leaf size=77

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

```
[Out] -a*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+1/2*b*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 14}

$$\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (b*x^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^2}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^2}\,dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^2}+b^2x\right)\,dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 0.49

$$\frac{(-2a+bx^3)\sqrt{(a+bx^3)^2}}{2x\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^2,x]
```

```
[Out] ((-2*a + b*x^3)*Sqrt[(a + b*x^3)^2])/(2*x*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.47

method	result	size
gosper	$-\frac{(-b\,x^3+2a)\sqrt{(b\,x^3+a)^2}}{2(b\,x^3+a)x}$	36
default	$-\frac{(-b\,x^3+2a)\sqrt{(b\,x^3+a)^2}}{2(b\,x^3+a)x}$	36
risch	$-\frac{a\sqrt{(b\,x^3+a)^2}}{x(b\,x^3+a)}+\frac{b\,x^2\sqrt{(b\,x^3+a)^2}}{2b\,x^3+2a}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/x
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.18

$$\frac{bx^3-2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^3 - 2*a)/x
```

Fricas [A]
time = 0.33, size = 14, normalized size = 0.18

$$\frac{bx^3-2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^3 - 2*a)/x
```

Sympy [**A**]
time = 0.02, size = 8, normalized size = 0.10

$$-\frac{a}{x} + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**2,x)
```

```
[Out] -a/x + b*x**2/2
```

Giac [**A**]
time = 6.41, size = 29, normalized size = 0.38

$$\frac{1}{2}bx^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{a\mathrm{sgn}(bx^3 + a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*sgn(b*x^3 + a) - a*sgn(b*x^3 + a)/x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{{\left(b\,x^3+a\right)}^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^2,x)
```

```
[Out] int(((a + b*x^3)^2)^(1/2)/x^2, x)
```

3.14 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3} \, dx$

Optimal. Leaf size=74

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -1/2*a*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+b*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 14}

$$\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (b*x*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(a + b*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^3} \, dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \frac{ab+b^2x^3}{x^3} \, dx}{ab+b^2x^3}\\
&= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(b^2+\frac{ab}{x^3}\right) \, dx}{ab+b^2x^3}\\
&= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.50

$$-\frac{(a-2bx^3)\sqrt{(a+bx^3)^2}}{2x^2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^3,x]
[Out] -1/2*((a - 2*b*x^3)*Sqrt[(a + b*x^3)^2])/(x^2*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 34, normalized size = 0.46

method	result	size
gosper	$-\frac{(-2bx^3+a)\sqrt{(bx^3+a)^2}}{2(bx^3+a)x^2}$	34
default	$-\frac{(-2bx^3+a)\sqrt{(bx^3+a)^2}}{2(bx^3+a)x^2}$	34
risch	$-\frac{a\sqrt{(bx^3+a)^2}}{2x^2(bx^3+a)}+\frac{bx\sqrt{(bx^3+a)^2}}{bx^3+a}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*(-2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/x^2
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.20

$$\frac{2\,bx^3-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] 1/2*(2*b*x^3 - a)/x^2
```

Fricas [A]
time = 0.33, size = 15, normalized size = 0.20

$$\frac{2\,bx^3-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^3 - a)/x^2
```

Sympy [**A**]
time = 0.02, size = 8, normalized size = 0.11

$$-\frac{a}{2x^2}+bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*x
```

Giac [**A**]
time = 5.77, size = 26, normalized size = 0.35

$$bx\mathrm{sgn}\left(bx^3+a\right)-\frac{a\mathrm{sgn}(bx^3+a)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] b*x*sgn(b*x^3 + a) - 1/2*a*sgn(b*x^3 + a)/x^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{{\left(b\,x^3+a\right)}^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^3,x)
```

```
[Out] int(((a + b*x^3)^2)^(1/2)/x^3, x)
```

3.15 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx$

Optimal. Leaf size=75

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\,\log(x)}{a+bx^3}$$

```
[Out] -1/3*a*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+b*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3
+a)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 14}

$$\frac{b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

```
[Out] -1/3*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]*Log[x])/(a + b*x^3)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^4}\,dx}{ab+b^2x^3}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^4}+\frac{b^2}{x}\right)dx}{ab+b^2x^3}$$
$$= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{b\sqrt{a^2+2abx^3+b^2x^6}\log(x)}{a+bx^3}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.52

$$-\frac{\sqrt{(a+bx^3)^2}\,(a-3bx^3\log(x))}{3x^3(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^4,x]
```

```
[Out] -1/3*(Sqrt[(a + b*x^3)^2]*(a - 3*b*x^3*Log[x]))/(x^3*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 38, normalized size = 0.51

method	result	size
default	$\frac{\sqrt{(b\,x^3+a)^2}\,(3b\ln(x)x^3-a)}{3x^3(b\,x^3+a)}$	38
risch	$-\frac{a\sqrt{(b\,x^3+a)^2}}{3x^3(b\,x^3+a)}+\frac{b\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((b*x^3+a)^2)^(1/2)*(3*b*ln(x)*x^3-a)/x^3/(b*x^3+a)
```

Maxima [A]
time = 0.28, size = 99, normalized size = 1.32

$$\frac{1}{3}(-1)^{2\,b^2x^3+2\,ab}\,b\log\left(2\,b^2x^3+2\,ab\right)-\frac{1}{3}(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*b*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3
 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) - 1/3*sqrt(b^2*x^6 + 2
*a*b*x^3 + a^2)/x^3
```

Fricas [A]
time = 0.36, size = 17, normalized size = 0.23

$$\frac{3\,bx^3\log\left(x\right)-a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x^3*log(x) - a)/x^3
```

Sympy [A]
time = 0.04, size = 10, normalized size = 0.13

$$-\frac{a}{3x^3}+b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**4,x)
```

```
[Out] -a/(3*x**3) + b*log(x)
```

Giac [A]
time = 5.05, size = 43, normalized size = 0.57

$$b\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{bx^3\mathrm{sgn}(bx^3+a)+a\mathrm{sgn}(bx^3+a)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] b*log(abs(x))*sgn(b*x^3 + a) - 1/3*(b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a)
)/x^3
```

Mupad [B]
time = 1.38, size = 112, normalized size = 1.49

$$\frac{\ln\left(a\,b+\sqrt{{\left(b\,x^3+a\right)}^2}\,\sqrt{b^2}+b^2\,x^3\right)\,\sqrt{b^2}}{3}-\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3}-\frac{a\,b\,\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)}{3\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^4,x)
```

```
[Out] (log(a*b + ((a + b*x^3)^2)^(1/2)*(b^2)^(1/2) + b^2*x^3)*(b^2)^(1/2))/3 - (a
^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*x^3) - (a*b*log(a*b + a^2/x^3 + ((a^2)^(
1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3))/(3*(a^2)^(1/2))
```

3.16 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx$

Optimal. Leaf size=77

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}$$

```
[Out] -1/4*a*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-b*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

Rubi [**A**]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = $\{1369, 14\}$

$$-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
```

```
[Out] -1/4*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^4*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^5}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^5}\,dx}{ab+b^2x^3}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^5}+\frac{b^2}{x^2}\right)dx}{ab+b^2x^3}$$
$$= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.48

$$-\frac{\sqrt{(a+bx^3)^2}\,(a+4bx^3)}{4x^4(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^5,x]
[Out] -1/4*(Sqrt[(a + b*x^3)^2]*(a + 4*b*x^3))/(x^4*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 34, normalized size = 0.44

method	result	size
gosper	$-\frac{(4b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{4(b\,x^3+a)x^4}$	34
default	$-\frac{(4b\,x^3+a)\sqrt{(b\,x^3+a)^2}}{4(b\,x^3+a)x^4}$	34
risch	$\frac{(-b\,x^3-\frac{a}{4})\sqrt{(b\,x^3+a)^2}}{x^4(b\,x^3+a)}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*(4*b*x^3+a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/x^4
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.17

$$-\frac{4\,bx^3+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b*x^3 + a)/x^4
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.17

$$-\frac{4\,bx^3+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*x^3 + a)/x^4
```

Sympy [A]
time = 0.04, size = 14, normalized size = 0.18

$$\frac{-a-4bx^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**5,x)
```

```
[Out] (-a - 4*b*x**3)/(4*x**4)
```

Giac [A]
time = 4.95, size = 30, normalized size = 0.39

$$-\frac{4\,bx^3\mathrm{sgn}(bx^3+a)+a\mathrm{sgn}(bx^3+a)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(4*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^4
```

Mupad [B]
time = 1.21, size = 33, normalized size = 0.43

$$-\frac{\left(4\,b\,x^3+a\right)\,\sqrt{{\left(b\,x^3+a\right)}^2}}{4\,x^4\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^5,x)
```

```
[Out] -((a + 4*b*x^3)*((a + b*x^3)^2)^(1/2))/(4*x^4*(a + b*x^3))
```

3.17 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

```
[Out] -1/5*a*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-1/2*b*((b*x^3+a)^2)^(1/2)/x^2/(b*x
^3+a)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1369, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

```
[Out] -1/5*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^6}\,dx}{ab+b^2x^3}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^6}+\frac{b^2}{x^3}\right)\,dx}{ab+b^2x^3}$$
$$= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\,\left(2a+5bx^3\right)}{10x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^6,x]
```

```
[Out] -1/10*(Sqrt[(a + b*x^3)^2]*(2*a + 5*b*x^3))/(x^5*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.46

method	result	size
risch	$\frac{\left(-\frac{b\,x^3}{2}-\frac{a}{5}\right)\sqrt{\left(b\,x^3+a\right)^2}}{x^5\left(b\,x^3+a\right)}$	35
gosper	$-\frac{\left(5b\,x^3+2a\right)\sqrt{\left(b\,x^3+a\right)^2}}{10\left(b\,x^3+a\right)x^5}$	36
default	$-\frac{\left(5b\,x^3+2a\right)\sqrt{\left(b\,x^3+a\right)^2}}{10\left(b\,x^3+a\right)x^5}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(5*b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)/x^5
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.19

$$-\frac{5\,bx^3+2\,a}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

Fricas [A]
time = 0.38, size = 15, normalized size = 0.19

$$-\frac{5\,bx^3+2\,a}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/10*(5*b*x^3 + 2*a)/x^5
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.19

$$\frac{-2a-5bx^3}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**6,x)
```

```
[Out] (-2*a - 5*b*x**3)/(10*x**5)
```

Giac [A]
time = 5.76, size = 31, normalized size = 0.39

$$-\frac{5\,bx^3\mathrm{sgn}(bx^3+a)+2\,a\mathrm{sgn}(bx^3+a)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] -1/10*(5*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^5
```

Mupad [B]
time = 1.18, size = 35, normalized size = 0.44

$$-\frac{\left(5\,b\,x^3+2\,a\right)\,\sqrt{{\left(b\,x^3+a\right)}^2}}{10\,x^5\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^6,x)
```

```
[Out] -((2*a + 5*b*x^3)*((a + b*x^3)^2)^(1/2))/(10*x^5*(a + b*x^3))
```

3.18 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7} \, dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

```
[Out] -1/6*a*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-1/3*b*((b*x^3+a)^2)^(1/2)/x^3/(b*x
^3+a)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
```

```
[Out] -1/6*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^6*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^7}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^7}\,dx}{ab+b^2x^3}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^7}+\frac{b^2}{x^4}\right)dx}{ab+b^2x^3}$$
$$= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.47

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(a+2bx^3\right)}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^7,x]
[Out] -1/6*(Sqrt[(a + b*x^3)^2]*(a + 2*b*x^3))/(x^6*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 34, normalized size = 0.43

method	result	size
gosper	$-\frac{\left(2b\,x^3+a\right)\sqrt{\left(b\,x^3+a\right)^2}}{6x^6\left(b\,x^3+a\right)}$	34
default	$-\frac{\left(2b\,x^3+a\right)\sqrt{\left(b\,x^3+a\right)^2}}{6x^6\left(b\,x^3+a\right)}$	34
risch	$\frac{\left(-\frac{b\,x^3}{3}-\frac{a}{6}\right)\sqrt{\left(b\,x^3+a\right)^2}}{x^6\left(b\,x^3+a\right)}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^7,x,method=_RETURNVERBOSE)
[Out] -1/6*(2*b*x^3+a)*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)
```

Maxima [A]
time = 0.28, size = 86, normalized size = 1.09

$$\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^2}{6\,a^2} + \frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b}{6\,ax^3} - \frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2/a^2 + 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
+ a^2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^6)
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.16

$$-\frac{2\,bx^3+a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*x^3 + a)/x^6
```

Sympy [A]
time = 0.05, size = 14, normalized size = 0.18

$$\frac{-a-2bx^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**7,x)
```

```
[Out] (-a - 2*b*x**3)/(6*x**6)
```

Giac [A]
time = 5.67, size = 30, normalized size = 0.38

$$-\frac{2\,bx^3\mathrm{sgn}(bx^3+a)+a\mathrm{sgn}(bx^3+a)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(2*b*x^3*sgn(b*x^3 + a) + a*sgn(b*x^3 + a))/x^6
```

Mupad [B]
time = 1.18, size = 33, normalized size = 0.42

$$-\frac{(2\,b\,x^3+a)\,\sqrt{(b\,x^3+a)^2}}{6\,x^6\,(b\,x^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^7,x)
```

```
[Out] -((a + 2*b*x^3)*((a + b*x^3)^2)^(1/2))/(6*x^6*(a + b*x^3))
```

3.19 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}$$

```
[Out] -1/7*a*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-1/4*b*((b*x^3+a)^2)^(1/2)/x^4/(b*x
^3+a)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1369, 14\}$

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

```
[Out] -1/7*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^7*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^8}\,dx}{ab+b^2x^3}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^8}+\frac{b^2}{x^5}\right)dx}{ab+b^2x^3}$$
$$= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a+bx^3)^2}\,(4a+7bx^3)}{28x^7(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^8,x]
```

```
[Out] -1/28*(Sqrt[(a + b*x^3)^2]*(4*a + 7*b*x^3))/(x^7*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.46

method	result	size
risch	$\frac{\left(-\frac{b\,x^3}{4}-\frac{a}{7}\right)\sqrt{\left(b\,x^3+a\right)^2}}{x^7(b\,x^3+a)}$	35
gosper	$-\frac{(7b\,x^3+4a)\sqrt{\left(b\,x^3+a\right)^2}}{28x^7(b\,x^3+a)}$	36
default	$-\frac{(7b\,x^3+4a)\sqrt{\left(b\,x^3+a\right)^2}}{28x^7(b\,x^3+a)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/28*(7*b*x^3+4*a)*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.19

$$-\frac{7\,bx^3+4\,a}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

Fricas [A]
time = 0.38, size = 15, normalized size = 0.19

$$-\frac{7\,bx^3+4\,a}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/28*(7*b*x^3 + 4*a)/x^7
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.19

$$\frac{-4a-7bx^3}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**8,x)
```

```
[Out] (-4*a - 7*b*x**3)/(28*x**7)
```

Giac [A]
time = 4.49, size = 31, normalized size = 0.39

$$-\frac{7\,bx^3\mathrm{sgn}(bx^3+a)+4\,a\mathrm{sgn}(bx^3+a)}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] -1/28*(7*b*x^3*sgn(b*x^3 + a) + 4*a*sgn(b*x^3 + a))/x^7
```

Mupad [B]
time = 1.27, size = 35, normalized size = 0.44

$$-\frac{(7\,b\,x^3+4\,a)\,\sqrt{(b\,x^3+a)^2}}{28\,x^7\,(b\,x^3+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^8,x)
```

```
[Out] -((4*a + 7*b*x^3)*((a + b*x^3)^2)^(1/2))/(28*x^7*(a + b*x^3))
```

3.20 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

```
[Out] -1/8*a*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-1/5*b*((b*x^3+a)^2)^(1/2)/x^5/(b*x
^3+a)
```

Rubi [A]
time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1369, 14\}$

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
```

```
[Out] -1/8*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^8*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^9}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^9}\,dx}{ab+b^2x^3}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^9}+\frac{b^2}{x^6}\right)dx}{ab+b^2x^3}$$
$$= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a+bx^3)^2}\,(5a+8bx^3)}{40x^8(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^9,x]
```

```
[Out] -1/40*(Sqrt[(a + b*x^3)^2]*(5*a + 8*b*x^3))/(x^8*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.46

method	result	size
risch	$\frac{\left(-\frac{b\,x^3}{5}-\frac{a}{8}\right)\sqrt{(b\,x^3+a)^2}}{x^8(b\,x^3+a)}$	35
gosper	$-\frac{(8b\,x^3+5a)\sqrt{(b\,x^3+a)^2}}{40x^8(b\,x^3+a)}$	36
default	$-\frac{(8b\,x^3+5a)\sqrt{(b\,x^3+a)^2}}{40x^8(b\,x^3+a)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(8*b*x^3+5*a)*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)
```

Maxima [A]
time = 0.27, size = 15, normalized size = 0.19

$$-\frac{8\,bx^3+5\,a}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/40*(8*b*x^3 + 5*a)/x^8
```

Fricas [A]
time = 0.34, size = 15, normalized size = 0.19

$$-\frac{8\,bx^3+5\,a}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/40*(8*b*x^3 + 5*a)/x^8
```

Sympy [A]
time = 0.05, size = 15, normalized size = 0.19

$$\frac{-5a-8bx^3}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**9,x)
```

```
[Out] (-5*a - 8*b*x**3)/(40*x**8)
```

Giac [A]
time = 4.19, size = 31, normalized size = 0.39

$$-\frac{8\,bx^3\mathrm{sgn}(bx^3+a)+5\,a\mathrm{sgn}(bx^3+a)}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/40*(8*b*x^3*sgn(b*x^3 + a) + 5*a*sgn(b*x^3 + a))/x^8
```

Mupad [B]
time = 1.16, size = 35, normalized size = 0.44

$$-\frac{\left(8\,b\,x^3+5\,a\right)\,\sqrt{{\left(b\,x^3+a\right)}^2}}{40\,x^8\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^9,x)
```

```
[Out] -((5*a + 8*b*x^3)*((a + b*x^3)^2)^(1/2))/(40*x^8*(a + b*x^3))
```

3.21 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

```
[Out] -1/9*a*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-1/6*b*((b*x^3+a)^2)^(1/2)/x^6/(b*x
^3+a)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 14}

$$-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
```

```
[Out] -1/9*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^9*(a + b*x^3)) - (b*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^{10}}\,dx}{ab+b^2x^3}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^{10}}+\frac{b^2}{x^7}\right)\,dx}{ab+b^2x^3}$$
$$= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\,(a+bx^3)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\,(a+bx^3)}$$

Mathematica [**A**]
time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a+bx^3)^2}\,(2a+3bx^3)}{18x^9\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^10,x]
[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(2*a + 3*b*x^3))/(x^9*(a + b*x^3))
```

Maple [**A**]
time = 0.02, size = 36, normalized size = 0.46

method	result	size
risch	$\frac{\left(-\frac{b\,x^3}{6}-\frac{a}{9}\right)\sqrt{\left(b\,x^3+a\right)^2}}{x^9\left(b\,x^3+a\right)}$	35
gosper	$-\frac{\left(3b\,x^3+2a\right)\sqrt{\left(b\,x^3+a\right)^2}}{18x^9\left(b\,x^3+a\right)}$	36
default	$-\frac{\left(3b\,x^3+2a\right)\sqrt{\left(b\,x^3+a\right)^2}}{18x^9\left(b\,x^3+a\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^10,x,method=_RETURNVERBOSE)
[Out] -1/18*(3*b*x^3+2*a)*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(53) = 106.
time = 0.28, size = 117, normalized size = 1.48

$$-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^3}{6\,a^3} - \frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^2}{6\,a^2x^3} + \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b}{6\,a^3x^6} - \frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}{9\,a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3/a^3 - 1/6*sqrt(b^2*x^6 + 2*a*b*x^3
 + a^2)*b^2/(a^2*x^3) + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b/(a^3*x^6) -
 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/(a^2*x^9)
```

Fricas [A]
time = 0.34, size = 15, normalized size = 0.19

$$-\frac{3\,bx^3+2\,a}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/18*(3*b*x^3 + 2*a)/x^9
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.19

$$\frac{-2a-3bx^3}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**10,x)
```

```
[Out] (-2*a - 3*b*x**3)/(18*x**9)
```

Giac [A]
time = 2.56, size = 31, normalized size = 0.39

$$-\frac{3\,bx^3\mathrm{sgn}(bx^3+a)+2\,a\mathrm{sgn}(bx^3+a)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/18*(3*b*x^3*sgn(b*x^3 + a) + 2*a*sgn(b*x^3 + a))/x^9
```

Mupad [B]
time = 1.15, size = 35, normalized size = 0.44

$$-\frac{\left(3\,b\,x^3+2\,a\right)\,\sqrt{{\left(b\,x^3+a\right)}^2}}{18\,x^9\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^10,x)
```

```
[Out] -((2*a + 3*b*x^3)*((a + b*x^3)^2)^(1/2))/(18*x^9*(a + b*x^3))
```

3.22 $\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

```
[Out] -1/10*a*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-1/7*b*((b*x^3+a)^2)^(1/2)/x^7/(b
*x^3+a)
```

Rubi [A]
time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 14}

$$-\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
```

```
[Out] -1/10*(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (b*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^3+b^2x^6}}{x^{11}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{ab+b^2x^3}{x^{11}}\,dx}{ab+b^2x^3}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{ab}{x^{11}}+\frac{b^2}{x^8}\right)dx}{ab+b^2x^3}$$
$$= -\frac{a\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)} - \frac{b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(7a+10bx^3\right)}{70x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]/x^11,x]
```

```
[Out] -1/70*(Sqrt[(a + b*x^3)^2]*(7*a + 10*b*x^3))/(x^10*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.46

method	result	size
risch	$\frac{\left(-\frac{b\,x^3}{7}-\frac{a}{10}\right)\sqrt{\left(b\,x^3+a\right)^2}}{x^{10}\left(b\,x^3+a\right)}$	35
gosper	$-\frac{\left(10b\,x^3+7a\right)\sqrt{\left(b\,x^3+a\right)^2}}{70x^{10}\left(b\,x^3+a\right)}$	36
default	$-\frac{\left(10b\,x^3+7a\right)\sqrt{\left(b\,x^3+a\right)^2}}{70x^{10}\left(b\,x^3+a\right)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)^2)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/70*(10*b*x^3+7*a)*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.19

$$-\frac{10\,bx^3+7\,a}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/70*(10*b*x^3 + 7*a)/x^10
```

Fricas [A]
time = 0.35, size = 15, normalized size = 0.19

$$-\frac{10\,bx^3+7\,a}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/70*(10*b*x^3 + 7*a)/x^10
```

Sympy [A]
time = 0.06, size = 15, normalized size = 0.19

$$\frac{-7a-10bx^3}{70x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)**2)**(1/2)/x**11,x)
```

```
[Out] (-7*a - 10*b*x**3)/(70*x**10)
```

Giac [A]
time = 5.00, size = 31, normalized size = 0.39

$$-\frac{10\,bx^3\mathrm{sgn}(bx^3+a)+7\,a\mathrm{sgn}(bx^3+a)}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)^2)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/70*(10*b*x^3*sgn(b*x^3 + a) + 7*a*sgn(b*x^3 + a))/x^10
```

Mupad [B]
time = 1.16, size = 35, normalized size = 0.44

$$-\frac{\left(10\,b\,x^3+7\,a\right)\,\sqrt{{\left(b\,x^3+a\right)}^2}}{70\,x^{10}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)^2)^(1/2)/x^11,x)
```

```
[Out] -((7*a + 10*b*x^3)*((a + b*x^3)^2)^(1/2))/(70*x^10*(a + b*x^3))
```

3.23 $\int x^9(a^2 + 2abx^3 + b^2x^6)^{3/2}\, dx$

Optimal. Leaf size=167

$$\frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\,(a+bx^3)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\,(a+bx^3)}+\frac{b^3x^{19}\sqrt{a^2+2abx^3}}{19\,(a+bx^3}$$

```
[Out] 1/10*a^3*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/13*a^2*b*x^13*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+3/16*a*b^2*x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/19*b^3*x^19*
((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{3ab^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\,(a+bx^3)}+\frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\,(a+bx^3)}+\frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a^2*b*x^13
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (3*a*b^2*x^16*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (b^3*x^19*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(19*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^9\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^3b^3x^9+3a^2b^4x^{12}+3ab^5x^{15}+b^6x^{18}\right)dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{a^3x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)} + \frac{3a^2bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)} + \frac{3ab^2x^{16}\sqrt{a}}{16}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{x^{10}\sqrt{\left(a+bx^3\right)^2}\left(1976a^3+4560a^2bx^3+3705ab^2x^6+1040b^3x^9\right)}{19760\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^3)^2]*(1976*a^3 + 4560*a^2*b*x^3 + 3705*a*b^2*x^6 + 104
0*b^3*x^9))/(19760*(a + b*x^3))
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.35

method	result	size
gosper	$\frac{x^{10}\left(1040b^3x^9+3705a\,b^2x^6+4560a^2b\,x^3+1976a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{19760\left(b\,x^3+a\right)^3}$	58
default	$\frac{x^{10}\left(1040b^3x^9+3705a\,b^2x^6+4560a^2b\,x^3+1976a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{19760\left(b\,x^3+a\right)^3}$	58
risch	$\frac{a^3x^{10}\sqrt{\left(b\,x^3+a\right)^2}}{10b\,x^3+10a}+\frac{3a^2b\,x^{13}\sqrt{\left(b\,x^3+a\right)^2}}{13\left(b\,x^3+a\right)}+\frac{3a\,b^2x^{16}\sqrt{\left(b\,x^3+a\right)^2}}{16\left(b\,x^3+a\right)}+\frac{b^3x^{19}\sqrt{\left(b\,x^3+a\right)^2}}{19b\,x^3+19a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/19760*x^10*(1040*b^3*x^9+3705*a*b^2*x^6+4560*a^2*b*x^3+1976*a^3)*((b*x^3+
a)^2)^(3/2)/(b*x^3+a)^3
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.21

$$\frac{1}{19}\,b^3x^{19}+\frac{3}{16}\,ab^2x^{16}+\frac{3}{13}\,a^2bx^{13}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

Fricas [A]
time = 0.33, size = 35, normalized size = 0.21

$$\frac{1}{19}\,b^3x^{19}+\frac{3}{16}\,ab^2x^{16}+\frac{3}{13}\,a^2bx^{13}+\frac{1}{10}\,a^3x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/19*b^3*x^19 + 3/16*a*b^2*x^16 + 3/13*a^2*b*x^13 + 1/10*a^3*x^10
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^9\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**9*((a + b*x**3)**2)**(3/2), x)
```

Giac [A]
time = 4.89, size = 67, normalized size = 0.40

$$\frac{1}{19}\,b^3x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{16}\,ab^2x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{13}\,a^2bx^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{10}\,a^3x^{10}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/19*b^3*x^19*sgn(b*x^3 + a) + 3/16*a*b^2*x^16*sgn(b*x^3 + a) + 3/13*a^2*b*
x^13*sgn(b*x^3 + a) + 1/10*a^3*x^10*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^9\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.24 $\int x^8(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal. Leaf size=119

$$\frac{a^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^3}-\frac{2a(a+bx^3)^4\sqrt{a^2+2abx^3+b^2x^6}}{15b^3}+\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^3}$$

```
[Out] 1/12*a^2*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/b^3-2/15*a*(b*x^3+a)^4*((b*x^3+a)^
2)^(1/2)/b^3+1/18*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 167, normalized size of antiderivative = 1.40, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$\frac{ab^2x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{5(a+bx^3)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{b^3x^{18}\sqrt{a^2+2abx^3+b^2x^6}}{18(a+bx^3)}+\frac{a^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (a^2*b*x^12*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (a*b^2*x^15*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^18*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(18*(a + b*x^3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^8 \left(ab+b^2x^3\right)^3 dx}{b^2 \left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x^2 \left(ab+b^2x\right)^3 dx, x, x^3\right)}{3b^2 \left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(a^3b^3x^2+3a^2b^4x^3+3ab^5x^4+b^6x^5\right) dx, x, x^3\right.}{3b^2 \left(ab+b^2x^3\right)}$$
$$= \frac{a^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)} + \frac{a^2bx^{12}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)} + \frac{ab^2x^{15}\sqrt{a^2+2}}{5\left(a+\right.}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.51

$$\frac{x^9\sqrt{\left(a+bx^3\right)^2}\left(20a^3+45a^2bx^3+36ab^2x^6+10b^3x^9\right)}{180\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^3)^2]*(20*a^3 + 45*a^2*b*x^3 + 36*a*b^2*x^6 + 10*b^3*x^9
))/(180*(a + b*x^3))
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.49

method	result	size
gosper	$\frac{x^9\left(10b^3x^9+36a\,b^2x^6+45a^2b\,x^3+20a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{180\left(b\,x^3+a\right)^3}$	58
default	$\frac{x^9\left(10b^3x^9+36a\,b^2x^6+45a^2b\,x^3+20a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{180\left(b\,x^3+a\right)^3}$	58
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,a^3x^9}{9b\,x^3+9a}+\frac{\sqrt{\left(b\,x^3+a\right)^2}\,a^2b\,x^{12}}{4b\,x^3+4a}+\frac{\sqrt{\left(b\,x^3+a\right)^2}\,a\,b^2x^{15}}{5b\,x^3+5a}+\frac{\sqrt{\left(b\,x^3+a\right)^2}\,b^3x^{18}}{18b\,x^3+18a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/180*x^9*(10*b^3*x^9+36*a*b^2*x^6+45*a^2*b*x^3+20*a^3)*((b*x^3+a)^2)^(3/2)
/(b*x^3+a)^3
```

Maxima [A]
time = 0.28, size = 114, normalized size = 0.96

$$\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^2x^3}{12\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}x^3}{18\,b^2}+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}a^3}{12\,b^3}-\frac{7\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}a}{90\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b*
x^3 + a^2)^(5/2)*x^3/b^2 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3/b^3 -
 7/90*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a/b^3
```

Fricas [A]
time = 0.34, size = 35, normalized size = 0.29

$$\frac{1}{18}\,b^3x^{18}+\frac{1}{5}\,ab^2x^{15}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^3*x^18 + 1/5*a*b^2*x^15 + 1/4*a^2*b*x^12 + 1/9*a^3*x^9
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**8*((a + b*x**3)**2)**(3/2), x)
```

Giac [A]
time = 5.58, size = 67, normalized size = 0.56

$$\frac{1}{18}\,b^3x^{18}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,ab^2x^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,a^2bx^{12}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{9}\,a^3x^9\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/18*b^3*x^18*sgn(b*x^3 + a) + 1/5*a*b^2*x^15*sgn(b*x^3 + a) + 1/4*a^2*b*x^
12*sgn(b*x^3 + a) + 1/9*a^3*x^9*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^8 \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.25 $\int x^7 (a^2+2abx^3+b^2x^6)^{3/2}\, dx$

Optimal. Leaf size=167

$$\frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}$$

```
[Out] 1/8*a^3*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/11*a^2*b*x^11*((b*x^3+a)^2)^(1/
2)/(b*x^3+a)+3/14*a*b^2*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/17*b^3*x^17*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{3ab^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a^2*b*x^11*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (3*a*b^2*x^14*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (b^3*x^17*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(17*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^7 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^7 \left(ab+b^2x^3\right)^3 dx}{b^2 \left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(a^3b^3x^7+3a^2b^4x^{10}+3ab^5x^{13}+b^6x^{16}\right) dx}{b^2 \left(ab+b^2x^3\right)}$$
$$= \frac{a^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8 \left(a+bx^3\right)} + \frac{3a^2bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11 \left(a+bx^3\right)} + \frac{3ab^2x^{14}\sqrt{a^2+}}{14 (a}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{x^8 \sqrt{\left(a+bx^3\right)^2} \left(1309a^3+2856a^2bx^3+2244ab^2x^6+616b^3x^9\right)}{10472 \left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^3)^2]*(1309*a^3 + 2856*a^2*b*x^3 + 2244*a*b^2*x^6 + 616*
b^3*x^9))/(10472*(a + b*x^3))
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.35

method	result	size
gosper	$\frac{x^{8} \left(616 b^{3} x^{9}+2244 a \,b^{2} x^{6}+2856 a^{2} b \,x^{3}+1309 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{10472 \left(b \,x^{3}+a \right)^{3}}$	58
default	$\frac{x^{8} \left(616 b^{3} x^{9}+2244 a \,b^{2} x^{6}+2856 a^{2} b \,x^{3}+1309 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{10472 \left(b \,x^{3}+a \right)^{3}}$	58
risch	$\frac{a^{3} x^{8} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{8 b \,x^{3}+8 a}+\frac{3 a^{2} b \,x^{11} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{11 \left(b \,x^{3}+a \right)}+\frac{3 a \,b^{2} x^{14} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{14 \left(b \,x^{3}+a \right)}+\frac{b^{3} x^{17} \sqrt{\left(b \,x^{3}+a \right)^{2}}}{17 b \,x^{3}+17 a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10472*x^8*(616*b^3*x^9+2244*a*b^2*x^6+2856*a^2*b*x^3+1309*a^3)*((b*x^3+a)
^2)^(3/2)/(b*x^3+a)^3
```

Maxima [A]
time = 0.30, size = 35, normalized size = 0.21

$$\frac{1}{17}\, b^3x^{17} + \frac{3}{14}\, ab^2x^{14} + \frac{3}{11}\, a^2bx^{11} + \frac{1}{8}\, a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

Fricas [A]
time = 0.35, size = 35, normalized size = 0.21

$$\frac{1}{17}\,b^3x^{17}+\frac{3}{14}\,ab^2x^{14}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^3*x^17 + 3/14*a*b^2*x^14 + 3/11*a^2*b*x^11 + 1/8*a^3*x^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**7*((a + b*x**3)**2)**(3/2), x)
```

Giac [A]
time = 4.78, size = 67, normalized size = 0.40

$$\frac{1}{17}\,b^3x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{14}\,ab^2x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{11}\,a^2bx^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{8}\,a^3x^8\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/17*b^3*x^17*sgn(b*x^3 + a) + 3/14*a*b^2*x^14*sgn(b*x^3 + a) + 3/11*a^2*b*
x^11*sgn(b*x^3 + a) + 1/8*a^3*x^8*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^7\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.26 $\int x^6(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}$$

```
[Out] 1/7*a^3*x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/10*a^2*b*x^10*((b*x^3+a)^2)^(1/
2)/(b*x^3+a)+3/13*a*b^2*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/16*b^3*x^16*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{3ab^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\left(a+bx^3\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\left(a+bx^3\right)}+\frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a^2*b*x^10*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (3*a*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (b^3*x^16*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(16*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^6 (ab+b^2x^3)^3 dx}{b^2 (ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int (a^3b^3x^6+3a^2b^4x^9+3ab^5x^{12}+b^6x^{15}) dx}{b^2 (ab+b^2x^3)}$$

$$= \frac{a^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)} + \frac{3a^2bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10(a+bx^3)} + \frac{3ab^2x^{13}\sqrt{a^2}}{13(}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{x^7\sqrt{(a+bx^3)^2} \left(1040a^3+2184a^2bx^3+1680ab^2x^6+455b^3x^9\right)}{7280 (a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^3)^2]*(1040*a^3 + 2184*a^2*b*x^3 + 1680*a*b^2*x^6 + 455*
b^3*x^9))/(7280*(a + b*x^3))
```

Maple [A]
time = 0.05, size = 58, normalized size = 0.35

method	result	size
gosper	$\frac{x^7\left(455b^3x^9+1680a\,b^2x^6+2184a^2b\,x^3+1040a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{7280\left(b\,x^3+a\right)^3}$	58
default	$\frac{x^7\left(455b^3x^9+1680a\,b^2x^6+2184a^2b\,x^3+1040a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{7280\left(b\,x^3+a\right)^3}$	58
risch	$\frac{a^3x^7\sqrt{\left(b\,x^3+a\right)^2}}{7b\,x^3+7a}+\frac{3a^2b\,x^{10}\sqrt{\left(b\,x^3+a\right)^2}}{10\left(b\,x^3+a\right)}+\frac{3a\,b^2x^{13}\sqrt{\left(b\,x^3+a\right)^2}}{13\left(b\,x^3+a\right)}+\frac{b^3x^{16}\sqrt{\left(b\,x^3+a\right)^2}}{16b\,x^3+16a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7280*x^7*(455*b^3*x^9+1680*a*b^2*x^6+2184*a^2*b*x^3+1040*a^3)*((b*x^3+a)^
2)^(3/2)/(b*x^3+a)^3
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.21

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{13}\,ab^2x^{13}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

Fricas [A]
time = 0.38, size = 35, normalized size = 0.21

$$\frac{1}{16}\,b^3x^{16}+\frac{3}{13}\,ab^2x^{13}+\frac{3}{10}\,a^2bx^{10}+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/16*b^3*x^16 + 3/13*a*b^2*x^13 + 3/10*a^2*b*x^10 + 1/7*a^3*x^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^6\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**6*((a + b*x**3)**2)**(3/2), x)
```

Giac [A]
time = 4.18, size = 67, normalized size = 0.40

$$\frac{1}{16}\,b^3x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{13}\,ab^2x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{10}\,a^2bx^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{7}\,a^3x^7\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/16*b^3*x^16*sgn(b*x^3 + a) + 3/13*a*b^2*x^13*sgn(b*x^3 + a) + 3/10*a^2*b*
x^10*sgn(b*x^3 + a) + 1/7*a^3*x^7*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.27 $\int x^5(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal. Leaf size=78

$$-\frac{a(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^2}+\frac{(a+bx^3)^4\sqrt{a^2+2abx^3+b^2x^6}}{15b^2}$$

```
[Out] -1/12*a*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/b^2+1/15*(b*x^3+a)^4*((b*x^3+a)^2)^
(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {1369, 272, 45}

$$\frac{(a+bx^3)^4\sqrt{a^2+2abx^3+b^2x^6}}{15b^2}-\frac{a(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -1/12*(a*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/b^2 + ((a + b*x^3)^
4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(15*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^5(ab+b^2x^3)^3\,dx}{b^2\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x(ab+b^2x)^3\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\left(-\frac{a(ab+b^2x)^3}{b}+\frac{(ab+b^2x)^4}{b^2}\right)\,dx,x,x^3\right)}{3b^2\left(ab+b^2x^3\right)}$$
$$=-\frac{a(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12b^2}+\frac{(a+bx^3)^4\sqrt{a^2+2abx^3+b^2x^6}}{15b^2}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.78

$$\frac{x^6\sqrt{(a+bx^3)^2}\,(10a^3+20a^2bx^3+15ab^2x^6+4b^3x^9)}{60\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^3)^2]*(10*a^3 + 20*a^2*b*x^3 + 15*a*b^2*x^6 + 4*b^3*x^9)
)/(60*(a + b*x^3))
```

Maple [A]
time = 0.04, size = 58, normalized size = 0.74

method	result	size
gosper	$\frac{x^{6}\left(4b^{3}x^{9}+15a\,b^{2}x^{6}+20a^{2}b\,x^{3}+10a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{60\left(b\,x^{3}+a\right)^{3}}$	58
default	$\frac{x^{6}\left(4b^{3}x^{9}+15a\,b^{2}x^{6}+20a^{2}b\,x^{3}+10a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{60\left(b\,x^{3}+a\right)^{3}}$	58
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{3}x^{6}}{6b\,x^{3}+6a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a^{2}b\,x^{9}}{3b\,x^{3}+3a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,a\,b^{2}x^{12}}{4b\,x^{3}+4a}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{3}x^{15}}{15b\,x^{3}+15a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/60*x^6*(4*b^3*x^9+15*a*b^2*x^6+20*a^2*b*x^3+10*a^3)*((b*x^3+a)^2)^(3/2)/(
b*x^3+a)^3
```

Maxima [A]
time = 0.27, size = 83, normalized size = 1.06

$$-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}ax^3}{12\,b}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}a^2}{12\,b^2}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*x^3/b - 1/12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(3/2)*a^2/b^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/b^2
```

Fricas [A]
time = 0.36, size = 35, normalized size = 0.45

$$\frac{1}{15}\,b^3x^{15}+\frac{1}{4}\,ab^2x^{12}+\frac{1}{3}\,a^2bx^9+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*b^3*x^15 + 1/4*a*b^2*x^12 + 1/3*a^2*b*x^9 + 1/6*a^3*x^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**5*((a + b*x**3)**2)**(3/2), x)
```

Giac [A]
time = 3.21, size = 45, normalized size = 0.58

$$\frac{1}{60}\,(4\,b^3x^{15}+15\,ab^2x^{12}+20\,a^2bx^9+10\,a^3x^6)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/60*(4*b^3*x^15 + 15*a*b^2*x^12 + 20*a^2*b*x^9 + 10*a^3*x^6)*sgn(b*x^3 + a
)
```

Mupad [B]
time = 1.25, size = 46, normalized size = 0.59

$$\frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}\,\left(-a^2+3\,a\,b\,x^3+4\,b^2\,x^6\right)}{60\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] ((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)*(4*b^2*x^6 - a^2 + 3*a*b*x^3))/(60*b^2)
```

3.28 $\int x^4(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{b^3x^{14}\sqrt{a^2+2abx^3}}{14\,(a+bx^3)}$$

```
[Out] 1/5*a^3*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/8*a^2*b*x^8*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+3/11*a*b^2*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/14*b^3*x^14*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{3ab^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a^2*b*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (3*a*b^2*x^11*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (b^3*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(14*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\int x^4(ab+b^2x^3)^3\,dx}{b^2\,(ab+b^2x^3)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\int\left(a^3b^3x^4+3a^2b^4x^7+3ab^5x^{10}+b^6x^{13}\right)dx}{b^2\,(ab+b^2x^3)}$$
$$=\frac{a^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{3a^2bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{3ab^2x^{11}\sqrt{a^2+}}{11\,(a-}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{x^5\sqrt{(a+bx^3)^2}\,(616a^3+1155a^2bx^3+840ab^2x^6+220b^3x^9)}{3080\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)^2]*(616*a^3 + 1155*a^2*b*x^3 + 840*a*b^2*x^6 + 220*b^
3*x^9))/(3080*(a + b*x^3))
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.35

method	result	size
gosper	$\frac{x^{5}\left(220b^{3}x^{9}+840a\,b^{2}x^{6}+1155a^{2}b\,x^{3}+616a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{3080\left(b\,x^{3}+a\right)^{3}}$	58
default	$\frac{x^{5}\left(220b^{3}x^{9}+840a\,b^{2}x^{6}+1155a^{2}b\,x^{3}+616a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{3080\left(b\,x^{3}+a\right)^{3}}$	58
risch	$\frac{a^{3}x^{5}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{5b\,x^{3}+5a}+\frac{3a^{2}b\,x^{8}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{8\left(b\,x^{3}+a\right)}+\frac{3a\,b^{2}x^{11}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{11\left(b\,x^{3}+a\right)}+\frac{b^{3}x^{14}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{14b\,x^{3}+14a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3080*x^5*(220*b^3*x^9+840*a*b^2*x^6+1155*a^2*b*x^3+616*a^3)*((b*x^3+a)^2)
^(3/2)/(b*x^3+a)^3
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.21

$$\frac{1}{14}\,b^3x^{14}+\frac{3}{11}\,ab^2x^{11}+\frac{3}{8}\,a^2bx^8+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Fricas [A]
time = 0.38, size = 35, normalized size = 0.21

$$\frac{1}{14}\,b^3x^{14}+\frac{3}{11}\,ab^2x^{11}+\frac{3}{8}\,a^2bx^8+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/14*b^3*x^14 + 3/11*a*b^2*x^11 + 3/8*a^2*b*x^8 + 1/5*a^3*x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**4*((a + b*x**3)**2)**(3/2), x)
```

Giac [A]
time = 3.70, size = 67, normalized size = 0.40

$$\frac{1}{14}\,b^3x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{11}\,ab^2x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{8}\,a^2bx^8\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,a^3x^5\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*sgn(b*x^3 + a) + 3/11*a*b^2*x^11*sgn(b*x^3 + a) + 3/8*a^2*b*x
^8*sgn(b*x^3 + a) + 1/5*a^3*x^5*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.29 $\int x^3(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\,(a+bx^3)}+\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\,(a+bx^3)}+\frac{b^3x^{13}\sqrt{a^2+2abx^3+}}{13\,(a+bx^3)}$$

```
[Out] 1/4*a^3*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/7*a^2*b*x^7*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+3/10*a*b^2*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/13*b^3*x^13*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1369, 276}

$$\frac{3ab^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\,(a+bx^3)}+\frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\,(a+bx^3)}+\frac{b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (3*a^2*b*x^7*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (3*a*b^2*x^10*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(13*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^3 \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^3(ab+b^2x^3)^3 \, dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(a^3b^3x^3+3a^2b^4x^6+3ab^5x^9+b^6x^{12}\right) dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{a^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)} + \frac{3a^2bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)} + \frac{3ab^2x^{10}\sqrt{a^2 +}}{10\left(a\right.}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{x^4\sqrt{\left(a+bx^3\right)^2}\left(455a^3+780a^2bx^3+546ab^2x^6+140b^3x^9\right)}{1820\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)^2]*(455*a^3 + 780*a^2*b*x^3 + 546*a*b^2*x^6 + 140*b^3
*x^9))/(1820*(a + b*x^3))
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.35

method	result	size
gosper	$\frac{x^4\left(140b^3x^9+546a\,b^2x^6+780a^2b\,x^3+455a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{1820\left(b\,x^3+a\right)^3}$	58
default	$\frac{x^4\left(140b^3x^9+546a\,b^2x^6+780a^2b\,x^3+455a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{1820\left(b\,x^3+a\right)^3}$	58
risch	$\frac{a^3x^4\sqrt{\left(b\,x^3+a\right)^2}}{4b\,x^3+4a}+\frac{3a^2b\,x^7\sqrt{\left(b\,x^3+a\right)^2}}{7\left(b\,x^3+a\right)}+\frac{3a\,b^2x^{10}\sqrt{\left(b\,x^3+a\right)^2}}{10\left(b\,x^3+a\right)}+\frac{b^3x^{13}\sqrt{\left(b\,x^3+a\right)^2}}{13b\,x^3+13a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1820*x^4*(140*b^3*x^9+546*a*b^2*x^6+780*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)^
(3/2)/(b*x^3+a)^3
```

Maxima [A]
time = 0.27, size = 35, normalized size = 0.21

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{7}\,a^2bx^7+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Fricas [A]
time = 0.36, size = 35, normalized size = 0.21

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{7}\,a^2bx^7+\frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*b^3*x^13 + 3/10*a*b^2*x^10 + 3/7*a^2*b*x^7 + 1/4*a^3*x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**3*((a + b*x**3)**2)**(3/2), x)
```

Giac [A]
time = 3.39, size = 67, normalized size = 0.40

$$\frac{1}{13}\,b^3x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{10}\,ab^2x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{7}\,a^2bx^7\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,a^3x^4\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*sgn(b*x^3 + a) + 3/10*a*b^2*x^10*sgn(b*x^3 + a) + 3/7*a^2*b*x
^7*sgn(b*x^3 + a) + 1/4*a^3*x^4*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.30 $\int x^2(a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

```
[Out] 1/12*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1366, 623}

$$\frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(12*b)
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \left(a^2+2abx+b^2x^2\right)^{3/2}\,dx,x,x^3\right)\\ &= \frac{(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{12b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 1.67

$$\frac{x^3\sqrt{(a+bx^3)^2}\,\left(4a^3+6a^2bx^3+4ab^2x^6+b^3x^9\right)}{12\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(4*a^3 + 6*a^2*b*x^3 + 4*a*b^2*x^6 + b^3*x^9))/(12
*(a + b*x^3))
```

Maple [A]
time = 0.05, size = 24, normalized size = 0.67

method	result	size
default	$\frac{\left(b\,x^{3}+a\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{12b}$	24
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(b\,x^{3}+a\right)^{3}}{12b}$	26
gosper	$\frac{x^{3}\left(b^{3}x^{9}+4a\,b^{2}x^{6}+6a^{2}b\,x^{3}+4a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{12\left(b\,x^{3}+a\right)^{3}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*(b*x^3+a)*((b*x^3+a)^2)^(3/2)/b
```

Maxima [A]
time = 0.28, size = 52, normalized size = 1.44

$$\frac{1}{12}\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}x^{3}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}a}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*x^3 + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(3/2)*a/b
```

Fricas [A]
time = 0.34, size = 35, normalized size = 0.97

$$\frac{1}{12}\,b^{3}x^{12}+\frac{1}{3}\,ab^{2}x^{9}+\frac{1}{2}\,a^{2}bx^{6}+\frac{1}{3}\,a^{3}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*b^3*x^12 + 1/3*a*b^2*x^9 + 1/2*a^2*b*x^6 + 1/3*a^3*x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\left(a + bx^3\right)^2 \right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x**3)**2)**(3/2), x)
```

Giac [A]
time = 3.38, size = 44, normalized size = 1.22

$$\frac{1}{12} \left(2 \left(bx^6 + 2 \, ax^3\right)a^2 + \left(bx^6 + 2 \, ax^3\right)^2 b\right) \mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*(2*(b*x^6 + 2*a*x^3)*a^2 + (b*x^6 + 2*a*x^3)^2*b)*sgn(b*x^3 + a)
```

Mupad [B]
time = 1.22, size = 36, normalized size = 1.00

$$\frac{\left(b^2 \, x^3 + a \, b\right) \, \left(a^2 + 2 \, a \, b \, x^3 + b^2 \, x^6\right)^{3/2}}{12 \, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] ((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2))/(12*b^2)
```

3.31 $\int x(a^2 + 2abx^3 + b^2x^6)^{3/2}\, dx$

Optimal. Leaf size=167

$$\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{b^3x^{11}\sqrt{a^2+2abx^3+}}{11\,(a+bx^3)}$$

```
[Out] 1/2*a^3*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3/5*a^2*b*x^5*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+3/8*a*b^2*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/11*b^3*x^11*((b*x^
3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.02, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1369, 276}

$$\frac{3ab^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a^2*b*x^5*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (3*a*b^2*x^8*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(11*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x\left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x\left(ab+b^2x^3\right)^3\,dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^3b^3x+3a^2b^4x^4+3ab^5x^7+b^6x^{10}\right)dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)} + \frac{3a^2bx^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)} + \frac{3ab^2x^8\sqrt{a^2+}}{8\left(a+\right.}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{x^2\sqrt{\left(a+bx^3\right)^2}\left(220a^3+264a^2bx^3+165ab^2x^6+40b^3x^9\right)}{440\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
[Out] (x^2*Sqrt[(a + b*x^3)^2]*(220*a^3 + 264*a^2*b*x^3 + 165*a*b^2*x^6 + 40*b^3*
x^9))/(440*(a + b*x^3))
```

Maple [A]
time = 0.06, size = 58, normalized size = 0.35

method	result	size
gosper	$\frac{x^{2}\left(40b^{3}x^{9}+165a\,b^{2}x^{6}+264a^{2}b\,x^{3}+220a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{440\left(b\,x^{3}+a\right)^{3}}$	58
default	$\frac{x^{2}\left(40b^{3}x^{9}+165a\,b^{2}x^{6}+264a^{2}b\,x^{3}+220a^{3}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{3}{2}}}{440\left(b\,x^{3}+a\right)^{3}}$	58
risch	$\frac{a^{3}x^{2}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{2b\,x^{3}+2a}+\frac{3a^{2}b\,x^{5}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{5\left(b\,x^{3}+a\right)}+\frac{3a\,b^{2}x^{8}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{8\left(b\,x^{3}+a\right)}+\frac{b^{3}x^{11}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{11b\,x^{3}+11a}$	116

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/440*x^2*(40*b^3*x^9+165*a*b^2*x^6+264*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(3
/2)/(b*x^3+a)^3
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.21

$$\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Fricas [A]
time = 0.36, size = 35, normalized size = 0.21

$$\frac{1}{11}\,b^3x^{11}+\frac{3}{8}\,ab^2x^8+\frac{3}{5}\,a^2bx^5+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] 1/11*b^3*x^11 + 3/8*a*b^2*x^8 + 3/5*a^2*b*x^5 + 1/2*a^3*x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x*((a + b*x**3)**2)**(3/2), x)
```

Giac [A]
time = 3.26, size = 67, normalized size = 0.40

$$\frac{1}{11}\,b^3x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{8}\,ab^2x^8\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{5}\,a^2bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^3x^2\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/11*b^3*x^11*sgn(b*x^3 + a) + 3/8*a*b^2*x^8*sgn(b*x^3 + a) + 3/5*a^2*b*x^5
*sgn(b*x^3 + a) + 1/2*a^3*x^2*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.32 $\int (a^2+2abx^3+b^2x^6)^{3/2}\,dx$

Optimal. Leaf size=162

$$\frac{a^3x(a^2+2abx^3+b^2x^6)^{3/2}}{(a+bx^3)^3}+\frac{3a^2bx^4(a^2+2abx^3+b^2x^6)^{3/2}}{4(a+bx^3)^3}+\frac{3ab^2x^7(a^2+2abx^3+b^2x^6)^{3/2}}{7(a+bx^3)^3}+\frac{b^3x^{10}(a^2+2abx$$

```
[Out] a^3*x*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/(b*x^3+a)^3+3/4*a^2*b*x^4*(b^2*x^6+2*a*
b*x^3+a^2)^(3/2)/(b*x^3+a)^3+3/7*a*b^2*x^7*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/(b
*x^3+a)^3+1/10*b^3*x^10*(b^2*x^6+2*a*b*x^3+a^2)^(3/2)/(b*x^3+a)^3
```

Rubi [A]
time = 0.02, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1357, 200}

$$\frac{3ab^2x^7(a^2+2abx^3+b^2x^6)^{3/2}}{7(a+bx^3)^3}+\frac{3a^2bx^4(a^2+2abx^3+b^2x^6)^{3/2}}{4(a+bx^3)^3}+\frac{b^3x^{10}(a^2+2abx^3+b^2x^6)^{3/2}}{10(a+bx^3)^3}+\frac{a^3x(a^2+2abx^3+b^2x^6)^{3/2}}{(a+bx^3)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(a + b*x^3)^3 + (3*a^2*b*x^4*(a^2
 + 2*a*b*x^3 + b^2*x^6)^(3/2))/(4*(a + b*x^3)^3) + (3*a*b^2*x^7*(a^2 + 2*a*
b*x^3 + b^2*x^6)^(3/2))/(7*(a + b*x^3)^3) + (b^3*x^10*(a^2 + 2*a*b*x^3 + b^
2*x^6)^(3/2))/(10*(a + b*x^3)^3)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx = \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}\int \left(2ab+2b^2x^3\right)^3\,dx}{\left(2ab+2b^2x^3\right)^3}$$
$$= \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}\int \left(8a^3b^3+24a^2b^4x^3+24ab^5x^6+8b^6x^9\right)dx}{\left(2ab+2b^2x^3\right)^3}$$
$$= \frac{a^3x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{\left(a+bx^3\right)^3} + \frac{3a^2bx^4\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{4\left(a+bx^3\right)^3} + \frac{3ab^2x^7(a^2+2a}{7\,(a+}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 0.36

$$\frac{x\sqrt{\left(a+bx^3\right)^2}\left(140a^3+105a^2bx^3+60ab^2x^6+14b^3x^9\right)}{140\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^3)^2]*(140*a^3 + 105*a^2*b*x^3 + 60*a*b^2*x^6 + 14*b^3*x^9
))/(140*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 56, normalized size = 0.35

method	result	size
gosper	$\frac{x\left(14b^3x^9+60a\,b^2x^6+105a^2b\,x^3+140a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{140\left(b\,x^3+a\right)^3}$	56
default	$\frac{x\left(14b^3x^9+60a\,b^2x^6+105a^2b\,x^3+140a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{140\left(b\,x^3+a\right)^3}$	56
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,b^3x^{10}}{10b\,x^3+10a}+\frac{3\sqrt{\left(b\,x^3+a\right)^2}\,a\,b^2x^7}{7\left(b\,x^3+a\right)}+\frac{3\sqrt{\left(b\,x^3+a\right)^2}\,a^2b\,x^4}{4\left(b\,x^3+a\right)}+\frac{\sqrt{\left(b\,x^3+a\right)^2}\,a^3x}{b\,x^3+a}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/140*x*(14*b^3*x^9+60*a*b^2*x^6+105*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/2)
/(b*x^3+a)^3
```

Maxima [A]
time = 0.29, size = 32, normalized size = 0.20

$$\frac{1}{10}\,b^3x^{10}+\frac{3}{7}\,ab^2x^7+\frac{3}{4}\,a^2bx^4+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Fricas [A]
time = 0.36, size = 32, normalized size = 0.20

$$\frac{1}{10}\,b^3x^{10} + \frac{3}{7}\,ab^2x^7 + \frac{3}{4}\,a^2bx^4 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*b^3*x^10 + 3/7*a*b^2*x^7 + 3/4*a^2*b*x^4 + a^3*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^3 + b^2x^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(3/2), x)
```

Giac [A]
time = 4.48, size = 64, normalized size = 0.40

$$\frac{1}{10}\,b^3x^{10}\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{7}\,ab^2x^7\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{4}\,a^2bx^4\mathrm{sgn}\left(bx^3 + a\right) + a^3x\mathrm{sgn}\left(bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/10*b^3*x^10*sgn(b*x^3 + a) + 3/7*a*b^2*x^7*sgn(b*x^3 + a) + 3/4*a^2*b*x^4
*sgn(b*x^3 + a) + a^3*x*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.33 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx$

Optimal. Leaf size=160

$$\frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\,(a+bx^3)}+\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] a^2*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a*b^2*x^6*((b*x^3+a)^2)^(1/2)/(
b*x^3+a)+1/9*b^3*x^9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a^3*ln(x)*((b*x^3+a)^2)^
(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\,(a+bx^3)}+\frac{a^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
```

```
[Out] (a^2*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (a*b^2*x^6*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^9*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(9*(a + b*x^3)) + (a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/
(a + b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```

```
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x}\,dx}{b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x}\,dx,x,x^3\right)}{3b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)\,dx,x,x^3\right)}{3b^2(ab+b^2x^3)}$$
$$= \frac{a^2bx^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{ab^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{b^3x^9\sqrt{a^2+2abx^3}}{9(a+bx}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 0.38

$$\frac{\sqrt{(a+bx^3)^2}\,(bx^3(18a^2+9abx^3+2b^2x^6)+18a^3\log(x))}{18(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x,x]
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(18*a^2 + 9*a*b*x^3 + 2*b^2*x^6) + 18*a^3*Log[x
]))/(18*(a + b*x^3))
```

Maple [A]
time = 0.04, size = 57, normalized size = 0.36

method	result	size
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{3}{2}}(2b^3x^9+9a\,b^2x^6+18a^2b\,x^3+18a^3\ln(x))}{18(b\,x^3+a)^3}$	57
risch	$\frac{\sqrt{(b\,x^3+a)^2}\;b(\frac{1}{9}b^2x^9+\frac{1}{2}ab\,x^6+a^2x^3)}{b\,x^3+a}+\frac{a^3\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/18*((b*x^3+a)^2)^(3/2)*(2*b^3*x^9+9*a*b^2*x^6+18*a^2*b*x^3+18*a^3*ln(x))/
(b*x^3+a)^3
```

Maxima [A]
time = 0.28, size = 152, normalized size = 0.95

$$\frac{1}{6}\sqrt{b^2x^6+2\,abx^3+a^2}\,abx^3+\frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^3\log\left(2\,b^2x^3+2\,ab\right)-\frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{1}{2}\sqrt{b^2x^6+2\,abx^3+a^2}\,a^2+\frac{1}{9}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*
a^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*log(2*a*b*x/a
bs(x) + 2*a^2/(x^2*abs(x))) + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2 + 1/9
*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)
```

Fricas [A]
time = 0.37, size = 32, normalized size = 0.20

$$\frac{1}{9}\,b^3x^9+\frac{1}{2}\,ab^2x^6+a^2bx^3+a^3\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/9*b^3*x^9 + 1/2*a*b^2*x^6 + a^2*b*x^3 + a^3*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x, x)
```

Giac [A]
time = 5.04, size = 65, normalized size = 0.41

$$\frac{1}{9}\,b^3x^9\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*sgn(b*x^3 + a) + 1/2*a*b^2*x^6*sgn(b*x^3 + a) + a^2*b*x^3*sgn(b
*x^3 + a) + a^3*log(abs(x))*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x, x)
```

3.34 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=165

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}$$

```
[Out] -a^3*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+3/2*a^2*b*x^2*((b*x^3+a)^2)^(1/2)/(b*x
^3+a)+3/5*a*b^2*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/8*b^3*x^8*((b*x^3+a)^2)
^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.03, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{3ab^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (3*a^2*b*x^2*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (3*a*b^2*x^5*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(8*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^2}\,dx}{b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^2}+3a^2b^4x+3ab^5x^4+b^6x^7\right)dx}{b^2(ab+b^2x^3)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}+\frac{3a^2bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{3ab^2x^5\sqrt{a^2+2a}}{5(a+b}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a+bx^3)^2}\,(-40a^3+60a^2bx^3+24ab^2x^6+5b^3x^9)}{40x(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-40*a^3 + 60*a^2*b*x^3 + 24*a*b^2*x^6 + 5*b^3*x^9))/(
40*x*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
gosper	$-\frac{(-5b^3x^9-24a\,b^2x^6-60a^2b\,x^3+40a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{40x(b\,x^3+a)^3}$	58
default	$-\frac{(-5b^3x^9-24a\,b^2x^6-60a^2b\,x^3+40a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{40x(b\,x^3+a)^3}$	58
risch	$\frac{\sqrt{(b\,x^3+a)^2}\;b(\frac{1}{8}b^2x^8+\frac{3}{5}ab\,x^5+\frac{3}{2}a^2x^2)}{b\,x^3+a}-\frac{a^3\sqrt{(b\,x^3+a)^2}}{x(b\,x^3+a)}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(-5*b^3*x^9-24*a*b^2*x^6-60*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)/x/(
b*x^3+a)^3
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.22

$$\frac{5\,b^3x^9+24\,ab^2x^6+60\,a^2bx^3-40\,a^3}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

Fricas [A]
time = 0.36, size = 37, normalized size = 0.22

$$\frac{5\,b^3x^9+24\,ab^2x^6+60\,a^2bx^3-40\,a^3}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/40*(5*b^3*x^9 + 24*a*b^2*x^6 + 60*a^2*b*x^3 - 40*a^3)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**2, x)
```

Giac [A]
time = 4.67, size = 67, normalized size = 0.41

$$\frac{1}{8}\,b^3x^8\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{5}\,ab^2x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{2}\,a^2bx^2\mathrm{sgn}\left(bx^3+a\right)-\frac{a^3\mathrm{sgn}(bx^3+a)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*b^3*x^8*sgn(b*x^3 + a) + 3/5*a*b^2*x^5*sgn(b*x^3 + a) + 3/2*a^2*b*x^2*s
gn(b*x^3 + a) - a^3*sgn(b*x^3 + a)/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^2, x)
```

3.35 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=163

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

```
[Out] -1/2*a^3*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+3*a^2*b*x*((b*x^3+a)^2)^(1/2)/(b
*x^3+a)+3/4*a*b^2*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/7*b^3*x^7*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3ab^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
```

```
[Out] -1/2*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (3*a^2*b*x*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (3*a*b^2*x^4*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(7*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^3}\,dx}{b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(3a^2b^4+\frac{a^3b^3}{x^3}+3ab^5x^3+b^6x^6\right)\,dx}{b^2(ab+b^2x^3)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{3a^2bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3ab^2x^4\sqrt{a^2+2abx}}{4(a+bx^3}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a+bx^3)^2}\,(-14a^3+84a^2bx^3+21ab^2x^6+4b^3x^9)}{28x^2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^3,x]
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^3 + 84*a^2*b*x^3 + 21*a*b^2*x^6 + 4*b^3*x^9))/(
28*x^2*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.36

method	result	size
gosper	$-\frac{(-4b^3x^9-21a\,b^2x^6-84a^2b\,x^3+14a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{28x^2(b\,x^3+a)^3}$	58
default	$-\frac{(-4b^3x^9-21a\,b^2x^6-84a^2b\,x^3+14a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{28x^2(b\,x^3+a)^3}$	58
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b(\frac{1}{7}b^2x^7+\frac{3}{4}ab\,x^4+3a^2x)}{b\,x^3+a}-\frac{a^3\sqrt{(b\,x^3+a)^2}}{2x^2(b\,x^3+a)}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/28*(-4*b^3*x^9-21*a*b^2*x^6-84*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/x^2
/(b*x^3+a)^3
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.23

$$\frac{4\,b^3x^9+21\,ab^2x^6+84\,a^2bx^3-14\,a^3}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

Fricas [A]
time = 0.34, size = 37, normalized size = 0.23

$$\frac{4\,b^3x^9+21\,ab^2x^6+84\,a^2bx^3-14\,a^3}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="fricas")
[Out] 1/28*(4*b^3*x^9 + 21*a*b^2*x^6 + 84*a^2*b*x^3 - 14*a^3)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**3,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**3, x)
```

Giac [A]
time = 5.50, size = 65, normalized size = 0.40

$$\frac{1}{7}\,b^3x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{3}{4}\,ab^2x^4\mathrm{sgn}\left(bx^3+a\right)+3\,a^2bx\mathrm{sgn}\left(bx^3+a\right)-\frac{a^3\mathrm{sgn}(bx^3+a)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^3,x, algorithm="giac")
[Out] 1/7*b^3*x^7*sgn(b*x^3 + a) + 3/4*a*b^2*x^4*sgn(b*x^3 + a) + 3*a^2*b*x*sgn(b
*x^3 + a) - 1/2*a^3*sgn(b*x^3 + a)/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^3, x)
```

3.36 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=161

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}+\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)}+\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -1/3*a^3*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+a*b^2*x^3*((b*x^3+a)^2)^(1/2)/(b
*x^3+a)+1/6*b^3*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3*a^2*b*ln(x)*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{6(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

```
[Out] -1/3*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (a*b^2*x^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^6*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(6*(a + b*x^3)) + (3*a^2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log
[x])/(a + b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```

```
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^4}\,dx}{b^2(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x^2}\,dx,x,x^3\right)}{3b^2(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(3ab^5+\frac{a^3b^3}{x^2}+\frac{3a^2b^4}{x}+b^6x\right)\,dx,x,x^3\right)}{3b^2(ab+b^2x^3)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}+\frac{ab^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^6\sqrt{a^2+2abx^3}}{6(a+bx^3}$$

Mathematica [A]
time = 0.01, size = 62, normalized size = 0.39

$$\frac{\sqrt{(a+bx^3)^2}\,(-2a^3+6ab^2x^6+b^3x^9+18a^2bx^3\log(x))}{6x^3(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^3 + 6*a*b^2*x^6 + b^3*x^9 + 18*a^2*b*x^3*Log[x])
)/(6*x^3*(a + b*x^3))
```

Maple [A]
time = 0.04, size = 59, normalized size = 0.37

method	result	size
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{3}{2}}(b^3x^9+6a\,b^2x^6+18a^2b\ln(x)x^3-2a^3)}{6(b\,x^3+a)^3x^3}$	59
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b(b\,x^3+3a)^2}{6b\,x^3+6a}-\frac{a^3\sqrt{(b\,x^3+a)^2}}{3x^3(b\,x^3+a)}+\frac{3a^2b\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*((b*x^3+a)^2)^(3/2)*(b^3*x^9+6*a*b^2*x^6+18*a^2*b*ln(x)*x^3-2*a^3)/(b*x
^3+a)^3/x^3
```

Maxima [A]
time = 0.28, size = 156, normalized size = 0.97

$$\frac{1}{2}\sqrt{b^2x^6+2\,abx^3+a^2}\,b^2x^3+(-1)^{2\,b^2x^3+2\,ab}a^2b\log\left(2\,b^2x^3+2\,ab\right)-(-1)^{2\,abx^3+2\,a^2}a^2b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{3}{2}\sqrt{b^2x^6+2\,abx^3+a^2}\,ab-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2*x^3 + (-1)^(2*b^2*x^3 + 2*a*b)*a^2*
b*log(2*b^2*x^3 + 2*a*b) - (-1)^(2*a*b*x^3 + 2*a^2)*a^2*b*log(2*a*b*x/abs(x
) + 2*a^2/(x^2*abs(x))) + 3/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b - 1/3*(b^
2*x^6 + 2*a*b*x^3 + a^2)^(3/2)/x^3
```

Fricas [A]
time = 0.38, size = 38, normalized size = 0.24

$$\frac{b^3x^9+6\,ab^2x^6+18\,a^2bx^3\log\left(x\right)-2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b^3*x^9 + 6*a*b^2*x^6 + 18*a^2*b*x^3*log(x) - 2*a^3)/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**4, x)
```

Giac [A]
time = 3.99, size = 85, normalized size = 0.53

$$\frac{1}{6}\,b^3x^6\mathrm{sgn}\left(bx^3+a\right)+ab^2x^3\mathrm{sgn}\left(bx^3+a\right)+3\,a^2b\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{3\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\mathrm{sgn}\left(bx^3+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*sgn(b*x^3 + a) + a*b^2*x^3*sgn(b*x^3 + a) + 3*a^2*b*log(abs(x))
*sgn(b*x^3 + a) - 1/3*(3*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^4, x)
```

3.37 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=165

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}$$

```
[Out] -1/4*a^3*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-3*a^2*b*((b*x^3+a)^2)^(1/2)/x/(b
*x^3+a)+3/2*a*b^2*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5*b^3*x^5*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{3ab^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

```
[Out] -1/4*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^4*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (3*a*b^2*x^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (b^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(5*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^5}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^5}\,dx}{b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^5}+\frac{3a^2b^4}{x^2}+3ab^5x+b^6x^4\right)dx}{b^2(ab+b^2x^3)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)} + \frac{3ab^2x^2\sqrt{a^2+2abx}}{2(a+bx^3}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a+bx^3)^2}\,(-5a^3-60a^2bx^3+30ab^2x^6+4b^3x^9)}{20x^4(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-5*a^3 - 60*a^2*b*x^3 + 30*a*b^2*x^6 + 4*b^3*x^9))/(2
0*x^4*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
gosper	$-\frac{(-4b^3x^9-30a\,b^2x^6+60a^2b\,x^3+5a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{20(b\,x^3+a)^3x^4}$	58
default	$-\frac{(-4b^3x^9-30a\,b^2x^6+60a^2b\,x^3+5a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{20(b\,x^3+a)^3x^4}$	58
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b^2(\frac{1}{5}b\,x^5+\frac{3}{2}a\,x^2)}{b\,x^3+a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-3a^2b\,x^3-\frac{1}{4}a^3)}{(b\,x^3+a)x^4}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(-4*b^3*x^9-30*a*b^2*x^6+60*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/(b*x
^3+a)^3/x^4
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.22

$$\frac{4\,b^3x^9+30\,ab^2x^6-60\,a^2bx^3-5\,a^3}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="maxima")
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

Fricas [A]
time = 0.38, size = 37, normalized size = 0.22

$$\frac{4\,b^3x^9 + 30\,ab^2x^6 - 60\,a^2bx^3 - 5\,a^3}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="fricas")
[Out] 1/20*(4*b^3*x^9 + 30*a*b^2*x^6 - 60*a^2*b*x^3 - 5*a^3)/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**5,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**5, x)
```

Giac [A]
time = 3.88, size = 69, normalized size = 0.42

$$\frac{1}{5}\,b^3x^5\mathrm{sgn}\left(bx^3 + a\right) + \frac{3}{2}\,ab^2x^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{12\,a^2bx^3\mathrm{sgn}(bx^3 + a) + a^3\mathrm{sgn}(bx^3 + a)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^5,x, algorithm="giac")
[Out] 1/5*b^3*x^5*sgn(b*x^3 + a) + 3/2*a*b^2*x^2*sgn(b*x^3 + a) - 1/4*(12*a^2*b*x
^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)}^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^5, x)
```

3.38 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=163

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\,(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\,(a+bx^3)}+\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}$$

```
[Out] -1/5*a^3*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-3/2*a^2*b*((b*x^3+a)^2)^(1/2)/x^
2/(b*x^3+a)+3*a*b^2*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*b^3*x^4*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{3ab^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\,(a+bx^3)}+\frac{b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\,(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] -1/5*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (3*a*b^2*x*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (b^3*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]
)/(4*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^6}\,dx}{b^2\,(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(3ab^5+\frac{a^3b^3}{x^6}+\frac{3a^2b^4}{x^3}+b^6x^3\right)dx}{b^2\,(ab+b^2x^3)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\,(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\,(a+bx^3)} + \frac{3ab^2x\sqrt{a^2+2abx^3}}{a+bx^3}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a+bx^3)^2}\,(-4a^3-30a^2bx^3+60ab^2x^6+5b^3x^9)}{20x^5\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-4*a^3 - 30*a^2*b*x^3 + 60*a*b^2*x^6 + 5*b^3*x^9))/(2
0*x^5*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.36

method	result	size
gosper	$-\frac{(-5b^3x^9-60a\,b^2x^6+30a^2b\,x^3+4a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{20(b\,x^3+a)^3x^5}$	58
default	$-\frac{(-5b^3x^9-60a\,b^2x^6+30a^2b\,x^3+4a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{20(b\,x^3+a)^3x^5}$	58
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b^2(\frac{1}{4}b\,x^4+3ax)}{b\,x^3+a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-\frac{3}{2}a^2b\,x^3-\frac{1}{5}a^3)}{(b\,x^3+a)x^5}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(-5*b^3*x^9-60*a*b^2*x^6+30*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/(b*x
^3+a)^3/x^5
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.23

$$\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="maxima")
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.23

$$\frac{5\,b^3x^9+60\,ab^2x^6-30\,a^2bx^3-4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="fricas")
[Out] 1/20*(5*b^3*x^9 + 60*a*b^2*x^6 - 30*a^2*b*x^3 - 4*a^3)/x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**6,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**6, x)
```

Giac [A]
time = 3.90, size = 68, normalized size = 0.42

$$\frac{1}{4}\,b^3x^4\mathrm{sgn}\left(bx^3+a\right)+3\,ab^2x\mathrm{sgn}\left(bx^3+a\right)-\frac{15\,a^2bx^3\mathrm{sgn}(bx^3+a)+2\,a^3\mathrm{sgn}(bx^3+a)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^6,x, algorithm="giac")
[Out] 1/4*b^3*x^4*sgn(b*x^3 + a) + 3*a*b^2*x*sgn(b*x^3 + a) - 1/10*(15*a^2*b*x^3*
sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^6, x)
```

3.39 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=162

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}\ \mathrm{lo}}{a+bx^3}$$

```
[Out] -1/6*a^3*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-a^2*b*((b*x^3+a)^2)^(1/2)/x^3/(b
*x^3+a)+1/3*b^3*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+3*a*b^2*ln(x)*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3\left(a+bx^3\right)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
```

```
[Out] -1/6*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^6*(a + b*x^3)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*x^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(3*(a + b*x^3)) + (3*a*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*L
og[x])/(a + b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```

[p - 1/2]

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^7}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^7}\,dx}{b^2(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x^3}\,dx,x,x^3\right)}{3b^2(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(b^6+\frac{a^3b^3}{x^3}+\frac{3a^2b^4}{x^2}+\frac{3ab^5}{x}\right)dx,x,x^3\right)}{3b^2(ab+b^2x^3)}\\ &= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{x^3(a+bx^3)}+\frac{b^3x^3\sqrt{a^2+2abx^3+}}{3(a+bx^3)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.38

$$-\frac{\sqrt{(a+bx^3)^2}\,(a^3+6a^2bx^3-2b^3x^9-18ab^2x^6\log(x))}{6x^6(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^7,x]
[Out] -1/6*(Sqrt[(a + b*x^3)^2]*(a^3 + 6*a^2*b*x^3 - 2*b^3*x^9 - 18*a*b^2*x^6*Log
[x]))/(x^6*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 60, normalized size = 0.37

method	result	size
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{3}{2}}(2b^3x^9+18a\,b^2\ln(x)x^6-6a^2b\,x^3-a^3)}{6(b\,x^3+a)^3x^6}$	60
risch	$\frac{b^3x^3\sqrt{(b\,x^3+a)^2}}{3b\,x^3+3a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-a^2b\,x^3-\frac{1}{6}a^3)}{(b\,x^3+a)x^6}+\frac{3a\,b^2\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
[Out] 1/6*((b*x^3+a)^2)^(3/2)*(2*b^3*x^9+18*a*b^2*ln(x)*x^6-6*a^2*b*x^3-a^3)/(b*x
^3+a)^3/x^6
```

Maxima [A]
time = 0.28, size = 220, normalized size = 1.36

$$\frac{\sqrt{b^2x^6+2\,abx^3+a^2}\,b^3x^3}{2\,a}+(-1)^{2\,b^2x^3+2\,ab}\,ab^2\log\left(2\,b^2x^3+2\,ab\right)-(-1)^{2\,abx^3+2\,a^2}\,ab^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{3}{2}\,\sqrt{b^2x^6+2\,abx^3+a^2}\,b^2+\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^2}{6\,a^2}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b}{6\,ax^3}-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3*x^3/a + (-1)^(2*b^2*x^3 + 2*a*b)*a*
b^2*log(2*b^2*x^3 + 2*a*b) - (-1)^(2*a*b*x^3 + 2*a^2)*a*b^2*log(2*a*b*x/abs
(x) + 2*a^2/(x^2*abs(x))) + 3/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^2 + 1/6*(
b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2/a^2 - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^
(3/2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^6)
```

Fricas [A]
time = 0.35, size = 39, normalized size = 0.24

$$\frac{2\,b^3x^9+18\,ab^2x^6\log\left(x\right)-6\,a^2bx^3-a^3}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^9 + 18*a*b^2*x^6*log(x) - 6*a^2*b*x^3 - a^3)/x^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**7, x)
```

Giac [A]
time = 3.52, size = 86, normalized size = 0.53

$$\frac{1}{3}\,b^3x^3\mathrm{sgn}\left(bx^3+a\right)+3\,ab^2\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{9\,ab^2x^6\mathrm{sgn}\left(bx^3+a\right)+6\,a^2bx^3\mathrm{sgn}\left(bx^3+a\right)+a^3\mathrm{sgn}\left(bx^3+a\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*sgn(b*x^3 + a) + 3*a*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(9*a*
b^2*x^6*sgn(b*x^3 + a) + 6*a^2*b*x^3*sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x
^6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^7, x)
```

3.40 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=165

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}+\frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}$$

```
[Out] -1/7*a^3*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-3/4*a^2*b*((b*x^3+a)^2)^(1/2)/x^
4/(b*x^3+a)-3*a*b^2*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+1/2*b^3*x^2*((b*x^3+a)^
2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}+\frac{b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

```
[Out] -1/7*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^7*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (b^3*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(2*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^8}\,dx}{b^2\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^8}+\frac{3a^2b^4}{x^5}+\frac{3ab^5}{x^2}+b^6x\right)dx}{b^2\left(ab+b^2x^3\right)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)} - \frac{3ab^2\sqrt{a^2+2abx^3}}{x\left(a+bx^3\right)}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(4a^3+21a^2bx^3+84ab^2x^6-14b^3x^9\right)}{28x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^8,x]
```

```
[Out] -1/28*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 21*a^2*b*x^3 + 84*a*b^2*x^6 - 14*b^3*x^
9))/(x^7*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 58, normalized size = 0.35

method	result	size
gosper	$-\frac{\left(-14b^3x^9+84a\,b^2x^6+21a^2b\,x^3+4a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{28\left(b\,x^3+a\right)^3x^7}$	58
default	$-\frac{\left(-14b^3x^9+84a\,b^2x^6+21a^2b\,x^3+4a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{28\left(b\,x^3+a\right)^3x^7}$	58
risch	$\frac{b^3x^2\sqrt{\left(b\,x^3+a\right)^2}}{2b\,x^3+2a}+\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(-3a\,b^2x^6-\frac{3}{4}a^2b\,x^3-\frac{1}{7}a^3\right)}{\left(b\,x^3+a\right)x^7}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/28*(-14*b^3*x^9+84*a*b^2*x^6+21*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/(b*
x^3+a)^3/x^7
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.22

$$\frac{14\,b^3x^9-84\,ab^2x^6-21\,a^2bx^3-4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

Fricas [A]
time = 0.36, size = 37, normalized size = 0.22

$$\frac{14\,b^3x^9 - 84\,ab^2x^6 - 21\,a^2bx^3 - 4\,a^3}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/28*(14*b^3*x^9 - 84*a*b^2*x^6 - 21*a^2*b*x^3 - 4*a^3)/x^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**8, x)
```

Giac [A]
time = 3.39, size = 70, normalized size = 0.42

$$\frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^3 + a\right) - \frac{84\,ab^2x^6\mathrm{sgn}(bx^3 + a) + 21\,a^2bx^3\mathrm{sgn}(bx^3 + a) + 4\,a^3\mathrm{sgn}(bx^3 + a)}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x^3 + a) - 1/28*(84*a*b^2*x^6*sgn(b*x^3 + a) + 21*a^2*b*x
^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^7
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)}^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^8, x)
```

3.41 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=162

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -1/8*a^3*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-3/5*a^2*b*((b*x^3+a)^2)^(1/2)/x^
5/(b*x^3+a)-3/2*a*b^2*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+b^3*x*((b*x^3+a)^2)
^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

```
[Out] -1/8*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^8*(a + b*x^3)) - (3*a^2*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(a + b*x^3)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^9}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{\left(ab+b^2x^3\right)^3}{x^9}\,dx}{b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^6+\frac{a^3b^3}{x^9}+\frac{3a^2b^4}{x^6}+\frac{3ab^5}{x^3}\right)\,dx}{b^2\left(ab+b^2x^3\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\left(a+bx^3\right)} - \frac{3ab^2\sqrt{a^2+2abx^3+}}{2x^2\left(a+bx^3\right)}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.38

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(5a^3+24a^2bx^3+60ab^2x^6-40b^3x^9\right)}{40x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^9,x]
```

```
[Out] -1/40*(Sqrt[(a + b*x^3)^2]*(5*a^3 + 24*a^2*b*x^3 + 60*a*b^2*x^6 - 40*b^3*x^
9))/(x^8*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.36

method	result	size
gosper	$-\frac{\left(-40b^3x^9+60a\,b^2x^6+24a^2b\,x^3+5a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{40x^8\left(b\,x^3+a\right)^3}$	58
default	$-\frac{\left(-40b^3x^9+60a\,b^2x^6+24a^2b\,x^3+5a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{40x^8\left(b\,x^3+a\right)^3}$	58
risch	$\frac{b^3x\sqrt{\left(b\,x^3+a\right)^2}}{b\,x^3+a}+\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(-\frac{3}{2}a\,b^2x^6-\frac{3}{5}a^2b\,x^3-\frac{1}{8}a^3\right)}{\left(b\,x^3+a\right)x^8}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/40*(-40*b^3*x^9+60*a*b^2*x^6+24*a^2*b*x^3+5*a^3)*((b*x^3+a)^2)^(3/2)/x^8
/(b*x^3+a)^3
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.23

$$\frac{40\,b^3x^9-60\,ab^2x^6-24\,a^2bx^3-5\,a^3}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

Fricas [A]
time = 0.37, size = 37, normalized size = 0.23

$$\frac{40\,b^3x^9-60\,ab^2x^6-24\,a^2bx^3-5\,a^3}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/40*(40*b^3*x^9 - 60*a*b^2*x^6 - 24*a^2*b*x^3 - 5*a^3)/x^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**9, x)
```

Giac [A]
time = 3.78, size = 67, normalized size = 0.41

$$b^3x\mathrm{sgn}\left(bx^3+a\right)-\frac{60\,ab^2x^6\mathrm{sgn}(bx^3+a)+24\,a^2bx^3\mathrm{sgn}(bx^3+a)+5\,a^3\mathrm{sgn}(bx^3+a)}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] b^3*x*sgn(b*x^3 + a) - 1/40*(60*a*b^2*x^6*sgn(b*x^3 + a) + 24*a^2*b*x^3*sgn
(b*x^3 + a) + 5*a^3*sgn(b*x^3 + a))/x^8
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^9,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^9, x)
```

3.42 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=161

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6(a+bx^3)}-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3(a+bx^3)}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}\,\log(x}{a+bx^3}$$

```
[Out] -1/9*a^3*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-1/2*a^2*b*((b*x^3+a)^2)^(1/2)/x^
6/(b*x^3+a)-a*b^2*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+b^3*ln(x)*((b*x^3+a)^2)
^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$-\frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6(a+bx^3)}-\frac{ab^2\sqrt{a^2+2abx^3+b^2x^6}}{x^3(a+bx^3)}+\frac{b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

```
[Out] -1/9*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^9*(a + b*x^3)) - (a^2*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^6*(a + b*x^3)) - (a*b^2*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log
[x])/(a + b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```

[p - 1/2]

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^{10}}\,dx}{b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x^4}\,dx,x,x^3\right)}{3b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(\frac{a^3b^3}{x^4}+\frac{3a^2b^4}{x^3}+\frac{3ab^5}{x^2}+\frac{b^6}{x}\right)dx,x,x^3\right)}{3b^2(ab+b^2x^3)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} - \frac{a^2b\sqrt{a^2+2abx^3+b^2x^6}}{2x^6(a+bx^3)} - \frac{ab^2\sqrt{a^2+2abx^3+}}{x^3(a+bx^3)}$$

Mathematica [A]
time = 0.01, size = 63, normalized size = 0.39

$$-\frac{\sqrt{(a+bx^3)^2}\,(a(2a^2+9abx^3+18b^2x^6)-18b^3x^9\log(x))}{18x^9(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^10,x]
```

```
[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(a*(2*a^2 + 9*a*b*x^3 + 18*b^2*x^6) - 18*b^3*x^9
*Log[x]))/(x^9*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 60, normalized size = 0.37

method	result	size
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{3}{2}}(18b^3\ln(x)x^9-18a\,b^2x^6-9a^2b\,x^3-2a^3)}{18(b\,x^3+a)^3x^9}$	60
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,(-a\,b^2x^6-\frac{1}{2}a^2b\,x^3-\frac{1}{9}a^3)}{(b\,x^3+a)x^9}+\frac{b^3\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*((b*x^3+a)^2)^(3/2)*(18*b^3*ln(x)*x^9-18*a*b^2*x^6-9*a^2*b*x^3-2*a^3)/
(b*x^3+a)^3/x^9
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(113) = 226$.
time = 0.29, size = 253, normalized size = 1.57

$$\frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,b^4x^3}{6\,a^2} + \frac{1}{3}\,(-1)^{2\,b^2x^3 + 2\,ab}\,b^3\log\left(2\,b^2x^3 + 2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3 + 2\,a^2}\,b^3\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right) + \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,b^3}{2\,a} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^3}{18\,a^3} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^2}{6\,a^2x^3} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b}{18\,a^3x^6} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}}{9\,a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*a
*b)*b^3*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^3*log(2*a*b
*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^3/a
 - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^3/a^3 - 1/6*(b^2*x^6 + 2*a*b*x^
3 + a^2)^(3/2)*b^2/(a^2*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^
3*x^6) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^9)
```

Fricas [A]
time = 0.36, size = 39, normalized size = 0.24

$$\frac{18\,b^3x^9\log\left(x\right) - 18\,ab^2x^6 - 9\,a^2bx^3 - 2\,a^3}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(18*b^3*x^9*log(x) - 18*a*b^2*x^6 - 9*a^2*b*x^3 - 2*a^3)/x^9
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**10, x)
```

Giac [A]
time = 3.76, size = 85, normalized size = 0.53

$$b^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3 + a\right) - \frac{11\,b^3x^9\mathrm{sgn}(bx^3 + a) + 18\,ab^2x^6\mathrm{sgn}(bx^3 + a) + 9\,a^2bx^3\mathrm{sgn}(bx^3 + a) + 2\,a^3\mathrm{sgn}(bx^3 + a)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x))*sgn(b*x^3 + a) - 1/18*(11*b^3*x^9*sgn(b*x^3 + a) + 18*a*b^2
*x^6*sgn(b*x^3 + a) + 9*a^2*b*x^3*sgn(b*x^3 + a) + 2*a^3*sgn(b*x^3 + a))/x^
9
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{3/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^10, x)
```

3.43 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}} \, dx$

Optimal. Leaf size=165

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}$$

```
[Out] -1/10*a^3*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-3/7*a^2*b*((b*x^3+a)^2)^(1/2)/
x^7/(b*x^3+a)-3/4*a*b^2*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-b^3*((b*x^3+a)^2)
^(1/2)/x/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

```
[Out] -1/10*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(x*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{11}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^{11}}\,dx}{b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{11}}+\frac{3a^2b^4}{x^8}+\frac{3ab^5}{x^5}+\frac{b^6}{x^2}\right)dx}{b^2(ab+b^2x^3)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3}}{4x^4(a+bx^3}$$

__Mathematica__ [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\,(14a^3+60a^2bx^3+105ab^2x^6+140b^3x^9)}{140x^{10}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^11,x]
```

```
[Out] -1/140*(Sqrt[(a + b*x^3)^2]*(14*a^3 + 60*a^2*b*x^3 + 105*a*b^2*x^6 + 140*b^
3*x^9))/(x^10*(a + b*x^3))
```

__Maple__ [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-b^3x^9-\frac{3}{4}a\,b^2x^6-\frac{3}{7}a^2b\,x^3-\frac{1}{10}a^3\right)}{(b\,x^3+a)x^{10}}$	57
gosper	$-\frac{(140b^3x^9+105a\,b^2x^6+60a^2b\,x^3+14a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{140x^{10}(b\,x^3+a)^3}$	58
default	$-\frac{(140b^3x^9+105a\,b^2x^6+60a^2b\,x^3+14a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{140x^{10}(b\,x^3+a)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/140*(140*b^3*x^9+105*a*b^2*x^6+60*a^2*b*x^3+14*a^3)*((b*x^3+a)^2)^(3/2)/
x^10/(b*x^3+a)^3
```

__Maxima__ [A]
time = 0.28, size = 37, normalized size = 0.22

$$-\frac{140\,b^3x^9+105\,ab^2x^6+60\,a^2bx^3+14\,a^3}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

Fricas [A]
time = 0.36, size = 37, normalized size = 0.22

$$-\frac{140\,b^3x^9+105\,ab^2x^6+60\,a^2bx^3+14\,a^3}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/140*(140*b^3*x^9 + 105*a*b^2*x^6 + 60*a^2*b*x^3 + 14*a^3)/x^10
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**11, x)
```

Giac [A]
time = 3.69, size = 69, normalized size = 0.42

$$-\frac{140\,b^3x^9\mathrm{sgn}(bx^3+a)+105\,ab^2x^6\mathrm{sgn}(bx^3+a)+60\,a^2bx^3\mathrm{sgn}(bx^3+a)+14\,a^3\mathrm{sgn}(bx^3+a)}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/140*(140*b^3*x^9*sgn(b*x^3 + a) + 105*a*b^2*x^6*sgn(b*x^3 + a) + 60*a^2*
b*x^3*sgn(b*x^3 + a) + 14*a^3*sgn(b*x^3 + a))/x^10
```

Mupad [B]
time = 1.21, size = 151, normalized size = 0.92

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^11,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^6 +
2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(7*x^7*(a + b*x^3))
```

3.44 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}$$

```
[Out] -1/11*a^3*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-3/8*a^2*b*((b*x^3+a)^2)^(1/2)/
x^8/(b*x^3+a)-3/5*a*b^2*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-1/2*b^3*((b*x^3+a
)^2)^(1/2)/x^2/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
```

```
[Out] -1/11*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^11*(a + b*x^3)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(5*x^5*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(2*x^2*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{12}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^{12}}\,dx}{b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{12}}+\frac{3a^2b^4}{x^9}+\frac{3ab^5}{x^6}+\frac{b^6}{x^3}\right)dx}{b^2(ab+b^2x^3)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+}}{5x^5(a+bx^3)}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\,(40a^3+165a^2bx^3+264ab^2x^6+220b^3x^9)}{440x^{11}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^12,x]
[Out] -1/440*(Sqrt[(a + b*x^3)^2]*(40*a^3 + 165*a^2*b*x^3 + 264*a*b^2*x^6 + 220*b
^3*x^9))/(x^11*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{2}b^3x^9-\frac{3}{5}a\,b^2x^6-\frac{3}{8}a^2b\,x^3-\frac{1}{11}a^3\right)}{(b\,x^3+a)x^{11}}$	57
gosper	$-\frac{(220b^3x^9+264a\,b^2x^6+165a^2b\,x^3+40a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{440x^{11}(b\,x^3+a)^3}$	58
default	$-\frac{(220b^3x^9+264a\,b^2x^6+165a^2b\,x^3+40a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{440x^{11}(b\,x^3+a)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x,method=_RETURNVERBOSE)
[Out] -1/440*(220*b^3*x^9+264*a*b^2*x^6+165*a^2*b*x^3+40*a^3)*((b*x^3+a)^2)^(3/2)
/x^11/(b*x^3+a)^3
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.22

$$-\frac{220\,b^3x^9+264\,ab^2x^6+165\,a^2bx^3+40\,a^3}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="maxima")
[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11
```

Fricas [A]
time = 0.36, size = 37, normalized size = 0.22

$$-\frac{220\,b^3x^9+264\,ab^2x^6+165\,a^2bx^3+40\,a^3}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="fricas")
[Out] -1/440*(220*b^3*x^9 + 264*a*b^2*x^6 + 165*a^2*b*x^3 + 40*a^3)/x^11
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**12,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**12, x)
```

Giac [A]
time = 4.20, size = 69, normalized size = 0.41

$$-\frac{220\,b^3x^9\mathrm{sgn}(bx^3+a)+264\,ab^2x^6\mathrm{sgn}(bx^3+a)+165\,a^2bx^3\mathrm{sgn}(bx^3+a)+40\,a^3\mathrm{sgn}(bx^3+a)}{440\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^12,x, algorithm="giac")
[Out] -1/440*(220*b^3*x^9*sgn(b*x^3 + a) + 264*a*b^2*x^6*sgn(b*x^3 + a) + 165*a^2
*b*x^3*sgn(b*x^3 + a) + 40*a^3*sgn(b*x^3 + a))/x^11
```

Mupad [B]
time = 1.22, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^2\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^12,x)
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(2*x^2*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(8*x^8*(a + b*x^3))
```

3.45 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=41

$$-\frac{(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

```
[Out] -1/12*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/a/x^12
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 270}

$$-\frac{(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

```
[Out] -1/12*((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^12)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{13}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^{13}}\,dx}{b^2(ab+b^2x^3)}\\ &= -\frac{(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{12ax^{12}}\end{aligned}$$

<u>**Mathematica**</u> [**A**]
time = 0.01, size = 59, normalized size = 1.44

$$-\frac{\sqrt{(a+bx^3)^2}\,(a^3+4a^2bx^3+6ab^2x^6+4b^3x^9)}{12x^{12}\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^13,x]
```

```
[Out] -1/12*(Sqrt[(a + b*x^3)^2]*(a^3 + 4*a^2*b*x^3 + 6*a*b^2*x^6 + 4*b^3*x^9))/(
x^12*(a + b*x^3))
```

<u>**Maple**</u> [**A**]
time = 0.02, size = 56, normalized size = 1.37

method	result	size
gosper	$-\frac{(4b^3x^9+6a\,b^2x^6+4a^2b\,x^3+a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{12x^{12}(b\,x^3+a)^3}$	56
default	$-\frac{(4b^3x^9+6a\,b^2x^6+4a^2b\,x^3+a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{12x^{12}(b\,x^3+a)^3}$	56
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{3}b^3x^9-\frac{1}{2}a\,b^2x^6-\frac{1}{3}a^2b\,x^3-\frac{1}{12}a^3\right)}{(b\,x^3+a)x^{12}}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(4*b^3*x^9+6*a*b^2*x^6+4*a^2*b*x^3+a^3)*((b*x^3+a)^2)^(3/2)/x^12/(b*x
^3+a)^3
```

<u>**Maxima**</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(28) = 56.
time = 0.28, size = 148, normalized size = 3.61

$$\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^4}{12\,a^4}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3}{12\,a^3x^3}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^2}{12\,a^4x^6}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b}{12\,a^3x^9}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}}{12\,a^2x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^4/a^4 + 1/12*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(3/2)*b^3/(a^3*x^3) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^
4*x^6) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^9) - 1/12*(b^2*x^6
 + 2*a*b*x^3 + a^2)^(5/2)/(a^2*x^12)
```

Fricas [A]
time = 0.34, size = 35, normalized size = 0.85

$$-\frac{4\,b^3x^9+6\,ab^2x^6+4\,a^2bx^3+a^3}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b^3*x^9 + 6*a*b^2*x^6 + 4*a^2*b*x^3 + a^3)/x^12
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**13, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(28) = 56.
time = 4.70, size = 68, normalized size = 1.66

$$-\frac{4\,b^3x^9\mathrm{sgn}(bx^3+a)+6\,ab^2x^6\mathrm{sgn}(bx^3+a)+4\,a^2bx^3\mathrm{sgn}(bx^3+a)+a^3\mathrm{sgn}(bx^3+a)}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(4*b^3*x^9*sgn(b*x^3 + a) + 6*a*b^2*x^6*sgn(b*x^3 + a) + 4*a^2*b*x^3*
sgn(b*x^3 + a) + a^3*sgn(b*x^3 + a))/x^12
```

Mupad [B]
time = 1.21, size = 151, normalized size = 3.68

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,x^{12}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3\,\left(b\,x^3+a\right)}-\frac{a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^6\,\left(b\,x^3+a\right)}-\frac{a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^9\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^13,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(2*x^6*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(3*x^9*(a + b*x^3))
```

3.46 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}$$

```
[Out] -1/13*a^3*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-3/10*a^2*b*((b*x^3+a)^2)^(1/2)
/x^10/(b*x^3+a)-3/7*a*b^2*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-1/4*b^3*((b*x^3
+a)^2)^(1/2)/x^4/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] -1/13*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^13*(a + b*x^3)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(4*x^4*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{14}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^{14}}\,dx}{b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{14}}+\frac{3a^2b^4}{x^{11}}+\frac{3ab^5}{x^8}+\frac{b^6}{x^5}\right)dx}{b^2(ab+b^2x^3)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3+}}{7x^7(a+bx^3)}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\,(140a^3+546a^2bx^3+780ab^2x^6+455b^3x^9)}{1820x^{13}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^14,x]
```

```
[Out] -1/1820*(Sqrt[(a + b*x^3)^2]*(140*a^3 + 546*a^2*b*x^3 + 780*a*b^2*x^6 + 455
*b^3*x^9))/(x^13*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{4}b^3x^9-\frac{3}{7}a\,b^2x^6-\frac{3}{10}a^2b\,x^3-\frac{1}{13}a^3\right)}{(b\,x^3+a)x^{13}}$	57
gosper	$-\frac{(455b^3x^9+780a\,b^2x^6+546a^2b\,x^3+140a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{1820x^{13}(b\,x^3+a)^3}$	58
default	$-\frac{(455b^3x^9+780a\,b^2x^6+546a^2b\,x^3+140a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{1820x^{13}(b\,x^3+a)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1820*(455*b^3*x^9+780*a*b^2*x^6+546*a^2*b*x^3+140*a^3)*((b*x^3+a)^2)^(3/
2)/x^13/(b*x^3+a)^3
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.22

$$-\frac{455\,b^3x^9+780\,ab^2x^6+546\,a^2bx^3+140\,a^3}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13
```

Fricas [A]
time = 0.38, size = 37, normalized size = 0.22

$$-\frac{455\,b^3x^9+780\,ab^2x^6+546\,a^2bx^3+140\,a^3}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/1820*(455*b^3*x^9 + 780*a*b^2*x^6 + 546*a^2*b*x^3 + 140*a^3)/x^13
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**14, x)
```

Giac [A]
time = 3.90, size = 69, normalized size = 0.41

$$-\frac{455\,b^3x^9\mathrm{sgn}(bx^3+a)+780\,ab^2x^6\mathrm{sgn}(bx^3+a)+546\,a^2bx^3\mathrm{sgn}(bx^3+a)+140\,a^3\mathrm{sgn}(bx^3+a)}{1820\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/1820*(455*b^3*x^9*sgn(b*x^3 + a) + 780*a*b^2*x^6*sgn(b*x^3 + a) + 546*a^
2*b*x^3*sgn(b*x^3 + a) + 140*a^3*sgn(b*x^3 + a))/x^13
```

Mupad [B]
time = 1.19, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^14,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(13*x^13*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(10*x^10*(a + b*x^3))
```

3.47 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\,(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\,(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\,(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\,(a+bx^3)}$$

```
[Out] -1/14*a^3*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-3/11*a^2*b*((b*x^3+a)^2)^(1/2)
/x^11/(b*x^3+a)-3/8*a*b^2*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-1/5*b^3*((b*x^3
+a)^2)^(1/2)/x^5/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\,(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\,(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\,(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
```

```
[Out] -1/14*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^14*(a + b*x^3)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(5*x^5*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{15}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^{15}}\,dx}{b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^3b^3}{x^{15}}+\frac{3a^2b^4}{x^{12}}+\frac{3ab^5}{x^9}+\frac{b^6}{x^6}\right)dx}{b^2(ab+b^2x^3)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{3ab^2\sqrt{a^2+2abx^3}}{8x^8(a+bx^3}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^3)^2}\,(220a^3+840a^2bx^3+1155ab^2x^6+616b^3x^9)}{3080x^{14}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^15,x]
[Out] -1/3080*(Sqrt[(a + b*x^3)^2]*(220*a^3 + 840*a^2*b*x^3 + 1155*a*b^2*x^6 + 61
6*b^3*x^9))/(x^14*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 58, normalized size = 0.35

method	result	size
risch	$\frac{\sqrt{(bx^3+a)^2}\,\left(-\frac{1}{14}a^3-\frac{3}{11}a^2bx^3-\frac{3}{8}ab^2x^6-\frac{1}{5}b^3x^9\right)}{(bx^3+a)x^{14}}$	57
gosper	$-\frac{(616b^3x^9+1155ab^2x^6+840a^2bx^3+220a^3)\left((bx^3+a)^2\right)^{\frac{3}{2}}}{3080x^{14}(bx^3+a)^3}$	58
default	$-\frac{(616b^3x^9+1155ab^2x^6+840a^2bx^3+220a^3)\left((bx^3+a)^2\right)^{\frac{3}{2}}}{3080x^{14}(bx^3+a)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x,method=_RETURNVERBOSE)
[Out] -1/3080*(616*b^3*x^9+1155*a*b^2*x^6+840*a^2*b*x^3+220*a^3)*((b*x^3+a)^2)^(3
/2)/x^14/(b*x^3+a)^3
```

Maxima [A]
time = 0.28, size = 37, normalized size = 0.22

$$-\frac{616\,b^3x^9+1155\,ab^2x^6+840\,a^2bx^3+220\,a^3}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="maxima")
[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14
```

Fricas [A]
time = 0.36, size = 37, normalized size = 0.22

$$-\frac{616\,b^3x^9+1155\,ab^2x^6+840\,a^2bx^3+220\,a^3}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="fricas")
[Out] -1/3080*(616*b^3*x^9 + 1155*a*b^2*x^6 + 840*a^2*b*x^3 + 220*a^3)/x^14
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**15,x)
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**15, x)
```

Giac [A]
time = 3.74, size = 69, normalized size = 0.41

$$-\frac{616\,b^3x^9\mathrm{sgn}(bx^3+a)+1155\,ab^2x^6\mathrm{sgn}(bx^3+a)+840\,a^2bx^3\mathrm{sgn}(bx^3+a)+220\,a^3\mathrm{sgn}(bx^3+a)}{3080\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^15,x, algorithm="giac")
[Out] -1/3080*(616*b^3*x^9*sgn(b*x^3 + a) + 1155*a*b^2*x^6*sgn(b*x^3 + a) + 840*a
^2*b*x^3*sgn(b*x^3 + a) + 220*a^3*sgn(b*x^3 + a))/x^14
```

Mupad [B]
time = 1.21, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{14\,x^{14}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^15,x)
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(14*x^14*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(11*x^11*(a + b*x^3))
```

3.48 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=84

$$-\frac{(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}+\frac{b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}$$

```
[Out] -1/15*(b*x^3+a)^3*((b*x^3+a)^2)^(1/2)/a/x^15+1/60*b*(b*x^3+a)^3*((b*x^3+a)^
2)^(1/2)/a^2/x^12
```

Rubi [A]

time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1369, 272, 47, 37}

$$\frac{b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}-\frac{(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

```
[Out] -1/15*((a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^15) + (b*(a + b*
x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(60*a^2*x^12)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{16}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^3}{x^{16}}\,dx}{b^2(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x^6}\,dx,x,x^3\right)}{3b^2(ab+b^2x^3)}$$
$$= -\frac{(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^3}{x^5}\,d\right.}{15ab(ab+b^2x^3)}$$
$$= -\frac{(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{15ax^{15}} + \frac{b(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}{60a^2x^{12}}$$

Mathematica [**A**]
time = 0.01, size = 61, normalized size = 0.73

$$-\frac{\sqrt{(a+bx^3)^2}\,(4a^3+15a^2bx^3+20ab^2x^6+10b^3x^9)}{60x^{15}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^16,x]
```

```
[Out] -1/60*(Sqrt[(a + b*x^3)^2]*(4*a^3 + 15*a^2*b*x^3 + 20*a*b^2*x^6 + 10*b^3*x^
9))/(x^15*(a + b*x^3))
```

Maple [**A**]
time = 0.03, size = 58, normalized size = 0.69

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{15}a^3-\frac{1}{4}a^2b\,x^3-\frac{1}{3}a\,b^2x^6-\frac{1}{6}b^3x^9\right)}{(b\,x^3+a)x^{15}}$	57
gosper	$-\frac{(10b^3x^9+20a\,b^2x^6+15a^2b\,x^3+4a^3)\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}{60x^{15}(b\,x^3+a)^3}$	58

default	$-\frac{\left(10 b^{3} x^{9}+20 a \,b^{2} x^{6}+15 a^{2} b \,x^{3}+4 a^{3}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{3}{2}}}{60 x^{15} \left(b \,x^{3}+a \right)^{3}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] -1/60*(10*b^3*x^9+20*a*b^2*x^6+15*a^2*b*x^3+4*a^3)*((b*x^3+a)^2)^(3/2)/x^15
/(b*x^3+a)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(58) = 116$.
time = 0.29, size = 179, normalized size = 2.13

$$-\frac{{\left(b^{2} x^{6} + 2 \, a b x^{3} + a^{2}\right)}^{\frac{3}{2}} b^{5}}{12 \, a^{5}} - \frac{{\left(b^{2} x^{6} + 2 \, a b x^{3} + a^{2}\right)}^{\frac{3}{2}} b^{4}}{12 \, a^{4} x^{3}} + \frac{{\left(b^{2} x^{6} + 2 \, a b x^{3} + a^{2}\right)}^{\frac{5}{2}} b^{3}}{12 \, a^{5} x^{6}} - \frac{{\left(b^{2} x^{6} + 2 \, a b x^{3} + a^{2}\right)}^{\frac{5}{2}} b^{2}}{12 \, a^{4} x^{9}} + \frac{{\left(b^{2} x^{6} + 2 \, a b x^{3} + a^{2}\right)}^{\frac{5}{2}} b}{12 \, a^{3} x^{12}} - \frac{{\left(b^{2} x^{6} + 2 \, a b x^{3} + a^{2}\right)}^{\frac{5}{2}}}{15 \, a^{2} x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] -1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^5/a^5 - 1/12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(3/2)*b^4/(a^4*x^3) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/(a
^5*x^6) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^4*x^9) + 1/12*(b^2*
x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a^3*x^12) - 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2
)^(5/2)/(a^2*x^15)
```

Fricas [A]
time = 0.35, size = 37, normalized size = 0.44

$$-\frac{10 \, b^{3} x^{9} + 20 \, a b^{2} x^{6} + 15 \, a^{2} b x^{3} + 4 \, a^{3}}{60 \, x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^3*x^9 + 20*a*b^2*x^6 + 15*a^2*b*x^3 + 4*a^3)/x^15
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b x^{3}\right)^{2}\right)^{\frac{3}{2}}}{x^{16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**16, x)
```

Giac [A]
time = 3.80, size = 69, normalized size = 0.82

$$-\frac{10\,b^3x^9\mathrm{sgn}(bx^3+a)+20\,ab^2x^6\mathrm{sgn}(bx^3+a)+15\,a^2bx^3\mathrm{sgn}(bx^3+a)+4\,a^3\mathrm{sgn}(bx^3+a)}{60\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/60*(10*b^3*x^9*sgn(b*x^3 + a) + 20*a*b^2*x^6*sgn(b*x^3 + a) + 15*a^2*b*x
^3*sgn(b*x^3 + a) + 4*a^3*sgn(b*x^3 + a))/x^15
```

Mupad [B]
time = 1.21, size = 151, normalized size = 1.80

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{15\,x^{15}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,\left(b\,x^3+a\right)}-\frac{a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^9\,\left(b\,x^3+a\right)}-\frac{a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^{12}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^16,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(15*x^15*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a*b^2*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(3*x^9*(a + b*x^3)) - (a^2*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(4*x^12*(a + b*x^3))
```

3.49 $\int \frac{(a^2+2abx^3+b^2x^6)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)}-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}$$

```
[Out] -1/16*a^3*((b*x^3+a)^2)^(1/2)/x^16/(b*x^3+a)-3/13*a^2*b*((b*x^3+a)^2)^(1/2)
/x^13/(b*x^3+a)-3/10*a*b^2*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-1/7*b^3*((b*x
^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)}-\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

```
[Out] -1/16*(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^16*(a + b*x^3)) - (3*a^2*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (3*a*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(10*x^10*(a + b*x^3)) - (b^3*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(7*x^7*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}{x^{17}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\int \frac{\left(ab+b^2x^3\right)^3}{x^{17}}\,dx}{b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\int \left(\frac{a^3b^3}{x^{17}}+\frac{3a^2b^4}{x^{14}}+\frac{3ab^5}{x^{11}}+\frac{b^6}{x^8}\right)dx}{b^2\left(ab+b^2x^3\right)}$$

$$= -\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)} - \frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)} - \frac{3ab^2\sqrt{a^2+2abx^3+}}{10x^{10}\left(a+bx^3\right.}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 0.37

$$-\frac{\sqrt{\left(a+bx^3\right)^2}\left(455a^3+1680a^2bx^3+2184ab^2x^6+1040b^3x^9\right)}{7280x^{16}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)/x^17,x]
```

```
[Out] -1/7280*(Sqrt[(a + b*x^3)^2]*(455*a^3 + 1680*a^2*b*x^3 + 2184*a*b^2*x^6 + 1
040*b^3*x^9))/(x^16*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 58, normalized size = 0.35

method	result	size
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(-\frac{1}{16}a^3-\frac{3}{13}a^2b\,x^3-\frac{3}{10}a\,b^2x^6-\frac{1}{7}b^3x^9\right)}{\left(b\,x^3+a\right)x^{16}}$	57
gosper	$-\frac{\left(1040b^3x^9+2184a\,b^2x^6+1680a^2b\,x^3+455a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{7280x^{16}\left(b\,x^3+a\right)^3}$	58
default	$-\frac{\left(1040b^3x^9+2184a\,b^2x^6+1680a^2b\,x^3+455a^3\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}{7280x^{16}\left(b\,x^3+a\right)^3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7280*(1040*b^3*x^9+2184*a*b^2*x^6+1680*a^2*b*x^3+455*a^3)*((b*x^3+a)^2)^
(3/2)/x^16/(b*x^3+a)^3
```

Maxima [A]
time = 0.34, size = 37, normalized size = 0.22

$$-\frac{1040\,b^3x^9+2184\,ab^2x^6+1680\,a^2bx^3+455\,a^3}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16
```

Fricas [A]
time = 0.38, size = 37, normalized size = 0.22

$$-\frac{1040\,b^3x^9 + 2184\,ab^2x^6 + 1680\,a^2bx^3 + 455\,a^3}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/7280*(1040*b^3*x^9 + 2184*a*b^2*x^6 + 1680*a^2*b*x^3 + 455*a^3)/x^16
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^3)^2\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(3/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(3/2)/x**17, x)
```

Giac [A]
time = 4.00, size = 69, normalized size = 0.41

$$-\frac{1040\,b^3x^9\mathrm{sgn}(bx^3+a) + 2184\,ab^2x^6\mathrm{sgn}(bx^3+a) + 1680\,a^2bx^3\mathrm{sgn}(bx^3+a) + 455\,a^3\mathrm{sgn}(bx^3+a)}{7280\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/7280*(1040*b^3*x^9*sgn(b*x^3 + a) + 2184*a*b^2*x^6*sgn(b*x^3 + a) + 1680
*a^2*b*x^3*sgn(b*x^3 + a) + 455*a^3*sgn(b*x^3 + a))/x^16
```

Mupad [B]
time = 1.22, size = 151, normalized size = 0.90

$$-\frac{a^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3+a\right)}-\frac{b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{3\,a\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{10\,x^{10}\,\left(b\,x^3+a\right)}-\frac{3\,a^2\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)/x^17,x)
```

```
[Out] - (a^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (b^3*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (3*a*b^2*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(10*x^10*(a + b*x^3)) - (3*a^2*b*(a^2 + b^2*x^6 + 2*a
*b*x^3)^(1/2))/(13*x^13*(a + b*x^3))
```

3.50 $\int x^{13}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2a\cdots}}{23\,(a+b\cdots}$$

```
[Out] 1/14*a^5*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/17*a^4*b*x^17*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+1/2*a^3*b^2*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/23*a^2*b^3
*x^23*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/26*a*b^4*x^26*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/29*b^5*x^29*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^{29}\sqrt{a^2+2abx^3+b^2x^6}}{29\,(a+bx^3)}+\frac{5ab^4x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\,(a+bx^3)}+\frac{10a^2b^3x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\,(a+bx^3)}+\frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}+\frac{a^3b^2x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (5*a^4*b*x^17
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^3*b^2*x^20*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^23*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(23*(a + b*x^3)) + (5*a*b^4*x^26*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(26*(a + b*x^3)) + (b^5*x^29*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(29*
(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^{13}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{13}\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}$$

$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^{13}+5a^4b^6x^{16}+10a^3b^7x^{19}+10a^2b^8x^{22}+5\right.}{b^4\left(ab+b^2x^3\right)}$$

$$=\frac{a^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5a^4bx^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^3b^2x^{20}\sqrt{a}}{2}$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 0.33

$$\frac{x^{14}\sqrt{\left(a+bx^3\right)^2}\left(147407a^5+606970a^4bx^3+1031849a^3b^2x^6+897260a^2b^3x^9+396865ab^4x^{12}+71162b^5x^{15}\right)}{2063698\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^14*Sqrt[(a + b*x^3)^2]*(147407*a^5 + 606970*a^4*b*x^3 + 1031849*a^3*b^2*
x^6 + 897260*a^2*b^3*x^9 + 396865*a*b^4*x^12 + 71162*b^5*x^15))/(2063698*(a
 + b*x^3))
```

Maple [A]
time = 0.06, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^{14}\left(71162b^5x^{15}+396865b^4a\,x^{12}+897260a^2b^3x^9+1031849b^2a^3x^6+606970a^4b\,x^3+147407a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{2063698\left(b\,x^3+a\right)^5}$
default	$\frac{x^{14}\left(71162b^5x^{15}+396865b^4a\,x^{12}+897260a^2b^3x^9+1031849b^2a^3x^6+606970a^4b\,x^3+147407a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{2063698\left(b\,x^3+a\right)^5}$
risch	$\frac{a^5x^{14}\sqrt{\left(b\,x^3+a\right)^2}}{14b\,x^3+14a}+\frac{5a^4b\,x^{17}\sqrt{\left(b\,x^3+a\right)^2}}{17\left(b\,x^3+a\right)}+\frac{a^3b^2x^{20}\sqrt{\left(b\,x^3+a\right)^2}}{2b\,x^3+2a}+\frac{10a^2b^3x^{23}\sqrt{\left(b\,x^3+a\right)^2}}{23\left(b\,x^3+a\right)}+\frac{5a\,b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2063698*x^14*(71162*b^5*x^15+396865*a*b^4*x^12+897260*a^2*b^3*x^9+1031849
*a^3*b^2*x^6+606970*a^4*b*x^3+147407*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.22

$$\frac{1}{29}\,b^5x^{29}+\frac{5}{26}\,ab^4x^{26}+\frac{10}{23}\,a^2b^3x^{23}+\frac{1}{2}\,a^3b^2x^{20}+\frac{5}{17}\,a^4bx^{17}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^2*x^20 + 5
/17*a^4*b*x^17 + 1/14*a^5*x^14
```

Fricas [A]
time = 0.38, size = 57, normalized size = 0.22

$$\frac{1}{29}\,b^5x^{29}+\frac{5}{26}\,ab^4x^{26}+\frac{10}{23}\,a^2b^3x^{23}+\frac{1}{2}\,a^3b^2x^{20}+\frac{5}{17}\,a^4bx^{17}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/29*b^5*x^29 + 5/26*a*b^4*x^26 + 10/23*a^2*b^3*x^23 + 1/2*a^3*b^2*x^20 + 5
/17*a^4*b*x^17 + 1/14*a^5*x^14
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{13}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**13*((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 3.34, size = 105, normalized size = 0.41

$$\frac{1}{29}\,b^5x^{29}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{26}\,ab^4x^{26}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{23}\,a^2b^3x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^3b^2x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{17}\,a^4bx^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{14}\,a^5x^{14}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/29*b^5*x^29*sgn(b*x^3 + a) + 5/26*a*b^4*x^26*sgn(b*x^3 + a) + 10/23*a^2*b
^3*x^23*sgn(b*x^3 + a) + 1/2*a^3*b^2*x^20*sgn(b*x^3 + a) + 5/17*a^4*b*x^17*
sgn(b*x^3 + a) + 1/14*a^5*x^14*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{13}\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^13*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.51 $\int x^{12}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\,(a+bx^3)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\,(a+bx^3)}+\frac{5a^2b^3x^{22}\sqrt{a^2+}}{11\,(a-}$$

```
[Out] 1/13*a^5*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/16*a^4*b*x^16*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+10/19*a^3*b^2*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/11*a^2*b^
3*x^22*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5*a*b^4*x^25*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/28*b^5*x^28*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^{28}\sqrt{a^2+2abx^3+b^2x^6}}{28\,(a+bx^3)}+\frac{ab^4x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{5a^2b^3x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16\,(a+bx^3)}+\frac{10a^3b^2x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^4*b*x^16
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*(a + b*x^3)) + (10*a^3*b^2*x^19*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a^2*b^3*x^22*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (a*b^4*x^25*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(5*(a + b*x^3)) + (b^5*x^28*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(28*
(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^{12}(a^2+2abx^3+b^2x^6)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{12}(ab+b^2x^3)^5\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (a^5b^5x^{12}+5a^4b^6x^{15}+10a^3b^7x^{18}+10a^2b^8x^{21}+5a}{b^4(ab+b^2x^3)}$$
$$= \frac{a^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16(a+bx^3)}+\frac{10a^3b^2x^{19}\sqrt{}}{19}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^{13}\sqrt{(a+bx^3)^2}\,(117040a^5+475475a^4bx^3+800800a^3b^2x^6+691600a^2b^3x^9+304304ab^4x^{12}+54340b^5x^{15})}{1521520(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^13*Sqrt[(a + b*x^3)^2]*(117040*a^5 + 475475*a^4*b*x^3 + 800800*a^3*b^2*x
^6 + 691600*a^2*b^3*x^9 + 304304*a*b^4*x^12 + 54340*b^5*x^15))/(1521520*(a
+ b*x^3))
```

Maple [A]
time = 0.06, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^{13}(54340b^5x^{15}+304304b^4a\,x^{12}+691600a^2b^3x^9+800800b^2a^3x^6+475475a^4b\,x^3+117040a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{1521520(b\,x^3+a)^5}$
default	$\frac{x^{13}(54340b^5x^{15}+304304b^4a\,x^{12}+691600a^2b^3x^9+800800b^2a^3x^6+475475a^4b\,x^3+117040a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{1521520(b\,x^3+a)^5}$
risch	$\frac{a^5x^{13}\sqrt{(b\,x^3+a)^2}}{13b\,x^3+13a}+\frac{5a^4b\,x^{16}\sqrt{(b\,x^3+a)^2}}{16(b\,x^3+a)}+\frac{10a^3b^2x^{19}\sqrt{(b\,x^3+a)^2}}{19(b\,x^3+a)}+\frac{5a^2b^3x^{22}\sqrt{(b\,x^3+a)^2}}{11(b\,x^3+a)}+\frac{a\,b^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1521520*x^13*(54340*b^5*x^15+304304*a*b^4*x^12+691600*a^2*b^3*x^9+800800*
a^3*b^2*x^6+475475*a^4*b*x^3+117040*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.27, size = 57, normalized size = 0.22

$$\frac{1}{28}\,b^5x^{28}+\frac{1}{5}\,ab^4x^{25}+\frac{5}{11}\,a^2b^3x^{22}+\frac{10}{19}\,a^3b^2x^{19}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^2*x^19 + 5
/16*a^4*b*x^16 + 1/13*a^5*x^13
```

Fricas [A]
time = 0.37, size = 57, normalized size = 0.22

$$\frac{1}{28}\,b^{5}x^{28}+\frac{1}{5}\,ab^{4}x^{25}+\frac{5}{11}\,a^{2}b^{3}x^{22}+\frac{10}{19}\,a^{3}b^{2}x^{19}+\frac{5}{16}\,a^{4}bx^{16}+\frac{1}{13}\,a^{5}x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/28*b^5*x^28 + 1/5*a*b^4*x^25 + 5/11*a^2*b^3*x^22 + 10/19*a^3*b^2*x^19 + 5
/16*a^4*b*x^16 + 1/13*a^5*x^13
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{12}\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**12*((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 3.76, size = 105, normalized size = 0.41

$$\frac{1}{28}\,b^{5}x^{28}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{1}{5}\,ab^{4}x^{25}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{5}{11}\,a^{2}b^{3}x^{22}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{10}{19}\,a^{3}b^{2}x^{19}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{5}{16}\,a^{4}bx^{16}\mathrm{sgn}\left(bx^{3}+a\right)+\frac{1}{13}\,a^{5}x^{13}\mathrm{sgn}\left(bx^{3}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/28*b^5*x^28*sgn(b*x^3 + a) + 1/5*a*b^4*x^25*sgn(b*x^3 + a) + 5/11*a^2*b^3
*x^22*sgn(b*x^3 + a) + 10/19*a^3*b^2*x^19*sgn(b*x^3 + a) + 5/16*a^4*b*x^16*
sgn(b*x^3 + a) + 1/13*a^5*x^13*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{12}\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^12*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.52 $\int x^{11}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=160

$$-\frac{a^3(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^4}+\frac{a^2(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{7b^4}-\frac{a(a+bx^3)^7\sqrt{a^2+2abx^3+b^2x^6}}{8b^4}+$$

```
[Out] -1/18*a^3*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^4+1/7*a^2*(b*x^3+a)^6*((b*x^3+a
)^2)^(1/2)/b^4-1/8*a*(b*x^3+a)^7*((b*x^3+a)^2)^(1/2)/b^4+1/27*(b*x^3+a)^8*(
(b*x^3+a)^2)^(1/2)/b^4
```

Rubi [A]
time = 0.08, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {1369, 272, 45}

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^8}{27b^4}-\frac{a\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^7}{8b^4}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^6}{7b^4}-\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^5}{18b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/18*(a^3*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/b^4 + (a^2*(a + b
*x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*b^4) - (a*(a + b*x^3)^7*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(8*b^4) + ((a + b*x^3)^8*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(27*b^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{11}\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int x^3\left(ab+b^2x\right)^5\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int\left(-\frac{a^3(ab+b^2x)^5}{b^3}+\frac{3a^2(ab+b^2x)^6}{b^4}-\frac{3a(ab+b^2x)^7}{b^5}\right.\right.}{3b^4\left(ab+b^2x^3\right)}$$
$$=-\frac{a^3\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^4}+\frac{a^2\left(a+bx^3\right)^6\sqrt{a^2+2abx^3+b^2x^6}}{7b^4}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.52

$$\frac{x^{12}\sqrt{\left(a+bx^3\right)^2}\left(126a^5+504a^4bx^3+840a^3b^2x^6+720a^2b^3x^9+315ab^4x^{12}+56b^5x^{15}\right)}{1512\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
[Out] (x^12*Sqrt[(a + b*x^3)^2]*(126*a^5 + 504*a^4*b*x^3 + 840*a^3*b^2*x^6 + 720*
a^2*b^3*x^9 + 315*a*b^4*x^12 + 56*b^5*x^15))/(1512*(a + b*x^3))
```

Maple [A]
time = 0.06, size = 80, normalized size = 0.50

method	result
gosper	$\frac{x^{12}\left(56b^5x^{15}+315b^4a\,x^{12}+720a^2b^3x^9+840b^2a^3x^6+504a^4b\,x^3+126a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{1512\left(b\,x^3+a\right)^5}$
default	$\frac{x^{12}\left(56b^5x^{15}+315b^4a\,x^{12}+720a^2b^3x^9+840b^2a^3x^6+504a^4b\,x^3+126a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{1512\left(b\,x^3+a\right)^5}$
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,b^5x^{27}}{27b\,x^3+27a}+\frac{5\sqrt{\left(b\,x^3+a\right)^2}\,b^4a\,x^{24}}{24\left(b\,x^3+a\right)}+\frac{10\sqrt{\left(b\,x^3+a\right)^2}\,a^2b^3x^{21}}{21\left(b\,x^3+a\right)}+\frac{5\sqrt{\left(b\,x^3+a\right)^2}\,b^2a^3x^{18}}{9\left(b\,x^3+a\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/1512*x^12*(56*b^5*x^15+315*a*b^4*x^12+720*a^2*b^3*x^9+840*a^3*b^2*x^6+504
*a^4*b*x^3+126*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.27, size = 145, normalized size = 0.91

$$\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}x^6}{27\,b^2}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}a^3x^3}{18\,b^3}-\frac{11\,(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}ax^3}{216\,b^3}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}a^4}{18\,b^4}+\frac{83\,(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}a^2}{1512\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/27*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*x^6/b^2 - 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(5/2)*a^3*x^3/b^3 - 11/216*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a*x^3/b
^3 - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^4/b^4 + 83/1512*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(7/2)*a^2/b^4
```

Fricas [A]
time = 0.35, size = 57, normalized size = 0.36

$$\frac{1}{27}\,b^5x^{27}+\frac{5}{24}\,ab^4x^{24}+\frac{10}{21}\,a^2b^3x^{21}+\frac{5}{9}\,a^3b^2x^{18}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/27*b^5*x^27 + 5/24*a*b^4*x^24 + 10/21*a^2*b^3*x^21 + 5/9*a^3*b^2*x^18 + 1
/3*a^4*b*x^15 + 1/12*a^5*x^12
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**11*((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 4.39, size = 105, normalized size = 0.66

$$\frac{1}{27}\,b^5x^{27}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{24}\,ab^4x^{24}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{21}\,a^2b^3x^{21}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{9}\,a^3b^2x^{18}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{3}\,a^4bx^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{12}\,a^5x^{12}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/27*b^5*x^27*sgn(b*x^3 + a) + 5/24*a*b^4*x^24*sgn(b*x^3 + a) + 10/21*a^2*b
^3*x^21*sgn(b*x^3 + a) + 5/9*a^3*b^2*x^18*sgn(b*x^3 + a) + 1/3*a^4*b*x^15*s
gn(b*x^3 + a) + 1/12*a^5*x^12*sgn(b*x^3 + a)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^{11} \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.53 $\int x^{10}(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}+\frac{a^2b^3x^{20}\sqrt{a^2+2a}}{2\,(a+b}$$

```
[Out] 1/11*a^5*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/14*a^4*b*x^14*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+10/17*a^3*b^2*x^17*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a^2*b^3
*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/23*a*b^4*x^23*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/26*b^5*x^26*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^{26}\sqrt{a^2+2abx^3+b^2x^6}}{26\,(a+bx^3)}+\frac{5ab^4x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\,(a+bx^3)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\,(a+bx^3)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^4*b*x^14
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)) + (10*a^3*b^2*x^17*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a^2*b^3*x^20*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^23*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(23*(a + b*x^3)) + (b^5*x^26*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(26*
(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule steps

$$\int x^{10}\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^{10}\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^{10}+5a^4b^6x^{13}+10a^3b^7x^{16}+10a^2b^8x^{19}+5\right.}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{a^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{10a^3b^2x^{17}}{1}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^{11}\sqrt{\left(a+bx^3\right)^2}\left(71162a^5+279565a^4bx^3+460460a^3b^2x^6+391391a^2b^3x^9+170170ab^4x^{12}+30107b^5x^{15}\right)}{782782\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^11*Sqrt[(a + b*x^3)^2]*(71162*a^5 + 279565*a^4*b*x^3 + 460460*a^3*b^2*x^
6 + 391391*a^2*b^3*x^9 + 170170*a*b^4*x^12 + 30107*b^5*x^15))/(782782*(a +
b*x^3))
```

Maple [A]
time = 0.06, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^{11}\left(30107b^5x^{15}+170170b^4a\,x^{12}+391391a^2b^3x^9+460460b^2a^3x^6+279565a^4b\,x^3+71162a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{782782\left(b\,x^3+a\right)^5}$
default	$\frac{x^{11}\left(30107b^5x^{15}+170170b^4a\,x^{12}+391391a^2b^3x^9+460460b^2a^3x^6+279565a^4b\,x^3+71162a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{782782\left(b\,x^3+a\right)^5}$
risch	$\frac{a^5x^{11}\sqrt{\left(b\,x^3+a\right)^2}}{11b\,x^3+11a}+\frac{5a^4b\,x^{14}\sqrt{\left(b\,x^3+a\right)^2}}{14\left(b\,x^3+a\right)}+\frac{10a^3b^2x^{17}\sqrt{\left(b\,x^3+a\right)^2}}{17\left(b\,x^3+a\right)}+\frac{a^2b^3x^{20}\sqrt{\left(b\,x^3+a\right)^2}}{2b\,x^3+2a}+\frac{5a\,b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/782782*x^11*(30107*b^5*x^15+170170*a*b^4*x^12+391391*a^2*b^3*x^9+460460*a
^3*b^2*x^6+279565*a^4*b*x^3+71162*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.22

$$\frac{1}{26}\,b^5x^{26}+\frac{5}{23}\,ab^4x^{23}+\frac{1}{2}\,a^2b^3x^{20}+\frac{10}{17}\,a^3b^2x^{17}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^2*x^17 + 5
/14*a^4*b*x^14 + 1/11*a^5*x^11
```

Fricas [A]
time = 0.39, size = 57, normalized size = 0.22

$$\frac{1}{26}\,b^5x^{26}+\frac{5}{23}\,ab^4x^{23}+\frac{1}{2}\,a^2b^3x^{20}+\frac{10}{17}\,a^3b^2x^{17}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/26*b^5*x^26 + 5/23*a*b^4*x^23 + 1/2*a^2*b^3*x^20 + 10/17*a^3*b^2*x^17 + 5
/14*a^4*b*x^14 + 1/11*a^5*x^11
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{10}\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**10*((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 4.06, size = 105, normalized size = 0.41

$$\frac{1}{26}\,b^5x^{26}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{23}\,ab^4x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^2b^3x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{17}\,a^3b^2x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{14}\,a^4bx^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{11}\,a^5x^{11}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/26*b^5*x^26*sgn(b*x^3 + a) + 5/23*a*b^4*x^23*sgn(b*x^3 + a) + 1/2*a^2*b^3
*x^20*sgn(b*x^3 + a) + 10/17*a^3*b^2*x^17*sgn(b*x^3 + a) + 5/14*a^4*b*x^14*
sgn(b*x^3 + a) + 1/11*a^5*x^11*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{10}\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^10*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.54 $\int x^9(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\,(a+bx^3)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{10a^2b^3x^{19}\sqrt{a^2+\cdots}}{19\,(a+\cdots}$$

```
[Out] 1/10*a^5*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/13*a^4*b*x^13*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+5/8*a^3*b^2*x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/19*a^2*b^3
*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/22*a*b^4*x^22*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+1/25*b^5*x^25*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^{25}\sqrt{a^2+2abx^3+b^2x^6}}{25\,(a+bx^3)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\,(a+bx^3)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\,(a+bx^3)}+\frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\,(a+bx^3)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3)) + (5*a^4*b*x^13
*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^3*b^2*x^16*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^2*b^3*x^19*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6])/(19*(a + b*x^3)) + (5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(22*(a + b*x^3)) + (b^5*x^25*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2
5*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^9 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^9(ab+b^2x^3)^5 \, dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int (a^5b^5x^9+5a^4b^6x^{12}+10a^3b^7x^{15}+10a^2b^8x^{18}+5ab^9}{b^4(ab+b^2x^3)}$$
$$= \frac{a^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10(a+bx^3)} + \frac{5a^4bx^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)} + \frac{5a^3b^2x^{16}\sqrt{a^2}}{8(a}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^{10}\sqrt{(a+bx^3)^2}\,(54340a^5+209000a^4bx^3+339625a^3b^2x^6+286000a^2b^3x^9+123500ab^4x^{12}+21736b^5x^{15})}{543400(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^3)^2]*(54340*a^5 + 209000*a^4*b*x^3 + 339625*a^3*b^2*x^
6 + 286000*a^2*b^3*x^9 + 123500*a*b^4*x^12 + 21736*b^5*x^15))/(543400*(a +
b*x^3))
```

Maple [A]
time = 0.06, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^{10}(21736b^5x^{15}+123500b^4a\,x^{12}+286000a^2b^3x^9+339625b^2a^3x^6+209000a^4b\,x^3+54340a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{543400(b\,x^3+a)^5}$
default	$\frac{x^{10}(21736b^5x^{15}+123500b^4a\,x^{12}+286000a^2b^3x^9+339625b^2a^3x^6+209000a^4b\,x^3+54340a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{543400(b\,x^3+a)^5}$
risch	$\frac{a^5x^{10}\sqrt{(b\,x^3+a)^2}}{10b\,x^3+10a}+\frac{5a^4b\,x^{13}\sqrt{(b\,x^3+a)^2}}{13(b\,x^3+a)}+\frac{5a^3b^2x^{16}\sqrt{(b\,x^3+a)^2}}{8(b\,x^3+a)}+\frac{10a^2b^3x^{19}\sqrt{(b\,x^3+a)^2}}{19(b\,x^3+a)}+\frac{5a\,b^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/543400*x^10*(21736*b^5*x^15+123500*a*b^4*x^12+286000*a^2*b^3*x^9+339625*a
^3*b^2*x^6+209000*a^4*b*x^3+54340*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.22

$$\frac{1}{25}\,b^5x^{25}+\frac{5}{22}\,ab^4x^{22}+\frac{10}{19}\,a^2b^3x^{19}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 + 5
/13*a^4*b*x^13 + 1/10*a^5*x^10
```

Fricas [A]
time = 0.36, size = 57, normalized size = 0.22

$$\frac{1}{25}\,b^5x^{25}+\frac{5}{22}\,ab^4x^{22}+\frac{10}{19}\,a^2b^3x^{19}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25*b^5*x^25 + 5/22*a*b^4*x^22 + 10/19*a^2*b^3*x^19 + 5/8*a^3*b^2*x^16 + 5
/13*a^4*b*x^13 + 1/10*a^5*x^10
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^9\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**9*((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 4.86, size = 105, normalized size = 0.41

$$\frac{1}{25}\,b^5x^{25}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{22}\,ab^4x^{22}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{19}\,a^2b^3x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^3b^2x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{13}\,a^4bx^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{10}\,a^5x^{10}\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/25*b^5*x^25*sgn(b*x^3 + a) + 5/22*a*b^4*x^22*sgn(b*x^3 + a) + 10/19*a^2*b
^3*x^19*sgn(b*x^3 + a) + 5/8*a^3*b^2*x^16*sgn(b*x^3 + a) + 5/13*a^4*b*x^13*
sgn(b*x^3 + a) + 1/10*a^5*x^10*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^9\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^9*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.55 $\int x^8(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=119

$$\frac{a^2(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^3}-\frac{2a(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^3}+\frac{(a+bx^3)^7\sqrt{a^2+2abx^3+b^2x^6}}{24b^3}$$

```
[Out] 1/18*a^2*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^3-2/21*a*(b*x^3+a)^6*((b*x^3+a)^
2)^(1/2)/b^3+1/24*(b*x^3+a)^7*((b*x^3+a)^2)^(1/2)/b^3
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {1369, 272, 45}

$$\frac{\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^7}{24b^3}-\frac{2a\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^6}{21b^3}+\frac{a^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^5}{18b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(18*b^3) - (2*a*(a + b*
x^3)^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^3) + ((a + b*x^3)^7*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(24*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^8 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^8 (ab+b^2x^3)^5 dx}{b^4 (ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x^2 (ab+b^2x)^5 dx, x, x^3\right)}{3b^4 (ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(\frac{a^2(ab+b^2x)^5}{b^2} - \frac{2a(ab+b^2x)^6}{b^3} + \frac{(ab+b^2x)^7}{b^4}\right) dx, x\right.}{3b^4 (ab+b^2x^3)}$$
$$= \frac{a^2(a+bx^3)^5 \sqrt{a^2+2abx^3+b^2x^6}}{18b^3} - \frac{2a(a+bx^3)^6 \sqrt{a^2+2abx^3+b^2x^6}}{21b^3}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.70

$$\frac{x^9 \sqrt{(a+bx^3)^2} \left(56a^5+210a^4bx^3+336a^3b^2x^6+280a^2b^3x^9+120ab^4x^{12}+21b^5x^{15}\right)}{504 (a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^3)^2]*(56*a^5 + 210*a^4*b*x^3 + 336*a^3*b^2*x^6 + 280*a^
2*b^3*x^9 + 120*a*b^4*x^12 + 21*b^5*x^15))/(504*(a + b*x^3))
```

Maple [A]
time = 0.06, size = 80, normalized size = 0.67

method	result
gosper	$\frac{x^9 (21b^5x^{15}+120b^4a\,x^{12}+280a^2b^3x^9+336b^2a^3x^6+210a^4b\,x^3+56a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{504(b\,x^3+a)^5}$
default	$\frac{x^9 (21b^5x^{15}+120b^4a\,x^{12}+280a^2b^3x^9+336b^2a^3x^6+210a^4b\,x^3+56a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{504(b\,x^3+a)^5}$
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b^5x^{24}}{24b\,x^3+24a} + \frac{5\sqrt{(b\,x^3+a)^2}\,b^4a\,x^{21}}{21(b\,x^3+a)} + \frac{5\sqrt{(b\,x^3+a)^2}\,a^2b^3x^{18}}{9(b\,x^3+a)} + \frac{2\sqrt{(b\,x^3+a)^2}\,b^2a^3x^{15}}{3(b\,x^3+a)} + \frac{5}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/504*x^9*(21*b^5*x^15+120*a*b^4*x^12+280*a^2*b^3*x^9+336*a^3*b^2*x^6+210*a
^4*b*x^3+56*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.28, size = 114, normalized size = 0.96

$$\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}a^2x^3}{18\,b^2}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}x^3}{24\,b^2}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}a^3}{18\,b^3}-\frac{3\,(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}a}{56\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^2*x^3/b^2 + 1/24*(b^2*x^6 + 2*a*b*
x^3 + a^2)^(7/2)*x^3/b^2 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a^3/b^3 -
 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*a/b^3
```

Fricas [A]
time = 0.37, size = 57, normalized size = 0.48

$$\frac{1}{24}\,b^5x^{24}+\frac{5}{21}\,ab^4x^{21}+\frac{5}{9}\,a^2b^3x^{18}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*b^5*x^24 + 5/21*a*b^4*x^21 + 5/9*a^2*b^3*x^18 + 2/3*a^3*b^2*x^15 + 5/1
2*a^4*b*x^12 + 1/9*a^5*x^9
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**8*((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 4.78, size = 105, normalized size = 0.88

$$\frac{1}{24}\,b^5x^{24}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{21}\,ab^4x^{21}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{9}\,a^2b^3x^{18}\mathrm{sgn}\left(bx^3+a\right)+\frac{2}{3}\,a^3b^2x^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{12}\,a^4bx^{12}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{9}\,a^5x^9\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24*sgn(b*x^3 + a) + 5/21*a*b^4*x^21*sgn(b*x^3 + a) + 5/9*a^2*b^3
*x^18*sgn(b*x^3 + a) + 2/3*a^3*b^2*x^15*sgn(b*x^3 + a) + 5/12*a^4*b*x^12*sg
n(b*x^3 + a) + 1/9*a^5*x^9*sgn(b*x^3 + a)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^8 \left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.56 $\int x^7(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2a}}{17\left(a+b\right.}$$

```
[Out] 1/8*a^5*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/11*a^4*b*x^11*((b*x^3+a)^2)^(1/
2)/(b*x^3+a)+5/7*a^3*b^2*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/17*a^2*b^3*x
^17*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*a*b^4*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3
+a)+1/23*b^5*x^23*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1369, 276}

$$\frac{b^5x^{23}\sqrt{a^2+2abx^3+b^2x^6}}{23\left(a+bx^3\right)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\left(a+bx^3\right)}+\frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a^4*b*x^11*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^3*b^2*x^14*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (10*a^2*b^3*x^17*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(17*(a + b*x^3)) + (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(4*(a + b*x^3)) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(23*(a
+ b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^7\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\int x^7\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\int \left(a^5b^5x^7+5a^4b^6x^{10}+10a^3b^7x^{13}+10a^2b^8x^{16}+5a\right.}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{a^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^3b^2x^{14}\sqrt{a}}{7(}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^8\sqrt{\left(a+bx^3\right)^2}\left(30107a^5+109480a^4bx^3+172040a^3b^2x^6+141680a^2b^3x^9+60214ab^4x^{12}+10472b^5x^{15}\right)}{240856\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^3)^2]*(30107*a^5 + 109480*a^4*b*x^3 + 172040*a^3*b^2*x^6
 + 141680*a^2*b^3*x^9 + 60214*a*b^4*x^12 + 10472*b^5*x^15))/(240856*(a + b*
x^3))
```

Maple [A]
time = 0.06, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^{8}\left(10472 b^{5} x^{15}+60214 b^{4} a\,x^{12}+141680 a^{2} b^{3} x^{9}+172040 b^{2} a^{3} x^{6}+109480 a^{4} b\,x^{3}+30107 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{240856\left(b\,x^{3}+a\right)^{5}}$
default	$\frac{x^{8}\left(10472 b^{5} x^{15}+60214 b^{4} a\,x^{12}+141680 a^{2} b^{3} x^{9}+172040 b^{2} a^{3} x^{6}+109480 a^{4} b\,x^{3}+30107 a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{240856\left(b\,x^{3}+a\right)^{5}}$
risch	$\frac{a^{5} x^{8}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{8 b\,x^{3}+8 a}+\frac{5 a^{4} b\,x^{11}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{11\left(b\,x^{3}+a\right)}+\frac{5 a^{3} b^{2} x^{14}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{7\left(b\,x^{3}+a\right)}+\frac{10 a^{2} b^{3} x^{17}\sqrt{\left(b\,x^{3}+a\right)^{2}}}{17\left(b\,x^{3}+a\right)}+\frac{a\,b^{4}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/240856*x^8*(10472*b^5*x^15+60214*a*b^4*x^12+141680*a^2*b^3*x^9+172040*a^3
*b^2*x^6+109480*a^4*b*x^3+30107*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.22

$$\frac{1}{23}\,b^5x^{23}+\frac{1}{4}\,ab^4x^{20}+\frac{10}{17}\,a^2b^3x^{17}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5/
11*a^4*b*x^11 + 1/8*a^5*x^8
```

Fricas [A]
time = 0.35, size = 57, normalized size = 0.22

$$\frac{1}{23}\,b^5x^{23}+\frac{1}{4}\,ab^4x^{20}+\frac{10}{17}\,a^2b^3x^{17}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/23*b^5*x^23 + 1/4*a*b^4*x^20 + 10/17*a^2*b^3*x^17 + 5/7*a^3*b^2*x^14 + 5/
11*a^4*b*x^11 + 1/8*a^5*x^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^7\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**7*((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 4.44, size = 105, normalized size = 0.41

$$\frac{1}{23}\,b^5x^{23}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,ab^4x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{17}\,a^2b^3x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^3b^2x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}\,a^4bx^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{8}\,a^5x^8\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/23*b^5*x^23*sgn(b*x^3 + a) + 1/4*a*b^4*x^20*sgn(b*x^3 + a) + 10/17*a^2*b^
3*x^17*sgn(b*x^3 + a) + 5/7*a^3*b^2*x^14*sgn(b*x^3 + a) + 5/11*a^4*b*x^11*s
gn(b*x^3 + a) + 1/8*a^5*x^8*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^7\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^7*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.57 $\int x^6(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\,(a+bx^3)}+\frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2}}{8\,(a+b}$$

```
[Out] 1/7*a^5*x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a^4*b*x^10*((b*x^3+a)^2)^(1/2
)/(b*x^3+a)+10/13*a^3*b^2*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/8*a^2*b^3*x^
16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/19*a*b^4*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3
+a)+1/22*b^5*x^22*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^{22}\sqrt{a^2+2abx^3+b^2x^6}}{22\,(a+bx^3)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19\,(a+bx^3)}+\frac{5a^2b^3x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\,(a+bx^3)}+\frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^4*b*x^10*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^3*b^2*x^13*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)) + (5*a^2*b^3*x^16*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(8*(a + b*x^3)) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(19*(a + b*x^3)) + (b^5*x^22*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(22*(a
+ b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^6 \left(a^2+2abx^3+b^2x^6\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int x^6 (ab+b^2x^3)^5 \, dx}{b^4 \left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(a^5b^5x^6+5a^4b^6x^9+10a^3b^7x^{12}+10a^2b^8x^{15}+5ab^9\right.}{b^4 \left(ab+b^2x^3\right)}$$
$$= \frac{a^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)} + \frac{a^4bx^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)} + \frac{10a^3b^2x^{13}\sqrt{a^2}}{13\left(a\right.}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^7\sqrt{(a+bx^3)^2}\,(21736a^5+76076a^4bx^3+117040a^3b^2x^6+95095a^2b^3x^9+40040ab^4x^{12}+6916b^5x^{15})}{152152\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
[Out] (x^7*Sqrt[(a + b*x^3)^2]*(21736*a^5 + 76076*a^4*b*x^3 + 117040*a^3*b^2*x^6
+ 95095*a^2*b^3*x^9 + 40040*a*b^4*x^12 + 6916*b^5*x^15))/(152152*(a + b*x^3
))
```

Maple [A]
time = 0.06, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^7\left(6916b^5x^{15}+40040b^4a\,x^{12}+95095a^2b^3x^9+117040b^2a^3x^6+76076a^4b\,x^3+21736a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{152152\left(b\,x^3+a\right)^5}$
default	$\frac{x^7\left(6916b^5x^{15}+40040b^4a\,x^{12}+95095a^2b^3x^9+117040b^2a^3x^6+76076a^4b\,x^3+21736a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{152152\left(b\,x^3+a\right)^5}$
risch	$\frac{a^5x^7\sqrt{\left(b\,x^3+a\right)^2}}{7b\,x^3+7a}+\frac{a^4b\,x^{10}\sqrt{\left(b\,x^3+a\right)^2}}{2b\,x^3+2a}+\frac{10a^3b^2x^{13}\sqrt{\left(b\,x^3+a\right)^2}}{13\left(b\,x^3+a\right)}+\frac{5a^2b^3x^{16}\sqrt{\left(b\,x^3+a\right)^2}}{8\left(b\,x^3+a\right)}+\frac{5a\,b^4x}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/152152*x^7*(6916*b^5*x^15+40040*a*b^4*x^12+95095*a^2*b^3*x^9+117040*a^3*b
^2*x^6+76076*a^4*b*x^3+21736*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.22

$$\frac{1}{22}\,b^5x^{22}+\frac{5}{19}\,ab^4x^{19}+\frac{5}{8}\,a^2b^3x^{16}+\frac{10}{13}\,a^3b^2x^{13}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 + 1
/2*a^4*b*x^10 + 1/7*a^5*x^7
```

Fricas [A]
time = 0.38, size = 57, normalized size = 0.22

$$\frac{1}{22}\,b^5x^{22}+\frac{5}{19}\,ab^4x^{19}+\frac{5}{8}\,a^2b^3x^{16}+\frac{10}{13}\,a^3b^2x^{13}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/22*b^5*x^22 + 5/19*a*b^4*x^19 + 5/8*a^2*b^3*x^16 + 10/13*a^3*b^2*x^13 + 1
/2*a^4*b*x^10 + 1/7*a^5*x^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^6\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**6*((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 5.35, size = 105, normalized size = 0.41

$$\frac{1}{22}\,b^5x^{22}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{19}\,ab^4x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^2b^3x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{13}\,a^3b^2x^{13}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^4bx^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{7}\,a^5x^7\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22*sgn(b*x^3 + a) + 5/19*a*b^4*x^19*sgn(b*x^3 + a) + 5/8*a^2*b^3
*x^16*sgn(b*x^3 + a) + 10/13*a^3*b^2*x^13*sgn(b*x^3 + a) + 1/2*a^4*b*x^10*s
gn(b*x^3 + a) + 1/7*a^5*x^7*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^6\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^6*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.58 $\int x^5(a^2 + 2abx^3 + b^2x^6)^{5/2} \, dx$

Optimal. Leaf size=78

$$-\frac{a(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^2}+\frac{(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^2}$$

```
[Out] -1/18*a*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/b^2+1/21*(b*x^3+a)^6*((b*x^3+a)^2)^
(1/2)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$\frac{(a+bx^3)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^2}-\frac{a(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/18*(a*(a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/b^2 + ((a + b*x^3)^
6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(21*b^2)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^5\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int x\left(ab+b^2x\right)^5\,dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^5}{b}+\frac{\left(ab+b^2x\right)^6}{b^2}\right)dx,x,x^3\right)}{3b^4\left(ab+b^2x^3\right)}$$
$$=-\frac{a\left(a+bx^3\right)^5\sqrt{a^2+2abx^3+b^2x^6}}{18b^2}+\frac{\left(a+bx^3\right)^6\sqrt{a^2+2abx^3+b^2x^6}}{21b^2}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 1.06

$$\frac{x^6\sqrt{\left(a+bx^3\right)^2}\left(21a^5+70a^4bx^3+105a^3b^2x^6+84a^2b^3x^9+35ab^4x^{12}+6b^5x^{15}\right)}{126\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
[Out] (x^6*Sqrt[(a + b*x^3)^2]*(21*a^5 + 70*a^4*b*x^3 + 105*a^3*b^2*x^6 + 84*a^2*
b^3*x^9 + 35*a*b^4*x^12 + 6*b^5*x^15))/(126*(a + b*x^3))
```

Maple [A]
time = 0.05, size = 80, normalized size = 1.03

method	result
gosper	$\frac{x^6\left(6b^5x^{15}+35b^4a\,x^{12}+84a^2b^3x^9+105b^2a^3x^6+70a^4b\,x^3+21a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{126\left(b\,x^3+a\right)^5}$
default	$\frac{x^6\left(6b^5x^{15}+35b^4a\,x^{12}+84a^2b^3x^9+105b^2a^3x^6+70a^4b\,x^3+21a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{126\left(b\,x^3+a\right)^5}$
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,a^5x^6}{6b\,x^3+6a}+\frac{5\sqrt{\left(b\,x^3+a\right)^2}\,b\,a^4x^9}{9\left(b\,x^3+a\right)}+\frac{5\sqrt{\left(b\,x^3+a\right)^2}\,b^2a^3x^{12}}{6\left(b\,x^3+a\right)}+\frac{2\sqrt{\left(b\,x^3+a\right)^2}\,a^2b^3x^{15}}{3\left(b\,x^3+a\right)}+\frac{5\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/126*x^6*(6*b^5*x^15+35*a*b^4*x^12+84*a^2*b^3*x^9+105*a^3*b^2*x^6+70*a^4*b
*x^3+21*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.27, size = 83, normalized size = 1.06

$$-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}ax^3}{18\,b}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}a^2}{18\,b^2}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{21\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*a*x^3/b - 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*a^2/b^2 + 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/b^2
```

Fricas [A]
time = 0.34, size = 57, normalized size = 0.73

$$\frac{1}{21}\,b^5x^{21}+\frac{5}{18}\,ab^4x^{18}+\frac{2}{3}\,a^2b^3x^{15}+\frac{5}{6}\,a^3b^2x^{12}+\frac{5}{9}\,a^4bx^9+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*b^5*x^21 + 5/18*a*b^4*x^18 + 2/3*a^2*b^3*x^15 + 5/6*a^3*b^2*x^12 + 5/9
*a^4*b*x^9 + 1/6*a^5*x^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**5*((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 4.88, size = 67, normalized size = 0.86

$$\frac{1}{126}\,\left(6\,b^5x^{21}+35\,ab^4x^{18}+84\,a^2b^3x^{15}+105\,a^3b^2x^{12}+70\,a^4bx^9+21\,a^5x^6\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/126*(6*b^5*x^21 + 35*a*b^4*x^18 + 84*a^2*b^3*x^15 + 105*a^3*b^2*x^12 + 70
*a^4*b*x^9 + 21*a^5*x^6)*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.59 $\int x^4(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}+\frac{5a^2b^3x^{14}\sqrt{a^2+2a}}{7\,(a+bx}$$

```
[Out] 1/5*a^5*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/8*a^4*b*x^8*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+10/11*a^3*b^2*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/7*a^2*b^3*x^1
4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/17*a*b^4*x^17*((b*x^3+a)^2)^(1/2)/(b*x^3+
a)+1/20*b^5*x^20*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^{20}\sqrt{a^2+2abx^3+b^2x^6}}{20\,(a+bx^3)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17\,(a+bx^3)}+\frac{5a^2b^3x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{7\,(a+bx^3)}+\frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\,(a+bx^3)}+\frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x^3)) + (5*a^4*b*x^8*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3)) + (10*a^3*b^2*x^11*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3)) + (5*a^2*b^3*x^14*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(17*(a + b*x^3)) + (b^5*x^20*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(20*(a
 + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx=\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\int x^4\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{\sqrt{a^2+2abx^3+b^2x^6}\,\int\left(a^5b^5x^4+5a^4b^6x^7+10a^3b^7x^{10}+10a^2b^8x^{13}+5ab\right.}{b^4\left(ab+b^2x^3\right)}$$
$$=\frac{a^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5a^4bx^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{10a^3b^2x^{11}\sqrt{a}}{11\,(}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^5\sqrt{\left(a+bx^3\right)^2}\left(10472a^5+32725a^4bx^3+47600a^3b^2x^6+37400a^2b^3x^9+15400ab^4x^{12}+2618b^5x^{15}\right)}{52360\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)^2]*(10472*a^5 + 32725*a^4*b*x^3 + 47600*a^3*b^2*x^6 +
 37400*a^2*b^3*x^9 + 15400*a*b^4*x^12 + 2618*b^5*x^15))/(52360*(a + b*x^3))
```

Maple [A]
time = 0.06, size = 80, normalized size = 0.31

method	result
gosper	$\frac{x^5\left(2618b^5x^{15}+15400b^4a\,x^{12}+37400a^2b^3x^9+47600b^2a^3x^6+32725a^4b\,x^3+10472a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{52360\left(b\,x^3+a\right)^5}$
default	$\frac{x^5\left(2618b^5x^{15}+15400b^4a\,x^{12}+37400a^2b^3x^9+47600b^2a^3x^6+32725a^4b\,x^3+10472a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{52360\left(b\,x^3+a\right)^5}$
risch	$\frac{a^5x^5\sqrt{\left(b\,x^3+a\right)^2}}{5b\,x^3+5a}+\frac{5a^4b\,x^8\sqrt{\left(b\,x^3+a\right)^2}}{8\left(b\,x^3+a\right)}+\frac{10a^3b^2x^{11}\sqrt{\left(b\,x^3+a\right)^2}}{11\left(b\,x^3+a\right)}+\frac{5a^2b^3x^{14}\sqrt{\left(b\,x^3+a\right)^2}}{7\left(b\,x^3+a\right)}+\frac{5a\,b^4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/52360*x^5*(2618*b^5*x^15+15400*a*b^4*x^12+37400*a^2*b^3*x^9+47600*a^3*b^2
*x^6+32725*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.22

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5
/8*a^4*b*x^8 + 1/5*a^5*x^5
```

Fricas [A]
time = 0.37, size = 57, normalized size = 0.22

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{17}\,ab^4x^{17}+\frac{5}{7}\,a^2b^3x^{14}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{8}\,a^4bx^8+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/20*b^5*x^20 + 5/17*a*b^4*x^17 + 5/7*a^2*b^3*x^14 + 10/11*a^3*b^2*x^11 + 5
/8*a^4*b*x^8 + 1/5*a^5*x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**4*((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 4.55, size = 105, normalized size = 0.41

$$\frac{1}{20}\,b^5x^{20}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{17}\,ab^4x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^2b^3x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{11}\,a^3b^2x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,a^4bx^8\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{5}\,a^5x^5\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20*sgn(b*x^3 + a) + 5/17*a*b^4*x^17*sgn(b*x^3 + a) + 5/7*a^2*b^3
*x^14*sgn(b*x^3 + a) + 10/11*a^3*b^2*x^11*sgn(b*x^3 + a) + 5/8*a^4*b*x^8*sg
n(b*x^3 + a) + 1/5*a^5*x^5*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.60 $\int x^3(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=252

$$\frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{10a^2b^3x^{13}\sqrt{a^2+2a}}{13(a+bx}$$

```
[Out] 1/4*a^5*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/7*a^4*b*x^7*((b*x^3+a)^2)^(1/2)
/(b*x^3+a)+a^3*b^2*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/13*a^2*b^3*x^13*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/16*a*b^4*x^16*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1
/19*b^5*x^19*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1369, 276}

$$\frac{b^5x^{19}\sqrt{a^2+2abx^3+b^2x^6}}{19(a+bx^3)}+\frac{5ab^4x^{16}\sqrt{a^2+2abx^3+b^2x^6}}{16(a+bx^3)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13(a+bx^3)}+\frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{a^3b^2x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3)) + (5*a^4*b*x^7*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3)) + (a^3*b^2*x^10*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (10*a^2*b^3*x^13*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])/(13*(a + b*x^3)) + (5*a*b^4*x^16*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(16*(a + b*x^3)) + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*(a + b*x
^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int x^3\left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int\left(a^5b^5x^3+5a^4b^6x^6+10a^3b^7x^9+10a^2b^8x^{12}+5ab^9x\right.}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{a^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}+\frac{5a^4bx^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}+\frac{a^3b^2x^{10}\sqrt{a^2+}}{a+}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^4\sqrt{\left(a+bx^3\right)^2}\left(6916a^5+19760a^4bx^3+27664a^3b^2x^6+21280a^2b^3x^9+8645ab^4x^{12}+1456b^5x^{15}\right)}{27664\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)^2]*(6916*a^5 + 19760*a^4*b*x^3 + 27664*a^3*b^2*x^6 +
21280*a^2*b^3*x^9 + 8645*a*b^4*x^12 + 1456*b^5*x^15))/(27664*(a + b*x^3))
```

Maple [A]
time = 0.06, size = 80, normalized size = 0.32

method	result
gosper	$\frac{x^4\left(1456b^5x^{15}+8645b^4a\,x^{12}+21280a^2b^3x^9+27664b^2a^3x^6+19760a^4b\,x^3+6916a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{27664\left(b\,x^3+a\right)^5}$
default	$\frac{x^4\left(1456b^5x^{15}+8645b^4a\,x^{12}+21280a^2b^3x^9+27664b^2a^3x^6+19760a^4b\,x^3+6916a^5\right)\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}{27664\left(b\,x^3+a\right)^5}$
risch	$\frac{a^5x^4\sqrt{\left(b\,x^3+a\right)^2}}{4b\,x^3+4a}+\frac{5a^4b\,x^7\sqrt{\left(b\,x^3+a\right)^2}}{7\left(b\,x^3+a\right)}+\frac{a^3b^2x^{10}\sqrt{\left(b\,x^3+a\right)^2}}{b\,x^3+a}+\frac{10a^2b^3x^{13}\sqrt{\left(b\,x^3+a\right)^2}}{13\left(b\,x^3+a\right)}+\frac{5a\,b^4x^{1}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/27664*x^4*(1456*b^5*x^15+8645*a*b^4*x^12+21280*a^2*b^3*x^9+27664*a^3*b^2*
x^6+19760*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.29, size = 56, normalized size = 0.22

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*a
^4*b*x^7 + 1/4*a^5*x^4
```

Fricas [**A**]
time = 0.38, size = 56, normalized size = 0.22

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{16}\,ab^4x^{16}+\frac{10}{13}\,a^2b^3x^{13}+a^3b^2x^{10}+\frac{5}{7}\,a^4bx^7+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/19*b^5*x^19 + 5/16*a*b^4*x^16 + 10/13*a^2*b^3*x^13 + a^3*b^2*x^10 + 5/7*a
^4*b*x^7 + 1/4*a^5*x^4
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**3*((a + b*x**3)**2)**(5/2), x)
```

Giac [**A**]
time = 8.08, size = 104, normalized size = 0.41

$$\frac{1}{19}\,b^5x^{19}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{16}\,ab^4x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{13}\,a^2b^3x^{13}\mathrm{sgn}\left(bx^3+a\right)+a^3b^2x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,a^4bx^7\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{4}\,a^5x^4\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19*sgn(b*x^3 + a) + 5/16*a*b^4*x^16*sgn(b*x^3 + a) + 10/13*a^2*b
^3*x^13*sgn(b*x^3 + a) + a^3*b^2*x^10*sgn(b*x^3 + a) + 5/7*a^4*b*x^7*sgn(b*
x^3 + a) + 1/4*a^5*x^4*sgn(b*x^3 + a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.61 $\int x^2(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^3)\,(a^2+2abx^3+b^2x^6)^{5/2}}{18b}$$

```
[Out] 1/18*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1366, 623}

$$\frac{(a+bx^3)\,(a^2+2abx^3+b^2x^6)^{5/2}}{18b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(18*b)
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx &= \frac{1}{3}\text{Subst}\left(\int\left(a^2+2abx+b^2x^2\right)^{5/2}\,dx,x,x^3\right)\\ &= \frac{(a+bx^3)\,(a^2+2abx^3+b^2x^6)^{5/2}}{18b}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.

time = 0.01, size = 82, normalized size = 2.28

$$\frac{x^3\sqrt{(a+bx^3)^2}\,(6a^5+15a^4bx^3+20a^3b^2x^6+15a^2b^3x^9+6ab^4x^{12}+b^5x^{15})}{18\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^3)^2]*(6*a^5 + 15*a^4*b*x^3 + 20*a^3*b^2*x^6 + 15*a^2*b^
3*x^9 + 6*a*b^4*x^12 + b^5*x^15))/(18*(a + b*x^3))
```

Maple [A]
time = 0.05, size = 24, normalized size = 0.67

method	result	size
default	$\frac{(b\,x^3+a)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{18b}$	24
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,(b\,x^3+a)^5}{18b}$	26
gosper	$\frac{x^3\,(b^5x^{15}+6b^4a\,x^{12}+15a^2b^3x^9+20b^2a^3x^6+15a^4b\,x^3+6a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{18(b\,x^3+a)^5}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*(b*x^3+a)*((b*x^3+a)^2)^(5/2)/b
```

Maxima [A]
time = 0.28, size = 52, normalized size = 1.44

$$\frac{1}{18}\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}x^3+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}a}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*x^3 + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(5/2)*a/b
```

Fricas [A]
time = 0.36, size = 57, normalized size = 1.58

$$\frac{1}{18}\,b^5x^{18}+\frac{1}{3}\,ab^4x^{15}+\frac{5}{6}\,a^2b^3x^{12}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{6}\,a^4bx^6+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/18*b^5*x^18 + 1/3*a*b^4*x^15 + 5/6*a^2*b^3*x^12 + 10/9*a^3*b^2*x^9 + 5/6*
a^4*b*x^6 + 1/3*a^5*x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**2*((a + b*x**3)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(32) = 64.
time = 4.19, size = 66, normalized size = 1.83

$$\frac{1}{18}\left(3\left(bx^6+2\,ax^3\right)a^4+3\left(bx^6+2\,ax^3\right)^2a^2b+\left(bx^6+2\,ax^3\right)^3b^2\right)\operatorname{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/18*(3*(b*x^6 + 2*a*x^3)*a^4 + 3*(b*x^6 + 2*a*x^3)^2*a^2*b + (b*x^6 + 2*a*
x^3)^3*b^2)*sgn(b*x^3 + a)
```

Mupad [B]
time = 1.24, size = 36, normalized size = 1.00

$$\frac{\left(b^2\,x^3+a\,b\right)\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{18\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] ((a*b + b^2*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2))/(18*b^2)
```

3.62 $\int x(a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=252

$$\frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2a\cdots}}{11(a+bx\cdots}$$

```
[Out] 1/2*a^5*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+a^4*b*x^5*((b*x^3+a)^2)^(1/2)/(b*
x^3+a)+5/4*a^3*b^2*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/11*a^2*b^3*x^11*((b
*x^3+a)^2)^(1/2)/(b*x^3+a)+5/14*a*b^4*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/
17*b^5*x^17*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.04, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1369, 276}

$$\frac{b^5x^{17}\sqrt{a^2+2abx^3+b^2x^6}}{17(a+bx^3)}+\frac{5ab^4x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14(a+bx^3)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11(a+bx^3)}+\frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^8\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (a^4*b*x^5*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^8*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(4*(a + b*x^3)) + (10*a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(11*(a + b*x^3)) + (5*a*b^4*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(14*(a + b*x^3)) + (b^5*x^17*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*(a + b*x^
3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule steps

$$\int x(a^2+2abx^3+b^2x^6)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\int x(ab+b^2x^3)^5\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\int (a^5b^5x+5a^4b^6x^4+10a^3b^7x^7+10a^2b^8x^{10}+5ab^9x^{13}}{b^4(ab+b^2x^3)}$$
$$= \frac{a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)} + \frac{a^4bx^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3} + \frac{5a^3b^2x^8\sqrt{a^2+2}}{4(a+b}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{x^2\sqrt{(a+bx^3)^2}\,(2618a^5+5236a^4bx^3+6545a^3b^2x^6+4760a^2b^3x^9+1870ab^4x^{12}+308b^5x^{15})}{5236(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^3)^2]*(2618*a^5 + 5236*a^4*b*x^3 + 6545*a^3*b^2*x^6 + 47
60*a^2*b^3*x^9 + 1870*a*b^4*x^12 + 308*b^5*x^15))/(5236*(a + b*x^3))
```

Maple [A]
time = 0.06, size = 80, normalized size = 0.32

method	result
gosper	$\frac{x^2(308b^5x^{15}+1870b^4a\,x^{12}+4760a^2b^3x^9+6545b^2a^3x^6+5236a^4b\,x^3+2618a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{5236(b\,x^3+a)^5}$
default	$\frac{x^2(308b^5x^{15}+1870b^4a\,x^{12}+4760a^2b^3x^9+6545b^2a^3x^6+5236a^4b\,x^3+2618a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{5236(b\,x^3+a)^5}$
risch	$\frac{a^5x^2\sqrt{(b\,x^3+a)^2}}{2b\,x^3+2a}+\frac{a^4b\,x^5\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}+\frac{5a^3b^2x^8\sqrt{(b\,x^3+a)^2}}{4(b\,x^3+a)}+\frac{10a^2b^3x^{11}\sqrt{(b\,x^3+a)^2}}{11(b\,x^3+a)}+\frac{5a\,b^4x^{14}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5236*x^2*(308*b^5*x^15+1870*a*b^4*x^12+4760*a^2*b^3*x^9+6545*a^3*b^2*x^6+
5236*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.22

$$\frac{1}{17}b^5x^{17}+\frac{5}{14}ab^4x^{14}+\frac{10}{11}a^2b^3x^{11}+\frac{5}{4}a^3b^2x^8+a^4bx^5+\frac{1}{2}a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a^
4*b*x^5 + 1/2*a^5*x^2
```

Fricas [A]
time = 0.35, size = 56, normalized size = 0.22

$$\frac{1}{17}\,b^5x^{17}+\frac{5}{14}\,ab^4x^{14}+\frac{10}{11}\,a^2b^3x^{11}+\frac{5}{4}\,a^3b^2x^8+a^4bx^5+\frac{1}{2}\,a^5x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/17*b^5*x^17 + 5/14*a*b^4*x^14 + 10/11*a^2*b^3*x^11 + 5/4*a^3*b^2*x^8 + a^
4*b*x^5 + 1/2*a^5*x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 4.44, size = 104, normalized size = 0.41

$$\frac{1}{17}\,b^5x^{17}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{14}\,ab^4x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{11}\,a^2b^3x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^3b^2x^8\mathrm{sgn}\left(bx^3+a\right)+a^4bx^5\mathrm{sgn}\left(bx^3+a\right)+\frac{1}{2}\,a^5x^2\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17*sgn(b*x^3 + a) + 5/14*a*b^4*x^14*sgn(b*x^3 + a) + 10/11*a^2*b
^3*x^11*sgn(b*x^3 + a) + 5/4*a^3*b^2*x^8*sgn(b*x^3 + a) + a^4*b*x^5*sgn(b*x
^3 + a) + 1/2*a^5*x^2*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.63 $\int (a^2+2abx^3+b^2x^6)^{5/2}\,dx$

Optimal. Leaf size=247

$$\frac{a^5x(a^2+2abx^3+b^2x^6)^{5/2}}{(a+bx^3)^5}+\frac{5a^4bx^4(a^2+2abx^3+b^2x^6)^{5/2}}{4\,(a+bx^3)^5}+\frac{10a^3b^2x^7(a^2+2abx^3+b^2x^6)^{5/2}}{7\,(a+bx^3)^5}+\frac{a^2b^3x^{10}(a^2+2a}{(a+b}$$

```
[Out] a^5*x*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+5/4*a^4*b*x^4*(b^2*x^6+2*a*
b*x^3+a^2)^(5/2)/(b*x^3+a)^5+10/7*a^3*b^2*x^7*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)
/(b*x^3+a)^5+a^2*b^3*x^10*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+5/13*a*
b^4*x^13*(b^2*x^6+2*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5+1/16*b^5*x^16*(b^2*x^6+2
*a*b*x^3+a^2)^(5/2)/(b*x^3+a)^5
```

Rubi [A]
time = 0.03, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1357, 200}

$$\frac{b^5x^{16}(a^2+2abx^3+b^2x^6)^{5/2}}{16\,(a+bx^3)^5}+\frac{5ab^4x^{13}(a^2+2abx^3+b^2x^6)^{5/2}}{13\,(a+bx^3)^5}+\frac{a^2b^3x^{10}(a^2+2abx^3+b^2x^6)^{5/2}}{(a+bx^3)^5}+\frac{a^5x(a^2+2abx^3+b^2x^6)^{5/2}}{(a+bx^3)^5}+\frac{5a^4bx^4(a^2+2abx^3+b^2x^6)^{5/2}}{4\,(a+bx^3)^5}+\frac{10a^3b^2x^7(a^2+2abx^3+b^2x^6)^{5/2}}{7\,(a+bx^3)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a^4*b*x^4*(a^2
 + 2*a*b*x^3 + b^2*x^6)^(5/2))/(4*(a + b*x^3)^5) + (10*a^3*b^2*x^7*(a^2 + 2
*a*b*x^3 + b^2*x^6)^(5/2))/(7*(a + b*x^3)^5) + (a^2*b^3*x^10*(a^2 + 2*a*b*x
^3 + b^2*x^6)^(5/2))/(a + b*x^3)^5 + (5*a*b^4*x^13*(a^2 + 2*a*b*x^3 + b^2*x
^6)^(5/2))/(13*(a + b*x^3)^5) + (b^5*x^16*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)
)/(16*(a + b*x^3)^5)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx = \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}\int \left(2ab+2b^2x^3\right)^5\,dx}{\left(2ab+2b^2x^3\right)^5}$$
$$= \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}\int \left(32a^5b^5+160a^4b^6x^3+320a^3b^7x^6+320a^2b^8x^9+1\right.}{\left(2ab+2b^2x^3\right)^5}$$
$$= \frac{a^5x(a^2+2abx^3+b^2x^6)^{5/2}}{(a+bx^3)^5} + \frac{5a^4bx^4(a^2+2abx^3+b^2x^6)^{5/2}}{4\,(a+bx^3)^5} + \frac{10a^3b^2x^7(a^2}{7\,(a}$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 0.33

$$\frac{x\sqrt{(a+bx^3)^2}\,(1456a^5+1820a^4bx^3+2080a^3b^2x^6+1456a^2b^3x^9+560ab^4x^{12}+91b^5x^{15})}{1456\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^3)^2]*(1456*a^5 + 1820*a^4*b*x^3 + 2080*a^3*b^2*x^6 + 1456
*a^2*b^3*x^9 + 560*a*b^4*x^12 + 91*b^5*x^15))/(1456*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 78, normalized size = 0.32

method	result
gosper	$\frac{x(91b^5x^{15}+560b^4a\,x^{12}+1456a^2b^3x^9+2080b^2a^3x^6+1820a^4b\,x^3+1456a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{1456(b\,x^3+a)^5}$
default	$\frac{x(91b^5x^{15}+560b^4a\,x^{12}+1456a^2b^3x^9+2080b^2a^3x^6+1820a^4b\,x^3+1456a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{1456(b\,x^3+a)^5}$
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,a^5x}{b\,x^3+a}+\frac{5\sqrt{(b\,x^3+a)^2}\,b\,a^4x^4}{4(b\,x^3+a)}+\frac{10\sqrt{(b\,x^3+a)^2}\,b^2a^3x^7}{7(b\,x^3+a)}+\frac{\sqrt{(b\,x^3+a)^2}\,a^2b^3x^{10}}{b\,x^3+a}+\frac{5\sqrt{(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1456*x*(91*b^5*x^15+560*a*b^4*x^12+1456*a^2*b^3*x^9+2080*a^3*b^2*x^6+1820
*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5
```

Maxima [A]
time = 0.31, size = 53, normalized size = 0.21

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{13}\,ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}\,a^3b^2x^7+\frac{5}{4}\,a^4bx^4+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4
*b*x^4 + a^5*x
```

Fricas [A]
time = 0.43, size = 53, normalized size = 0.21

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{13}\,ab^4x^{13}+a^2b^3x^{10}+\frac{10}{7}\,a^3b^2x^7+\frac{5}{4}\,a^4bx^4+a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*b^5*x^16 + 5/13*a*b^4*x^13 + a^2*b^3*x^10 + 10/7*a^3*b^2*x^7 + 5/4*a^4
*b*x^4 + a^5*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(5/2), x)
```

Giac [A]
time = 3.93, size = 101, normalized size = 0.41

$$\frac{1}{16}\,b^5x^{16}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{13}\,ab^4x^{13}\mathrm{sgn}\left(bx^3+a\right)+a^2b^3x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{7}\,a^3b^2x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^4bx^4\mathrm{sgn}\left(bx^3+a\right)+a^5x\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16*sgn(b*x^3 + a) + 5/13*a*b^4*x^13*sgn(b*x^3 + a) + a^2*b^3*x^1
0*sgn(b*x^3 + a) + 10/7*a^3*b^2*x^7*sgn(b*x^3 + a) + 5/4*a^4*b*x^4*sgn(b*x^
3 + a) + a^5*x*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.64 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx$

Optimal. Leaf size=251

$$\frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}+\frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}+\frac{5ab^4x^{12}\sqrt{a^2+}}{12(a}$$

```
[Out] 5/3*a^4*b*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/3*a^3*b^2*x^6*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+10/9*a^2*b^3*x^9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/12*a*b^4*x^
12*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/15*b^5*x^15*((b*x^3+a)^2)^(1/2)/(b*x^3+a
)+a^5*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.05, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$\frac{b^5x^{15}\sqrt{a^2+2abx^3+b^2x^6}}{15(a+bx^3)}+\frac{5ab^4x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12(a+bx^3)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^3+b^2x^6}}{9(a+bx^3)}+\frac{a^5\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}+\frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
```

```
[Out] (5*a^4*b*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^3*b^2*
x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (10*a^2*b^3*x^9*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (5*a*b^4*x^12*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(12*(a + b*x^3)) + (b^5*x^15*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(15*(a + b*x^3)) + (a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x}\,dx}{b^4(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x}\,dx, x, x^3\right)}{3b^4(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \left(5a^4b^6+\frac{a^5b^5}{x}+10a^3b^7x+10a^2b^8x^2+5ab^9x^3\right.\right.}{3b^4(ab+b^2x^3)}$$

$$= \frac{5a^4bx^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}+\frac{5a^3b^2x^6\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}+\frac{10a^2b^3x^9\sqrt{a^2}}{9(a}$$

Mathematica [A]
time = 0.02, size = 82, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\,(bx^3(300a^4+300a^3bx^3+200a^2b^2x^6+75ab^3x^9+12b^4x^{12})+180a^5\log(x))}{180(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x,x]
[Out] (Sqrt[(a + b*x^3)^2]*(b*x^3*(300*a^4 + 300*a^3*b*x^3 + 200*a^2*b^2*x^6 + 75
*a*b^3*x^9 + 12*b^4*x^12) + 180*a^5*Log[x]))/(180*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 79, normalized size = 0.31

method	result	size
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{5}{2}}(12b^5x^{15}+75b^4a\,x^{12}+200a^2b^3x^9+300b^2a^3x^6+300a^4b\,x^3+180a^5\ln(x))}{180(b\,x^3+a)^5}$	79
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b(\frac{1}{15}b^4x^{15}+\frac{5}{12}a\,b^3x^{12}+\frac{10}{9}a^2b^2x^9+\frac{5}{3}a^3b\,x^6+\frac{5}{3}a^4x^3)}{b\,x^3+a}+\frac{a^5\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

[Out] 1/180*((b*x^3+a)^2)^(5/2)*(12*b^5*x^15+75*b^4*a*x^12+200*a^2*b^3*x^9+300*b^2*a^3*x^6+300*a^4*b*x^3+180*a^5*ln(x))/(b*x^3+a)^5

Maxima [A]
time = 0.29, size = 206, normalized size = 0.82

$$\frac{1}{6}\sqrt{b^2x^6+2abx^3+a^2}\,a^3bx^3+\frac{1}{3}(-1)^{2b^2x^3+2ab}a^5\log\left(2b^2x^3+2ab\right)-\frac{1}{3}(-1)^{2abx^3+2a^2}a^5\log\left(\frac{2abx}{|x|}+\frac{2a^2}{x^2|x|}\right)+\frac{1}{12}\left(b^2x^6+2abx^3+a^2\right)^{\frac{3}{2}}abx^3+\frac{1}{2}\sqrt{b^2x^6+2abx^3+a^2}\,a^4+\frac{7}{36}\left(b^2x^6+2abx^3+a^2\right)^{\frac{3}{2}}a^2+\frac{1}{15}\left(b^2x^6+2abx^3+a^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="maxima")

[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b*x^3 + 1/3*(-1)^(2*b^2*x^3 + 2*a*b)*a^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^5*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a*b*x^3 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^4 + 7/36*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2 + 1/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)

Fricas [A]
time = 0.38, size = 55, normalized size = 0.22

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{12}\,ab^4x^{12}+\frac{10}{9}\,a^2b^3x^9+\frac{5}{3}\,a^3b^2x^6+\frac{5}{3}\,a^4bx^3+a^5\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="fricas")

[Out] 1/15*b^5*x^15 + 5/12*a*b^4*x^12 + 10/9*a^2*b^3*x^9 + 5/3*a^3*b^2*x^6 + 5/3*a^4*b*x^3 + a^5*log(x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x, x)

Giac [A]
time = 3.35, size = 104, normalized size = 0.41

$$\frac{1}{15}\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{12}\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{9}\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{3}\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{3}\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+a^5\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15*sgn(b*x^3 + a) + 5/12*a*b^4*x^12*sgn(b*x^3 + a) + 10/9*a^2*b^
3*x^9*sgn(b*x^3 + a) + 5/3*a^3*b^2*x^6*sgn(b*x^3 + a) + 5/3*a^4*b*x^3*sgn(b
*x^3 + a) + a^5*log(abs(x))*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x, x)
```

3.65 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=251

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^8\sqrt{a^2+2abx}}{4\left(a+bx^3\right.}$$

```
[Out] -a^5*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+5/2*a^4*b*x^2*((b*x^3+a)^2)^(1/2)/(b*x
^3+a)+2*a^3*b^2*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/4*a^2*b^3*x^8*((b*x^3+a
)^2)^(1/2)/(b*x^3+a)+5/11*a*b^4*x^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/14*b^5
*x^14*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.04, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^{14}\sqrt{a^2+2abx^3+b^2x^6}}{14\left(a+bx^3\right)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5a^2b^3x^8\sqrt{a^2+2abx^3+b^2x^6}}{4\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{2a^3b^2x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3))) + (5*a^4*b*x^2*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (2*a^3*b^2*x^5*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(4*(a + b*x^3)) + (5*a*b^4*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(1
1*(a + b*x^3)) + (b^5*x^14*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*(a + b*x^3)
)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^2}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^2}+5a^4b^6x+10a^3b^7x^4+10a^2b^8x^7+5ab^9x^{10}+b^1\right.}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}+\frac{5a^4bx^2\sqrt{a^2+2abx^3+b^2x^6}}{2(a+bx^3)}+\frac{2a^3b^2x^5\sqrt{a^2+2a}}{a+bx^}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\,(-308a^5+770a^4bx^3+616a^3b^2x^6+385a^2b^3x^9+140ab^4x^{12}+22b^5x^{15})}{308x(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-308*a^5 + 770*a^4*b*x^3 + 616*a^3*b^2*x^6 + 385*a^2*
b^3*x^9 + 140*a*b^4*x^12 + 22*b^5*x^15))/(308*x*(a + b*x^3))
```

Maple [A]
time = 0.04, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{(-22b^5x^{15}-140b^4a\,x^{12}-385a^2b^3x^9-616b^2a^3x^6-770a^4b\,x^3+308a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{308x(b\,x^3+a)^5}$	80
default	$-\frac{(-22b^5x^{15}-140b^4a\,x^{12}-385a^2b^3x^9-616b^2a^3x^6-770a^4b\,x^3+308a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{308x(b\,x^3+a)^5}$	80
risch	$\frac{\sqrt{(b\,x^3+a)^2}\;b\left(\frac{1}{14}b^4x^{14}+\frac{5}{11}a\,b^3x^{11}+\frac{5}{4}a^2b^2x^8+2a^3b\,x^5+\frac{5}{2}a^4x^2\right)}{b\,x^3+a}-\frac{a^5\sqrt{(b\,x^3+a)^2}}{x(b\,x^3+a)}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/308*(-22*b^5*x^15-140*a*b^4*x^12-385*a^2*b^3*x^9-616*a^3*b^2*x^6-770*a^4
*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x/(b*x^3+a)^5
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.24

$$\frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 7
70*a^4*b*x^3 - 308*a^5)/x
```

Fricas [A]
time = 0.34, size = 59, normalized size = 0.24

$$\frac{22\,b^5x^{15}+140\,ab^4x^{12}+385\,a^2b^3x^9+616\,a^3b^2x^6+770\,a^4bx^3-308\,a^5}{308\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/308*(22*b^5*x^15 + 140*a*b^4*x^12 + 385*a^2*b^3*x^9 + 616*a^3*b^2*x^6 + 7
70*a^4*b*x^3 - 308*a^5)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**2, x)
```

Giac [A]
time = 4.15, size = 105, normalized size = 0.42

$$\frac{1}{14}\,b^5x^{14}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{11}\,ab^4x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,a^2b^3x^8\mathrm{sgn}\left(bx^3+a\right)+2\,a^3b^2x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^4bx^2\mathrm{sgn}\left(bx^3+a\right)-\frac{a^5\mathrm{sgn}\left(bx^3+a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/14*b^5*x^14*sgn(b*x^3 + a) + 5/11*a*b^4*x^11*sgn(b*x^3 + a) + 5/4*a^2*b^3
*x^8*sgn(b*x^3 + a) + 2*a^3*b^2*x^5*sgn(b*x^3 + a) + 5/2*a^4*b*x^2*sgn(b*x^
3 + a) - a^5*sgn(b*x^3 + a)/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^2, x)
```

3.66 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=251

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^2b^3x^7\sqrt{a^2+2abx^3}}{7\left(a+bx^3\right)}$$

```
[Out] -1/2*a^5*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+5*a^4*b*x*((b*x^3+a)^2)^(1/2)/(b
*x^3+a)+5/2*a^3*b^2*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10/7*a^2*b^3*x^7*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)+1/2*a*b^4*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/13
*b^5*x^13*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^{13}\sqrt{a^2+2abx^3+b^2x^6}}{13\left(a+bx^3\right)}+\frac{ab^4x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}+\frac{10a^2b^3x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}+\frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

```
[Out] -1/2*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (5*a^4*b*x*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^3*b^2*x^4*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(2*(a + b*x^3)) + (10*a^2*b^3*x^7*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(7*(a + b*x^3)) + (a*b^4*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
2*(a + b*x^3)) + (b^5*x^13*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*(a + b*x^3)
)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^3}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(5a^4b^6+\frac{a^5b^5}{x^3}+10a^3b^7x^3+10a^2b^8x^6+5ab^9x^9+b^{10}\right.}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)}+\frac{5a^4bx\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^3b^2x^4\sqrt{a^2+2a}}{2(a+b}$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\,(-91a^5+910a^4bx^3+455a^3b^2x^6+260a^2b^3x^9+91ab^4x^{12}+14b^5x^{15})}{182x^2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-91*a^5 + 910*a^4*b*x^3 + 455*a^3*b^2*x^6 + 260*a^2*b
^3*x^9 + 91*a*b^4*x^12 + 14*b^5*x^15))/(182*x^2*(a + b*x^3))
```

Maple [A]
time = 0.04, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{\left(-14b^5x^{15}-91b^4a\,x^{12}-260a^2b^3x^9-455b^2a^3x^6-910a^4b\,x^3+91a^5\right)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{182x^2(b\,x^3+a)^5}$	80
default	$-\frac{\left(-14b^5x^{15}-91b^4a\,x^{12}-260a^2b^3x^9-455b^2a^3x^6-910a^4b\,x^3+91a^5\right)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{182x^2(b\,x^3+a)^5}$	80
risch	$\frac{\sqrt{(b\,x^3+a)^2}\;b\left(\frac{1}{13}b^4x^{13}+\frac{1}{2}a\,b^3x^{10}+\frac{10}{7}a^2b^2x^7+\frac{5}{2}a^3b\,x^4+5a^4x\right)}{b\,x^3+a}-\frac{a^5\sqrt{(b\,x^3+a)^2}}{2x^2(b\,x^3+a)}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/182*(-14*b^5*x^15-91*a*b^4*x^12-260*a^2*b^3*x^9-455*a^3*b^2*x^6-910*a^4*
b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^2/(b*x^3+a)^5
```

Maxima [A]
time = 0.30, size = 59, normalized size = 0.24

$$\frac{14\,b^5x^{15}+91\,ab^4x^{12}+260\,a^2b^3x^9+455\,a^3b^2x^6+910\,a^4bx^3-91\,a^5}{182\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 91
0*a^4*b*x^3 - 91*a^5)/x^2
```

Fricas [A]
time = 0.35, size = 59, normalized size = 0.24

$$\frac{14\,b^5x^{15} + 91\,ab^4x^{12} + 260\,a^2b^3x^9 + 455\,a^3b^2x^6 + 910\,a^4bx^3 - 91\,a^5}{182\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/182*(14*b^5*x^15 + 91*a*b^4*x^12 + 260*a^2*b^3*x^9 + 455*a^3*b^2*x^6 + 91
0*a^4*b*x^3 - 91*a^5)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**3, x)
```

Giac [A]
time = 3.51, size = 103, normalized size = 0.41

$$\frac{1}{13}\,b^5x^{13}\mathrm{sgn}\left(bx^3+a\right) + \frac{1}{2}\,ab^4x^{10}\mathrm{sgn}\left(bx^3+a\right) + \frac{10}{7}\,a^2b^3x^7\mathrm{sgn}\left(bx^3+a\right) + \frac{5}{2}\,a^3b^2x^4\mathrm{sgn}\left(bx^3+a\right) + 5\,a^4bx\mathrm{sgn}\left(bx^3+a\right) - \frac{a^5\mathrm{sgn}\left(bx^3+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13*sgn(b*x^3 + a) + 1/2*a*b^4*x^10*sgn(b*x^3 + a) + 10/7*a^2*b^3
*x^7*sgn(b*x^3 + a) + 5/2*a^3*b^2*x^4*sgn(b*x^3 + a) + 5*a^4*b*x*sgn(b*x^3
+ a) - 1/2*a^5*sgn(b*x^3 + a)/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^3, x)
```

3.67 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=252

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5ab^4x^9\sqrt{a^2+2a}}{9\left(a+bx\right.}$$

```
[Out] -1/3*a^5*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+10/3*a^3*b^2*x^3*((b*x^3+a)^2)^(
1/2)/(b*x^3+a)+5/3*a^2*b^3*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/9*a*b^4*x^9*
((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/12*b^5*x^12*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5
*a^4*b*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.05, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$\frac{b^5x^{12}\sqrt{a^2+2abx^3+b^2x^6}}{12\left(a+bx^3\right)}+\frac{5ab^4x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{5a^4b\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

```
[Out] -1/3*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^3*(a + b*x^3)) + (10*a^3*b^2*
x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a^2*b^3*x^6*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^9*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(9*(a + b*x^3)) + (b^5*x^12*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])/(12*(a + b*x^3)) + (5*a^4*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
 b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^4}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^4}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{(ab+b^2x)^5}{x^2}\,dx, x, x^3\right)}{3b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \left(10a^3b^7+\frac{a^5b^5}{x^2}+\frac{5a^4b^6}{x}+10a^2b^8x+5ab^9x^2+b\right.\right.}{3b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^3+b^2x^6}}{3(a+bx^3)}+\frac{5a^2b^3x^6\sqrt{a^2+}}{3(a+}$$

Mathematica [A]
time = 0.02, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^3)^2}\,(-12a^5+120a^3b^2x^6+60a^2b^3x^9+20ab^4x^{12}+3b^5x^{15}+180a^4bx^3\log(x))}{36x^3(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-12*a^5 + 120*a^3*b^2*x^6 + 60*a^2*b^3*x^9 + 20*a*b^4
*x^12 + 3*b^5*x^15 + 180*a^4*b*x^3*Log[x]))/(36*x^3*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 82, normalized size = 0.33

method	result	size
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{5}{2}}(3b^5x^{15}+20b^4a\,x^{12}+60a^2b^3x^9+120b^2a^3x^6+180b\,a^4\ln(x)x^3-12a^5)}{36x^3(b\,x^3+a)^5}$	82
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b^2(\frac{1}{12}b^3x^{12}+\frac{5}{9}a\,b^2x^9+\frac{5}{3}a^2b\,x^6+\frac{10}{3}a^3x^3)}{b\,x^3+a}-\frac{a^5\sqrt{(b\,x^3+a)^2}}{3x^3(b\,x^3+a)}+\frac{5a^4b\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+20*b^4*a*x^12+60*a^2*b^3*x^9+120*b^2*a
^3*x^6+180*b*a^4*ln(x)*x^3-12*a^5)/x^3/(b*x^3+a)^5
```

Maxima [A]
time = 0.32, size = 214, normalized size = 0.85

$$\frac{5}{6}\sqrt{b^2x^6+2\,abx^3+a^2}\,a^2b^2x^3+\frac{5}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^4b\log\left(2\,b^2x^3+2\,ab\right)-\frac{5}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^4b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{5}{12}\,\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}b^2x^3+\frac{5}{2}\sqrt{b^2x^6+2\,abx^3+a^2}\,a^3b+\frac{35}{36}\,\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{3}{2}}ab-\frac{\left(b^2x^6+2\,abx^3+a^2\right)^{\frac{5}{2}}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2*x^3 + 5/3*(-1)^(2*b^2*x^3 + 2*a
*b)*a^4*b*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^4*b*log(2
*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2
)*b^2*x^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3*b + 35/36*(b^2*x^6 + 2*
a*b*x^3 + a^2)^(3/2)*a*b - 1/3*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)/x^3
```

Fricas [A]
time = 0.38, size = 61, normalized size = 0.24

$$\frac{3\,b^5x^{15}+20\,ab^4x^{12}+60\,a^2b^3x^9+120\,a^3b^2x^6+180\,a^4bx^3\log\left(x\right)-12\,a^5}{36\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/36*(3*b^5*x^15 + 20*a*b^4*x^12 + 60*a^2*b^3*x^9 + 120*a^3*b^2*x^6 + 180*a
^4*b*x^3*log(x) - 12*a^5)/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**4, x)
```

Giac [A]
time = 3.80, size = 124, normalized size = 0.49

$$\frac{1}{12}\,b^5x^{12}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{9}\,ab^4x^9\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{3}\,a^2b^3x^6\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{3}\,a^3b^2x^3\mathrm{sgn}\left(bx^3+a\right)+5\,a^4b\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{5\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+a^5\mathrm{sgn}\left(bx^3+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*b^5*x^12*sgn(b*x^3 + a) + 5/9*a*b^4*x^9*sgn(b*x^3 + a) + 5/3*a^2*b^3*x
^6*sgn(b*x^3 + a) + 10/3*a^3*b^2*x^3*sgn(b*x^3 + a) + 5*a^4*b*log(abs(x))*s
gn(b*x^3 + a) - 1/3*(5*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^4, x)
```

3.68 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^5}\,dx$

Optimal. Leaf size=249

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^3}}{a+bx^3}$$

```
[Out] -1/4*a^5*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-5*a^4*b*((b*x^3+a)^2)^(1/2)/x/(b
*x^3+a)+5*a^3*b^2*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+2*a^2*b^3*x^5*((b*x^3+a
)^2)^(1/2)/(b*x^3+a)+5/8*a*b^4*x^8*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/11*b^5*x
^11*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^{11}\sqrt{a^2+2abx^3+b^2x^6}}{11\left(a+bx^3\right)}+\frac{5ab^4x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\left(a+bx^3\right)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] -1/4*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^4*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^3*b^2*x^2*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (2*a^2*b^3*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^
2*x^6])/(a + b*x^3) + (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a +
 b*x^3)) + (b^5*x^11*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^5}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^5}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^2}+10a^3b^7x+10a^2b^8x^4+5ab^9x^7+b^{10}x^{10}\right.}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x(a+bx^3)}+\frac{5a^3b^2x^2\sqrt{a^2+2abx}}{a+bx^3}$$

Mathematica [A]
time = 0.02, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\,(-22a^5-440a^4bx^3+440a^3b^2x^6+176a^2b^3x^9+55ab^4x^{12}+8b^5x^{15})}{88x^4(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-22*a^5 - 440*a^4*b*x^3 + 440*a^3*b^2*x^6 + 176*a^2*b
^3*x^9 + 55*a*b^4*x^12 + 8*b^5*x^15))/(88*x^4*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{(-8b^5x^{15}-55b^4a\,x^{12}-176a^2b^3x^9-440b^2a^3x^6+440a^4b\,x^3+22a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{88x^4(b\,x^3+a)^5}$	80
default	$-\frac{(-8b^5x^{15}-55b^4a\,x^{12}-176a^2b^3x^9-440b^2a^3x^6+440a^4b\,x^3+22a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{88x^4(b\,x^3+a)^5}$	80
risch	$\frac{\sqrt{(b\,x^3+a)^2}\;b^2(\frac{1}{11}b^3x^{11}+\frac{5}{8}a\,b^2x^8+2a^2b\,x^5+5a^3x^2)}{b\,x^3+a}+\frac{\sqrt{(b\,x^3+a)^2}\;(-5a^4b\,x^3-\frac{1}{4}a^5)}{(b\,x^3+a)x^4}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/88*(-8*b^5*x^15-55*a*b^4*x^12-176*a^2*b^3*x^9-440*a^3*b^2*x^6+440*a^4*b*
x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^4/(b*x^3+a)^5
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.24

$$\frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*
a^4*b*x^3 - 22*a^5)/x^4
```

Fricas [A]
time = 0.35, size = 59, normalized size = 0.24

$$\frac{8\,b^5x^{15}+55\,ab^4x^{12}+176\,a^2b^3x^9+440\,a^3b^2x^6-440\,a^4bx^3-22\,a^5}{88\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/88*(8*b^5*x^15 + 55*a*b^4*x^12 + 176*a^2*b^3*x^9 + 440*a^3*b^2*x^6 - 440*
a^4*b*x^3 - 22*a^5)/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**5, x)
```

Giac [A]
time = 3.04, size = 107, normalized size = 0.43

$$\frac{1}{11}\,b^5x^{11}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{8}\,ab^4x^8\mathrm{sgn}\left(bx^3+a\right)+2\,a^2b^3x^5\mathrm{sgn}\left(bx^3+a\right)+5\,a^3b^2x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{20\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+a^5\mathrm{sgn}\left(bx^3+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11*sgn(b*x^3 + a) + 5/8*a*b^4*x^8*sgn(b*x^3 + a) + 2*a^2*b^3*x^5
*sgn(b*x^3 + a) + 5*a^3*b^2*x^2*sgn(b*x^3 + a) - 1/4*(20*a^4*b*x^3*sgn(b*x^
3 + a) + a^5*sgn(b*x^3 + a))/x^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^5, x)
```

3.69 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=251

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\,(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\,(a+bx^3)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+}}{2\,(a+bx^3)}$$

```
[Out] -1/5*a^5*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5/2*a^4*b*((b*x^3+a)^2)^(1/2)/x^
2/(b*x^3+a)+10*a^3*b^2*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/2*a^2*b^3*x^4*((b*
x^3+a)^2)^(1/2)/(b*x^3+a)+5/7*a*b^4*x^7*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/10*
b^5*x^10*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.04, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^{10}\sqrt{a^2+2abx^3+b^2x^6}}{10\,(a+bx^3)}+\frac{5ab^4x^7\sqrt{a^2+2abx^3+b^2x^6}}{7\,(a+bx^3)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^3+b^2x^6}}{2\,(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5\,(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\,(a+bx^3)}+\frac{10a^3b^2x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] -1/5*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a + b*x^3)) + (10*a^3*b^2*x*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6])/(a + b*x^3) + (5*a^2*b^3*x^4*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6])/(2*(a + b*x^3)) + (5*a*b^4*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(7*(a + b*x^3)) + (b^5*x^10*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(10*(a + b*x^3
))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^6}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^6}\,dx}{b^4(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(10a^3b^7+\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^3}+10a^2b^8x^3+5ab^9x^6+b^{10}x^9\right.}{b^4(ab+b^2x^3)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^2(a+bx^3)} + \frac{10a^3b^2x\sqrt{a^2+2ab}}{a+bx^3}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\,(-14a^5-175a^4bx^3+700a^3b^2x^6+175a^2b^3x^9+50ab^4x^{12}+7b^5x^{15})}{70x^5(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-14*a^5 - 175*a^4*b*x^3 + 700*a^3*b^2*x^6 + 175*a^2*b
^3*x^9 + 50*a*b^4*x^12 + 7*b^5*x^15))/(70*x^5*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{(-7b^5x^{15}-50b^4a\,x^{12}-175a^2b^3x^9-700b^2a^3x^6+175a^4b\,x^3+14a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{70(b\,x^3+a)^5x^5}$	80
default	$-\frac{(-7b^5x^{15}-50b^4a\,x^{12}-175a^2b^3x^9-700b^2a^3x^6+175a^4b\,x^3+14a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{70(b\,x^3+a)^5x^5}$	80
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b^2(\frac{1}{10}b^3x^{10}+\frac{5}{7}a\,b^2x^7+\frac{5}{2}a^2b\,x^4+10a^3x)}{b\,x^3+a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-\frac{5}{2}a^4b\,x^3-\frac{1}{5}a^5)}{(b\,x^3+a)x^5}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/70*(-7*b^5*x^15-50*a*b^4*x^12-175*a^2*b^3*x^9-700*a^3*b^2*x^6+175*a^4*b*
x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^5
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*
a^4*b*x^3 - 14*a^5)/x^5
```

Fricas [A]
time = 0.36, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{15}+50\,ab^4x^{12}+175\,a^2b^3x^9+700\,a^3b^2x^6-175\,a^4bx^3-14\,a^5}{70\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/70*(7*b^5*x^15 + 50*a*b^4*x^12 + 175*a^2*b^3*x^9 + 700*a^3*b^2*x^6 - 175*
a^4*b*x^3 - 14*a^5)/x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**6, x)
```

Giac [A]
time = 3.94, size = 106, normalized size = 0.42

$$\frac{1}{10}\,b^5x^{10}\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{7}\,ab^4x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,a^2b^3x^4\mathrm{sgn}\left(bx^3+a\right)+10\,a^3b^2x\mathrm{sgn}\left(bx^3+a\right)-\frac{25\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a^5\mathrm{sgn}\left(bx^3+a\right)}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*sgn(b*x^3 + a) + 5/7*a*b^4*x^7*sgn(b*x^3 + a) + 5/2*a^2*b^3*x
^4*sgn(b*x^3 + a) + 10*a^3*b^2*x*sgn(b*x^3 + a) - 1/10*(25*a^4*b*x^3*sgn(b*
x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^6, x)
```

3.70 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=252

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^3}}{6\left(a+bx^3\right)}$$

```
[Out] -1/6*a^5*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-5/3*a^4*b*((b*x^3+a)^2)^(1/2)/x^
3/(b*x^3+a)+10/3*a^2*b^3*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5/6*a*b^4*x^6*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/9*b^5*x^9*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10*a^
3*b^2*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.05, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$\frac{b^5x^9\sqrt{a^2+2abx^3+b^2x^6}}{9\left(a+bx^3\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{10a^3b^2\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
```

```
[Out] -1/6*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^6*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (10*a^2*b^3*x^3*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(3*(a + b*x^3)) + (5*a*b^4*x^6*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(6*(a + b*x^3)) + (b^5*x^9*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(9*(a + b*x^3)) + (10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^7}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^7}\,dx}{b^4(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{(ab+b^2x)^5}{x^3}\,dx, x, x^3\right)}{3b^4(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \left(10a^2b^8+\frac{a^5b^5}{x^3}+\frac{5a^4b^6}{x^2}+\frac{10a^3b^7}{x}+5ab^9x+b^{10}x\right.\right.}{3b^4(ab+b^2x^3)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{6x^6(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)} + \frac{10a^2b^3x^3\sqrt{a^2+2ab}}{3(a+bx^3}$$

Mathematica [A]
time = 0.02, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^3)^2}\,(-3a^5-30a^4bx^3+60a^2b^3x^9+15ab^4x^{12}+2b^5x^{15}+180a^3b^2x^6\log(x))}{18x^6(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^7,x]
[Out] (Sqrt[(a + b*x^3)^2]*(-3*a^5 - 30*a^4*b*x^3 + 60*a^2*b^3*x^9 + 15*a*b^4*x^1
2 + 2*b^5*x^15 + 180*a^3*b^2*x^6*Log[x]))/(18*x^6*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 82, normalized size = 0.33

method	result	siz
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{5}{2}}(2b^5x^{15}+15b^4a\,x^{12}+60a^2b^3x^9+180b^2a^3\ln(x)x^6-30a^4b\,x^3-3a^5)}{18(b\,x^3+a)^5x^6}$	82
risch	$\frac{\sqrt{(b\,x^3+a)^2}\;b^3(\frac{1}{9}b^2x^9+\frac{5}{6}ab\,x^6+\frac{10}{3}a^2x^3)}{b\,x^3+a}+\frac{\sqrt{(b\,x^3+a)^2}\;(-\frac{5}{3}a^4b\,x^3-\frac{1}{6}a^5)}{(b\,x^3+a)x^6}+\frac{10a^3b^2\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

[Out] 1/18*((b*x^3+a)^2)^(5/2)*(2*b^5*x^15+15*b^4*a*x^12+60*a^2*b^3*x^9+180*b^2*a^3*ln(x)*x^6-30*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^6

Maxima [A]
time = 0.29, size = 282, normalized size = 1.12

$$\frac{5}{3}\sqrt{b^2x^6+2\,abx^3+a^2}\,ab^3x^3+\frac{10}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^3b^2\log\left(2\,b^2x^3+2\,ab\right)-\frac{10}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^3b^2\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{5\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3x^3}{6\,a}+5\sqrt{b^2x^6+2\,abx^3+a^2}\,a^2b^2+\frac{35}{18}\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^2+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^2}{6\,a^2}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b}{2\,ax^3}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="maxima")

[Out] 5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a*b)*a^3*b^2*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^3*b^2*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^3*x^3/a + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*b^2 + 35/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2 + 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/a^2 - 1/2*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b/(a*x^3) - 1/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^6)

Fricas [A]
time = 0.40, size = 61, normalized size = 0.24

$$\frac{2\,b^5x^{15}+15\,ab^4x^{12}+60\,a^2b^3x^9+180\,a^3b^2x^6\log\left(x\right)-30\,a^4bx^3-3\,a^5}{18\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="fricas")

[Out] 1/18*(2*b^5*x^15 + 15*a*b^4*x^12 + 60*a^2*b^3*x^9 + 180*a^3*b^2*x^6*log(x) - 30*a^4*b*x^3 - 3*a^5)/x^6

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**7,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**7, x)

Giac [A]
time = 3.96, size = 126, normalized size = 0.50

$$\frac{1}{9}\,b^5x^9\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{6}\,ab^4x^6\mathrm{sgn}\left(bx^3+a\right)+\frac{10}{3}\,a^2b^3x^3\mathrm{sgn}\left(bx^3+a\right)+10\,a^3b^2\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{30\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+10\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+a^5\mathrm{sgn}\left(bx^3+a\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9*sgn(b*x^3 + a) + 5/6*a*b^4*x^6*sgn(b*x^3 + a) + 10/3*a^2*b^3*x^
3*sgn(b*x^3 + a) + 10*a^3*b^2*log(abs(x))*sgn(b*x^3 + a) - 1/6*(30*a^3*b^2*
x^6*sgn(b*x^3 + a) + 10*a^4*b*x^3*sgn(b*x^3 + a) + a^5*sgn(b*x^3 + a))/x^6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^7, x)
```

3.71 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx$

Optimal. Leaf size=248

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+}}{a+bx^3}$$

```
[Out] -1/7*a^5*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/4*a^4*b*((b*x^3+a)^2)^(1/2)/x^
4/(b*x^3+a)-10*a^3*b^2*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+5*a^2*b^3*x^2*((b*x^
3+a)^2)^(1/2)/(b*x^3+a)+a*b^4*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/8*b^5*x^8
*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.04, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^8\sqrt{a^2+2abx^3+b^2x^6}}{8\,(a+bx^3)}+\frac{ab^4x^5\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\,(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\,(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
```

```
[Out] -1/7*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^7*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a^2*b^3*x^2*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])/(a + b*x^3) + (a*b^4*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a +
 b*x^3) + (b^5*x^8*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^8}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^8}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^8}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^2}+10a^2b^8x+5ab^9x^4+b^{10}x^7\right) dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3}}{x(a+bx^3)}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^3)^2}\,(-8a^5-70a^4bx^3-560a^3b^2x^6+280a^2b^3x^9+56ab^4x^{12}+7b^5x^{15})}{56x^7(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^8,x]
[Out] (Sqrt[(a + b*x^3)^2]*(-8*a^5 - 70*a^4*b*x^3 - 560*a^3*b^2*x^6 + 280*a^2*b^3
*x^9 + 56*a*b^4*x^12 + 7*b^5*x^15))/(56*x^7*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{(-7b^5x^{15}-56b^4a\,x^{12}-280a^2b^3x^9+560b^2a^3x^6+70a^4b\,x^3+8a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{56x^7(b\,x^3+a)^5}$	80
default	$-\frac{(-7b^5x^{15}-56b^4a\,x^{12}-280a^2b^3x^9+560b^2a^3x^6+70a^4b\,x^3+8a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{56x^7(b\,x^3+a)^5}$	80
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b^3(\frac{1}{8}b^2x^8+ab\,x^5+5a^2x^2)}{b\,x^3+a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-10b^2a^3x^6-\frac{5}{4}a^4b\,x^3-\frac{1}{7}a^5)}{(b\,x^3+a)x^7}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
[Out] -1/56*(-7*b^5*x^15-56*a*b^4*x^12-280*a^2*b^3*x^9+560*a^3*b^2*x^6+70*a^4*b*x
^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^7/(b*x^3+a)^5
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a
^4*b*x^3 - 8*a^5)/x^7
```

Fricas [A]
time = 0.35, size = 59, normalized size = 0.24

$$\frac{7\,b^5x^{15}+56\,ab^4x^{12}+280\,a^2b^3x^9-560\,a^3b^2x^6-70\,a^4bx^3-8\,a^5}{56\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^5*x^15 + 56*a*b^4*x^12 + 280*a^2*b^3*x^9 - 560*a^3*b^2*x^6 - 70*a
^4*b*x^3 - 8*a^5)/x^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**8, x)
```

Giac [A]
time = 3.13, size = 107, normalized size = 0.43

$$\frac{1}{8}\,b^5x^8\mathrm{sgn}\left(bx^3+a\right)+ab^4x^5\mathrm{sgn}\left(bx^3+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{280\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+35\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+4\,a^5\mathrm{sgn}\left(bx^3+a\right)}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8*sgn(b*x^3 + a) + a*b^4*x^5*sgn(b*x^3 + a) + 5*a^2*b^3*x^2*sgn(b
*x^3 + a) - 1/28*(280*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(b*x^3 +
 a) + 4*a^5*sgn(b*x^3 + a))/x^7
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^8, x)
```

3.72 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9} \, dx$

Optimal. Leaf size=247

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)}+\frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}$$

```
[Out] -1/8*a^5*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-a^4*b*((b*x^3+a)^2)^(1/2)/x^5/(b
*x^3+a)-5*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+10*a^2*b^3*x*((b*x^3+a)
^2)^(1/2)/(b*x^3+a)+5/4*a*b^4*x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/7*b^5*x^7
*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^7\sqrt{a^2+2abx^3+b^2x^6}}{7(a+bx^3)}+\frac{5ab^4x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{10a^2b^3x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

```
[Out] -1/8*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^8*(a + b*x^3)) - (a^4*b*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(x^2*(a + b*x^3)) + (10*a^2*b^3*x*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(a + b*x^3) + (5*a*b^4*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a
+ b*x^3)) + (b^5*x^7*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^9}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^9}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(10a^2b^8+\frac{a^5b^5}{x^9}+\frac{5a^4b^6}{x^6}+\frac{10a^3b^7}{x^3}+5ab^9x^3+b^{10}x^6\right)\,dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3+\ldots}}{x^2(a+bx^3)}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.34

$$\frac{\sqrt{(a+bx^3)^2}\,(-7a^5-56a^4bx^3-280a^3b^2x^6+560a^2b^3x^9+70ab^4x^{12}+8b^5x^{15})}{56x^8(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^9,x]
```

```
[Out] (Sqrt[(a + b*x^3)^2]*(-7*a^5 - 56*a^4*b*x^3 - 280*a^3*b^2*x^6 + 560*a^2*b^3
*x^9 + 70*a*b^4*x^12 + 8*b^5*x^15))/(56*x^8*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{(-8b^5x^{15}-70b^4a\,x^{12}-560a^2b^3x^9+280b^2a^3x^6+56a^4b\,x^3+7a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{56(b\,x^3+a)^5x^8}$	80
default	$-\frac{(-8b^5x^{15}-70b^4a\,x^{12}-560a^2b^3x^9+280b^2a^3x^6+56a^4b\,x^3+7a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{56(b\,x^3+a)^5x^8}$	80
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b^3\left(\frac{1}{7}b^2x^7+\frac{5}{4}ab\,x^4+10a^2x\right)}{b\,x^3+a}+\frac{\sqrt{(b\,x^3+a)^2}\,\left(-5b^2a^3x^6-a^4b\,x^3-\frac{1}{8}a^5\right)}{(b\,x^3+a)x^8}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/56*(-8*b^5*x^15-70*a*b^4*x^12-560*a^2*b^3*x^9+280*a^3*b^2*x^6+56*a^4*b*x
^3+7*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^8
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.24

$$\frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a
^4*b*x^3 - 7*a^5)/x^8
```

Fricas [A]
time = 0.36, size = 59, normalized size = 0.24

$$\frac{8\,b^5x^{15}+70\,ab^4x^{12}+560\,a^2b^3x^9-280\,a^3b^2x^6-56\,a^4bx^3-7\,a^5}{56\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/56*(8*b^5*x^15 + 70*a*b^4*x^12 + 560*a^2*b^3*x^9 - 280*a^3*b^2*x^6 - 56*a
^4*b*x^3 - 7*a^5)/x^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**9, x)
```

Giac [A]
time = 2.90, size = 105, normalized size = 0.43

$$\frac{1}{7}\,b^5x^7\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{4}\,ab^4x^4\mathrm{sgn}\left(bx^3+a\right)+10\,a^2b^3x\mathrm{sgn}\left(bx^3+a\right)-\frac{40\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+8\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+a^5\mathrm{sgn}\left(bx^3+a\right)}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7*sgn(b*x^3 + a) + 5/4*a*b^4*x^4*sgn(b*x^3 + a) + 10*a^2*b^3*x*sg
n(b*x^3 + a) - 1/8*(40*a^3*b^2*x^6*sgn(b*x^3 + a) + 8*a^4*b*x^3*sgn(b*x^3 +
 a) + a^5*sgn(b*x^3 + a))/x^8
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^9,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^9, x)
```

3.73 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx$

Optimal. Leaf size=252

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^3+}}{3\left(a+bx^3\right)}$$

```
[Out] -1/9*a^5*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-5/6*a^4*b*((b*x^3+a)^2)^(1/2)/x^
6/(b*x^3+a)-10/3*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+5/3*a*b^4*x^3*((
b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/6*b^5*x^6*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+10*a^
2*b^3*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.05, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$\frac{b^5x^6\sqrt{a^2+2abx^3+b^2x^6}}{6\left(a+bx^3\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{10a^2b^3\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
```

```
[Out] -1/9*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^9*(a + b*x^3)) - (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(6*x^6*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (5*a*b^4*x^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(3*(a + b*x^3)) + (b^5*x^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/
(6*(a + b*x^3)) + (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{10}}\,dx}{b^4\,(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{(ab+b^2x)^5}{x^4}\,dx, x, x^3\right)}{3b^4\,(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \left(5ab^9+\frac{a^5b^5}{x^4}+\frac{5a^4b^6}{x^3}+\frac{10a^3b^7}{x^2}+\frac{10a^2b^8}{x}+b^{10}x\right)\right.}{3b^4\,(ab+b^2x^3)}$$

$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\,(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{6x^6\,(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3}}{3x^3\,(a+bx^3}$$

Mathematica [A]
time = 0.02, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^3)^2}\,(-2a^5-15a^4bx^3-60a^3b^2x^6+30ab^4x^{12}+3b^5x^{15}+180a^2b^3x^9\log(x))}{18x^9\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^10,x]
[Out] (Sqrt[(a + b*x^3)^2]*(-2*a^5 - 15*a^4*b*x^3 - 60*a^3*b^2*x^6 + 30*a*b^4*x^1
2 + 3*b^5*x^15 + 180*a^2*b^3*x^9*Log[x]))/(18*x^9*(a + b*x^3))
```

Maple [A]
time = 0.06, size = 82, normalized size = 0.33

method	result	size
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{5}{2}}(3b^5x^{15}+30b^4a\,x^{12}+180a^2b^3\ln(x)x^9-60b^2a^3x^6-15a^4b\,x^3-2a^5)}{18(b\,x^3+a)^5x^9}$	82
risch	$\frac{\sqrt{(b\,x^3+a)^2}\;b^3(b\,x^3+5a)^2}{6b\,x^3+6a}+\frac{\sqrt{(b\,x^3+a)^2}\;(-\frac{10}{3}b^2a^3x^6-\frac{5}{6}a^4b\,x^3-\frac{1}{9}a^5)}{(b\,x^3+a)x^9}+\frac{10a^2b^3\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

[Out] 1/18*((b*x^3+a)^2)^(5/2)*(3*b^5*x^15+30*b^4*a*x^12+180*a^2*b^3*ln(x)*x^9-60*b^2*a^3*x^6-15*a^4*b*x^3-2*a^5)/(b*x^3+a)^5/x^9

Maxima [A]
time = 0.28, size = 313, normalized size = 1.24

$$\frac{5}{3}\sqrt{b^2x^6+2\,abx^3+a^2}\,b^4x^3+\frac{10}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,a^2b^3\log\left(2\,b^2x^3+2\,ab\right)-\frac{10}{3}\,(-1)^{2\,abx^3+2\,a^2}\,a^2b^3\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{5\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^4x^3}{6\,a^2}+5\,\sqrt{b^2x^6+2\,abx^3+a^2}\,ab^3+\frac{35\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^3}{18\,a}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^3}{18\,a^3}-\frac{11\,(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^2}{18\,a^2x^3}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{18\,a^3x^6}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{9\,a^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="maxima")

[Out] 5/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4*x^3 + 10/3*(-1)^(2*b^2*x^3 + 2*a*b)*a^2*b^3*log(2*b^2*x^3 + 2*a*b) - 10/3*(-1)^(2*a*b*x^3 + 2*a^2)*a^2*b^3*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/6*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^4*x^3/a^2 + 5*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a*b^3 + 35/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^3/a + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/a^3 - 11/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^2/(a^2*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^6) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^9)

Fricas [A]
time = 0.42, size = 61, normalized size = 0.24

$$\frac{3\,b^5x^{15}+30\,ab^4x^{12}+180\,a^2b^3x^9\log\left(x\right)-60\,a^3b^2x^6-15\,a^4bx^3-2\,a^5}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="fricas")

[Out] 1/18*(3*b^5*x^15 + 30*a*b^4*x^12 + 180*a^2*b^3*x^9*log(x) - 60*a^3*b^2*x^6 - 15*a^4*b*x^3 - 2*a^5)/x^9

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**10,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**10, x)

Giac [A]
time = 3.88, size = 127, normalized size = 0.50

$$\frac{1}{6}\,b^5x^6\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{3}\,ab^4x^3\mathrm{sgn}\left(bx^3+a\right)+10\,a^2b^3\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{110\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+60\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+15\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+2\,a^5\mathrm{sgn}\left(bx^3+a\right)}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6*sgn(b*x^3 + a) + 5/3*a*b^4*x^3*sgn(b*x^3 + a) + 10*a^2*b^3*log(
abs(x))*sgn(b*x^3 + a) - 1/18*(110*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*
x^6*sgn(b*x^3 + a) + 15*a^4*b*x^3*sgn(b*x^3 + a) + 2*a^5*sgn(b*x^3 + a))/x^
9
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^{5/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^10, x)
```

3.74 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{11}}\,dx$

Optimal. Leaf size=253

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b}}{x\left(a+bx^3\right)}$$

```
[Out] -1/10*a^5*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-5/7*a^4*b*((b*x^3+a)^2)^(1/2)/
x^7/(b*x^3+a)-5/2*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-10*a^2*b^3*((b*
x^3+a)^2)^(1/2)/x/(b*x^3+a)+5/2*a*b^4*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/5
*b^5*x^5*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1369, 276\}$

$$\frac{b^5x^5\sqrt{a^2+2abx^3+b^2x^6}}{5\left(a+bx^3\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

```
[Out] -1/10*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(x*(a + b*x^3)) + (5*a*b^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(2*(a + b*x^3)) + (b^5*x^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*(a + b*x
^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{11}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{11}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{11}}+\frac{5a^4b^6}{x^8}+\frac{10a^3b^7}{x^5}+\frac{10a^2b^8}{x^2}+5ab^9x+b^{10}x^4\right)dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{10x^{10}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3 -}}{2x^4(a+bx^3)}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\,(7a^5+50a^4bx^3+175a^3b^2x^6+700a^2b^3x^9-175ab^4x^{12}-14b^5x^{15})}{70x^{10}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^11,x]
```

```
[Out] -1/70*(Sqrt[(a + b*x^3)^2]*(7*a^5 + 50*a^4*b*x^3 + 175*a^3*b^2*x^6 + 700*a^
2*b^3*x^9 - 175*a*b^4*x^12 - 14*b^5*x^15))/(x^10*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{(-14b^5x^{15}-175b^4a\,x^{12}+700a^2b^3x^9+175b^2a^3x^6+50a^4b\,x^3+7a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{70(b\,x^3+a)^5x^{10}}$	80
default	$-\frac{(-14b^5x^{15}-175b^4a\,x^{12}+700a^2b^3x^9+175b^2a^3x^6+50a^4b\,x^3+7a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{70(b\,x^3+a)^5x^{10}}$	80
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b^4(\frac{1}{5}b\,x^5+\frac{5}{2}a\,x^2)}{b\,x^3+a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-10a^2b^3x^9-\frac{5}{2}b^2a^3x^6-\frac{5}{7}a^4b\,x^3-\frac{1}{10}a^5)}{(b\,x^3+a)x^{10}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/70*(-14*b^5*x^15-175*a*b^4*x^12+700*a^2*b^3*x^9+175*a^3*b^2*x^6+50*a^4*b
*x^3+7*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^10
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.23

$$\frac{14\,b^5x^{15}+175\,ab^4x^{12}-700\,a^2b^3x^9-175\,a^3b^2x^6-50\,a^4bx^3-7\,a^5}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] 1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 50
*a^4*b*x^3 - 7*a^5)/x^10
```

Fricas [**A**]
time = 0.36, size = 59, normalized size = 0.23

$$\frac{14\,b^5x^{15}+175\,ab^4x^{12}-700\,a^2b^3x^9-175\,a^3b^2x^6-50\,a^4bx^3-7\,a^5}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/70*(14*b^5*x^15 + 175*a*b^4*x^12 - 700*a^2*b^3*x^9 - 175*a^3*b^2*x^6 - 50
*a^4*b*x^3 - 7*a^5)/x^10
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**11, x)
```

Giac [**A**]
time = 4.25, size = 108, normalized size = 0.43

$$\frac{1}{5}\,b^5x^5\mathrm{sgn}\left(bx^3+a\right)+\frac{5}{2}\,ab^4x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{700\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+175\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+50\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+7\,a^5\mathrm{sgn}\left(bx^3+a\right)}{70\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5*sgn(b*x^3 + a) + 5/2*a*b^4*x^2*sgn(b*x^3 + a) - 1/70*(700*a^2*b
^3*x^9*sgn(b*x^3 + a) + 175*a^3*b^2*x^6*sgn(b*x^3 + a) + 50*a^4*b*x^3*sgn(b
*x^3 + a) + 7*a^5*sgn(b*x^3 + a))/x^10
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^11,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^11, x)
```

3.75 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx$

Optimal. Leaf size=247

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)}$$

```
[Out] -1/11*a^5*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-5/8*a^4*b*((b*x^3+a)^2)^(1/2)/
x^8/(b*x^3+a)-2*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5*a^2*b^3*((b*x^3
+a)^2)^(1/2)/x^2/(b*x^3+a)+5*a*b^4*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+1/4*b^5*
x^4*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.04, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^4\sqrt{a^2+2abx^3+b^2x^6}}{4(a+bx^3)}+\frac{5ab^4x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^2(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{2a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^5(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] -1/11*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^11*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*(a + b*x^3)) - (2*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6])/(x^2*(a + b*x^3)) + (5*a*b^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
/(a + b*x^3) + (b^5*x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{12}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{12}}\,dx}{b^4\,(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(5ab^9+\frac{a^5b^5}{x^{12}}+\frac{5a^4b^6}{x^9}+\frac{10a^3b^7}{x^6}+\frac{10a^2b^8}{x^3}+b^{10}x^3\right)dx}{b^4\,(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\,(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\,(a+bx^3)} - \frac{2a^3b^2\sqrt{a^2+2abx^3}}{x^5\,(a+bx^3}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^3)^2}\,(8a^5+55a^4bx^3+176a^3b^2x^6+440a^2b^3x^9-440ab^4x^{12}-22b^5x^{15})}{88x^{11}\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^12,x]
```

```
[Out] -1/88*(Sqrt[(a + b*x^3)^2]*(8*a^5 + 55*a^4*b*x^3 + 176*a^3*b^2*x^6 + 440*a^
2*b^3*x^9 - 440*a*b^4*x^12 - 22*b^5*x^15))/(x^11*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{(-22b^5x^{15}-440b^4a\,x^{12}+440a^2b^3x^9+176b^2a^3x^6+55a^4b\,x^3+8a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{88x^{11}(b\,x^3+a)^5}$	80
default	$-\frac{(-22b^5x^{15}-440b^4a\,x^{12}+440a^2b^3x^9+176b^2a^3x^6+55a^4b\,x^3+8a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{88x^{11}(b\,x^3+a)^5}$	80
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,b^4(\frac{1}{4}b\,x^4+5ax)}{b\,x^3+a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-5a^2b^3x^9-2b^2a^3x^6-\frac{5}{8}a^4b\,x^3-\frac{1}{11}a^5)}{(b\,x^3+a)x^{11}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

```
[Out] -1/88*(-22*b^5*x^15-440*a*b^4*x^12+440*a^2*b^3*x^9+176*a^3*b^2*x^6+55*a^4*b
*x^3+8*a^5)*((b*x^3+a)^2)^(5/2)/x^11/(b*x^3+a)^5
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.24

$$\frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] 1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 55
*a^4*b*x^3 - 8*a^5)/x^11
```

Fricas [A]
time = 0.37, size = 59, normalized size = 0.24

$$\frac{22\,b^5x^{15}+440\,ab^4x^{12}-440\,a^2b^3x^9-176\,a^3b^2x^6-55\,a^4bx^3-8\,a^5}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] 1/88*(22*b^5*x^15 + 440*a*b^4*x^12 - 440*a^2*b^3*x^9 - 176*a^3*b^2*x^6 - 55
*a^4*b*x^3 - 8*a^5)/x^11
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**12, x)
```

Giac [A]
time = 4.57, size = 106, normalized size = 0.43

$$\frac{1}{4}\,b^5x^4\mathrm{sgn}\left(bx^3+a\right)+5\,ab^4x\mathrm{sgn}\left(bx^3+a\right)-\frac{440\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+176\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+55\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+8\,a^5\mathrm{sgn}\left(bx^3+a\right)}{88\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4*sgn(b*x^3 + a) + 5*a*b^4*x*sgn(b*x^3 + a) - 1/88*(440*a^2*b^3*x
^9*sgn(b*x^3 + a) + 176*a^3*b^2*x^6*sgn(b*x^3 + a) + 55*a^4*b*x^3*sgn(b*x^3
 + a) + 8*a^5*sgn(b*x^3 + a))/x^11
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^12,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^12, x)
```

3.76 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{13}}\,dx$

Optimal. Leaf size=252

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b}}{3x^3\left(a+bx^3\right)}$$

```
[Out] -1/12*a^5*((b*x^3+a)^2)^(1/2)/x^12/(b*x^3+a)-5/9*a^4*b*((b*x^3+a)^2)^(1/2)/
x^9/(b*x^3+a)-5/3*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-10/3*a^2*b^3*((
b*x^3+a)^2)^(1/2)/x^3/(b*x^3+a)+1/3*b^5*x^3*((b*x^3+a)^2)^(1/2)/(b*x^3+a)+5
*a*b^4*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.04, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$\frac{b^5x^3\sqrt{a^2+2abx^3+b^2x^6}}{3\left(a+bx^3\right)}+\frac{5ab^4\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^3\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{3x^6\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
```

```
[Out] -1/12*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^12*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6])/(3*x^3*(a + b*x^3)) + (b^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^
6])/(3*(a + b*x^3)) + (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a +
 b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{13}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{13}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{(ab+b^2x)^5}{x^5}\,dx, x, x^3\right)}{3b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \left(b^{10}+\frac{a^5b^5}{x^5}+\frac{5a^4b^6}{x^4}+\frac{10a^3b^7}{x^3}+\frac{10a^2b^8}{x^2}+\frac{5ab^9}{x}\right) dx\right.}{3b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3}}{3x^6(a+bx^3)}$$

Mathematica [A]
time = 0.01, size = 85, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^3)^2}\,(3a^5+20a^4bx^3+60a^3b^2x^6+120a^2b^3x^9-12b^5x^{15}-180ab^4x^{12}\log(x))}{36x^{12}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^13,x]
[Out] -1/36*(Sqrt[(a + b*x^3)^2]*(3*a^5 + 20*a^4*b*x^3 + 60*a^3*b^2*x^6 + 120*a^2
*b^3*x^9 - 12*b^5*x^15 - 180*a*b^4*x^12*Log[x]))/(x^12*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 82, normalized size = 0.33

method	result	size
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{5}{2}}(12b^5x^{15}+180b^4a\ln(x)x^{12}-120a^2b^3x^9-60b^2a^3x^6-20a^4b\,x^3-3a^5)}{36(b\,x^3+a)^5x^{12}}$	82
risch	$\frac{b^5x^3\sqrt{(b\,x^3+a)^2}}{3b\,x^3+3a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-\frac{10}{3}a^2b^3x^9-\frac{5}{3}b^2a^3x^6-\frac{5}{9}a^4b\,x^3-\frac{1}{12}a^5)}{(b\,x^3+a)x^{12}}+\frac{5a\,b^4\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x,method=_RETURNVERBOSE)
```

[Out] 1/36*((b*x^3+a)^2)^(5/2)*(12*b^5*x^15+180*b^4*a*ln(x)*x^12-120*a^2*b^3*x^9-60*b^2*a^3*x^6-20*a^4*b*x^3-3*a^5)/(b*x^3+a)^5/x^12

Maxima [A]
time = 0.29, size = 342, normalized size = 1.36

$$\frac{5\sqrt{b^2x^6+2\,abx^3+a^2}\,b^5x^3}{6\,a}+\frac{5}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,ab^4\log\left(2\,b^2x^3+2\,ab\right)-\frac{5}{3}\,(-1)^{2\,abx^3+2\,a^2}\,ab^4\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)+\frac{5\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^5x^3}{12\,a^3}+\frac{5}{2}\,\sqrt{b^2x^6+2\,abx^3+a^2}\,b^4+\frac{35\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}b^4}{36\,a^2}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^4}{9\,a^4}-\frac{2\,(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^3}{9\,a^3x^3}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^2}{9\,a^4x^6}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{36\,a^3x^9}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{12\,a^2x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="maxima")

[Out] 5/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5*x^3/a + 5/3*(-1)^(2*b^2*x^3 + 2*a*b)*a*b^4*log(2*b^2*x^3 + 2*a*b) - 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*a*b^4*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x))) + 5/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^5*x^3/a^3 + 5/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^4 + 35/36*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^4/a^2 + 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/a^4 - 2/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^3/(a^3*x^3) - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^6) + 1/36*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^9) - 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^12)

Fricas [A]
time = 0.36, size = 61, normalized size = 0.24

$$\frac{12\,b^5x^{15}+180\,ab^4x^{12}\log\left(x\right)-120\,a^2b^3x^9-60\,a^3b^2x^6-20\,a^4bx^3-3\,a^5}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="fricas")

[Out] 1/36*(12*b^5*x^15 + 180*a*b^4*x^12*log(x) - 120*a^2*b^3*x^9 - 60*a^3*b^2*x^6 - 20*a^4*b*x^3 - 3*a^5)/x^12

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**13,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**13, x)

Giac [A]
time = 5.02, size = 125, normalized size = 0.50

$$\frac{1}{3}\,b^5x^3\mathrm{sgn}\left(bx^3+a\right)+5\,ab^4\log\left(|x|\right)\mathrm{sgn}\left(bx^3+a\right)-\frac{125\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+120\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+60\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+20\,a^4bx^3\mathrm{sgn}(bx^3+a)+3\,a^5\mathrm{sgn}(bx^3+a)}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^13,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3*sgn(b*x^3 + a) + 5*a*b^4*log(abs(x))*sgn(b*x^3 + a) - 1/36*(125
*a*b^4*x^12*sgn(b*x^3 + a) + 120*a^2*b^3*x^9*sgn(b*x^3 + a) + 60*a^3*b^2*x^
6*sgn(b*x^3 + a) + 20*a^4*b*x^3*sgn(b*x^3 + a) + 3*a^5*sgn(b*x^3 + a))/x^12
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^13, x)
```

3.77 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{14}}\,dx$

Optimal. Leaf size=253

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2}}{2x^4\left(a+bx^3\right)}$$

```
[Out] -1/13*a^5*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-1/2*a^4*b*((b*x^3+a)^2)^(1/2)/
x^10/(b*x^3+a)-10/7*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/2*a^2*b^3*(
(b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-5*a*b^4*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)+1/
2*b^5*x^2*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.04, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x^2\sqrt{a^2+2abx^3+b^2x^6}}{2\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{2x^4\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```

```
[Out] -1/13*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^13*(a + b*x^3)) - (a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^10*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])/(2*x^4*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])/(x*(a + b*x^3)) + (b^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*(a + b*x^
3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{14}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{14}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{14}}+\frac{5a^4b^6}{x^{11}}+\frac{10a^3b^7}{x^8}+\frac{10a^2b^8}{x^5}+\frac{5ab^9}{x^2}+b^{10}x\right)dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3 -}}{7x^7(a+bx^3)}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\,(14a^5+91a^4bx^3+260a^3b^2x^6+455a^2b^3x^9+910ab^4x^{12}-91b^5x^{15})}{182x^{13}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^14,x]
```

```
[Out] -1/182*(Sqrt[(a + b*x^3)^2]*(14*a^5 + 91*a^4*b*x^3 + 260*a^3*b^2*x^6 + 455*
a^2*b^3*x^9 + 910*a*b^4*x^12 - 91*b^5*x^15))/(x^13*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{(-91b^5x^{15}+910b^4a\,x^{12}+455a^2b^3x^9+260b^2a^3x^6+91a^4b\,x^3+14a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{182(b\,x^3+a)^5x^{13}}$	80
default	$-\frac{(-91b^5x^{15}+910b^4a\,x^{12}+455a^2b^3x^9+260b^2a^3x^6+91a^4b\,x^3+14a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{182(b\,x^3+a)^5x^{13}}$	80
risch	$\frac{b^5x^2\sqrt{(b\,x^3+a)^2}}{2b\,x^3+2a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-5b^4a\,x^{12}-\frac{5}{2}a^2b^3x^9-\frac{10}{7}b^2a^3x^6-\frac{1}{2}a^4b\,x^3-\frac{1}{13}a^5)}{(b\,x^3+a)x^{13}}$	100

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x,method=_RETURNVERBOSE)
```

```
[Out] -1/182*(-91*b^5*x^15+910*a*b^4*x^12+455*a^2*b^3*x^9+260*a^3*b^2*x^6+91*a^4*
b*x^3+14*a^5)*((b*x^3+a)^2)^(5/2)/(b*x^3+a)^5/x^13
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.23

$$\frac{91\,b^5x^{15}-910\,ab^4x^{12}-455\,a^2b^3x^9-260\,a^3b^2x^6-91\,a^4bx^3-14\,a^5}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 - 9
1*a^4*b*x^3 - 14*a^5)/x^13
```

Fricas [A]
time = 0.35, size = 59, normalized size = 0.23

$$\frac{91\,b^5x^{15}-910\,ab^4x^{12}-455\,a^2b^3x^9-260\,a^3b^2x^6-91\,a^4bx^3-14\,a^5}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="fricas")
```

```
[Out] 1/182*(91*b^5*x^15 - 910*a*b^4*x^12 - 455*a^2*b^3*x^9 - 260*a^3*b^2*x^6 - 9
1*a^4*b*x^3 - 14*a^5)/x^13
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**14, x)
```

Giac [A]
time = 4.32, size = 108, normalized size = 0.43

$$\frac{1}{2}\,b^5x^2\mathrm{sgn}\left(bx^3+a\right)-\frac{910\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+455\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+260\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+91\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+14\,a^5\mathrm{sgn}\left(bx^3+a\right)}{182\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2*sgn(b*x^3 + a) - 1/182*(910*a*b^4*x^12*sgn(b*x^3 + a) + 455*a^2
*b^3*x^9*sgn(b*x^3 + a) + 260*a^3*b^2*x^6*sgn(b*x^3 + a) + 91*a^4*b*x^3*sgn
(b*x^3 + a) + 14*a^5*sgn(b*x^3 + a))/x^13
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^14,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^14, x)
```

3.78 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{15}} \, dx$

Optimal. Leaf size=248

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}$$

```
[Out] -1/14*a^5*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-5/11*a^4*b*((b*x^3+a)^2)^(1/2)
/x^11/(b*x^3+a)-5/4*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-2*a^2*b^3*((b
*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)-5/2*a*b^4*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)+
b^5*x*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$\frac{b^5x\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

```
[Out] -1/14*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^14*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(x^5*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])/(2*x^2*(a + b*x^3)) + (b^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a + b*x^3
)
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{15}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{15}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(b^{10}+\frac{a^5b^5}{x^{15}}+\frac{5a^4b^6}{x^{12}}+\frac{10a^3b^7}{x^9}+\frac{10a^2b^8}{x^6}+\frac{5ab^9}{x^3}\right)dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3}}{4x^8(a+bx^3}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\,(22a^5+140a^4bx^3+385a^3b^2x^6+616a^2b^3x^9+770ab^4x^{12}-308b^5x^{15})}{308x^{14}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^15,x]
```

```
[Out] -1/308*(Sqrt[(a + b*x^3)^2]*(22*a^5 + 140*a^4*b*x^3 + 385*a^3*b^2*x^6 + 616
*a^2*b^3*x^9 + 770*a*b^4*x^12 - 308*b^5*x^15))/(x^14*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 80, normalized size = 0.32

method	result	size
gosper	$-\frac{(-308b^5x^{15}+770b^4a\,x^{12}+616a^2b^3x^9+385b^2a^3x^6+140a^4b\,x^3+22a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{308x^{14}(b\,x^3+a)^5}$	80
default	$-\frac{(-308b^5x^{15}+770b^4a\,x^{12}+616a^2b^3x^9+385b^2a^3x^6+140a^4b\,x^3+22a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{308x^{14}(b\,x^3+a)^5}$	80
risch	$\frac{b^5x\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}+\frac{\sqrt{(b\,x^3+a)^2}\,(-\frac{5}{2}b^4a\,x^{12}-2a^2b^3x^9-\frac{5}{4}b^2a^3x^6-\frac{5}{11}a^4b\,x^3-\frac{1}{14}a^5)}{(b\,x^3+a)x^{14}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x,method=_RETURNVERBOSE)
```

```
[Out] -1/308*(-308*b^5*x^15+770*a*b^4*x^12+616*a^2*b^3*x^9+385*a^3*b^2*x^6+140*a^
4*b*x^3+22*a^5)*((b*x^3+a)^2)^(5/2)/x^14/(b*x^3+a)^5
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.24

$$\frac{308\,b^5x^{15}-770\,ab^4x^{12}-616\,a^2b^3x^9-385\,a^3b^2x^6-140\,a^4bx^3-22\,a^5}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="maxima")
```

```
[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 -
140*a^4*b*x^3 - 22*a^5)/x^14
```

Fricas [A]
time = 0.36, size = 59, normalized size = 0.24

$$\frac{308\,b^5x^{15} - 770\,ab^4x^{12} - 616\,a^2b^3x^9 - 385\,a^3b^2x^6 - 140\,a^4bx^3 - 22\,a^5}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="fricas")
```

```
[Out] 1/308*(308*b^5*x^15 - 770*a*b^4*x^12 - 616*a^2*b^3*x^9 - 385*a^3*b^2*x^6 -
140*a^4*b*x^3 - 22*a^5)/x^14
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^{\frac{5}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**15, x)
```

Giac [A]
time = 3.49, size = 105, normalized size = 0.42

$$b^5x\text{sgn}\left(bx^3+a\right) - \frac{770\,ab^4x^{12}\text{sgn}\left(bx^3+a\right) + 616\,a^2b^3x^9\text{sgn}\left(bx^3+a\right) + 385\,a^3b^2x^6\text{sgn}\left(bx^3+a\right) + 140\,a^4bx^3\text{sgn}\left(bx^3+a\right) + 22\,a^5\text{sgn}\left(bx^3+a\right)}{308\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^15,x, algorithm="giac")
```

```
[Out] b^5*x*sgn(b*x^3 + a) - 1/308*(770*a*b^4*x^12*sgn(b*x^3 + a) + 616*a^2*b^3*x
^9*sgn(b*x^3 + a) + 385*a^3*b^2*x^6*sgn(b*x^3 + a) + 140*a^4*b*x^3*sgn(b*x^
3 + a) + 22*a^5*sgn(b*x^3 + a))/x^14
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^15,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^15, x)
```

3.79 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}} \, dx$

Optimal. Leaf size=251

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b}}{3x^6(a+bx^3)}$$

```
[Out] -1/15*a^5*((b*x^3+a)^2)^(1/2)/x^15/(b*x^3+a)-5/12*a^4*b*((b*x^3+a)^2)^(1/2)
/x^12/(b*x^3+a)-10/9*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^9/(b*x^3+a)-5/3*a^2*b^3*
((b*x^3+a)^2)^(1/2)/x^6/(b*x^3+a)-5/3*a*b^4*((b*x^3+a)^2)^(1/2)/x^3/(b*x^3+
a)+b^5*ln(x)*((b*x^3+a)^2)^(1/2)/(b*x^3+a)
```

Rubi [A]

time = 0.05, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$\frac{b^5\log(x)\sqrt{a^2+2abx^3+b^2x^6}}{a+bx^3}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{3x^3(a+bx^3)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{3x^6(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{9x^9(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
```

```
[Out] -1/15*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^15*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(12*x^12*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(9*x^9*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(3*x^6*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(3*x^3*(a + b*x^3)) + (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]*Log[x])/(a
 + b*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{16}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{16}}\,dx}{b^4(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{(ab+b^2x)^5}{x^6}\,dx, x, x^3\right)}{3b^4(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \left(\frac{a^5b^5}{x^6}+\frac{5a^4b^6}{x^5}+\frac{10a^3b^7}{x^4}+\frac{10a^2b^8}{x^3}+\frac{5ab^9}{x^2}+\frac{b^{10}}{x}\right) dx\right.}{3b^4(ab+b^2x^3)}\\ &= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{15x^{15}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{12x^{12}(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3}}{9x^9(a+bx^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 85, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^3)^2}\,(a(12a^4+75a^3bx^3+200a^2b^2x^6+300ab^3x^9+300b^4x^{12})-180b^5x^{15}\log(x))}{180x^{15}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^16,x]
[Out] -1/180*(Sqrt[(a + b*x^3)^2]*(a*(12*a^4 + 75*a^3*b*x^3 + 200*a^2*b^2*x^6 + 3
00*a*b^3*x^9 + 300*b^4*x^12) - 180*b^5*x^15*Log[x]))/(x^15*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 82, normalized size = 0.33

method	result	size
default	$\frac{\left((b\,x^3+a)^2\right)^{\frac{5}{2}}(180b^5\ln(x)x^{15}-300b^4a\,x^{12}-300a^2b^3x^9-200b^2a^3x^6-75a^4b\,x^3-12a^5)}{180(b\,x^3+a)^5x^{15}}$	82
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,(-\frac{1}{15}a^5-\frac{5}{12}a^4b\,x^3-\frac{10}{9}b^2a^3x^6-\frac{5}{3}a^2b^3x^9-\frac{5}{3}b^4a\,x^{12})}{(b\,x^3+a)x^{15}}+\frac{b^5\ln(x)\sqrt{(b\,x^3+a)^2}}{b\,x^3+a}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x,method=_RETURNVERBOSE)
```

```
[Out] 1/180*((b*x^3+a)^2)^(5/2)*(180*b^5*ln(x)*x^15-300*b^4*a*x^12-300*a^2*b^3*x^
9-200*b^2*a^3*x^6-75*a^4*b*x^3-12*a^5)/(b*x^3+a)^5/x^15
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(175) = 350$.
time = 0.30, size = 374, normalized size = 1.49

$$\frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,b^6x^3}{6\,a^2} + \frac{1}{3}\,(-1)^{2\,b^2x^3+2\,ab}\,b^5\log\left(2\,b^2x^3 + 2\,ab\right) - \frac{1}{3}\,(-1)^{2\,abx^3+2\,a^2}\,b^5\log\left(\frac{2\,abx}{|x|} + \frac{2\,a^2}{x^2|x|}\right) + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^6x^3}{12\,a^4} + \frac{\sqrt{b^2x^6 + 2\,abx^3 + a^2}\,b^5}{2\,a} + \frac{7\,(b^2x^6 + 2\,abx^3 + a^2)^{\frac{3}{2}}b^5}{36\,a^3} - \frac{2\,(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^5}{45\,a^5} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{5}{2}}b^4}{9\,a^4x^3} + \frac{2\,(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^3}{45\,a^5x^6} - \frac{11\,(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b^2}{180\,a^4x^9} + \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}b}{20\,a^3x^{12}} - \frac{(b^2x^6 + 2\,abx^3 + a^2)^{\frac{7}{2}}}{15\,a^2x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^6*x^3/a^2 + 1/3*(-1)^(2*b^2*x^3 + 2*a
*b)*b^5*log(2*b^2*x^3 + 2*a*b) - 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b^5*log(2*a*b
*x/abs(x) + 2*a^2/(x^2*abs(x))) + 1/12*(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^
6*x^3/a^4 + 1/2*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*b^5/a + 7/36*(b^2*x^6 + 2*a
*b*x^3 + a^2)^(3/2)*b^5/a^3 - 2/45*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^5/a^
5 - 1/9*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^4/(a^4*x^3) + 2/45*(b^2*x^6 + 2
*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^6) - 11/180*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7
/2)*b^2/(a^4*x^9) + 1/20*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^12) - 1
/15*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^15)
```

Fricas [A]
time = 0.43, size = 61, normalized size = 0.24

$$\frac{180\,b^5x^{15}\log\left(x\right) - 300\,ab^4x^{12} - 300\,a^2b^3x^9 - 200\,a^3b^2x^6 - 75\,a^4bx^3 - 12\,a^5}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] 1/180*(180*b^5*x^15*log(x) - 300*a*b^4*x^12 - 300*a^2*b^3*x^9 - 200*a^3*b^2
*x^6 - 75*a^4*b*x^3 - 12*a^5)/x^15
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a + bx^3\right)^2\right)^{\frac{5}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**16, x)
```

Giac [A]
time = 4.55, size = 123, normalized size = 0.49

$$b^5 \log(|x|) \operatorname{sgn}(bx^3+a) - \frac{137\,b^5x^{15}\operatorname{sgn}(bx^3+a) + 300\,ab^4x^{12}\operatorname{sgn}(bx^3+a) + 300\,a^2b^3x^9\operatorname{sgn}(bx^3+a) + 200\,a^3b^2x^6\operatorname{sgn}(bx^3+a) + 75\,a^4bx^3\operatorname{sgn}(bx^3+a) + 12\,a^5\operatorname{sgn}(bx^3+a)}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] b^5*log(abs(x))*sgn(b*x^3 + a) - 1/180*(137*b^5*x^15*sgn(b*x^3 + a) + 300*a
*b^4*x^12*sgn(b*x^3 + a) + 300*a^2*b^3*x^9*sgn(b*x^3 + a) + 200*a^3*b^2*x^6
*sgn(b*x^3 + a) + 75*a^4*b*x^3*sgn(b*x^3 + a) + 12*a^5*sgn(b*x^3 + a))/x^15
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^16, x)
```

3.80 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{17}}\,dx$

Optimal. Leaf size=251

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2}}{7x^7\left(a+bx^3\right)}$$

```
[Out] -1/16*a^5*((b*x^3+a)^2)^(1/2)/x^16/(b*x^3+a)-5/13*a^4*b*((b*x^3+a)^2)^(1/2)
/x^13/(b*x^3+a)-a^3*b^2*((b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-10/7*a^2*b^3*((b
*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)-5/4*a*b^4*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)-
b^5*((b*x^3+a)^2)^(1/2)/x/(b*x^3+a)
```

Rubi [A]

time = 0.04, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{x\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{4x^4\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] -1/16*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^16*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])/(7*x^7*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])/(4*x^4*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x*(a + b*x^
3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{17}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{17}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{17}}+\frac{5a^4b^6}{x^{14}}+\frac{10a^3b^7}{x^{11}}+\frac{10a^2b^8}{x^8}+\frac{5ab^9}{x^5}+\frac{b^{10}}{x^2}\right)dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)} - \frac{a^3b^2\sqrt{a^2+2abx^3+}}{x^{10}(a+bx^3)}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\,(91a^5+560a^4bx^3+1456a^3b^2x^6+2080a^2b^3x^9+1820ab^4x^{12}+1456b^5x^{15})}{1456x^{16}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^17,x]
```

```
[Out] -1/1456*(Sqrt[(a + b*x^3)^2]*(91*a^5 + 560*a^4*b*x^3 + 1456*a^3*b^2*x^6 + 2
080*a^2*b^3*x^9 + 1820*a*b^4*x^12 + 1456*b^5*x^15))/(x^16*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 80, normalized size = 0.32

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{16}a^5-\frac{5}{13}a^4b\,x^3-b^2a^3x^6-\frac{10}{7}a^2b^3x^9-\frac{5}{4}b^4a\,x^{12}-b^5x^{15}\right)}{(b\,x^3+a)x^{16}}$	79
gosper	$-\frac{(1456b^5x^{15}+1820b^4a\,x^{12}+2080a^2b^3x^9+1456b^2a^3x^6+560a^4b\,x^3+91a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{1456x^{16}(b\,x^3+a)^5}$	80
default	$-\frac{(1456b^5x^{15}+1820b^4a\,x^{12}+2080a^2b^3x^9+1456b^2a^3x^6+560a^4b\,x^3+91a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{1456x^{16}(b\,x^3+a)^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x,method=_RETURNVERBOSE)
```

```
[Out] -1/1456*(1456*b^5*x^15+1820*a*b^4*x^12+2080*a^2*b^3*x^9+1456*a^3*b^2*x^6+56
0*a^4*b*x^3+91*a^5)*((b*x^3+a)^2)^(5/2)/x^16/(b*x^3+a)^5
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.24

$$-\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="maxima")

[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2*x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16

Fricas [A]
time = 0.36, size = 59, normalized size = 0.24

$$-\frac{1456\,b^5x^{15}+1820\,ab^4x^{12}+2080\,a^2b^3x^9+1456\,a^3b^2x^6+560\,a^4bx^3+91\,a^5}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="fricas")

[Out] -1/1456*(1456*b^5*x^15 + 1820*a*b^4*x^12 + 2080*a^2*b^3*x^9 + 1456*a^3*b^2*x^6 + 560*a^4*b*x^3 + 91*a^5)/x^16

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**17,x)

[Out] Integral(((a + b*x**3)**2)**(5/2)/x**17, x)

Giac [A]
time = 4.30, size = 107, normalized size = 0.43

$$-\frac{1456\,b^5x^{15}\mathrm{sgn}(bx^3+a)+1820\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+2080\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+1456\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+560\,a^4bx^3\mathrm{sgn}(bx^3+a)+91\,a^5\mathrm{sgn}(bx^3+a)}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^17,x, algorithm="giac")

[Out] -1/1456*(1456*b^5*x^15*sgn(b*x^3 + a) + 1820*a*b^4*x^12*sgn(b*x^3 + a) + 2080*a^2*b^3*x^9*sgn(b*x^3 + a) + 1456*a^3*b^2*x^6*sgn(b*x^3 + a) + 560*a^4*b*x^3*sgn(b*x^3 + a) + 91*a^5*sgn(b*x^3 + a))/x^16

Mupad [B]
time = 1.26, size = 231, normalized size = 0.92

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^{10}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^17,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^6 +
2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(13*x^13*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1
/2))/(7*x^7*(a + b*x^3)) - (a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(x^1
0*(a + b*x^3))
```

3.81 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}} \, dx$

Optimal. Leaf size=253

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b}}{4x^8\left(a+bx^3\right)}$$

```
[Out] -1/17*a^5*((b*x^3+a)^2)^(1/2)/x^17/(b*x^3+a)-5/14*a^4*b*((b*x^3+a)^2)^(1/2)
/x^14/(b*x^3+a)-10/11*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-5/4*a^2*b^
3*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)-a*b^4*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)
-1/2*b^5*((b*x^3+a)^2)^(1/2)/x^2/(b*x^3+a)
```

Rubi [A]

time = 0.04, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{2x^2\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{x^5\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{4x^8\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

```
[Out] -1/17*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^17*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(14*x^14*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a
*b*x^3 + b^2*x^6])/(4*x^8*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(x^5*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(2*x^2*(a +
 b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{18}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{18}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{18}}+\frac{5a^4b^6}{x^{15}}+\frac{10a^3b^7}{x^{12}}+\frac{10a^2b^8}{x^9}+\frac{5ab^9}{x^6}+\frac{b^{10}}{x^3}\right)dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{14x^{14}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3}}{11x^{11}(a+bx}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\,(308a^5+1870a^4bx^3+4760a^3b^2x^6+6545a^2b^3x^9+5236ab^4x^{12}+2618b^5x^{15})}{5236x^{17}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^18,x]
```

```
[Out] -1/5236*(Sqrt[(a + b*x^3)^2]*(308*a^5 + 1870*a^4*b*x^3 + 4760*a^3*b^2*x^6 +
 6545*a^2*b^3*x^9 + 5236*a*b^4*x^12 + 2618*b^5*x^15))/(x^17*(a + b*x^3))
```

Maple [A]
time = 0.02, size = 80, normalized size = 0.32

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,(-\frac{1}{17}a^5-\frac{5}{14}a^4b\,x^3-\frac{10}{11}b^2a^3x^6-\frac{5}{4}a^2b^3x^9-b^4a\,x^{12}-\frac{1}{2}b^5x^{15})}{(b\,x^3+a)x^{17}}$	79
gosper	$-\frac{(2618b^5x^{15}+5236b^4a\,x^{12}+6545a^2b^3x^9+4760b^2a^3x^6+1870a^4b\,x^3+308a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{5236x^{17}(b\,x^3+a)^5}$	80
default	$-\frac{(2618b^5x^{15}+5236b^4a\,x^{12}+6545a^2b^3x^9+4760b^2a^3x^6+1870a^4b\,x^3+308a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{5236x^{17}(b\,x^3+a)^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5236*(2618*b^5*x^15+5236*a*b^4*x^12+6545*a^2*b^3*x^9+4760*a^3*b^2*x^6+18
70*a^4*b*x^3+308*a^5)*((b*x^3+a)^2)^(5/2)/x^17/(b*x^3+a)^5
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.23

$$-\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2*
x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

Fricas [A]
time = 0.38, size = 59, normalized size = 0.23

$$-\frac{2618\,b^5x^{15}+5236\,ab^4x^{12}+6545\,a^2b^3x^9+4760\,a^3b^2x^6+1870\,a^4bx^3+308\,a^5}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] -1/5236*(2618*b^5*x^15 + 5236*a*b^4*x^12 + 6545*a^2*b^3*x^9 + 4760*a^3*b^2*
x^6 + 1870*a^4*b*x^3 + 308*a^5)/x^17
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{18}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**18,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**18, x)
```

Giac [A]
time = 4.37, size = 107, normalized size = 0.42

$$-\frac{2618\,b^5x^{15}\mathrm{sgn}(bx^3+a)+5236\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+6545\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+4760\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+1870\,a^4bx^3\mathrm{sgn}(bx^3+a)+308\,a^5\mathrm{sgn}(bx^3+a)}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] -1/5236*(2618*b^5*x^15*sgn(b*x^3 + a) + 5236*a*b^4*x^12*sgn(b*x^3 + a) + 65
45*a^2*b^3*x^9*sgn(b*x^3 + a) + 4760*a^3*b^2*x^6*sgn(b*x^3 + a) + 1870*a^4*
b*x^3*sgn(b*x^3 + a) + 308*a^5*sgn(b*x^3 + a))/x^17
```

Mupad [B]
time = 1.32, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^2\,\left(b\,x^3+a\right)}-\frac{a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^5\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{14\,x^{14}\,\left(b\,x^3+a\right)}-\frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^8\,\left(b\,x^3+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^18,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(17*x^17*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(2*x^2*(a + b*x^3)) - (a*b^4*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(x^5*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(14*x^14*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(4*x^8*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(1
1*x^11*(a + b*x^3))
```

3.82 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx$

Optimal. Leaf size=41

$$-\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

```
[Out] -1/18*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^18
```

Rubi [A]

time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 270}

$$-\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

```
[Out] -1/18*((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^18)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{19}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{19}}\,dx}{b^4(ab+b^2x^3)}\\ &= -\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{18ax^{18}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 81, normalized size = 1.98

$$-\frac{\sqrt{(a+bx^3)^2}\,(a^5+6a^4bx^3+15a^3b^2x^6+20a^2b^3x^9+15ab^4x^{12}+6b^5x^{15})}{18x^{18}\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^19,x]
```

```
[Out] -1/18*(Sqrt[(a + b*x^3)^2]*(a^5 + 6*a^4*b*x^3 + 15*a^3*b^2*x^6 + 20*a^2*b^3
*x^9 + 15*a*b^4*x^12 + 6*b^5*x^15))/(x^18*(a + b*x^3))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(28) = 56.
time = 0.03, size = 78, normalized size = 1.90

method	result	size
gosper	$-\frac{(6b^5x^{15}+15b^4a\,x^{12}+20a^2b^3x^9+15b^2a^3x^6+6a^4b\,x^3+a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{18x^{18}(b\,x^3+a)^5}$	78
default	$-\frac{(6b^5x^{15}+15b^4a\,x^{12}+20a^2b^3x^9+15b^2a^3x^6+6a^4b\,x^3+a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{18x^{18}(b\,x^3+a)^5}$	78
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{18}a^5-\frac{1}{3}a^4b\,x^3-\frac{5}{6}b^2a^3x^6-\frac{10}{9}a^2b^3x^9-\frac{5}{6}b^4a\,x^{12}-\frac{1}{3}b^5x^{15}\right)}{(b\,x^3+a)x^{18}}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*(6*b^5*x^15+15*a*b^4*x^12+20*a^2*b^3*x^9+15*a^3*b^2*x^6+6*a^4*b*x^3+a
^5)*((b*x^3+a)^2)^(5/2)/x^18/(b*x^3+a)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(28) = 56.
time = 0.28, size = 210, normalized size = 5.12

$$\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^6}{18\,a^6}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^5}{18\,a^5x^3}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^4}{18\,a^6x^6}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^3}{18\,a^5x^9}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^2}{18\,a^4x^{12}}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{18\,a^3x^{15}}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{18\,a^2x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^6/a^6 + 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(5/2)*b^5/(a^5*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^
6*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^9) - 1/18*(b^2*x
^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(7/2)*b/(a^3*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^18)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(28) = 56$.
time = 0.36, size = 57, normalized size = 1.39

$$-\frac{6\,b^5x^{15}+15\,ab^4x^{12}+20\,a^2b^3x^9+15\,a^3b^2x^6+6\,a^4bx^3+a^5}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/18*(6*b^5*x^15 + 15*a*b^4*x^12 + 20*a^2*b^3*x^9 + 15*a^3*b^2*x^6 + 6*a^4
*b*x^3 + a^5)/x^18
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**19,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**19, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(28) = 56$.
time = 3.46, size = 106, normalized size = 2.59

$$-\frac{6\,b^5x^{15}\mathrm{sgn}\left(bx^3+a\right)+15\,ab^4x^{12}\mathrm{sgn}\left(bx^3+a\right)+20\,a^2b^3x^9\mathrm{sgn}\left(bx^3+a\right)+15\,a^3b^2x^6\mathrm{sgn}\left(bx^3+a\right)+6\,a^4bx^3\mathrm{sgn}\left(bx^3+a\right)+a^5\mathrm{sgn}\left(bx^3+a\right)}{18\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^19,x, algorithm="giac")
```

```
[Out] -1/18*(6*b^5*x^15*sgn(b*x^3 + a) + 15*a*b^4*x^12*sgn(b*x^3 + a) + 20*a^2*b^
3*x^9*sgn(b*x^3 + a) + 15*a^3*b^2*x^6*sgn(b*x^3 + a) + 6*a^4*b*x^3*sgn(b*x^
3 + a) + a^5*sgn(b*x^3 + a))/x^18
```

Mupad [B]
time = 1.22, size = 231, normalized size = 5.63

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{18\,x^{18}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^3\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,\left(b\,x^3+a\right)}-\frac{a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\,\left(b\,x^3+a\right)}-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,\left(b\,x^3+a\right)}-\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^{12}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^19,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(3*x^3*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (a^4*b*(a^2 + b^2*x^6 + 2*a*b*x
^3)^(1/2))/(3*x^15*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(
1/2))/(9*x^9*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(
6*x^12*(a + b*x^3))
```

3.83 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{20}}\,dx$

Optimal. Leaf size=253

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}-\frac{a^2b^3\sqrt{a^2+2abx^3+b^2}}{x^{10}(a+bx^3)}$$

```
[Out] -1/19*a^5*((b*x^3+a)^2)^(1/2)/x^19/(b*x^3+a)-5/16*a^4*b*((b*x^3+a)^2)^(1/2)
/x^16/(b*x^3+a)-10/13*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-a^2*b^3*((
b*x^3+a)^2)^(1/2)/x^10/(b*x^3+a)-5/7*a*b^4*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a
)-1/4*b^5*((b*x^3+a)^2)^(1/2)/x^4/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{4x^4(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{7x^7(a+bx^3)}-\frac{a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{x^{10}(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
```

```
[Out] -1/19*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^19*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(16*x^16*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a^2*b^3*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6])/(x^10*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6])/(7*x^7*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^4*(a
+ b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{20}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{20}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{20}}+\frac{5a^4b^6}{x^{17}}+\frac{10a^3b^7}{x^{14}}+\frac{10a^2b^8}{x^{11}}+\frac{5ab^9}{x^8}+\frac{b^{10}}{x^5}\right)dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{16x^{16}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3}}{13x^{13}(a+bx}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\,(1456a^5+8645a^4bx^3+21280a^3b^2x^6+27664a^2b^3x^9+19760ab^4x^{12}+6916b^5x^{15})}{27664x^{19}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^20,x]
[Out] -1/27664*(Sqrt[(a + b*x^3)^2]*(1456*a^5 + 8645*a^4*b*x^3 + 21280*a^3*b^2*x^
6 + 27664*a^2*b^3*x^9 + 19760*a*b^4*x^12 + 6916*b^5*x^15))/(x^19*(a + b*x^3
))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.32

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{19}a^5-\frac{5}{16}a^4b\,x^3-\frac{10}{13}b^2a^3x^6-a^2b^3x^9-\frac{5}{7}b^4a\,x^{12}-\frac{1}{4}b^5x^{15}\right)}{(b\,x^3+a)x^{19}}$	79
gosper	$-\frac{(6916b^5x^{15}+19760b^4a\,x^{12}+27664a^2b^3x^9+21280b^2a^3x^6+8645a^4b\,x^3+1456a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{27664x^{19}(b\,x^3+a)^5}$	80
default	$-\frac{(6916b^5x^{15}+19760b^4a\,x^{12}+27664a^2b^3x^9+21280b^2a^3x^6+8645a^4b\,x^3+1456a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{27664x^{19}(b\,x^3+a)^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x,method=_RETURNVERBOSE)
[Out] -1/27664*(6916*b^5*x^15+19760*a*b^4*x^12+27664*a^2*b^3*x^9+21280*a^3*b^2*x^
6+8645*a^4*b*x^3+1456*a^5)*((b*x^3+a)^2)^(5/2)/x^19/(b*x^3+a)^5
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.23

$$-\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="maxima")
```

```
[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3*
b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

Fricas [A]
time = 0.39, size = 59, normalized size = 0.23

$$-\frac{6916\,b^5x^{15}+19760\,ab^4x^{12}+27664\,a^2b^3x^9+21280\,a^3b^2x^6+8645\,a^4bx^3+1456\,a^5}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/27664*(6916*b^5*x^15 + 19760*a*b^4*x^12 + 27664*a^2*b^3*x^9 + 21280*a^3*
b^2*x^6 + 8645*a^4*b*x^3 + 1456*a^5)/x^19
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{20}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**20,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**20, x)
```

Giac [A]
time = 4.08, size = 107, normalized size = 0.42

$$-\frac{6916\,b^5x^{15}\mathrm{sgn}(bx^3+a)+19760\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+27664\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+21280\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+8645\,a^4bx^3\mathrm{sgn}(bx^3+a)+1456\,a^5\mathrm{sgn}(bx^3+a)}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^20,x, algorithm="giac")
```

```
[Out] -1/27664*(6916*b^5*x^15*sgn(b*x^3 + a) + 19760*a*b^4*x^12*sgn(b*x^3 + a) +
27664*a^2*b^3*x^9*sgn(b*x^3 + a) + 21280*a^3*b^2*x^6*sgn(b*x^3 + a) + 8645*
a^4*b*x^3*sgn(b*x^3 + a) + 1456*a^5*sgn(b*x^3 + a))/x^19
```

Mupad [B]
time = 1.31, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{19\,x^{19}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^4\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{16\,x^{16}\,\left(b\,x^3+a\right)}-\frac{a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^{10}\,\left(b\,x^3+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^20,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(19*x^19*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(4*x^4*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(16*x^16*(a + b*x^3)) - (a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(
1/2))/(x^10*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(
13*x^13*(a + b*x^3))
```

3.84 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}(a+bx^3)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b}}{11x^{11}(a+bx^3)}$$

```
[Out] -1/20*a^5*((b*x^3+a)^2)^(1/2)/x^20/(b*x^3+a)-5/17*a^4*b*((b*x^3+a)^2)^(1/2)
/x^17/(b*x^3+a)-5/7*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-10/11*a^2*b^
3*((b*x^3+a)^2)^(1/2)/x^11/(b*x^3+a)-5/8*a*b^4*((b*x^3+a)^2)^(1/2)/x^8/(b*x
^3+a)-1/5*b^5*((b*x^3+a)^2)^(1/2)/x^5/(b*x^3+a)
```

Rubi [A]

time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{5x^5(a+bx^3)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{8x^8(a+bx^3)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}(a+bx^3)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}(a+bx^3)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

```
[Out] -1/20*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^20*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(11*x^11*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(8*x^8*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(5*x^5
*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{21}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{21}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{21}}+\frac{5a^4b^6}{x^{18}}+\frac{10a^3b^7}{x^{15}}+\frac{10a^2b^8}{x^{12}}+\frac{5ab^9}{x^9}+\frac{b^{10}}{x^6}\right)dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{20x^{20}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3 -}}{7x^{14}(a+bx^3}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\,(2618a^5+15400a^4bx^3+37400a^3b^2x^6+47600a^2b^3x^9+32725ab^4x^{12}+10472b^5x^{15})}{52360x^{20}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^21,x]
```

```
[Out] -1/52360*(Sqrt[(a + b*x^3)^2]*(2618*a^5 + 15400*a^4*b*x^3 + 37400*a^3*b^2*x
^6 + 47600*a^2*b^3*x^9 + 32725*a*b^4*x^12 + 10472*b^5*x^15))/(x^20*(a + b*x
^3))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{20}a^5-\frac{5}{17}a^4b\,x^3-\frac{5}{7}b^2a^3x^6-\frac{10}{11}a^2b^3x^9-\frac{5}{8}b^4a\,x^{12}-\frac{1}{5}b^5x^{15}\right)}{(b\,x^3+a)x^{20}}$	79
gosper	$-\frac{(10472b^5x^{15}+32725b^4a\,x^{12}+47600a^2b^3x^9+37400b^2a^3x^6+15400a^4b\,x^3+2618a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{52360x^{20}(b\,x^3+a)^5}$	80
default	$-\frac{(10472b^5x^{15}+32725b^4a\,x^{12}+47600a^2b^3x^9+37400b^2a^3x^6+15400a^4b\,x^3+2618a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{52360x^{20}(b\,x^3+a)^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x,method=_RETURNVERBOSE)
```

```
[Out] -1/52360*(10472*b^5*x^15+32725*a*b^4*x^12+47600*a^2*b^3*x^9+37400*a^3*b^2*x
^6+15400*a^4*b*x^3+2618*a^5)*((b*x^3+a)^2)^(5/2)/x^20/(b*x^3+a)^5
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.23

$$-\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="maxima")
```

```
[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^3
*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

Fricas [A]
time = 0.39, size = 59, normalized size = 0.23

$$-\frac{10472\,b^5x^{15}+32725\,ab^4x^{12}+47600\,a^2b^3x^9+37400\,a^3b^2x^6+15400\,a^4bx^3+2618\,a^5}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="fricas")
```

```
[Out] -1/52360*(10472*b^5*x^15 + 32725*a*b^4*x^12 + 47600*a^2*b^3*x^9 + 37400*a^3
*b^2*x^6 + 15400*a^4*b*x^3 + 2618*a^5)/x^20
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{21}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**21,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**21, x)
```

Giac [A]
time = 5.56, size = 107, normalized size = 0.42

$$-\frac{10472\,b^5x^{15}\mathrm{sgn}(bx^3+a)+32725\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+47600\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+37400\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+15400\,a^4bx^3\mathrm{sgn}(bx^3+a)+2618\,a^5\mathrm{sgn}(bx^3+a)}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^21,x, algorithm="giac")
```

```
[Out] -1/52360*(10472*b^5*x^15*sgn(b*x^3 + a) + 32725*a*b^4*x^12*sgn(b*x^3 + a) +
 47600*a^2*b^3*x^9*sgn(b*x^3 + a) + 37400*a^3*b^2*x^6*sgn(b*x^3 + a) + 1540
0*a^4*b*x^3*sgn(b*x^3 + a) + 2618*a^5*sgn(b*x^3 + a))/x^20
```

Mupad [B]
time = 1.24, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{20\,x^{20}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{5\,x^5\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)}-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}-\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^{14}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^21,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(20*x^20*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(5*x^5*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(17*x^17*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3
)^(1/2))/(11*x^11*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(7*x^14*(a + b*x^3))
```

3.85 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{22}}\,dx$

Optimal. Leaf size=84

$$-\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}}+\frac{b(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}$$

```
[Out] -1/21*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^21+1/126*b*(b*x^3+a)^5*((b*x^3+a)
^2)^(1/2)/a^2/x^18
```

Rubi [A]

time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1369, 272, 47, 37}

$$\frac{b(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}-\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] -1/21*((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^21) + (b*(a + b*
x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(126*a^2*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{22}}\,dx &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{22}}\,dx}{b^4(ab+b^2x^3)}\\ &= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^8}\,dx,x,x^3\right)}{3b^4(ab+b^2x^3)}\\ &= -\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\text{Subst}\left(\int \frac{(ab+b^2x)^5}{x^7}\,a\right.}{21ab^3(ab+b^2x^3)}\\ &= -\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{21ax^{21}} + \frac{b(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{126a^2x^{18}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.99

$$-\frac{\sqrt{(a+bx^3)^2}\,(6a^5+35a^4bx^3+84a^3b^2x^6+105a^2b^3x^9+70ab^4x^{12}+21b^5x^{15})}{126x^{21}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^22,x]
```

```
[Out] -1/126*(Sqrt[(a + b*x^3)^2]*(6*a^5 + 35*a^4*b*x^3 + 84*a^3*b^2*x^6 + 105*a^
2*b^3*x^9 + 70*a*b^4*x^12 + 21*b^5*x^15))/(x^21*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.95

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{1}{21}a^5-\frac{5}{18}a^4b\,x^3-\frac{2}{3}b^2a^3x^6-\frac{5}{6}a^2b^3x^9-\frac{5}{9}b^4a\,x^{12}-\frac{1}{6}b^5x^{15}\right)}{(b\,x^3+a)x^{21}}$	79
gosper	$-\frac{(21b^5x^{15}+70b^4a\,x^{12}+105a^2b^3x^9+84b^2a^3x^6+35a^4b\,x^3+6a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{126x^{21}(b\,x^3+a)^5}$	80

default	$-\frac{\left(21 b^{5} x^{15}+70 b^{4} a \,x^{12}+105 a^{2} b^{3} x^{9}+84 b^{2} a^{3} x^{6}+35 a^{4} b \,x^{3}+6 a^{5}\right) \left(\left(b \,x^{3}+a \right)^{2}\right)^{\frac{5}{2}}}{126 x^{21} \left(b \,x^{3}+a \right)^{5}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x,method=_RETURNVERBOSE)
```

```
[Out] -1/126*(21*b^5*x^15+70*a*b^4*x^12+105*a^2*b^3*x^9+84*a^3*b^2*x^6+35*a^4*b*x
^3+6*a^5)*((b*x^3+a)^2)^(5/2)/x^21/(b*x^3+a)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(58) = 116$.
time = 0.29, size = 241, normalized size = 2.87

$$-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^7}{18\,a^7}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{5}{2}}b^6}{18\,a^6x^3}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^5}{18\,a^7x^6}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^4}{18\,a^6x^9}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^3}{18\,a^5x^{12}}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b^2}{18\,a^4x^{15}}+\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}b}{18\,a^3x^{18}}-\frac{(b^2x^6+2\,abx^3+a^2)^{\frac{7}{2}}}{21\,a^2x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="maxima")
```

```
[Out] -1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^7/a^7 - 1/18*(b^2*x^6 + 2*a*b*x^3
 + a^2)^(5/2)*b^6/(a^6*x^3) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/(a
^7*x^6) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^9) + 1/18*(b^2*
x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^3/(a^5*x^12) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a
^2)^(7/2)*b^2/(a^4*x^15) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^
18) - 1/21*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^21)
```

Fricas [A]
time = 0.36, size = 59, normalized size = 0.70

$$-\frac{21\,b^5x^{15}+70\,ab^4x^{12}+105\,a^2b^3x^9+84\,a^3b^2x^6+35\,a^4bx^3+6\,a^5}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="fricas")
```

```
[Out] -1/126*(21*b^5*x^15 + 70*a*b^4*x^12 + 105*a^2*b^3*x^9 + 84*a^3*b^2*x^6 + 35
*a^4*b*x^3 + 6*a^5)/x^21
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**22,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**22, x)
```

Giac [A]
time = 4.29, size = 107, normalized size = 1.27

$$-\frac{21\,b^5x^{15}\mathrm{sgn}(bx^3+a)+70\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+105\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+84\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+35\,a^4bx^3\mathrm{sgn}(bx^3+a)+6\,a^5\mathrm{sgn}(bx^3+a)}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^22,x, algorithm="giac")
```

```
[Out] -1/126*(21*b^5*x^15*sgn(b*x^3 + a) + 70*a*b^4*x^12*sgn(b*x^3 + a) + 105*a^2
*b^3*x^9*sgn(b*x^3 + a) + 84*a^3*b^2*x^6*sgn(b*x^3 + a) + 35*a^4*b*x^3*sgn(
b*x^3 + a) + 6*a^5*sgn(b*x^3 + a))/x^21
```

Mupad [B]
time = 1.22, size = 231, normalized size = 2.75

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{21\,x^{21}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^6\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{18\,x^{18}\,\left(b\,x^3+a\right)}-\frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,x^{12}\,\left(b\,x^3+a\right)}-\frac{2\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^22,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(6*x^6*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(18*x^18*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)
^(1/2))/(6*x^12*(a + b*x^3)) - (2*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
)/(3*x^15*(a + b*x^3))
```

3.86 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{23}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b}}{13x^{13}\left(a+bx^3\right)}$$

```
[Out] -1/22*a^5*((b*x^3+a)^2)^(1/2)/x^22/(b*x^3+a)-5/19*a^4*b*((b*x^3+a)^2)^(1/2)
/x^19/(b*x^3+a)-5/8*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^16/(b*x^3+a)-10/13*a^2*b^
3*((b*x^3+a)^2)^(1/2)/x^13/(b*x^3+a)-1/2*a*b^4*((b*x^3+a)^2)^(1/2)/x^10/(b*
x^3+a)-1/7*b^5*((b*x^3+a)^2)^(1/2)/x^7/(b*x^3+a)
```

Rubi [A]

time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{7x^7\left(a+bx^3\right)}-\frac{ab^4\sqrt{a^2+2abx^3+b^2x^6}}{2x^{10}\left(a+bx^3\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{13x^{13}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}\left(a+bx^3\right)}-\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}\left(a+bx^3\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{8x^{16}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
```

```
[Out] -1/22*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^22*(a + b*x^3)) - (5*a^4*b*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(19*x^19*(a + b*x^3)) - (5*a^3*b^2*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6])/(8*x^16*(a + b*x^3)) - (10*a^2*b^3*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6])/(13*x^13*(a + b*x^3)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6])/(2*x^10*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(7*x^7*
(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{23}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{23}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{23}}+\frac{5a^4b^6}{x^{20}}+\frac{10a^3b^7}{x^{17}}+\frac{10a^2b^8}{x^{14}}+\frac{5ab^9}{x^{11}}+\frac{b^{10}}{x^8}\right)dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{22x^{22}(a+bx^3)} - \frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}}{19x^{19}(a+bx^3)} - \frac{5a^3b^2\sqrt{a^2+2abx^3-}}{8x^{16}(a+bx^3}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\,(6916a^5+40040a^4bx^3+95095a^3b^2x^6+117040a^2b^3x^9+76076ab^4x^{12}+21736b^5x^{15})}{152152x^{22}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^23,x]
[Out] -1/152152*(Sqrt[(a + b*x^3)^2]*(6916*a^5 + 40040*a^4*b*x^3 + 95095*a^3*b^2*
x^6 + 117040*a^2*b^3*x^9 + 76076*a*b^4*x^12 + 21736*b^5*x^15))/(x^22*(a + b
*x^3))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.31

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,(-\frac{1}{22}a^5-\frac{5}{19}a^4b\,x^3-\frac{5}{8}b^2a^3x^6-\frac{10}{13}a^2b^3x^9-\frac{1}{2}b^4a\,x^{12}-\frac{1}{7}b^5x^{15})}{(b\,x^3+a)x^{22}}$	79
gosper	$-\frac{(21736b^5x^{15}+76076b^4a\,x^{12}+117040a^2b^3x^9+95095b^2a^3x^6+40040a^4b\,x^3+6916a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{152152x^{22}(b\,x^3+a)^5}$	80
default	$-\frac{(21736b^5x^{15}+76076b^4a\,x^{12}+117040a^2b^3x^9+95095b^2a^3x^6+40040a^4b\,x^3+6916a^5)\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}{152152x^{22}(b\,x^3+a)^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x,method=_RETURNVERBOSE)
[Out] -1/152152*(21736*b^5*x^15+76076*a*b^4*x^12+117040*a^2*b^3*x^9+95095*a^3*b^2
*x^6+40040*a^4*b*x^3+6916*a^5)*((b*x^3+a)^2)^(5/2)/x^22/(b*x^3+a)^5
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.23

$$-\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="maxima")
```

```
[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*a
^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

Fricas [A]
time = 0.36, size = 59, normalized size = 0.23

$$-\frac{21736\,b^5x^{15}+76076\,ab^4x^{12}+117040\,a^2b^3x^9+95095\,a^3b^2x^6+40040\,a^4bx^3+6916\,a^5}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="fricas")
```

```
[Out] -1/152152*(21736*b^5*x^15 + 76076*a*b^4*x^12 + 117040*a^2*b^3*x^9 + 95095*a
^3*b^2*x^6 + 40040*a^4*b*x^3 + 6916*a^5)/x^22
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{23}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**23,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**23, x)
```

Giac [A]
time = 3.85, size = 107, normalized size = 0.42

$$-\frac{21736\,b^5x^{15}\mathrm{sgn}(bx^3+a)+76076\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+117040\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+95095\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+40040\,a^4bx^3\mathrm{sgn}(bx^3+a)+6916\,a^5\mathrm{sgn}(bx^3+a)}{152152\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^23,x, algorithm="giac")
```

```
[Out] -1/152152*(21736*b^5*x^15*sgn(b*x^3 + a) + 76076*a*b^4*x^12*sgn(b*x^3 + a)
+ 117040*a^2*b^3*x^9*sgn(b*x^3 + a) + 95095*a^3*b^2*x^6*sgn(b*x^3 + a) + 40
040*a^4*b*x^3*sgn(b*x^3 + a) + 6916*a^5*sgn(b*x^3 + a))/x^22
```

Mupad [B]
time = 1.22, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{22\,x^{22}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^7\,\left(b\,x^3+a\right)}-\frac{a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{2\,x^{10}\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{19\,x^{19}\,\left(b\,x^3+a\right)}-\frac{10\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{13\,x^{13}\,\left(b\,x^3+a\right)}-\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^{16}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^23,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(22*x^22*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(7*x^7*(a + b*x^3)) - (a*b^4*(a^2 + b^2*x^6
+ 2*a*b*x^3)^(1/2))/(2*x^10*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a*b*
x^3)^(1/2))/(19*x^19*(a + b*x^3)) - (10*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)
^(1/2))/(13*x^13*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2
))/(8*x^16*(a + b*x^3))
```

3.87 $\int \frac{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}{x^{24}}\,dx$

Optimal. Leaf size=255

$$-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2}}{7x^{14}\left(a+bx^3\right)}$$

```
[Out] -1/23*a^5*((b*x^3+a)^2)^(1/2)/x^23/(b*x^3+a)-1/4*a^4*b*((b*x^3+a)^2)^(1/2)/
x^20/(b*x^3+a)-10/17*a^3*b^2*((b*x^3+a)^2)^(1/2)/x^17/(b*x^3+a)-5/7*a^2*b^3
*((b*x^3+a)^2)^(1/2)/x^14/(b*x^3+a)-5/11*a*b^4*((b*x^3+a)^2)^(1/2)/x^11/(b*
x^3+a)-1/8*b^5*((b*x^3+a)^2)^(1/2)/x^8/(b*x^3+a)
```

Rubi [A]
time = 0.04, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 276}

$$-\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}}{8x^8\left(a+bx^3\right)}-\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}}{11x^{11}\left(a+bx^3\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^3+b^2x^6}}{7x^{14}\left(a+bx^3\right)}-\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}\left(a+bx^3\right)}-\frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}\left(a+bx^3\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}}{17x^{17}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

```
[Out] -1/23*(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(x^23*(a + b*x^3)) - (a^4*b*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6])/(4*x^20*(a + b*x^3)) - (10*a^3*b^2*Sqrt[a^2 +
 2*a*b*x^3 + b^2*x^6])/(17*x^17*(a + b*x^3)) - (5*a^2*b^3*Sqrt[a^2 + 2*a*b*
x^3 + b^2*x^6])/(7*x^14*(a + b*x^3)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6])/(11*x^11*(a + b*x^3)) - (b^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(8*x^8*
(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{24}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{24}}\,dx}{b^4(ab+b^2x^3)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(\frac{a^5b^5}{x^{24}}+\frac{5a^4b^6}{x^{21}}+\frac{10a^3b^7}{x^{18}}+\frac{10a^2b^8}{x^{15}}+\frac{5ab^9}{x^{12}}+\frac{b^{10}}{x^9}\right)dx}{b^4(ab+b^2x^3)}$$
$$= -\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}}{23x^{23}(a+bx^3)} - \frac{a^4b\sqrt{a^2+2abx^3+b^2x^6}}{4x^{20}(a+bx^3)} - \frac{10a^3b^2\sqrt{a^2+2abx^3 -}}{17x^{17}(a+bx^3}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^3)^2}\,(10472a^5+60214a^4bx^3+141680a^3b^2x^6+172040a^2b^3x^9+109480ab^4x^{12}+30107b^5x^{15})}{240856x^{23}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^24,x]
```

```
[Out] -1/240856*(Sqrt[(a + b*x^3)^2]*(10472*a^5 + 60214*a^4*b*x^3 + 141680*a^3*b^
2*x^6 + 172040*a^2*b^3*x^9 + 109480*a*b^4*x^12 + 30107*b^5*x^15))/(x^23*(a
+ b*x^3))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.31

method	result	size
risch	$\frac{\sqrt{(b x^3+a)^2}\,(-\frac{1}{23}a^5-\frac{1}{4}a^4 b x^3-\frac{10}{17}b^2a^3x^6-\frac{5}{7}a^2b^3x^9-\frac{5}{11}b^4 a x^{12}-\frac{1}{8}b^5x^{15})}{(b x^3+a)x^{23}}$	79
gosper	$-\frac{(30107b^5x^{15}+109480b^4 a x^{12}+172040a^2b^3x^9+141680b^2a^3x^6+60214a^4 b x^3+10472a^5)\left((b x^3+a)^2\right)^{\frac{5}{2}}}{240856x^{23}(b x^3+a)^5}$	80
default	$-\frac{(30107b^5x^{15}+109480b^4 a x^{12}+172040a^2b^3x^9+141680b^2a^3x^6+60214a^4 b x^3+10472a^5)\left((b x^3+a)^2\right)^{\frac{5}{2}}}{240856x^{23}(b x^3+a)^5}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x,method=_RETURNVERBOSE)
```

```
[Out] -1/240856*(30107*b^5*x^15+109480*a*b^4*x^12+172040*a^2*b^3*x^9+141680*a^3*b
^2*x^6+60214*a^4*b*x^3+10472*a^5)*((b*x^3+a)^2)^(5/2)/x^23/(b*x^3+a)^5
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.23

$$-\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 141680
*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

Fricas [A]
time = 0.35, size = 59, normalized size = 0.23

$$-\frac{30107\,b^5x^{15}+109480\,ab^4x^{12}+172040\,a^2b^3x^9+141680\,a^3b^2x^6+60214\,a^4bx^3+10472\,a^5}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/240856*(30107*b^5*x^15 + 109480*a*b^4*x^12 + 172040*a^2*b^3*x^9 + 141680
*a^3*b^2*x^6 + 60214*a^4*b*x^3 + 10472*a^5)/x^23
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}{x^{24}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**24,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**24, x)
```

Giac [A]
time = 5.66, size = 107, normalized size = 0.42

$$-\frac{30107\,b^5x^{15}\mathrm{sgn}(bx^3+a)+109480\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+172040\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+141680\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+60214\,a^4bx^3\mathrm{sgn}(bx^3+a)+10472\,a^5\mathrm{sgn}(bx^3+a)}{240856\,x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^24,x, algorithm="giac")
```

```
[Out] -1/240856*(30107*b^5*x^15*sgn(b*x^3 + a) + 109480*a*b^4*x^12*sgn(b*x^3 + a)
 + 172040*a^2*b^3*x^9*sgn(b*x^3 + a) + 141680*a^3*b^2*x^6*sgn(b*x^3 + a) +
60214*a^4*b*x^3*sgn(b*x^3 + a) + 10472*a^5*sgn(b*x^3 + a))/x^23
```

Mupad [B]
time = 1.23, size = 231, normalized size = 0.91

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{23\,x^{23}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{8\,x^8\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{11\,x^{11}\,\left(b\,x^3+a\right)}-\frac{a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{4\,x^{20}\,\left(b\,x^3+a\right)}-\frac{5\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{7\,x^{14}\,\left(b\,x^3+a\right)}-\frac{10\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{17\,x^{17}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^24,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(23*x^23*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(8*x^8*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(11*x^11*(a + b*x^3)) - (a^4*b*(a^2 + b^2*x^6 + 2*a*b
*x^3)^(1/2))/(4*x^20*(a + b*x^3)) - (5*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^
(1/2))/(7*x^14*(a + b*x^3)) - (10*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)
)/(17*x^17*(a + b*x^3))
```

3.88 $\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}} dx$

Optimal. Leaf size=128

$$-\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}}+\frac{b(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}}-\frac{b^2(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{504a^3x^{18}}$$

```
[Out] -1/24*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a/x^24+1/84*b*(b*x^3+a)^5*((b*x^3+a)^
2)^(1/2)/a^2/x^21-1/504*b^2*(b*x^3+a)^5*((b*x^3+a)^2)^(1/2)/a^3/x^18
```

Rubi [A]
time = 0.04, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1369, 272, 47, 37}

$$-\frac{\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^5}{24ax^{24}}+\frac{b\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^5}{84a^2x^{21}}-\frac{b^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^5}{504a^3x^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

```
[Out] -1/24*((a + b*x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(a*x^24) + (b*(a + b*
x^3)^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(84*a^2*x^21) - (b^2*(a + b*x^3)^5*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(504*a^3*x^18)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1))), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^{5/2}}{x^{25}}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \frac{(ab+b^2x^3)^5}{x^{25}}\,dx}{b^4(ab+b^2x^3)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{(ab+b^2x)^5}{x^9}\,dx, x, x^3\right)}{3b^4(ab+b^2x^3)}$$

$$= -\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} - \frac{\sqrt{a^2+2abx^3+b^2x^6}\,\mathrm{Subst}\left(\int \frac{(ab+b^2x)^5}{x^8}\,d\right.}{12ab^3(ab+b^2x^3)}$$

$$= -\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} + \frac{b(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}} + \frac{\sqrt{a^2}}{}$$

$$= -\frac{(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{24ax^{24}} + \frac{b(a+bx^3)^5\sqrt{a^2+2abx^3+b^2x^6}}{84a^2x^{21}} - \frac{b^2(a}{}$$

Mathematica [A]
time = 0.01, size = 83, normalized size = 0.65

$$-\frac{\sqrt{(a+bx^3)^2}\,(21a^5+120a^4bx^3+280a^3b^2x^6+336a^2b^3x^9+210ab^4x^{12}+56b^5x^{15})}{504x^{24}(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)/x^25,x]
```

```
[Out] -1/504*(Sqrt[(a + b*x^3)^2]*(21*a^5 + 120*a^4*b*x^3 + 280*a^3*b^2*x^6 + 336
*a^2*b^3*x^9 + 210*a*b^4*x^12 + 56*b^5*x^15))/(x^24*(a + b*x^3))
```

Maple [A]
time = 0.03, size = 80, normalized size = 0.62

method	result	size
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{1}{24}a^{5}-\frac{1}{9}b^{5}x^{15}-\frac{5}{12}b^{4}a\,x^{12}-\frac{2}{3}a^{2}b^{3}x^{9}-\frac{5}{9}b^{2}a^{3}x^{6}-\frac{5}{21}a^{4}b\,x^{3}\right)}{\left(b\,x^{3}+a\right)x^{24}}$	79
gosper	$-\frac{\left(56b^{5}x^{15}+210b^{4}a\,x^{12}+336a^{2}b^{3}x^{9}+280b^{2}a^{3}x^{6}+120a^{4}b\,x^{3}+21a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{504x^{24}\left(b\,x^{3}+a\right)^{5}}$	80
default	$-\frac{\left(56b^{5}x^{15}+210b^{4}a\,x^{12}+336a^{2}b^{3}x^{9}+280b^{2}a^{3}x^{6}+120a^{4}b\,x^{3}+21a^{5}\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{\frac{5}{2}}}{504x^{24}\left(b\,x^{3}+a\right)^{5}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x,method=_RETURNVERBOSE)
```

```
[Out] -1/504*(56*b^5*x^15+210*a*b^4*x^12+336*a^2*b^3*x^9+280*a^3*b^2*x^6+120*a^4*
b*x^3+21*a^5)*((b*x^3+a)^2)^(5/2)/x^24/(b*x^3+a)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(89) = 178$.
time = 0.30, size = 272, normalized size = 2.12

$$\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{8}}{18\,a^{8}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{5}{2}}b^{7}}{18\,a^{7}x^{3}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{6}}{18\,a^{8}x^{6}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{5}}{18\,a^{7}x^{9}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{4}}{18\,a^{6}x^{12}}+\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{3}}{18\,a^{5}x^{15}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b^{2}}{18\,a^{4}x^{18}}+\frac{3\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}b}{56\,a^{3}x^{21}}-\frac{\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{7}{2}}}{24\,a^{2}x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="maxima")
```

```
[Out] 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2)*b^8/a^8 + 1/18*(b^2*x^6 + 2*a*b*x^3
+ a^2)^(5/2)*b^7/(a^7*x^3) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^6/(a^
8*x^6) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^5/(a^7*x^9) - 1/18*(b^2*x
^6 + 2*a*b*x^3 + a^2)^(7/2)*b^4/(a^6*x^12) + 1/18*(b^2*x^6 + 2*a*b*x^3 + a^
2)^(7/2)*b^3/(a^5*x^15) - 1/18*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b^2/(a^4*x
^18) + 3/56*(b^2*x^6 + 2*a*b*x^3 + a^2)^(7/2)*b/(a^3*x^21) - 1/24*(b^2*x^6
+ 2*a*b*x^3 + a^2)^(7/2)/(a^2*x^24)
```

Fricas [A]
time = 0.37, size = 59, normalized size = 0.46

$$-\frac{56\,b^{5}x^{15}+210\,ab^{4}x^{12}+336\,a^{2}b^{3}x^{9}+280\,a^{3}b^{2}x^{6}+120\,a^{4}bx^{3}+21\,a^{5}}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="fricas")
```

```
[Out] -1/504*(56*b^5*x^15 + 210*a*b^4*x^12 + 336*a^2*b^3*x^9 + 280*a^3*b^2*x^6 +
120*a^4*b*x^3 + 21*a^5)/x^24
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^3)^2\right)^{\frac{5}{2}}}{x^{25}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**(5/2)/x**25,x)
```

```
[Out] Integral(((a + b*x**3)**2)**(5/2)/x**25, x)
```

Giac [A]
time = 3.26, size = 107, normalized size = 0.84

$$-\frac{56\,b^5x^{15}\mathrm{sgn}(bx^3+a)+210\,ab^4x^{12}\mathrm{sgn}(bx^3+a)+336\,a^2b^3x^9\mathrm{sgn}(bx^3+a)+280\,a^3b^2x^6\mathrm{sgn}(bx^3+a)+120\,a^4bx^3\mathrm{sgn}(bx^3+a)+21\,a^5\mathrm{sgn}(bx^3+a)}{504\,x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^(5/2)/x^25,x, algorithm="giac")
```

```
[Out] -1/504*(56*b^5*x^15*sgn(b*x^3 + a) + 210*a*b^4*x^12*sgn(b*x^3 + a) + 336*a^
2*b^3*x^9*sgn(b*x^3 + a) + 280*a^3*b^2*x^6*sgn(b*x^3 + a) + 120*a^4*b*x^3*s
gn(b*x^3 + a) + 21*a^5*sgn(b*x^3 + a))/x^24
```

Mupad [B]
time = 1.22, size = 231, normalized size = 1.80

$$-\frac{a^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{24\,x^{24}\,\left(b\,x^3+a\right)}-\frac{b^5\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^9\,\left(b\,x^3+a\right)}-\frac{5\,a\,b^4\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,x^{12}\,\left(b\,x^3+a\right)}-\frac{5\,a^4\,b\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{21\,x^{21}\,\left(b\,x^3+a\right)}-\frac{2\,a^2\,b^3\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,x^{15}\,\left(b\,x^3+a\right)}-\frac{5\,a^3\,b^2\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{9\,x^{18}\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)/x^25,x)
```

```
[Out] - (a^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(24*x^24*(a + b*x^3)) - (b^5*(a^2
 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(9*x^9*(a + b*x^3)) - (5*a*b^4*(a^2 + b^2*x^
6 + 2*a*b*x^3)^(1/2))/(12*x^12*(a + b*x^3)) - (5*a^4*b*(a^2 + b^2*x^6 + 2*a
*b*x^3)^(1/2))/(21*x^21*(a + b*x^3)) - (2*a^2*b^3*(a^2 + b^2*x^6 + 2*a*b*x^
3)^(1/2))/(3*x^15*(a + b*x^3)) - (5*a^3*b^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/
2))/(9*x^18*(a + b*x^3))
```

3.89 $\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=240

$$\frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a^{2/3}(a+b}{}$$

```
[Out] 1/2*x^2*(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)+1/3*a^(2/3)*(b*x^3+a)*ln(a^(1/3)+b^
(1/3)*x)/b^(5/3)/((b*x^3+a)^2)^(1/2)-1/6*a^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/b^(5/3)/((b*x^3+a)^2)^(1/2)+1/3*a^(2/3)*(b*x^3+a)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)*3^(1/2)/((b*x^3+
a)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1369, 327, 298, 31, 648, 631, 210, 642}

$$\frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}(a+bx^3)\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{a^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x^2*(a + b*x^3))/(2*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x
^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(5/3)*Sqr
t[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (a^(2/3)*(a + b*x^3)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(ab+b^2x^3)\int \frac{x^4}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a(ab+b^2x^3))\int \frac{x}{ab+b^2x^3}\,dx}{b\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a^{2/3}(ab+b^2x^3))\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a^{2/3}(ab+}{3}$$
$$= \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a^{2/3}(ab+b^2x}{6b^{8/3}\sqrt{}}$$
$$= \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{a^{2/3}(a+bx^3)\,\mathrm{l}}{6b^{5/3}}$$
$$= \frac{x^2(a+bx^3)}{2b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{a^{2/3}(a+bx^3}{3b^{5/3}\sqrt{a^2}}$$

Mathematica [A]
time = 0.04, size = 131, normalized size = 0.55

$$\frac{(a+bx^3)\left(3b^{2/3}x^2+2\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6b^{5/3}\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(3*b^(2/3)*x^2 + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x] - a^(2/3)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(5/3)*Sqrt[(a + b*x^3)^2])
```

Maple [A]
time = 0.14, size = 113, normalized size = 0.47

method	result	size
risch	$\frac{x^2\sqrt{(b\,x^3+a)^2}}{2(b\,x^3+a)b} - \frac{\sqrt{(b\,x^3+a)^2}\,a\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\ln(x-\_R)}{\_R}\right)}{3(b\,x^3+a)b^2}$	77

default	$\frac{(b\,x^{3}+a)\left(3x^{2}b\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}\,a+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)a-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)a\right)}{6\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(b*x^3+a)*(3*x^2*b*(a/b)^(1/3)+2*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/
(a/b)^(1/3))*3^(1/2)*a+2*ln(x+(a/b)^(1/3))*a-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*a)/((b*x^3+a)^2)^(1/2)/b^2/(a/b)^(1/3)
```

Maxima [A]
time = 0.49, size = 109, normalized size = 0.45

$$\frac{x^{2}}{2\,b}-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{2}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{2}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{2}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(b^2*(a/b)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a
/b)^(1/3)) + 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(1/3))
```

Fricas [A]
time = 0.41, size = 123, normalized size = 0.51

$$\frac{3\,x^{2}-2\,\sqrt{3}\,\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)-\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(ax^{2}-bx\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}+a\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\right)+2\left(\frac{a^{2}}{b^{2}}\right)^{\frac{1}{3}}\log\left(ax+b\left(\frac{a^{2}}{b^{2}}\right)^{\frac{2}{3}}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(3*x^2 - 2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^
(1/3) - sqrt(3)*a)/a) - (a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a
*(a^2/b^2)^(1/3)) + 2*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b
```

Sympy [A]
time = 0.07, size = 32, normalized size = 0.13

$$\operatorname{RootSum}\left(27t^{3}b^{5}-a^{2},\left(t\mapsto t\log\left(\frac{9t^{2}b^{3}}{a}+x\right)\right)\right)+\frac{x^{2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*b**5 - a**2, Lambda(_t, _t*log(9*_t**2*b**3/a + x))) + x**
2/(2*b)
```

Giac [A]
time = 3.38, size = 146, normalized size = 0.61

$$\frac{x^2\operatorname{sgn}(bx^3+a)}{2\,b}+\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}(bx^3+a)}{3\,b}+\frac{\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}(bx^3+a)}{3\,b^3}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sgn}(bx^3+a)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*sgn(b*x^3 + a)/b + 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn(
b*x^3 + a)/b + 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^3 - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-
a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x^4/((a + b*x^3)^2)^(1/2), x)
```

3.90 $\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=235

$$\frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\,(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{a}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\,(a+bx^3)}{6b}$$

```
[Out] x*(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)-1/3*a^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*
x)/b^(4/3)/((b*x^3+a)^2)^(1/2)+1/6*a^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/b^(4/3)/((b*x^3+a)^2)^(1/2)+1/3*a^(1/3)*(b*x^3+a)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(4/3)*3^(1/2)/((b*x^3+a)^2)^
(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1369, 327, 206, 31, 648, 631, 210, 642}

$$\frac{\sqrt[3]{a}\,(a+bx^3)\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{a}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{a}\,(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(a + b*x^3))/(b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*
ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(4/3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) - (a^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/
(3*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (a^(1/3)*(a + b*x^3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(ab+b^2x^3)\int \frac{x^3}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a(ab+b^2x^3))\int \frac{1}{ab+b^2x^3}\,dx}{b\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{a}\,(ab+b^2x^3)\right)\int \frac{1}{\sqrt[3]{a}\,\sqrt[3]{b}+b^{2/3}x}\,dx}{3b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{a}\,(ab+b\right.}{3b^{5/}}$$
$$= \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(\sqrt[3]{a}\,(ab+b^2x^3)\right)}{6b^{7/3}\sqrt{a}}$$
$$= \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}\,(a+bx^3)\log}{6b^{4/3}\sqrt{}}$$
$$= \frac{x(a+bx^3)}{b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{a}\,(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{a}\,(a+bx^3)\,\mathrm{l}}{3b^{4/3}\sqrt{a^2+}}$$

Mathematica [A]
time = 0.02, size = 128, normalized size = 0.54

$$\frac{(a+bx^3)\left(6\sqrt[3]{b}\,x+2\sqrt{3}\,\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{6b^{4/3}\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(6*b^(1/3)*x + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(
1/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x] + a^(1/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(6*b^(4/3)*Sqrt[(a + b*x^3)^2])
```

Maple [A]
time = 0.13, size = 110, normalized size = 0.47

method	result	size
risch	$\frac{x\sqrt{(b\,x^3+a)^2}}{(b\,x^3+a)b} - \frac{\sqrt{(b\,x^3+a)^2}\;a\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{3(b\,x^3+a)b^2}$	74

default	$\frac{(b\,x^3+a)\left(6xb\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}\,a-2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)a+\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)a\right)}{6\sqrt{(b\,x^3+a)^2}\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	110

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(b*x^3+a)*(6*x*b*(a/b)^(2/3)+2*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a
/b)^(1/3))*3^(1/2)*a-2*ln(x+(a/b)^(1/3))*a+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)*a)/((b*x^3+a)^2)^(1/2)/b^2/(a/b)^(2/3)
```

Maxima [A]
time = 0.53, size = 106, normalized size = 0.45

$$\frac{x}{b}-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/b - 1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^
2*(a/b)^(2/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*(a/b)^(2
/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

Fricas [A]
time = 0.37, size = 106, normalized size = 0.45

$$\frac{2\,\sqrt{3}\,\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+6\,x}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - sqrt(3
)*a)/a) - (-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) + 2*(-a/b)^
(1/3)*log(x - (-a/b)^(1/3)) + 6*x)/b
```

Sympy [A]
time = 0.06, size = 22, normalized size = 0.09

$$\operatorname{RootSum}\left(27t^3b^4+a,\left(t\mapsto t\log\left(-3tb+x\right)\right)\right)+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*b**4 + a, Lambda(_t, _t*log(-3*_t*b + x))) + x/b
```

Giac [A]
time = 2.79, size = 143, normalized size = 0.61

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}(bx^3+a)}{3\,b}+\frac{x\operatorname{sgn}(bx^3+a)}{b}-\frac{\sqrt{3}\,(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}(bx^3+a)}{3\,b^2}-\frac{(-ab^2)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sgn}(bx^3+a)}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*x^3 + a)/b + x*sgn(b*x^3
+ a)/b - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))
/(-a/b)^(1/3))*sgn(b*x^3 + a)/b^2 - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))*sgn(b*x^3 + a)/b^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x^3/((a + b*x^3)^2)^(1/2), x)
```

3.91 $\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/3*(b*x^3+a)*ln(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1366, 622, 31}

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 622

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx,x,x^3\right)$$
$$= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.80

$$\frac{(a+bx^3)\log(a+bx^3)}{3b\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] ((a + b*x^3)*Log[a + b*x^3])/(3*b*Sqrt[(a + b*x^3)^2])
```

Maple [A]
time = 0.12, size = 32, normalized size = 0.73

method	result	size
default	$\frac{(b\,x^3+a)\ln(b\,x^3+a)}{3b\sqrt{(b\,x^3+a)^2}}$	32
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\ln(b\,x^3+a)}{3(b\,x^3+a)b}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*(b*x^3+a)*ln(b*x^3+a)/b/((b*x^3+a)^2)^(1/2)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 0.34

$$\frac{\log\left(x^3+\frac{a}{b}\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^3 + a/b)/b
```

Fricas [A]
time = 0.37, size = 13, normalized size = 0.30

$$\frac{\log\left(bx^3+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*log(b*x^3 + a)/b
```

Sympy [A]
time = 0.05, size = 10, normalized size = 0.23

$$\frac{\log\left(a+bx^3\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x**3+a)**2)**(1/2),x)
[Out] log(a + b*x**3)/(3*b)
```

Giac [A]
time = 3.70, size = 22, normalized size = 0.50

$$\frac{\log\left(|bx^3+a|\right)\operatorname{sgn}(bx^3+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/3*log(abs(b*x^3 + a))*sgn(b*x^3 + a)/b
```

Mupad [B]
time = 1.39, size = 33, normalized size = 0.75

$$\frac{\ln\left(b^2\,x^3+a\,b\right)\,\mathrm{sign}(2\,b^2\,x^3+2\,a\,b)}{3\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^2)^(1/2),x)
[Out] (log(a*b + b^2*x^3)*sign(2*a*b + 2*b^2*x^3))/(3*(b^2)^(1/2))
```

3.92 $\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=202

$$-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -1/3*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)+1/
6*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(2/3)/((b*x
^3+a)^2)^(1/2)-1/3*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/
2))/a^(1/3)/b^(2/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1369, 298, 31, 648, 631, 210, 642}

$$-\frac{(a+bx^3)\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) - ((a + b*x^3)*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(1/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a +
 b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3)*b^(2/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(ab+b^2x^3)\int \frac{x}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{(ab+b^2x^3)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3\sqrt[3]{a}\,b\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\int \frac{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}\,bx+b^{4/3}x^2}\,dx}{3\sqrt[3]{a}\,b\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\int \frac{-\sqrt[3]{a}\,b+2b^{4/3}x}{a^{2/3}b^{2/3}-\sqrt[3]{a}\,bx+b^{4/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab}{}$$

$$= -\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= -\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3\sqrt[3]{a}\,b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+b}{6}$$

Mathematica [A]
time = 0.02, size = 109, normalized size = 0.54

$$\frac{(a+bx^3)\left(-2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{6\sqrt[3]{a}\,b^{2/3}\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((a + b*x^3)*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Lo
g[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(
6*a^(1/3)*b^(2/3)*Sqrt[(a + b*x^3)^2])
```

Maple [A]
time = 0.14, size = 97, normalized size = 0.48

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{3(b\,x^3+a)b}$	47

default	$-\frac{\left(b\,x^{3}+a\right)\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{\left(b\,x^{3}+a\right)^{2}}\,b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b*x^3+a)*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3)
)+2*ln(x+(a/b)^(1/3))-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))/((b*x^3+a)^2)^(1/2
)/b/(a/b)^(1/3)
```

Maxima [A]
time = 0.49, size = 98, normalized size = 0.49

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(1
/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3)) - 1/3*log
(x + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

Fricas [A]
time = 0.38, size = 304, normalized size = 1.50

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^{2}x^{3}-ab+3\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^{2}\right)^{\frac{2}{3}}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}-3\left(-ab^{2}\right)^{\frac{2}{3}}x}{bx^{3}+a}\right)+\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}bx+\left(-ab^{2}\right)^{\frac{2}{3}}\right)-2\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(bx-\left(-ab^{2}\right)^{\frac{1}{3}}\right)}{6\,ab^{2}},\ \frac{6\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx+\left(-ab^{2}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}}{b}\right)+\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}bx+\left(-ab^{2}\right)^{\frac{2}{3}}\right)-2\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(bx-\left(-ab^{2}\right)^{\frac{1}{3}}\right)}{6\,ab^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(
1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/
a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + (-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^
2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3))
)/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2
*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*log(b^2*
x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x - (-a
*b^2)^(1/3)))/(a*b^2)]
```

Sympy [A]
time = 0.05, size = 24, normalized size = 0.12

$$\operatorname{RootSum}\left(27t^3ab^2+1,\left(t\mapsto t\log\left(9t^2ab+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 + 1, Lambda(_t, _t*log(9*_t**2*a*b + x)))
```

Giac [A]
time = 4.32, size = 124, normalized size = 0.61

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)\operatorname{sgn}(bx^3+a)}{3\left(-ab^2\right)^{\frac{1}{3}}}-\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)\operatorname{sgn}(bx^3+a)}{6\left(-ab^2\right)^{\frac{1}{3}}}-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)\operatorname{sgn}(bx^3+a)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))*sgn(b*x^3
 + a)/(-a*b^2)^(1/3) - 1/6*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))*sgn(b*x
^3 + a)/(-a*b^2)^(1/3) - 1/3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))*sgn(b*
x^3 + a)/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(x/((a + b*x^3)^2)^(1/2), x)
```

3.93 $\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=202

$$-\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}\ldots\right.}{6a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/3*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(1/3)/((b*x^3+a)^2)^(1/2)-1/6
*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(1/3)/((b*x^
3+a)^2)^(1/2)-1/3*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(2/3)/b^(1/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1357, 206, 31, 648, 631, 210, 642}

$$-\frac{(a+bx^3)\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -(((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + ((a + b*x^3)*Log[a^(1/3
) + b^(1/3)*x])/(3*a^(2/3)*b^(1/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a +
 b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(1/3)*
Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(2ab+2b^2x^3)\int \frac{1}{2ab+2b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{(2ab+2b^2x^3)\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}+\sqrt[3]{2}\,b^{2/3}x}\,dx}{3\;2^{2/3}a^{2/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(2ab+2b^2x^3)\int \frac{2\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{b}}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{}}}{3\;2^{2/3}a^{2/3}b^{2/3}\sqrt{a^2+2abx}}$$

$$= \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(2ab+2b^2x^3)\int \frac{-2^{2/3}\sqrt[3]{a}\,b+2\;2^{2/3}b^{4/3}x}{2^{2/3}a^{2/3}b^{2/3}-2^{2/3}\sqrt[3]{a}\,bx+2^{2/3}b^{4/3}}}{12a^{2/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= -\frac{(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{2/3}\sqrt[3]{b}\,\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+}{}$$

__Mathematica__ [A]
time = 0.01, size = 109, normalized size = 0.54

$$-\frac{(a+bx^3)\left(2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{6a^{2/3}\sqrt[3]{b}\,\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] -1/6*((a + b*x^3)*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] -
2*Log[a^(1/3) + b^(1/3)*x] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
))/(a^(2/3)*b^(1/3)*Sqrt[(a + b*x^3)^2])
```

__Maple__ [A]
time = 0.13, size = 97, normalized size = 0.48

method	result	size
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{3(b\,x^3+a)b}$	47

default	$\frac{(b\,x^3+a)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)-\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\right)}{6\sqrt{(b\,x^3+a)^2}\,b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(b*x^3+a)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3)
)+2*ln(x+(a/b)^(1/3))-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)))/((b*x^3+a)^2)^(1/2
)/b/(a/b)^(2/3)
```

Maxima [A]
time = 0.49, size = 98, normalized size = 0.49

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^(2
/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3)) + 1/3*log
(x + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

Fricas [A]
time = 0.42, size = 299, normalized size = 1.48

$$\left[\frac{3\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a^2b)^{\frac{2}{3}}x+(a^2b)^{\frac{1}{3}}a\right)+2\,(a^2b)^{\frac{2}{3}}\log\left(abx+(a^2b)^{\frac{2}{3}}\right)}{6\,a^2b},\ \frac{6\sqrt{\frac{1}{3}}\,ab\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\,\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a^2b)^{\frac{2}{3}}x+(a^2b)^{\frac{1}{3}}a\right)+2\,(a^2b)^{\frac{2}{3}}\log\left(abx+(a^2b)^{\frac{2}{3}}\right)}{6\,a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^
(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a
^2*b), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*
b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*lo
g(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*x
+ (a^2*b)^(2/3)))/(a^2*b)]
```

Sympy [A]
time = 0.06, size = 20, normalized size = 0.10

$$\operatorname{RootSum}\left(27t^3a^2b-1,\left(t\mapsto t\log\left(3ta+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**2*b - 1, Lambda(_t, _t*log(3*_t*a + x)))
```

Giac [A]
time = 4.91, size = 122, normalized size = 0.60

$$-\frac{1}{6}\left(\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}\right)\operatorname{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a - 2*sqrt(3)*(-a*b^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) - (-a*b^2)^(1
/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b))*sgn(b*x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2)^(1/2),x)
```

```
[Out] int(1/((a + b*x^3)^2)^(1/2), x)
```

3.94 $\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=80

$$\frac{(a+bx^3)\log(x)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (b*x^3+a)*ln(x)/a/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a/((b*x^3+a
)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1369, 272, 36, 29, 31}

$$\frac{\log(x)\left(a+bx^3\right)}{a\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] ((a + b*x^3)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log
[a + b*x^3])/(3*a*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{(ab+b^2x^3)\int \frac{1}{x(ab+b^2x^3)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{x(ab+b^2x)}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)}{3ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(b(ab+b^2x^3))\,\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{(a+bx^3)\log(x)}{a\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(a+bx^3)\log\left(a+bx^3\right)}{3a\sqrt{a^2+2abx^3+b^2x^6}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 0.52

$$\frac{(a+bx^3)\left(3\log(x)-\log\left(a+bx^3\right)\right)}{3a\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
[Out] ((a + b*x^3)*(3*Log[x] - Log[a + b*x^3]))/(3*a*Sqrt[(a + b*x^3)^2])
```

Maple [A]
time = 0.15, size = 37, normalized size = 0.46

method	result	size
default	$-\frac{(b\,x^3+a)\left(\ln(b\,x^3+a)-3\ln(x)\right)}{3\sqrt{(b\,x^3+a)^2}\;a}$	37
risch	$-\frac{\sqrt{(b\,x^3+a)^2}\;\ln(b\,x^3+a)}{3(b\,x^3+a)a}+\frac{\sqrt{(b\,x^3+a)^2}\;\ln(x)}{(b\,x^3+a)a}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x^3+a)*(ln(b*x^3+a)-3*ln(x))/((b*x^3+a)^2)^(1/2)/a
```

Maxima [A]
time = 0.27, size = 43, normalized size = 0.54

$$-\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a
```

Fricas [A]
time = 0.37, size = 18, normalized size = 0.22

$$-\frac{\log\left(bx^3+a\right)-3\,\log\left(x\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(log(b*x^3 + a) - 3*log(x))/a
```

Sympy [A]
time = 0.10, size = 15, normalized size = 0.19

$$\frac{\log\left(x\right)}{a}-\frac{\log\left(\frac{a}{b}+x^3\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] log(x)/a - log(a/b + x**3)/(3*a)
```

Giac [A]
time = 4.63, size = 32, normalized size = 0.40

$$-\frac{1}{3}\left(\frac{\log\left(|bx^3+a|\right)}{a}-\frac{3\,\log\left(|x|\right)}{a}\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(log(abs(b*x^3 + a))/a - 3*log(abs(x))/a)*sgn(b*x^3 + a)
```

Mupad [B]
time = 1.39, size = 48, normalized size = 0.60

$$-\frac{\ln\left(a\,b+\frac{a^2}{x^3}+\frac{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{x^3}\right)}{3\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] -log(a*b + a^2/x^3 + ((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/x^3)/(
3*(a^2)^(1/2))
```

3.95 $\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=238

$$-\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\,(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}\,(a+$$

```
[Out] (-b*x^3-a)/a/x/((b*x^3+a)^2)^(1/2)+1/3*b^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)
*x)/a^(4/3)/((b*x^3+a)^2)^(1/2)-1/6*b^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/a^(4/3)/((b*x^3+a)^2)^(1/2)+1/3*b^(1/3)*(b*x^3+a)*arct
an(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)*3^(1/2)/((b*x^3+a)^2)
^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1369, 331, 298, 31, 648, 631, 210, 642}

$$-\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\,(a+bx^3)\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\sqrt[3]{b}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\sqrt[3]{b}\,(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -((a + b*x^3)/(a*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])) + (b^(1/3)*(a + b*x^3)
*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(4/3)*Sqrt[a
^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])
/(3*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(1/3)*(a + b*x^3)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*Sqrt[a^2 + 2*a*b*x^3 +
b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(ab+b^2x^3)\int \frac{1}{x^2(ab+b^2x^3)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(b(ab+b^2x^3))\int \frac{x}{ab+b^2x^3}\,dx}{a\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2x^3)\int \frac{1}{\sqrt[3]{a}\sqrt[3]{b}+b^{2/3}x}\,dx}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(ab+b^2x^3)\int}{3a^{4/3}\sqrt{a}}$$
$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(ab+b^2x^3)}{6a^{4/3}b^{2/}}$$
$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\sqrt[3]{b}(a+bx}{6a}$$
$$= -\frac{a+bx^3}{ax\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\sqrt[3]{b}(a+}{3a^{4/3}}$$

Mathematica [A]
time = 0.02, size = 133, normalized size = 0.56

$$-\frac{(a+bx^3)\left(6\sqrt[3]{a}-2\sqrt{3}\sqrt[3]{b}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{b}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+\sqrt[3]{b}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)\right)}{6a^{4/3}x\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/6*((a + b*x^3)*(6*a^(1/3) - 2*Sqrt[3]*b^(1/3)*x*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]] - 2*b^(1/3)*x*Log[a^(1/3) + b^(1/3)*x] + b^(1/3)*x*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(4/3)*x*Sqrt[(a + b*x^3)^2]
)
```

Maple [A]
time = 0.14, size = 111, normalized size = 0.47

method	result	size
risch	$-\frac{\sqrt{(bx^3+a)^2}}{(bx^3+a)ax}+\frac{\sqrt{(bx^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(a^4\_Z^3-b)}\_R\ln\left(\left(-4\_R^3a^4+3b\right)x-a^3\_R^2\right)\right)}{3bx^3+3a}$	93

default	$-\frac{(b\,x^{3}+a)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)x-2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)x+\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x+6\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\sqrt{(b\,x^{3}+a)^{2}}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}a x}$	111

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b*x^3+a)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3
))*x-2*ln(x+(a/b)^(1/3))*x+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x+6*(a/b)^(1/3
))/((b*x^3+a)^2)^(1/2)/(a/b)^(1/3)/a/x
```

Maxima [A]
time = 0.52, size = 106, normalized size = 0.45

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
1/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/3)) + 1/3*lo
g(x + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 1/(a*x)
```

Fricas [A]
time = 0.38, size = 103, normalized size = 0.43

$$-\frac{2\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-2\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)+6}{6\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3
)) + x*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 2*x*(b/a)
^(1/3)*log(b*x + a*(b/a)^(2/3)) + 6)/(a*x)
```

Sympy [A]
time = 0.08, size = 29, normalized size = 0.12

$$\operatorname{RootSum}\left(27t^{3}a^{4}-b,\left(t\mapsto t\log\left(\frac{9t^{2}a^{3}}{b}+x\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**4 - b, Lambda(_t, _t*log(9*_t**2*a**3/b + x))) - 1/(a*x
)
```

Giac [A]
time = 4.52, size = 131, normalized size = 0.55

$$\frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{2\,\sqrt{3}\,(-ab^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b}-\frac{(-ab^2)^{\frac{2}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b}-\frac{6}{ax}\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 2*sqrt(3)*(-a*b^2)^(
2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - (-a*b^
2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 6/(a*x))*sgn(b*
x^3 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*((a + b*x^3)^2)^(1/2)), x)
```

3.96 $\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=243

$$\frac{-a-bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}(a+$$

```
[Out] 1/2*(-b*x^3-a)/a/x^2/((b*x^3+a)^2)^(1/2)-1/3*b^(2/3)*(b*x^3+a)*ln(a^(1/3)+b
^(1/3)*x)/a^(5/3)/((b*x^3+a)^2)^(1/2)+1/6*b^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1
/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/((b*x^3+a)^2)^(1/2)+1/3*b^(2/3)*(b*x^3+a
)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)*3^(1/2)/((b*x^3
+a)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 240, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1369, 331, 206, 31, 648, 631, 210, 642}

$$-\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}(a+bx^3)\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/2*(a + b*x^3)/(a*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*
x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)
*x])/(3*a^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b^(2/3)*(a + b*x^3)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(5/3)*Sqrt[a^2 + 2*a*b*x^
3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(ab+b^2x^3)\int \frac{1}{x^3(ab+b^2x^3)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(b(ab+b^2x^3))\int \frac{1}{ab+b^2x^3}\,dx}{a\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{b}\,(ab+b^2x^3)\right)\int \frac{1}{\sqrt[3]{a}\,\sqrt[3]{b}+b^{2/3}x}\,dx}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(\sqrt[3]{\right.}{}$$
$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(ab+b^2}{6a^{5/3}}$$
$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}(a+}{}$$
$$= -\frac{a+bx^3}{2ax^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{b^{2/3}(a+bx^3)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b^{2/3}(a}{3a}$$

Mathematica [A]
time = 0.02, size = 140, normalized size = 0.58

$$-\frac{(a+bx^3)\left(3a^{2/3}-2\sqrt{3}\,b^{2/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2b^{2/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{6a^{5/3}x^2\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/6*((a + b*x^3)*(3*a^(2/3) - 2*Sqrt[3]*b^(2/3)*x^2*ArcTan[(1 - (2*b^(1/3)
*x)/a^(1/3))/Sqrt[3]] + 2*b^(2/3)*x^2*Log[a^(1/3) + b^(1/3)*x] - b^(2/3)*x^
2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(a^(5/3)*x^2*Sqrt[(a + b
*x^3)^2])
```

Maple [A]
time = 0.13, size = 118, normalized size = 0.49

method	result	size
risch	$-\frac{\sqrt{(b\,x^3+a)^2}}{2(b\,x^3+a)a\,x^2}+\frac{\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^5\_Z^3+b^2\right)}\_R\ln\left(\left(-4\_R^3a^5-3b^2\right)x-a^2b\_R\right)\right)}{3b\,x^3+3a}$	94

default	$-\frac{(b\,x^{3}+a)\left(-2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)x^{2}+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)x^{2}-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)x^{2}+3\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,x^{2}}$	118

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b*x^3+a)*(-2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3
))*x^2+2*ln(x+(a/b)^(1/3))*x^2-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*x^2+3*(a/b
)^(2/3))/((b*x^3+a)^2)^(1/2)/(a/b)^(2/3)/a/x^2
```

Maxima [A]
time = 0.49, size = 106, normalized size = 0.44

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{1}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)^(
2/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/3)) - 1/3*lo
g(x + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 1/2/(a*x^2)
```

Fricas [A]
time = 0.39, size = 143, normalized size = 0.59

$$\frac{2\,\sqrt{3}\,x^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-x^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}+abx\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)+2\,x^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\right)-3}{6\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*x^2*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2
/3) - sqrt(3)*b)/b) - x^2*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(
1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x^2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2
)^(1/3)) - 3)/(a*x^2)
```

Sympy [A]
time = 0.09, size = 32, normalized size = 0.13

$$\operatorname{RootSum}\left(27t^{3}a^{5}+b^{2},\left(t\mapsto t\log\left(-\frac{3ta^{2}}{b}+x\right)\right)\right)-\frac{1}{2ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] RootSum(27*_t**3*a**5 + b**2, Lambda(_t, _t*log(-3*_t*a**2/b + x))) - 1/(2*
a*x**2)
```

Giac [A]
time = 4.14, size = 125, normalized size = 0.51

$$\frac{1}{6}\left(\frac{2\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}-\frac{2\,\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2}-\frac{3}{ax^2}\right)\operatorname{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - 2*sqrt(3)*(-a*b^2)^(
1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - (-a*b^2)^(
1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/(a*x^2))*sgn(b*x^3 +
a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{{\left(b\,x^3+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] int(1/(x^3*((a + b*x^3)^2)^(1/2)), x)
```

3.97 $\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=125

$$\frac{-a-bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b(a+bx^3)\log(x)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/3*(-b*x^3-a)/a/x^3/((b*x^3+a)^2)^(1/2)-b*(b*x^3+a)*ln(x)/a^2/((b*x^3+a)^2
)^(1/2)+1/3*b*(b*x^3+a)*ln(b*x^3+a)/a^2/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 46}

$$-\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b\log(x)\left(a+bx^3\right)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)\log\left(a+bx^3\right)}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/3*(a + b*x^3)/(a*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (b*(a + b*x^3)*L
og[x])/(a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (b*(a + b*x^3)*Log[a + b*x^3
])/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt{a^2+2abx^3+b^2x^6}}\,dx &= \frac{(ab+b^2x^3)\int \frac{1}{x^4(ab+b^2x^3)}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\ &= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \frac{1}{x^2(ab+b^2x)}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\ &= \frac{(ab+b^2x^3)\,\text{Subst}\left(\int \left(\frac{1}{abx^2}-\frac{1}{a^2x}+\frac{b}{a^2(a+bx)}\right)\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}\\ &= -\frac{a+bx^3}{3ax^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b(a+bx^3)\log(x)}{a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)\log(a}{3a^2\sqrt{a^2+2abx^3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.43

$$-\frac{(a+bx^3)\left(a+3bx^3\log(x)-bx^3\log\left(a+bx^3\right)\right)}{3a^2x^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]),x]
```

```
[Out] -1/3*((a + b*x^3)*(a + 3*b*x^3*Log[x] - b*x^3*Log[a + b*x^3]))/(a^2*x^3*Sqr
t[(a + b*x^3)^2])
```

Maple [A]
time = 0.16, size = 52, normalized size = 0.42

method	result	size
default	$\frac{(b\,x^3+a)\left(b\ln\left(b\,x^3+a\right)x^3-3b\ln(x)x^3-a\right)}{3\sqrt{\left(b\,x^3+a\right)^2}\;a^2x^3}$	52
risch	$-\frac{\sqrt{\left(b\,x^3+a\right)^2}}{3(b\,x^3+a)a\,x^3}-\frac{\sqrt{\left(b\,x^3+a\right)^2}\;b\ln(x)}{(b\,x^3+a)a^2}+\frac{\sqrt{\left(b\,x^3+a\right)^2}\;b\ln\left(-b\,x^3-a\right)}{3(b\,x^3+a)a^2}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/((b*x^3+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^3+a)*(b*ln(b*x^3+a)*x^3-3*b*ln(x)*x^3-a)/((b*x^3+a)^2)^(1/2)/a^2/x
^3
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.58

$$\frac{(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^2}-\frac{\sqrt{b^2x^6+2\,abx^3+a^2}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^2
 - 1/3*sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)/(a^2*x^3)
```

Fricas [A]
time = 0.36, size = 33, normalized size = 0.26

$$\frac{bx^3\log\left(bx^3+a\right)-3\,bx^3\log\left(x\right)-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x^3*log(b*x^3 + a) - 3*b*x^3*log(x) - a)/(a^2*x^3)
```

Sympy [A]
time = 0.15, size = 31, normalized size = 0.25

$$-\frac{1}{3ax^3}-\frac{b\log\left(x\right)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/((b*x**3+a)**2)**(1/2),x)
```

```
[Out] -1/(3*a*x**3) - b*log(x)/a**2 + b*log(a/b + x**3)/(3*a**2)
```

Giac [A]
time = 4.11, size = 50, normalized size = 0.40

$$\frac{1}{3}\left(\frac{b\log\left(|bx^3+a|\right)}{a^2}-\frac{3\,b\log\left(|x|\right)}{a^2}+\frac{bx^3-a}{a^2x^3}\right)\mathrm{sgn}\left(bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/((b*x^3+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(b*log(abs(b*x^3 + a))/a^2 - 3*b*log(abs(x))/a^2 + (b*x^3 - a)/(a^2*x^3
))*sgn(b*x^3 + a)
```

Mupad [B]
time = 1.40, size = 75, normalized size = 0.60

$$\frac{a\,b\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^3}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}\right)}{3\,{\left(a^2\right)}^{3/2}}-\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*((a + b*x^3)^2)^(1/2)),x)
```

```
[Out] (a*b*atanh((a^2 + a*b*x^3)/((a^2)^(1/2)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)))
)/(3*(a^2)^(3/2)) - (a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(3*a^2*x^3)
```

3.98 $\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=280

$$\frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)}{27a^{4/3}b^{5/3}}$$

```
[Out] 1/9*x^2/a/b/((b*x^3+a)^2)^(1/2)-1/6*x^2/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-1/2
7*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)+1/54*
(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(5/3)/((b*x^3
+a)^2)^(1/2)-1/27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(4/3)/b^(5/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1369, 294, 296, 298, 31, 648, 631, 210, 642}

$$\frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] x^2/(9*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(6*b*(a + b*x^3)*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqr
t[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 - ((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(54*a^(4/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^4}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}} \\
&= -\frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)\int \frac{x}{\left(ab+b^2x^3\right)^2}\,dx}{3\sqrt{a^2+2abx^3+b^2x^6}} \\
&= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(ab+b^2x^3\right)}{9ab\sqrt{a^2+2a}} \\
&= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(ab+b^2x^3\right)}{27a^{4/3}b^2\sqrt{a}} \\
&= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\mathrm{lo}}{27a^{4/3}b^{5/3}\sqrt{}} \\
&= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\mathrm{lo}}{27a^{4/3}b^{5/3}\sqrt{}} \\
&= \frac{x^2}{9ab\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\left(a+bx^3\right)\mathrm{t}}{9\sqrt{3}\,a^{4/3}b^{5/}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 235, normalized size = 0.84

$$
\frac{-3a^{4/3}b^{2/3}x^2+6\sqrt[3]{a}\,b^{5/3}x^5-2\sqrt{3}\left(a+bx^3\right)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\left(a+bx^3\right)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{4/3}b^{5/3}\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (-3*a^(4/3)*b^(2/3)*x^2 + 6*a^(1/3)*b^(5/3)*x^5 - 2*Sqrt[3]*(a + b*x^3)^2*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(a + b*x^3)^2*Log[a^(1/3) +
b^(1/3)*x] + a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*a*b*x^3
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + b^2*x^6*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(4/3)*b^(5/3)*(a + b*x^3)*Sqrt[(a + b*
x^3)^2])
```

Maple [A]
time = 0.03, size = 301, normalized size = 1.08

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{x^{5}}{9a}-\frac{x^{2}}{18b}\right)}{\left(b\,x^{3}+a\right)^{3}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{27\left(b\,x^{3}+a\right)a\,b^{2}}$
default	$-\frac{\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{2}x^{6}+2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{2}x^{6}-\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{2}x^{6}-6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}x^{5}+4\sqrt{3}\,\arctan\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/54*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*b^2*x^6
+2*ln(x+(a/b)^(1/3))*b^2*x^6-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*b^2*x^6-6*(a
/b)^(1/3)*b^2*x^5+4*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/
3))*a*b*x^3+4*ln(x+(a/b)^(1/3))*a*b*x^3-2*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
*a*b*x^3+3*(a/b)^(1/3)*a*b*x^2+2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/
3))/(a/b)^(1/3))*a^2+2*ln(x+(a/b)^(1/3))*a^2-ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*a^2)*(b*x^3+a)/(a/b)^(1/3)/b^2/a/((b*x^3+a)^2)^(3/2)
```

Maxima [A]
time = 0.49, size = 149, normalized size = 0.53

$$\frac{2\,bx^{5}-ax^{2}}{18\,\left(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b\right)}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^{2}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^{2}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^{2}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(2*b*x^5 - a*x^2)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) + 1/
54*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/27*log(x
+ (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

Fricas [A]
time = 0.36, size = 512, normalized size = 1.83

$$\left[\frac{6\,ab^{3}x^{5}-3\,a^{2}b^{2}x^{2}+3\,\sqrt{\frac{1}{3}}\,\left(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^{2}x^{3}-ab+3\,\sqrt{\frac{1}{3}}\,\left(abx+2\,\left(-ab^{2}\right)^{\frac{2}{3}}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}-3\,\left(-ab^{2}\right)^{\frac{2}{3}}x}{bx^{3}+a}\right)+\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}bx+\left(-ab^{2}\right)^{\frac{2}{3}}\right)-2\,\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(bx-\left(-ab^{2}\right)^{\frac{1}{3}}\right)}{54\,\left(a^{2}b^{5}x^{6}+2\,a^{3}b^{4}x^{3}+a^{4}b^{3}\right)},\ \frac{6\,ab^{3}x^{5}-3\,a^{2}b^{2}x^{2}+6\,\sqrt{\frac{1}{3}}\,\left(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\,\arctan\left(\frac{\sqrt{\frac{1}{3}}\,\left(2\,bx+\left(-ab^{2}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}}{b}\right)+\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}bx+\left(-ab^{2}\right)^{\frac{2}{3}}\right)-2\,\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(bx-\left(-ab^{2}\right)^{\frac{1}{3}}\right)}{54\,\left(a^{2}b^{5}x^{6}+2\,a^{3}b^{4}x^{3}+a^{4}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3
 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
+ 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b
^2)^(2/3)*x)/(b*x^3 + a)) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(
b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 + 2*a*b*x^3 + a
^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3
+ a^4*b^3), 1/54*(6*a*b^3*x^5 - 3*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*
a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*
b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^
2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^5*x^6 + 2
*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**4/((a + b*x**3)**2)**(3/2), x)
```

Giac [A]
time = 4.10, size = 185, normalized size = 0.66

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}\,ab\mathrm{sgn}\left(bx^3+a\right)}-\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}\,ab\mathrm{sgn}\left(bx^3+a\right)}-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^2b\mathrm{sgn}\left(bx^3+a\right)}+\frac{2\,bx^5-ax^2}{18\left(bx^3+a\right)^2ab\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] 1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2
)^(1/3)*a*b*sgn(b*x^3 + a)) - 1/54*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/((-a*b^2)^(1/3)*a*b*sgn(b*x^3 + a)) - 1/27*(-a/b)^(2/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b*sgn(b*x^3 + a)) + 1/18*(2*b*x^5 - a*x^2)/((b*x^3 + a)^2*a*b
*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.99 $\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=276

$$\frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+b}{27a^{5/3}b}$$

```
[Out] 1/18*x/a/b/((b*x^3+a)^2)^(1/2)-1/6*x/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+1/27*(
b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(4/3)/((b*x^3+a)^2)^(1/2)-1/54*(b*
x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(4/3)/((b*x^3+a)
^2)^(1/2)-1/27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(5/3)/b^(4/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1369, 294, 205, 206, 31, 648, 631, 210, 642}

$$\frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] x/(18*a*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(6*b*(a + b*x^3)*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3
]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) +
((a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(54*a^(5/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```

[p - 1/2]

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}\\
&= -\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(ab+b^2x^3\right)\int \frac{1}{\left(ab+b^2x^3\right)^2}\,dx}{6\sqrt{a^2+2abx^3+b^2x^6}}\\
&= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(ab+b^2x}{9ab\sqrt{a^2+}}\\
&= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(ab+b^2x}{27a^{5/3}b^{5/3}}\\
&= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3}{27a^{5/3}b^{4/3}}\\
&= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+bx^3}{27a^{5/3}b^{4/3}}\\
&= \frac{x}{18ab\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{(a+bx^3}{9\sqrt{3}\,a^{5/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 235, normalized size = 0.85

$$\frac{-6a^{5/3}\sqrt[3]{b}\,x+3a^{2/3}b^{4/3}x^4-2\sqrt{3}\left(a+bx^3\right)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\left(a+bx^3\right)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-2abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{5/3}b^{4/3}\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (-6*a^(5/3)*b^(1/3)*x + 3*a^(2/3)*b^(4/3)*x^4 - 2*Sqrt[3]*(a + b*x^3)^2*Arc
Tan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(a + b*x^3)^2*Log[a^(1/3) + b^
(1/3)*x] - a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 2*a*b*x^3*L
og[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - b^2*x^6*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(5/3)*b^(4/3)*(a + b*x^3)*Sqrt[(a + b*x^
3)^2])
```

Maple [A]
time = 0.03, size = 297, normalized size = 1.08

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{x^{4}}{18a}-\frac{x}{9b}\right)}{\left(b\,x^{3}+a\right)^{3}}+\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{27\left(b\,x^{3}+a\right)a\,b^{2}}$
default	$-\frac{\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{2}x^{6}-2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{2}x^{6}+\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{2}x^{6}-3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}x^{4}+4\sqrt{3}\,\arctan\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/54*(2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*b^2*x^6
-2*ln(x+(a/b)^(1/3))*b^2*x^6+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*b^2*x^6-3*(a
/b)^(2/3)*b^2*x^4+4*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/
3))*a*b*x^3-4*ln(x+(a/b)^(1/3))*a*b*x^3+2*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
*a*b*x^3+6*(a/b)^(2/3)*a*b*x+2*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3)
)/(a/b)^(1/3))*a^2-2*ln(x+(a/b)^(1/3))*a^2+ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)*a^2)*(b*x^3+a)/(a/b)^(2/3)/b^2/a/((b*x^3+a)^2)^(3/2)
```

Maxima [A]
time = 0.51, size = 146, normalized size = 0.53

$$\frac{bx^{4}-2\,ax}{18\,(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b)}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,ab^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(b*x^4 - 2*a*x)/(a*b^3*x^6 + 2*a^2*b^2*x^3 + a^3*b) + 1/27*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/54
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/27*log(x +
(a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

Fricas [A]
time = 0.38, size = 503, normalized size = 1.82

$$\left[\frac{3\,a^{2}b^{2}x^{4}-6\,a^{3}bx+3\,\sqrt{\frac{1}{3}}\,(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^{3}-3\,(a^{2}b)^{\frac{1}{3}}ax-a^{2}+3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)-(b^{2}x^{6}+2\,abx^{3}+a^{2})(a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(a^{2}b)^{\frac{2}{3}}x+(a^{2}b)^{\frac{1}{3}}a\right)+2\,(b^{2}x^{6}+2\,abx^{3}+a^{2})(a^{2}b)^{\frac{2}{3}}\log\left(abx+(a^{2}b)^{\frac{2}{3}}\right)}{54\,(a^{3}b^{4}x^{6}+2\,a^{4}b^{3}x^{3}+a^{5}b^{2})},\ \frac{3\,a^{2}b^{2}x^{4}-6\,a^{3}bx+6\,\sqrt{\frac{1}{3}}\,(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\,\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{a^{2}}\right)-(b^{2}x^{6}+2\,abx^{3}+a^{2})(a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(a^{2}b)^{\frac{2}{3}}x+(a^{2}b)^{\frac{1}{3}}a\right)+2\,(b^{2}x^{6}+2\,abx^{3}+a^{2})(a^{2}b)^{\frac{2}{3}}\log\left(abx+(a^{2}b)^{\frac{2}{3}}\right)}{54\,(a^{3}b^{4}x^{6}+2\,a^{4}b^{3}x^{3}+a^{5}b^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 3*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^3 +
 a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 +
 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^
(1/3)/b))/(b*x^3 + a)) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*
x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a
^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*
b^2), 1/54*(3*a^2*b^2*x^4 - 6*a^3*b*x + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*
x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a
^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(a^
2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*(b^2*x^6 +
2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^3*b^4*x^6 + 2
*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**2)**(3/2), x)
```

Giac [A]
time = 3.83, size = 176, normalized size = 0.64

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a\mathrm{sgn}\left(bx^3+a\right)}-\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a\mathrm{sgn}\left(bx^3+a\right)}-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^2b\mathrm{sgn}\left(bx^3+a\right)}+\frac{bx^4-2ax}{18\left(bx^3+a\right)^2ab\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^
2)^(2/3)*a*sgn(b*x^3 + a)) - 1/54*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/
((-a*b^2)^(2/3)*a*sgn(b*x^3 + a)) - 1/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1
/3)))/(a^2*b*sgn(b*x^3 + a)) + 1/18*(b*x^4 - 2*a*x)/((b*x^3 + a)^2*a*b*sgn(
b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.100 $\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] -1/6/b/(b*x^3+a)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1366, 621}

$$-\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -1/6*1/(b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 621

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx,x,x^3\right)\\ &= -\frac{1}{6b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+bx^3}{6b\left((a+bx^3)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] -1/6*(a + b*x^3)/(b*((a + b*x^3)^2)^(3/2))
```

Maple [A]
time = 0.02, size = 24, normalized size = 0.63

method	result	size
gosper	$-\frac{b\,x^3+a}{6b\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}$	24
default	$-\frac{b\,x^3+a}{6b\left((b\,x^3+a)^2\right)^{\frac{3}{2}}}$	24
risch	$-\frac{\sqrt{(b\,x^3+a)^2}}{6(b\,x^3+a)^3 b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b*x^3+a)/b/((b*x^3+a)^2)^(3/2)
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.42

$$-\frac{1}{6\left(x^3+\frac{a}{b}\right)^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6/((x^3 + a/b)^2*b^3)
```

Fricas [A]
time = 0.36, size = 26, normalized size = 0.68

$$-\frac{1}{6\left(b^3x^6+2\,ab^2x^3+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
[Out] -1/6/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral(x**2/((a + b*x**3)**2)**(3/2), x)
```

Giac [**A**]
time = 3.57, size = 24, normalized size = 0.63

$$-\frac{1}{6\left(bx^3+a\right)^2b\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] -1/6/((b*x^3 + a)^2*b*sgn(b*x^3 + a))
```

Mupad [**B**]
time = 1.19, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{6\,b\,{\left(b\,x^3+a\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] -(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(6*b*(a + b*x^3)^3)
```

3.101 $\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=277

$$\frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\left(a+bx^3\right)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2(a+bx^3}{27a^{7/3}b^{2/3}}$$

```
[Out] 2/9*x^2/a^2/((b*x^3+a)^2)^(1/2)+1/6*x^2/a/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-2/2
7*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)+1/27*
(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(2/3)/((b*x^3
+a)^2)^(1/2)-2/27*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2
))/a^(7/3)/b^(2/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1369, 296, 298, 31, 648, 631, 210, 642}

$$\frac{x^2}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\left(a+bx^3\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{7/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (2*x^2)/(9*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(6*a*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x
)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (2*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(7/3)*b^(2/3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(27*a^(7/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
```

```
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^3))\int \frac{x}{(ab+b^2x^3)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(2b(ab+b^2x^3))\int \frac{x}{(ab+b^2x^3)^2}\,dx}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(2(ab+b^2x^3}{9a^2\sqrt{a^2+2}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{(2(ab+b^2x^3}{27a^{7/3}b\sqrt{}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\,l}{27a^{7/3}b^{2/3}\sqrt{}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)\,l}{27a^{7/3}b^{2/3}\sqrt{}}$$

$$= \frac{2x^2}{9a^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{6a(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{2(a+bx^3)}{9\sqrt{3}\,a^{7/3}b^{2/}}$$

Mathematica [A]
time = 0.05, size = 237, normalized size = 0.86

$$\frac{21a^{4/3}b^{2/3}x^2+12\sqrt[3]{a}\,b^{5/3}x^5-4\sqrt{3}\,(a+bx^3)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-4(a+bx^3)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+2a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+4abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+2b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{7/3}b^{2/3}(a+bx^3)\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (21*a^(4/3)*b^(2/3)*x^2 + 12*a^(1/3)*b^(5/3)*x^5 - 4*Sqrt[3]*(a + b*x^3)^2*
ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 4*(a + b*x^3)^2*Log[a^(1/3) +
 b^(1/3)*x] + 2*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 4*a*b*
x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 2*b^2*x^6*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(2/3)*(a + b*x^3)*Sqrt[(a
 + b*x^3)^2])
```

Maple [A]
time = 0.03, size = 301, normalized size = 1.09

method	result
risch	$\frac{\sqrt{(b\,x^3+a)^2}\left(\frac{2b\,x^5}{9a^2}+\frac{7x^2}{18a}\right)}{(b\,x^3+a)^3}+\frac{2\sqrt{(b\,x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}(b\,\_Z^3+a)}\frac{\ln(x-\_R)}{\_R}\right)}{27(b\,x^3+a)a^2b}$
default	$\frac{\left(-4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^2x^6-4\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^2x^6+2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^2x^6+12\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2x^5-8\sqrt{3}\,\mathrm{arcta}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(-4*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*b^2*x^6
-4*ln(x+(a/b)^(1/3))*b^2*x^6+2*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*b^2*x^6+12
*(a/b)^(1/3)*b^2*x^5-8*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^
(1/3))*a*b*x^3-8*ln(x+(a/b)^(1/3))*a*b*x^3+4*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*a*b*x^3+21*(a/b)^(1/3)*a*b*x^2-4*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)
^(1/3))/(a/b)^(1/3))*a^2-4*ln(x+(a/b)^(1/3))*a^2+2*ln(x^2-(a/b)^(1/3)*x+(a/
b)^(2/3))*a^2)*(b*x^3+a)/(a/b)^(1/3)/b/a^2/((b*x^3+a)^2)^(3/2)
```

Maxima [A]
time = 0.50, size = 147, normalized size = 0.53

$$\frac{4\,bx^5+7\,ax^2}{18\,(a^2b^2x^6+2\,a^3bx^3+a^4)}+\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(4*b*x^5 + 7*a*x^2)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 2/27*sqrt(3)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/3)) + 1/
27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(1/3)) - 2/27*log(x
+ (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

Fricas [A]
time = 0.38, size = 514, normalized size = 1.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 6*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x
^3 + a^3*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*
x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a
*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*
log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b^2*x^6 + 2*a*b*x^3
 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^4*x^6 + 2*a^4*b^3*
x^3 + a^5*b^2), 1/54*(12*a*b^3*x^5 + 21*a^2*b^2*x^2 + 12*sqrt(1/3)*(a*b^3*x
^6 + 2*a^2*b^2*x^3 + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x
 + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^6 + 2*a*b*x^3 + a^
2)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 4*(b
^2*x^6 + 2*a*b*x^3 + a^2)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^
4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2)**(3/2), x)
```

Giac [**A**]
time = 4.06, size = 173, normalized size = 0.62

$$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^2\mathrm{sgn}\left(bx^3+a\right)}-\frac{\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^2\mathrm{sgn}\left(bx^3+a\right)}-\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3\mathrm{sgn}\left(bx^3+a\right)}+\frac{4\,bx^5+7\,ax^2}{18\left(bx^3+a\right)^2a^2\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2
)^(1/3)*a^2*sgn(b*x^3 + a)) - 1/27*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))
/((-a*b^2)^(1/3)*a^2*sgn(b*x^3 + a)) - 2/27*(-a/b)^(2/3)*log(abs(x - (-a/b)
^(1/3)))/(a^3*sgn(b*x^3 + a)) + 1/18*(4*b*x^5 + 7*a*x^2)/((b*x^3 + a)^2*a^2
*sgn(b*x^3 + a))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.102 $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=286

$$\frac{x(a+bx^3)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5x(a+bx^3)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5(a+bx^3)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}\,\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5(a+bx^3)}{27a^{8/3}\sqrt[3]{b}\,(}$$

```
[Out] 1/6*x*(b*x^3+a)/a/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)+5/18*x*(b*x^3+a)^2/a^2/(b^2
*x^6+2*a*b*x^3+a^2)^(3/2)+5/27*(b*x^3+a)^3*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^
(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)-5/54*(b*x^3+a)^3*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)-5/27*(b*x
^3+a)^3*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(1/3)/(
b^2*x^6+2*a*b*x^3+a^2)^(3/2)*3^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1357, 205, 206, 31, 648, 631, 210, 642}

$$\frac{5x(a+bx^3)^2}{18a^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{x(a+bx^3)}{6a\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5(a+bx^3)^3\,\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}\,\left(a^2+2abx^3+b^2x^6\right)^{3/2}}+\frac{5(a+bx^3)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{8/3}\sqrt[3]{b}\,\left(a^2+2abx^3+b^2x^6\right)^{3/2}}-\frac{5(a+bx^3)^3\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}\,\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

```
[Out] (x*(a + b*x^3))/(6*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*x*(a + b*x^3)^
2)/(18*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(8/3)*b^(1/3)*(a^2 +
2*a*b*x^3 + b^2*x^6)^(3/2)) + (5*(a + b*x^3)^3*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(8/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)) - (5*(a + b*x^3)^3*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a^2 + 2*a
*b*x^3 + b^2*x^6)^(3/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{(2ab+2b^2x^3)^3 \int \frac{1}{(2ab+2b^2x^3)^3}\,dx}{(a^2+2abx^3+b^2x^6)^{3/2}}$$

$$= \frac{x(a+bx^3)}{6a\,(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{\left(5(2ab+2b^2x^3)^3\right)\int \frac{1}{(2ab+2b^2x^3)^2}\,dx}{12ab\,(a^2+2abx^3+b^2x^6)^{3/2}}$$

$$= \frac{x(a+bx^3)}{6a\,(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{5x(a+bx^3)^2}{18a^2\,(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{\left(5(2ab+2b^2x^3\right.}{36a^2b^2\,(a^2+2}$$

$$= \frac{x(a+bx^3)}{6a\,(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{5x(a+bx^3)^2}{18a^2\,(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{\left(5(2ab+2b^2x^3\right.}{108\ 2^{2/3}a^{8/3}}$$

$$= \frac{x(a+bx^3)}{6a\,(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{5x(a+bx^3)^2}{18a^2\,(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{5(a+bx^3)^3\log}{27a^{8/3}\sqrt[3]{b}\,(a^2+}$$

$$= \frac{x(a+bx^3)}{6a\,(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{5x(a+bx^3)^2}{18a^2\,(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{5(a+bx^3)^3\log}{27a^{8/3}\sqrt[3]{b}\,(a^2+}$$

$$= \frac{x(a+bx^3)}{6a\,(a^2+2abx^3+b^2x^6)^{3/2}} + \frac{5x(a+bx^3)^2}{18a^2\,(a^2+2abx^3+b^2x^6)^{3/2}} - \frac{5(a+bx^3)^3\tan}{9\sqrt{3}\,a^{8/3}\sqrt[3]{b}\,(a}$$

Mathematica [A]
time = 0.05, size = 235, normalized size = 0.82

$$\frac{24a^{5/3}\sqrt[3]{b}\,x+15a^{2/3}b^{4/3}x^4-10\sqrt{3}\,(a+bx^3)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+10(a+bx^3)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-5a^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-10abx^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-5b^2x^6\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{8/3}\sqrt[3]{b}\,(a+bx^3)\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-3/2),x]
```

```
[Out] (24*a^(5/3)*b^(1/3)*x + 15*a^(2/3)*b^(4/3)*x^4 - 10*Sqrt[3]*(a + b*x^3)^2*A
rcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*(a + b*x^3)^2*Log[a^(1/3) +
 b^(1/3)*x] - 5*a^2*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 10*a*b
*x^3*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 5*b^2*x^6*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(1/3)*(a + b*x^3)*Sqrt[(
a + b*x^3)^2])
```

Maple [A]
time = 0.03, size = 299, normalized size = 1.05

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{5 b\,x^{4}}{18 a^{2}}+\frac{4 x}{9 a}\right)}{\left(b\,x^{3}+a\right)^{3}}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{27\left(b\,x^{3}+a\right)a^{2}b}$
default	$\left(-10\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{2}x^{6}+10\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{2}x^{6}-5\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{2}x^{6}+15\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}x^{4}-20\sqrt{3}\,\mathrm{arct}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(-10*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*b^2*x^
6+10*ln(x+(a/b)^(1/3))*b^2*x^6-5*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*b^2*x^6+
15*(a/b)^(2/3)*b^2*x^4-20*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/
b)^(1/3))*a*b*x^3+20*ln(x+(a/b)^(1/3))*a*b*x^3-10*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))*a*b*x^3+24*(a/b)^(2/3)*a*b*x-10*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(
a/b)^(1/3))/(a/b)^(1/3))*a^2+10*ln(x+(a/b)^(1/3))*a^2-5*ln(x^2-(a/b)^(1/3)*
x+(a/b)^(2/3))*a^2)*(b*x^3+a)/(a/b)^(2/3)/b/a^2/((b*x^3+a)^2)^(3/2)
```

Maxima [A]
time = 0.49, size = 145, normalized size = 0.51

$$\frac{5\,bx^{4}+8\,ax}{18\,(a^{2}b^{2}x^{6}+2\,a^{3}bx^{3}+a^{4})}+\frac{5\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{2}b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^{2}b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{2}b\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*(5*b*x^4 + 8*a*x)/(a^2*b^2*x^6 + 2*a^3*b*x^3 + a^4) + 5/27*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(2/3)) - 5/54
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(2/3)) + 5/27*log(x +
(a/b)^(1/3))/(a^2*b*(a/b)^(2/3))
```

Fricas [A]
time = 0.38, size = 499, normalized size = 1.74

$$\left[\frac{15\,a^{2}b^{2}x^{4}+24\,a^{3}bx+15\,\sqrt{\frac{1}{3}}\,(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^{3}-3\,(a^{2}b)^{\frac{1}{3}}ax-a^{2}+3\,\sqrt{\frac{1}{3}}\,\left(2\,abx^{2}+(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)-5\,(b^{2}x^{6}+2\,abx^{3}+a^{2})(a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(a^{2}b)^{\frac{2}{3}}x+(a^{2}b)^{\frac{1}{3}}a\right)+10\,(b^{2}x^{6}+2\,abx^{3}+a^{2})(a^{2}b)^{\frac{2}{3}}\log\left(abx+(a^{2}b)^{\frac{2}{3}}\right)}{54\,(a^{4}b^{3}x^{6}+2\,a^{5}b^{2}x^{3}+a^{6}b)},\ \frac{15\,a^{2}b^{2}x^{4}+24\,a^{3}bx+30\,\sqrt{\frac{1}{3}}\,(ab^{3}x^{6}+2\,a^{2}b^{2}x^{3}+a^{3}b)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\,\arctan\left(\frac{\sqrt{\frac{1}{3}}\,\left(2\,(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{a^{2}}\right)-5\,(b^{2}x^{6}+2\,abx^{3}+a^{2})(a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(a^{2}b)^{\frac{2}{3}}x+(a^{2}b)^{\frac{1}{3}}a\right)+10\,(b^{2}x^{6}+2\,abx^{3}+a^{2})(a^{2}b)^{\frac{2}{3}}\log\left(abx+(a^{2}b)^{\frac{2}{3}}\right)}{54\,(a^{4}b^{3}x^{6}+2\,a^{5}b^{2}x^{3}+a^{6}b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 15*sqrt(1/3)*(a*b^3*x^6 + 2*a^2*b^2*x^
3 + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) - 5*(b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log
(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(b^2*x^6 + 2*a*b*x^3 + a
^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^3*x^6 + 2*a^5*b^2*x^3
+ a^6*b), 1/54*(15*a^2*b^2*x^4 + 24*a^3*b*x + 30*sqrt(1/3)*(a*b^3*x^6 + 2*a
^2*b^2*x^3 + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)
*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^6 + 2*a*b*x^3 +
 a^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 10*(
b^2*x^6 + 2*a*b*x^3 + a^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b
^3*x^6 + 2*a^5*b^2*x^3 + a^6*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-3/2), x)
```

Giac [A]
time = 2.99, size = 177, normalized size = 0.62

$$-\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^3\mathrm{sgn}\left(bx^3+a\right)}+\frac{5\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\mathrm{sgn}\left(bx^3+a\right)}+\frac{5\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,a^3b\mathrm{sgn}\left(bx^3+a\right)}+\frac{5\,bx^4+8\,ax}{18\left(bx^3+a\right)^2a^2\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
[Out] -5/27*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*sgn(b*x^3 + a)) + 5/27*s
qrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))
/(a^3*b*sgn(b*x^3 + a)) + 5/54*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(a^3*b*sgn(b*x^3 + a)) + 1/18*(5*b*x^4 + 8*a*x)/((b*x^3 + a)^2*
a^2*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.103 $\int \frac{1}{x(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\left(a+bx^3\right)\log(x)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+b\right.}{3a^3\sqrt{a^2+2abx^3+b^2\right.}$$

```
[Out] 1/3/a^2/((b*x^3+a)^2)^(1/2)+1/6/a/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+(b*x^3+a)*l
n(x)/a^3/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a^3/((b*x^3+a)^2)^(1
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 46}

$$\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 1/(3*a^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*(a + b*x^3)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]) + ((a + b*x^3)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - ((a + b*x^3)*Log[a + b*x^3])/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```

```
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2(ab+b^2x^3)\right)\int \frac{1}{x(ab+b^2x^3)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(b^2(ab+b^2x^3)\right)\text{Subst}\left(\int \frac{1}{x(ab+b^2x)^3}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{\left(b^2(ab+b^2x^3)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)dx,\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{1}{3a^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6a\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{(a+}{a^3\sqrt{a^2+}}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 0.50

$$\frac{a(3a+2bx^3)+6(a+bx^3)^2\log(x)-2(a+bx^3)^2\log\left(a+bx^3\right)}{6a^3\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
[Out] (a*(3*a + 2*b*x^3) + 6*(a + b*x^3)^2*Log[x] - 2*(a + b*x^3)^2*Log[a + b*x^3
])/(6*a^3*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A]
time = 0.05, size = 107, normalized size = 0.73

method	result	size
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(\frac{b\,x^3}{3a^2}+\frac{1}{2a}\right)}{\left(b\,x^3+a\right)^3}+\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\ln(x)}{\left(b\,x^3+a\right)a^3}-\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\ln\left(b\,x^3+a\right)}{3\left(b\,x^3+a\right)a^3}$	97
default	$\frac{\left(6\ln(x)b^2x^6-2\ln\left(b\,x^3+a\right)b^2x^6+12\ln(x)ab\,x^3-4\ln\left(b\,x^3+a\right)ab\,x^3+2ab\,x^3+6a^2\ln(x)-2\ln\left(b\,x^3+a\right)a^2+3a^2\right)\left(b\,x^3+a\right)}{6a^3\left(\left(b\,x^3+a\right)^2\right)^{\frac{3}{2}}}$	107

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(6*ln(x)*b^2*x^6-2*ln(b*x^3+a)*b^2*x^6+12*ln(x)*a*b*x^3-4*ln(b*x^3+a)*a
*b*x^3+2*a*b*x^3+6*a^2*ln(x)-2*ln(b*x^3+a)*a^2+3*a^2)*(b*x^3+a)/a^3/((b*x^3
+a)^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 88, normalized size = 0.60

$$-\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^3}+\frac{1}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}\,a^2}+\frac{1}{6\,\left(x^3+\frac{a}{b}\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^3
+ 1/3/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2) + 1/6/((x^3 + a/b)^2*a*b^2)
```

Fricas [A]
time = 0.37, size = 90, normalized size = 0.61

$$\frac{2\,abx^3+3\,a^2-2\,(b^2x^6+2\,abx^3+a^2)\log\left(bx^3+a\right)+6\,(b^2x^6+2\,abx^3+a^2)\log\left(x\right)}{6\,(a^3b^2x^6+2\,a^4bx^3+a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*b*x^3 + 3*a^2 - 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*log(b*x^3 + a) + 6*(
b^2*x^6 + 2*a*b*x^3 + a^2)*log(x))/(a^3*b^2*x^6 + 2*a^4*b*x^3 + a^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**3)**2)**(3/2)), x)
```

Giac [A]
time = 3.39, size = 87, normalized size = 0.59

$$-\frac{\log\left(|bx^3+a|\right)}{3\,a^3\mathrm{sgn}\left(bx^3+a\right)}+\frac{\log\left(|x|\right)}{a^3\mathrm{sgn}\left(bx^3+a\right)}+\frac{3\,b^2x^6+8\,abx^3+6\,a^2}{6\,(bx^3+a)^2a^3\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(b*x^3 + a))/(a^3*sgn(b*x^3 + a)) + log(abs(x))/(a^3*sgn(b*x^3
+ a)) + 1/6*(3*b^2*x^6 + 8*a*b*x^3 + 6*a^2)/((b*x^3 + a)^2*a^3*sgn(b*x^3 +
a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

3.104 $\int \frac{1}{x^2(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{14(a+bx^3)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{14\sqrt[3]{b}\,(a+bx^3}{9\sqrt{3}\,a^{10/3}}$$

```
[Out] 7/18/a^2/x/((b*x^3+a)^2)^(1/2)+1/6/a/x/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-14/9*(
b*x^3+a)/a^3/x/((b*x^3+a)^2)^(1/2)+14/27*b^(1/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/
3)*x)/a^(10/3)/((b*x^3+a)^2)^(1/2)-7/27*b^(1/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/((b*x^3+a)^2)^(1/2)+14/27*b^(1/3)*(b*x^3+
a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)*3^(1/2)/((b*x
^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1369, 296, 331, 298, 31, 648, 631, 210, 642}

$$\frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}+\frac{14\sqrt[3]{b}\,(a+bx^3)\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{14\sqrt[3]{b}\,(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{7\sqrt[3]{b}\,(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{10/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{14(a+bx^3)}{9a^3x\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 7/(18*a^2*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x*(a + b*x^3)*Sqrt[a^
2 + 2*a*b*x^3 + b^2*x^6]) - (14*(a + b*x^3))/(9*a^3*x*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) + (14*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (14*b^
(1/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]) - (7*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(27*a^(10/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
```

```
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7b\left(ab+b^2x^3\right)\right)\int \frac{1}{x^2\left(ab+b^2x^3\right)^2}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(14(a \cdots}{9a \cdots}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\cdots}{9a^3x \cdots}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\cdots}{9a^3x \cdots}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\cdots}{9a^3x \cdots}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\cdots}{9a^3x \cdots}$$

$$= \frac{7}{18a^2x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{\cdots}{9a^3x \cdots}$$

Mathematica [A]
time = 0.05, size = 260, normalized size = 0.82

$$\frac{-54a^{7/3}-147a^{4/3}bx^3-84\sqrt[3]{a}\,b^2x^6+28\sqrt{3}\sqrt[3]{b}\,x(a+bx^3)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)+28\sqrt[3]{b}\,x(a+bx^3)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-14a^2\sqrt[3]{b}\,x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-28ab^{4/3}x^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-14b^{7/3}x^7\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{10/3}x\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-54*a^(7/3) - 147*a^(4/3)*b*x^3 - 84*a^(1/3)*b^2*x^6 + 28*Sqrt[3]*b^(1/3)*
x*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 28*b^(1/3)*x*
(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] - 14*a^2*b^(1/3)*x*Log[a^(2/3) - a^(
```

```
1/3)*b^(1/3)*x + b^(2/3)*x^2] - 28*a*b^(4/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2] - 14*b^(7/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(54*a^(10/3)*x*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A]
time = 0.06, size = 316, normalized size = 1.00

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{14 b^{2} x^{6}}{9 a^{3}}-\frac{49 b\,x^{3}}{18 a^{2}}-\frac{1}{a}\right)}{\left(b\,x^{3}+a\right)^{3} x}+\frac{14\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(a^{10}\_Z^{3}-b\right)}\_R\ln\left(\left(-4\_R^{3} a^{10}+3 b\right) x-a^{7}\_\right.\right.}{27\left(b\,x^{3}+a\right)}$
default	$-\frac{\left(-28\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2 x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) b^{2} x^{7}-28\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) b^{2} x^{7}+14\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) b^{2} x^{7}+84\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2} x^{6}-56\sqrt{3}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/54*(-28*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*b^2*x
^7-28*ln(x+(a/b)^(1/3))*b^2*x^7+14*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*b^2*x^
7+84*(a/b)^(1/3)*b^2*x^6-56*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(
a/b)^(1/3))*a*b*x^4-56*ln(x+(a/b)^(1/3))*a*b*x^4+28*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))*a*b*x^4+147*(a/b)^(1/3)*a*b*x^3-28*3^(1/2)*arctan(1/3*3^(1/2)*(-
2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^2*x-28*ln(x+(a/b)^(1/3))*a^2*x+14*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))*a^2*x+54*(a/b)^(1/3)*a^2)*(b*x^3+a)/x/(a/b)^(1/3)/
a^3/((b*x^3+a)^2)^(3/2)
```

Maxima [A]
time = 0.51, size = 148, normalized size = 0.47

$$-\frac{28\,b^{2}x^{6}+49\,abx^{3}+18\,a^{2}}{18\,\left(a^{3}b^{2}x^{7}+2\,a^{4}bx^{4}+a^{5}x\right)}-\frac{14\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{3}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\,\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^{3}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{3}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/18*(28*b^2*x^6 + 49*a*b*x^3 + 18*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x
) - 14/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*
(a/b)^(1/3)) - 7/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1/3)
) + 14/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

Fricas [A]
time = 0.35, size = 201, normalized size = 0.64

$$-\frac{84\,b^{2}x^{6}+147\,abx^{3}+28\,\sqrt{3}\,(b^{2}x^{7}+2\,abx^{4}+a^{2}x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+14\,(b^{2}x^{7}+2\,abx^{4}+a^{2}x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-28\,(b^{2}x^{7}+2\,abx^{4}+a^{2}x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)+54\,a^{2}}{54\,(a^{3}b^{2}x^{7}+2\,a^{4}bx^{4}+a^{5}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/54*(84*b^2*x^6 + 147*a*b*x^3 + 28*sqrt(3)*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*
(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^7 +
 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)
) - 28*(b^2*x^7 + 2*a*b*x^4 + a^2*x)*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3)) +
 54*a^2)/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left((a + bx^3)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**3)**2)**(3/2)), x)
```

Giac [A]
time = 4.01, size = 201, normalized size = 0.64

$$\frac{14\, b\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\, a^4 \mathrm{sgn}\left(bx^3 + a\right)} + \frac{14\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\, a^4 b \mathrm{sgn}\left(bx^3 + a\right)} - \frac{7\left(-ab^2\right)^{\frac{2}{3}} \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\, a^4 b \mathrm{sgn}\left(bx^3 + a\right)} - \frac{10\, b^2 x^5 + 13\, abx^2}{18\left(bx^3 + a\right)^2 a^3 \mathrm{sgn}\left(bx^3 + a\right)} - \frac{1}{a^3 x \mathrm{sgn}\left(bx^3 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 14/27*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*sgn(b*x^3 + a)) + 14/2
7*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/(a^4*b*sgn(b*x^3 + a)) - 7/27*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/(a^4*b*sgn(b*x^3 + a)) - 1/18*(10*b^2*x^5 + 13*a*b*x^2)/((b*
x^3 + a)^2*a^3*sgn(b*x^3 + a)) - 1/(a^3*x*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \left(a^2 + 2\, a\, b\, x^3 + b^2\, x^6\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

3.105 $\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=316

$$\frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{20b^{2/3}(a+}{9\sqrt{3}\,a^{11}}$$

```
[Out] 4/9/a^2/x^2/((b*x^3+a)^2)^(1/2)+1/6/a/x^2/(b*x^3+a)/((b*x^3+a)^2)^(1/2)-10/
9*(b*x^3+a)/a^3/x^2/((b*x^3+a)^2)^(1/2)-20/27*b^(2/3)*(b*x^3+a)*ln(a^(1/3)+
b^(1/3)*x)/a^(11/3)/((b*x^3+a)^2)^(1/2)+10/27*b^(2/3)*(b*x^3+a)*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/((b*x^3+a)^2)^(1/2)+20/27*b^(2/3)*(
b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)*3^(1/2)
/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1369, 296, 331, 206, 31, 648, 631, 210, 642}

$$\frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6ax^2\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}+\frac{20b^{2/3}\left(a+bx^3\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{20b^{2/3}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{10b^{2/3}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{11/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{10\left(a+bx^3\right)}{9a^3x^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] 4/(9*a^2*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(6*a*x^2*(a + b*x^3)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6]) - (10*(a + b*x^3))/(9*a^3*x^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + (20*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(
Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (
20*b^(2/3)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + (10*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/(27*a^(11/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
```

```
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx = \frac{\left(b^2(ab+b^2x^3)\right)\int \frac{1}{x^3(ab+b^2x^3)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(4b(ab+b^2x^3)\right)\int \frac{1}{x^3(ab+b^2x^3)^2}\,dx}{3a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(20}{9}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{9a^3}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{9a^3}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{9a^3}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{9a^3}$$

$$= \frac{4}{9a^2x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{6ax^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{9a^3}$$

Mathematica [A]
time = 0.06, size = 266, normalized size = 0.84

$$\frac{-27a^{8/3}-96a^{5/3}bx^3-60a^{2/3}b^2x^6+40\sqrt{3}\,b^{2/3}x^2(a+bx^3)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)-40b^{2/3}x^2(a+bx^3)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+20a^2b^{2/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+40ab^{5/3}x^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)+20b^{8/3}x^8\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{11/3}x^2\left(a+bx^3\right)\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-27*a^(8/3) - 96*a^(5/3)*b*x^3 - 60*a^(2/3)*b^2*x^6 + 40*Sqrt[3]*b^(2/3)*x
^2*(a + b*x^3)^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 40*b^(2/3)*x
^2*(a + b*x^3)^2*Log[a^(1/3) + b^(1/3)*x] + 20*a^2*b^(2/3)*x^2*Log[a^(2/3)
```

```
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + 40*a*b^(5/3)*x^5*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2] + 20*b^(8/3)*x^8*Log[a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2])/(54*a^(11/3)*x^2*(a + b*x^3)*Sqrt[(a + b*x^3)^2])
```

Maple [A]

time = 0.04, size = 322, normalized size = 1.02

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(-\frac{10 b^{2} x^{6}}{9 a^{3}}-\frac{16 b\,x^{3}}{9 a^{2}}-\frac{1}{2 a}\right)}{\left(b\,x^{3}+a\right)^{3} x^{2}}+\frac{20\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^{3}+b^{2}\right)}\_R\ln\left(\left(\left(-4\_R^{3}a^{11}-3b^{2}\right)x-a^{4}b\right.\right.\right.}{27\left(b\,x^{3}+a\right)}$
default	$\frac{\left(40\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)\sqrt{3}\,b^{2}x^{8}-40\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{2}x^{8}+20\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{2}x^{8}-60\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}x^{6}+80\arctan\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/54*(40*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*3^(1/2)*b^2*x^8
-40*ln(x+(a/b)^(1/3))*b^2*x^8+20*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*b^2*x^8-
60*(a/b)^(2/3)*b^2*x^6+80*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3)
)*3^(1/2)*a*b*x^5-80*ln(x+(a/b)^(1/3))*a*b*x^5+40*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))*a*b*x^5-96*(a/b)^(2/3)*a*b*x^3+40*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(
1/3))/(a/b)^(1/3))*3^(1/2)*a^2*x^2-40*ln(x+(a/b)^(1/3))*a^2*x^2+20*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))*a^2*x^2-27*(a/b)^(2/3)*a^2)*(b*x^3+a)/x^2/(a/b)^(
2/3)/a^3/((b*x^3+a)^2)^(3/2)
```

Maxima [A]

time = 0.49, size = 150, normalized size = 0.47

$$-\frac{20\,b^{2}x^{6}+32\,abx^{3}+9\,a^{2}}{18\,\left(a^{3}b^{2}x^{8}+2\,a^{4}bx^{5}+a^{5}x^{2}\right)}-\frac{20\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{3}\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{10\,\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^{3}\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{20\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\,a^{3}\,\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/18*(20*b^2*x^6 + 32*a*b*x^3 + 9*a^2)/(a^3*b^2*x^8 + 2*a^4*b*x^5 + a^5*x^
2) - 20/27*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3
*(a/b)^(2/3)) + 10/27*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/
3)) - 20/27*log(x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

Fricas [A]

time = 0.40, size = 242, normalized size = 0.77

$$-\frac{60\,b^{2}x^{6}+96\,abx^{3}-40\,\sqrt{3}\,\left(b^{2}x^{8}+2\,abx^{5}+a^{2}x^{2}\right)\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)+20\,\left(b^{2}x^{8}+2\,abx^{5}+a^{2}x^{2}\right)\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}+abx\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)-40\,\left(b^{2}x^{8}+2\,abx^{5}+a^{2}x^{2}\right)\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\right)+27\,a^{2}}{54\,\left(a^{3}b^{2}x^{8}+2\,a^{4}bx^{5}+a^{5}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/54*(60*b^2*x^6 + 96*a*b*x^3 - 40*sqrt(3)*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)
*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b
) + 20*(b^2*x^8 + 2*a*b*x^5 + a^2*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x
*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - 40*(b^2*x^8 + 2*a*b*x^5 + a^2*x
^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) + 27*a^2)/(a^3*b^2*x^8 +
 2*a^4*b*x^5 + a^5*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**3)**2)**(3/2)), x)
```

Giac [A]
time = 4.19, size = 184, normalized size = 0.58

$$\frac{20\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4\mathrm{sgn}\left(bx^3+a\right)}-\frac{20\,\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4\mathrm{sgn}\left(bx^3+a\right)}-\frac{10\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\,a^4\mathrm{sgn}\left(bx^3+a\right)}-\frac{20\,b^2x^6+32\,abx^3+9\,a^2}{18\left(bx^4+ax\right)^2a^3\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 20/27*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*sgn(b*x^3 + a)) - 20/2
7*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/
3))/(a^4*sgn(b*x^3 + a)) - 10/27*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/(a^4*sgn(b*x^3 + a)) - 1/18*(20*b^2*x^6 + 32*a*b*x^3 + 9*a^2)
/((b*x^4 + a*x)^2*a^3*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

3.106 $\int \frac{1}{x^4(a^2+2abx^3+b^2x^6)^{3/2}}\,dx$

Optimal. Leaf size=188

$$-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b(a+bx^3)}{a^4\sqrt{a^2+2abx}}$$

[Out] -2/3*b/a^3/((b*x^3+a)^2)^(1/2)-1/6*b/a^2/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+1/3*(-b*x^3-a)/a^3/x^3/((b*x^3+a)^2)^(1/2)-3*b*(b*x^3+a)*ln(x)/a^4/((b*x^3+a)^2)^(1/2)+b*(b*x^3+a)*ln(b*x^3+a)/a^4/((b*x^3+a)^2)^(1/2)

Rubi [A]
time = 0.07, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 46}

$$-\frac{b}{6a^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{3b\log(x)\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{b(a+bx^3)\log\left(a+bx^3\right)}{a^4\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^3x^3\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-2*b)/(3*a^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(6*a^2*(a + b*x^3)*Sqrt[
a^2 + 2*a*b*x^3 + b^2*x^6]) - (a + b*x^3)/(3*a^3*x^3*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]) - (3*b*(a + b*x^3)*Log[x])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 + (b*(a + b*x^3)*Log[a + b*x^3])/(a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```

[p - 1/2]

Rubi steps

$$\int \frac{1}{x^4 (a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^3)) \int \frac{1}{x^4(ab+b^2x^3)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(b^2(ab+b^2x^3))\,\mathrm{Subst}\left(\int \frac{1}{x^2(ab+b^2x)^3}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(b^2(ab+b^2x^3))\,\mathrm{Subst}\left(\int \left(\frac{1}{a^3b^3x^2} - \frac{3}{a^4b^2x} + \frac{1}{a^2b(a+bx)^3} + \frac{2}{a^3b(a+bx)^2} + \frac{3}{a^4b(a+b}\right.\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= -\frac{2b}{3a^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{b}{6a^2(a+bx^3)\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{3a^3x^3}$$

Mathematica [A]
time = 0.02, size = 97, normalized size = 0.52

$$\frac{-a(2a^2+9abx^3+6b^2x^6) - 18bx^3(a+bx^3)^2\log(x) + 6bx^3(a+bx^3)^2\log(a+bx^3)}{6a^4x^3(a+bx^3)\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2)),x]
```

```
[Out] (-(a*(2*a^2 + 9*a*b*x^3 + 6*b^2*x^6)) - 18*b*x^3*(a + b*x^3)^2*Log[x] + 6*b
*x^3*(a + b*x^3)^2*Log[a + b*x^3])/(6*a^4*x^3*(a + b*x^3)*Sqrt[(a + b*x^3)^
2])
```

Maple [A]
time = 0.04, size = 133, normalized size = 0.71

method	result
risch	$\frac{\sqrt{(bx^3+a)^2}\left(-\frac{b^2x^6}{a^3}-\frac{3bx^3}{2a^2}-\frac{1}{3a}\right)}{(bx^3+a)^3x^3} - \frac{3\sqrt{(bx^3+a)^2}\,b\ln(x)}{(bx^3+a)a^4} + \frac{\sqrt{(bx^3+a)^2}\,b\ln(-bx^3-a)}{(bx^3+a)a^4}$
default	$\frac{(6\ln(bx^3+a)b^3x^9-18b^3\ln(x)x^9+12\ln(bx^3+a)ab^2x^6-36\ln(x)ab^2x^6-6ab^2x^6+6\ln(bx^3+a)a^2bx^3-18a^2b\ln(x)x^3-9a^2bx^3-2a^3)}{6x^3a^4\left((bx^3+a)^2\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(6*ln(b*x^3+a)*b^3*x^9-18*b^3*ln(x)*x^9+12*ln(b*x^3+a)*a*b^2*x^6-36*ln(
x)*a*b^2*x^6-6*a*b^2*x^6+6*ln(b*x^3+a)*a^2*b*x^3-18*a^2*b*ln(x)*x^3-9*a^2*b
*x^3-2*a^3)*(b*x^3+a)/x^3/a^4/((b*x^3+a)^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 117, normalized size = 0.62

$$\frac{(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{a^4}-\frac{b}{\sqrt{b^2x^6+2\,abx^3+a^2}\,a^3}-\frac{1}{6\,\left(x^3+\frac{a}{b}\right)^2a^2b}-\frac{1}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^4 - b
/(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^3) - 1/6/((x^3 + a/b)^2*a^2*b) - 1/3/(s
qrt(b^2*x^6 + 2*a*b*x^3 + a^2)*a^2*x^3)
```

Fricas [A]
time = 0.37, size = 119, normalized size = 0.63

$$-\frac{6\,ab^2x^6+9\,a^2bx^3+2\,a^3-6\,(b^3x^9+2\,ab^2x^6+a^2bx^3)\log\left(bx^3+a\right)+18\,(b^3x^9+2\,ab^2x^6+a^2bx^3)\log\left(x\right)}{6\,(a^4b^2x^9+2\,a^5bx^6+a^6x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(6*a*b^2*x^6 + 9*a^2*b*x^3 + 2*a^3 - 6*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*
x^3)*log(b*x^3 + a) + 18*(b^3*x^9 + 2*a*b^2*x^6 + a^2*b*x^3)*log(x))/(a^4*b
^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^4\left(\left(a+bx^3\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**3)**2)**(3/2)), x)
```

Giac [A]
time = 5.02, size = 120, normalized size = 0.64

$$\frac{b\log\left(|bx^3+a|\right)}{a^4\mathrm{sgn}\left(bx^3+a\right)}-\frac{3\,b\log\left(|x|\right)}{a^4\mathrm{sgn}\left(bx^3+a\right)}-\frac{9\,b^3x^6+22\,ab^2x^3+14\,a^2b}{6\,(bx^3+a)^2a^4\mathrm{sgn}\left(bx^3+a\right)}+\frac{3\,bx^3-a}{3\,a^4x^3\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x^3 + a))/(a^4*sgn(b*x^3 + a)) - 3*b*log(abs(x))/(a^4*sgn(b*x^3
 + a)) - 1/6*(9*b^3*x^6 + 22*a*b^2*x^3 + 14*a^2*b)/((b*x^3 + a)^2*a^4*sgn(b
*x^3 + a)) + 1/3*(3*b*x^3 - a)/(a^4*x^3*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2)), x)
```

3.107 $\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+1$$

```
[Out] 5/486*x/a^2/b^2/((b*x^3+a)^2)^(1/2)-1/12*x^4/b/(b*x^3+a)^3/((b*x^3+a)^2)^(1
/2)-1/27*x/b^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/162*x/a/b^2/(b*x^3+a)/((b*
x^3+a)^2)^(1/2)+5/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(7/3)/((b*x
^3+a)^2)^(1/2)-5/1458*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a
^(8/3)/b^(7/3)/((b*x^3+a)^2)^(1/2)-5/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(7/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1369, 294, 205, 206, 31, 648, 631, 210, 642}

$$\frac{x}{162ab^2\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5\left(a+bx^3\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{8/3}b^{7/3}\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (5*x)/(486*a^2*b^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^4/(12*b*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(27*b^2*(a + b*x^3)^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + x/(162*a*b^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x
^6]) - (5*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2
43*Sqrt[3]*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*(a + b*x^3
)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(14
58*a^(8/3)*b^(7/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^6}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^6}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x^3}{\left(ab+b^2x^3\right)^4}\,dx}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{27b^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{27b^2\,(}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{27b^2\,(}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{27b^2\,(}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{27b^2\,(}$$

$$= \frac{5x}{486a^2b^2\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^4}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} - \frac{}{27b^2\,(}$$

Mathematica [A]
time = 0.09, size = 218, normalized size = 0.61

$$\frac{\left(a+bx^3\right)\left(243a\sqrt[3]{b}\,x-351\sqrt[3]{b}\,x\left(a+bx^3\right)+\frac{18\sqrt[3]{b}\,x\left(a+bx^3\right)^2}{a}+\frac{30\sqrt[3]{b}\,x\left(a+bx^3\right)^3}{a^2}+\frac{20\sqrt{3}\left(a+bx^3\right)^4\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{8/3}}+\frac{20\left(a+bx^3\right)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{8/3}}-\frac{10\left(a+bx^3\right)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{8/3}}\right)}{2916b^{7/3}\left(\left(a+bx^3\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a*b^(1/3)*x - 351*b^(1/3)*x*(a + b*x^3) + (18*b^(1/3)*x*(
a + b*x^3)^2)/a + (30*b^(1/3)*x*(a + b*x^3)^3)/a^2 + (20*Sqrt[3]*(a + b*x^3
)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(8/3) + (20*(a +
b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) - (10*(a + b*x^3)^4*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(8/3)))/(2916*b^(7/3)*((a + b*x^3)^2)
^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(235) = 470$.
time = 0.04, size = 519, normalized size = 1.45

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{5 b\,x^{10}}{486 a^{2}}+\frac{x^{7}}{27 a}-\frac{25 x^{4}}{324 b}-\frac{5 a x}{243 b^{2}}\right)}{\left(b\,x^{3}+a\right)^{5}}+\frac{5\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^{2}}\right)}{729\left(b\,x^{3}+a\right)a^{2}b^{3}}$
default	$-\frac{\left(20\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^{4}x^{12}-20\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^{4}x^{12}+10\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^{4}x^{12}-30\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{4}x^{10}+80\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2916*(20*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*b^4*
x^12-20*ln(x+(a/b)^(1/3))*b^4*x^12+10*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*b^4
*x^12-30*(a/b)^(2/3)*b^4*x^10+80*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/
3))/(a/b)^(1/3))*a*b^3*x^9-80*ln(x+(a/b)^(1/3))*a*b^3*x^9+40*ln(x^2-(a/b)^(
1/3)*x+(a/b)^(2/3))*a*b^3*x^9-108*(a/b)^(2/3)*a*b^3*x^7+120*3^(1/2)*arctan(
1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^2*b^2*x^6-120*ln(x+(a/b)^(1/3
))*a^2*b^2*x^6+60*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^2*b^2*x^6+225*(a/b)^(
2/3)*a^2*b^2*x^4+80*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/
3))*a^3*b*x^3-80*ln(x+(a/b)^(1/3))*a^3*b*x^3+40*ln(x^2-(a/b)^(1/3)*x+(a/b)^
(2/3))*a^3*b*x^3+60*(a/b)^(2/3)*a^3*b*x+20*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x
+(a/b)^(1/3))/(a/b)^(1/3))*a^4-20*ln(x+(a/b)^(1/3))*a^4+10*ln(x^2-(a/b)^(1/
3)*x+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(2/3)/b^3/a^2/((b*x^3+a)^2)^(5/2)
```

Maxima [A]
time = 0.51, size = 195, normalized size = 0.54

$$\frac{10\,b^{3}x^{10}+36\,ab^{2}x^{7}-75\,a^{2}bx^{4}-20\,a^{3}x}{972\,\left(a^{2}b^{6}x^{12}+4\,a^{3}b^{5}x^{9}+6\,a^{4}b^{4}x^{6}+4\,a^{5}b^{3}x^{3}+a^{6}b^{2}\right)}+\frac{5\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^{2}b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^{2}b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^{2}b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/972*(10*b^3*x^10 + 36*a*b^2*x^7 - 75*a^2*b*x^4 - 20*a^3*x)/(a^2*b^6*x^12 + 4*a^3*b^5*x^9 + 6*a^4*b^4*x^6 + 4*a^5*b^3*x^3 + a^6*b^2) + 5/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3)) - 5/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^3*(a/b)^(2/3)) + 5/729*log(x + (a/b)^(1/3))/(a^2*b^3*(a/b)^(2/3))

Fricas [A]
time = 0.37, size = 723, normalized size = 2.01

$$\left[\frac{30a^2b^4x^{10}+108a^3b^3x^7-225a^4b^2x^4-60a^5bx+30\sqrt{\frac{1}{3}}(ab^5x^{12}+4a^2b^4x^9+6a^3b^3x^6+4a^4b^2x^3+a^5b)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3-3(a^2b)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-10(b^4x^{12}+4ab^3x^9+6a^2b^2x^6+4a^3bx^3+a^4)(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a^2b)^{\frac{2}{3}}x+(a^2b)^{\frac{1}{3}}a\right)+20(b^4x^{12}+4ab^3x^9+6a^2b^2x^6+4a^3bx^3+a^4)(a^2b)^{\frac{2}{3}}\log\left(abx+(a^2b)^{\frac{2}{3}}\right)}{2916(a^4b^7x^{12}+4a^5b^6x^9+6a^6b^5x^6+4a^7b^4x^3+a^8b^3)}, \frac{30a^2b^4x^{10}+108a^3b^3x^7-225a^4b^2x^4-60a^5bx+60\sqrt{\frac{1}{3}}(ab^5x^{12}+4a^2b^4x^9+6a^3b^3x^6+4a^4b^2x^3+a^5b)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-10(b^4x^{12}+4ab^3x^9+6a^2b^2x^6+4a^3bx^3+a^4)(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a^2b)^{\frac{2}{3}}x+(a^2b)^{\frac{1}{3}}a\right)+20(b^4x^{12}+4ab^3x^9+6a^2b^2x^6+4a^3bx^3+a^4)(a^2b)^{\frac{2}{3}}\log\left(abx+(a^2b)^{\frac{2}{3}}\right)}{2916(a^4b^7x^{12}+4a^5b^6x^9+6a^6b^5x^6+4a^7b^4x^3+a^8b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] [1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7 - 225*a^4*b^2*x^4 - 60*a^5*b*x + 30*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(30*a^2*b^4*x^10 + 108*a^3*b^3*x^7 - 225*a^4*b^2*x^4 - 60*a^5*b*x + 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 10*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)

[Out] Integral(x**6/((a + b*x**3)**2)**(5/2), x)

Giac [A]
time = 4.77, size = 205, normalized size = 0.57

$$-\frac{5 \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458 \left(-ab^2\right)^{\frac{2}{3}} a^2 b \operatorname{sgn}\left(bx^3 + a\right)} - \frac{5 \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729 a^3 b^2 \operatorname{sgn}\left(bx^3 + a\right)} + \frac{5 \sqrt{3} \left(-ab^2\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729 a^3 b^3 \operatorname{sgn}\left(bx^3 + a\right)} + \frac{10 b^3 x^{10} + 36 ab^2 x^7 - 75 a^2 b x^4 - 20 a^3 x}{972 \left(bx^3 + a\right)^4 a^2 b^2 \operatorname{sgn}\left(bx^3 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -5/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2*b*sgn(
b*x^3 + a)) - 5/729*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b^2*sgn(b*
x^3 + a)) + 5/729*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a^3*b^3*sgn(b*x^3 + a)) + 1/972*(10*b^3*x^10 + 36*a*b^
2*x^7 - 75*a^2*b*x^4 - 20*a^3*x)/((b*x^3 + a)^4*a^2*b^2*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(a^2 + 2\, a\, b\, x^3 + b^2\, x^6\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^6/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.108 $\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{a}{12b^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 1/12*a/b^2/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)-1/9/b^2/(b*x^3+a)^2/((b*x^3+a)^2
)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 45}

$$\frac{a}{12b^2(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{1}{9b^2(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] a/(12*b^2*(a + b*x^3)^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - 1/(9*b^2*(a + b*
x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^5}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(b^4(ab+b^2x^3))\int \frac{x^5}{(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(b^4(ab+b^2x^3))\,\mathrm{Subst}\left(\int \frac{x}{(ab+b^2x)^5}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{(b^4(ab+b^2x^3))\,\mathrm{Subst}\left(\int \left(-\frac{a}{b^6(a+bx)^5}+\frac{1}{b^6(a+bx)^4}\right)\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$
$$= \frac{a}{12b^2\,(a+bx^3)^3\,\sqrt{a^2+2abx^3+b^2x^6}} - \frac{1}{9b^2\,(a+bx^3)^2\,\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.50

$$\frac{-a-4bx^3}{36b^2\,(a+bx^3)^3\,\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
[Out] (-a - 4*b*x^3)/(36*b^2*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

Maple [A]
time = 0.03, size = 32, normalized size = 0.41

method	result	size
gosper	$-\frac{(b\,x^3+a)\,(4b\,x^3+a)}{36b^2\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}$	32
default	$-\frac{(b\,x^3+a)\,(4b\,x^3+a)}{36b^2\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}$	32
risch	$\frac{\sqrt{(b\,x^3+a)^2}\,\left(-\frac{x^3}{9b}-\frac{a}{36b^2}\right)}{(b\,x^3+a)^5}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/36*(b*x^3+a)*(4*b*x^3+a)/b^2/((b*x^3+a)^2)^(5/2)
```

Maxima [A]
time = 0.28, size = 43, normalized size = 0.55

$$-\frac{1}{9\left(b^{2}x^{6}+2\,abx^{3}+a^{2}\right)^{\frac{3}{2}}b^{2}}+\frac{a}{12\left(x^{3}+\frac{a}{b}\right)^{4}b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] -1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*b^2) + 1/12*a/((x^3 + a/b)^4*b^6)
```

Fricas [A]
time = 0.34, size = 58, normalized size = 0.74

$$-\frac{4\,bx^{3}+a}{36\left(b^{6}x^{12}+4\,ab^{5}x^{9}+6\,a^{2}b^{4}x^{6}+4\,a^{3}b^{3}x^{3}+a^{4}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
[Out] -1/36*(4*b*x^3 + a)/(b^6*x^12 + 4*a*b^5*x^9 + 6*a^2*b^4*x^6 + 4*a^3*b^3*x^3
 + a^4*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{5}}{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
[Out] Integral(x**5/((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 4.31, size = 32, normalized size = 0.41

$$-\frac{4\,bx^{3}+a}{36\left(bx^{3}+a\right)^{4}b^{2}\mathrm{sgn}\left(bx^{3}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
[Out] -1/36*(4*b*x^3 + a)/((b*x^3 + a)^4*b^2*sgn(b*x^3 + a))
```

Mupad [B]
time = 1.28, size = 42, normalized size = 0.54

$$-\frac{\left(4\,b\,x^3+a\right)\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{36\,b^2\,{\left(b\,x^3+a\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
[Out] -((a + 4*b*x^3)*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2))/(36*b^2*(a + b*x^3)^5)
```

3.109 $\int \frac{x^4}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=368

$$\frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{}{32}$$

```
[Out] 7/243*x^2/a^3/b/((b*x^3+a)^2)^(1/2)-1/12*x^2/b/(b*x^3+a)^3/((b*x^3+a)^2)^(1
/2)+1/54*x^2/a/b/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+7/324*x^2/a^2/b/(b*x^3+a)/
((b*x^3+a)^2)^(1/2)-7/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(5/3)/
((b*x^3+a)^2)^(1/2)+7/1458*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/a^(10/3)/b^(5/3)/((b*x^3+a)^2)^(1/2)-7/729*(b*x^3+a)*arctan(1/3*(a^(1/3
)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/b^(5/3)*3^(1/2)/((b*x^3+a)^2)^(1/2
)
```

Rubi [A]

time = 0.13, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1369, 294, 296, 298, 31, 648, 631, 210, 642}

$$\frac{7x^2}{324a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{7(a+bx^3)\,\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{243\sqrt{3}\,a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{7(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{729a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{1458a^{10/3}b^{5/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (7*x^2)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x^2/(12*b*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(54*a*b*(a + b*x^3)^2*Sqrt[a^2 +
2*a*b*x^3 + b^2*x^6]) + (7*x^2)/(324*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3
 + b^2*x^6]) - (7*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (7*(
a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + (7*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(1458*a^(10/3)*b^(5/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^4}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{x^4}{\left(ab+b^2x^3\right)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2\left(ab+b^2x^3\right)\right)\int \frac{x}{\left(ab+b^2x^3\right)^4}\,dx}{6\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} \cdots$$

$$= -\frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{54ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}} \cdots$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

$$= \frac{7x^2}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x^2}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \cdots$$

Mathematica [A]
time = 0.09, size = 229, normalized size = 0.62

$$\frac{(a+bx^3)\left(-243a^{10/3}b^{2/3}x^2+54a^{7/3}b^{2/3}x^2(a+bx^3)+63a^{4/3}b^{2/3}x^2(a+bx^3)^2+84\sqrt[3]{a}\,b^{2/3}x^2(a+bx^3)^3+28\sqrt{3}\,(a+bx^3)^4\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)-28(a+bx^3)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+14(a+bx^3)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{2916a^{10/3}b^{5/3}\left((a+bx^3)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(10/3)*b^(2/3)*x^2 + 54*a^(7/3)*b^(2/3)*x^2*(a + b*x^3
) + 63*a^(4/3)*b^(2/3)*x^2*(a + b*x^3)^2 + 84*a^(1/3)*b^(2/3)*x^2*(a + b*x^
3)^3 + 28*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))] - 28*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] + 14*(a + b*x^3)^4*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(10/3)*b^(5/3)*((a + b
*x^3)^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(244) = 488$.
time = 0.04, size = 521, normalized size = 1.42

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{7 b^{2} x^{11}}{243 a^{3}}+\frac{35 b\,x^{8}}{324 a^{2}}+\frac{4 x^{5}}{27 a}-\frac{7 x^{2}}{486 b}\right)}{\left(b\,x^{3}+a\right)^{5}}+\frac{7 \sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{729 \left(b\,x^{3}+a\right) b^{2} a^{3}}$
default	$-\frac{\left(28\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) b^{4} x^{12}+28\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) b^{4} x^{12}-14\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) b^{4} x^{12}-84\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{4} x^{11}+112 \cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2916*(28*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*b^4*
x^12+28*ln(x+(a/b)^(1/3))*b^4*x^12-14*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*b^4
*x^12-84*(a/b)^(1/3)*b^4*x^11+112*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1
/3))/(a/b)^(1/3))*a*b^3*x^9+112*ln(x+(a/b)^(1/3))*a*b^3*x^9-56*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))*a*b^3*x^9-315*(a/b)^(1/3)*a*b^3*x^8+168*3^(1/2)*arcta
n(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^2*b^2*x^6+168*ln(x+(a/b)^(1
/3))*a^2*b^2*x^6-84*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^2*b^2*x^6-432*(a/b)
^(1/3)*a^2*b^2*x^5+112*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^
(1/3))*a^3*b*x^3+112*ln(x+(a/b)^(1/3))*a^3*b*x^3-56*ln(x^2-(a/b)^(1/3)*x+(a
/b)^(2/3))*a^3*b*x^3+42*(a/b)^(1/3)*a^3*b*x^2+28*3^(1/2)*arctan(1/3*3^(1/2)
*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^4+28*ln(x+(a/b)^(1/3))*a^4-14*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(1/3)/b^2/a^3/((b*x^3+a)^2)^(5
/2)
```

Maxima [A]
time = 0.50, size = 195, normalized size = 0.53

$$\frac{28\,b^{3}x^{11}+105\,ab^{2}x^{8}+144\,a^{2}bx^{5}-14\,a^{3}x^{2}}{972\,\left(a^{3}b^{5}x^{12}+4\,a^{4}b^{4}x^{9}+6\,a^{5}b^{3}x^{6}+4\,a^{6}b^{2}x^{3}+a^{7}b\right)}+\frac{7\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^{3}b^{2}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{7\,\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^{3}b^{2}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^{3}b^{2}\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(28*b^3*x^11 + 105*a*b^2*x^8 + 144*a^2*b*x^5 - 14*a^3*x^2)/(a^3*b^5*x
^12 + 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 7/729*sqrt(3
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3))
 + 7/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(1/3)) - 7/
729*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(1/3))
```

Fricas [A]
time = 0.36, size = 734, normalized size = 1.99

$$\left[\frac{84\,ab^{5}x^{11}+315\,a^{2}b^{4}x^{8}+432\,a^{3}b^{3}x^{5}-42\,a^{4}b^{2}x^{2}+42\,\sqrt{\frac{1}{3}}\left(ab^{5}x^{12}+4\,a^{2}b^{4}x^{9}+6\,a^{3}b^{3}x^{6}+4\,a^{4}b^{2}x^{3}+a^{5}b\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^{2}x^{3}-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^{2}\right)^{\frac{2}{3}}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}-3\left(-ab^{2}\right)^{\frac{2}{3}}x}{bx^{3}+a}\right)+14\left(b^{4}x^{12}+4\,ab^{3}x^{9}+6\,a^{2}b^{2}x^{6}+4\,a^{3}bx^{3}+a^{4}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}bx+\left(-ab^{2}\right)^{\frac{2}{3}}\right)-28\left(b^{4}x^{12}+4\,ab^{3}x^{9}+6\,a^{2}b^{2}x^{6}+4\,a^{3}bx^{3}+a^{4}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(bx-\left(-ab^{2}\right)^{\frac{1}{3}}\right)}{2916\left(a^{4}b^{7}x^{12}+4\,a^{5}b^{6}x^{9}+6\,a^{6}b^{5}x^{6}+4\,a^{7}b^{4}x^{3}+a^{8}b^{3}\right)},\ \frac{84\,ab^{5}x^{11}+315\,a^{2}b^{4}x^{8}+432\,a^{3}b^{3}x^{5}-42\,a^{4}b^{2}x^{2}+84\,\sqrt{\frac{1}{3}}\left(ab^{5}x^{12}+4\,a^{2}b^{4}x^{9}+6\,a^{3}b^{3}x^{6}+4\,a^{4}b^{2}x^{3}+a^{5}b\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx+\left(-ab^{2}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}}{b}\right)+14\left(b^{4}x^{12}+4\,ab^{3}x^{9}+6\,a^{2}b^{2}x^{6}+4\,a^{3}bx^{3}+a^{4}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x^{2}+\left(-ab^{2}\right)^{\frac{1}{3}}bx+\left(-ab^{2}\right)^{\frac{2}{3}}\right)-28\left(b^{4}x^{12}+4\,ab^{3}x^{9}+6\,a^{2}b^{2}x^{6}+4\,a^{3}bx^{3}+a^{4}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(bx-\left(-ab^{2}\right)^{\frac{1}{3}}\right)}{2916\left(a^{4}b^{7}x^{12}+4\,a^{5}b^{6}x^{9}+6\,a^{6}b^{5}x^{6}+4\,a^{7}b^{4}x^{3}+a^{8}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(84*a*b^5*x^11 + 315*a^2*b^4*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2
 + 42*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3
 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x
+ 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b
^2)^(2/3)*x)/(b*x^3 + a)) + 14*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*
a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)
^(2/3)) - 28*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(
-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*
a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*b^3), 1/2916*(84*a*b^5*x^11 + 315*a^2*b^4
*x^8 + 432*a^3*b^3*x^5 - 42*a^4*b^2*x^2 + 84*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*
b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2)^(1/3)/a)*ar
ctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 14*(b^
4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 28*(b^4*x^12 + 4*a*b^3*x
^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(
1/3)))/(a^4*b^7*x^12 + 4*a^5*b^6*x^9 + 6*a^6*b^5*x^6 + 4*a^7*b^4*x^3 + a^8*
b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 4.47, size = 207, normalized size = 0.56

$$\frac{7\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}b\mathrm{sgn}\left(bx^{3}+a\right)}-\frac{7\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\left(-ab^{2}\right)^{\frac{1}{3}}a^{3}b\mathrm{sgn}\left(bx^{3}+a\right)}-\frac{7\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^{4}b\mathrm{sgn}\left(bx^{3}+a\right)}+\frac{28\,b^{3}x^{11}+105\,ab^{2}x^{8}+144\,a^{2}bx^{5}-14\,a^{3}x^{2}}{972\left(bx^{3}+a\right)^{4}a^{3}b\mathrm{sgn}\left(bx^{3}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 7/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^
2)^(1/3)*a^3*b*sgn(b*x^3 + a)) - 7/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(1/3)*a^3*b*sgn(b*x^3 + a)) - 7/729*(-a/b)^(2/3)*log(abs(x
- (-a/b)^(1/3)))/(a^4*b*sgn(b*x^3 + a)) + 1/972*(28*b^3*x^11 + 105*a*b^2*x^
8 + 144*a^2*b*x^5 - 14*a^3*x^2)/((b*x^3 + a)^4*a^3*b*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^4/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.110 $\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=360

$$\frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+$$

```
[Out] 5/243*x/a^3/b/((b*x^3+a)^2)^(1/2)-1/12*x/b/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+
1/108*x/a/b/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/81*x/a^2/b/(b*x^3+a)/((b*x^3+
a)^2)^(1/2)+10/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(4/3)/((b*x^3
+a)^2)^(1/2)-5/729*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1
1/3)/b^(4/3)/((b*x^3+a)^2)^(1/2)-10/729*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(11/3)/b^(4/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1369, 294, 205, 206, 31, 648, 631, 210, 642}

$$\frac{x}{81a^2b\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x}{108ab\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{x}{12b\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{10(a+bx^3)\,\mathrm{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{10(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{11/3}b^{4/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (5*x)/(243*a^3*b*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - x/(12*b*(a + b*x^3)^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x/(108*a*b*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b
*x^3 + b^2*x^6]) + x/(81*a^2*b*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
 - (10*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*
Sqrt[3]*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (10*(a + b*x^3)
*Log[a^(1/3) + b^(1/3)*x])/(729*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) - (5*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(72
9*a^(11/3)*b^(4/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^3}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^3)\right)\int \frac{x^3}{(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{12b\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(b^2(ab+b^2x^3)\right)\int \frac{1}{(ab+b^2x^3)^4}\,dx}{12\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{x}{12b\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab\,(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x}}$$

$$= -\frac{x}{12b\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x}{108ab\,(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x}}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108ab\,(}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108ab\,(}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108ab\,(}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108ab\,(}$$

$$= \frac{5x}{243a^3b\sqrt{a^2+2abx^3+b^2x^6}} - \frac{x}{12b\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{}{108ab\,(}$$

Mathematica [A]
time = 0.09, size = 221, normalized size = 0.61

$$\frac{(a+bx^3)\left(-243a^{11/3}\sqrt[3]{b}\,x+27a^{8/3}\sqrt[3]{b}\,x(a+bx^3)+36a^{5/3}\sqrt[3]{b}\,x(a+bx^3)^2+60a^{2/3}\sqrt[3]{b}\,x(a+bx^3)^3+40\sqrt{3}\,(a+bx^3)^4\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)+40(a+bx^3)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-20(a+bx^3)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{2916a^{11/3}b^{4/3}\left((a+bx^3)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(11/3)*b^(1/3)*x + 27*a^(8/3)*b^(1/3)*x*(a + b*x^3) +
36*a^(5/3)*b^(1/3)*x*(a + b*x^3)^2 + 60*a^(2/3)*b^(1/3)*x*(a + b*x^3)^3 + 4
0*Sqrt[3]*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
+ 40*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3)*x] - 20*(a + b*x^3)^4*Log[a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]))/(2916*a^(11/3)*b^(4/3)*((a + b*x^3)^2)
^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(236) = 472$.

time = 0.04, size = 519, normalized size = 1.44

method	result
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(\frac{5 b^2 x^{10}}{243 a^3}+\frac{2 b\,x^7}{27 a^2}+\frac{31 x^4}{324 a}-\frac{10 x}{243 b}\right)}{\left(b\,x^3+a\right)^5}+\frac{10\sqrt{\left(b\,x^3+a\right)^2}\,\left(\sum_{\_R=\mathrm{RootOf}\left(b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{\_R^2}\right)}{729\left(b\,x^3+a\right)b^2 a^3}$
default	$-\frac{\left(40\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{12}-40\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{12}+20\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{12}-60\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x^{10}+160\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2916*(40*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*b^4*
x^12-40*ln(x+(a/b)^(1/3))*b^4*x^12+20*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*b^4
*x^12-60*(a/b)^(2/3)*b^4*x^10+160*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1
/3))/(a/b)^(1/3))*a*b^3*x^9-160*ln(x+(a/b)^(1/3))*a*b^3*x^9+80*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))*a*b^3*x^9-216*(a/b)^(2/3)*a*b^3*x^7+240*3^(1/2)*arcta
n(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^2*b^2*x^6-240*ln(x+(a/b)^(1
/3))*a^2*b^2*x^6+120*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^2*b^2*x^6-279*(a/b
)^(2/3)*a^2*b^2*x^4+160*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)
^(1/3))*a^3*b*x^3-160*ln(x+(a/b)^(1/3))*a^3*b*x^3+80*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))*a^3*b*x^3+120*(a/b)^(2/3)*a^3*b*x+40*3^(1/2)*arctan(1/3*3^(1/2)
*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^4-40*ln(x+(a/b)^(1/3))*a^4+20*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(2/3)/b^2/a^3/((b*x^3+a)^2)^(5
/2)
```

Maxima [A]

time = 0.52, size = 193, normalized size = 0.54

$$\frac{20\,b^3x^{10}+72\,ab^2x^7+93\,a^2bx^4-40\,a^3x}{972\left(a^3b^5x^{12}+4\,a^4b^4x^9+6\,a^5b^3x^6+4\,a^6b^2x^3+a^7b\right)}+\frac{10\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{10\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")

[Out] 1/972*(20*b^3*x^10 + 72*a*b^2*x^7 + 93*a^2*b*x^4 - 40*a^3*x)/(a^3*b^5*x^12 + 4*a^4*b^4*x^9 + 6*a^5*b^3*x^6 + 4*a^6*b^2*x^3 + a^7*b) + 10/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3)) - 5/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b^2*(a/b)^(2/3)) + 10/729*log(x + (a/b)^(1/3))/(a^3*b^2*(a/b)^(2/3))

Fricas [A]
time = 0.37, size = 723, normalized size = 2.01

$$\left[\frac{60\,a^{2}b^{4}x^{10}+216\,a^{3}b^{3}x^{7}+279\,a^{4}b^{2}x^{4}-120\,a^{5}bx+60\,\sqrt{\frac{1}{3}}\,(ab^{5}x^{12}+4\,a^{2}b^{4}x^{9}+6\,a^{3}b^{3}x^{6}+4\,a^{4}b^{2}x^{3}+a^{5}b)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}-3\,(a^{2}b)^{\frac{1}{3}}ax-a^{2}+3\,\sqrt{\frac{1}{3}}\left(2\,abx^{2}+(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)-20\,(b^{4}x^{12}+4\,ab^{3}x^{9}+6\,a^{2}b^{2}x^{6}+4\,a^{3}bx^{3}+a^{4})(a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(a^{2}b)^{\frac{2}{3}}x+(a^{2}b)^{\frac{1}{3}}a\right)+40\,(b^{4}x^{12}+4\,ab^{3}x^{9}+6\,a^{2}b^{2}x^{6}+4\,a^{3}bx^{3}+a^{4})(a^{2}b)^{\frac{2}{3}}\log\left(abx+(a^{2}b)^{\frac{2}{3}}\right)}{2916\,(a^{5}b^{6}x^{12}+4\,a^{6}b^{5}x^{9}+6\,a^{7}b^{4}x^{6}+4\,a^{8}b^{3}x^{3}+a^{9}b^{2})},\ \frac{60\,a^{2}b^{4}x^{10}+216\,a^{3}b^{3}x^{7}+279\,a^{4}b^{2}x^{4}-120\,a^{5}bx+120\,\sqrt{\frac{1}{3}}\,(ab^{5}x^{12}+4\,a^{2}b^{4}x^{9}+6\,a^{3}b^{3}x^{6}+4\,a^{4}b^{2}x^{3}+a^{5}b)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^{2}b)^{\frac{2}{3}}x-(a^{2}b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^{2}b)^{\frac{1}{3}}}{b}}}{a^{2}}\right)-20\,(b^{4}x^{12}+4\,ab^{3}x^{9}+6\,a^{2}b^{2}x^{6}+4\,a^{3}bx^{3}+a^{4})(a^{2}b)^{\frac{2}{3}}\log\left(abx^{2}-(a^{2}b)^{\frac{2}{3}}x+(a^{2}b)^{\frac{1}{3}}a\right)+40\,(b^{4}x^{12}+4\,ab^{3}x^{9}+6\,a^{2}b^{2}x^{6}+4\,a^{3}bx^{3}+a^{4})(a^{2}b)^{\frac{2}{3}}\log\left(abx+(a^{2}b)^{\frac{2}{3}}\right)}{2916\,(a^{5}b^{6}x^{12}+4\,a^{6}b^{5}x^{9}+6\,a^{7}b^{4}x^{6}+4\,a^{8}b^{3}x^{3}+a^{9}b^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")

[Out] [1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^7 + 279*a^4*b^2*x^4 - 120*a^5*b*x + 60*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(60*a^2*b^4*x^10 + 216*a^3*b^3*x^7 + 279*a^4*b^2*x^4 - 120*a^5*b*x + 120*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 20*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 40*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)

[Out] Integral(x**3/((a + b*x**3)**2)**(5/2), x)

Giac [A]
time = 4.94, size = 199, normalized size = 0.55

$$-\frac{10\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\left(-ab^2\right)^{\frac{2}{3}}a^3\mathrm{sgn}\left(bx^3+a\right)}-\frac{5\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\left(-ab^2\right)^{\frac{2}{3}}a^3\mathrm{sgn}\left(bx^3+a\right)}-\frac{10\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^4b\mathrm{sgn}\left(bx^3+a\right)}+\frac{20\,b^3x^{10}+72\,ab^2x^7+93\,a^2bx^4-40\,a^3x}{972\left(bx^3+a\right)^4a^3b\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -10/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*
b^2)^(2/3)*a^3*sgn(b*x^3 + a)) - 5/729*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3))/((-a*b^2)^(2/3)*a^3*sgn(b*x^3 + a)) - 10/729*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/(a^4*b*sgn(b*x^3 + a)) + 1/972*(20*b^3*x^10 + 72*a*b^2*x^7 +
 93*a^2*b*x^4 - 40*a^3*x)/((b*x^3 + a)^4*a^3*b*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x^3/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.111 $\int \frac{x^2}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

```
[Out] -1/12/b/(b*x^3+a)/(b^2*x^6+2*a*b*x^3+a^2)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1366, 621}

$$-\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] -1/12*1/(b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2))
```

Rule 621

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, x^3\right)\\ &= -\frac{1}{12b\left(a+bx^3\right)\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 27, normalized size = 0.71

$$-\frac{a+bx^3}{12b\left((a+bx^3)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
[Out] -1/12*(a + b*x^3)/(b*((a + b*x^3)^2)^(5/2))
```

Maple [A]
time = 0.03, size = 24, normalized size = 0.63

method	result	size
gosper	$-\frac{b\,x^3+a}{12b\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}$	24
default	$-\frac{b\,x^3+a}{12b\left((b\,x^3+a)^2\right)^{\frac{5}{2}}}$	24
risch	$-\frac{\sqrt{(b\,x^3+a)^2}}{12(b\,x^3+a)^5 b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/12*(b*x^3+a)/b/((b*x^3+a)^2)^(5/2)
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.42

$$-\frac{1}{12\left(x^3+\frac{a}{b}\right)^4 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] -1/12/((x^3 + a/b)^4*b^5)
```

Fricas [A]
time = 0.33, size = 48, normalized size = 1.26

$$-\frac{1}{12\left(b^5x^{12}+4\,ab^4x^9+6\,a^2b^3x^6+4\,a^3b^2x^3+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12/(b^5*x^12 + 4*a*b^4*x^9 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^3 + a^4*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 5.91, size = 24, normalized size = 0.63

$$-\frac{1}{12\left(bx^3+a\right)^4 b\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12/((b*x^3 + a)^4*b*sgn(b*x^3 + a))
```

Mupad [B]
time = 1.26, size = 34, normalized size = 0.89

$$-\frac{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}{12\,b\,\left(b\,x^3+a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] -(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2)/(12*b*(a + b*x^3)^5)
```

3.112 $\int \frac{x}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{}{3}$$

```
[Out] 35/243*x^2/a^4/((b*x^3+a)^2)^(1/2)+1/12*x^2/a/(b*x^3+a)^3/((b*x^3+a)^2)^(1/
2)+5/54*x^2/a^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+35/324*x^2/a^3/(b*x^3+a)/((
b*x^3+a)^2)^(1/2)-35/729*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)/b^(2/3)/(
(b*x^3+a)^2)^(1/2)+35/1458*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/a^(13/3)/b^(2/3)/((b*x^3+a)^2)^(1/2)-35/729*(b*x^3+a)*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)/b^(2/3)*3^(1/2)/((b*x^3+a)^2)^(1/
2)
```

Rubi [A]
time = 0.13, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1369, 296, 298, 31, 648, 631, 210, 642}

$$\frac{5x^2}{54a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{x^2}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35\left(a+bx^3\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}\,a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{35\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{729a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{1458a^{13/3}b^{2/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{35x^2}{324a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (35*x^2)/(243*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + x^2/(12*a*(a + b*x^3)^
3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*x^2)/(54*a^2*(a + b*x^3)^2*Sqrt[a^2
 + 2*a*b*x^3 + b^2*x^6]) + (35*x^2)/(324*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x
^3 + b^2*x^6]) - (35*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(243*Sqrt[3]*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (
35*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6]) + (35*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2])/(1458*a^(13/3)*b^(2/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(b^4(ab+b^2x^3))\int \frac{x}{(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{12a\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{(5b^3(ab+b^2x^3))\int \frac{x}{(ab+b^2x^3)^4}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{12a\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x^2}{54a^2\,(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{x^2}{12a\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{5x^2}{54a^2\,(a+bx^3)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\quad}{54a^2\,(}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\quad}{54a^2\,(}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\quad}{54a^2\,(}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\quad}{54a^2\,(}$$

$$= \frac{35x^2}{243a^4\sqrt{a^2+2abx^3+b^2x^6}} + \frac{x^2}{12a\,(a+bx^3)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\quad}{54a^2\,(}$$

Mathematica [A]
time = 0.08, size = 219, normalized size = 0.61

$$\frac{(a+bx^3)\left(243a^{10/3}x^2+270a^{7/3}x^2(a+bx^3)+315a^{4/3}x^2(a+bx^3)^2+420\sqrt[3]{a}\,x^2(a+bx^3)^3+\frac{140\sqrt{3}\,(a+bx^3)^4\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{2/3}}-\frac{140(a+bx^3)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{b^{2/3}}+\frac{70(a+bx^3)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{b^{2/3}}\right)}{2916a^{13/3}\left((a+bx^3)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a^(10/3)*x^2 + 270*a^(7/3)*x^2*(a + b*x^3) + 315*a^(4/3)*
x^2*(a + b*x^3)^2 + 420*a^(1/3)*x^2*(a + b*x^3)^3 + (140*Sqrt[3]*(a + b*x^3
)^4*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(2/3) - (140*(a +
 b*x^3)^4*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + (70*(a + b*x^3)^4*Log[a^(2/3)
```

```
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3)))/(2916*a^(13/3)*((a + b*x^3)^
2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(235) = 470$.
time = 0.03, size = 521, normalized size = 1.45

method	result
risch	$\frac{\sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\frac{35 b^{3} x^{11}}{243 a^{4}}+\frac{175 b^{2} x^{8}}{324 a^{3}}+\frac{20 b\,x^{5}}{27 a^{2}}+\frac{104 x^{2}}{243 a}\right)}{\left(b\,x^{3}+a\right)^{5}}+\frac{35 \sqrt{\left(b\,x^{3}+a\right)^{2}}\,\left(\sum_{\_R=\mathrm{RootOf}\left(b\,\_Z^{3}+a\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{729 \left(b\,x^{3}+a\right) b\,a^{4}}$
default	$\left(-140\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) b^{4} x^{12}-140\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) b^{4} x^{12}+70\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) b^{4} x^{12}+420\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{4} x^{11}-560$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2916*(-140*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*b^4
*x^12-140*ln(x+(a/b)^(1/3))*b^4*x^12+70*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*b
^4*x^12+420*(a/b)^(1/3)*b^4*x^11-560*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)
^(1/3))/(a/b)^(1/3))*a*b^3*x^9-560*ln(x+(a/b)^(1/3))*a*b^3*x^9+280*ln(x^2-(
a/b)^(1/3)*x+(a/b)^(2/3))*a*b^3*x^9+1575*(a/b)^(1/3)*a*b^3*x^8-840*3^(1/2)*
arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^2*b^2*x^6-840*ln(x+(a/
b)^(1/3))*a^2*b^2*x^6+420*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^2*b^2*x^6+216
0*(a/b)^(1/3)*a^2*b^2*x^5-560*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))
/(a/b)^(1/3))*a^3*b*x^3-560*ln(x+(a/b)^(1/3))*a^3*b*x^3+280*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))*a^3*b*x^3+1248*(a/b)^(1/3)*a^3*b*x^2-140*3^(1/2)*arctan(
1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^4-140*ln(x+(a/b)^(1/3))*a^4+7
0*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(1/3)/b/a^4/((b*x^
3+a)^2)^(5/2)
```

Maxima [A]
time = 0.51, size = 191, normalized size = 0.53

$$\frac{140\,b^{3}x^{11}+525\,ab^{2}x^{8}+720\,a^{2}bx^{5}+416\,a^{3}x^{2}}{972\,\left(a^{4}b^{4}x^{12}+4\,a^{5}b^{3}x^{9}+6\,a^{6}b^{2}x^{6}+4\,a^{7}bx^{3}+a^{8}\right)}+\frac{35\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^{4}b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{35\,\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^{4}b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{35\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^{4}b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(140*b^3*x^11 + 525*a*b^2*x^8 + 720*a^2*b*x^5 + 416*a^3*x^2)/(a^4*b^4
*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 35/729*sqrt(3)
```

```
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(1/3)) +
35/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(1/3)) - 35/729
*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(1/3))
```

Fricas [A]
time = 0.38, size = 734, normalized size = 2.04

$$\left[\frac{420\,ab^5x^{11}+1575\,a^2b^4x^8+2160\,a^3b^3x^5+1248\,a^4b^2x^2+210\,\sqrt{\frac{1}{3}}\left(ab^5x^{12}+4\,a^2b^4x^9+6\,a^3b^3x^6+4\,a^4b^2x^3+a^5b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\,\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\,\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)+70\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2+\left(-ab^2\right)^{\frac{1}{3}}bx+\left(-ab^2\right)^{\frac{2}{3}}\right)-140\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx-\left(-ab^2\right)^{\frac{1}{3}}\right)}{2916\left(a^5b^6x^{12}+4\,a^6b^5x^9+6\,a^7b^4x^6+4\,a^8b^3x^3+a^9b^2\right)},\ \frac{420\,ab^5x^{11}+1575\,a^2b^4x^8+2160\,a^3b^3x^5+1248\,a^4b^2x^2+420\,\sqrt{\frac{1}{3}}\left(ab^5x^{12}+4\,a^2b^4x^9+6\,a^3b^3x^6+4\,a^4b^2x^3+a^5b\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,bx+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)+70\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^2+\left(-ab^2\right)^{\frac{1}{3}}bx+\left(-ab^2\right)^{\frac{2}{3}}\right)-140\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx-\left(-ab^2\right)^{\frac{1}{3}}\right)}{2916\left(a^5b^6x^{12}+4\,a^6b^5x^9+6\,a^7b^4x^6+4\,a^8b^3x^3+a^9b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(420*a*b^5*x^11 + 1575*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^
2*x^2 + 210*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b
^2*x^3 + a^5*b)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(
a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3
*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^
6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-
a*b^2)^(2/3)) - 140*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 +
 a^4)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x
^9 + 6*a^7*b^4*x^6 + 4*a^8*b^3*x^3 + a^9*b^2), 1/2916*(420*a*b^5*x^11 + 157
5*a^2*b^4*x^8 + 2160*a^3*b^3*x^5 + 1248*a^4*b^2*x^2 + 420*sqrt(1/3)*(a*b^5*
x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt(-(-a*b^2
)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a
)/b) + 70*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*
b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 140*(b^4*x^
12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a^5*b^6*x^12 + 4*a^6*b^5*x^9 + 6*a^7*b^4*x^6 + 4*a^8*
b^3*x^3 + a^9*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2)**(5/2), x)
```

Giac [A]
time = 5.61, size = 195, normalized size = 0.54

$$\frac{35\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,\left(-ab^2\right)^{\frac{1}{3}}a^4\mathrm{sgn}\left(bx^3+a\right)}-\frac{35\,\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,\left(-ab^2\right)^{\frac{1}{3}}a^4\mathrm{sgn}\left(bx^3+a\right)}-\frac{35\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^5\mathrm{sgn}\left(bx^3+a\right)}+\frac{140\,b^3x^{11}+525\,ab^2x^8+720\,a^2bx^5+416\,a^3x^2}{972\left(bx^3+a\right)^4a^4\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 35/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b
^2)^(1/3)*a^4*sgn(b*x^3 + a)) - 35/1458*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(1/3)*a^4*sgn(b*x^3 + a)) - 35/729*(-a/b)^(2/3)*log(abs(x -
 (-a/b)^(1/3)))/(a^5*sgn(b*x^3 + a)) + 1/972*(140*b^3*x^11 + 525*a*b^2*x^8
+ 720*a^2*b*x^5 + 416*a^3*x^2)/((b*x^3 + a)^4*a^4*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(x/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.113 $\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=364

$$\frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x(a+bx^3)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{55x(a+b\cdots}{243a^4\left(a^2+2abx^3\cdots\right.}$$

```
[Out] 1/12*x*(b*x^3+a)/a/(b^2*x^6+2*a*b*x^3+a^2)^(5/2)+11/108*x*(b*x^3+a)^2/a^2/(
b^2*x^6+2*a*b*x^3+a^2)^(5/2)+11/81*x*(b*x^3+a)^3/a^3/(b^2*x^6+2*a*b*x^3+a^2
)^(5/2)+55/243*x*(b*x^3+a)^4/a^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2)+110/729*(b*x
^3+a)^5*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(5/2
)-55/729*(b*x^3+a)^5*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)/b^(
1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(5/2)-110/729*(b*x^3+a)^5*arctan(1/3*(a^(1/3)-
2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)/b^(1/3)/(b^2*x^6+2*a*b*x^3+a^2)^(5/2
)*3^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1357, 205, 206, 31, 648, 631, 210, 642}

$$\frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}}-\frac{110(a+bx^3)^5\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}\,a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{110(a+bx^3)^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}-\frac{55(a+bx^3)^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{729a^{14/3}\sqrt[3]{b}\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{55x(a+bx^3)^4}{243a^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}+\frac{11x(a+bx^3)^3}{81a^3\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

```
[Out] (x*(a + b*x^3))/(12*a*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3
)^2)/(108*a^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (11*x*(a + b*x^3)^3)/(81
*a^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) + (55*x*(a + b*x^3)^4)/(243*a^4*(a^
2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (110*(a + b*x^3)^5*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(243*Sqrt[3]*a^(14/3)*b^(1/3)*(a^2 + 2*a*b*x^3
 + b^2*x^6)^(5/2)) + (110*(a + b*x^3)^5*Log[a^(1/3) + b^(1/3)*x])/(729*a^(1
4/3)*b^(1/3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)) - (55*(a + b*x^3)^5*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(729*a^(14/3)*b^(1/3)*(a^2 + 2*a*b
*x^3 + b^2*x^6)^(5/2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(2ab+2b^2x^3\right)^5 \int \frac{1}{\left(2ab+2b^2x^3\right)^5}\,dx}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(11\left(2ab+2b^2x^3\right)^5\right)\int \frac{1}{\left(2ab+2b^2x^3\right)^4}\,dx}{24ab\left(a^2+2abx^3+b^2x^6\right)^{5/2}}$$

$$= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{\left(11(2ab+2b\right.}{54a^2b^2\left(a^2\right.}$$

$$= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a}{81a^3\left(a^2+2a\right.}$$

$$= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a}{81a^3\left(a^2+2a\right.}$$

$$= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a}{81a^3\left(a^2+2a\right.}$$

$$= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a}{81a^3\left(a^2+2a\right.}$$

$$= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a}{81a^3\left(a^2+2a\right.}$$

$$= \frac{x(a+bx^3)}{12a\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a+bx^3)^2}{108a^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}} + \frac{11x(a}{81a^3\left(a^2+2a\right.}$$

Mathematica [A]
time = 0.07, size = 211, normalized size = 0.58

$$\frac{(a+bx^3)\left(243a^{11/3}x+297a^{8/3}x(a+bx^3)+396a^{5/3}x(a+bx^3)^2+660a^{2/3}x(a+bx^3)^3+\frac{440\sqrt{3}\,(a+bx^3)^4\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt[3]{b}}+\frac{440(a+bx^3)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}}-\frac{220(a+bx^3)^4\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}}\right)}{2916a^{14/3}\left((a+bx^3)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^(-5/2),x]
```

```
[Out] ((a + b*x^3)*(243*a^(11/3)*x + 297*a^(8/3)*x*(a + b*x^3) + 396*a^(5/3)*x*(a
 + b*x^3)^2 + 660*a^(2/3)*x*(a + b*x^3)^3 + (440*Sqrt[3]*(a + b*x^3)^4*ArcT
an[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/b^(1/3) + (440*(a + b*x^3)^
```

```
4*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) - (220*(a + b*x^3)^4*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3)))/(2916*a^(14/3)*((a + b*x^3)^2)^(5/2
))
```

Maple [A]
time = 0.03, size = 519, normalized size = 1.43

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\,\left(\frac{55 b^3 x^{10}}{243 a^4}+\frac{22 b^2 x^7}{27 a^3}+\frac{341 b x^4}{324 a^2}+\frac{133 x}{243 a}\right)}{(b x^3+a)^5}+\frac{110\sqrt{(b x^3+a)^2}\,\left(\sum_{\_R=\mathrm{RootOf}(b\_Z^3+a)}\frac{\ln(x-\_R)}{\_R^2}\right)}{729 (b x^3+a) b a^4}$
default	$\left(-440\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right) b^4 x^{12}+440\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right) b^4 x^{12}-220\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right) b^4 x^{12}+660\left(\frac{a}{b}\right)^{\frac{2}{3}} b^4 x^{10}-17$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2916*(-440*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*b^4
*x^12+440*ln(x+(a/b)^(1/3))*b^4*x^12-220*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*
b^4*x^12+660*(a/b)^(2/3)*b^4*x^10-1760*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/
b)^(1/3))/(a/b)^(1/3))*a*b^3*x^9+1760*ln(x+(a/b)^(1/3))*a*b^3*x^9-880*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*b^3*x^9+2376*(a/b)^(2/3)*a*b^3*x^7-2640*3^(1
/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^2*b^2*x^6+2640*ln(
x+(a/b)^(1/3))*a^2*b^2*x^6-1320*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^2*b^2*x
^6+3069*(a/b)^(2/3)*a^2*b^2*x^4-1760*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)
^(1/3))/(a/b)^(1/3))*a^3*b*x^3+1760*ln(x+(a/b)^(1/3))*a^3*b*x^3-880*ln(x^2-
(a/b)^(1/3)*x+(a/b)^(2/3))*a^3*b*x^3+1596*(a/b)^(2/3)*a^3*b*x-440*3^(1/2)*a
rctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^4+440*ln(x+(a/b)^(1/3))
*a^4-220*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^4)*(b*x^3+a)/(a/b)^(2/3)/b/a^4
/((b*x^3+a)^2)^(5/2)
```

Maxima [A]
time = 0.51, size = 189, normalized size = 0.52

$$\frac{220\,b^3x^{10}+792\,ab^2x^7+1023\,a^2bx^4+532\,a^3x}{972\,(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8)}+\frac{110\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{55\,\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{110\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^4b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/972*(220*b^3*x^10 + 792*a*b^2*x^7 + 1023*a^2*b*x^4 + 532*a^3*x)/(a^4*b^4*
x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8) + 110/729*sqrt(3)
```

```
*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^4*b*(a/b)^(2/3)) -
55/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^4*b*(a/b)^(2/3)) + 110/729
*log(x + (a/b)^(1/3))/(a^4*b*(a/b)^(2/3))
```

Fricas [A]
time = 0.37, size = 719, normalized size = 1.98

$$\left[\frac{660\,a^2b^4x^{10}+2376\,a^3b^3x^7+3069\,a^4b^2x^4+1596\,a^5bx+660\,\sqrt{\frac{1}{3}}\left(ab^5x^{12}+4\,a^2b^4x^9+6\,a^3b^3x^6+4\,a^4b^2x^3+a^5b\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\,(a^2b)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)-220\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a^2b)^{\frac{2}{3}}x+(a^2b)^{\frac{1}{3}}a\right)+440\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)(a^2b)^{\frac{2}{3}}\log\left(abx+(a^2b)^{\frac{2}{3}}\right)}{2916\left(a^6b^5x^{12}+4\,a^7b^4x^9+6\,a^8b^3x^6+4\,a^9b^2x^3+a^{10}b\right)},\ \frac{660\,a^2b^4x^{10}+2376\,a^3b^3x^7+3069\,a^4b^2x^4+1596\,a^5bx+1320\,\sqrt{\frac{1}{3}}\left(ab^5x^{12}+4\,a^2b^4x^9+6\,a^3b^3x^6+4\,a^4b^2x^3+a^5b\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(a^2b)^{\frac{2}{3}}x-(a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-220\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)(a^2b)^{\frac{2}{3}}\log\left(abx^2-(a^2b)^{\frac{2}{3}}x+(a^2b)^{\frac{1}{3}}a\right)+440\left(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4\right)(a^2b)^{\frac{2}{3}}\log\left(abx+(a^2b)^{\frac{2}{3}}\right)}{2916\left(a^6b^5x^{12}+4\,a^7b^4x^9+6\,a^8b^3x^6+4\,a^9b^2x^3+a^{10}b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2916*(660*a^2*b^4*x^10 + 2376*a^3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*
b*x + 660*sqrt(1/3)*(a*b^5*x^12 + 4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2
*x^3 + a^5*b)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x -
 a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a
^2*b)^(1/3)/b))/(b*x^3 + a)) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6
+ 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 440*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)
*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 +
6*a^8*b^3*x^6 + 4*a^9*b^2*x^3 + a^10*b), 1/2916*(660*a^2*b^4*x^10 + 2376*a^
3*b^3*x^7 + 3069*a^4*b^2*x^4 + 1596*a^5*b*x + 1320*sqrt(1/3)*(a*b^5*x^12 +
4*a^2*b^4*x^9 + 6*a^3*b^3*x^6 + 4*a^4*b^2*x^3 + a^5*b)*sqrt((a^2*b)^(1/3)/b
)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)
/b)/a^2) - 220*(b^4*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 440*(b^4*
x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*(a^2*b)^(2/3)*log(a
*b*x + (a^2*b)^(2/3)))/(a^6*b^5*x^12 + 4*a^7*b^4*x^9 + 6*a^8*b^3*x^6 + 4*a^
9*b^2*x^3 + a^10*b)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2abx^3+b^2x^6\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**(-5/2), x)
```

Giac [A]
time = 3.23, size = 199, normalized size = 0.55

$$-\frac{110\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^5\mathrm{sgn}\left(bx^3+a\right)}+\frac{110\,\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^5b\,\mathrm{sgn}\left(bx^3+a\right)}+\frac{55\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^5b\,\mathrm{sgn}\left(bx^3+a\right)}+\frac{220\,b^3x^{10}+792\,ab^2x^7+1023\,a^2bx^4+532\,a^3x}{972\left(bx^3+a\right)^4a^4\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -110/729*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^5*sgn(b*x^3 + a)) + 110
/729*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/(a^5*b*sgn(b*x^3 + a)) + 55/729*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^5*b*sgn(b*x^3 + a)) + 1/972*(220*b^3*x^10 + 792*a*b^
2*x^7 + 1023*a^2*b*x^4 + 532*a^3*x)/((b*x^3 + a)^4*a^4*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.114 $\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{}{6a^3\left(\cdots\right.}$$

```
[Out] 1/3/a^4/((b*x^3+a)^2)^(1/2)+1/12/a/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+1/9/a^2/
(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+1/6/a^3/(b*x^3+a)/((b*x^3+a)^2)^(1/2)+(b*x^
3+a)*ln(x)/a^5/((b*x^3+a)^2)^(1/2)-1/3*(b*x^3+a)*ln(b*x^3+a)/a^5/((b*x^3+a)
^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 46}

$$\frac{1}{9a^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{\log(x)\left(a+bx^3\right)}{a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{6a^3\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 1/(3*a^4*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*(a + b*x^3)^3*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]) + 1/(9*a^2*(a + b*x^3)^2*Sqrt[a^2 + 2*a*b*x^3 + b^2
*x^6]) + 1/(6*a^3*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + ((a + b*x^
3)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - ((a + b*x^3)*Log[a + b*x
^3])/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
```

```
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4\left(ab+b^2x^3\right)\right)\int \frac{1}{x(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \frac{1}{x(ab+b^2x)^5}\,dx,x,x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4\left(ab+b^2x^3\right)\right)\text{Subst}\left(\int \left(\frac{1}{a^5b^5x}-\frac{1}{ab^4(a+bx)^5}-\frac{1}{a^2b^4(a+bx)^4}-\frac{1}{a^3b^4(a+bx)^3}-\frac{1}{a^4b^4}\right.\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{3a^4\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12a\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{}{9a^2\left(a+\right.}$$

Mathematica [A]
time = 0.03, size = 96, normalized size = 0.43

$$\frac{a(25a^3+52a^2bx^3+42ab^2x^6+12b^3x^9)+36(a+bx^3)^4\log(x)-12(a+bx^3)^4\log\left(a+bx^3\right)}{36a^5\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
[Out] (a*(25*a^3 + 52*a^2*b*x^3 + 42*a*b^2*x^6 + 12*b^3*x^9) + 36*(a + b*x^3)^4*L
og[x] - 12*(a + b*x^3)^4*Log[a + b*x^3])/(36*a^5*(a + b*x^3)^3*Sqrt[(a + b*
x^3)^2])
```

Maple [A]
time = 0.04, size = 193, normalized size = 0.87

method	result
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(\frac{b^3x^9}{3a^4}+\frac{7b^2x^6}{6a^3}+\frac{13b\,x^3}{9a^2}+\frac{25}{36a}\right)}{\left(b\,x^3+a\right)^5}+\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\ln(x)}{\left(b\,x^3+a\right)a^5}-\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\ln\left(b\,x^3+a\right)}{3\left(b\,x^3+a\right)a^5}$
default	$-\frac{(12\ln(b\,x^3+a)b^4x^{12}-36\ln(x)b^4x^{12}+48\ln(b\,x^3+a)a\,b^3x^9-144\ln(x)a\,b^3x^9-12a\,b^3x^9+72\ln(b\,x^3+a)a^2b^2x^6-216\ln(x)a^2b^2x^6-42a}{36a^5\left(\left(b\,x^3+a\right)^2\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*(12*ln(b*x^3+a)*b^4*x^12-36*ln(x)*b^4*x^12+48*ln(b*x^3+a)*a*b^3*x^9-1
44*ln(x)*a*b^3*x^9-12*a*b^3*x^9+72*ln(b*x^3+a)*a^2*b^2*x^6-216*ln(x)*a^2*b^
2*x^6-42*a^2*b^2*x^6+48*ln(b*x^3+a)*a^3*b*x^3-144*ln(x)*a^3*b*x^3-52*a^3*b*
x^3+12*ln(b*x^3+a)*a^4-36*a^4*ln(x)-25*a^4)*(b*x^3+a)/a^5/((b*x^3+a)^2)^(5/
2)
```

Maxima [A]
time = 0.28, size = 132, normalized size = 0.59

$$-\frac{(-1)^{2\,abx^3+2\,a^2}\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^5}+\frac{1}{9\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}a^2}+\frac{1}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}\,a^4}+\frac{1}{6\,\left(x^3+\frac{a}{b}\right)^2a^3b^2}+\frac{1}{12\,\left(x^3+\frac{a}{b}\right)^4ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-1)^(2*a*b*x^3 + 2*a^2)*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^5
+ 1/9/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2) + 1/3/(sqrt(b^2*x^6 + 2*a*b*x
^3 + a^2)*a^4) + 1/6/((x^3 + a/b)^2*a^3*b^2) + 1/12/((x^3 + a/b)^4*a*b^4)
```

Fricas [A]
time = 0.36, size = 178, normalized size = 0.80

$$\frac{12\,ab^3x^9+42\,a^2b^2x^6+52\,a^3bx^3+25\,a^4-12\,(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4)\log\left(bx^3+a\right)+36\,(b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4)\log\left(x\right)}{36\,(a^5b^4x^{12}+4\,a^6b^3x^9+6\,a^7b^2x^6+4\,a^8bx^3+a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/36*(12*a*b^3*x^9 + 42*a^2*b^2*x^6 + 52*a^3*b*x^3 + 25*a^4 - 12*(b^4*x^12
+ 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(b*x^3 + a) + 36*(b^4
*x^12 + 4*a*b^3*x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4)*log(x))/(a^5*b^4*x
^12 + 4*a^6*b^3*x^9 + 6*a^7*b^2*x^6 + 4*a^8*b*x^3 + a^9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**3)**2)**(5/2)), x)
```

Giac [A]
time = 3.23, size = 109, normalized size = 0.49

$$-\frac{\log\left(\left|bx^{3}+a\right|\right)}{3\,a^{5}\mathrm{sgn}\left(bx^{3}+a\right)}+\frac{\log\left(\left|x\right|\right)}{a^{5}\mathrm{sgn}\left(bx^{3}+a\right)}+\frac{25\,b^{4}x^{12}+112\,ab^{3}x^{9}+192\,a^{2}b^{2}x^{6}+152\,a^{3}bx^{3}+50\,a^{4}}{36\,\left(bx^{3}+a\right)^{4}a^{5}\mathrm{sgn}\left(bx^{3}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(b*x^3 + a))/(a^5*sgn(b*x^3 + a)) + log(abs(x))/(a^5*sgn(b*x^3
+ a)) + 1/36*(25*b^4*x^12 + 112*a*b^3*x^9 + 192*a^2*b^2*x^6 + 152*a^3*b*x^3
 + 50*a^4)/((b*x^3 + a)^4*a^5*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

3.115 $\int \frac{1}{x^2\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=398

$$\frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] 455/972/a^4/x/((b*x^3+a)^2)^(1/2)+1/12/a/x/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)+
13/108/a^2/x/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+65/324/a^3/x/(b*x^3+a)/((b*x^3
+a)^2)^(1/2)-455/243*(b*x^3+a)/a^5/x/((b*x^3+a)^2)^(1/2)+455/729*b^(1/3)*(b
*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(16/3)/((b*x^3+a)^2)^(1/2)-455/1458*b^(1/3)
*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(16/3)/((b*x^3+a)^2)
^(1/2)+455/729*b^(1/3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3
^(1/2))/a^(16/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1369, 296, 331, 298, 31, 648, 631, 210, 642}

$$\frac{13}{108a^2x\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^2}+\frac{1}{12ax\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)^3}+\frac{455\sqrt[3]{b}\left(a+bx^3\right)\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{243\sqrt{3}a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455\sqrt[3]{b}\left(a+bx^3\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{729a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{455\sqrt[3]{b}\left(a+bx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{1458a^{16/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{455\left(a+bx^3\right)}{243a^5x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}}+\frac{65}{324a^3x\sqrt{a^2+2abx^3+b^2x^6}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 455/(972*a^4*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x*(a + b*x^3)^3*S
qrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 13/(108*a^2*x*(a + b*x^3)^2*Sqrt[a^2 + 2*
a*b*x^3 + b^2*x^6]) + 65/(324*a^3*x*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*
x^6]) - (455*(a + b*x^3))/(243*a^5*x*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (45
5*b^(1/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(2
43*Sqrt[3]*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (455*b^(1/3)*(a + b*
x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) - (455*b^(1/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(1458*a^(16/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^3)\right)\int \frac{1}{x^2(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(13b^3(ab+b^2x^3)\right)\int \frac{1}{x^2(ab+b^2x^3)^4}\,d}{12a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3}}$$

$$= \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{13}{108a^2x\left(a+bx^3\right)^2\sqrt{a^2+2abx^3}}$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{455}{972a^4x\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} +$$

Mathematica [A]

time = 0.08, size = 242, normalized size = 0.61

$$\frac{(a+bx^3)\left(-243a^{10/3}bx^2-594a^{7/3}bx^2(a+bx^3)-1179a^{4/3}bx^2(a+bx^3)^2-2544\sqrt[3]{a}\,bx^2(a+bx^3)^3-\frac{2916\sqrt[3]{a}\,(a+bx^3)^4}{x}-1820\sqrt{3}\,\sqrt[3]{b}\,(a+bx^3)^4\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)+1820\sqrt[3]{b}\,(a+bx^3)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-910\sqrt[3]{b}\,(a+bx^3)^4\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{2916a^{16/3}\left((a+bx^3)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(10/3)*b*x^2 - 594*a^(7/3)*b*x^2*(a + b*x^3) - 1179*a^
(4/3)*b*x^2*(a + b*x^3)^2 - 2544*a^(1/3)*b*x^2*(a + b*x^3)^3 - (2916*a^(1/3
)*(a + b*x^3)^4)/x - 1820*Sqrt[3]*b^(1/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) +
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 1820*b^(1/3)*(a + b*x^3)^4*Log[a^(1/3) +
b^(1/3)*x] - 910*b^(1/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2]))/(2916*a^(16/3)*((a + b*x^3)^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(261) = 522$.
time = 0.05, size = 536, normalized size = 1.35

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(-\frac{455 b^4 x^{12}}{243 a^5}-\frac{2275 b^3 x^9}{324 a^4}-\frac{260 b^2 x^6}{27 a^3}-\frac{1352 b x^3}{243 a^2}-\frac{1}{a}\right)}{(b x^3+a)^5 x}+\frac{455\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^{16}\_Z^3-b\right)}\_R\ln\left(\left(-\cdots\right.\right.\right.}{729(b x^3+a)}$
default	$-\frac{\left(-1820\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{13}-1820\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{13}+910\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{13}+5460\left(\frac{a}{b}\right)^{\frac{1}{3}}b^4x^{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2916*(-1820*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*b
^4*x^13-1820*ln(x+(a/b)^(1/3))*b^4*x^13+910*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))*b^4*x^13+5460*(a/b)^(1/3)*b^4*x^12-7280*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x
+(a/b)^(1/3))/(a/b)^(1/3))*a*b^3*x^10-7280*ln(x+(a/b)^(1/3))*a*b^3*x^10+364
0*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*b^3*x^10+20475*(a/b)^(1/3)*a*b^3*x^9-
10920*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^2*b^2*x^
7-10920*ln(x+(a/b)^(1/3))*a^2*b^2*x^7+5460*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)*a^2*b^2*x^7+28080*(a/b)^(1/3)*a^2*b^2*x^6-7280*3^(1/2)*arctan(1/3*3^(1/2)
*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^3*b*x^4-7280*ln(x+(a/b)^(1/3))*a^3*b*x^4
+3640*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^3*b*x^4+16224*(a/b)^(1/3)*a^3*b*x
^3-1820*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^4*x-18
20*ln(x+(a/b)^(1/3))*a^4*x+910*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^4*x+2916
*(a/b)^(1/3)*a^4)*(b*x^3+a)/x/(a/b)^(1/3)/a^5/((b*x^3+a)^2)^(5/2)
```

Maxima [A]
time = 0.51, size = 192, normalized size = 0.48

$$-\frac{1820\,b^4x^{12}+6825\,ab^3x^9+9360\,a^2b^2x^6+5408\,a^3bx^3+972\,a^4}{972\left(a^5b^4x^{13}+4\,a^6b^3x^{10}+6\,a^7b^2x^7+4\,a^8bx^4+a^9x\right)}-\frac{455\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{455\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{455\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/972*(1820*b^4*x^12 + 6825*a*b^3*x^9 + 9360*a^2*b^2*x^6 + 5408*a^3*b*x^3
+ 972*a^4)/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^8*b*x^4 + a
^9*x) - 455/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a^5*(a/b)^(1/3)) - 455/1458*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*(
a/b)^(1/3)) + 455/729*log(x + (a/b)^(1/3))/(a^5*(a/b)^(1/3))
```

Fricas [A]
time = 0.40, size = 311, normalized size = 0.78

$$-\frac{5460\,b^{4}x^{12}+20475\,ab^{3}x^{9}+28080\,a^{2}b^{2}x^{6}+16224\,a^{3}bx^{3}+2916\,a^{4}+1820\,\sqrt{3}\,(b^{4}x^{13}+4\,ab^{3}x^{10}+6\,a^{2}b^{2}x^{7}+4\,a^{3}bx^{4}+a^{4}x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\,\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\,\sqrt{3}\right)+910\,(b^{4}x^{13}+4\,ab^{3}x^{10}+6\,a^{2}b^{2}x^{7}+4\,a^{3}bx^{4}+a^{4}x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^{2}-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-1820\,(b^{4}x^{13}+4\,ab^{3}x^{10}+6\,a^{2}b^{2}x^{7}+4\,a^{3}bx^{4}+a^{4}x)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx+a\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{2916\,(a^{5}b^{4}x^{13}+4\,a^{6}b^{3}x^{10}+6\,a^{7}b^{2}x^{7}+4\,a^{8}bx^{4}+a^{9}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2916*(5460*b^4*x^12 + 20475*a*b^3*x^9 + 28080*a^2*b^2*x^6 + 16224*a^3*b*
x^3 + 2916*a^4 + 1820*sqrt(3)*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*
a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(
3)) + 910*(b^4*x^13 + 4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(
b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) - 1820*(b^4*x^13 +
4*a*b^3*x^10 + 6*a^2*b^2*x^7 + 4*a^3*b*x^4 + a^4*x)*(b/a)^(1/3)*log(b*x + a
*(b/a)^(2/3)))/(a^5*b^4*x^13 + 4*a^6*b^3*x^10 + 6*a^7*b^2*x^7 + 4*a^8*b*x^4
 + a^9*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**3)**2)**(5/2)), x)
```

Giac [A]
time = 2.94, size = 223, normalized size = 0.56

$$\frac{455\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^{6}\mathrm{sgn}\left(bx^{3}+a\right)}+\frac{455\,\sqrt{3}\,(-ab^{2})^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^{6}b\mathrm{sgn}\left(bx^{3}+a\right)}-\frac{455\,(-ab^{2})^{\frac{2}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{1458\,a^{6}b\mathrm{sgn}\left(bx^{3}+a\right)}-\frac{1}{a^{5}x\mathrm{sgn}\left(bx^{3}+a\right)}-\frac{848\,b^{4}x^{11}+2937\,ab^{3}x^{8}+3528\,a^{2}b^{2}x^{5}+1520\,a^{3}bx^{2}}{972\,(bx^{3}+a)^{4}a^{5}\mathrm{sgn}\left(bx^{3}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 455/729*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^6*sgn(b*x^3 + a)) + 45
5/729*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(a^6*b*sgn(b*x^3 + a)) - 455/1458*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^6*b*sgn(b*x^3 + a)) - 1/(a^5*x*sgn(b*x^3 + a)) -
1/972*(848*b^4*x^11 + 2937*a*b^3*x^8 + 3528*a^2*b^2*x^5 + 1520*a^3*b*x^2)/(
(b*x^3 + a)^4*a^5*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

3.116 $\int \frac{1}{x^3(a^2+2abx^3+b^2x^6)^{5/2}}\,dx$

Optimal. Leaf size=398

$$\frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}+\frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2}}$$

```
[Out] 154/243/a^4/x^2/((b*x^3+a)^2)^(1/2)+1/12/a/x^2/(b*x^3+a)^3/((b*x^3+a)^2)^(1
/2)+7/54/a^2/x^2/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)+77/324/a^3/x^2/(b*x^3+a)/(
(b*x^3+a)^2)^(1/2)-385/243*(b*x^3+a)/a^5/x^2/((b*x^3+a)^2)^(1/2)-770/729*b^
(2/3)*(b*x^3+a)*ln(a^(1/3)+b^(1/3)*x)/a^(17/3)/((b*x^3+a)^2)^(1/2)+385/729*
b^(2/3)*(b*x^3+a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(17/3)/((b*x^
3+a)^2)^(1/2)+770/729*b^(2/3)*(b*x^3+a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/a^(17/3)*3^(1/2)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {1369, 296, 331, 206, 31, 648, 631, 210, 642}

$$\frac{7}{54a^2x^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^2}+\frac{1}{12ax^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)^3}+\frac{770b^{2/3}(a+bx^3)\,\text{ArcTan}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{243\sqrt{3}\,a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{770b^{2/3}(a+bx^3)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}+\frac{385b^{2/3}(a+bx^3)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{729a^{17/3}\sqrt{a^2+2abx^3+b^2x^6}}-\frac{385(a+bx^3)}{243a^5x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}}+\frac{77}{324a^3x^2\sqrt{a^2+2abx^3+b^2x^6}\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] 154/(243*a^4*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 1/(12*a*x^2*(a + b*x^3)
^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + 7/(54*a^2*x^2*(a + b*x^3)^2*Sqrt[a^2
+ 2*a*b*x^3 + b^2*x^6]) + 77/(324*a^3*x^2*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3
+ b^2*x^6]) - (385*(a + b*x^3))/(243*a^5*x^2*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6
]) + (770*b^(2/3)*(a + b*x^3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/
3))])/(243*Sqrt[3]*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (770*b^(2/3)
*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 +
 b^2*x^6]) + (385*b^(2/3)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(729*a^(17/3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^3 \left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^3)\right)\int \frac{1}{x^3(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{\left(7b^3(ab+b^2x^3)\right)\int \frac{1}{x^3(ab+b^2x^3)^4}\,dx}{6a\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3}}$$

$$= \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} + \frac{7}{54a^2x^2\left(a+bx^3\right)^2\sqrt{a^2+2abx^3}}$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} +$$

$$= \frac{154}{243a^4x^2\sqrt{a^2+2abx^3+b^2x^6}} + \frac{1}{12ax^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}} +$$

Mathematica [A]

time = 0.09, size = 234, normalized size = 0.59

$$\frac{(a+bx^3)\left(-243a^{11/3}bx-621a^{8/3}bx(a+bx^3)-1314a^{5/3}bx(a+bx^3)^2-3162a^{2/3}bx(a+bx^3)^3-\frac{1458a^{2/3}(a+bx^3)^4}{x^2}-3080\sqrt{3}\,b^{2/3}(a+bx^3)^4\tan^{-1}\left(\frac{-\sqrt[3]{a}+2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)-3080b^{2/3}(a+bx^3)^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+1540b^{2/3}(a+bx^3)^4\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)\right)}{2916a^{17/3}\left((a+bx^3)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] ((a + b*x^3)*(-243*a^(11/3)*b*x - 621*a^(8/3)*b*x*(a + b*x^3) - 1314*a^(5/3
)*b*x*(a + b*x^3)^2 - 3162*a^(2/3)*b*x*(a + b*x^3)^3 - (1458*a^(2/3)*(a + b
*x^3)^4)/x^2 - 3080*Sqrt[3]*b^(2/3)*(a + b*x^3)^4*ArcTan[(-a^(1/3) + 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))] - 3080*b^(2/3)*(a + b*x^3)^4*Log[a^(1/3) + b^(1/3
)*x] + 1540*b^(2/3)*(a + b*x^3)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2]))/(2916*a^(17/3)*((a + b*x^3)^2)^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(261) = 522$.
time = 0.04, size = 542, normalized size = 1.36

method	result
risch	$\frac{\sqrt{(b x^3+a)^2}\left(-\frac{385 b^4 x^{12}}{243 a^5}-\frac{154 b^3 x^9}{27 a^4}-\frac{2387 b^2 x^6}{324 a^3}-\frac{931 b x^3}{243 a^2}-\frac{1}{2a}\right)}{(b x^3+a)^5 x^2}+\frac{770\sqrt{(b x^3+a)^2}\left(\sum_{\_R=\mathrm{RootOf}\left(a^{17}\_Z^3+b^2\right)}\_R\ln((\cdots\right.}{729(b x^3+a)}$
default	$-\frac{\left(-3080\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)b^4x^{14}+3080\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)b^4x^{14}-1540\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)b^4x^{14}+4620\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4x\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2916*(-3080*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*b
^4*x^14+3080*ln(x+(a/b)^(1/3))*b^4*x^14-1540*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/
3))*b^4*x^14+4620*(a/b)^(2/3)*b^4*x^12-12320*3^(1/2)*arctan(1/3*3^(1/2)*(-2
*x+(a/b)^(1/3))/(a/b)^(1/3))*a*b^3*x^11+12320*ln(x+(a/b)^(1/3))*a*b^3*x^11-
6160*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*b^3*x^11+16632*(a/b)^(2/3)*a*b^3*x
^9-18480*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^2*b^2
*x^8+18480*ln(x+(a/b)^(1/3))*a^2*b^2*x^8-9240*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))*a^2*b^2*x^8+21483*(a/b)^(2/3)*a^2*b^2*x^6-12320*3^(1/2)*arctan(1/3*3^(
1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^3*b*x^5+12320*ln(x+(a/b)^(1/3))*a^3*
b*x^5-6160*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^3*b*x^5+11172*(a/b)^(2/3)*a^
3*b*x^3-3080*3^(1/2)*arctan(1/3*3^(1/2)*(-2*x+(a/b)^(1/3))/(a/b)^(1/3))*a^4
*x^2+3080*ln(x+(a/b)^(1/3))*a^4*x^2-1540*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*
a^4*x^2+1458*(a/b)^(2/3)*a^4)*(b*x^3+a)/x^2/(a/b)^(2/3)/a^5/((b*x^3+a)^2)^(
5/2)
```

Maxima [A]

time = 0.52, size = 194, normalized size = 0.49

$$-\frac{1540\,b^{4}x^{12}+5544\,ab^{3}x^{9}+7161\,a^{2}b^{2}x^{6}+3724\,a^{3}bx^{3}+486\,a^{4}}{972\,\left(a^{5}b^{4}x^{14}+4\,a^{6}b^{3}x^{11}+6\,a^{7}b^{2}x^{8}+4\,a^{8}bx^{5}+a^{9}x^{2}\right)}-\frac{770\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{385\,\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{770\,\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{729\,a^{5}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/972*(1540*b^4*x^12 + 5544*a*b^3*x^9 + 7161*a^2*b^2*x^6 + 3724*a^3*b*x^3
+ 486*a^4)/(a^5*b^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a
^9*x^2) - 770/729*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3
))/(a^5*(a/b)^(2/3)) + 385/729*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^5*
(a/b)^(2/3)) - 770/729*log(x + (a/b)^(1/3))/(a^5*(a/b)^(2/3))
```

Fricas [A]
time = 0.37, size = 352, normalized size = 0.88

$$-\frac{4620\,b^{4}x^{12}+16632\,ab^{3}x^{9}+21483\,a^{2}b^{2}x^{6}+11172\,a^{3}bx^{3}+1458\,a^{4}-3080\,\sqrt{3}\left(b^{4}x^{14}+4\,ab^{3}x^{11}+6\,a^{2}b^{2}x^{8}+4\,a^{3}bx^{5}+a^{4}x^{2}\right)\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,ax\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)+1540\,\left(b^{4}x^{14}+4\,ab^{3}x^{11}+6\,a^{2}b^{2}x^{8}+4\,a^{3}bx^{5}+a^{4}x^{2}\right)\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(b^{2}x^{2}+abx\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}+a^{2}\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{2}{3}}\right)-3080\,\left(b^{4}x^{14}+4\,ab^{3}x^{11}+6\,a^{2}b^{2}x^{8}+4\,a^{3}bx^{5}+a^{4}x^{2}\right)\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\log\left(bx-a\left(-\frac{b^{2}}{a^{2}}\right)^{\frac{1}{3}}\right)}{2916\,\left(a^{5}b^{4}x^{14}+4\,a^{6}b^{3}x^{11}+6\,a^{7}b^{2}x^{8}+4\,a^{8}bx^{5}+a^{9}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2916*(4620*b^4*x^12 + 16632*a*b^3*x^9 + 21483*a^2*b^2*x^6 + 11172*a^3*b*
x^3 + 1458*a^4 - 3080*sqrt(3)*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*
a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^
(2/3) - sqrt(3)*b)/b) + 1540*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*a
^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) +
 a^2*(-b^2/a^2)^(2/3)) - 3080*(b^4*x^14 + 4*a*b^3*x^11 + 6*a^2*b^2*x^8 + 4*
a^3*b*x^5 + a^4*x^2)*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)))/(a^5*b
^4*x^14 + 4*a^6*b^3*x^11 + 6*a^7*b^2*x^8 + 4*a^8*b*x^5 + a^9*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(\left(a+bx^{3}\right)^{2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**3)**2)**(5/2)), x)
```

Giac [A]
time = 3.58, size = 215, normalized size = 0.54

$$\frac{770\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{729\,a^{6}\mathrm{sgn}\left(bx^{3}+a\right)}-\frac{770\,\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{729\,a^{6}\mathrm{sgn}\left(bx^{3}+a\right)}-\frac{385\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{729\,a^{6}\mathrm{sgn}\left(bx^{3}+a\right)}-\frac{1}{2\,a^{5}x^{2}\mathrm{sgn}\left(bx^{3}+a\right)}-\frac{1054\,b^{4}x^{10}+3600\,ab^{3}x^{7}+4245\,a^{2}b^{2}x^{4}+1780\,a^{3}bx}{972\,\left(bx^{3}+a\right)^{4}a^{5}\mathrm{sgn}\left(bx^{3}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 770/729*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^6*sgn(b*x^3 + a)) - 77
0/729*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(a^6*sgn(b*x^3 + a)) - 385/729*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^6*sgn(b*x^3 + a)) - 1/2/(a^5*x^2*sgn(b*x^3 + a)) - 1
/972*(1054*b^4*x^10 + 3600*a*b^3*x^7 + 4245*a^2*b^2*x^4 + 1780*a^3*b*x)/((b
*x^3 + a)^4*a^5*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

3.117 $\int \frac{1}{x^4 \left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{9a^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}-\frac{}{2a}$$

```
[Out] -4/3*b/a^5/((b*x^3+a)^2)^(1/2)-1/12*b/a^2/(b*x^3+a)^3/((b*x^3+a)^2)^(1/2)-2
/9*b/a^3/(b*x^3+a)^2/((b*x^3+a)^2)^(1/2)-1/2*b/a^4/(b*x^3+a)/((b*x^3+a)^2)^
(1/2)+1/3*(-b*x^3-a)/a^5/x^3/((b*x^3+a)^2)^(1/2)-5*b*(b*x^3+a)*ln(x)/a^6/((
b*x^3+a)^2)^(1/2)+5/3*b*(b*x^3+a)*ln(b*x^3+a)/a^6/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1369, 272, 46}

$$-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{5b\log(x)\left(a+bx^3\right)}{a^6\sqrt{a^2+2abx^3+b^2x^6}}+\frac{5b\left(a+bx^3\right)\log\left(a+bx^3\right)}{3a^6\sqrt{a^2+2abx^3+b^2x^6}}-\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{a+bx^3}{3a^5x^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{2a^4\left(a+bx^3\right)\sqrt{a^2+2abx^3+b^2x^6}}-\frac{2b}{9a^3\left(a+bx^3\right)^2\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-4*b)/(3*a^5*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - b/(12*a^2*(a + b*x^3)^3*Sq
rt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (2*b)/(9*a^3*(a + b*x^3)^2*Sqrt[a^2 + 2*a*
b*x^3 + b^2*x^6]) - b/(2*a^4*(a + b*x^3)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) -
 (a + b*x^3)/(3*a^5*x^3*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) - (5*b*(a + b*x^3)
*Log[x])/(a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6]) + (5*b*(a + b*x^3)*Log[a + b
*x^3])/(3*a^6*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^3)\right)\int \frac{1}{x^4(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4(ab+b^2x^3)\right)\text{Subst}\left(\int \frac{1}{x^2(ab+b^2x)^5}\,dx, x, x^3\right)}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= \frac{\left(b^4(ab+b^2x^3)\right)\text{Subst}\left(\int \left(\frac{1}{a^5b^5x^2}-\frac{5}{a^6b^4x}+\frac{1}{a^2b^3(a+bx)^5}+\frac{2}{a^3b^3(a+bx)^4}+\frac{3}{a^4b^3(a+\cdots}\right.\right.}{3\sqrt{a^2+2abx^3+b^2x^6}}$$

$$= -\frac{4b}{3a^5\sqrt{a^2+2abx^3+b^2x^6}}-\frac{b}{12a^2\left(a+bx^3\right)^3\sqrt{a^2+2abx^3+b^2x^6}}-\frac{\cdots}{9a^3\,(\cdots}$$

Mathematica [A]
time = 0.03, size = 119, normalized size = 0.44

$$\frac{-a(12a^4+125a^3bx^3+260a^2b^2x^6+210ab^3x^9+60b^4x^{12})-180bx^3(a+bx^3)^4\log(x)+60bx^3(a+bx^3)^4\log\left(a+bx^3\right)}{36a^6x^3\left(a+bx^3\right)^3\sqrt{\left(a+bx^3\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2)),x]
```

```
[Out] (-(a*(12*a^4 + 125*a^3*b*x^3 + 260*a^2*b^2*x^6 + 210*a*b^3*x^9 + 60*b^4*x^1
2)) - 180*b*x^3*(a + b*x^3)^4*Log[x] + 60*b*x^3*(a + b*x^3)^4*Log[a + b*x^3
])/(36*a^6*x^3*(a + b*x^3)^3*Sqrt[(a + b*x^3)^2])
```

Maple [A]
time = 0.04, size = 219, normalized size = 0.81

method	result
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(-\frac{1}{3a}-\frac{125b\,x^3}{36a^2}-\frac{65b^2x^6}{9a^3}-\frac{35b^3x^9}{6a^4}-\frac{5b^4x^{12}}{3a^5}\right)}{\left(b\,x^3+a\right)^5x^3}-\frac{5\sqrt{\left(b\,x^3+a\right)^2}\,b\ln(x)}{\left(b\,x^3+a\right)a^6}+\frac{5\sqrt{\left(b\,x^3+a\right)^2}\,b\ln(-b\,x^3-a\cdots}{3\left(b\,x^3+a\right)a^6}$
default	$\frac{\left(60\ln\left(b\,x^3+a\right)b^5x^{15}-180b^5\ln(x)x^{15}+240\ln\left(b\,x^3+a\right)a\,b^4x^{12}-720\ln(x)a\,b^4x^{12}-60b^4a\,x^{12}+360\ln\left(b\,x^3+a\right)a^2b^3x^9-1080\ln(x)a^2b^3x^9\cdots\right.}{36x^3a^6\left(\left(b\cdots\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/36*(60*ln(b*x^3+a)*b^5*x^15-180*b^5*ln(x)*x^15+240*ln(b*x^3+a)*a*b^4*x^12
-720*ln(x)*a*b^4*x^12-60*b^4*a*x^12+360*ln(b*x^3+a)*a^2*b^3*x^9-1080*ln(x)*
a^2*b^3*x^9-210*a^2*b^3*x^9+240*ln(b*x^3+a)*a^3*b^2*x^6-720*ln(x)*a^3*b^2*x
^6-260*b^2*a^3*x^6+60*ln(b*x^3+a)*a^4*b*x^3-180*b*a^4*ln(x)*x^3-125*a^4*b*x
^3-12*a^5)*(b*x^3+a)/x^3/a^6/((b*x^3+a)^2)^(5/2)
```

Maxima [A]
time = 0.29, size = 163, normalized size = 0.61

$$\frac{5\,(-1)^{2\,abx^3+2\,a^2}\,b\log\left(\frac{2\,abx}{|x|}+\frac{2\,a^2}{x^2|x|}\right)}{3\,a^6}-\frac{5\,b}{9\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}a^3}-\frac{5\,b}{3\,\sqrt{b^2x^6+2\,abx^3+a^2}\,a^5}-\frac{1}{3\,(b^2x^6+2\,abx^3+a^2)^{\frac{3}{2}}a^2x^3}-\frac{5}{6\,\left(x^3+\frac{a}{b}\right)^2a^4b}-\frac{1}{12\,\left(x^3+\frac{a}{b}\right)^4a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/3*(-1)^(2*a*b*x^3 + 2*a^2)*b*log(2*a*b*x/abs(x) + 2*a^2/(x^2*abs(x)))/a^6
 - 5/9*b/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^3) - 5/3*b/(sqrt(b^2*x^6 + 2*
a*b*x^3 + a^2)*a^5) - 1/3/((b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2)*a^2*x^3) - 5/6
/((x^3 + a/b)^2*a^4*b) - 1/12/((x^3 + a/b)^4*a^2*b^3)
```

Fricas [A]
time = 0.35, size = 207, normalized size = 0.77

$$-\frac{60\,ab^4x^{12}+210\,a^2b^3x^9+260\,a^3b^2x^6+125\,a^4bx^3+12\,a^5-60\,(b^5x^{15}+4\,ab^4x^{12}+6\,a^2b^3x^9+4\,a^3b^2x^6+a^4bx^3)\log\left(bx^3+a\right)+180\,(b^5x^{15}+4\,ab^4x^{12}+6\,a^2b^3x^9+4\,a^3b^2x^6+a^4bx^3)\log\left(x\right)}{36\,(a^6b^4x^{15}+4\,a^7b^3x^{12}+6\,a^8b^2x^9+4\,a^9bx^6+a^{10}x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/36*(60*a*b^4*x^12 + 210*a^2*b^3*x^9 + 260*a^3*b^2*x^6 + 125*a^4*b*x^3 +
12*a^5 - 60*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4*a^3*b^2*x^6 + a^4*
b*x^3)*log(b*x^3 + a) + 180*(b^5*x^15 + 4*a*b^4*x^12 + 6*a^2*b^3*x^9 + 4*a^
3*b^2*x^6 + a^4*b*x^3)*log(x))/(a^6*b^4*x^15 + 4*a^7*b^3*x^12 + 6*a^8*b^2*x
^9 + 4*a^9*b*x^6 + a^10*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**3)**2)**(5/2)), x)
```

Giac [A]
time = 4.36, size = 143, normalized size = 0.53

$$\frac{5\,b\log\left(|bx^3+a|\right)}{3\,a^6\mathrm{sgn}\left(bx^3+a\right)}-\frac{5\,b\log\left(|x|\right)}{a^6\mathrm{sgn}\left(bx^3+a\right)}+\frac{5\,bx^3-a}{3\,a^6x^3\mathrm{sgn}\left(bx^3+a\right)}-\frac{125\,b^5x^{12}+548\,ab^4x^9+912\,a^2b^3x^6+688\,a^3b^2x^3+202\,a^4b}{36\,\left(bx^3+a\right)^4a^6\mathrm{sgn}\left(bx^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 5/3*b*log(abs(b*x^3 + a))/(a^6*sgn(b*x^3 + a)) - 5*b*log(abs(x))/(a^6*sgn(b
*x^3 + a)) + 1/3*(5*b*x^3 - a)/(a^6*x^3*sgn(b*x^3 + a)) - 1/36*(125*b^5*x^1
2 + 548*a*b^4*x^9 + 912*a^2*b^3*x^6 + 688*a^3*b^2*x^3 + 202*a^4*b)/((b*x^3
+ a)^4*a^6*sgn(b*x^3 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)),x)
```

```
[Out] int(1/(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2)), x)
```

3.118 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2} \, dx$

Optimal. Leaf size=313

$$\frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{5a^4b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}+\frac{10a^3b^2(dx)^{7+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^7(7+m)\left(a+bx^3\right)}+$$

```
[Out] a^5*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+5*a^4*b*(d*x)^(4+m)*(
(b*x^3+a)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)+10*a^3*b^2*(d*x)^(7+m)*((b*x^3+a)^2)
^(1/2)/d^7/(7+m)/(b*x^3+a)+10*a^2*b^3*(d*x)^(10+m)*((b*x^3+a)^2)^(1/2)/d^10
/(10+m)/(b*x^3+a)+5*a*b^4*(d*x)^(13+m)*((b*x^3+a)^2)^(1/2)/d^13/(13+m)/(b*x
^3+a)+b^5*(d*x)^(16+m)*((b*x^3+a)^2)^(1/2)/d^16/(16+m)/(b*x^3+a)
```

Rubi [A]

time = 0.09, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 276}

$$\frac{b^5\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+16}}{d^{16}(m+16)\left(a+bx^3\right)}+\frac{5ab^4\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+13}}{d^{13}(m+13)\left(a+bx^3\right)}+\frac{10a^2b^3\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+10}}{d^{10}(m+10)\left(a+bx^3\right)}+\frac{a^5\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+1}}{d(m+1)\left(a+bx^3\right)}+\frac{5a^4b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\left(a+bx^3\right)}+\frac{10a^3b^2\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+7}}{d^7(m+7)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3))
 + (5*a^4*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a
+ b*x^3)) + (10*a^3*b^2*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7
*(7 + m)*(a + b*x^3)) + (10*a^2*b^3*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b
^2*x^6])/(d^10*(10 + m)*(a + b*x^3)) + (5*a*b^4*(d*x)^(13 + m)*Sqrt[a^2 + 2
*a*b*x^3 + b^2*x^6])/(d^13*(13 + m)*(a + b*x^3)) + (b^5*(d*x)^(16 + m)*Sqrt
[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^16*(16 + m)*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{5/2}\,dx = \frac{\sqrt{a^2+2abx^3+b^2x^6}\int (dx)^m \left(ab+b^2x^3\right)^5\,dx}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{\sqrt{a^2+2abx^3+b^2x^6}\int \left(a^5b^5(dx)^m+\frac{5a^4b^6(dx)^{3+m}}{d^3}+\frac{10a^3b^7(dx)^{6+m}}{d^6}+\frac{10a^2}{}\right.}{b^4\left(ab+b^2x^3\right)}$$
$$= \frac{a^5(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)}+\frac{5a^4b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)}+$$

Mathematica [A]
time = 0.19, size = 111, normalized size = 0.35

$$\frac{x(dx)^m\left(\left(a+bx^3\right)^2\right)^{5/2}\left(\frac{a^5}{1+m}+\frac{5a^4bx^3}{4+m}+\frac{10a^3b^2x^6}{7+m}+\frac{10a^2b^3x^9}{10+m}+\frac{5ab^4x^{12}}{13+m}+\frac{b^5x^{15}}{16+m}\right)}{\left(a+bx^3\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^3)^2)^(5/2)*(a^5/(1 + m) + (5*a^4*b*x^3)/(4 + m) + (10
*a^3*b^2*x^6)/(7 + m) + (10*a^2*b^3*x^9)/(10 + m) + (5*a*b^4*x^12)/(13 + m)
 + (b^5*x^15)/(16 + m)))/(a + b*x^3)^5
```

Maple [A]
time = 0.02, size = 453, normalized size = 1.45

method	result
gosper	$\frac{x\left(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714m\,x^{15}b^5+2555a\,b^4m^3x^{12}+3640b^5x^{15}+10a^2\right.}{}$
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(b^5m^5x^{15}+35b^5m^4x^{15}+445b^5m^3x^{15}+5a\,b^4m^5x^{12}+2485b^5m^2x^{15}+190a\,b^4m^4x^{12}+5714m\,x^{15}b^5+2555a\,b^4m^3x^{12}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^5*m^5*x^15+35*b^5*m^4*x^15+445*b^5*m^3*x^15+5*a*b^4*m^5*x^12+2485*b^5*
m^2*x^15+190*a*b^4*m^4*x^12+5714*b^5*m*x^15+2555*a*b^4*m^3*x^12+3640*b^5*x^
15+10*a^2*b^3*m^5*x^9+14810*a*b^4*m^2*x^12+410*a^2*b^3*m^4*x^9+34840*a*b^4*
m*x^12+5950*a^2*b^3*m^3*x^9+22400*a*b^4*x^12+10*a^3*b^2*m^5*x^6+36550*a^2*b
^3*m^2*x^9+440*a^3*b^2*m^4*x^6+89240*a^2*b^3*m*x^9+6970*a^3*b^2*m^3*x^6+582
40*a^2*b^3*x^9+5*a^4*b*m^5*x^3+47260*a^3*b^2*m^2*x^6+235*a^4*b*m^4*x^3+1239
```

```
20*a^3*b^2*m*x^6+4085*a^4*b*m^3*x^3+83200*a^3*b^2*x^6+a^5*m^5+31685*a^4*b*m
^2*x^3+50*a^5*m^4+100630*a^4*b*m*x^3+955*a^5*m^3+72800*a^4*b*x^3+8650*a^5*m
^2+36824*a^5*m+58240*a^5)*(d*x)^m*((b*x^3+a)^2)^(5/2)/(1+m)/(4+m)/(7+m)/(10
+m)/(13+m)/(16+m)/(b*x^3+a)^5
```

Maxima [A]
time = 0.28, size = 243, normalized size = 0.78

$$\frac{\left(\left(m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640\right)b^5d^mx^{16}+5\left(m^5+38\,m^4+511\,m^3+2962\,m^2+6968\,m+4480\right)ab^4d^mx^{13}+10\left(m^5+41\,m^4+595\,m^3+3655\,m^2+8924\,m+5824\right)a^2b^3d^mx^{10}+10\left(m^5+44\,m^4+697\,m^3+4726\,m^2+12392\,m+8320\right)a^3b^2d^mx^7+5\left(m^5+47\,m^4+817\,m^3+6337\,m^2+20126\,m+14560\right)a^4bd^mx^4+\left(m^5+50\,m^4+955\,m^3+8650\,m^2+36824\,m+58240\right)a^5d^mx\right)x^m}{m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] ((m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)*b^5*d^m*x^16 + 5*(m^5
+ 38*m^4 + 511*m^3 + 2962*m^2 + 6968*m + 4480)*a*b^4*d^m*x^13 + 10*(m^5 + 4
1*m^4 + 595*m^3 + 3655*m^2 + 8924*m + 5824)*a^2*b^3*d^m*x^10 + 10*(m^5 + 44
*m^4 + 697*m^3 + 4726*m^2 + 12392*m + 8320)*a^3*b^2*d^m*x^7 + 5*(m^5 + 47*m
^4 + 817*m^3 + 6337*m^2 + 20126*m + 14560)*a^4*b*d^m*x^4 + (m^5 + 50*m^4 +
955*m^3 + 8650*m^2 + 36824*m + 58240)*a^5*d^m*x)*x^m/(m^6 + 51*m^5 + 1005*m
^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

Fricas [A]
time = 0.38, size = 369, normalized size = 1.18

$$\frac{\left(\left(b^5m^5+35\,b^5m^4+445\,b^5m^3+2485\,b^5m^2+5714\,b^5m+3640\,b^5\right)x^{16}+5\left(ab^4m^5+38\,ab^4m^4+511\,ab^4m^3+2962\,ab^4m^2+6968\,ab^4m+4480\,ab^4\right)x^{13}+10\left(a^2b^3m^5+41\,a^2b^3m^4+595\,a^2b^3m^3+3655\,a^2b^3m^2+8924\,a^2b^3m+5824\,a^2b^3\right)x^{10}+10\left(a^3b^2m^5+44\,a^3b^2m^4+697\,a^3b^2m^3+4726\,a^3b^2m^2+12392\,a^3b^2m+8320\,a^3b^2\right)x^7+5\left(a^4bm^5+47\,a^4bm^4+817\,a^4bm^3+6337\,a^4bm^2+20126\,a^4bm+14560\,a^4b\right)x^4+\left(a^5m^5+50\,a^5m^4+955\,a^5m^3+8650\,a^5m^2+36824\,a^5m+58240\,a^5\right)x\right)(dx)^m}{m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 35*b^5*m^4 + 445*b^5*m^3 + 2485*b^5*m^2 + 5714*b^5*m + 3640*b^5
)*x^16 + 5*(a*b^4*m^5 + 38*a*b^4*m^4 + 511*a*b^4*m^3 + 2962*a*b^4*m^2 + 696
8*a*b^4*m + 4480*a*b^4)*x^13 + 10*(a^2*b^3*m^5 + 41*a^2*b^3*m^4 + 595*a^2*b
^3*m^3 + 3655*a^2*b^3*m^2 + 8924*a^2*b^3*m + 5824*a^2*b^3)*x^10 + 10*(a^3*b
^2*m^5 + 44*a^3*b^2*m^4 + 697*a^3*b^2*m^3 + 4726*a^3*b^2*m^2 + 12392*a^3*b^
2*m + 8320*a^3*b^2)*x^7 + 5*(a^4*b*m^5 + 47*a^4*b*m^4 + 817*a^4*b*m^3 + 633
7*a^4*b*m^2 + 20126*a^4*b*m + 14560*a^4*b)*x^4 + (a^5*m^5 + 50*a^5*m^4 + 95
5*a^5*m^3 + 8650*a^5*m^2 + 36824*a^5*m + 58240*a^5)*x)*(d*x)^m/(m^6 + 51*m^
5 + 1005*m^4 + 9605*m^3 + 45474*m^2 + 95064*m + 58240)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(\left(a+bx^3\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**3)**2)**(5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 900 vs. 2(247) = 494.
time = 4.22, size = 900, normalized size = 2.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^5*m^5*x^16*sgn(b*x^3 + a) + 35*(d*x)^m*b^5*m^4*x^16*sgn(b*x^3 +
a) + 445*(d*x)^m*b^5*m^3*x^16*sgn(b*x^3 + a) + 5*(d*x)^m*a*b^4*m^5*x^13*sgn
(b*x^3 + a) + 2485*(d*x)^m*b^5*m^2*x^16*sgn(b*x^3 + a) + 190*(d*x)^m*a*b^4*
m^4*x^13*sgn(b*x^3 + a) + 5714*(d*x)^m*b^5*m*x^16*sgn(b*x^3 + a) + 2555*(d*
x)^m*a*b^4*m^3*x^13*sgn(b*x^3 + a) + 3640*(d*x)^m*b^5*x^16*sgn(b*x^3 + a) +
 10*(d*x)^m*a^2*b^3*m^5*x^10*sgn(b*x^3 + a) + 14810*(d*x)^m*a*b^4*m^2*x^13*
sgn(b*x^3 + a) + 410*(d*x)^m*a^2*b^3*m^4*x^10*sgn(b*x^3 + a) + 34840*(d*x)^
m*a*b^4*m*x^13*sgn(b*x^3 + a) + 5950*(d*x)^m*a^2*b^3*m^3*x^10*sgn(b*x^3 + a
) + 22400*(d*x)^m*a*b^4*x^13*sgn(b*x^3 + a) + 10*(d*x)^m*a^3*b^2*m^5*x^7*sg
n(b*x^3 + a) + 36550*(d*x)^m*a^2*b^3*m^2*x^10*sgn(b*x^3 + a) + 440*(d*x)^m*
a^3*b^2*m^4*x^7*sgn(b*x^3 + a) + 89240*(d*x)^m*a^2*b^3*m*x^10*sgn(b*x^3 + a
) + 6970*(d*x)^m*a^3*b^2*m^3*x^7*sgn(b*x^3 + a) + 58240*(d*x)^m*a^2*b^3*x^1
0*sgn(b*x^3 + a) + 5*(d*x)^m*a^4*b*m^5*x^4*sgn(b*x^3 + a) + 47260*(d*x)^m*a
^3*b^2*m^2*x^7*sgn(b*x^3 + a) + 235*(d*x)^m*a^4*b*m^4*x^4*sgn(b*x^3 + a) +
123920*(d*x)^m*a^3*b^2*m*x^7*sgn(b*x^3 + a) + 4085*(d*x)^m*a^4*b*m^3*x^4*sg
n(b*x^3 + a) + 83200*(d*x)^m*a^3*b^2*x^7*sgn(b*x^3 + a) + (d*x)^m*a^5*m^5*x
*sgn(b*x^3 + a) + 31685*(d*x)^m*a^4*b*m^2*x^4*sgn(b*x^3 + a) + 50*(d*x)^m*a
^5*m^4*x*sgn(b*x^3 + a) + 100630*(d*x)^m*a^4*b*m*x^4*sgn(b*x^3 + a) + 955*(
d*x)^m*a^5*m^3*x*sgn(b*x^3 + a) + 72800*(d*x)^m*a^4*b*x^4*sgn(b*x^3 + a) +
8650*(d*x)^m*a^5*m^2*x*sgn(b*x^3 + a) + 36824*(d*x)^m*a^5*m*x*sgn(b*x^3 + a
) + 58240*(d*x)^m*a^5*x*sgn(b*x^3 + a))/(m^6 + 51*m^5 + 1005*m^4 + 9605*m^3
 + 45474*m^2 + 95064*m + 58240)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.119 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2}\,dx$

Optimal. Leaf size=205

$$\frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\,(a+bx^3)}+\frac{3a^2b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\,(a+bx^3)}+\frac{3ab^2(dx)^{7+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^7(7+m)\,(a+bx^3)}+\frac{b^3\cdots}{\cdots}$$

```
[Out] a^3*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+3*a^2*b*(d*x)^(4+m)*(
(b*x^3+a)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)+3*a*b^2*(d*x)^(7+m)*((b*x^3+a)^2)^(1
/2)/d^7/(7+m)/(b*x^3+a)+b^3*(d*x)^(10+m)*((b*x^3+a)^2)^(1/2)/d^10/(10+m)/(b
*x^3+a)
```

Rubi [A]
time = 0.06, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 276}

$$\frac{3ab^2\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+7}}{d^7(m+7)\,(a+bx^3)}+\frac{3a^2b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\,(a+bx^3)}+\frac{b^3\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+10}}{d^{10}(m+10)\,(a+bx^3)}+\frac{a^3\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+1}}{d(m+1)\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3))
 + (3*a^2*b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a
+ b*x^3)) + (3*a*b^2*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^7*(7
 + m)*(a + b*x^3)) + (b^3*(d*x)^(10 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(
d^10*(10 + m)*(a + b*x^3))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^3+b^2x^6} \int (dx)^m \left(ab+b^2x^3\right)^3 dx}{b^2\left(ab+b^2x^3\right)}$$

$$= \frac{\sqrt{a^2+2abx^3+b^2x^6} \int \left(a^3b^3(dx)^m + \frac{3a^2b^4(dx)^{3+m}}{d^3} + \frac{3ab^5(dx)^{6+m}}{d^6} + \frac{b^6(dx)^9}{d^9}\right.}{b^2\left(ab+b^2x^3\right)}$$

$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\left(a+bx^3\right)} + \frac{3a^2b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\left(a+bx^3\right)} +$$

Mathematica [A]
time = 0.07, size = 131, normalized size = 0.64

$$\frac{x(dx)^m\sqrt{\left(a+bx^3\right)^2}\left(a^3\left(280+138m+21m^2+m^3\right)+3a^2b\left(70+87m+18m^2+m^3\right)x^3+3ab^2\left(40+54m+15m^2+m^3\right)x^6+b^3\left(28+39m+12m^2+m^3\right)x^9\right)}{(1+m)(4+m)(7+m)(10+m)\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a^3*(280 + 138*m + 21*m^2 + m^3) + 3*a^2*b*
(70 + 87*m + 18*m^2 + m^3)*x^3 + 3*a*b^2*(40 + 54*m + 15*m^2 + m^3)*x^6 + b
^3*(28 + 39*m + 12*m^2 + m^3)*x^9))/((1 + m)*(4 + m)*(7 + m)*(10 + m)*(a +
b*x^3))
```

Maple [A]
time = 0.03, size = 199, normalized size = 0.97

method	result
gosper	$\frac{x\left(b^3m^3x^9+12b^3m^2x^9+39m\,x^9b^3+3a\,b^2m^3x^6+28b^3x^9+45a\,b^2m^2x^6+162m\,x^6a\,b^2+3a^2b\,m^3x^3+120a\,b^2x^6+54a^2b\,m^2x^3+261m\,x^3a^2b\right.}{(10+m)(7+m)(4+m)(1+m)\left(b\,x^3+a\right)^3}$
risch	$\frac{\sqrt{\left(b\,x^3+a\right)^2}\,\left(b^3m^3x^9+12b^3m^2x^9+39m\,x^9b^3+3a\,b^2m^3x^6+28b^3x^9+45a\,b^2m^2x^6+162m\,x^6a\,b^2+3a^2b\,m^3x^3+120a\,b^2x^6+54a^2b\,m\right.}{\left(b\,x^3+a\right)(10+m)(7+m)(4+m)(1+m)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b^3*m^3*x^9+12*b^3*m^2*x^9+39*b^3*m*x^9+3*a*b^2*m^3*x^6+28*b^3*x^9+45*a*
b^2*m^2*x^6+162*a*b^2*m*x^6+3*a^2*b*m^3*x^3+120*a*b^2*x^6+54*a^2*b*m^2*x^3+
261*a^2*b*m*x^3+a^3*m^3+210*a^2*b*x^3+21*a^3*m^2+138*a^3*m+280*a^3)*(d*x)^m
*((b*x^3+a)^2)^(3/2)/(10+m)/(7+m)/(4+m)/(1+m)/(b*x^3+a)^3
```

Maxima [A]
time = 0.29, size = 119, normalized size = 0.58

$$\frac{\left(\left(m^3+12\,m^2+39\,m+28\right)b^3d^mx^{10}+3\left(m^3+15\,m^2+54\,m+40\right)ab^2d^mx^7+3\left(m^3+18\,m^2+87\,m+70\right)a^2bd^mx^4+\left(m^3+21\,m^2+138\,m+280\right)a^3d^mx\right)x^m}{m^4+22\,m^3+159\,m^2+418\,m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 12*m^2 + 39*m + 28)*b^3*d^m*x^10 + 3*(m^3 + 15*m^2 + 54*m + 40)*a*b
^2*d^m*x^7 + 3*(m^3 + 18*m^2 + 87*m + 70)*a^2*b*d^m*x^4 + (m^3 + 21*m^2 + 1
38*m + 280)*a^3*d^m*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Fricas [A]
time = 0.35, size = 159, normalized size = 0.78

$$\frac{\left(\left(b^{3} m^{3}+12 \, b^{3} m^{2}+39 \, b^{3} m+28 \, b^{3}\right) x^{10}+3 \,\left(a b^{2} m^{3}+15 \, a b^{2} m^{2}+54 \, a b^{2} m+40 \, a b^{2}\right) x^{7}+3 \,\left(a^{2} b m^{3}+18 \, a^{2} b m^{2}+87 \, a^{2} b m+70 \, a^{2} b\right) x^{4}+\left(a^{3} m^{3}+21 \, a^{3} m^{2}+138 \, a^{3} m+280 \, a^{3}\right) x\right) \left(d x\right)^{m}}{m^{4}+22 \, m^{3}+159 \, m^{2}+418 \, m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 12*b^3*m^2 + 39*b^3*m + 28*b^3)*x^10 + 3*(a*b^2*m^3 + 15*a*b^2*
m^2 + 54*a*b^2*m + 40*a*b^2)*x^7 + 3*(a^2*b*m^3 + 18*a^2*b*m^2 + 87*a^2*b*m
 + 70*a^2*b)*x^4 + (a^3*m^3 + 21*a^3*m^2 + 138*a^3*m + 280*a^3)*x)*(d*x)^m/
(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d x\right)^{m} \left(\left(a + b x^{3}\right)^{2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**3)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(161) = 322.
time = 3.02, size = 384, normalized size = 1.87

$$\frac{\left(d x\right)^{m} b^{3} m^{3} x^{10} \mathrm{sgn}\left(b x^{3}+a\right)+12 \, \left(d x\right)^{m} b^{3} m^{2} x^{10} \mathrm{sgn}\left(b x^{3}+a\right)+39 \, \left(d x\right)^{m} b^{3} m x^{10} \mathrm{sgn}\left(b x^{3}+a\right)+3 \, \left(d x\right)^{m} a b^{2} m^{3} x^{7} \mathrm{sgn}\left(b x^{3}+a\right)+28 \, \left(d x\right)^{m} b^{3} x^{10} \mathrm{sgn}\left(b x^{3}+a\right)+45 \, \left(d x\right)^{m} a b^{2} m^{2} x^{7} \mathrm{sgn}\left(b x^{3}+a\right)+162 \, \left(d x\right)^{m} a b^{2} m x^{7} \mathrm{sgn}\left(b x^{3}+a\right)+3 \, \left(d x\right)^{m} a^{2} b m^{3} x^{4} \mathrm{sgn}\left(b x^{3}+a\right)+120 \, \left(d x\right)^{m} a b^{2} x^{7} \mathrm{sgn}\left(b x^{3}+a\right)+54 \, \left(d x\right)^{m} a^{2} b m^{2} x^{4} \mathrm{sgn}\left(b x^{3}+a\right)+261 \, \left(d x\right)^{m} a^{2} b m x^{4} \mathrm{sgn}\left(b x^{3}+a\right)+\left(d x\right)^{m} a^{3} m^{3} x \mathrm{sgn}\left(b x^{3}+a\right)+210 \, \left(d x\right)^{m} a^{2} b x^{4} \mathrm{sgn}\left(b x^{3}+a\right)+21 \, \left(d x\right)^{m} a^{3} m^{2} x \mathrm{sgn}\left(b x^{3}+a\right)+138 \, \left(d x\right)^{m} a^{3} m x \mathrm{sgn}\left(b x^{3}+a\right)+280 \, \left(d x\right)^{m} a^{3} x \mathrm{sgn}\left(b x^{3}+a\right)}{m^{4}+22 \, m^{3}+159 \, m^{2}+418 \, m+280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b^3*m^3*x^10*sgn(b*x^3 + a) + 12*(d*x)^m*b^3*m^2*x^10*sgn(b*x^3 +
a) + 39*(d*x)^m*b^3*m*x^10*sgn(b*x^3 + a) + 3*(d*x)^m*a*b^2*m^3*x^7*sgn(b*x
^3 + a) + 28*(d*x)^m*b^3*x^10*sgn(b*x^3 + a) + 45*(d*x)^m*a*b^2*m^2*x^7*sgn
(b*x^3 + a) + 162*(d*x)^m*a*b^2*m*x^7*sgn(b*x^3 + a) + 3*(d*x)^m*a^2*b*m^3*
x^4*sgn(b*x^3 + a) + 120*(d*x)^m*a*b^2*x^7*sgn(b*x^3 + a) + 54*(d*x)^m*a^2*
b*m^2*x^4*sgn(b*x^3 + a) + 261*(d*x)^m*a^2*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m
```

```
*a^3*m^3*x*sgn(b*x^3 + a) + 210*(d*x)^m*a^2*b*x^4*sgn(b*x^3 + a) + 21*(d*x)
^m*a^3*m^2*x*sgn(b*x^3 + a) + 138*(d*x)^m*a^3*m*x*sgn(b*x^3 + a) + 280*(d*x
)^m*a^3*x*sgn(b*x^3 + a))/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.120 $\int (dx)^m \sqrt{a^2+2abx^3+b^2x^6}\, dx$

Optimal. Leaf size=97

$$\frac{a(dx)^{1+m}\sqrt{a^2+2abx^3+b^2x^6}}{d(1+m)\,(a+bx^3)}+\frac{b(dx)^{4+m}\sqrt{a^2+2abx^3+b^2x^6}}{d^4(4+m)\,(a+bx^3)}$$

```
[Out] a*(d*x)^(1+m)*((b*x^3+a)^2)^(1/2)/d/(1+m)/(b*x^3+a)+b*(d*x)^(4+m)*((b*x^3+a
)^2)^(1/2)/d^4/(4+m)/(b*x^3+a)
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 14}

$$\frac{b\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+4}}{d^4(m+4)\,(a+bx^3)}+\frac{a\sqrt{a^2+2abx^3+b^2x^6}\,(dx)^{m+1}}{d(m+1)\,(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d*(1 + m)*(a + b*x^3)) +
 (b*(d*x)^(4 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])/(d^4*(4 + m)*(a + b*x^3)
)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int (dx)^m \sqrt{a^2 + 2abx^3 + b^2x^6}\, dx = \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int (dx)^m (ab + b^2x^3)\, dx}{ab + b^2x^3}$$
$$= \frac{\sqrt{a^2 + 2abx^3 + b^2x^6} \int \left(ab(dx)^m + \frac{b^2(dx)^{3+m}}{d^3}\right) dx}{ab + b^2x^3}$$
$$= \frac{a(dx)^{1+m}\sqrt{a^2 + 2abx^3 + b^2x^6}}{d(1+m)(a + bx^3)} + \frac{b(dx)^{4+m}\sqrt{a^2 + 2abx^3 + b^2x^6}}{d^4(4+m)(a + bx^3)}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.55

$$\frac{x(dx)^m \sqrt{(a + bx^3)^2}\, (a(4+m) + b(1+m)x^3)}{(1+m)(4+m)(a + bx^3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^3)^2]*(a*(4 + m) + b*(1 + m)*x^3))/((1 + m)*(4 + m
)*(a + b*x^3))
```

Maple [A]
time = 0.01, size = 56, normalized size = 0.58

method	result	size
gosper	$\frac{x(bm\,x^3+b\,x^3+am+4a)(dx)^m\sqrt{(b\,x^3+a)^2}}{(4+m)(1+m)(b\,x^3+a)}$	56
risch	$\frac{x(bm\,x^3+b\,x^3+am+4a)(dx)^m\sqrt{(b\,x^3+a)^2}}{(4+m)(1+m)(b\,x^3+a)}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(b*m*x^3+b*x^3+a*m+4*a)*(d*x)^m*((b*x^3+a)^2)^(1/2)/(4+m)/(1+m)/(b*x^3+a)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.36

$$\frac{(bd^m(m+1)x^4 + ad^m(m+4)x)x^m}{m^2 + 5\,m + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b*d^m*(m + 1)*x^4 + a*d^m*(m + 4)*x)*x^m/(m^2 + 5*m + 4)
```

Fricas [A]
time = 0.37, size = 35, normalized size = 0.36

$$\frac{((bm+b)x^4+(am+4\,a)x)(dx)^m}{m^2+5\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^4 + (a*m + 4*a)*x)*(d*x)^m/(m^2 + 5*m + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{(a+bx^3)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**3)**2), x)
```

Giac [A]
time = 2.41, size = 83, normalized size = 0.86

$$\frac{(dx)^m\,bmx^4\mathrm{sgn}(bx^3+a)+(dx)^m\,bx^4\mathrm{sgn}(bx^3+a)+(dx)^m\,amx\mathrm{sgn}(bx^3+a)+4\,(dx)^m\,ax\mathrm{sgn}(bx^3+a)}{m^2+5\,m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="giac")
```

```
[Out] ((d*x)^m*b*m*x^4*sgn(b*x^3 + a) + (d*x)^m*b*x^4*sgn(b*x^3 + a) + (d*x)^m*a*
m*x*sgn(b*x^3 + a) + 4*(d*x)^m*a*x*sgn(b*x^3 + a))/(m^2 + 5*m + 4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2), x)
```

3.121 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx$

Optimal. Leaf size=73

$$\frac{(dx)^{1+m}\left(a+bx^3\right)\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (d*x)^(1+m)*(b*x^3+a)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/d/(1
+m)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{1369, 371\}$

$$\frac{\left(a+bx^3\right)(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a^2+2abx^3+b^2x^6}}\,dx = \frac{(ab+b^2x^3)\int \frac{(dx)^m}{ab+b^2x^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}} = \frac{(dx)^{1+m}(a+bx^3)\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 0.85

$$\frac{x(dx)^m(a+bx^3)\,{}_2F_1\left(1,\frac{1+m}{3};1+\frac{1+m}{3};-\frac{bx^3}{a}\right)}{a(1+m)\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6],x]
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[1, (1 + m)/3, 1 + (1 + m)/3, -((b*
x^3)/a)])/(a*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{b^2x^6+2ab\,x^3+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="fricas")
[Out] integral((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{(a+bx^3)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(1/2),x)
[Out] Integral((d*x)**m/sqrt((a + b*x**3)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m/sqrt(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{a^2+2\,a\,b\,x^3+b^2\,x^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2),x)
[Out] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(1/2), x)
```

3.122 $\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{(dx)^{1+m}\left(a+bx^3\right)\,{}_2F_1\left(3,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (d*x)^(1+m)*(b*x^3+a)*hypergeom([3, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^3/d/
(1+m)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 371}

$$\frac{\left(a+bx^3\right)(dx)^{m+1}\,{}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^3))\int \frac{(dx)^m}{(ab+b^2x^3)^3}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}} = \frac{(dx)^{1+m}\,(a+bx^3)\;{}_2F_1\left(3,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A]
time = 0.03, size = 60, normalized size = 0.82

$$\frac{x(dx)^m\,(a+bx^3)\;{}_2F_1\left(3,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^3(1+m)\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(3/2),x]
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3)
/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b^2x^6+2ab\,x^3+a^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m/(b^4*x^12 + 4*a*b^3*x^9 +
6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**3)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2),x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(3/2), x)
```

3.123 $\int \frac{(dx)^m}{\left(a^2+2abx^3+b^2x^6\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{(dx)^{1+m}\left(a+bx^3\right)\,{}_2F_1\left(5,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

```
[Out] (d*x)^(1+m)*(b*x^3+a)*hypergeom([5, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^5/d/
(1+m)/((b*x^3+a)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 371}

$$\frac{\left(a+bx^3\right)(dx)^{m+1}\,{}_2F_1\left(5,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^3+b^2x^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b*
x^3)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^3 + b^2*x^6])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(dx)^m}{(a^2+2abx^3+b^2x^6)^{5/2}}\,dx = \frac{(b^4(ab+b^2x^3))\int \frac{(dx)^m}{(ab+b^2x^3)^5}\,dx}{\sqrt{a^2+2abx^3+b^2x^6}} = \frac{(dx)^{1+m}(a+bx^3)\,{}_2F_1\left(5,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^5d(1+m)\sqrt{a^2+2abx^3+b^2x^6}}$$

Mathematica [A]
time = 0.04, size = 60, normalized size = 0.82

$$\frac{x(dx)^m(a+bx^3)\,{}_2F_1\left(5,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a^5(1+m)\sqrt{(a+bx^3)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^3 + b^2*x^6)^(5/2),x]
[Out] (x*(d*x)^m*(a + b*x^3)*Hypergeometric2F1[5, (1 + m)/3, (4 + m)/3, -((b*x^3)
/a)])/(a^5*(1 + m)*Sqrt[(a + b*x^3)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(b^2x^6+2ab\,x^3+a^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
[Out] int((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x)^m/(b^6*x^18 + 6*a*b^5*x^15 +
 15*a^2*b^4*x^12 + 20*a^3*b^3*x^9 + 15*a^4*b^2*x^6 + 6*a^5*b*x^3 + a^6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left((a+bx^3)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**6+2*a*b*x**3+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**3)**2)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^6+2*a*b*x^3+a^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2),x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^6 + 2*a*b*x^3)^(5/2), x)
```

3.124 $\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p \, dx$

Optimal. Leaf size=77

$$\frac{(dx)^{1+m}\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{1+m}{3},-2p;\frac{4+m}{3};-\frac{bx^3}{a}\right)}{d(1+m)}$$

```
[Out] (d*x)^(1+m)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-2*p, 1/3+1/3*m],[4/3+1/3*
m],-b*x^3/a)/d/(1+m)/((1+b*x^3/a)^(2*p))
```

Rubi [A]
time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1370, 371}

$$\frac{(dx)^{m+1}\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{m+1}{3},-2p;\frac{m+4}{3};-\frac{bx^3}{a}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[(1 + m)/3, -
2*p, (4 + m)/3, -((b*x^3)/a)])/(d*(1 + m)*(1 + (b*x^3)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^3+b^2x^6\right)^p\,dx = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int (dx)^m\left(1+\frac{bx^3}{a}\right)^{2p}dx$$
$$= \frac{(dx)^{1+m}\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{1+m}{3},-2p;\frac{4+m}{3};-\frac{bx^3}{a}\right)}{d(1+m)}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.86

$$\frac{x(dx)^m\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}{}_2F_1\left(\frac{1+m}{3},-2p;1+\frac{1+m}{3};-\frac{bx^3}{a}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x*(d*x)^m*((a + b*x^3)^2)^p*Hypergeometric2F1[(1 + m)/3, -2*p, 1 + (1 + m)
/3, -((b*x^3)/a)])/((1 + m)*(1 + (b*x^3)/a)^(2*p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx)^m\left(b^2x^6+2ab\,x^3+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(\left(a + bx^3\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
[Out] Integral((d*x)**m*((a + b*x**3)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \left(a^2 + 2\,a\,b\,x^3 + b^2\,x^6\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
[Out] int((d*x)^m*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.125 $\int x^{11}(a^2+2abx^3+b^2x^6)^p \, dx$

Optimal. Leaf size=172

$$-\frac{a^3(a+bx^3)(a^2+2abx^3+b^2x^6)^p}{3b^4(1+2p)}+\frac{a^2(a+bx^3)^2(a^2+2abx^3+b^2x^6)^p}{2b^4(1+p)}-\frac{a(a+bx^3)^3(a^2+2abx^3+b^2x^6)^p}{b^4(3+2p)}+($$

```
[Out] -1/3*a^3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(1+2*p)+1/2*a^2*(b*x^3+a)^
2*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(1+p)-a*(b*x^3+a)^3*(b^2*x^6+2*a*b*x^3+a^2)
^p/b^4/(3+2*p)+1/6*(b*x^3+a)^4*(b^2*x^6+2*a*b*x^3+a^2)^p/b^4/(2+p)
```

Rubi [A]
time = 0.08, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1370, 272, 45}

$$\frac{(a+bx^3)^4(a^2+2abx^3+b^2x^6)^p}{6b^4(p+2)}-\frac{a(a+bx^3)^3(a^2+2abx^3+b^2x^6)^p}{b^4(2p+3)}+\frac{a^2(a+bx^3)^2(a^2+2abx^3+b^2x^6)^p}{2b^4(p+1)}-\frac{a^3(a+bx^3)(a^2+2abx^3+b^2x^6)^p}{3b^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] -1/3*(a^3*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^4*(1 + 2*p)) + (a^2
*(a + b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(2*b^4*(1 + p)) - (a*(a + b*x
^3)^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^4*(3 + 2*p)) + ((a + b*x^3)^4*(a^2
+ 2*a*b*x^3 + b^2*x^6)^p)/(6*b^4*(2 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int x^{11}\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^{11}\left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\
&= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x^3\left(1+\frac{bx}{a}\right)^{2p}\,dx, x,\right.\\
&= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^3\left(1+\frac{bx}{a}\right)^{2p}}{b^3}+\frac{3}{}\right.\right.\\
&= -\frac{a^3(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^4(1+2p)}+\frac{a^2(a+bx^3)^2\left(a^2+2abx^3+b^2x^6\right)^p}{2b^4(1+p)}-
\end{aligned}
$$

Mathematica [A]
time = 0.07, size = 110, normalized size = 0.64

$$
\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\left(-3a^3+3a^2b(1+2p)x^3-3ab^2(1+3p+2p^2)\,x^6+b^3(3+11p+12p^2+4p^3)\,x^9\right)}{6b^4(1+p)(2+p)(1+2p)(3+2p)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^3 - 3*a*b^2*(1
 + 3*p + 2*p^2)*x^6 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x^9))/(6*b^4*(1 + p)*
(2 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A]
time = 0.02, size = 150, normalized size = 0.87

method	result
gosper	$-\frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p\left(-4b^3p^3x^9-12b^3p^2x^9-11b^3p\,x^9-3b^3x^9+6a\,b^2p^2x^6+9a\,b^2p\,x^6+3a\,b^2x^6-6a^2bp\,x^3-3a^2b\,x^3+3a^3\right)\left(b\,x^3+a\right)}{6b^4\left(4p^4+20p^3+35p^2+25p+6\right)}$
risch	$-\frac{\left(-4b^4p^3x^{12}-12b^4p^2x^{12}-11b^4p\,x^{12}-4a\,b^3p^3x^9-3b^4x^{12}-6a\,b^3p^2x^9-2ap\,x^9b^3+6a^2b^2p^2x^6+3a^2p\,x^6b^2-6a^3p\,x^3b+3a^4\right)\left(\left(b\,x^3+a\right)\right.}{6(3+2p)(2+p)(1+p)(1+2p)b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(b^2*x^6+2*a*b*x^3+a^2)^p*(-4*b^3*p^3*x^9-12*b^3*p^2*x^9-11*b^3*p*x^9-
3*b^3*x^9+6*a*b^2*p^2*x^6+9*a*b^2*p*x^6+3*a*b^2*x^6-6*a^2*b*p*x^3-3*a^2*b*x
^3+3*a^3)*(b*x^3+a)/b^4/(4*p^4+20*p^3+35*p^2+25*p+6)
```

Maxima [A]
time = 0.27, size = 115, normalized size = 0.67

$$\frac{((4\,p^3+12\,p^2+11\,p+3)b^4x^{12}+2\,(2\,p^3+3\,p^2+p)ab^3x^9-3\,(2\,p^2+p)a^2b^2x^6+6\,a^3bpx^3-3\,a^4)(bx^3+a)^{2\,p}}{6\,(4\,p^4+20\,p^3+35\,p^2+25\,p+6)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/6*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^12 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^9
 - 3*(2*p^2 + p)*a^2*b^2*x^6 + 6*a^3*b*p*x^3 - 3*a^4)*(b*x^3 + a)^(2*p)/((4
*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

Fricas [A]
time = 0.37, size = 163, normalized size = 0.95

$$\frac{((4\,b^4p^3+12\,b^4p^2+11\,b^4p+3\,b^4)x^{12}+2\,(2\,ab^3p^3+3\,ab^3p^2+ab^3p)x^9+6\,a^3bpx^3-3\,(2\,a^2b^2p^2+a^2b^2p)x^6-3\,a^4)(b^2x^6+2\,abx^3+a^2)^p}{6\,(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/6*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^12 + 2*(2*a*b^3*p^3 + 3*
a*b^3*p^2 + a*b^3*p)*x^9 + 6*a^3*b*p*x^3 - 3*(2*a^2*b^2*p^2 + a^2*b^2*p)*x^
6 - 3*a^4)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^4*p^4 + 20*b^4*p^3 + 35*b^4*p
^2 + 25*b^4*p + 6*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**12*(a**2)**p/12, Eq(b, 0)), (6*a**3*log(x - (-a/b)**(1/3))/(1
8*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 6*a**3*l
og(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**3*b**4 + 54*a**2*b*
*5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 12*a**3*log(2)/(18*a**3*b**4 + 5
4*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 11*a**3/(18*a**3*b**4 +
 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a**2*b*x**3*log(x
- (-a/b)**(1/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b*
*7*x**9) + 18*a**2*b*x**3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))
/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a*
*2*b*x**3*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b*
*7*x**9) + 27*a**2*b*x**3/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**
```

```
6 + 18*b**7*x**9) + 18*a*b**2*x**6*log(x - (-a/b)**(1/3))/(18*a**3*b**4 + 5
4*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*x**6*log(4*x*
*2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3
 + 54*a*b**6*x**6 + 18*b**7*x**9) - 36*a*b**2*x**6*log(2)/(18*a**3*b**4 + 5
4*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 18*a*b**2*x**6/(18*a**3
*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) + 6*b**3*x**9*lo
g(x - (-a/b)**(1/3))/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 1
8*b**7*x**9) + 6*b**3*x**9*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3)
)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**7*x**9) - 12*b
**3*x**9*log(2)/(18*a**3*b**4 + 54*a**2*b**5*x**3 + 54*a*b**6*x**6 + 18*b**
7*x**9), Eq(p, -2)), (Integral(x**11/((a + b*x**3)**2)**(3/2), x), Eq(p, -3
/2)), (6*a**3*log(x - (-a/b)**(1/3))/(6*a*b**4 + 6*b**5*x**3) + 6*a**3*log(
4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a*b**4 + 6*b**5*x**3) - 12
*a**3*log(2)/(6*a*b**4 + 6*b**5*x**3) + 6*a**3/(6*a*b**4 + 6*b**5*x**3) + 6
*a**2*b*x**3*log(x - (-a/b)**(1/3))/(6*a*b**4 + 6*b**5*x**3) + 6*a**2*b*x**
3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(6*a*b**4 + 6*b**5*x**3
) - 12*a**2*b*x**3*log(2)/(6*a*b**4 + 6*b**5*x**3) - 3*a*b**2*x**6/(6*a*b**
4 + 6*b**5*x**3) + b**3*x**9/(6*a*b**4 + 6*b**5*x**3), Eq(p, -1)), (Integra
l(x**11/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (-3*a**4*(a**2 + 2*a*b*x**
3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*
p + 36*b**4) + 6*a**3*b*p*x**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*
p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) - 6*a**2*b**2*
p**2*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3
+ 210*b**4*p**2 + 150*b**4*p + 36*b**4) - 3*a**2*b**2*p*x**6*(a**2 + 2*a*b*
x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b*
*4*p + 36*b**4) + 4*a*b**3*p**3*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24
*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 6*a*b*
*3*p**2*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p*
*3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 2*a*b**3*p*x**9*(a**2 + 2*a*b*
x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b*
*4*p + 36*b**4) + 4*b**4*p**3*x**12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*
b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 12*b**4
*p**2*x**12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**
3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4) + 11*b**4*p*x**12*(a**2 + 2*a*b*x
**3 + b**2*x**6)**p/(24*b**4*p**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**
4*p + 36*b**4) + 3*b**4*x**12*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(24*b**4*p
**4 + 120*b**4*p**3 + 210*b**4*p**2 + 150*b**4*p + 36*b**4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(166) = 332.

time = 3.49, size = 375, normalized size = 2.18

$$\frac{4\,(b^2x^6+2\,abx^3+a^2)^p b^4p^3x^{12}+12\,(b^2x^6+2\,abx^3+a^2)^p b^4p^2x^{12}+11\,(b^2x^6+2\,abx^3+a^2)^p b^4px^{12}+4\,(b^2x^6+2\,abx^3+a^2)^p ab^3p^3x^9+3\,(b^2x^6+2\,abx^3+a^2)^p b^4x^{12}+6\,(b^2x^6+2\,abx^3+a^2)^p ab^3p^2x^9+2\,(b^2x^6+2\,abx^3+a^2)^p ab^3px^9-6\,(b^2x^6+2\,abx^3+a^2)^p a^2b^2p^2x^6-3\,(b^2x^6+2\,abx^3+a^2)^p a^2b^2px^6+6\,(b^2x^6+2\,abx^3+a^2)^p a^3bpx^3-3\,(b^2x^6+2\,abx^3+a^2)^p a^4}{6\,(4\,b^4p^4+20\,b^4p^3+35\,b^4p^2+25\,b^4p+6\,b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/6*(4*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p^3*x^12 + 12*(b^2*x^6 + 2*a*b*x^3
 + a^2)^p*b^4*p^2*x^12 + 11*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^4*p*x^12 + 4*(b
^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^3*x^9 + 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p
*b^4*x^12 + 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^3*p^2*x^9 + 2*(b^2*x^6 + 2*
a*b*x^3 + a^2)^p*a*b^3*p*x^9 - 6*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2*p^2*
x^6 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b^2*p*x^6 + 6*(b^2*x^6 + 2*a*b*x^
3 + a^2)^p*a^3*b*p*x^3 - 3*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^4)/(4*b^4*p^4 +
20*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

Mupad [B]
time = 1.31, size = 207, normalized size = 1.20

$$(a^2+2\,a\,b\,x^3+b^2\,x^6)^p\,\left(\frac{x^{12}\,(4\,p^3+12\,p^2+11\,p+3)}{6\,(4\,p^4+20\,p^3+35\,p^2+25\,p+6)}-\frac{a^4}{2\,b^4\,(4\,p^4+20\,p^3+35\,p^2+25\,p+6)}+\frac{a^3\,p\,x^3}{b^3\,(4\,p^4+20\,p^3+35\,p^2+25\,p+6)}+\frac{a\,p\,x^9\,(2\,p^2+3\,p+1)}{3\,b\,(4\,p^4+20\,p^3+35\,p^2+25\,p+6)}-\frac{a^2\,p\,x^6\,(2\,p+1)}{2\,b^2\,(4\,p^4+20\,p^3+35\,p^2+25\,p+6)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^12*(11*p + 12*p^2 + 4*p^3 + 3))/(6*(25*p
+ 35*p^2 + 20*p^3 + 4*p^4 + 6)) - a^4/(2*b^4*(25*p + 35*p^2 + 20*p^3 + 4*p^
4 + 6)) + (a^3*p*x^3)/(b^3*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) + (a*p*x^9
*(3*p + 2*p^2 + 1))/(3*b*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)) - (a^2*p*x^6
*(2*p + 1))/(2*b^2*(25*p + 35*p^2 + 20*p^3 + 4*p^4 + 6)))
```

3.126 $\int x^8(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal. Leaf size=130

$$\frac{a^2(a+bx^3)(a^2+2abx^3+b^2x^6)^p}{3b^3(1+2p)}-\frac{a(a+bx^3)^2(a^2+2abx^3+b^2x^6)^p}{3b^3(1+p)}+\frac{(a+bx^3)^3(a^2+2abx^3+b^2x^6)^p}{3b^3(3+2p)}$$

```
[Out] 1/3*a^2*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(1+2*p)-1/3*a*(b*x^3+a)^2*(
b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(1+p)+1/3*(b*x^3+a)^3*(b^2*x^6+2*a*b*x^3+a^2)^
p/b^3/(3+2*p)
```

Rubi [A]
time = 0.06, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1370, 272, 45}

$$\frac{(a+bx^3)^3(a^2+2abx^3+b^2x^6)^p}{3b^3(2p+3)}-\frac{a(a+bx^3)^2(a^2+2abx^3+b^2x^6)^p}{3b^3(p+1)}+\frac{a^2(a+bx^3)(a^2+2abx^3+b^2x^6)^p}{3b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (a^2*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + 2*p)) - (a*(a +
 b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(1 + p)) + ((a + b*x^3)^3*(
a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b^3*(3 + 2*p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int x^8\left(a^2+2abx^3+b^2x^6\right)^p\,dx = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^8\left(1+\frac{bx^3}{a}\right)^{2p}\,dx$$
$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x^2\left(1+\frac{bx}{a}\right)^{2p}\,dx,x,x^3$$
$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(1+\frac{bx}{a}\right)^{2p}}{b^2}-\frac{2a^2(1$$
$$=\frac{a^2(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(1+2p)}-\frac{a(a+bx^3)^2\left(a^2+2abx^3+b^2x^6\right)^p}{3b^3(1+p)}+\frac{(a+$$

Mathematica [A]
time = 0.05, size = 77, normalized size = 0.59

$$\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\left(a^2-ab(1+2p)x^3+b^2\left(1+3p+2p^2\right)x^6\right)}{3b^3(1+p)(1+2p)(3+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(a^2 - a*b*(1 + 2*p)*x^3 + b^2*(1 + 3*p + 2*
p^2)*x^6))/(3*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A]
time = 0.02, size = 96, normalized size = 0.74

method	result	size
gosper	$\frac{(b\,x^3+a)\left(2b^2p^2x^6+3b^2p\,x^6+b^2x^6-2abp\,x^3-ab\,x^3+a^2\right)\left(b^2x^6+2ab\,x^3+a^2\right)^p}{3b^3\left(4p^3+12p^2+11p+3\right)}$	96
risch	$\frac{\left(2b^3p^2x^9+3b^3p\,x^9+b^3x^9+2a\,b^2p^2x^6+a\,b^2p\,x^6-2a^2bp\,x^3+a^3\right)\left(\left(b\,x^3+a\right)^2\right)^p}{3(1+p)(3+2p)(1+2p)b^3}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(b*x^3+a)*(2*b^2*p^2*x^6+3*b^2*p*x^6+b^2*x^6-2*a*b*p*x^3-a*b*x^3+a^2)*(
b^2*x^6+2*a*b*x^3+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

Maxima [A]
time = 0.28, size = 79, normalized size = 0.61

$$\frac{\left((2\,p^2+3\,p+1)b^3x^9+(2\,p^2+p)ab^2x^6-2\,a^2bpx^3+a^3\right)(bx^3+a)^{2\,p}}{3\,(4\,p^3+12\,p^2+11\,p+3)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/3*((2*p^2 + 3*p + 1)*b^3*x^9 + (2*p^2 + p)*a*b^2*x^6 - 2*a^2*b*p*x^3 + a^
3)*(b*x^3 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Fricas [A]
time = 0.35, size = 108, normalized size = 0.83

$$\frac{((2\,b^3p^2+3\,b^3p+b^3)x^9-2\,a^2bpx^3+(2\,ab^2p^2+ab^2p)x^6+a^3)(b^2x^6+2\,abx^3+a^2)^p}{3\,(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/3*((2*b^3*p^2 + 3*b^3*p + b^3)*x^9 - 2*a^2*b*p*x^3 + (2*a*b^2*p^2 + a*b^2
*p)*x^6 + a^3)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(4*b^3*p^3 + 12*b^3*p^2 + 11*b
^3*p + 3*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^9(a^2)^p}{9} & \text{for}\ b=0 \\ \int \frac{x^8}{\left((a+bx^3)^2\right)^{\frac{3}{2}}}\,dx & \text{for}\ p=-\frac{3}{2} \\ -\frac{2a^2\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^3+3b^4x^3}-\frac{2a^2\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^3+3b^4x^3}-\frac{2a^2}{3ab^3+3b^4x^3}+\frac{4a^2\log(2)}{3ab^3+3b^4x^3}-\frac{2abx^3\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^3+3b^4x^3}-\frac{2abx^3\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^3+3b^4x^3}+\frac{4abx^3\log(2)}{3ab^3+3b^4x^3}+\frac{b^2x^6}{3ab^3+3b^4x^3} & \text{for}\ p=-1 \\ \int \frac{x^8}{\sqrt{(a+bx^3)^2}}\,dx & \text{for}\ p=-\frac{1}{2} \\ \frac{a^3(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}-\frac{2a^2bpx^3(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{2ab^2p^2x^6(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{ab^2px^6(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{2b^3p^2x^9(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{3b^3px^9(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3}+\frac{b^3x^9(a^2+2abx^3+b^2x^6)^p}{12b^3p^3+36b^3p^2+33b^3p+9b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**9*(a**2)**p/9, Eq(b, 0)), (Integral(x**8/((a + b*x**3)**2)**(
3/2), x), Eq(p, -3/2)), (-2*a**2*log(x - (-a/b)**(1/3))/(3*a*b**3 + 3*b**4*
x**3) - 2*a**2*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**3
+ 3*b**4*x**3) - 2*a**2/(3*a*b**3 + 3*b**4*x**3) + 4*a**2*log(2)/(3*a*b**3
+ 3*b**4*x**3) - 2*a*b*x**3*log(x - (-a/b)**(1/3))/(3*a*b**3 + 3*b**4*x**3)
 - 2*a*b*x**3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**3 +
 3*b**4*x**3) + 4*a*b*x**3*log(2)/(3*a*b**3 + 3*b**4*x**3) + b**2*x**6/(3*a
*b**3 + 3*b**4*x**3), Eq(p, -1)), (Integral(x**8/sqrt((a + b*x**3)**2), x),
 Eq(p, -1/2)), (a**3*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*
b**3*p**2 + 33*b**3*p + 9*b**3) - 2*a**2*b*p*x**3*(a**2 + 2*a*b*x**3 + b**2
*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + 2*a*b**2*p**
2*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33
*b**3*p + 9*b**3) + a*b**2*p*x**6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b*
*3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3) + 2*b**3*p**2*x**9*(a**2 + 2*a
```

```
*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3)
+ 3*b**3*p*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**3*p**3 + 36*b**3*
p**2 + 33*b**3*p + 9*b**3) + b**3*x**9*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(
12*b**3*p**3 + 36*b**3*p**2 + 33*b**3*p + 9*b**3), True))
```

Giac [A]
time = 3.64, size = 235, normalized size = 1.81

$$\frac{2\,(b^2x^6+2\,abx^3+a^2)^pb^3p^2x^9+3\,(b^2x^6+2\,abx^3+a^2)^pb^3px^9+(b^2x^6+2\,abx^3+a^2)^pb^3x^9+2\,(b^2x^6+2\,abx^3+a^2)^pab^2p^2x^6+(b^2x^6+2\,abx^3+a^2)^pab^2px^6-2\,(b^2x^6+2\,abx^3+a^2)^pa^2bpx^3+(b^2x^6+2\,abx^3+a^2)^pa^3}{3\,(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/3*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*p^2*x^9 + 3*(b^2*x^6 + 2*a*b*x^3 +
 a^2)^p*b^3*p*x^9 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^3*x^9 + 2*(b^2*x^6 + 2*
a*b*x^3 + a^2)^p*a*b^2*p^2*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b^2*p*x^6
- 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a^2*b*p*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)
^p*a^3)/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Mupad [B]
time = 1.22, size = 137, normalized size = 1.05

$$(a^2+2\,a\,b\,x^3+b^2\,x^6)^p\,\left(\frac{x^9\,\left(\frac{2\,p^2}{3}+p+\frac{1}{3}\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{a^3}{3\,b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{2\,a^2\,p\,x^3}{3\,b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)}+\frac{a\,p\,x^6\,\left(2\,p+1\right)}{3\,b\,\left(4\,p^3+12\,p^2+11\,p+3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^9*(p + (2*p^2)/3 + 1/3))/(11*p + 12*p^2 +
 4*p^3 + 3) + a^3/(3*b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (2*a^2*p*x^3)/(3*b^
2*(11*p + 12*p^2 + 4*p^3 + 3)) + (a*p*x^6*(2*p + 1))/(3*b*(11*p + 12*p^2 +
4*p^3 + 3)))
```

3.127 $\int x^5(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal. Leaf size=84

$$-\frac{a(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(1+2p)}+\frac{\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(1+p)}$$

```
[Out] -1/3*a*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b^2/(1+2*p)+1/6*(b*x^3+a)^2*(b^2
*x^6+2*a*b*x^3+a^2)^p/b^2/(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.125$, Rules used = {1370, 272, 45}

$$\frac{\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(p+1)}-\frac{a(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] -1/3*(a*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(b^2*(1 + 2*p)) + ((a +
b*x^3)^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(6*b^2*(1 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int x^5\left(a^2+2abx^3+b^2x^6\right)^p\,dx=\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^5\left(1+\frac{bx^3}{a}\right)^{2p}dx$$
$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{2p}dx,x,x^3\right)$$
$$=\frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b}+\frac{a\left(1-\right.}{}\right.\right.$$
$$=-\frac{a(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p}{3b^2(1+2p)}+\frac{\left(a+bx^3\right)^2\left(a^2+2abx^3+b^2x^6\right)^p}{6b^2(1+p)}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.61

$$\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\left(-a+b(1+2p)x^3\right)}{6b^2(1+p)(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p*(-a + b*(1 + 2*p)*x^3))/(6*b^2*(1 + p)*(1 +
2*p))
```

Maple [A]
time = 0.03, size = 58, normalized size = 0.69

method	result	size
risch	$-\frac{\left(-2b^2p\,x^6-b^2x^6-2abp\,x^3+a^2\right)\left(\left(b\,x^3+a\right)^2\right)^p}{6b^2(1+p)(1+2p)}$	58
gosper	$-\frac{\left(b^2x^6+2ab\,x^3+a^2\right)^p\left(-2x^3pb-b\,x^3+a\right)\left(b\,x^3+a\right)}{6b^2(2p^2+3p+1)}$	60
norman	$\frac{x^6\mathrm{e}^{p\ln\left(b^2x^6+2ab\,x^3+a^2\right)}}{6p+6}-\frac{a^2\mathrm{e}^{p\ln\left(b^2x^6+2ab\,x^3+a^2\right)}}{6b^2(2p^2+3p+1)}+\frac{pa\,x^3\mathrm{e}^{p\ln\left(b^2x^6+2ab\,x^3+a^2\right)}}{3b(2p^2+3p+1)}$	120

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*(-2*b^2*p*x^6-b^2*x^6-2*a*b*p*x^3+a^2)/b^2/(1+p)/(1+2*p)*((b*x^3+a)^2)
^p
```

Maxima [A]
time = 0.28, size = 54, normalized size = 0.64

$$\frac{(b^2(2\,p+1)x^6+2\,abpx^3-a^2)(bx^3+a)^{2\,p}}{6\,(2\,p^2+3\,p+1)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/6*(b^2*(2*p + 1)*x^6 + 2*a*b*p*x^3 - a^2)*(b*x^3 + a)^(2*p)/((2*p^2 + 3*p
 + 1)*b^2)
```

Fricas [A]
time = 0.35, size = 70, normalized size = 0.83

$$\frac{((2\,b^2p+b^2)x^6+2\,abpx^3-a^2)(b^2x^6+2\,abx^3+a^2)^p}{6\,(2\,b^2p^2+3\,b^2p+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/6*((2*b^2*p + b^2)*x^6 + 2*a*b*p*x^3 - a^2)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p
/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^6(a^2)^p}{6} & \text{for}\ b=0 \\ \frac{a\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x^3}+\frac{a\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2+3b^3x^3}-\frac{2a\log(2)}{3ab^2+3b^3x^3}+\frac{a}{3ab^2+3b^3x^3}+\frac{bx^3\log\left(x-\sqrt[3]{-\frac{a}{b}}\right)}{3ab^2+3b^3x^3}+\frac{bx^3\log\left(4x^2+4x\sqrt[3]{-\frac{a}{b}}+4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3ab^2+3b^3x^3}-\frac{2bx^3\log(2)}{3ab^2+3b^3x^3} & \text{for}\ p=-1 \\ \int\frac{x^5}{\sqrt{(a+bx^3)^2}}\,dx & \text{for}\ p=-\frac{1}{2} \\ -\frac{a^2(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2}+\frac{2abpx^3(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2}+\frac{2b^2px^6(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2}+\frac{b^2x^6(a^2+2abx^3+b^2x^6)^p}{12b^2p^2+18b^2p+6b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**6*(a**2)**p/6, Eq(b, 0)), (a*log(x - (-a/b)**(1/3))/(3*a*b**2
 + 3*b**3*x**3) + a*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*
b**2 + 3*b**3*x**3) - 2*a*log(2)/(3*a*b**2 + 3*b**3*x**3) + a/(3*a*b**2 + 3
*b**3*x**3) + b*x**3*log(x - (-a/b)**(1/3))/(3*a*b**2 + 3*b**3*x**3) + b*x*
*3*log(4*x**2 + 4*x*(-a/b)**(1/3) + 4*(-a/b)**(2/3))/(3*a*b**2 + 3*b**3*x**
3) - 2*b*x**3*log(2)/(3*a*b**2 + 3*b**3*x**3), Eq(p, -1)), (Integral(x**5/s
qrt((a + b*x**3)**2), x), Eq(p, -1/2)), (-a**2*(a**2 + 2*a*b*x**3 + b**2*x*
*6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + 2*a*b*p*x**3*(a**2 + 2*a*b*x**
3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + 2*b**2*p*x**6*(a**2
```

```
 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2) + b**2*x*
*6*(a**2 + 2*a*b*x**3 + b**2*x**6)**p/(12*b**2*p**2 + 18*b**2*p + 6*b**2),
True))
```

Giac [A]
time = 3.03, size = 132, normalized size = 1.57

$$\frac{2\,(b^2x^6+2\,abx^3+a^2)^pb^2px^6+(b^2x^6+2\,abx^3+a^2)^pb^2x^6+2\,(b^2x^6+2\,abx^3+a^2)^pabpx^3-(b^2x^6+2\,abx^3+a^2)^pa^2}{6\,(2\,b^2p^2+3\,b^2p+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/6*(2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*b^2*p*x^6 + (b^2*x^6 + 2*a*b*x^3 + a^2
)^p*b^2*x^6 + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)^p*a*b*p*x^3 - (b^2*x^6 + 2*a*b*
x^3 + a^2)^p*a^2)/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Mupad [B]
time = 1.19, size = 85, normalized size = 1.01

$$\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,\left(\frac{x^6\,\left(2\,p+1\right)}{6\,\left(2\,p^2+3\,p+1\right)}-\frac{a^2}{6\,b^2\,\left(2\,p^2+3\,p+1\right)}+\frac{a\,p\,x^3}{3\,b\,\left(2\,p^2+3\,p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (a^2 + b^2*x^6 + 2*a*b*x^3)^p*((x^6*(2*p + 1))/(6*(3*p + 2*p^2 + 1)) - a^2/
(6*b^2*(3*p + 2*p^2 + 1)) + (a*p*x^3)/(3*b*(3*p + 2*p^2 + 1)))
```

3.128 $\int x^4(a^2 + 2abx^3 + b^2x^6)^p \, dx$

Optimal. Leaf size=60

$$\frac{1}{5}x^5\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\ {}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

```
[Out] 1/5*x^5*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([5/3, -2*p],[8/3],-b*x^3/a)/((1
+b*x^3/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1370, 371}

$$\frac{1}{5}x^5\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\ {}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^5*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*x
^3)/a)])/(5*(1 + (b*x^3)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^4\left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= \frac{1}{5}x^5\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\ {}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.05, size = 51, normalized size = 0.85

$$\frac{1}{5}x^5\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}{}_2F_1\left(\frac{5}{3},-2p;\frac{8}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
[Out] (x^5*((a + b*x^3)^2)^p*Hypergeometric2F1[5/3, -2*p, 8/3, -((b*x^3)/a)])/(5*
(1 + (b*x^3)/a)^(2*p))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^4\left(b^2x^6+2ab\,x^3+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
[Out] int(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\left(\left(a+bx^3\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Integral(x**4*((a + b*x**3)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^4\,\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] int(x^4*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.129 $\int x^3(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal. Leaf size=60

$$\frac{1}{4}x^4\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

```
[Out] 1/4*x^4*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([4/3, -2*p],[7/3],-b*x^3/a)/((1
+b*x^3/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1370, 371}

$$\frac{1}{4}x^4\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^4*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*x
^3)/a)])/(4*(1 + (b*x^3)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x^3\left(1+\frac{bx^3}{a}\right)^{2p}\,dx\\ &= \frac{1}{4}x^4\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.85

$$\frac{1}{4}x^4\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}{}_2F_1\left(\frac{4}{3},-2p;\frac{7}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^4*((a + b*x^3)^2)^p*Hypergeometric2F1[4/3, -2*p, 7/3, -((b*x^3)/a)])/(4*
(1 + (b*x^3)/a)^(2*p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^3\left(b^2x^6+2ab\,x^3+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(\left(a+bx^3\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Integral(x**3*((a + b*x**3)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] int(x^3*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.130 $\int x^2(a^2+2abx^3+b^2x^6)^p\,dx$

Optimal. Leaf size=41

$$\frac{(a+bx^3)\,(a^2+2abx^3+b^2x^6)^p}{3b(1+2p)}$$

```
[Out] 1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p/b/(1+2*p)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1366, 623}

$$\frac{(a+bx^3)\,(a^2+2abx^3+b^2x^6)^p}{3b(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] ((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p)/(3*b*(1 + 2*p))
```

Rule 623

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)*
((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] /; FreeQ[{a, b, c, p}, x] && EqQ[
b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \frac{1}{3}\text{Subst}\left(\int\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^3\right)\\ &= \frac{(a+bx^3)\,(a^2+2abx^3+b^2x^6)^p}{3b(1+2p)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 0.78

$$\frac{(a+bx^3)\left((a+bx^3)^2\right)^p}{3b(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
[Out] ((a + b*x^3)*((a + b*x^3)^2)^p)/(3*b*(1 + 2*p))
```

Maple [A]
time = 0.02, size = 31, normalized size = 0.76

method	result	size
risch	$\frac{\left(b\,x^{3}+a\right)\left(\left(b\,x^{3}+a\right)^{2}\right)^{p}}{3b(1+2p)}$	31
gosper	$\frac{\left(b\,x^{3}+a\right)\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)^{p}}{3b(1+2p)}$	40
norman	$\frac{x^{3}\mathrm{e}^{p\ln\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)}}{6p+3}+\frac{a\,\mathrm{e}^{p\ln\left(b^{2}x^{6}+2ab\,x^{3}+a^{2}\right)}}{3b(1+2p)}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x,method=_RETURNVERBOSE)
[Out] 1/3*(b*x^3+a)/b/(1+2*p)*((b*x^3+a)^2)^p
```

Maxima [A]
time = 0.28, size = 30, normalized size = 0.73

$$\frac{(bx^3+a)(bx^3+a)^{2p}}{3\,b(2\,p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
[Out] 1/3*(b*x^3 + a)*(b*x^3 + a)^(2*p)/(b*(2*p + 1))
```

Fricas [A]
time = 0.45, size = 37, normalized size = 0.90

$$\frac{(bx^3+a)(b^2x^6+2\,abx^3+a^2)^p}{3\,(2\,bp+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] 1/3*(b*x^3 + a)*(b^2*x^6 + 2*a*b*x^3 + a^2)^p/(2*b*p + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^3}{3\sqrt{a^2}} & \text{for } b = 0 \wedge p = -\frac{1}{2} \\ \frac{x^3(a^2)^p}{3} & \text{for } b = 0 \\ \int \frac{x^2}{\sqrt{(a+bx^3)^2}}\,dx & \text{for } p = -\frac{1}{2} \\ \frac{a(a^2+2abx^3+b^2x^6)^p}{6bp+3b} + \frac{bx^3(a^2+2abx^3+b^2x^6)^p}{6bp+3b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Piecewise((x**3/(3*sqrt(a**2)), Eq(b, 0) & Eq(p, -1/2)), (x**3*(a**2)**p/3,
 Eq(b, 0)), (Integral(x**2/sqrt((a + b*x**3)**2), x), Eq(p, -1/2)), (a*(a**
2 + 2*a*b*x**3 + b**2*x**6)**p/(6*b*p + 3*b) + b*x**3*(a**2 + 2*a*b*x**3 +
b**2*x**6)**p/(6*b*p + 3*b), True))
```

Giac [A]
time = 3.59, size = 58, normalized size = 1.41

$$\frac{(b^2x^6 + 2\,abx^3 + a^2)^p bx^3 + (b^2x^6 + 2\,abx^3 + a^2)^p a}{3\,(2\,bp + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/3*((b^2*x^6 + 2*a*b*x^3 + a^2)^p*b*x^3 + (b^2*x^6 + 2*a*b*x^3 + a^2)^p*a)
/(2*b*p + b)
```

Mupad [B]
time = 1.16, size = 46, normalized size = 1.12

$$\left(\frac{x^3}{3\,(2\,p+1)} + \frac{a}{3\,b\,(2\,p+1)}\right)\,(a^2 + 2\,a\,b\,x^3 + b^2\,x^6)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] (x^3/(3*(2*p + 1)) + a/(3*b*(2*p + 1)))*(a^2 + b^2*x^6 + 2*a*b*x^3)^p
```

3.131 $\int x(a^2+2abx^3+b^2x^6)^p \, dx$

Optimal. Leaf size=58

$$\frac{x^2(a+bx^3)\,(a^2+2abx^3+b^2x^6)^p\ {}_2F_1\left(1,\frac{5}{3}+2p;\frac{5}{3};-\frac{bx^3}{a}\right)}{2a}$$

```
[Out] 1/2*x^2*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 5/3+2*p],[5/3],-b
*x^3/a)/a
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1370, 371}

$$\frac{1}{2}x^2\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\ {}_2F_1\left(\frac{2}{3},-2p;\frac{5}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^2*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*x
^3)/a)])/(2*(1 + (b*x^3)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int x\left(a^2+2abx^3+b^2x^6\right)^p\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int x\left(1+\frac{bx^3}{a}\right)^{2p}dx\\ &= \frac{1}{2}x^2\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\ {}_2F_1\left(\frac{2}{3},-2p;\frac{5}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.88

$$\frac{1}{2}x^2\left(\left(a+bx^3\right)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}{}_2F_1\left(\frac{2}{3},-2p;\frac{5}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x^2*((a + b*x^3)^2)^p*Hypergeometric2F1[2/3, -2*p, 5/3, -((b*x^3)/a)])/(2*
(1 + (b*x^3)/a)^(2*p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x\left(b^2x^6+2ab\,x^3+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a+bx^3\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b**2*x**6+2*a*b*x**3+a**2)**p,x)
```

```
[Out] Integral(x*((a + b*x**3)**2)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\,{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
```

```
[Out] int(x*(a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.132 $\int (a^2 + 2abx^3 + b^2x^6)^p \, dx$

Optimal. Leaf size=53

$$\frac{x(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(1,\frac{4}{3}+2p;\frac{4}{3};-\frac{bx^3}{a}\right)}{a}$$

```
[Out] x*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 4/3+2*p],[4/3],-b*x^3/a
)/a
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1357, 252, 251}

$$x\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(\frac{1}{3},-2p;\frac{4}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (x*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1/3, -2*p, 4/3, -((b*x^3
)/a)])/(1 + (b*x^3)/a)^(2*p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+2abx^3+b^2x^6\right)^p\,dx = \left(\left(2ab+2b^2x^3\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \left(2ab+2b^2x^3\right)^{2p}\,dx$$
$$= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\right)\int \left(1+\frac{bx^3}{a}\right)^{2p}\,dx$$
$$= x\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(\frac{1}{3},-2p;\frac{4}{3};-\frac{bx^3}{a}\right)$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.12, size = 204, normalized size = 3.85

$$\frac{4^{-p}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\,x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-2p}\left(\frac{i\left(1+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a}}\right)}{3i+\sqrt{3}}\right)^{-2p}\left(\left(a+bx^3\right)^2\right)^p F_1\left(1+2p;-2p,-2p;2(1+p);-\frac{i\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{3}\sqrt[3]{a}},\frac{i+\sqrt{3}-\frac{2i\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{3i+\sqrt{3}}\right)}{\sqrt[3]{b}\,(1+2p)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*((a + b*x^3)^2)^p*AppellF1[1 + 2*p, -2*p,
 -2*p, 2*(1 + p), ((-I)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(Sqrt[3]*a^(1/3))
, (I + Sqrt[3] - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(4^p*b^(1/3)*
(1 + 2*p)*((a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^(2*
p)*((I*(1 + (b^(1/3)*x)/a^(1/3)))/(3*I + Sqrt[3]))^(2*p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(b^2x^6+2ab\,x^3+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} + 2abx^{3} + b^{2}x^{6}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p,x)
[Out] Integral((a**2 + 2*a*b*x**3 + b**2*x**6)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p, x)
```

3.133 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x} \, dx$

Optimal. Leaf size=63

$$-\frac{(a+bx^3)(a^2+2abx^3+b^2x^6)^p \ {}_2F_1\left(1,1+2p;2(1+p);1+\frac{bx^3}{a}\right)}{3a(1+2p)}$$

```
[Out] -1/3*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([1, 1+2*p],[2+2*p],1+b*x
^3/a)/a/(1+2*p)
```

Rubi [A]
time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1370, 272, 67}

$$-\frac{(a+bx^3)(a^2+2abx^3+b^2x^6)^p \ {}_2F_1\left(1,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
```

```
[Out] -1/3*((a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[1, 1 + 2*
p, 2*(1 + p), 1 + (b*x^3)/a])/(a*(1 + 2*p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\begin{aligned}\int \frac{(a^2+2abx^3+b^2x^6)^p}{x}\,dx &= \left(\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x}\,dx\\ &= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x}\,dx,x,x^3\right)\\ &= -\frac{(a+bx^3)\,(a^2+2abx^3+b^2x^6)^p\,{}_2F_1\left(1,1+2p;2(1+p);1+\frac{bx^3}{a}\right)}{3a(1+2p)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.86

$$-\frac{(a+bx^3)\left((a+bx^3)^2\right)^p\,{}_2F_1\left(1,1+2p;2+2p;1+\frac{bx^3}{a}\right)}{3a(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x,x]
[Out] -1/3*((a + b*x^3)*((a + b*x^3)^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 + 2*p,
1 + (b*x^3)/a])/(a*(1 + 2*p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^{3}\right)^{2}\right)^{p}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x,x)
[Out] Integral(((a + b*x**3)**2)**p/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^p}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x, x)
```

3.134 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2} dx$

Optimal. Leaf size=58

$$-\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\ {}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

```
[Out] -(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-1/3, -2*p],[2/3],-b*x^3/a)/x/((1+b*x
^3/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1370, 371}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\ {}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
```

```
[Out] -(((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x^
3)/a)])/(x*(1 + (b*x^3)/a)^(2*p)))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^2}\,dx = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^2}\,dx$$
$$= -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\ {}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.84

$$-\frac{\left((a+bx^3)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\ {}_2F_1\left(-\frac{1}{3},-2p;\frac{2}{3};-\frac{bx^3}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^2,x]
[Out] -((((a + b*x^3)^2)^p*Hypergeometric2F1[-1/3, -2*p, 2/3, -((b*x^3)/a)])/(x*(
1 + (b*x^3)/a)^(2*p)))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + bx^3\right)^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**2,x)
[Out] Integral(((a + b*x**3)**2)**p/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^2,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^2,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^2, x)
```

3.135 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3} dx$

Optimal. Leaf size=60

$$-\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

```
[Out] -1/2*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-2/3, -2*p],[1/3],-b*x^3/a)/x^2/(
(1+b*x^3/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1370, 371}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}\left(a^2+2abx^3+b^2x^6\right)^p {}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
```

```
[Out] -1/2*((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((b
*x^3)/a)])/(x^2*(1 + (b*x^3)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^3}\,dx = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^3}\,dx$$
$$= -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\,{}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.85

$$-\frac{\left((a+bx^3)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}\,{}_2F_1\left(-\frac{2}{3},-2p;\frac{1}{3};-\frac{bx^3}{a}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^3,x]
[Out] -1/2*(((a + b*x^3)^2)^p*Hypergeometric2F1[-2/3, -2*p, 1/3, -((b*x^3)/a)])/(
x^2*(1 + (b*x^3)/a)^(2*p))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^3)^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**3,x)
```

```
[Out] Integral(((a + b*x**3)**2)**p/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^3,x)
```

```
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^3, x)
```

3.136 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx$

Optimal. Leaf size=64

$$\frac{b(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(2,1+2p;2(1+p);1+\frac{bx^3}{a}\right)}{3a^2(1+2p)}$$

[Out] 1/3*b*(b*x^3+a)*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([2, 1+2*p],[2+2*p],1+b*x^3/a)/a^2/(1+2*p)

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1370, 272, 67}

$$\frac{b(a+bx^3)\left(a^2+2abx^3+b^2x^6\right)^p\,{}_2F_1\left(2,2p+1;2(p+1);\frac{bx^3}{a}+1\right)}{3a^2(2p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]

[Out] (b*(a + b*x^3)*(a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[2, 1 + 2*p, 2*(1 + p), 1 + (b*x^3)/a])/(3*a^2*(1 + 2*p))

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^4}\,dx = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^4}\,dx$$
$$= \frac{1}{3}\left(\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x^2}\,dx, x, x^3\right)$$
$$= \frac{b(a+bx^3)(a^2+2abx^3+b^2x^6)^p\,{}_2F_1\left(2,1+2p;2(1+p);1+\frac{bx^3}{a}\right)}{3a^2(1+2p)}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.86

$$\frac{b(a+bx^3)\left((a+bx^3)^2\right)^p\,{}_2F_1\left(2,1+2p;2+2p;1+\frac{bx^3}{a}\right)}{3a^2(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^4,x]
```

```
[Out] (b*(a + b*x^3)*((a + b*x^3)^2)^p*Hypergeometric2F1[2, 1 + 2*p, 2 + 2*p, 1 +
 (b*x^3)/a])/(3*a^2*(1 + 2*p))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

```
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^3)^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**4,x)
[Out] Integral(((a + b*x**3)**2)**p/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^4,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^4,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^4, x)
```

3.137 $\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\,{}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

```
[Out] -1/4*(b^2*x^6+2*a*b*x^3+a^2)^p*hypergeom([-4/3, -2*p],[-1/3],-b*x^3/a)/x^4/
((1+b*x^3/a)^(2*p))
```

Rubi [A]
time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1370, 371}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\,{}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
```

```
[Out] -1/4*((a^2 + 2*a*b*x^3 + b^2*x^6)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -((
b*x^3)/a)])/(x^4*(1 + (b*x^3)/a)^(2*p))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(a^2+2abx^3+b^2x^6)^p}{x^5}\,dx = \left(\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{2p}}{x^5}\,dx$$
$$= -\frac{\left(1+\frac{bx^3}{a}\right)^{-2p}(a^2+2abx^3+b^2x^6)^p\,{}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

Mathematica [A]
time = 0.06, size = 51, normalized size = 0.85

$$-\frac{\left((a+bx^3)^2\right)^p\left(1+\frac{bx^3}{a}\right)^{-2p}{}_2F_1\left(-\frac{4}{3},-2p;-\frac{1}{3};-\frac{bx^3}{a}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^3 + b^2*x^6)^p/x^5,x]
[Out] -1/4*(((a + b*x^3)^2)^p*Hypergeometric2F1[-4/3, -2*p, -1/3, -((b*x^3)/a)])/
(x^4*(1 + (b*x^3)/a)^(2*p))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(b^2x^6+2ab\,x^3+a^2)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
[Out] int((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="maxima")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="fricas")
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^3)^2\right)^p}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**6+2*a*b*x**3+a**2)**p/x**5,x)
[Out] Integral(((a + b*x**3)**2)**p/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^6+2*a*b*x^3+a^2)^p/x^5,x, algorithm="giac")
[Out] integrate((b^2*x^6 + 2*a*b*x^3 + a^2)^p/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a^2+2\,a\,b\,x^3+b^2\,x^6\right)}^p}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^5,x)
[Out] int((a^2 + b^2*x^6 + 2*a*b*x^3)^p/x^5, x)
```

3.138 $\int \frac{x^8}{a+bx^3+cx^6} dx$

Optimal. Leaf size=81

$$\frac{x^3}{3c} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} - \frac{b\log(a+bx^3+cx^6)}{6c^2}$$

```
[Out] 1/3*x^3/c-1/6*b*ln(c*x^6+b*x^3+a)/c^2-1/3*(-2*a*c+b^2)*arctanh((2*c*x^3+b)/
(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1371, 717, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}} - \frac{b\log(a+bx^3+cx^6)}{6c^2} + \frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^3 + c*x^6),x]
```

```
[Out] x^3/(3*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^3\right)\\
&= \frac{x^3}{3c}+\frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^3\right)}{3c}\\
&= \frac{x^3}{3c}-\frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2}+\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6c^2}\\
&= \frac{x^3}{3c}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3c^2}\\
&= \frac{x^3}{3c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^3+cx^6\right)}{6c^2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 0.96

$$\frac{2cx^3+\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+bx^3+cx^6\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*x^3 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^3 + c*x^6])/(6*c^2)
```

Maple [A]
time = 0.05, size = 83, normalized size = 1.02

method	result
default	$\frac{x^3}{3c}+\frac{-\frac{b\ln\left(c\,x^6+b\,x^3+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{3c}$
risch	$\frac{x^3}{3c}-\frac{2\ln\left(\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-\left(4ac-b^2\right)\left(2ac-b^2\right)^2}\,b\right)x^3+2\sqrt{-\left(4ac-b^2\right)\left(2ac-b^2\right)^2}\,a\right)ab\right.}{3c\left(4ac-b^2\right)}+-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3/c+1/3/c*(-1/2*b/c*ln(c*x^6+b*x^3+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 254, normalized size = 3.14

$$\left[\frac{2\,(b^2c-4\,ac^2)x^3-(b^2-2\,ac)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-(b^3-4\,abc)\log\left(cx^6+bx^3+a\right)}{6\,(b^2c^2-4\,ac^3)},\ \frac{2\,(b^2c-4\,ac^2)x^3-2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)\log\left(cx^6+bx^3+a\right)}{6\,(b^2c^2-4\,ac^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c - 4*a*c^2)*x^3 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b
```

```
*x^3 + a)) - (b^3 - 4*a*b*c)*log(c*x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1
/6*(2*(b^2*c - 4*a*c^2)*x^3 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^6 +
 b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(75) = 150$.
time = 1.68, size = 316, normalized size = 3.90

$$\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)\log\left(x^3+\frac{-ab-12ac^2\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)+3b^2c\left(-\frac{b}{6c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)\log\left(x^3+\frac{-ab-12ac^2\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)+3b^2c\left(-\frac{b}{6c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{6c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right)+\frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))*
log(x**3 + (-a*b - 12*a*c**2*(-b/(6*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(6*c**2*(4*a*c - b**2))) + 3*b**2*c*(-b/(6*c**2) - sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(6*c**2) + s
qrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a*c - b**2)))*log(x**3 + (-a*b
 - 12*a*c**2*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(6*c**2*(4*a
*c - b**2))) + 3*b**2*c*(-b/(6*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
6*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**3/(3*c)
```

Giac [A]
time = 3.39, size = 75, normalized size = 0.93

$$\frac{x^3}{3\,c}-\frac{b\log\left(cx^6+bx^3+a\right)}{6\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3/c - 1/6*b*log(c*x^6 + b*x^3 + a)/c^2 + 1/3*(b^2 - 2*a*c)*arctan((2*
c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B]
time = 1.98, size = 1758, normalized size = 21.70

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3 + c*x^6),x)
```

[Out] x^3/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3 - 12*a*b*c))/(2*(36*a*c^3 - 9*b^2*c^2)) + (atan((4*c^3*x^3*(4*a*c - b^2)^(3/2)*((b*((b^5 + a^2*b*c^2 - 2*a*b^3*c)/c^3 + ((3*b^3 - 12*a*b*c)*((6*a^2*c^4 + 12*b^4*c^2 - 18*a*b^2*c^3)/c^3 + ((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 - 9*b^2*c^2)) - ((((2*a*c - b^2)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(6*c^2*(4*a*c - b^2)^(1/2)) + (9*b^2*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (3*b^2*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(4*c*(4*a*c - b^2)*(36*a*c^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(6*c^2*(4*a*c - b^2)^(1/2)) + (9*b^2*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 - 9*b^2*c^2)) - (b^2*(2*a*c - b^2)^3)/(4*c^3*(4*a*c - b^2)^(3/2)) + (((6*a^2*c^4 + 12*b^4*c^2 - 18*a*b^2*c^3)/c^3 + ((3*b^3 - 12*a*b*c)*((45*b^3*c^4 - 36*a*b*c^5)/c^3 + (27*b^2*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))))/(b^6 - 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c) - (c^2*(2*a*c - b^2)*(4*a*c - b^2)*(((3*b^3 - 12*a*b*c)*((((36*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (9*a*b*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((15*a*b^3*c^2 - 12*a^2*b*c^3)/c^3 - ((3*b^3 - 12*a*b*c)*((36*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) + (a*b*(2*a*c - b^2)^3)/(2*c^3*(4*a*c - b^2)^(3/2))))/(a^2*(b^6 - 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c)) + (b*c^2*(4*a*c - b^2)^(3/2)*((a*b^4 - a^2*b^2*c)/c^3 + ((3*b^3 - 12*a*b*c)*((15*a*b^3*c^2 - 12*a^2*b*c^3)/c^3 - ((3*b^3 - 12*a*b*c)*((36*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2)))/(2*(36*a*c^3 - 9*b^2*c^2))))/(2*(36*a*c^3 - 9*b^2*c^2)) + (((((36*a^2*c^5 - 72*a*b^2*c^4)/c^3 - (54*a*b*c^3*(3*b^3 - 12*a*b*c))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (9*a*b*c*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c - b^2))/(6*c^2*(4*a*c - b^2)^(1/2)) - (3*a*b*(3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2)/(2*c*(4*a*c - b^2)*(36*a*c^3 - 9*b^2*c^2))))/(a^2*(b^6 - 8*a^3*c^3 + 12*a^2*b^2*c^2 - 6*a*b^4*c)))*(2*a*c - b^2))/(3*c^2*(4*a*c - b^2)^(1/2))

3.139 $\int \frac{x^5}{a+bx^3+cx^6} \, dx$

Optimal. Leaf size=63

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

```
[Out] 1/6*ln(c*x^6+b*x^3+a)/c+1/3*b*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c/(-4
*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1371, 648, 632, 212, 642}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^3+cx^6\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3 + c*x^6),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^3 + c*x^6]/(6*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6c}\\
&= \frac{\log\left(a+bx^3+cx^6\right)}{6c} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3c}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^3+cx^6\right)}{6c}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.98

$$
\frac{-\frac{2b\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log\left(a+bx^3+cx^6\right)}{6c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^3 + c*x^6])/(6*c)
```

Maple [A]
time = 0.04, size = 60, normalized size = 0.95

method	result

default	$\frac{\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{6c}-\frac{b\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{3c\sqrt{4ac-b^{2}}}$
risch	$\frac{2\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,b\right)x^{3}+2\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,a\right)a}{3(4ac-b^{2})}-\frac{\ln\left(\left(-4abc+b^{3}+\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,b\right)x^{3}\cdots\right.}{6c(4ac-b^{2})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln(c*x^6+b*x^3+a)/c-1/3*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c
-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^{2}-4\,ac}\,b\log\left(\frac{2\,c^{2}x^{6}+2\,bcx^{3}+b^{2}-2\,ac+\left(2\,cx^{3}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{6}+bx^{3}+a}\right)+\left(b^{2}-4\,ac\right)\log\left(cx^{6}+bx^{3}+a\right)}{6\,\left(b^{2}c-4\,ac^{2}\right)},\ \frac{2\,\sqrt{-b^{2}+4\,ac}\,b\arctan\left(-\frac{\left(2\,cx^{3}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)+\left(b^{2}-4\,ac\right)\log\left(cx^{6}+bx^{3}+a\right)}{6\,\left(b^{2}c-4\,ac^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x
^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) + (b^2 - 4*a*c)*log(c*x^6 +
 b*x^3 + a))/(b^2*c - 4*a*c^2), 1/6*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^6 + b*x^3
+ a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.
time = 0.82, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^{2}}}{6c\left(4ac-b^{2}\right)}+\frac{1}{6c}\right)\log\left(x^{3}+\frac{-12ac\left(-\frac{b\sqrt{-4ac+b^{2}}}{6c(4ac-b^{2})}+\frac{1}{6c}\right)+2a+3b^{2}\left(-\frac{b\sqrt{-4ac+b^{2}}}{6c(4ac-b^{2})}+\frac{1}{6c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^{2}}}{6c\left(4ac-b^{2}\right)}+\frac{1}{6c}\right)\log\left(x^{3}+\frac{-12ac\left(\frac{b\sqrt{-4ac+b^{2}}}{6c(4ac-b^{2})}+\frac{1}{6c}\right)+2a+3b^{2}\left(\frac{b\sqrt{-4ac+b^{2}}}{6c(4ac-b^{2})}+\frac{1}{6c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**6+b*x**3+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c
*(-b*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(-b
*sqrt(-4*a*c + b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b) + (b*sqrt(-4*a*c +
 b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c))*log(x**3 + (-12*a*c*(b*sqrt(-4*a*c +
 b**2)/(6*c*(4*a*c - b**2)) + 1/(6*c)) + 2*a + 3*b**2*(b*sqrt(-4*a*c + b**2
)/(6*c*(4*a*c - b**2)) + 1/(6*c)))/b)
```

Giac [A]
time = 5.28, size = 59, normalized size = 0.94

$$-\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,c}+\frac{\log\left(cx^6+bx^3+a\right)}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
6*log(c*x^6 + b*x^3 + a)/c
```

Mupad [B]
time = 1.80, size = 1199, normalized size = 19.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3 + c*x^6),x)
```

```
[Out] (log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) + (b*ata
n((4*x^3*((b*(b^2 - ((12*b^2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2
))/(36*a*c^2 - 9*b^2*c))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)))*(12*a*
c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(45*b^2*c^2 - (27*b^2*c^3*(12
*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b^3*c^
2*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(6*c*(4*
a*c - b^2)^(1/2)) + (3*b^4*c*(12*a*c - 3*b^2))/(4*(36*a*c^2 - 9*b^2*c)*(4*a
*c - b^2))))/(4*a^2*c) + ((2*a*c - b^2)*(b^5/(4*(4*a*c - b^2)^(3/2)) + ((12
*a*c - 3*b^2)*((b*(45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9
*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*b^3*c^2*(12*a*c - 3*b^2))/(2*(36*a
*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) - (b*(12*b^
```

```
2*c - ((45*b^2*c^2 - (27*b^2*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))*(1
2*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c))))/(6*c*(4*a*c - b^2)^(1/2))))/(4*a
^2*c*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^(3/2))/b^3 + ((4*a*c - b^2)^(3/2)*
(a*b + ((((72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))
*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - 15*a*b*c)*(12*a*c - 3*b^2))/(
2*(36*a*c^2 - 9*b^2*c)) - (b*((b*(72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2)
)/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) - (9*a*b^2*c^2*(12*a*c -
 3*b^2))/((36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2))))/(6*c*(4*a*c - b^2)^(1
/2)) + (3*a*b^3*c*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)))
)/(a^2*b^2*c) + ((2*a*c - b^2)*(4*a*c - b^2)*((a*b^4)/(2*(4*a*c - b^2)^(3/2
)) + (((b*(72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))
)/(6*c*(4*a*c - b^2)^(1/2)) - (9*a*b^2*c^2*(12*a*c - 3*b^2))/((36*a*c^2 - 9
*b^2*c)*(4*a*c - b^2)^(1/2)))*(12*a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) +
(b*(((72*a*b*c^2 - (54*a*b*c^3*(12*a*c - 3*b^2))/(36*a*c^2 - 9*b^2*c))*(12*
a*c - 3*b^2))/(2*(36*a*c^2 - 9*b^2*c)) - 15*a*b*c))/(6*c*(4*a*c - b^2)^(1/2
))))/(a^2*b^3*c)))/(3*c*(4*a*c - b^2)^(1/2))
```

3.140 $\int \frac{x^2}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=38

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

```
[Out] -2/3*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1366, 632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3 + c*x^6),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*Sqrt[b^2 - 4*a*c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^2}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)$$
$$= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.11

$$\frac{2\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{3\sqrt{-b^2+4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3 + c*x^6),x]
[Out] (2*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/(3*Sqrt[-b^2 + 4*a*c])
```

Maple [A]
time = 0.02, size = 37, normalized size = 0.97

method	result	size
default	$\frac{2\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}$	37
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^2}\right)x^3-2a\right)}{3\sqrt{-4ac+b^2}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^2}\right)x^3+2a\right)}{3\sqrt{-4ac+b^2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 129, normalized size = 3.39

$$\left[\frac{\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac-(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{3\,\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{3\,(b^2-4\,ac)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*
a*c))/(c*x^6 + b*x^3 + a))/sqrt(b^2 - 4*a*c), -2/3*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(37) = 74.
time = 0.35, size = 131, normalized size = 3.45

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^3+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**3 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3 + sqrt(-1/(4*a*c - b**2))*log(x**3 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/3
```

Giac [A]
time = 5.21, size = 36, normalized size = 0.95

$$\frac{2\,\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

[Out] 2/3*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)

Mupad [B]
time = 1.23, size = 174, normalized size = 4.58

$$-\frac{2\,\mathrm{atan}\left(\frac{\frac{x^3\,{\left(4\,a\,c-b^2\right)}^4}{2}+a\,b\,{\left(4\,a\,c-b^2\right)}^3+a\,b^3\,{\left(4\,a\,c-b^2\right)}^2+b^2\,x^3\,{\left(4\,a\,c-b^2\right)}^3+\frac{b^4\,x^3\,{\left(4\,a\,c-b^2\right)}^2}{2}}{b^2\,\left(32\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}-4\,a^2\,b^2\,c\,\sqrt{4\,a\,c-b^2}\right)-64\,a^4\,c^3\,\sqrt{4\,a\,c-b^2}}\right)}{3\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*x^3 + c*x^6),x)

[Out] -(2*atan(((x^3*(4*a*c - b^2)^4)/2 + a*b*(4*a*c - b^2)^3 + a*b^3*(4*a*c - b^2)^2 + b^2*x^3*(4*a*c - b^2)^3 + (b^4*x^3*(4*a*c - b^2)^2)/2)/(b^2*(32*a^3*c^2*(4*a*c - b^2)^(1/2) - 4*a^2*b^2*c*(4*a*c - b^2)^(1/2)) - 64*a^4*c^3*(4*a*c - b^2)^(1/2))))/(3*(4*a*c - b^2)^(1/2))

3.141 $\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}$$

```
[Out] ln(x)/a-1/6*ln(c*x^6+b*x^3+a)/a+1/3*b*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2
))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1371, 719, 29, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^3 + c*x^6]/(6*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx,x,x^3\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)}{3a}+\frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,x^3\right)}{3a}\\
&= \frac{\log(x)}{a}-\frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}+\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^3\right)}{3a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^3+cx^6\right)}{6a}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.02, size = 66, normalized size = 0.96

$$\frac{\log(x)}{a}-\frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] Log[x]/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]*#
1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

Maple [A]
time = 0.03, size = 65, normalized size = 0.94

method	result	size
default	$-\frac{\frac{\ln\left(c\,x^6+b\,x^3+a\right)}{2}+\frac{b\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{3a}+\frac{\ln(x)}{a}$	65
risch	$\frac{\ln(x)}{a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^2c-a\,b^2\right)\_Z^2+\left(4ac-b^2\right)\_Z+c\right)}\_R\ln\left(\left(\left(-14ac+4b^2\right)\_R-7c\right)x^3+ab\_R-3b\right)\right)}{3}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a*(1/2*ln(c*x^6+b*x^3+a)+b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c
-b^2)^(1/2)))+ln(x)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.46, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-(b^2-4\,ac)\log\left(cx^6+bx^3+a\right)+6\,(b^2-4\,ac)\log(x)}{6\,(ab^2-4\,a^2c)},\frac{2\,\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^2-4\,ac)\log\left(cx^6+bx^3+a\right)+6\,(b^2-4\,ac)\log(x)}{6\,(ab^2-4\,a^2c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] [1/6*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^2 - 4*a*c)*log(c*x^6 + b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/6*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log(c*x^6 + b*x^3 + a) + 6*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.
time = 16.28, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{6a\,(4ac-b^2)}-\frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)+3ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{6a\,(4ac-b^2)}-\frac{1}{6a}\right)\log\left(x^3+\frac{-12a^2c\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)+3ab^2\left(\frac{b\sqrt{-4ac+b^2}}{6a(4ac-b^2)}-\frac{1}{6a}\right)-2ac+b^2}{bc}\right)+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x**6+b*x**3+a),x)

[Out] (-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-12*a**2*c*(-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(-b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c)) + (b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a))*log(x**3 + (-12*a**2*c*(b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) + 3*a*b**2*(b*sqrt(-4*a*c + b**2)/(6*a*(4*a*c - b**2)) - 1/(6*a)) - 2*a*c + b**2)/(b*c)) + log(x)/a

Giac [A]
time = 4.17, size = 66, normalized size = 0.96

$$-\frac{b\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,a}-\frac{\log\left(cx^6+bx^3+a\right)}{6\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] -1/3*b*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/6*log(c*x^6 + b*x^3 + a)/a + log(abs(x))/a

Mupad [B]
time = 1.92, size = 1362, normalized size = 19.74

$$\frac{\ln(x)}{a}+\frac{\ln\left(c\,x^6+b\,x^3+a\right)\,\left(12\,a\,c-3\,b^2\right)}{2\,\left(9\,a\,b^2-36\,a^2\,c\right)}-\frac{b\,\mathrm{atan}\left(\text{[illegible]}\right)}{\text{[illegible]}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log(x)/a + (log(a + b*x^3 + c*x^6)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c
)) - (b*atan((3*(4*a*c - b^2)^2*(4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(27*
b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(216*a
^3*(4*a*c - b^2)^(3/2)) + (9*b^4*c^3*(12*a*c - 3*b^2)^3)/(16*(9*a*b^2 - 36*
a^2*c)^3*(4*a*c - b^2)^(1/2)) - (3*b^6*c^3*(12*a*c - 3*b^2))/(16*a^2*(9*a*b
^2 - 36*a^2*c)*(4*a*c - b^2)^(3/2)) - (b*(12*a*c - 3*b^2)^2*(27*b^3*c^3 - (
27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(8*a*(9*a*b^2 - 3
6*a^2*c)^2*(4*a*c - b^2)^(1/2))))/(b^3*c^6*(49*a*c - 12*b^2)) - (3*(4*a*c -
 b^2)^(3/2)*(4*b^5 + 29*a^2*b*c^2 - 23*a*b^3*c)*(((12*a*c - 3*b^2)^3*(27*b^
3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(8*(9*a*
b^2 - 36*a^2*c)^3) - (b^7*c^3)/(48*a^3*(4*a*c - b^2)^2) - (b^2*(12*a*c - 3*
b^2)*(27*b^3*c^3 - (27*a*b^3*c^3*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))
))/(24*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)) + (9*b^5*c^3*(12*a*c - 3*b^2
)^2)/(16*a*(9*a*b^2 - 36*a^2*c)^2*(4*a*c - b^2))))/(b^3*c^6*(49*a*c - 12*b^
2)) + (48*a^4*x^3*(((4*b^4 + 7*a^2*c^2 - 15*a*b^2*c)*((b^3*(63*b^2*c^4 - ((
108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c))))/(
216*a^3*(4*a*c - b^2)^(3/2)) + (b*(108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3
*b^2)^3)/(48*a*(9*a*b^2 - 36*a^2*c)^3*(4*a*c - b^2)^(1/2)) - (b^3*(108*b^4*
c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2))/(144*a^3*(9*a*b^2 - 36*a^2*c)*(4*a*c
 - b^2)^(3/2)) - (b*(63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c -
3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))*(12*a*c - 3*b^2)^2)/(8*a*(9*a*b^2 - 36*a^
2*c)^2*(4*a*c - b^2)^(1/2))))/(16*a^4*c^3*(49*a*c - 12*b^2)) - ((4*b^5 + 29
*a^2*b*c^2 - 23*a*b^3*c)*(((63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b^2*c^4)*(12
*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))*(12*a*c - 3*b^2)^3)/(8*(9*a*b^2 -
36*a^2*c)^3) - (b^4*(108*b^4*c^3 - 378*a*b^2*c^4))/(1296*a^4*(4*a*c - b^2)^
2) + (b^2*(108*b^4*c^3 - 378*a*b^2*c^4)*(12*a*c - 3*b^2)^2)/(48*a^2*(9*a*b^
2 - 36*a^2*c)^2*(4*a*c - b^2)) - (b^2*(63*b^2*c^4 - ((108*b^4*c^3 - 378*a*b
^2*c^4)*(12*a*c - 3*b^2))/(2*(9*a*b^2 - 36*a^2*c)))*(12*a*c - 3*b^2))/(24*a
^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2))))/(16*a^4*c^3*(4*a*c - b^2)^(1/2)*(4
9*a*c - 12*b^2)))*(4*a*c - b^2)^2)/(b^3*c^3)))/(3*a*(4*a*c - b^2)^(1/2))
```

3.142 $\int \frac{1}{x^4(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{3ax^3}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}$$

```
[Out] -1/3/a/x^3-b*ln(x)/a^2+1/6*b*ln(c*x^6+b*x^3+a)/a^2-1/3*(-2*a*c+b^2)*arctanh
((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1371, 723, 814, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{b\log(x)}{a^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/3*1/(a*x^3) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(
3*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^3 + c*x^6])/(6*a
^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4 (a+bx^3+cx^6)} \, dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2 (a+bx+cx^2)} \, dx, x, x^3\right)$$
$$= -\frac{1}{3ax^3} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)} \, dx, x, x^3\right)}{3a}$$
$$= -\frac{1}{3ax^3} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right) dx, x, x^3\right)}{3a}$$
$$= -\frac{1}{3ax^3} - \frac{b\log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2} \, dx, x, x^3\right)}{3a^2}$$
$$= -\frac{1}{3ax^3} - \frac{b\log(x)}{a^2} + \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2} \, dx, x, x^3\right)}{6a^2} + \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2} \, d\right.}{6a^2}$$
$$= -\frac{1}{3ax^3} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^3+cx^6)}{6a^2} - \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2} \, dx, x, b\right.}{3a^2}$$
$$= -\frac{1}{3ax^3} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^3+cx^6)}{6a^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 92, normalized size = 1.03

$$-\frac{1}{3ax^3} - \frac{b\log(x)}{a^2} + \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&, \frac{b^2\log(x-\#1)-ac\log(x-\#1)+bc\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/3*1/(a*x^3) - (b*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6 & , (b^2*Log[
x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a^2
)
```

Maple [A]
time = 0.04, size = 85, normalized size = 0.96

method	result
default	$-\frac{-\frac{b\ln\left(c\,x^6+b\,x^3+a\right)}{2}+\frac{2\left(ac-\frac{b^2}{2}\right)\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{3a^2}-\frac{1}{3a\,x^3}-\frac{b\ln(x)}{a^2}$

risch	$-\frac{1}{3a\,x^3}-\frac{b\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c-a^2b^2\right)\_Z^2+\left(-4abc+b^3\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(-14a^3c+4a^2b^2\right)\_R^2+6\_Rabc-3c^2\right)x^3+\right.\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^2*(-1/2*b*ln(c*x^6+b*x^3+a)+2*(a*c-1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))-1/3/a/x^3-b*ln(x)/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.43, size = 293, normalized size = 3.29

$$\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\,x^3\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)-(b^3-4\,abc)x^3\log\left(cx^6+bx^3+a\right)+6\,(b^3-4\,abc)x^3\log\left(x\right)+2\,ab^2-8\,a^2c}{6\,(a^2b^2-4\,a^3c)x^3},\ -\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\,x^3\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)x^3\log\left(cx^6+bx^3+a\right)+6\,(b^3-4\,abc)x^3\log\left(x\right)+2\,ab^2-8\,a^2c}{6\,(a^2b^2-4\,a^3c)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b*c*x^3 + b^2
 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) - (b^3 - 4
*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3), -1/6*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^3*log(c*x^6 + b*x^3 + a) + 6*(b^3 - 4*a*b*c)*x^3*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.38, size = 93, normalized size = 1.04

$$\frac{b\log\left(cx^6+bx^3+a\right)}{6\,a^2}-\frac{b\log\left(|x|\right)}{a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,a^2}+\frac{bx^3-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*b*log(c*x^6 + b*x^3 + a)/a^2 - b*log(abs(x))/a^2 + 1/3*(b^2 - 2*a*c)*ar
ctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/3*(b*x^
3 - a)/(a^2*x^3)
```

Mupad [B]
time = 2.03, size = 2500, normalized size = 28.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (atan((48*a^8*x^3*((((4*b^5 + 9*a^2*b*c^2 - 16*a*b^3*c)*(((3*b^3 - 12*a*b*c)
*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4
 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*
c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c
- b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*(4*a*c - b^2)^(1/2)*(36
*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) + (((42*a^3*c^6 + 33*a^2*
b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 -
((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c -
 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b
^2)^(1/2))))/(2*(36*a^3*c - 9*a^2*b^2)) - ((((((2*a*c - b^2)*((252*a^4*b*c^
5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^
2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*
b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*
(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c -
 b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)^2*(108*a^4*b^4*c^3 - 378*a
^5*b^2*c^4))/(72*a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/
(6*a^2*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*((42*a^3*
c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^
3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^
4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a
```

^2*b^2)) + (12*b*c^6)/a^3))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((3*b^3 - 12*a*b*
c)*(2*a*c - b^2)^3*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(432*a^10*(4*a*c -
b^2)^(3/2)*(36*a^3*c - 9*a^2*b^2))))/(16*a^4*c^3*(a^2*c^2 - 12*b^4 + 48*a*b
^2*c)) + ((4*b^6 - 2*a^3*c^3 + 33*a^2*b^2*c^2 - 24*a*b^4*c)*(((3*b^3 - 12*a
*b*c)*(((((2*a*c - b^2)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 1
2*a*b*c)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2)
)))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^
4*b^4*c^3 - 378*a^5*b^2*c^4))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2
b^2)))(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - ((3*b^3 - 12*a*b*c)*(2
*a*c - b^2)^2*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(72*a^8*(4*a*c - b^2)*(3
6*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - c^7/a^4 - ((3*b^3 - 12
*a*b*c)*(((3*b^3 - 12*a*b*c)*((42*a^3*c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 -
 12*a*b*c)*((252*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108
*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^
3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) + (12*b*c^6)/a^3))/(2*(36*a^
3*c - 9*a^2*b^2)) + ((2*a*c - b^2)*(((3*b^3 - 12*a*b*c)*(((2*a*c - b^2)*((2
52*a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 -
 378*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)^(1
/2)) - ((3*b^3 - 12*a*b*c)*(2*a*c - b^2)*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4
))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a
^2*b^2)) + (((42*a^3*c^6 + 33*a^2*b^2*c^5)/a^4 + ((3*b^3 - 12*a*b*c)*((252*
a^4*b*c^5 - 18*a^3*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(108*a^4*b^4*c^3 - 37
8*a^5*b^2*c^4))/(2*a^4*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))
)*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2))))/(6*a^2*(4*a*c - b^2)^(1/2))
 + ((2*a*c - b^2)^4*(108*a^4*b^4*c^3 - 378*a^5*b^2*c^4))/(1296*a^12*(4*a*c -
 b^2)^2)))/(16*a^4*c^3*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 12*b^4 + 48*a*b^2*c))
)*(4*a*c - b^2)^2)/(8*a^3*c^6 - b^6*c^3 + 6*a*b^4*c^4 - 12*a^2*b^2*c^5) + (
3*a^4*(4*a*c - b^2)^2*(4*b^5 + 9*a^2*b*c^2 - 16*a*b^3*c)*(((3*b^3 - 12*a*b*
c)*(((3*b^3 - 12*a*b*c)*((((27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^
3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c - 9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2
*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^2))/(4*a*(
4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2)) - (
(2*a*c - b^2)*((9*a^3*b*c^5 - 27*a^2*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*((2
7*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*
(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(6*a^2*(4*a*c - b^2)
^(1/2))))/(2*(36*a^3*c - 9*a^2*b^2)) - (((a^2*c^6 - 9*a*b^2*c^5)/a^4 + ((3*
b^3 - 12*a*b*c)*((9*a^3*b*c^5 - 27*a^2*b^3*c^4)/a^4 - ((3*b^3 - 12*a*b*c)*(
(27*a^3*b^4*c^3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(
2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a^2*b^2))))/(2*(36*a^3*c - 9*a
^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (((((((27*a^3*b^4*c^
3 - 27*a^4*b^2*c^4)/a^4 - (27*a*b^3*c^3*(3*b^3 - 12*a*b*c))/(2*(36*a^3*c -
9*a^2*b^2)))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (9*b^3*c^3*(3*b^3
 - 12*a*b*c)*(2*a*c - b^2))/(4*a*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)
))*(2*a*c - b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) - (3*b^3*c^3*(3*b^3 - 12*a*b*
c)*(2*a*c - b^2)^2)/(8*a^3*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(2*a*c -

```
b^2))/(6*a^2*(4*a*c - b^2)^(1/2)) + (b^3*c^3*(3*b^3 - 12*a*b*c)*(2*a*c - b^
2)^3)/(16*a^5*(4*a*c - b^2)^(3/2)*(36*a^3*c - 9...
```

3.143 $\int \frac{x^7}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=636

$$\frac{x^2}{2c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x^2/c+1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b+(2*a*c-
b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/12*
ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-
4*a*c+b^2)^(1/2))^(2/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)
/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*
a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/
c^(5/3)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-
4*a*c+b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3
)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*
x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a*c+b^2
)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)+1/6*arct
an(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-2
*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)^(1/3)
```

Rubi [A]
time = 0.78, antiderivative size = 636, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1381, 1524, 298, 31, 648, 631, 210, 642}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] x^2/(2*c) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c
^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)
*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Ar
cTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2
^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((b - (b^2 - 2*a*c)
/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((b + (b^2 - 2*a*c)/Sq
```

```
rt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3
*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqr
t[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2
^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^7}{a+bx^3+cx^6}\,dx = \frac{x^2}{2c} - \frac{\int \frac{x(2a+2bx^3)}{a+bx^3+cx^6}\,dx}{2c}$$

$$= \frac{x^2}{2c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-\cdots}}}{2c}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\,2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{\cdots}{\left(b-\sqrt{\cdots}\right.}}{3\,2\cdots}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\cdots}{3\,2\cdots}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\cdots}{3\,2\cdots}$$

$$= \frac{x^2}{2c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\cdots}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{\cdots}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 70, normalized size = 0.11

$$\frac{3x^2-2\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^3 + c*x^6),x]
```

```
[Out] (3*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#
1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 61, normalized size = 0.10

method	result	size
default	$\frac{x^2}{2c}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)}\frac{\left(\_R^4 b+\_R a\right)\ln\left(x-\_R\right)}{2\_R^5 c+b\_R^2}}{3c}$	61
risch	$\frac{x^2}{2c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)}\frac{\left(-\_R^4 b-\_R a\right)\ln\left(x-\_R\right)}{2\_R^5 c+b\_R^2}}{3c}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c-1/3/c*sum((_R^4*b+_R*a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6
*c+_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^4 + a*x)/(c*x^6 + b*x^3 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6271 vs. 2(498) = 996.
time = 2.18, size = 6271, normalized size = 9.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/6*(4*sqrt(3)*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*
c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2
*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5
 - 4*a*c^6))^(1/3)*arctan(-1/6*(sqrt(2)*(1/2)^(1/3)*sqrt(2*(a^6*b^10 - 10*a
```

^7*b^8*c + 35*a^8*b^6*c^2 - 50*a^9*b^4*c^3 + 25*a^10*b^2*c^4)*x^2 + (1/2)^(
2/3)*((a^3*b^13*c^5 - 18*a^4*b^11*c^6 + 130*a^5*b^9*c^7 - 477*a^6*b^7*c^8 +
924*a^7*b^5*c^9 - 880*a^8*b^3*c^10 + 320*a^9*b*c^11)*x*sqrt((b^10 - 10*a*b
^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) - (a^3*b^15 - 17*a^4*b^13*c + 117
*a^5*b^11*c^2 - 415*a^6*b^9*c^3 + 795*a^7*b^7*c^4 - 775*a^8*b^5*c^5 + 300*a
^9*b^3*c^6)*x)*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10
- 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10
- 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2
/3) - (1/2)^(1/3)*(a^5*b^12 - 14*a^6*b^10*c + 75*a^7*b^8*c^2 - 190*a^8*b^6*
c^3 + 225*a^9*b^4*c^4 - 100*a^10*b^2*c^5 - (a^5*b^10*c^5 - 13*a^6*b^8*c^6 +
61*a^7*b^6*c^7 - 120*a^8*b^4*c^8 + 80*a^9*b^2*c^9)*sqrt((b^10 - 10*a*b^8*c
+ 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c
^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*c^2 + (b^2*c
^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 2
5*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))
/(b^2*c^5 - 4*a*c^6))^(1/3))*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^6 + 32*a^2*b^2*
c^7 - 32*a^3*c^8)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3
+ 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^1
3)) - sqrt(3)*(b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3))*((b^4 -
3*a*b^2*c + a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*
b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3) - 2*(1/2)^(1/3)*(sq
rt(3)*(a^3*b^11*c^5 - 15*a^4*b^9*c^6 + 87*a^5*b^7*c^7 - 242*a^6*b^5*c^8 + 3
20*a^7*b^3*c^9 - 160*a^8*b*c^10)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
- 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)) - sqrt(3)*(a^3*b^13 - 14*a^4*b^11*c + 75*a^5*b^9*c^2 -
190*a^6*b^7*c^3 + 225*a^7*b^5*c^4 - 100*a^8*b^3*c^5)*x)*((b^4 - 3*a*b^2*c
+ a^2*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 -
50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^1
2 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3) + 2*sqrt(3)*(a^5*b^10 - 10*a^
6*b^8*c + 35*a^7*b^6*c^2 - 50*a^8*b^4*c^3 + 25*a^9*b^2*c^4))/(a^5*b^10 - 10
*a^6*b^8*c + 35*a^7*b^6*c^2 - 50*a^8*b^4*c^3 + 25*a^9*b^2*c^4)) - 4*sqrt(3)
*(1/2)^(1/3)*c*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10
- 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10
- 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1
/3)*arctan(-1/6*(sqrt(2)*(1/2)^(1/3)*sqrt(2*(a^6*b^10 - 10*a^7*b^8*c + 35*a
^8*b^6*c^2 - 50*a^9*b^4*c^3 + 25*a^10*b^2*c^4)*x^2 - (1/2)^(2/3)*((a^3*b^13
*c^5 - 18*a^4*b^11*c^6 + 130*a^5*b^9*c^7 - 477*a^6*b^7*c^8 + 924*a^7*b^5*c^
9 - 880*a^8*b^3*c^10 + 320*a^9*b*c^11)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b
^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^
2*b^2*c^12 - 64*a^3*c^13)) + (a^3*b^15 - 17*a^4*b^13*c + 117*a^5*b^11*c^2 -
415*a^6*b^9*c^3 + 795*a^7*b^7*c^4 - 775*a^8*b^5*c^5 + 300*a^9*b^3*c^6)*x)*
((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c +
35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^1

```
1 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(2/3) - (1/2)^(1/
3)*(a^5*b^12 - 14*a^6*b^10*c + 75*a^7*b^8*c^2 - 190*a^8*b^6*c^3 + 225*a^9*b
^4*c^4 - 100*a^10*b^2*c^5 + (a^5*b^10*c^5 - 13*a^6*b^8*c^6 + 61*a^7*b^6*c^7
 - 120*a^8*b^4*c^8 + 80*a^9*b^2*c^9)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c
^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^
2*c^12 - 64*a^3*c^13)))*((b^4 - 3*a*b^2*c + a^2*c^2 - (b^2*c^5 - 4*a*c^6)*s
qrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/
(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^2*c^5 - 4*a
*c^6))^(1/3))*(sqrt(3)*(b^6*c^5 - 10*a*b^4*c^6 + 32*a^2*b^2*c^7 - 32*a^3*c^
8)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + sqrt(3)*(
b^8 - 9*a*b^6*c + 25*a^2*b^4*c^2 - 20*a^3*b^2*c^3))*((b^4 - 3*a*b^2*c + a^2
*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^
3*b^4*c^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 6
4*a^3*c^13)))/(b^2*c^5 - 4*a*c^6))^(1/3) - 2*(1/2)^(1/3)*(sqrt(3)*(a^3*b^11
*c^5 - 15*a^4*b^9*c^6 + 87*a^5*b^7*c^7 - 242*a^6*b^5*c^8 + 320*a^7*b^3*c^9
- 160*a^8*b*c^10)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c
^3 + 25*a^4*b^2*c^4)/(b^6*c^10 - 12*a*b^4*c^11 ...
```

Sympy [A]
time = 140.56, size = 279, normalized size = 0.44

$$\operatorname{RootSum}\left(t^{6}\cdot\left(46656a^{3}c^{8}-34992a^{2}b^{2}c^{7}+8748ab^{4}c^{6}-729b^{6}c^{5}\right)+t^{3}\cdot\left(432a^{4}c^{4}-1512a^{3}b^{2}c^{3}+1107a^{2}b^{4}c^{2}-297ab^{6}c+27b^{8}\right)+a^{5},\left(t\mapsto t\log\left(x+\frac{-15552t^{5}a^{4}c^{9}+27216t^{5}a^{3}b^{2}c^{8}-14580t^{5}a^{2}b^{4}c^{7}+3159t^{5}ab^{6}c^{6}-243t^{5}b^{8}c^{5}-72t^{2}a^{5}c^{5}+594t^{2}a^{4}b^{2}c^{4}-864t^{2}a^{3}b^{4}c^{3}+468t^{2}a^{2}b^{6}c^{2}-108t^{2}ab^{8}c+9t^{2}b^{10}}{5a^{5}bc^{2}-5a^{4}b^{3}c+a^{3}b^{5}}\right)\right)\right)+\frac{x^{2}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**8 - 34992*a**2*b**2*c**7 + 8748*a*b**4*c**6 -
729*b**6*c**5) + _t**3*(432*a**4*c**4 - 1512*a**3*b**2*c**3 + 1107*a**2*b**
4*c**2 - 297*a*b**6*c + 27*b**8) + a**5, Lambda(_t, _t*log(x + (-15552*_t**
5*a**4*c**9 + 27216*_t**5*a**3*b**2*c**8 - 14580*_t**5*a**2*b**4*c**7 + 315
9*_t**5*a*b**6*c**6 - 243*_t**5*b**8*c**5 - 72*_t**2*a**5*c**5 + 594*_t**2*
a**4*b**2*c**4 - 864*_t**2*a**3*b**4*c**3 + 468*_t**2*a**2*b**6*c**2 - 108*
_t**2*a*b**8*c + 9*_t**2*b**10)/(5*a**5*b*c**2 - 5*a**4*b**3*c + a**3*b**5)
))) + x**2/(2*c)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^7/(c*x^6 + b*x^3 + a), x)
```

Mupad [B]
time = 12.15, size = 2500, normalized size = 3.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1
/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1
/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))
^(2/3))/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*
c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) -
5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/6 - (9*a*
b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4
 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^
6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(
1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/18 + (a^4*x*(a*c - b^2))/c^2)*(-(b^8 +
16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c -
 b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))
)^(1/3) + log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c)
+ (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c -
b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*
(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c
- b^2)^3))^(2/3))/2)*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 4
1*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))
/6 - (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 +
16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c -
 b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/18 + (a^4*x*(a*c - b^2))/c^2)
*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a
^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*
(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2
*b^2*c^7)))^(1/3) + x^2/(2*c) - log((a^4*x*(a*c - b^2))/c^2 - (2^(1/3)*(3^(
1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^2*c*x*(b^4 + 8*a^2*c^2 - 6*a*
b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^8 + 16*a
^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 1
1*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2
)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4
*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2
*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*
(4*a*c - b^2)^3))^(1/3))/12 + (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b^2*c^2 - 8
```

```
*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2
*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/
2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/36)*
((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) +
 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 - b^6*c^5 +
 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log((a^4*x*(a*c - b^2))/c^2 - (2^
(1/3)*(3^(1/2)*1i + 1)*((2^(2/3)*(3^(1/2)*1i - 1)*(27*a^2*c*x*(b^4 + 8*a^2*
c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(
b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b
^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4
*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8 + 16*a^4*c^4
+ b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6
*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1
/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12 - (9*a*b*(b^6 - 12*a^3*c^3 + 19*a^2*b
^2*c^2 - 8*a*b^4*c))/c^2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2
) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(
2/3))/36)*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^
3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^8 -
 b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) - log((a^4*x*(a*c - b^2))
/c^2 - (2^(1/3)*(3^(1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i + 1)*(27*a^2*c*x*(b^
4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c -
b^2)^2*(-(b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2
- 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*
b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^8 + 1
6*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3
- 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c -
b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/12...
```

3.144 $\int \frac{x^6}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=631

$$\frac{x}{c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

```
[Out] x/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b+(2*a*c-b^2)/(
-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(
2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+
b^2)^(1/2))^(2/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-
4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^
2)^(1/2))^(1/3))*3^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3
)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+
b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(
-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(
-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a*c+b^2)/(-4*
a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3
*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-2*a*c+b
^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3
)
```

Rubi [A]

time = 0.65, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1381, 1436, 206, 31, 648, 631, 210, 642}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\,\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^3 + c*x^6),x]
```

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
```

```
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 -
 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^6}{a+bx^3+cx^6}\,dx = \frac{x}{c} - \frac{\int \frac{a+bx^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-\cdots}}}{2c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \cdots}{3\sqrt[3]{\cdots}}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\cdots}{3\sqrt[3]{2}\cdots}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\cdots}{3\sqrt[3]{2}\cdots}$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-}\cdots}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b\cdots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 70, normalized size = 0.11

$$\frac{x}{c} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^3 + c*x^6),x]
```

```
[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/
(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 59, normalized size = 0.09

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)}\frac{\left(-\_R^3 b-a\right)\ln\left(x-\_R\right)}{2\_R^5 c+b\_R^2}}{3c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)}\frac{\left(-\_R^3 b-a\right)\ln\left(x-\_R\right)}{2\_R^5 c+b\_R^2}}{3c}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*
b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5841 vs. 2(495) = 990.
time = 1.72, size = 5841, normalized size = 9.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*ar
ctan(1/6*(sqrt(2)*(1/2)^(2/3)*sqrt(2*(a^2*b^8 - 8*a^3*b^6*c + 20*a^4*b^4*c^
```

2 - 16*a^5*b^2*c^3 + 4*a^6*c^4)*x^2 + (1/2)^(2/3)*(b^12 - 14*a*b^10*c + 76*
a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*
c^6 - (b^11*c^4 - 16*a*b^9*c^5 + 98*a^2*b^7*c^6 - 280*a^3*b^5*c^7 + 352*a^4
*b^3*c^8 - 128*a^5*b*c^9)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b
^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11
)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b
^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c
^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^9*c^4 -
 12*a^2*b^7*c^5 + 50*a^3*b^5*c^6 - 80*a^4*b^3*c^7 + 32*a^5*b*c^8)*x*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - (a*b^10 - 12*a^2*b^8*c + 52*
a^3*b^6*c^2 - 96*a^4*b^4*c^3 + 68*a^5*b^2*c^4 - 16*a^6*c^5)*x)*(-(b^3 - 2*a
*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^1
1)))/(b^2*c^4 - 4*a*c^5))^(1/3))*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*
b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*
c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10
 - 64*a^3*c^11)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*
c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c
^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - 2*(1/2)^
(2/3)*(sqrt(3)*(a*b^12*c^4 - 17*a^2*b^10*c^5 + 114*a^3*b^8*c^6 - 378*a^4*b^
6*c^7 + 632*a^5*b^4*c^8 - 480*a^6*b^2*c^9 + 128*a^7*c^10)*x*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c
^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(a*b^13 - 15*a^2*b^11*c + 88
*a^3*b^9*c^2 - 252*a^4*b^7*c^3 + 356*a^5*b^5*c^4 - 220*a^6*b^3*c^5 + 48*a^7
*b*c^6)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 2
0*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^
2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - 2*sqrt(3)*(a^4*b^8
 - 8*a^5*b^6*c + 20*a^6*b^4*c^2 - 16*a^7*b^2*c^3 + 4*a^8*c^4))/(a^4*b^8 - 8
*a^5*b^6*c + 20*a^6*b^4*c^2 - 16*a^7*b^2*c^3 + 4*a^8*c^4)) - 4*sqrt(3)*(1/2
)^(1/3)*c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2
*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan(1/6*(sqrt(2)*(
1/2)^(2/3)*sqrt(2*(a^2*b^8 - 8*a^3*b^6*c + 20*a^4*b^4*c^2 - 16*a^5*b^2*c^3
 + 4*a^6*c^4)*x^2 + (1/2)^(2/3)*(b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 - 200*a
^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6 + (b^11*c^4 - 1
6*a*b^9*c^5 + 98*a^2*b^7*c^6 - 280*a^3*b^5*c^7 + 352*a^4*b^3*c^8 - 128*a^5*
b*c^9)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)
/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*
c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2
*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11))
)/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^9*c^4 - 12*a^2*b^7*c^5 + 5
0*a^3*b^5*c^6 - 80*a^4*b^3*c^7 + 32*a^5*b*c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2

```
*b^2*c^10 - 64*a^3*c^11)) + (a*b^10 - 12*a^2*b^8*c + 52*a^3*b^6*c^2 - 96*a^
4*b^4*c^3 + 68*a^5*b^2*c^4 - 16*a^6*c^5)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4
*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4
)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a
*c^5))^(1/3))*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b
^2*c^7 + 64*a^4*c^8)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^
3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) +
sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)
)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - 2*(1/2)^(2/3)*(sqrt(3)*(a*b
^12*c^4 - 17*a^2*b^10*c^5 + 114*a^3*b^8*c^6 - 378*a^4*b^6*c^7 + 632*a^5*b^4
*c^8 - 480*a^6*b^2*c^9 + 128*a^7*c^10)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)) + sqrt(3)*(a*b^13 - 15*a^2*b^11*c + 88*a^3*b^9*c^2 - 252*
a^4*b^7*c^3 + 356*a^5*b^5*c^4 - 220*a^6*b^3*c^5 + 48*a^7*b*c^6)*x)*(-(b^3 -
 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*
a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^...
```

Sympy [**A**]
time = 50.67, size = 196, normalized size = 0.31

$$\mathrm{RootSum}\left(t^6\cdot\left(46656a^3c^7-34992a^2b^2c^6+8748ab^4c^5-729b^6c^4\right)+t^3\cdot\left(864a^3bc^3-864a^2b^3c^2+270ab^5c-27b^7\right)+a^4,\left(t\mapsto t\log\left(x+\frac{1296t^4a^2bc^6-648t^4ab^3c^5+81t^4b^5c^4-12ta^3c^3+39ta^2b^2c^2-21tab^4c+3tb^6}{2a^3c^2-4a^2b^2c+ab^4}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5*c
 - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*_t*
*4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**
2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4))) +
 x/c
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^6/(c*x^6 + b*x^3 + a), x)
```

Mupad [**B**]
time = 3.40, size = 2280, normalized size = 3.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a*
c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(
4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^
(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + x/c + log((3*
a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*((b^7 + b^4*(-(4*a*c
- b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) -
b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*((b^7 + b^4*(-(4*a*c -
b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7
- b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*
a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i - 1)*((b^7 + b^4*(-(4*a*
c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^
2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
- b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2)
- b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2
)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 - log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(3^(1/2)*1i
 + 1)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*
(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2
)))*((3^(1/2)*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*
c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*
a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5
- 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c -
 (3*2^(2/3)*a*(3^(1/2)*1i - 1)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a
^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c
 - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 +
2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a
*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)
^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^
7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) - log((3*a^2*x*(b^4 +
```

```
2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i + 1)*(-(b^4*(-(4*a*c -
 b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a
*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/
2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 +
1/2)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(
1/3)
```

3.145 $\int \frac{x^4}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}}$$

```
[Out] 1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b-(-4*a*c+b^2)^(1/2
))^(2/3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(
1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(
b-(-4*a*c+b^2)^(1/2))^(2/3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)+1/6*arctan(1
/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b-(-4*a*c
+b^2)^(1/2))^(2/3)*2^(1/3)/c^(2/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)-1/6*ln(2^(1/3
)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(b+(-4*a*c+b^2)^(1/2))^(2/3)*2^(1
/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*
x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-4*a*c+b^2
)^(1/2))^(2/3)*2^(1/3)/c^(2/3)/(-4*a*c+b^2)^(1/2)-1/6*arctan(1/3*(1-2*2^(1/
3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-4*a*c+b^2)^(1/2))^
(2/3)*2^(1/3)/c^(2/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1388, 298, 31, 648, 631, 210, 642}

$$\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}+\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}+b\right)^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{2/3}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[
b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c])
- ((b + Sqrt[b^2 - 4*a*c])^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*Sqrt[b^2 - 4*a*c]
) + ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^2 - 4*
a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2
/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) - ((b - Sqrt[b^2 - 4*a*c])^(2/3)*Log[(b - Sq
rt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x +
2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^
```

```
2 - 4*a*c])^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*Sqrt[
b^2 - 4*a*c])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
```

```
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{a+bx^3+cx^6}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\ 2^{2/3}\sqrt[3]{c}\,\sqrt{b^2-4ac}} - \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\int \frac{\ }{\left(b-\sqrt{\ }\right.}}{3}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{3}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{3}$$

$$= \frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}\tan^{-1}}{2^{2/3}\sqrt{3}\,c^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 44, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)\#1^2}{b+2c\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^3 + c*x^6),x]
```

[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1^2)/(b + 2*c*#1^3) &]/3

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 43, normalized size = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	43

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)

[Out] 1/3*sum(_R^4/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")

[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4107 vs. 2(421) = 842.
time = 0.63, size = 4107, normalized size = 7.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] 2/3*sqrt(3)*(1/2)^(1/3)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^
2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^
2 - 4*a*c^3))^(1/3)*arctan(1/6*(sqrt(2)*(1/2)^(1/3)*sqrt(2*(a^2*b^4 - 4*a^3
*b^2*c + 4*a^4*c^2)*x^2 + (1/2)^(2/3)*((a*b^8*c^2 - 14*a^2*b^6*c^3 + 72*a^3
*b^4*c^4 - 160*a^4*b^2*c^5 + 128*a^5*c^6)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c
^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - (a*b^7 - 8*a^
2*b^5*c + 20*a^3*b^3*c^2 - 16*a^4*b*c^3)*x)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^
4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^

3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 2*(1/2)^(1/3)*(a^2*b^6*c^2 - 10*a
^3*b^4*c^3 + 32*a^4*b^2*c^4 - 32*a^5*c^5)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2
)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))*(-((b^2*c^2 - 4*a
*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b
^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3))*(sqrt(3)*(b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 1
2*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - sqrt(3)*(b^4 - 6*a*b^2*c + 8*
a^2*c^2))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^
4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^
(1/3) + 2*(1/2)^(1/3)*(sqrt(3)*(a*b^7*c^2 - 10*a^2*b^5*c^3 + 32*a^3*b^3*c^4
 - 32*a^4*b*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c
^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - sqrt(3)*(a*b^6 - 8*a^2*b^4*c + 20*a^3*
b^2*c^2 - 16*a^4*c^3)*x)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a
^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c
^2 - 4*a*c^3))^(1/3) + 2*sqrt(3)*(a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2))/(a^2*
b^4 - 4*a^3*b^2*c + 4*a^4*c^2)) - 2/3*sqrt(3)*(1/2)^(1/3)*(((b^2*c^2 - 4*a*
c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^
2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3)*arctan(1/6*(sqrt(2)*(1
/2)^(1/3)*sqrt(2*(a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*x^2 - (1/2)^(2/3)*((a*
b^8*c^2 - 14*a^2*b^6*c^3 + 72*a^3*b^4*c^4 - 160*a^4*b^2*c^5 + 128*a^5*c^6)*
x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c
^6 - 64*a^3*c^7)) + (a*b^7 - 8*a^2*b^5*c + 20*a^3*b^3*c^2 - 16*a^4*b*c^3)*x
)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*
b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(2/3) + 2
(1/2)^(1/3)(a^2*b^6*c^2 - 10*a^3*b^4*c^3 + 32*a^4*b^2*c^4 - 32*a^5*c^5)*s
qrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6
 - 64*a^3*c^7))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^
6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^
3))^(1/3))*(sqrt(3)*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*sqrt((b^4 - 4*a*
b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7))
 + sqrt(3)*(b^4 - 6*a*b^2*c + 8*a^2*c^2))*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 -
4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^
7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3) + 2*(1/2)^(1/3)*(sqrt(3)*(a*b^7*c^2 - 1
0*a^2*b^5*c^3 + 32*a^3*b^3*c^4 - 32*a^4*b*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*
a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + sqrt(3)*
(a*b^6 - 8*a^2*b^4*c + 20*a^3*b^2*c^2 - 16*a^4*c^3)*x)*(((b^2*c^2 - 4*a*c^3
)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c
^6 - 64*a^3*c^7)) - b)/(b^2*c^2 - 4*a*c^3))^(1/3) - 2*sqrt(3)*(a^2*b^4 - 4*
a^3*b^2*c + 4*a^4*c^2))/(a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)) - 1/6*(1/2)^(1
/3)*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12
*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(1/3)*
log(16*(a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*x^2 + 8*(1/2)^(2/3)*((a*b^8*c^2
 - 14*a^2*b^6*c^3 + 72*a^3*b^4*c^4 - 160*a^4*b^2*c^5 + 128*a^5*c^6)*x*sqrt((
b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*
a^3*c^7)) - (a*b^7 - 8*a^2*b^5*c + 20*a^3*b^3*c^2 - 16*a^4*b*c^3)*x)*(-((b^

```
2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5
 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(2/3) - 16*(1/2)
^(1/3)*(a^2*b^6*c^2 - 10*a^3*b^4*c^3 + 32*a^4*b^2*c^4 - 32*a^5*c^5)*sqrt((b
^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a
^3*c^7))*(-((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4
 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) + b)/(b^2*c^2 - 4*a*c^3))^(
1/3)) - 1/6*(1/2)^(1/3)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)) - b)/(b^2*c^2
 - 4*a*c^3))^(1/3)*log(16*(a^2*b^4 - 4*a^3*b^2*c + 4*a^4*c^2)*x^2 - 8*(1/2)
^(2/3)*((a*b^8*c^2 - 14*a^2*b^6*c^3 + 72*a^3*b^4*c^4 - 160*a^4*b^2*c^5 + 12
8*a^5*c^6)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*c^4 - 12*a*b^4*c^5 + 4
8*a^2*b^2*c^6 - 64*a^3*c^7)) + (a*b^7 - 8*a^2*b^5*c + 20*a^3*b^3*c^2 - 16*a
^4*b*c^3)*x)*(((b^2*c^2 - 4*a*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(b^6*
c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^...
```

Sympy [A]
time = 1.44, size = 175, normalized size = 0.31

$$\operatorname{RootSum}\left(t^{6}\cdot\left(46656a^{3}c^{5}-34992a^{2}b^{2}c^{4}+8748ab^{4}c^{3}-729b^{6}c^{2}\right)+t^{3}\left(-432a^{2}bc^{2}+216ab^{3}c-27b^{5}\right)+a^{2},\left(t\mapsto t\log\left(x+\frac{15552t^{5}a^{3}c^{5}-11664t^{5}a^{2}b^{2}c^{4}+2916t^{5}ab^{4}c^{3}-243t^{5}b^{6}c^{2}-108t^{2}a^{2}bc^{2}+63t^{2}ab^{3}c-9t^{2}b^{5}}{2a^{2}c-ab^{2}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**5 - 34992*a**2*b**2*c**4 + 8748*a*b**4*c**3 -
729*b**6*c**2) + _t**3*(-432*a**2*b*c**2 + 216*a*b**3*c - 27*b**5) + a**2,
Lambda(_t, _t*log(x + (15552*_t**5*a**3*c**5 - 11664*_t**5*a**2*b**2*c**4 +
 2916*_t**5*a*b**4*c**3 - 243*_t**5*b**6*c**2 - 108*_t**2*a**2*b*c**2 + 63*
_t**2*a*b**3*c - 9*_t**2*b**5)/(2*a**2*c - a*b**2))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^4/(c*x^6 + b*x^3 + a), x)
```

Mupad [B]
time = 8.11, size = 2695, normalized size = 4.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^
3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c - b^2) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2
)^2*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c
*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/2)*((b^5 + b^2*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)
^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/6 - 45*a^2*b^2*c^2 + 9*a*b^4*c))/18 +
 a^2*b*c*x)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c
 - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3
 - 48*a^2*b^2*c^4)))^(1/3) + log((2^(1/3)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*
c - b^2)^3))^(2/3)*(36*a^3*c^3 - (2^(2/3)*(54*a^2*c^3*x*(4*a*c - b^2) - (27
*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^
3))^(2/3))/2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3
*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/6 - 45*a
^2*b^2*c^2 + 9*a*b^4*c))/18 + a^2*b*c*x)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3
*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) - log((2^(1/3)*(3^(
1/2)*1i - 1)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*
c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^
3 - 45*a^2*b^2*c^2 + 9*a*b^4*c + (2^(2/3)*(3^(1/2)*1i + 1)*(54*a^2*c^3*x*(4
*a*c - b^2) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c -
b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/4)*((b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2
*(4*a*c - b^2)^3))^(1/3))/12))/36 + a^2*b*c*x)*((3^(1/2)*1i)/2 + 1/2)*((b^5
 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4
)))^(1/3) + log((2^(1/3)*(3^(1/2)*1i + 1)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*
c - b^2)^3))^(2/3)*(36*a^3*c^3 - 45*a^2*b^2*c^2 + 9*a*b^4*c - (2^(2/3)*(3^(
1/2)*1i - 1)*(54*a^2*c^3*x*(4*a*c - b^2) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i
+ 1)*(4*a*c - b^2)^2*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 -
8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/4
)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(
-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/12))/36 - a^2*b*c*x)
*((3^(1/2)*1i)/2 - 1/2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 +
12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) - log((2^(1/3)*(3^(1/2)*1i - 1)*((b^
5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*
c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3 - 45*a^2*b^2*c^
2 + 9*a*b^4*c + (2^(2/3)*(3^(1/2)*1i + 1)*(54*a^2*c^3*x*(4*a*c - b^2) - (27
*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^5 - b^2*(-(4*a*c - b^
2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c
```

```
^2*(4*a*c - b^2)^3))^(2/3))/4)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^
2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3)
)^(1/3))/12))/36 + a^2*b*c*x)*((3^(1/2)*1i)/2 + 1/2)*((b^5 - b^2*(-(4*a*c -
 b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))
/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)))^(1/3) + log((
2^(1/3)*(3^(1/2)*1i + 1)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^
2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3
)*(36*a^3*c^3 - 45*a^2*b^2*c^2 + 9*a*b^4*c - (2^(2/3)*(3^(1/2)*1i - 1)*(54*
a^2*c^3*x*(4*a*c - b^2) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2
)^2*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c
*(-(4*a*c - b^2)^3)^(1/2))/(c^2*(4*a*c - b^2)^3))^(2/3))/4)*((b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)
^(1/2))/(c^2*(4*a*c - b^2)^3))^(1/3))/12))/36 - a^2*b*c*x)*((3^(1/2)*1i)/2
- 1/2)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*
a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48
*a^2*b^2*c^4)))^(1/3)
```

3.146 $\int \frac{x^3}{a+bx^3+cx^6} \, dx$

Optimal. Leaf size=558

$$\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}}-$$

```
[Out] -1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b-(-4*a*c+b^2)^(1/
2))^(1/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^
(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*
(b-(-4*a*c+b^2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)+1/6*arctan(
1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b-(-4*a*
c+b^2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)+1/6*ln(2^(1/
3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(b+(-4*a*c+b^2)^(1/2))^(1/3)*2^(
2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)
*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-4*a*c+b^
2)^(1/2))^(1/3)*2^(2/3)/c^(1/3)/(-4*a*c+b^2)^(1/2)-1/6*arctan(1/3*(1-2*2^(1
/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-4*a*c+b^2)^(1/2))
^(1/3)*2^(2/3)/c^(1/3)*3^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1388, 206, 31, 648, 631, 210, 642}

$$\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[3]{\sqrt{b^2-4ac}+b}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[
b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c])
 - ((b + Sqrt[b^2 - 4*a*c])^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*Sqrt[b^2 - 4*a*c]
) - ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b + Sqrt[b^2 - 4*
a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1
/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) + ((b - Sqrt[b^2 - 4*a*c])^(1/3)*Log[(b - Sq
rt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x +
2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[b^2 - 4*a*c]) - ((b + Sqrt[b^
```

```
2 - 4*a*c])^(1/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b +
Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*Sqrt[
b^2 - 4*a*c])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
```

```
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^3}{a+bx^3+cx^6}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\int \frac{}{\left(b-\sqrt{b^2\right.}}}{3}$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\,\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\,\log}{3\sqrt[3]{2}}$$

$$= -\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\,\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\sqrt{b^2-4ac}} + \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\,\log}{3\sqrt[3]{2}}$$

$$= \frac{\sqrt[3]{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2-4ac}} - \frac{\sqrt[3]{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(1-\right.}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\sqrt{b^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 42, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)\#1}{b+2c\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^3 + c*x^6),x]
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (Log[x - #1]*#1)/(b + 2*c*#1^3) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 43, normalized size = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2571 vs. 2(421) = 842.
time = 0.42, size = 2571, normalized size = 4.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*arctan(1/3
*(sqrt(3)*a*b^2 - (1/2)^(2/3)*sqrt(b^2*x^2 - (1/2)^(1/3)*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))*x*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3) + (1/2)^(2/3)*(b^4 - 4*a
*b^2*c)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c
^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(2/3))*(sqrt(3)*(b^6*c - 12*a*b^4
*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a
```

```
^2*b^2*c^4 - 64*a^3*c^5)) - sqrt(3)*(b^4 - 4*a*b^2*c))*(((b^2*c - 4*a*c^2)*
sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*
c - 4*a*c^2))^(2/3) + (1/2)^(2/3)*(sqrt(3)*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))*x - sqrt(3)*(b^5 - 4*a*b^3*c)*x)*(((b^2*c - 4*a*c^2)*sqrt(b^2/
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c
^2))^(2/3))/(a*b^2)) + 2/3*sqrt(3)*(1/2)^(1/3)*(-((b^2*c - 4*a*c^2)*sqrt(b^
2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a
*c^2))^(1/3)*arctan(-1/3*(sqrt(3)*a*b^2 + (1/2)^(2/3)*sqrt(b^2*x^2 + (1/2)^
(1/3)*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3
 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*x*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3)
 + (1/2)^(2/3)*(b^4 - 4*a*b^2*c)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(2/3))*
(sqrt(3)*(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)*sqrt(b^2/(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + sqrt(3)*(b^4 - 4*a*b^
2*c))*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^
4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(2/3) - (1/2)^(2/3)*(sqrt(3)*(b^7*
c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*sqrt(b^2/(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*x + sqrt(3)*(b^5 - 4*a*b^3*c)*x)*(-
((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(2/3))/(a*b^2)) - 1/6*(1/2)^(1/3)*(((b^2*c
 - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)
) + 1)/(b^2*c - 4*a*c^2))^(1/3)*log(4*b^2*x^2 - 4*(1/2)^(1/3)*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5))*x*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3) + 4*(1/2)^(2/3)*(b^
4 - 4*a*b^2*c)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(2/3)) - 1/6*(1/2)^(1/3)*(
-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3)*log(4*b^2*x^2 + 4*(1/2)^(1/3)*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*x*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3) + 4*(1/2)^
(2/3)*(b^4 - 4*a*b^2*c)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c
^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(2/3)) + 1/3*(1/
2)^(1/3)*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3)*log((1/2)^(1/3)*(b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5))*(((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^
2*b^2*c^4 - 64*a^3*c^5)) + 1)/(b^2*c - 4*a*c^2))^(1/3) + b*x) + 1/3*(1/2)^(
1/3)*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3)*log(-(1/2)^(1/3)*(b^4*c - 8*a
*b^2*c^2 + 16*a^2*c^3)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))*(-((b^2*c - 4*a*c^2)*sqrt(b^2/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2
```

```
*b^2*c^4 - 64*a^3*c^5)) - 1)/(b^2*c - 4*a*c^2))^(1/3) + b*x)
```

Sympy [A]
time = 0.99, size = 122, normalized size = 0.22

$$\operatorname{RootSum}\left(t^{6}\cdot\left(46656a^{3}c^{4}-34992a^{2}b^{2}c^{3}+8748ab^{4}c^{2}-729b^{6}c\right)+t^{3}\cdot\left(432a^{2}c^{2}-216ab^{2}c+27b^{4}\right)+a,\left(t\mapsto t\log\left(x+\frac{2592t^{4}a^{2}c^{3}-1296t^{4}ab^{2}c^{2}+162t^{4}b^{4}c+12tac-3tb^{2}}{b}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**4 - 34992*a**2*b**2*c**3 + 8748*a*b**4*c**2 -
729*b**6*c) + _t**3*(432*a**2*c**2 - 216*a*b**2*c + 27*b**4) + a, Lambda(_t
, _t*log(x + (2592*_t**4*a**2*c**3 - 1296*_t**4*a*b**2*c**2 + 162*_t**4*b**
4*c + 12*_t*a*c - 3*_t*b**2)/b)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a), x)
```

Mupad [B]
time = 7.71, size = 2129, normalized size = 3.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(2/3)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/
(c*(4*a*c - b^2)^3))^(1/3)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (9*2^(1/3)*a*c^3*(
4*a*c - b^2)^2*(x - (2^(2/3)*b*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2
*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/2)*(-(b*(-(4*a*c - b^2)^3)^(1
/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/2))/6 + 3*a
*c^2*x*(2*a*c - b^2))*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a
*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3) +
log((2^(2/3)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (9*2^(1/3)*a*c^3*(x - (2^(2/3)*b
*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b
^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a
^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/2)*((b*(-(4*a*c - b^2)^3)^(
1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/6 + 3*a*c^
2*x*(2*a*c - b^2))*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b
```

```
^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3) + lo
g((2^(2/3)*(3^(1/2)*1i - 1)*(36*a^2*b*c^3 - 9*a*b^3*c^2 + (2^(1/3)*(3^(1/2)
*1i + 1)*(81*a*c^3*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)
*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2
*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16
*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/36)*(-(b*(-(4*a*c - b^2)^
3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/12 - 3
*a*c^2*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 - 1/2)*((b*(-(4*a*c - b^2)^3)^(1/2)
 + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 4
8*a^2*b^2*c^3)))^(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*(9*a*b^3*c^2 - 36*a^
2*b*c^3 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*a*c^3*x*(4*a*c - b^2)^2 + (81*2^(2/
3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) +
 b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b*(-(4*a*c
 - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3)
)/36)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(c*(4*a
*c - b^2)^3))^(1/3))/12 - 3*a*c^2*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(
(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(b^6*c - 64
*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3) + log((2^(2/3)*(3^(1/2)*1
i - 1)*(36*a^2*b*c^3 - 9*a*b^3*c^2 + (2^(1/3)*(81*a*c^3*x*(4*a*c - b^2)^2 -
 (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3
)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/4)*(3^(
1/2)*1i + 1)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(
c*(4*a*c - b^2)^3))^(2/3))/36)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*
c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/12 - 3*a*c^2*x*(2*a*c - b^2))*
((3^(1/2)*1i)/2 - 1/2)*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8
*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))^(1/3)
- log((2^(2/3)*(3^(1/2)*1i + 1)*(9*a*b^3*c^2 - 36*a^2*b*c^3 + (2^(1/3)*(81*
a*c^3*x*(4*a*c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2
)^2*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c
- b^2)^3))^(1/3))/4)*(3^(1/2)*1i - 1)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 -
16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(2/3))/36)*((b*(-(4*a*c - b^2)
^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(c*(4*a*c - b^2)^3))^(1/3))/12 -
3*a*c^2*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(-(b*(-(4*a*c - b^2)^3)^(1/
2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(54*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3)))^(1/3)
```

3.147 $\int \frac{x}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$-\frac{\sqrt[3]{2}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b+\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\cdots}\right.}{3\sqrt{b^2-4ac}\cdots}$$

```
[Out] -1/3*2^(1/3)*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))/(b-
(-4*a*c+b^2)^(1/2))^(1/3)/(-4*a*c+b^2)^(1/2)+1/6*c^(1/3)*ln(2^(2/3)*c^(2/3)
*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^
(2/3))*2^(1/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)/(-4*a*c+b^2)^(1/2)-1/3*2^(1/3)*
c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(
1/2))*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)/(-4*a*c+b^2)^(1/2)+1/3*2^(1/3)*c
^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))/(-4*a*c+b^2)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/6*c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c
^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*2^(1/3)
/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)+1/3*2^(1/3)*c^(1/3)*arctan
(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/
(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Rubi [A]
time = 0.27, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1389, 298, 31, 648, 631, 210, 642}

$$-\frac{\sqrt[3]{2}\sqrt[3]{c}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\,2^{2/3}\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{2}\sqrt[3]{c}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt{b^2-4ac}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3 + c*x^6),x]
```

```
[Out] -((2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)
)) + (2^(1/3)*c^(1/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a
*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(1
/3)) - (2^(1/3)*c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)
*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (2^(1/3)*c^(1/3)
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2
/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2
```

```
])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^
(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(2/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
```

```
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x}{a+bx^3+cx^6}\,dx = \frac{c\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{\left(\sqrt[3]{2}\,c^{2/3}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(\sqrt[3]{2}\,c^{2/3}\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-}}}{\sqrt[3]{2}}}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\,\sqrt[3]{}}{}}}{3\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{}}}$$

$$= -\frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\right.}{3\sqrt{b^2-4ac}\,\sqrt[3]{b+\sqrt{b^2-}}}$$

$$= -\frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\right.}{3\sqrt{b^2-4ac}\,\sqrt[3]{b+\sqrt{b^2-}}}$$

$$= -\frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt{b^2-4ac}\,\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{2}\,\sqrt[3]{c}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt{b^2-4ac}\,\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 43, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)}{b\#1+2c\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^3 + c*x^6),x]
[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1 + 2*c*#1^4) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 41, normalized size = 0.07

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)} \frac{\_R \ln\left(x-\_R\right)}{2\_R^5 c+b\_R^2}}{3}$	41
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)} \frac{\_R \ln\left(x-\_R\right)}{2\_R^5 c+b\_R^2}}{3}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*sum(_R/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] integrate(x/(c*x^6 + b*x^3 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2980 vs. 2(421) = 842.
time = 0.43, size = 2980, normalized size = 5.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*arctan(1/
3*(sqrt(3)*sqrt(2)*(1/2)^(1/3)*sqrt(2*b^2*c^2*x^2 + (1/2)^(2/3)*((a*b^7*c -
 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*sqrt(b^2/(a^2*b^6 - 12*a^3
*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x - (b^5*c - 4*a*b^3*c^2)*x)*(-((a*b
^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^
3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3) + (1/2)^(1/3)*(b^4*c - 4*a*b^2*c^2 - (a*b
```

^6*c - 8*a^2*b^4*c^2 + 16*a^3*b^2*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a
^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3))*(a*
b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3))*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c
 + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3) - 2*sqrt(3)*
(1/2)^(1/3)*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*sqrt(b^2/(a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/
(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2
*c))^(1/3) - sqrt(3)*b^2*c)/(b^2*c)) + 2/3*sqrt(3)*(1/2)^(1/3)*(((a*b^2 - 4
*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) -
1)/(a*b^2 - 4*a^2*c))^(1/3)*arctan(1/3*(sqrt(3)*sqrt(2)*(1/2)^(1/3)*sqrt(2*
b^2*c^2*x^2 - (1/2)^(2/3)*((a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*
a^4*b*c^4)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))
*x + (b^5*c - 4*a*b^3*c^2)*x)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^
3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(2/3) + (1/
2)^(1/3)*(b^4*c - 4*a*b^2*c^2 + (a*b^6*c - 8*a^2*b^4*c^2 + 16*a^3*b^2*c^3)*
sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(((a*b^2
- 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))
 - 1)/(a*b^2 - 4*a^2*c))^(1/3))*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*sqrt(b^2
/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*(((a*b^2 - 4*a^2*c
)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*
b^2 - 4*a^2*c))^(1/3) - 2*sqrt(3)*(1/2)^(1/3)*(a*b^5*c - 8*a^2*b^3*c^2 + 16
*a^3*b*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)*x*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3) + sqrt(3)*b^2*c)/(b^2*c)) - 1/6
(1/2)^(1/3)(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4
*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(16*b^2*c^2*x^2 +
8*(1/2)^(2/3)*((a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*s
qrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*x - (b^5*c
- 4*a*b^3*c^2)*x)*(-((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*a^2*c))^(2/3) + 8*(1/2)^(1/3)*
(b^4*c - 4*a*b^2*c^2 - (a*b^6*c - 8*a^2*b^4*c^2 + 16*a^3*b^2*c^3)*sqrt(b^2/
(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*
c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a
*b^2 - 4*a^2*c))^(1/3)) - 1/6*(1/2)^(1/3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2
*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))
^(1/3)*log(16*b^2*c^2*x^2 - 8*(1/2)^(2/3)*((a*b^7*c - 12*a^2*b^5*c^2 + 48*a
^3*b^3*c^3 - 64*a^4*b*c^4)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3))*x + (b^5*c - 4*a*b^3*c^2)*x)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/
(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2
c))^(2/3) + 8(1/2)^(1/3)*(b^4*c - 4*a*b^2*c^2 + (a*b^6*c - 8*a^2*b^4*c^2
+ 16*a^3*b^2*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*
c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)) + 1/3*(1/2)^(1/3)*(-((a*b

```
^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^
3)) + 1)/(a*b^2 - 4*a^2*c))^(1/3)*log(2*b*c*x + (1/2)^(2/3)*(b^4 - 4*a*b^2*
c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*(-((a*b^2 - 4*a^2*c)*sqrt(b
^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) + 1)/(a*b^2 - 4*
a^2*c))^(2/3)) + 1/3*(1/2)^(1/3)*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(1/3)*lo
g(2*b*c*x + (1/2)^(2/3)*(b^4 - 4*a*b^2*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b
^2*c^2 - 64*a^4*c^3)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3)))*(((a*b^2 - 4*a^2*c)*sqrt(b^2/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3)) - 1)/(a*b^2 - 4*a^2*c))^(2/3))
```

Sympy [A]
time = 0.81, size = 158, normalized size = 0.28

$$\operatorname{RootSum}\left(t^{6}\cdot\left(46656a^{4}c^{3}-34992a^{3}b^{2}c^{2}+8748a^{2}b^{4}c-729ab^{6}\right)+t^{3}\left(-432a^{2}c^{2}+216ab^{2}c-27b^{4}\right)+c,\left(t\mapsto t\log\left(x+\frac{-15552t^{5}a^{4}c^{3}+11664t^{5}a^{3}b^{2}c^{2}-2916t^{5}a^{2}b^{4}c+243t^{5}ab^{6}+72t^{2}a^{2}c^{2}-54t^{2}ab^{2}c+9t^{2}b^{4}}{bc}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**4*c**3 - 34992*a**3*b**2*c**2 + 8748*a**2*b**4*c -
729*a*b**6) + _t**3*(-432*a**2*c**2 + 216*a*b**2*c - 27*b**4) + c, Lambda(_
t, _t*log(x + (-15552*_t**5*a**4*c**3 + 11664*_t**5*a**3*b**2*c**2 - 2916*_
t**5*a**2*b**4*c + 243*_t**5*a*b**6 + 72*_t**2*a**2*c**2 - 54*_t**2*a*b**2*
c + 9*_t**2*b**4)/(b*c))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a), x)
```

Mupad [B]
time = 5.39, size = 1543, normalized size = 2.77

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3 + c*x^6),x)
```

```
[Out] log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*
(4*a*c - b^2)^2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c
```

```
)/(a*(4*a*c - b^2)^3))^(2/3))/2)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2
*c^2 - 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*(-(b*(-(4*a*c - b^2)^3)^(1/2) +
b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a
^3*b^2*c^2)))^(1/3) + log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c)
+ (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 -
 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/2)*(b*(-(4*a*c - b^2)^
3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*((b*(-(4*
a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3
 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/3) - log(c^4*x - ((27*c^3*x*(b^4 + 8
*a^2*c^2 - 6*a*b^2*c) + (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^
2*((b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(a*(4*a*c -
b^2)^3))^(2/3))/4)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2
*c))/(54*a*(4*a*c - b^2)^3))*((3^(1/2)*1i)/2 + 1/2)*(-(b*(-(4*a*c - b^2)^3)
^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4
*c + 48*a^3*b^2*c^2)))^(1/3) + log(c^4*x - ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*
a*b^2*c) - (27*2^(1/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((b*(-(4*a*
c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3
))/4)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(54*a*(4
*a*c - b^2)^3))*((3^(1/2)*1i)/2 - 1/2)*(-(b*(-(4*a*c - b^2)^3)^(1/2) + b^4
+ 16*a^2*c^2 - 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b
^2*c^2)))^(1/3) - log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) + (2
7*2^(1/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^
(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/4)*(b*(-(
4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^
3))*((3^(1/2)*1i)/2 + 1/2)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2
+ 8*a*b^2*c)/(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/
3) + log(c^4*x + ((27*c^3*x*(b^4 + 8*a^2*c^2 - 6*a*b^2*c) - (27*2^(1/3)*a*b
*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b*(-(4*a*c - b^2)^3)^(1/2) - b^4 -
 16*a^2*c^2 + 8*a*b^2*c)/(a*(4*a*c - b^2)^3))^(2/3))/4)*(b*(-(4*a*c - b^2)^
3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(54*a*(4*a*c - b^2)^3))*((3^(1/2)
*1i)/2 - 1/2)*((b*(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c)/
(54*(a*b^6 - 64*a^4*c^3 - 12*a^2*b^4*c + 48*a^3*b^2*c^2)))^(1/3)
```

3.148 $\int \frac{1}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=558

$$-\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{}}\right.}{3\sqrt{b^2-4ac}\,(b}$$

```
[Out] 1/3*2^(2/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))/(b-(
-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/6*c^(2/3)*ln(2^(2/3)*c^(2/3)*
x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(
2/3))*2^(2/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/3*2^(2/3)*c
^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1
/2))*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/3*2^(2/3)*c^
(2/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))/(-4*a*c+b^2)^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^
(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*2^(2/3)/
(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/3*2^(2/3)*c^(2/3)*arctan(
1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/(
-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A]
time = 0.35, antiderivative size = 558, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1361, 206, 31, 648, 631, 210, 642}

$$-\frac{2^{2/3}c^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{3\sqrt[3]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(-1),x]
```

```
[Out] -((2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c]
)^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)
)) + (2^(2/3)*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a
*c])^(1/3))/Sqrt[3]])/(Sqrt[3]*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2
/3)) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)
*x])/(3*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (2^(2/3)*c^(2/3)
*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*Sqrt[b^2 - 4*a*
c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(2
/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2
```

```
])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*L
og[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^
(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(3*2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2
- 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
```

```
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{a+bx^3+cx^6}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2^{2/3}c\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(2^{2/3}c\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}\ \cdots}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b\cdots}}}{\sqrt[3]{2}}\cdots}}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac\cdots}\right.}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\cdots\right.}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac\cdots}\right.}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\cdots\right.}{3\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac\cdots}\right.}$$

$$= -\frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{2^{2/3}c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right.}{\sqrt{3}\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 45, normalized size = 0.08

$$\frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{\log(x-\#1)}{b\#1^2+2c\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(-1),x]
[Out] RootSum[a + b*#1^3 + c*#1^6 & , Log[x - #1]/(b*#1^2 + 2*c*#1^5) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 40, normalized size = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}\right)}{3}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] 1/3*sum(1/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] integrate(1/(c*x^6 + b*x^3 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4279 vs. 2(421) = 842.
time = 0.60, size = 4279, normalized size = 7.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] 2/3*sqrt(3)*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2
 - 4*a^3*c))^(1/3)*arctan(-1/6*(sqrt(2)*(1/2)^(2/3)*sqrt(2*(b^4*c^2 - 4*a*b
^2*c^3 + 4*a^2*c^4)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 -
56*a^3*b^2*c^3 + 32*a^4*c^4 - (a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 16
0*a^5*b^3*c^3 + 128*a^6*b*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6
- 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)*sqrt(
```

(b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(2/3) - (1/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*x)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3))*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2))*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(2/3) + 2*(1/2)^(2/3)*(sqrt(3)*(a^2*b^8*c - 14*a^3*b^6*c^2 + 72*a^4*b^4*c^3 - 160*a^5*b^2*c^4 + 128*a^6*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - sqrt(3)*(b^7*c - 8*a*b^5*c^2 + 20*a^2*b^3*c^3 - 16*a^3*b*c^4)*x)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(2/3) + 2*sqrt(3)*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4))/(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)) - 2/3*sqrt(3)*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3)*arctan(-1/6*(sqrt(2)*(1/2)^(2/3)*sqrt(2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2 + (1/2)^(2/3)*(b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4 + (a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) + (1/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*x)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3))*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2))*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) + 2*(1/2)^(2/3)*(sqrt(3)*(a^2*b^8*c - 14*a^3*b^6*c^2 + 72*a^4*b^4*c^3 - 160*a^5*b^2*c^4 + 128*a^6*c^5)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + sqrt(3)*(b^7*c - 8*a*b^5*c^2 + 20*a^2*b^3*c^3 - 16*a^3*b*c^4)*x)*(-((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(2/3) - 2*sqrt(3)*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4))/(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)) - 1/6*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)*log(16*(b^4*c^2 - 4*a*b^2*

```
c^3 + 4*a^2*c^4)*x^2 + 8*(1/2)^(2/3)*(b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 5
6*a^3*b^2*c^3 + 32*a^4*c^4 - (a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160
*a^5*b^3*c^3 + 128*a^6*b*c^4)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 -
 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)))*(((a^2*b^2 - 4*a^3*c)*sqrt((
b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*
a^7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(2/3) - 8*(1/2)^(1/3)*((a^2*b^7*c - 10*
a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*x*sqrt((b^4 - 4*a*b^2*c + 4*a^
2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)) - (b^6*c - 8
*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*x)*(((a^2*b^2 - 4*a^3*c)*sqrt((b^
4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^
7*c^3)) + b)/(a^2*b^2 - 4*a^3*c))^(1/3)) - 1/6*(1/2)^(1/3)*(-((a^2*b^2 - 4*
a^3*c)*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/(a^4*b^6 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2 - 64*a^7*c^3)) - b)/(a^2*b^2 - 4*a^3*c))^(1/3)*log(16*(b^4*c^2 - 4*
a*b^2*c^3 + 4*a^2*c^4)*x^2 + 8*(1/2)^(2/3)*(b^8 - 10*a*b^6*c + 36*a^2*b^4*c
^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4 + (a^2*b^9 - 1...
```

Sympy [A]
time = 3.69, size = 155, normalized size = 0.28

$$\operatorname{RootSum}\left(t^{6}\cdot\left(46656a^{5}c^{3}-34992a^{4}b^{2}c^{2}+8748a^{3}b^{4}c-729a^{2}b^{6}\right)+t^{3}\cdot\left(432a^{2}bc^{2}-216ab^{3}c+27b^{5}\right)+c^{2},\left(t\mapsto t\log\left(x+\frac{-1296t^{4}a^{4}bc^{2}+648t^{4}a^{3}b^{3}c-81t^{4}a^{2}b^{5}+12ta^{2}c^{2}-15tab^{2}c+3tb^{4}}{2ac^{2}-b^{2}c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**5*c**3 - 34992*a**4*b**2*c**2 + 8748*a**3*b**4*c -
729*a**2*b**6) + _t**3*(432*a**2*b*c**2 - 216*a*b**3*c + 27*b**5) + c**2, L
ambda(_t, _t*log(x + (-1296*_t**4*a**4*b*c**2 + 648*_t**4*a**3*b**3*c - 81*
_t**4*a**2*b**5 + 12*_t*a**2*c**2 - 15*_t*a*b**2*c + 3*_t*b**4)/(2*a*c**2 -
 b**2*c))))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^6 + b*x^3 + a), x)
```

Mupad [B]
time = 8.49, size = 2597, normalized size = 4.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3 + c*x^6),x)

[Out] log(6*c^5*x + (2^(2/3)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)
*(36*a*c^5 - 9*b^2*c^4 + (9*2^(1/3)*b*c^3*(x + (2^(2/3)*a*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^
(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1
/2))/(a^2*(4*a*c - b^2)^3))^(2/3))/2))/6)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*b
^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*x + (2
^(2/3)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2
*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(36*a*c^5 - 9*b
^2*c^4 + (9*2^(1/3)*b*c^3*(x + (2^(2/3)*a*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a
*c - b^2)^3))^(1/3))/2)*(4*a*c - b^2)^2*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c
 - b^2)^3))^(2/3))/2))/6)*((b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c
^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*b^6 - 64*a^5*c^3
- 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*x - (2^(2/3)*(3^(1/2)*
1i - 1)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c -
2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(9*b^2*c^4 - 3
6*a*c^5 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*b*c^3*x*(4*a*c - b^2)^2 + (81*2^(2/
3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4
*a*c - b^2)^3))^(1/3))/4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(2
/3))/36))/12)*((3^(1/2)*1i)/2 - 1/2)*((b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^2*b^6 -
64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) - log(6*c^5*x - (2^(2/3
)*(3^(1/2)*1i + 1)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8
*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3)*(36
*a*c^5 - 9*b^2*c^4 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*b*c^3*x*(4*a*c - b^2)^2
- (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*(-(b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/
2))/(a^2*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c -
b^2)^3))^(2/3))/36))/12)*((3^(1/2)*1i)/2 + 1/2)*((b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c - 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(54*
(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) + log(6*c^5*
x - (2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^
2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3)
)^(1/3)*(9*b^2*c^4 - 36*a*c^5 + (2^(1/3)*(3^(1/2)*1i + 1)*(81*b*c^3*x*(4*a*
c - b^2)^2 + (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c -
```

```
b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5 - b^2*(-(4*a*c - b^2
)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^
2*(4*a*c - b^2)^3))^(2/3))/36))/12)*((3^(1/2)*1i)/2 - 1/2)*((b^5 - b^2*(-(4
*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^
(1/2))/(54*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)))^(1/3) -
 log(6*c^5*x - (2^(2/3)*(3^(1/2)*1i + 1)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*
c - b^2)^3))^(1/3)*(36*a*c^5 - 9*b^2*c^4 + (2^(1/3)*(3^(1/2)*1i - 1)*(81*b*
c^3*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^
2*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*
(-(4*a*c - b^2)^3)^(1/2))/(a^2*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c - b^2)^3)
^(1/2))/(a^2*(4*a*c - b^2)^3))^(2/3))/36))/12)*((3^(1/2)*1i)/2 + 1/2)*((b^5
 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c + 2*a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(54*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
)))^(1/3)
```

3.149 $\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=610

$$-\frac{1}{ax}+\frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/a/x+1/6*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(1+b/
(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/12*c^(1/3)*ln(
2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a
*c+b^2)^(1/2))^(2/3))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b-(-4*a*c+b^2)^(1
/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1
/2))^(1/3))*3^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a*3^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/3)+1/6*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(
1/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*
c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/
3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b+(-4*
a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*
a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))*2^(1/3)/a*3^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Rubi [A]
time = 0.47, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1382, 1524, 298, 31, 648, 631, 210, 642}

$$\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(1/(a*x)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1
/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*a*(b - Sqr
t[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3
]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log
[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt
[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^
2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c]
```

```
)^(1/3)) - (c^(1/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(
2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^
2])/(6*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(1 - b/Sqrt[b^2
- 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2
 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a
*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)}\,dx = -\frac{1}{ax}+\frac{\int \frac{x\left(-b-cx^3\right)}{a+bx^3+cx^6}\,dx}{a}$$

$$= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right.\right.}{}$$

$$= -\frac{1}{ax}+\frac{\left(c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(c^{2/3}\left(1+\right.\right.}{}$$

$$= -\frac{1}{ax}+\frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\right.}{}$$

$$= -\frac{1}{ax}+\frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\right.}{}$$

$$= -\frac{1}{ax}+\frac{\sqrt[3]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(1-\frac{b}{\sqrt{b^2}}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 71, normalized size = 0.12

$$-\frac{1}{ax}-\frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3 + c*x^6)),x]
[Out] -(1/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x - #1]
*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 61, normalized size = 0.10

method	result
default	$-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)} \frac{\left(\_R^4 c+\_R b\right)\ln\left(x-\_R\right)}{2\_R^5 c+b\_R^2}}{3a}-\frac{1}{ax}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^7c^3-48b^2c^2a^6+12b^4c\,a^5-b^6a^4\right)\_Z^6+\left(-32a^3b\,c^3+32a^2b^3c^2-10a\,b^5c+b^7\right)\_Z^3+c^4\right)} \_R\ln\left(\left(\left(224a^7c^3-176\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] -1/3/a*sum((_R^4*c+_R*b)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b
+a))-1/a/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] -integrate((c*x^4 + b*x)/(c*x^6 + b*x^3 + a), x)/a - 1/(a*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5924 vs. $2(471) = 942$.
time = 1.43, size = 5924, normalized size = 9.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((
b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 1
2*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*a
rctan(1/6*(sqrt(2)*(1/2)^(1/3)*sqrt(2*(b^8*c^6 - 8*a*b^6*c^7 + 20*a^2*b^4*c
^8 - 16*a^3*b^2*c^9 + 4*a^4*c^10)*x^2 + (1/2)^(2/3)*((a^4*b^12*c^3 - 17*a^5
```

*b^10*c^4 + 114*a^6*b^8*c^5 - 378*a^7*b^6*c^6 + 632*a^8*b^4*c^7 - 480*a^9*b
^2*c^8 + 128*a^10*c^9)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^
2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)
) - (b^13*c^3 - 15*a*b^11*c^4 + 88*a^2*b^9*c^5 - 252*a^3*b^7*c^6 + 356*a^4*
b^5*c^7 - 220*a^5*b^3*c^8 + 48*a^6*b*c^9)*x)*((b^3 - 2*a*b*c + (a^4*b^2 - 4
*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4
)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a
^5*c))^(2/3) - (1/2)^(1/3)*(b^11*c^4 - 12*a*b^9*c^5 + 52*a^2*b^7*c^6 - 96*a
^3*b^5*c^7 + 68*a^4*b^3*c^8 - 16*a^5*b*c^9 - (a^4*b^10*c^4 - 14*a^5*b^8*c^5
 + 74*a^6*b^6*c^6 - 180*a^7*b^4*c^7 + 192*a^8*b^2*c^8 - 64*a^9*c^9)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12
*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c + (a^4*b^2 -
4*a^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^
4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*
a^5*c))^(1/3))*(sqrt(3)*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*sqrt((b^8 -
8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*
b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) - sqrt(3)*(b^6 - 8*a*b^4*c + 18*a^2
*b^2*c^2 - 8*a^3*c^3))*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*
a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^
4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3) - 2*(1/2)
^(1/3)*(sqrt(3)*(a^4*b^9*c^3 - 12*a^5*b^7*c^4 + 50*a^6*b^5*c^5 - 80*a^7*b^3
*c^6 + 32*a^8*b*c^7)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*
c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3))
- sqrt(3)*(b^10*c^3 - 12*a*b^8*c^4 + 52*a^2*b^6*c^5 - 96*a^3*b^4*c^6 + 68*a
^4*b^2*c^7 - 16*a^5*c^8)*x)*((b^3 - 2*a*b*c + (a^4*b^2 - 4*a^5*c)*sqrt((b^8
 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a
^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3) + 2*
sqrt(3)*(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - 16*a^3*b^2*c^7 + 4*a^4*c^
8))/(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - 16*a^3*b^2*c^7 + 4*a^4*c^8))
- 4*sqrt(3)*(1/2)^(1/3)*a*x*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8
 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a
^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3)*arct
an(1/6*(sqrt(2)*(1/2)^(1/3)*sqrt(2*(b^8*c^6 - 8*a*b^6*c^7 + 20*a^2*b^4*c^8
- 16*a^3*b^2*c^9 + 4*a^4*c^10)*x^2 - (1/2)^(2/3)*((a^4*b^12*c^3 - 17*a^5*b^
10*c^4 + 114*a^6*b^8*c^5 - 378*a^7*b^6*c^6 + 632*a^8*b^4*c^7 - 480*a^9*b^2*
c^8 + 128*a^10*c^9)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c
^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) +
 (b^13*c^3 - 15*a*b^11*c^4 + 88*a^2*b^9*c^5 - 252*a^3*b^7*c^6 + 356*a^4*b^5
*c^7 - 220*a^5*b^3*c^8 + 48*a^6*b*c^9)*x)*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^
5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(
a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*
c))^(2/3) - (1/2)^(1/3)*(b^11*c^4 - 12*a*b^9*c^5 + 52*a^2*b^7*c^6 - 96*a^3*
b^5*c^7 + 68*a^4*b^3*c^8 - 16*a^5*b*c^9 + (a^4*b^10*c^4 - 14*a^5*b^8*c^5 +
74*a^6*b^6*c^6 - 180*a^7*b^4*c^7 + 192*a^8*b^2*c^8 - 64*a^9*c^9)*sqrt((b^8
- 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^

```
9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a
^5*c)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/
(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5
*c))^(1/3))*(sqrt(3)*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*sqrt((b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4
*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) + sqrt(3)*(b^6 - 8*a*b^4*c + 18*a^2*b^
2*c^2 - 8*a^3*c^3))*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 - 8*a*b
^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c
 + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3) - 2*(1/2)^(1
/3)*(sqrt(3)*(a^4*b^9*c^3 - 12*a^5*b^7*c^4 + 50*a^6*b^5*c^5 - 80*a^7*b^3*c^
6 + 32*a^8*b*c^7)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)) + s
qrt(3)*(b^10*c^3 - 12*a*b^8*c^4 + 52*a^2*b^6*c^5 - 96*a^3*b^4*c^6 + 68*a^4*
b^2*c^7 - 16*a^5*c^8)*x)*((b^3 - 2*a*b*c - (a^4*b^2 - 4*a^5*c)*sqrt((b^8 -
8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(a^8*b^6 - 12*a^9*
b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)))/(a^4*b^2 - 4*a^5*c))^(1/3) - 2*sqr
t(3)*(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - ...
```

Sympy [A]
time = 2.26, size = 252, normalized size = 0.41

$$\operatorname{RootSum}\left(t^{6}\cdot\left(46656a^{7}c^{3}-34992a^{6}b^{2}c^{2}+8748a^{5}b^{4}c-729a^{4}b^{6}\right)+t^{3}\left(-864a^{3}bc^{3}+864a^{2}b^{3}c^{2}-270ab^{5}c+27b^{7}\right)+c^{4},\left(t\mapsto t\log\left(x+\frac{-15552t^{5}a^{8}c^{4}+27216t^{5}a^{7}b^{2}c^{3}-14580t^{5}a^{6}b^{4}c^{2}+3159t^{5}a^{5}b^{6}c-243t^{5}a^{4}b^{8}+252t^{2}a^{4}bc^{4}-567t^{2}a^{3}b^{3}c^{3}+378t^{2}a^{2}b^{5}c^{2}-99t^{2}ab^{7}c+9t^{2}b^{9}}{2a^{2}c^{5}-4ab^{2}c^{4}+b^{4}c^{3}}\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] RootSum(_t**6*(46656*a**7*c**3 - 34992*a**6*b**2*c**2 + 8748*a**5*b**4*c -
729*a**4*b**6) + _t**3*(-864*a**3*b*c**3 + 864*a**2*b**3*c**2 - 270*a*b**5*
c + 27*b**7) + c**4, Lambda(_t, _t*log(x + (-15552*_t**5*a**8*c**4 + 27216*
_t**5*a**7*b**2*c**3 - 14580*_t**5*a**6*b**4*c**2 + 3159*_t**5*a**5*b**6*c
- 243*_t**5*a**4*b**8 + 252*_t**2*a**4*b*c**4 - 567*_t**2*a**3*b**3*c**3 +
378*_t**2*a**2*b**5*c**2 - 99*_t**2*a*b**7*c + 9*_t**2*b**9)/(2*a**2*c**5 -
 4*a*b**2*c**4 + b**4*c**3)))) - 1/(a*x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^2), x)
```

Mupad [B]
time = 6.89, size = 2978, normalized size = 4.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(27*a^7*c^3*x*(b
^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(4*a*
c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^
3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^7 + b^4*(-(4*a*c -
 b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b
^2)^3))^(1/3))/6)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*
a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*
b^2*c^2)))^(1/3) + log(36*a^9*c^6 + 9*a^7*b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/
3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/
3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3
*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c -
 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*((b^4
*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2
*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
)/(a^4*(4*a*c - b^2)^3))^(1/3))/6)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 +
32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b
^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a
^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - 1/(a*x) + log(36*a^9*c^6 + 9*a^7*b^4*c
^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(3^(1/2)*1i - 1)*(27*a^7*c^3*x*(b^6 - 8*a^3*
c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) - (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i + 1)
*(4*a*c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*
a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7 + b^4*(-(4
*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a
*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 - 1/2)*((b^7 + b^4*(-(4*a*c - b^2)
^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1
/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^
7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - log(36*a^9*c^6 + 9*a^7*b^4
*c^4 - 45*a^8*b^2*c^5 + (2^(2/3)*(3^(1/2)*1i + 1)*(27*a^7*c^3*x*(b^6 - 8*a^
3*c^3 + 18*a^2*b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i -
1)*(4*a*c - b^2)^2*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 3
2*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c
*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(-(b^7 + b^4*(-
(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4
*a*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^
2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^
```

```
(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*
a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) - log(36*a^9*c^6 + 9*a^7*b
^4*c^4 - 45*a^8*b^2*c^5 + (2^(2/3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2*
b^2*c^2 - 8*a*b^4*c) + (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2
)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 +
2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(3^(1/2)*1i + 1)*((b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4
*a*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 + 1/2)*(-(b^4*(-(4*a*c - b^2)^3)
^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64
*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3) + log(36*a^9*c^6 + 9*a^7*
b^4*c^4 - 45*a^8*b^2*c^5 - (2^(2/3)*(27*a^7*c^3*x*(b^6 - 8*a^3*c^3 + 18*a^2
*b^2*c^2 - 8*a*b^4*c) - (27*2^(1/3)*a^10*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^
2)^2*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 +
 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2
)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/4)*(3^(1/2)*1i - 1)*((b^4*(-(4*a*
c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(
4*a*c - b^2)^3))^(1/3))/12)*((3^(1/2)*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)^3
)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 6
4*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)))^(1/3)
```

3.150 $\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=612

$$-\frac{1}{2ax^2}+\frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}}{\sqrt[3]{b+\sqrt{}}}}{\sqrt{}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,a\left(b+\sqrt{b^2-4ac}\right)^{2/}}$$

```
[Out] -1/2/a/x^2-1/6*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(
1+b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*c^(2/3)
*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(
-4*a*c+b^2)^(1/2))^(2/3))*(1+b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b-(-4*a*c+b^2
)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2
)^(1/2))^(1/3))*3^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a*3^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(2/3)-1/6*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2
))^(1/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1
/12*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))
^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(1-b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b+
(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+
(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))*2^(2/3)/a*3^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A]
time = 0.58, antiderivative size = 612, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$,
Rules used = {1382, 1436, 206, 31, 648, 631, 210, 642}

$$\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/2*1/(a*x^2) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*
c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b -
 Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sq
rt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])
*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]])/(3*2^(1/3)*a*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(1 - b/Sqrt[b^2 - 4*a*c])*Log[(b + Sqr
t[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x]])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*
```

```
a*c])^(2/3)) + (c^(2/3)*(1 + b/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c
])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3
)*x^2])/(6*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(1 - b/Sqrt[
b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt
[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 -
 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^3+cx^6)}\,dx = -\frac{1}{2ax^2} + \frac{\int \frac{-2b-2cx^3}{a+bx^3+cx^6}\,dx}{2a}$$

$$= -\frac{1}{2ax^2} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\cdots}{\cdots}$$

$$= -\frac{1}{2ax^2} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(c\left(1+\frac{\cdots}{\sqrt{\cdots}}\right)\right)\cdots}{\cdots}$$

$$= -\frac{1}{2ax^2} - \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(\cdots\right)}{\cdots}$$

$$= -\frac{1}{2ax^2} - \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(\cdots\right)}{\cdots}$$

$$= -\frac{1}{2ax^2} + \frac{c^{2/3}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(1-\frac{\cdots}{\sqrt{\cdots}}\right)\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 75, normalized size = 0.12

$$-\frac{1}{2ax^2} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/2*1/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*Log[x - #1] + c*Log[x -
 #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 62, normalized size = 0.10

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^6+b\_Z^3+a\right)}\frac{\left(-c\_R^3-b\right)\ln\left(x-\_R\right)}{2\_R^5c+b\_R^2}}{3a}-\frac{1}{2a\,x^2}$
risch	$-\frac{1}{2a\,x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64c^3a^8-48a^7b^2c^2+12a^6b^4c-a^5b^6\right)\_Z^6+\left(-16a^4c^4+56a^3b^2c^3-41b^4c^2a^2+11b^6ca-b^8\right)\_Z^3+c^5\right)}\_R\ln\left(\left(\left((224c\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*sum((-_R^3*c-b)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a)
)-1/2/a/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^3 + b)/(c*x^6 + b*x^3 + a), x)/a - 1/2/(a*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6396 vs. 2(471) = 942.
time = 1.51, size = 6396, normalized size = 10.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 -
4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4
*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5
*b^2 - 4*a^6*c))^(1/3)*arctan(1/6*(sqrt(2)*(1/2)^(2/3)*sqrt(2*(b^10*c^4 - 1
0*a*b^8*c^5 + 35*a^2*b^6*c^6 - 50*a^3*b^4*c^7 + 25*a^4*b^2*c^8)*x^2 + (1/2)
```

^(2/3)*(b^16 - 18*a*b^14*c + 133*a^2*b^12*c^2 - 518*a^3*b^10*c^3 + 1135*a^4
*b^8*c^4 - 1380*a^5*b^6*c^5 + 850*a^6*b^4*c^6 - 200*a^7*b^2*c^7 - (a^5*b^14
 - 19*a^6*b^12*c + 147*a^7*b^10*c^2 - 590*a^8*b^8*c^3 + 1288*a^9*b^6*c^4 -
1440*a^10*b^4*c^5 + 640*a^11*b^2*c^6)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*
c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)
*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4
)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4
*a^6*c))^(2/3) + (1/2)^(1/3)*((a^5*b^11*c^2 - 15*a^6*b^9*c^3 + 87*a^7*b^7*c
^4 - 242*a^8*b^5*c^5 + 320*a^9*b^3*c^6 - 160*a^10*b*c^7)*x*sqrt((b^10 - 10*
a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*
a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - (b^13*c^2 - 14*a*b^11*c^3 +
75*a^2*b^9*c^4 - 190*a^3*b^7*c^5 + 225*a^4*b^5*c^6 - 100*a^5*b^3*c^7)*x)*(-
(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c +
35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c
 + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3))*(sqrt(3)*(a
^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*b^2*c^3 + 64*a^9*c^4)*sqrt
((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^
10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - sqrt(3)*(b^10 -
12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60*a^4*b^2*c^4))*(-(b^4 - 3*
a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^
6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^1
2*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3) - 2*(1/2)^(2/3)*(sqrt
(3)*(a^5*b^13*c^2 - 18*a^6*b^11*c^3 + 130*a^7*b^9*c^4 - 477*a^8*b^7*c^5 + 9
24*a^9*b^5*c^6 - 880*a^10*b^3*c^7 + 320*a^11*b*c^8)*x*sqrt((b^10 - 10*a*b^8
*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*
b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - sqrt(3)*(b^15*c^2 - 17*a*b^13*c^3
 + 117*a^2*b^11*c^4 - 415*a^3*b^9*c^5 + 795*a^4*b^7*c^6 - 775*a^5*b^5*c^7 +
 300*a^6*b^3*c^8)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 + (a^5*b^2 - 4*a^6*c)*sqr
t((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6
*c))^(2/3) - 2*sqrt(3)*(b^10*c^5 - 10*a*b^8*c^6 + 35*a^2*b^6*c^7 - 50*a^3*b
^4*c^8 + 25*a^4*b^2*c^9))/(b^10*c^5 - 10*a*b^8*c^6 + 35*a^2*b^6*c^7 - 50*a^
3*b^4*c^8 + 25*a^4*b^2*c^9)) - 4*sqrt(3)*(1/2)^(1/3)*a*x^2*(-(b^4 - 3*a*b^2
*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2
 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2
*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(1/3)*arctan(1/6*(sqrt(2)*(1/2)^
(2/3)*sqrt(2*(b^10*c^4 - 10*a*b^8*c^5 + 35*a^2*b^6*c^6 - 50*a^3*b^4*c^7 + 2
5*a^4*b^2*c^8)*x^2 + (1/2)^(2/3)*(b^16 - 18*a*b^14*c + 133*a^2*b^12*c^2 - 5
18*a^3*b^10*c^3 + 1135*a^4*b^8*c^4 - 1380*a^5*b^6*c^5 + 850*a^6*b^4*c^6 - 2
00*a^7*b^2*c^7 + (a^5*b^14 - 19*a^6*b^12*c + 147*a^7*b^10*c^2 - 590*a^8*b^8
*c^3 + 1288*a^9*b^6*c^4 - 1440*a^10*b^4*c^5 + 640*a^11*b^2*c^6)*sqrt((b^10
- 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6
- 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*(-(b^4 - 3*a*b^2*c + a^2
*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^

```
3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(2/3) - (1/2)^(1/3)*((a^5*b^11*c^2 - 15*
a^6*b^9*c^3 + 87*a^7*b^7*c^4 - 242*a^8*b^5*c^5 + 320*a^9*b^3*c^6 - 160*a^10
*b*c^7)*x*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^
4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) + (b
^13*c^2 - 14*a*b^11*c^3 + 75*a^2*b^9*c^4 - 190*a^3*b^7*c^5 + 225*a^4*b^5*c^
6 - 100*a^5*b^3*c^7)*x)*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*
sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)
/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*
a^6*c))^(1/3))*(sqrt(3)*(a^5*b^8 - 13*a^6*b^6*c + 60*a^7*b^4*c^2 - 112*a^8*
b^2*c^3 + 64*a^9*c^4)*sqrt((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4
*c^3 + 25*a^4*b^2*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)) + sqrt(3)*(b^10 - 12*a*b^8*c + 52*a^2*b^6*c^2 - 95*a^3*b^4*c^3 + 60
*a^4*b^2*c^4))*(-(b^4 - 3*a*b^2*c + a^2*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^1
0 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 50*a^3*b^4*c^3 + 25*a^4*b^2*c^4)/(a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^2 - 4*a^6*c))^(
2/3) - 2*(1/2)^(2/3)*(sqrt(3)*(a^5*b^13*c^2 - 1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)*x^3), x)
```

Mupad [**B**]
time = 10.65, size = 2500, normalized size = 4.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3 + c*x^6)),x)
```

[Out] log((2^(2/3)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4
*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) -
5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(72*a^8*b
c^6 + (2^(1/3)(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^1
0*b*c^3*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) +
41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3
))/2)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 -
56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^
3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/18 + 9*a^6*b^5*
c^4 - 54*a^7*b^3*c^5))/6 - 3*a^6*c^6*x*(2*a*c - b^2))*(-(b^8 + 16*a^4*c^4 +
b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*
c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/
2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) + lo
g((2^(2/3)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c
^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5
*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3)*(72*a^8*b*c
^6 + (2^(1/3)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^10*
b*c^3*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 4
1*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(1/3))
/2)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56
*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*
c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3))/18 + 9*a^6*b^5*c^
4 - 54*a^7*b^3*c^5))/6 - 3*a^6*c^6*x*(2*a*c - b^2))*(-(b^8 + 16*a^4*c^4 - b
^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c
- 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)
)/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) - 1/(2
*a*x^2) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c -
b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2
*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c
- b^2)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 - (2^(1/3)
*(3^(1/2)*1i + 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a
^10*b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*
c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*
c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*
a*c - b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2)
+ 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2
)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/
3))/36))/12 - 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 - 1/2)*(-(b^8 + 16
*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 -
11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b
^2)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^
(1/3) - log((2^(2/3)*(3^(1/2)*1i + 1)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b
^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(

```
-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c -
 b^2)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 + (2^(1/3)*(
3^(1/2)*1i - 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 - (81*2^(2/3)*a^1
0*b*c^3*(3^(1/2)*1i + 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c
- b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*
c - b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) +
41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))^(2/3)
)/36))/12 + 3*a^6*c^6*x*(2*a*c - b^2))*((3^(1/2)*1i)/2 + 1/2)*(-(b^8 + 16*a
^4*c^4 + b^5*(-(4*a*c - b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 1
1*a*b^6*c + 5*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^3*c*(-(4*a*c - b^2
)^3)^(1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1
/3) + log((2^(2/3)*(3^(1/2)*1i - 1)*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c - b^2
)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*(-(
4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b
^2)^3))^(1/3)*(72*a^8*b*c^6 + 9*a^6*b^5*c^4 - 54*a^7*b^3*c^5 - (2^(1/3)*(3^
(1/2)*1i + 1)*(81*a^8*c^3*x*(a*c - b^2)*(4*a*c - b^2)^2 + (81*2^(2/3)*a^10*
b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((b^8 + 16*a^4*c^4 - b^5*(-(4*a*c -
b^2)^3)^(1/2) + 41*a^2*b^4*c^2 - 56*a^3*b^2*c^3 - 11*a*b^6*c - 5*a^2*b*c^2*
(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c
- b^2)^3))^(1/3))/4)*((b^8 + 16*a^4*c^4 - b^5*(...
```

3.151 $\int \frac{x^{11}}{3+4x^3+x^6} \, dx$

Optimal. Leaf size=35

$$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)$$

```
[Out] -4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 715, 646, 31}

$$\frac{x^6}{6}-\frac{4x^3}{3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{9}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
```

```
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{3+4x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{3+4x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-4+x+\frac{12+13x}{3+4x+x^2}\right)\,dx, x, x^3\right)\\
&= -\frac{4x^3}{3}+\frac{x^6}{6}+\frac{1}{3}\text{Subst}\left(\int \frac{12+13x}{3+4x+x^2}\,dx, x, x^3\right)\\
&= -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right)+\frac{9}{2}\text{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right)\\
&= -\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{1}{6}\log\left(1+x^3\right)+\frac{9}{2}\log\left(3+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(3 + 4*x^3 + x^6),x]
[Out] (-4*x^3)/3 + x^6/6 - Log[1 + x^3]/6 + (9*Log[3 + x^3])/2
```

Maple [A]
time = 0.03, size = 28, normalized size = 0.80

method	result	size
default	$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{\ln(x^3+1)}{6}+\frac{9\ln(x^3+3)}{2}$	28
risch	$\frac{x^6}{6}-\frac{4x^3}{3}+\frac{8}{3}-\frac{\ln(x^3+1)}{6}+\frac{9\ln(x^3+3)}{2}$	29
norman	$-\frac{4x^3}{3}+\frac{x^6}{6}-\frac{\ln(1+x)}{6}+\frac{9\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
[Out] -4/3*x^3+1/6*x^6-1/6*ln(x^3+1)+9/2*ln(x^3+3)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 0.77

$$\frac{1}{6}x^6 - \frac{4}{3}x^3 + \frac{9}{2}\log\left(x^3+3\right) - \frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Fricas [A]
time = 0.34, size = 27, normalized size = 0.77

$$\frac{1}{6}x^6 - \frac{4}{3}x^3 + \frac{9}{2}\log\left(x^3+3\right) - \frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Sympy [A]
time = 0.04, size = 29, normalized size = 0.83

$$\frac{x^6}{6} - \frac{4x^3}{3} - \frac{\log\left(x^3+1\right)}{6} + \frac{9\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**6+4*x**3+3),x)
[Out] x**6/6 - 4*x**3/3 - log(x**3 + 1)/6 + 9*log(x**3 + 3)/2
```

Giac [A]
time = 3.22, size = 29, normalized size = 0.83

$$\frac{1}{6}x^6 - \frac{4}{3}x^3 + \frac{9}{2}\log\left(\left|x^3+3\right|\right) - \frac{1}{6}\log\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/6*x^6 - 4/3*x^3 + 9/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

Mupad [B]
time = 1.25, size = 27, normalized size = 0.77

$$\frac{9\,\ln\left(x^3+3\right)}{2} - \frac{\ln\left(x^3+1\right)}{6} - \frac{4\,x^3}{3} + \frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(4*x^3 + x^6 + 3),x)
[Out] (9*log(x^3 + 3))/2 - log(x^3 + 1)/6 - (4*x^3)/3 + x^6/6
```

3.152 $\int \frac{x^8}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=28

$$\frac{x^3}{3}+\frac{1}{6}\log\left(1+x^3\right)-\frac{3}{2}\log\left(3+x^3\right)$$

```
[Out] 1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 717, 646, 31}

$$\frac{x^3}{3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{3}{2}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(3 + 4*x^3 + x^6),x]
```

```
[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```

```
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{3+4x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{3+4x+x^2}\,dx, x, x^3\right)\\ &= \frac{x^3}{3}+\frac{1}{3}\text{Subst}\left(\int \frac{-3-4x}{3+4x+x^2}\,dx, x, x^3\right)\\ &= \frac{x^3}{3}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right)-\frac{3}{2}\text{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right)\\ &= \frac{x^3}{3}+\frac{1}{6}\log\left(1+x^3\right)-\frac{3}{2}\log\left(3+x^3\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 28, normalized size = 1.00

$$\frac{x^3}{3}+\frac{1}{6}\log\left(1+x^3\right)-\frac{3}{2}\log\left(3+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(3 + 4*x^3 + x^6),x]
[Out] x^3/3 + Log[1 + x^3]/6 - (3*Log[3 + x^3])/2
```

Maple [A]
time = 0.02, size = 23, normalized size = 0.82

method	result	size
default	$\frac{x^3}{3}+\frac{\ln(x^3+1)}{6}-\frac{3\ln(x^3+3)}{2}$	23
risch	$\frac{x^3}{3}+\frac{\ln(x^3+1)}{6}-\frac{3\ln(x^3+3)}{2}$	23
norman	$\frac{x^3}{3}+\frac{\ln(1+x)}{6}-\frac{3\ln(x^3+3)}{2}+\frac{\ln(x^2-x+1)}{6}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3+1/6*ln(x^3+1)-3/2*ln(x^3+3)
```

Maxima [A]
time = 0.31, size = 22, normalized size = 0.79

$$\frac{1}{3}\,x^3-\frac{3}{2}\,\log\left(x^3+3\right)+\frac{1}{6}\,\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Fricas [A]
time = 0.35, size = 22, normalized size = 0.79

$$\frac{1}{3}\,x^{3}-\frac{3}{2}\,\log\left(x^{3}+3\right)+\frac{1}{6}\,\log\left(x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 - 3/2*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Sympy [A]
time = 0.04, size = 22, normalized size = 0.79

$$\frac{x^{3}}{3}+\frac{\log\left(x^{3}+1\right)}{6}-\frac{3\log\left(x^{3}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**6+4*x**3+3),x)
```

```
[Out] x**3/3 + log(x**3 + 1)/6 - 3*log(x**3 + 3)/2
```

Giac [A]
time = 3.23, size = 24, normalized size = 0.86

$$\frac{1}{3}\,x^{3}-\frac{3}{2}\,\log\left(\left|x^{3}+3\right|\right)+\frac{1}{6}\,\log\left(\left|x^{3}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 3/2*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

Mupad [B]
time = 0.05, size = 22, normalized size = 0.79

$$\frac{\ln\left(x^{3}+1\right)}{6}-\frac{3\,\ln\left(x^{3}+3\right)}{2}+\frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x^3 + 1)/6 - (3*log(x^3 + 3))/2 + x^3/3
```

3.153 $\int \frac{x^5}{3+4x^3+x^6} dx$

Optimal. Leaf size=21

$$-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{2}\log\left(3+x^3\right)$$

```
[Out] -1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1371, 646, 31}

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(3 + 4*x^3 + x^6),x]
```

```
[Out] -1/6*Log[1 + x^3] + Log[3 + x^3]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{3+4x^3+x^6}\,dx = \frac{1}{3}\mathrm{Subst}\left(\int \frac{x}{3+4x+x^2}\,dx, x, x^3\right)$$
$$= -\left(\frac{1}{6}\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right)\right) + \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right)$$
$$= -\frac{1}{6}\log\left(1+x^3\right) + \frac{1}{2}\log\left(3+x^3\right)$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$-\frac{1}{6}\log\left(1+x^3\right) + \frac{1}{2}\log\left(3+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(3 + 4*x^3 + x^6),x]
[Out] -1/6*Log[1 + x^3] + Log[3 + x^3]/2
```

Maple [A]
time = 0.03, size = 18, normalized size = 0.86

method	result	size
default	$-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{2}$	18
risch	$-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{2}$	18
norman	$-\frac{\ln(1+x)}{6}+\frac{\ln(x^3+3)}{2}-\frac{\ln(x^2-x+1)}{6}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(x^3+1)+1/2*ln(x^3+3)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.81

$$\frac{1}{2}\log\left(x^3+3\right) - \frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Fricas [A]
time = 0.41, size = 17, normalized size = 0.81

$$\frac{1}{2}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/2*log(x^3 + 3) - 1/6*log(x^3 + 1)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.71

$$-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6+4*x**3+3),x)
[Out] -log(x**3 + 1)/6 + log(x**3 + 3)/2
```

Giac [A]
time = 4.81, size = 19, normalized size = 0.90

$$\frac{1}{2}\log\left(\left|x^3+3\right|\right)-\frac{1}{6}\log\left(\left|x^3+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/2*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1))
```

Mupad [B]
time = 0.05, size = 17, normalized size = 0.81

$$\frac{\ln\left(x^3+3\right)}{2}-\frac{\ln\left(x^3+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(4*x^3 + x^6 + 3),x)
[Out] log(x^3 + 3)/2 - log(x^3 + 1)/6
```

3.154 $\int \frac{x^2}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{3}\tanh^{-1}\left(2+x^3\right)$$

```
[Out] -1/3*arctanh(x^3+2)
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 2.10, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1366, 630, 31}

$$\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{6}\log\left(x^3+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(3 + 4*x^3 + x^6),x]
```

```
[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{3+4x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{3+4x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right)\\
&= \frac{1}{6}\log\left(1+x^3\right)-\frac{1}{6}\log\left(3+x^3\right)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 0.00, size = 21, normalized size = 2.10

$$\frac{1}{6}\log\left(1+x^3\right)-\frac{1}{6}\log\left(3+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(3 + 4*x^3 + x^6),x]
```

```
[Out] Log[1 + x^3]/6 - Log[3 + x^3]/6
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.
time = 0.02, size = 18, normalized size = 1.80

method	result	size
default	$\frac{\ln(x^3+1)}{6}-\frac{\ln(x^3+3)}{6}$	18
risch	$\frac{\ln(x^3+1)}{6}-\frac{\ln(x^3+3)}{6}$	18
norman	$\frac{\ln(1+x)}{6}-\frac{\ln(x^3+3)}{6}+\frac{\ln(x^2-x+1)}{6}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*ln(x^3+1)-1/6*ln(x^3+3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.
time = 0.31, size = 17, normalized size = 1.70

$$-\frac{1}{6}\log\left(x^3+3\right)+\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.
time = 0.34, size = 17, normalized size = 1.70

$$-\frac{1}{6}\log\left(x^3+3\right)+\frac{1}{6}\log\left(x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/6*log(x^3 + 3) + 1/6*log(x^3 + 1)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 1.50

$$\frac{\log\left(x^{3}+1\right)}{6}-\frac{\log\left(x^{3}+3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+4*x**3+3),x)
```

```
[Out] log(x**3 + 1)/6 - log(x**3 + 3)/6
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.
time = 4.27, size = 19, normalized size = 1.90

$$-\frac{1}{6}\log\left(\left|x^{3}+3\right|\right)+\frac{1}{6}\log\left(\left|x^{3}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/6*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1))
```

Mupad [B]
time = 0.38, size = 16, normalized size = 1.60

$$\frac{\operatorname{atanh}\left(\frac{9}{2\,\left(8\,x^{3}+6\right)}+\frac{5}{4}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(4*x^3 + x^6 + 3),x)
```

```
[Out] atanh(9/(2*(8*x^3 + 6)) + 5/4)/3
```

3.155 $\int \frac{1}{x(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{3}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{18}\log\left(3+x^3\right)$$

[Out] 1/3*ln(x)-1/6*ln(x^3+1)+1/18*ln(x^3+3)

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1371, 719, 29, 646, 31}

$$-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{18}\log\left(x^3+3\right)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(3 + 4*x^3 + x^6)),x]

[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(3+4x^3+x^6)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(3+4x+x^2)}\,dx, x, x^3\right)\\ &= \frac{1}{9}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right) + \frac{1}{9}\text{Subst}\left(\int \frac{-4-x}{3+4x+x^2}\,dx, x, x^3\right)\\ &= \frac{\log(x)}{3} + \frac{1}{18}\text{Subst}\left(\int \frac{1}{3+x}\,dx, x, x^3\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^3\right)\\ &= \frac{\log(x)}{3} - \frac{1}{6}\log\left(1+x^3\right) + \frac{1}{18}\log\left(3+x^3\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$\frac{\log(x)}{3} - \frac{1}{6}\log\left(1+x^3\right) + \frac{1}{18}\log\left(3+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(3 + 4*x^3 + x^6)),x]
[Out] Log[x]/3 - Log[1 + x^3]/6 + Log[3 + x^3]/18
```

Maple [A]
time = 0.02, size = 31, normalized size = 1.15

method	result	size
risch	$\frac{\ln(x)}{3} - \frac{\ln(x^3+1)}{6} + \frac{\ln(x^3+3)}{18}$	22
default	$\frac{\ln(x^3+3)}{18} + \frac{\ln(x)}{3} - \frac{\ln(1+x)}{6} - \frac{\ln(x^2-x+1)}{6}$	31
norman	$\frac{\ln(x^3+3)}{18} + \frac{\ln(x)}{3} - \frac{\ln(1+x)}{6} - \frac{\ln(x^2-x+1)}{6}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
[Out] 1/18*ln(x^3+3)+1/3*ln(x)-1/6*ln(1+x)-1/6*ln(x^2-x+1)
```

Maxima [A]
time = 0.29, size = 23, normalized size = 0.85

$$\frac{1}{18}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{9}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="maxima")
[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/9*log(x^3)
```

Fricas [A]
time = 0.35, size = 21, normalized size = 0.78

$$\frac{1}{18}\log\left(x^3+3\right)-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{3}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="fricas")
[Out] 1/18*log(x^3 + 3) - 1/6*log(x^3 + 1) + 1/3*log(x)
```

Sympy [A]
time = 0.05, size = 20, normalized size = 0.74

$$\frac{\log\left(x\right)}{3}-\frac{\log\left(x^3+1\right)}{6}+\frac{\log\left(x^3+3\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6+4*x**3+3),x)
[Out] log(x)/3 - log(x**3 + 1)/6 + log(x**3 + 3)/18
```

Giac [A]
time = 3.56, size = 24, normalized size = 0.89

$$\frac{1}{18}\log\left(\left|x^3+3\right|\right)-\frac{1}{6}\log\left(\left|x^3+1\right|\right)+\frac{1}{3}\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6+4*x^3+3),x, algorithm="giac")
[Out] 1/18*log(abs(x^3 + 3)) - 1/6*log(abs(x^3 + 1)) + 1/3*log(abs(x))
```

Mupad [B]
time = 1.26, size = 21, normalized size = 0.78

$$\frac{\ln\left(x^3+3\right)}{18}-\frac{\ln\left(x^3+1\right)}{6}+\frac{\ln\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(4*x^3 + x^6 + 3)),x)
[Out] log(x^3 + 3)/18 - log(x^3 + 1)/6 + log(x)/3
```

3.156 $\int \frac{1}{x^4(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{9x^3}-\frac{4\log(x)}{9}+\frac{1}{6}\log\left(1+x^3\right)-\frac{1}{54}\log\left(3+x^3\right)$$

```
[Out] -1/9/x^3-4/9*ln(x)+1/6*ln(x^3+1)-1/54*ln(x^3+3)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1371, 723, 814}

$$-\frac{1}{9x^3}+\frac{1}{6}\log\left(x^3+1\right)-\frac{1}{54}\log\left(x^3+3\right)-\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/9*1/x^3 - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4(3+4x^3+x^6)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(3+4x+x^2)}\,dx, x, x^3\right)$$
$$= -\frac{1}{9x^3} + \frac{1}{9}\text{Subst}\left(\int \frac{-4-x}{x(3+4x+x^2)}\,dx, x, x^3\right)$$
$$= -\frac{1}{9x^3} + \frac{1}{9}\text{Subst}\left(\int \left(-\frac{4}{3x} + \frac{3}{2(1+x)} - \frac{1}{6(3+x)}\right)dx, x, x^3\right)$$
$$= -\frac{1}{9x^3} - \frac{4\log(x)}{9} + \frac{1}{6}\log\left(1+x^3\right) - \frac{1}{54}\log\left(3+x^3\right)$$

Mathematica [**A**]
time = 0.00, size = 34, normalized size = 1.00

$$-\frac{1}{9x^3} - \frac{4\log(x)}{9} + \frac{1}{6}\log\left(1+x^3\right) - \frac{1}{54}\log\left(3+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(3 + 4*x^3 + x^6)),x]
[Out] -1/9*1/x^3 - (4*Log[x])/9 + Log[1 + x^3]/6 - Log[3 + x^3]/54
```

Maple [**A**]
time = 0.03, size = 36, normalized size = 1.06

method	result	size
risch	$-\frac{1}{9x^3}-\frac{4\ln(x)}{9}+\frac{\ln(x^3+1)}{6}-\frac{\ln(x^3+3)}{54}$	27
default	$-\frac{\ln(x^3+3)}{54}-\frac{1}{9x^3}-\frac{4\ln(x)}{9}+\frac{\ln(1+x)}{6}+\frac{\ln(x^2-x+1)}{6}$	36
norman	$-\frac{\ln(x^3+3)}{54}-\frac{1}{9x^3}-\frac{4\ln(x)}{9}+\frac{\ln(1+x)}{6}+\frac{\ln(x^2-x+1)}{6}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
[Out] -1/54*ln(x^3+3)-1/9/x^3-4/9*ln(x)+1/6*ln(1+x)+1/6*ln(x^2-x+1)
```

Maxima [**A**]
time = 0.29, size = 28, normalized size = 0.82

$$-\frac{1}{9\,x^3} - \frac{1}{54}\log\left(x^3+3\right) + \frac{1}{6}\log\left(x^3+1\right) - \frac{4}{27}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/9/x^3 - 1/54*log(x^3 + 3) + 1/6*log(x^3 + 1) - 4/27*log(x^3)
```

Fricas [A]
time = 0.39, size = 35, normalized size = 1.03

$$-\frac{x^3 \log\left(x^3+3\right) - 9\,x^3 \log\left(x^3+1\right) + 24\,x^3 \log\left(x\right) + 6}{54\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/54*(x^3*log(x^3 + 3) - 9*x^3*log(x^3 + 1) + 24*x^3*log(x) + 6)/x^3
```

Sympy [A]
time = 0.06, size = 29, normalized size = 0.85

$$-\frac{4\log\left(x\right)}{9} + \frac{\log\left(x^3+1\right)}{6} - \frac{\log\left(x^3+3\right)}{54} - \frac{1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6+4*x**3+3),x)
```

```
[Out] -4*log(x)/9 + log(x**3 + 1)/6 - log(x**3 + 3)/54 - 1/(9*x**3)
```

Giac [A]
time = 4.19, size = 36, normalized size = 1.06

$$\frac{4\,x^3-3}{27\,x^3} - \frac{1}{54}\,\log\left(\left|x^3+3\right|\right) + \frac{1}{6}\,\log\left(\left|x^3+1\right|\right) - \frac{4}{9}\,\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/27*(4*x^3 - 3)/x^3 - 1/54*log(abs(x^3 + 3)) + 1/6*log(abs(x^3 + 1)) - 4/9
*log(abs(x))
```

Mupad [B]
time = 1.23, size = 26, normalized size = 0.76

$$\frac{\ln\left(x^3+1\right)}{6} - \frac{\ln\left(x^3+3\right)}{54} - \frac{4\,\ln\left(x\right)}{9} - \frac{1}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x^3 + 1)/6 - log(x^3 + 3)/54 - (4*log(x))/9 - 1/(9*x^3)
```

3.157 $\int \frac{1}{x^7(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\log(x)}{27}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{162}\log\left(3+x^3\right)$$

```
[Out] -1/18/x^6+4/27/x^3+13/27*ln(x)-1/6*ln(x^3+1)+1/162*ln(x^3+3)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1371, 723, 814}

$$-\frac{1}{18x^6}+\frac{4}{27x^3}-\frac{1}{6}\log\left(x^3+1\right)+\frac{1}{162}\log\left(x^3+3\right)+\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/18*1/x^6 + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/1
62
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(3+4x^3+x^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3\left(3+4x+x^2\right)}\,dx, x, x^3\right) \\
&= -\frac{1}{18x^6}+\frac{1}{9}\text{Subst}\left(\int \frac{-4-x}{x^2\left(3+4x+x^2\right)}\,dx, x, x^3\right) \\
&= -\frac{1}{18x^6}+\frac{1}{9}\text{Subst}\left(\int \left(-\frac{4}{3x^2}+\frac{13}{9x}-\frac{3}{2(1+x)}+\frac{1}{18(3+x)}\right)dx, x, x^3\right) \\
&= -\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\log(x)}{27}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{162}\log\left(3+x^3\right)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 41, normalized size = 1.00

$$-\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\log(x)}{27}-\frac{1}{6}\log\left(1+x^3\right)+\frac{1}{162}\log\left(3+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/18*1/x^6 + 4/(27*x^3) + (13*Log[x])/27 - Log[1 + x^3]/6 + Log[3 + x^3]/1
62
```

Maple [A]
time = 0.03, size = 41, normalized size = 1.00

method	result	size
risch	$\frac{-\frac{1}{18}+\frac{4x^3}{27}}{x^6}+\frac{13\ln(x)}{27}-\frac{\ln(x^3+1)}{6}+\frac{\ln(x^3+3)}{162}$	33
default	$\frac{\ln(x^3+3)}{162}-\frac{1}{18x^6}+\frac{4}{27x^3}+\frac{13\ln(x)}{27}-\frac{\ln(1+x)}{6}-\frac{\ln(x^2-x+1)}{6}$	41
norman	$\frac{-\frac{1}{18}+\frac{4x^3}{27}}{x^6}+\frac{13\ln(x)}{27}-\frac{\ln(1+x)}{6}+\frac{\ln(x^3+3)}{162}-\frac{\ln(x^2-x+1)}{6}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/162*ln(x^3+3)-1/18/x^6+4/27/x^3+13/27*ln(x)-1/6*ln(1+x)-1/6*ln(x^2-x+1)
```

Maxima [A]
time = 0.28, size = 35, normalized size = 0.85

$$\frac{8\,x^3-3}{54\,x^6}+\frac{1}{162}\,\log\left(x^3+3\right)-\frac{1}{6}\,\log\left(x^3+1\right)+\frac{13}{81}\,\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/54*(8*x^3 - 3)/x^6 + 1/162*log(x^3 + 3) - 1/6*log(x^3 + 1) + 13/81*log(x^
3)
```

Fricas [A]
time = 0.35, size = 40, normalized size = 0.98

$$\frac{x^6 \log(x^3+3) - 27\,x^6 \log(x^3+1) + 78\,x^6 \log(x) + 24\,x^3 - 9}{162\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/162*(x^6*log(x^3 + 3) - 27*x^6*log(x^3 + 1) + 78*x^6*log(x) + 24*x^3 - 9)
/x^6
```

Sympy [A]
time = 0.07, size = 34, normalized size = 0.83

$$\frac{13\log(x)}{27} - \frac{\log(x^3+1)}{6} + \frac{\log(x^3+3)}{162} + \frac{8x^3-3}{54x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**6+4*x**3+3),x)
```

```
[Out] 13*log(x)/27 - log(x**3 + 1)/6 + log(x**3 + 3)/162 + (8*x**3 - 3)/(54*x**6)
```

Giac [A]
time = 4.41, size = 41, normalized size = 1.00

$$-\frac{13\,x^6 - 8\,x^3 + 3}{54\,x^6} + \frac{1}{162}\log\left(\left|x^3+3\right|\right) - \frac{1}{6}\log\left(\left|x^3+1\right|\right) + \frac{13}{27}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/54*(13*x^6 - 8*x^3 + 3)/x^6 + 1/162*log(abs(x^3 + 3)) - 1/6*log(abs(x^3
+ 1)) + 13/27*log(abs(x))
```

Mupad [B]
time = 0.04, size = 32, normalized size = 0.78

$$\frac{\ln(x^3+3)}{162} - \frac{\ln(x^3+1)}{6} + \frac{13\,\ln(x)}{27} + \frac{\frac{4\,x^3}{27} - \frac{1}{18}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x^3 + 3)/162 - log(x^3 + 1)/6 + (13*log(x))/27 + ((4*x^3)/27 - 1/18)/x^
6
```

3.158 $\int \frac{x^{10}}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=124

$$-2x^2+\frac{x^5}{5}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x\right.$$

```
[Out] -2*x^2+1/5*x^5-9/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-3/
2*3^(2/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+3/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x
^2)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1381, 1516, 1524, 298, 31, 648, 632, 210, 642, 631}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{x^5}{5}-2x^2-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{3}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)+\frac{1}{6}\log(x+1)-\frac{3}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] -2*x^2 + x^5/5 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (9*3^(1/6)*ArcTan[
(3^(1/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3*3^(2/3)*Log[3^(1/3) + x])/2
 - Log[1 - x + x^2]/12 + (3*3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{3+4x^3+x^6}\,dx &= \frac{x^5}{5}-\frac{1}{5}\int \frac{x^4(15+20x^3)}{3+4x^3+x^6}\,dx\\
&= -2x^2+\frac{x^5}{5}+\frac{1}{10}\int \frac{x(120+130x^3)}{3+4x^3+x^6}\,dx\\
&= -2x^2+\frac{x^5}{5}-\frac{1}{2}\int \frac{x}{1+x^3}\,dx+\frac{27}{2}\int \frac{x}{3+x^3}\,dx\\
&= -2x^2+\frac{x^5}{5}+\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx-\frac{1}{2}\left(3\ 3^{2/3}\right)\int \frac{1}{\sqrt[3]{3}+x}\,dx+\frac{1}{2}(3\ 3^{2\\
&= -2x^2+\frac{x^5}{5}+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x\\
&= -2x^2+\frac{x^5}{5}+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{3}{4}3^{2/3}\log\left(3\\
&= -2x^2+\frac{x^5}{5}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{9}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{3}{2}3^{2/3}\log\left(\right.\end{aligned}$$

Mathematica [A]
time = 0.04, size = 118, normalized size = 0.95

$$\frac{1}{60}\left(-120x^2+12x^5-270\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-10\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+10\log(1+x)-90\ 3^{2/3}\log\left(3+3^{2/3}x\right)-5\log\left(1-x+x^2\right)+45\ 3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-120*x^2 + 12*x^5 - 270*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 10*Sqrt[
3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Log[1 + x] - 90*3^(2/3)*Log[3 + 3^(2/3)*
x] - 5*Log[1 - x + x^2] + 45*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/60
```

Maple [A]
time = 0.03, size = 94, normalized size = 0.76

method	result
risch	$\frac{x^5}{5}-2x^2+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3+9\right)}\_R\ln\left(\_R^2+3x\right)\right)}{2}+\frac{\ln(1+x)}{6}-\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$
default	$\frac{x^5}{5}-2x^2-\frac{3\,3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}+\frac{3\,3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}+\frac{9\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+\frac{\ln(1+x)}{6}-\frac{\ln(x^2-x+1)}{12}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5-2*x^2-3/2*3^(2/3)*ln(3^(1/3)+x)+3/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^
2)+9/2*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)-1/12*ln(x^
2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 94, normalized size = 0.76

$$\frac{1}{5}x^5-2x^2+\frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{3}{2}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{9}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)*
log(x + 3^(1/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3^(1/6)*
arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1
)
```

Fricas [A]
time = 0.37, size = 102, normalized size = 0.82

$$\frac{1}{5}x^5-2x^2+\frac{3}{2}\sqrt{3}(-9)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\sqrt{3}\left(2(-9)^{\frac{1}{3}}x+3\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{3}{4}(-9)^{\frac{1}{3}}\log\left(3x^2-(-9)^{\frac{2}{3}}x-3(-9)^{\frac{1}{3}}\right)+\frac{3}{2}(-9)^{\frac{1}{3}}\log\left(3x+(-9)^{\frac{2}{3}}\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/2*sqrt(3)*(-9)^(1/3)*arctan(1/9*sqrt(3)*(2*(-9)^(1/3)*x
 + 3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/4*(-9)^(1/3)*log(3*x
^2 - (-9)^(2/3)*x - 3*(-9)^(1/3)) + 3/2*(-9)^(1/3)*log(3*x + (-9)^(2/3)) -
1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.31, size = 144, normalized size = 1.16

$$\frac{x^5}{5}-2x^2+\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{3872\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^5}{3281}+\frac{3188648\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^2}{88587}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{3188648\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^2}{88587}+\frac{3872\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^5}{3281}\right)+\mathrm{RootSum}\left(8t^3+243,\left(t\mapsto t\log\left(\frac{3872t^5}{3281}+\frac{3188648t^2}{88587}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(x**6+4*x**3+3),x)
```

```
[Out] x**5/5 - 2*x**2 + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 3872*(-1/12
 - sqrt(3)*I/12)**5/3281 + 3188648*(-1/12 - sqrt(3)*I/12)**2/88587) + (-1/1
2 + sqrt(3)*I/12)*log(x + 3188648*(-1/12 + sqrt(3)*I/12)**2/88587 + 3872*(-
1/12 + sqrt(3)*I/12)**5/3281) + RootSum(8*_t**3 + 243, Lambda(_t, _t*log(38
72*_t**5/3281 + 3188648*_t**2/88587 + x)))
```

Giac [A]
time = 5.47, size = 96, normalized size = 0.77

$$\frac{1}{5}x^5-2x^2+\frac{3}{4}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{3}{2}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{9}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/5*x^5 - 2*x^2 + 3/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 3/2*3^(2/3)*
log(abs(x + 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9/2*3^(
1/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(
abs(x + 1))
```

Mupad [B]
time = 0.24, size = 124, normalized size = 1.00

$$\frac{\ln(x+1)}{6}-\frac{3\,3^{2/3}\,\ln\left(x+3^{1/3}\right)}{2}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-2x^2+\frac{x^5}{5}-\frac{3(-1)^{1/3}\,\ln\left(x-\frac{(-1)^{1/3}3^{1/3}}{2}-\frac{(-1)^{1/6}3^{5/6}}{2}+\frac{3^{1/3}}{2}\right)\left(3^{2/3}+3^{1/6}\,3\mathrm{i}\right)}{4}+\frac{3(-1)^{1/3}\,3^{2/3}\,\ln\left(x+(-1)^{2/3}3^{1/3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x + 1)/6 - (3*3^(2/3)*log(x + 3^(1/3)))/2 + log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 + 1/12) - 2*x^2 + x^5/5 - (3*(-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 -
((-1)^(1/6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/4 + (3*(-1)^(1/
3)*3^(2/3)*log(x + (-1)^(2/3)*3^(1/3)))/2
```

3.159 $\int \frac{x^9}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=122

$$-4x+\frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{3}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+\right.$$

```
[Out] -4*x+1/4*x^4-3/2*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+3/2*
3^(1/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-3/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2
)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1381, 1516, 1436, 206, 31, 648, 632, 210, 642, 631}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{3}{2}3^{5/6}\text{ArcTan}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{x^4}{4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{3}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-4x-\frac{1}{6}\log(x+1)+\frac{3}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] -4*x + x^4/4 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3*3^(5/6)*ArcTan[(3
^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3*3^(1/3)*Log[3^(1/3) + x])/2 +
 Log[1 - x + x^2]/12 - (3*3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
```

```
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^9}{3+4x^3+x^6}\,dx = \frac{x^4}{4} - \frac{1}{4}\int \frac{x^3(12+16x^3)}{3+4x^3+x^6}\,dx$$

$$= -4x + \frac{x^4}{4} + \frac{1}{4}\int \frac{48+52x^3}{3+4x^3+x^6}\,dx$$

$$= -4x + \frac{x^4}{4} - \frac{1}{2}\int \frac{1}{1+x^3}\,dx + \frac{27}{2}\int \frac{1}{3+x^3}\,dx$$

$$= -4x + \frac{x^4}{4} - \frac{1}{6}\int \frac{1}{1+x}\,dx - \frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx + \frac{1}{2}\left(3\sqrt[3]{3}\right)\int \frac{1}{\sqrt[3]{3}+x}\,dx + \frac{1}{2}\left(3\sqrt[3]{\right.$$

$$= -4x + \frac{x^4}{4} - \frac{1}{6}\log(1+x) + \frac{3}{2}\sqrt[3]{3}\,\log\left(\sqrt[3]{3}+x\right) + \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{}{1-x}$$

$$= -4x + \frac{x^4}{4} - \frac{1}{6}\log(1+x) + \frac{3}{2}\sqrt[3]{3}\,\log\left(\sqrt[3]{3}+x\right) + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{3}{4}\sqrt[3]{3}\,\log\left(\right.$$

$$= -4x + \frac{x^4}{4} + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{3}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) - \frac{1}{6}\log(1+x) + \frac{3}{2}\sqrt[3]{3}\,\log\left(\right.$$

Mathematica [A]
time = 0.02, size = 114, normalized size = 0.93

$$\frac{1}{12}\left(-48x+3x^4-18\,3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\,\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\log(1+x)+18\sqrt[3]{3}\,\log\left(3+3^{2/3}x\right)+\log\left(1-x+x^2\right)-9\sqrt[3]{3}\,\log\left(3-3^{2/3}x+\sqrt[3]{3}\,x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-48*x + 3*x^4 - 18*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*Arc
Tan[(-1 + 2*x)/Sqrt[3]] - 2*Log[1 + x] + 18*3^(1/3)*Log[3 + 3^(2/3)*x] + Lo
g[1 - x + x^2] - 9*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [A]
time = 0.03, size = 92, normalized size = 0.75

method	result
risch	$\frac{x^4}{4}-4x+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^3-3\right)}\_R\ln\left(x+\_R\right)\right)}{2}+\frac{\ln\left(4x^2-4x+4\right)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}-\frac{\ln(1+x)}{6}$
default	$\frac{x^4}{4}-4x+\frac{3\,3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}-\frac{3\,3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}+\frac{3\,3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}-\frac{\ln(1+x)}{6}+\frac{\ln(x^2-x+1)}{12}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)

[Out] 1/4*x^4-4*x+3/2*3^(1/3)*ln(3^(1/3)+x)-3/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
+3/2*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)+1/12*ln(x^2-
x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))

Maxima [A]
time = 0.51, size = 92, normalized size = 0.75

$$\frac{1}{4}x^4+\frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)-4x+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="maxima")

[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)

Fricas [A]
time = 0.38, size = 90, normalized size = 0.74

$$\frac{1}{4}x^4+\frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{2}{3}\cdot 3^{\frac{1}{6}}x-\frac{1}{3}\sqrt{3}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)-4x+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="fricas")

[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(2/3*3^(1/6)*x - 1/3*sqrt(3)) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(x + 3^(1/3)) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)

Sympy [C] Result contains complex when optimal does not.
time = 0.31, size = 129, normalized size = 1.06

$$\frac{x^4}{4}-4x-\frac{\log(x+1)}{6}+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{9841}{19692}-\frac{9841\sqrt{3}\,i}{19692}+\frac{360\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^4}{547}\right)+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{9841}{19692}+\frac{360\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^4}{547}+\frac{9841\sqrt{3}\,i}{19692}\right)+\mathrm{RootSum}\left(8t^3-81,\left(t\mapsto t\log\left(\frac{360t^4}{547}-\frac{9841t}{1641}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**6+4*x**3+3),x)
```

```
[Out] x**4/4 - 4*x - log(x + 1)/6 + (1/12 + sqrt(3)*I/12)*log(x - 9841/19692 - 98
41*sqrt(3)*I/19692 + 360*(1/12 + sqrt(3)*I/12)**4/547) + (1/12 - sqrt(3)*I/
12)*log(x - 9841/19692 + 360*(1/12 - sqrt(3)*I/12)**4/547 + 9841*sqrt(3)*I/
19692) + RootSum(8*_t**3 - 81, Lambda(_t, _t*log(360*_t**4/547 - 9841*_t/16
41 + x)))
```

Giac [A]
time = 4.30, size = 94, normalized size = 0.77

$$\frac{1}{4}\,x^{4}+\frac{3}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-\frac{3}{4}\cdot 3^{\frac{1}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{3}{2}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)-4\,x+\frac{1}{12}\,\log\left(x^{2}-x+1\right)-\frac{1}{6}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 3/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*x - 1)) - 3/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 3
/2*3^(1/3)*log(abs(x + 3^(1/3))) - 4*x + 1/12*log(x^2 - x + 1) - 1/6*log(ab
s(x + 1))
```

Mupad [B]
time = 1.42, size = 119, normalized size = 0.98

$$\frac{3\,3^{1/3}\,\ln\left(x+3^{1/3}\right)}{2}-\frac{\ln\left(x+1\right)}{6}-4\,x+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{1}{12}+\frac{\sqrt{3}\,1{}\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-\frac{1}{12}+\frac{\sqrt{3}\,1{}\mathrm{i}}{12}\right)+\frac{x^4}{4}-\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1{}\mathrm{i}}{2}\right)\,\left(\frac{3\,3^{1/3}}{4}+\frac{3^{5/6}\,3{}\mathrm{i}}{4}\right)+3^{1/3}\,\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1{}\mathrm{i}}{2}\right)\,\left(-\frac{3}{4}+\frac{\sqrt{3}\,3{}\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3*3^(1/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - 4*x + log(x - (3^(1/2)*1i)/
2 - 1/2)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)
*1i)/12 - 1/12) + x^4/4 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*((3*3^(1/3))/
4 + (3^(5/6)*3i)/4) + 3^(1/3)*log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*((3^(1/2)
*3i)/4 - 3/4)
```

3.160 $\int \frac{x^7}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=119

$$\frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{4}3$$

[Out] 1/2*x^2+3/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/2*3^(2/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1381, 1524, 298, 31, 648, 632, 210, 642, 631}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{x^2}{2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{4}3^{2/3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)-\frac{1}{6}\log(x+1)+\frac{1}{2}3^{2/3}\log\left(x+\sqrt[3]{3}\right)$$

Antiderivative was successfully verified.

[In] Int[x^7/(3 + 4*x^3 + x^6),x]

[Out] x^2/2 - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)])/2 - Log[1 + x]/6 + (3^(2/3)*Log[3^(1/3) + x])/2 + Log[1 - x + x^2]/12 - (3^(2/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^7}{3+4x^3+x^6}\,dx = \frac{x^2}{2}-\frac{1}{2}\int \frac{x(6+8x^3)}{3+4x^3+x^6}\,dx$$
$$= \frac{x^2}{2}+\frac{1}{2}\int \frac{x}{1+x^3}\,dx-\frac{9}{2}\int \frac{x}{3+x^3}\,dx$$
$$= \frac{x^2}{2}-\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx+\frac{1}{2}3^{2/3}\int \frac{1}{\sqrt[3]{3}+x}\,dx-\frac{1}{2}3^{2/3}\int \frac{\sqrt[3]{3}-}{3^{2/3}-\sqrt[3]{3}}$$
$$= \frac{x^2}{2}-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx$$
$$= \frac{x^2}{2}-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{4}3^{2/3}\log\left(3^{2/3}-\sqrt[3]{}\right.$$
$$= \frac{x^2}{2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{3}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{6}\log(1+x)+\frac{1}{2}3^{2/3}\log\left(\sqrt[3]{3}+x\right.$$

Mathematica [A]
time = 0.02, size = 111, normalized size = 0.93

$$\frac{1}{12}\left(6x^2+18\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\log(1+x)+6\,3^{2/3}\log\left(3+3^{2/3}x\right)+\log\left(1-x+x^2\right)-3\,3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}\,x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(3 + 4*x^3 + x^6),x]
```

```
[Out] (6*x^2 + 18*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1
+ 2*x)/Sqrt[3]] - 2*Log[1 + x] + 6*3^(2/3)*Log[3 + 3^(2/3)*x] + Log[1 - x +
 x^2] - 3*3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [A]
time = 0.03, size = 89, normalized size = 0.75

method	result
risch	$\frac{x^2}{2}+\frac{\ln(4x^2-4x+4)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3-9)}\_R\ln\left(\_R^2+3x\right)\right)}{2}-\frac{\ln(1+x)}{6}$
default	$\frac{x^2}{2}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}-\frac{3\,3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}-\frac{\ln(1+x)}{6}+\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\,\arctan}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+1/2*3^(2/3)*ln(3^(1/3)+x)-1/4*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-3/2
*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)+1/12*ln(x^2-x+1)
+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 89, normalized size = 0.75

$$\frac{1}{2}\,x^{2}-\frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-\frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^{2}-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(x +
3^(1/3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*arctan(1
/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Fricas [A]
time = 0.36, size = 99, normalized size = 0.83

$$\frac{1}{2}\,x^{2}-\frac{1}{2}\cdot 9^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{2}{9}\cdot 9^{\frac{1}{3}}\sqrt{3}\,x-\frac{1}{3}\,\sqrt{3}\right)+\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-\frac{1}{4}\cdot 9^{\frac{1}{3}}\log\left(3\,x^{2}-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+\frac{1}{2}\cdot 9^{\frac{1}{3}}\log\left(3\,x+9^{\frac{2}{3}}\right)+\frac{1}{12}\log\left(x^{2}-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*9^(1/3)*sqrt(3)*arctan(2/9*9^(1/3)*sqrt(3)*x - 1/3*sqrt(3)) +
 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*9^(1/3)*log(3*x^2 - 9^(2/3
)*x + 3*9^(1/3)) + 1/2*9^(1/3)*log(3*x + 9^(2/3)) + 1/12*log(x^2 - x + 1) -
 1/6*log(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.31, size = 134, normalized size = 1.13

$$\frac{x^{2}}{2}-\frac{\log\left(x+1\right)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{6562\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{2}}{183}-\frac{1872\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{5}}{61}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{1872\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{5}}{61}+\frac{6562\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{2}}{183}\right)+\operatorname{RootSum}\left(8t^{3}-9,\left(t\mapsto t\log\left(-\frac{1872t^{5}}{61}+\frac{6562t^{2}}{183}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**6+4*x**3+3),x)
```

```
[Out] x**2/2 - log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 6562*(1/12 - sqrt(3)*
I/12)**2/183 - 1872*(1/12 - sqrt(3)*I/12)**5/61) + (1/12 + sqrt(3)*I/12)*lo
g(x - 1872*(1/12 + sqrt(3)*I/12)**5/61 + 6562*(1/12 + sqrt(3)*I/12)**2/183)
 + RootSum(8*_t**3 - 9, Lambda(_t, _t*log(-1872*_t**5/61 + 6562*_t**2/183 +
 x)))
```

Giac [A]
time = 3.66, size = 91, normalized size = 0.76

$$\frac{1}{2}\,x^{2} - \frac{1}{4}\cdot 3^{\frac{2}{3}}\log\left(x^{2} - 3^{\frac{1}{3}}x + 3^{\frac{2}{3}}\right) + \frac{1}{2}\cdot 3^{\frac{2}{3}}\log\left(\left|x + 3^{\frac{1}{3}}\right|\right) + \frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x - 1)\right) - \frac{3}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) + \frac{1}{12}\,\log\left(x^{2} - x + 1\right) - \frac{1}{6}\,\log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/4*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/2*3^(2/3)*log(abs(
x + 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 3/2*3^(1/6)*arc
tan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x +
1))
```

Mupad [B]
time = 0.19, size = 118, normalized size = 0.99

$$\frac{3^{2/3}\,\ln\left(x + 3^{1/3}\right)}{2} - \frac{\ln\left(x + 1\right)}{6} - \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) + \frac{x^{2}}{2} - \ln\left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{4} - \frac{3^{1/6}\,3\mathrm{i}}{4}\right) - \ln\left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{4} + \frac{3^{1/6}\,3\mathrm{i}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3^(2/3)*log(x + 3^(1/3)))/2 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
+ 1/12) + x^2/2 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(2/3)/4 - (3^(1/6)
*3i)/4) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/3)/4 + (3^(1/6)*3i)/4)
```

3.161 $\int \frac{x^6}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=113

$$x-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{1}{4}\sqrt[3]{3}$$

[Out] x+1/2*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/2*3^(1/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1381, 1436, 206, 31, 648, 632, 210, 642, 631}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\text{ArcTan}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{4}\sqrt[3]{3}\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)+x+\frac{1}{6}\log(x+1)-\frac{1}{2}\sqrt[3]{3}\log\left(x+\sqrt[3]{3}\right)$$

Antiderivative was successfully verified.

[In] Int[x^6/(3 + 4*x^3 + x^6),x]

[Out] x - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + (3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)])/2 + Log[1 + x]/6 - (3^(1/3)*Log[3^(1/3) + x])/2 - Log[1 - x + x^2]/12 + (3^(1/3)*Log[3^(2/3) - 3^(1/3)*x + x^2])/4

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^6}{3+4x^3+x^6}\,dx = x-\int \frac{3+4x^3}{3+4x^3+x^6}\,dx$$
$$= x+\frac{1}{2}\int \frac{1}{1+x^3}\,dx-\frac{9}{2}\int \frac{1}{3+x^3}\,dx$$
$$= x+\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx-\frac{1}{2}\sqrt[3]{3}\int \frac{1}{\sqrt[3]{3}+x}\,dx-\frac{1}{2}\sqrt[3]{3}\int \frac{2\sqrt[3]{3}}{3^{2/3}-\sqrt[3]{}}$$
$$= x+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx$$
$$= x+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right)-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{1}{4}\sqrt[3]{3}\log\left(3^{2/3}-\sqrt[3]{}\right.$$
$$= x-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{1}{2}3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{1}{2}\sqrt[3]{3}\log\left(\sqrt[3]{3}+x\right.$$

Mathematica [A]
time = 0.02, size = 111, normalized size = 0.98

$$\frac{1}{12}\left(12x+6\,3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+2\log(1+x)-6\sqrt[3]{3}\log\left(3+3^{2/3}x\right)-\log\left(1-x+x^2\right)+3\sqrt[3]{3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(3 + 4*x^3 + x^6),x]
```

```
[Out] (12*x + 6*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 2*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] + 2*Log[1 + x] - 6*3^(1/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x
^2] + 3*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [A]
time = 0.03, size = 85, normalized size = 0.75

method	result
risch	$x+\frac{\ln(1+x)}{6}-\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(\_Z^3+3)}\_R\ln\left(x-\_R\right)\right)}{2}$
default	$x-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{2}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{4}-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+\frac{\ln(1+x)}{6}-\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\,\arctan}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] x-1/2*3^(1/3)*ln(3^(1/3)+x)+1/4*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/2*3^(5/
6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)-1/12*ln(x^2-x+1)+1/6*3
^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 85, normalized size = 0.75

$$-\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)+x-\frac{1}{12}\log\left(x^{2}-x+1\right)+\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1/3
)*log(x + 3^(1/3)) + x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Fricas [A]
time = 0.37, size = 88, normalized size = 0.78

$$\frac{1}{2}\sqrt{3}(-3)^{\frac{1}{3}}\arctan\left(\frac{1}{9}\sqrt{3}\left(2(-3)^{\frac{2}{3}}x-3\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{4}(-3)^{\frac{1}{3}}\log\left(x^{2}+(-3)^{\frac{1}{3}}x+(-3)^{\frac{2}{3}}\right)+\frac{1}{2}(-3)^{\frac{1}{3}}\log\left(x-(-3)^{\frac{1}{3}}\right)+x-\frac{1}{12}\log\left(x^{2}-x+1\right)+\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*(-3)^(1/3)*arctan(1/9*sqrt(3)*(2*(-3)^(2/3)*x - 3)) + 1/6*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/4*(-3)^(1/3)*log(x^2 + (-3)^(1/3)*x +
(-3)^(2/3)) + 1/2*(-3)^(1/3)*log(x - (-3)^(1/3)) + x - 1/12*log(x^2 - x + 1
) + 1/6*log(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.31, size = 126, normalized size = 1.12

$$x+\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{121}{246}-\frac{121\sqrt{3}i}{246}+\frac{864\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^{4}}{41}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x-\frac{121}{246}+\frac{864\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^{4}}{41}+\frac{121\sqrt{3}i}{246}\right)+\operatorname{RootSum}\left(8t^{3}+3,\left(t\mapsto t\log\left(\frac{864t^{4}}{41}+\frac{242t}{41}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6+4*x**3+3),x)
```

```
[Out] x + log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 121/246 - 121*sqrt(3)*I/2
46 + 864*(-1/12 - sqrt(3)*I/12)**4/41) + (-1/12 + sqrt(3)*I/12)*log(x - 121
/246 + 864*(-1/12 + sqrt(3)*I/12)**4/41 + 121*sqrt(3)*I/246) + RootSum(8*_t
**3 + 3, Lambda(_t, _t*log(864*_t**4/41 + 242*_t/41 + x)))
```

Giac [A]
time = 3.17, size = 87, normalized size = 0.77

$$-\frac{1}{2}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\cdot 3^{\frac{1}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{2}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+x-\frac{1}{12}\log\left(x^{2}-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/2*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) + 1/4*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/2*3^(1/3
)*log(abs(x + 3^(1/3))) + x - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

Mupad [B]
time = 0.16, size = 104, normalized size = 0.92

$$x+\frac{\ln\left(x+1\right)}{6}-\frac{3^{1/3}\,\ln\left(x+3^{1/3}\right)}{2}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{4}-\frac{3^{5/6}\,1\mathrm{i}}{4}\right)+\frac{{\left(-1\right)}^{1/3}\,3^{1/3}\,\ln\left(x-{\left(-1\right)}^{1/3}\,3^{1/3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(4*x^3 + x^6 + 3),x)
```

```
[Out] x + log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/2 - log(x - (3^(1/2)*1i)/2 -
1/2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)
/12 - 1/12) + log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/4 - (3^(5/6)*1i)
/4) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/2
```

3.162 $\int \frac{x^4}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\right.}{4\sqrt[3]{}}$$

```
[Out] -1/2*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/6*3^(2/3)*ln(3
^(1/3)+x)-1/12*ln(x^2-x+1)+1/12*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/6*arcta
n(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$,
Rules used = {1388, 298, 31, 648, 631, 210, 642, 632}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2}\sqrt[6]{3}\,\text{ArcTan}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{4\sqrt[3]{3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{2\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(3 + 4*x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - (3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(
5/6)])/2 + Log[1 + x]/6 - Log[3^(1/3) + x]/(2*3^(1/3)) - Log[1 - x + x^2]/1
2 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{3+4x^3+x^6}\,dx = -\left(\frac{1}{2}\int \frac{x}{1+x^3}\,dx\right)+\frac{3}{2}\int \frac{x}{3+x^3}\,dx$$

$$= \frac{1}{6}\int \frac{1}{1+x}\,dx - \frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx - \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{2\sqrt[3]{3}} + \frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}\,x+x^2}\,dx}{2\sqrt[3]{3}}$$

$$= \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}} - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{3}{4}\int \frac{}{3^{2/}}$$

$$= \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\log\left(3^{2/3}-\sqrt[3]{3}\,x+x^2\right)}{4\sqrt[3]{3}} + \frac{1}{2}\mathrm{Su}$$

$$= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{2}\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{2\sqrt[3]{3}} - \frac{1}{12}\log$$

Mathematica [A]
time = 0.02, size = 107, normalized size = 0.96

$$\frac{1}{12}\left(-6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+2\log(1+x)-2\,3^{2/3}\log\left(3+3^{2/3}x\right)-\log\left(1-x+x^2\right)+3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}\,x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(3 + 4*x^3 + x^6),x]
```

```
[Out] (-6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] + 2*Log[1 + x] - 2*3^(2/3)*Log[3 + 3^(2/3)*x] - Log[1 - x + x^2] +
3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [A]
time = 0.03, size = 84, normalized size = 0.75

method	result
risch	$\frac{\ln(1+x)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^3+1\right)}\_R\ln\left(3\_R^2+x\right)\right)}{2}-\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$
default	$-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{6}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{12}+\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{2}+\frac{\ln(1+x)}{6}-\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x}{}\right)}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*3^(2/3)*ln(3^(1/3)+x)+1/12*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/2*3^(1/
6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)-1/12*ln(x^2-x+1)-1/6*3
^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 84, normalized size = 0.75

$$\frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(x + 3^(1/3))
- 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3*3^(1/6
)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Fricas [A]
time = 0.37, size = 106, normalized size = 0.95

$$-\frac{1}{12}\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-3^{\frac{1}{3}}(-1)^{\frac{2}{3}}x+x^2-3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(3^{\frac{1}{3}}(-1)^{\frac{2}{3}}+x\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{2}\cdot 3^{\frac{1}{6}}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2(-1)^{\frac{1}{3}}x+3^{\frac{1}{3}}\right)\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/12*3^(2/3)*(-1)^(1/3)*log(-3^(1/3)*(-1)^(2/3)*x + x^2 - 3^(2/3)*(-1)^(1/
3)) + 1/6*3^(2/3)*(-1)^(1/3)*log(3^(1/3)*(-1)^(2/3) + x) - 1/6*sqrt(3)*arct
an(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*(-1)^(1/3)*arctan(1/3*3^(1/6)*(2*(-
1)^(1/3)*x + 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.30, size = 134, normalized size = 1.20

$$\frac{\log\left(x+1\right)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{2592\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^5}{5}+\frac{168\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^2}{5}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{168\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^2}{5}+\frac{2592\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^5}{5}\right)+\operatorname{RootSum}\left(24t^3+1,\left(t\mapsto t\log\left(\frac{2592t^5}{5}+\frac{168t^2}{5}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x + 2592*(-1/12 - sqrt(3)*I/12)**
5/5 + 168*(-1/12 - sqrt(3)*I/12)**2/5) + (-1/12 + sqrt(3)*I/12)*log(x + 168
*(-1/12 + sqrt(3)*I/12)**2/5 + 2592*(-1/12 + sqrt(3)*I/12)**5/5) + RootSum(
24*_t**3 + 1, Lambda(_t, _t*log(2592*_t**5/5 + 168*_t**2/5 + x)))
```

Giac [A]
time = 7.43, size = 86, normalized size = 0.77

$$\frac{1}{12}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{2}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/12*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/6*3^(2/3)*log(abs(x + 3^(1/
3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/2*3^(1/6)*arctan(1/3*3
^(1/6)*(2*x - 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

Mupad [B]
time = 1.37, size = 114, normalized size = 1.02

$$\frac{\ln(x+1)}{6}-\frac{3^{2/3}\,\ln\left(x+3^{1/3}\right)}{6}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{(-1)^{1/3}\,\ln\left(x-\frac{(-1)^{1/3}\,3^{1/3}}{2}-\frac{(-1)^{1/6}\,3^{5/6}}{2}+\frac{3^{1/3}}{2}\right)\left(3^{2/3}+3^{1/6}\,3\mathrm{i}\right)}{12}+\frac{(-1)^{1/3}\,3^{2/3}\,\ln\left(x+(-1)^{2/3}\,3^{1/3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x + 1)/6 - (3^(2/3)*log(x + 3^(1/3)))/6 + log(x - (3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
+ 1/12) - ((-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^(1/6)*3^(5/6))
/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/12 + ((-1)^(1/3)*3^(2/3)*log(x + (-
1)^(2/3)*3^(1/3)))/6
```

3.163 $\int \frac{x^3}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}\,x+\right.}{4\ 3^{2/3}}$$

```
[Out] -1/6*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/6*3^(1/3)*ln(3
^(1/3)+x)+1/12*ln(x^2-x+1)-1/12*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/6*arcta
n(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1388, 206, 31, 648, 631, 210, 642, 632}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{4\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{2\ 3^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(3 + 4*x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2*
3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(2*3^(2/3)) + Log[1 - x + x^2]/1
2 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(4*3^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^3}{3+4x^3+x^6}\,dx = -\left(\frac{1}{2}\int \frac{1}{1+x^3}\,dx\right) + \frac{3}{2}\int \frac{1}{3+x^3}\,dx$$

$$= -\left(\frac{1}{6}\int \frac{1}{1+x}\,dx\right) - \frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx + \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{2\ 3^{2/3}} + \frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{2\ 3^{2/3}}$$

$$= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}} + \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{\int \frac{-}{3^{2/}}}{}$$

$$= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}x+x^2\right)}{4\ 3^{2/3}} +$$

$$= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\sqrt[6]{3}} - \frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{2\ 3^{2/3}} + \frac{1}{12}\log(1-x$$

Mathematica [A]
time = 0.02, size = 106, normalized size = 0.95

$$\frac{1}{12}\left(-2 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) - 2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\log(1+x) + 2\sqrt[3]{3}\log\left(3+3^{2/3}x\right) + \log\left(1-x+x^2\right) - \sqrt[3]{3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(3 + 4*x^3 + x^6),x]
[Out] (-2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/S
qrt[3]] - 2*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + Log[1 - x + x^2] -
3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/12
```

Maple [A]
time = 0.03, size = 84, normalized size = 0.75

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^3-1\right)}\_R\ln\left(x+3\_R\right)\right)}{2} - \frac{\ln(1+x)}{6} + \frac{\ln(x^2-x+1)}{12} - \frac{\sqrt{3}\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$
default	$\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{6} - \frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{12} + \frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2 3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{6} - \frac{\ln(1+x)}{6} + \frac{\ln(x^2-x+1)}{12} - \frac{\sqrt{3}\arctan\left(\frac{(2x-}{}\right.}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*3^(1/3)*ln(3^(1/3)+x)-1/12*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1/6*3^(5/6
)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)+1/12*ln(x^2-x+1)-1/6*3^
(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 84, normalized size = 0.75

$$\frac{1}{6}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-\frac{1}{12}\cdot 3^{\frac{1}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{12}\,\log\left(x^{2}-x+1\right)-\frac{1}{6}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1/3
)*log(x + 3^(1/3)) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Fricas [A]
time = 0.36, size = 102, normalized size = 0.91

$$\frac{1}{6}\cdot 9^{\frac{1}{6}}\sqrt{3}\,\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}\,x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{36}\cdot 9^{\frac{2}{3}}\log\left(3\,x^{2}-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+\frac{1}{18}\cdot 9^{\frac{2}{3}}\log\left(3\,x+9^{\frac{2}{3}}\right)-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{12}\,\log\left(x^{2}-x+1\right)-\frac{1}{6}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/6*9^(1/6)*sqrt(3)*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*9^(1/3)*sq
rt(3))) - 1/36*9^(2/3)*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 1/18*9^(2/3)*lo
g(3*x + 9^(2/3)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/12*log(x^2
 - x + 1) - 1/6*log(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.30, size = 110, normalized size = 0.98

$$-\frac{\log\left(x+1\right)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{4}+\frac{\sqrt{3}\,i}{4}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{1}{4}+648\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{4}-\frac{\sqrt{3}\,i}{4}\right)+\operatorname{RootSum}\left(72t^{3}-1,\left(t\mapsto t\log\left(648t^{4}-3t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 - sqrt(3)*I/1
2)**4 + sqrt(3)*I/4) + (1/12 + sqrt(3)*I/12)*log(x - 1/4 + 648*(1/12 + sqrt
(3)*I/12)**4 - sqrt(3)*I/4) + RootSum(72*_t**3 - 1, Lambda(_t, _t*log(648*_
t**4 - 3*_t + x)))
```

Giac [A]
time = 3.18, size = 86, normalized size = 0.77

$$\frac{1}{6}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-\frac{1}{12}\cdot 3^{\frac{1}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{12}\,\log\left(x^{2}-x+1\right)-\frac{1}{6}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/6*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x - 1)) - 1/12*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/6*3^(1/3
)*log(abs(x + 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))
```

Mupad [B]
time = 1.36, size = 113, normalized size = 1.01

$$\frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{6}-\frac{\ln\left(x+1\right)}{6}+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{12}+\frac{3^{5/6}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{1/3}}{12}-\frac{3^{5/6}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3^(1/3)*log(x + 3^(1/3)))/6 - log(x + 1)/6 + log(x - (3^(1/2)*1i)/2 - 1/2)
*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
- 1/12) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(1/3)/12 + (3^(5/6)*1i)/12
) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/12 - (3^(5/6)*1i)/12)
```

3.164 $\int \frac{x}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}\,x+\right.}{12\sqrt[3]{3}}$$

```
[Out] 1/6*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/18*3^(2/3)*ln(3
^(1/3)+x)+1/12*ln(x^2-x+1)-1/36*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*arcta
n(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$,
Rules used = {1389, 298, 31, 648, 632, 210, 642, 631}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{12\sqrt[3]{3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{6\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(3 + 4*x^3 + x^6),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(2
*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(6*3^(1/3)) + Log[1 - x + x^2]/
12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x}{3+4x^3+x^6}\,dx = \frac{1}{2}\int \frac{x}{1+x^3}\,dx - \frac{1}{2}\int \frac{x}{3+x^3}\,dx$$

$$= -\left(\frac{1}{6}\int \frac{1}{1+x}\,dx\right) + \frac{1}{6}\int \frac{1}{1-x+x^2}\,dx + \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{6\sqrt[3]{3}} - \frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}\,x+x^2}\,dx}{6\sqrt[3]{3}}$$

$$= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{}{3}$$

$$= -\frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x+x^2\right) - \frac{\log\left(3^{2/3}-\sqrt[3]{3}\,x+x^2\right)}{12\sqrt[3]{3}} - \frac{1}{2}$$

$$= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{2\ 3^{5/6}} - \frac{1}{6}\log(1+x) + \frac{\log\left(\sqrt[3]{3}+x\right)}{6\sqrt[3]{3}} + \frac{1}{12}\log\left(1-x\right.$$

Mathematica [A]
time = 0.02, size = 108, normalized size = 0.96

$$\frac{1}{36}\left(6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right) + 6\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right) - 6\log(1+x) + 2\ 3^{2/3}\log\left(3+3^{2/3}x\right) + 3\log\left(1-x+x^2\right) - 3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}\,x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(3 + 4*x^3 + x^6),x]
```

```
[Out] (6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] - 6*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] + 3*Log[1 - x + x^2] -
 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

Maple [A]
time = 0.02, size = 84, normalized size = 0.75

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^3-1\right)} \_R\ln\left(3\_R^2+x\right)\right)}{6} + \frac{\ln\left(4x^2-4x+4\right)}{12} + \frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6} - \frac{\ln(1+x)}{6}$
default	$\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{18} - \frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{36} - \frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{6} - \frac{\ln(1+x)}{6} + \frac{\ln\left(x^2-x+1\right)}{12} + \frac{\sqrt{3}\,\arctan\left(\frac{(2x-1}{}\right.}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*3^(2/3)*ln(3^(1/3)+x)-1/36*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*3^(1/
6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)+1/12*ln(x^2-x+1)+1/6*3
^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 84, normalized size = 0.75

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^{2}-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3)
) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1/3*3^(1
/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Fricas [A]
time = 0.39, size = 84, normalized size = 0.75

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(-\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^{2}-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(x + 3^(1/3)
) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*3^(1/6)*arctan(-1/3*3^(
1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.13, size = 119, normalized size = 1.06

$$-\frac{\log\left(x+1\right)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+90\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{2}+11664\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{5}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+11664\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{5}+90\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{2}\right)+\operatorname{RootSum}\left(648t^{3}-1,\left(t\mapsto t\log\left(11664t^{5}+90t^{2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 90*(1/12 - sqrt(3)*I/12)**2 +
 11664*(1/12 - sqrt(3)*I/12)**5) + (1/12 + sqrt(3)*I/12)*log(x + 11664*(1/1
2 + sqrt(3)*I/12)**5 + 90*(1/12 + sqrt(3)*I/12)**2) + RootSum(648*_t**3 - 1
, Lambda(_t, _t*log(11664*_t**5 + 90*_t**2 + x)))
```

Giac [A]
time = 3.85, size = 86, normalized size = 0.77

$$-\frac{1}{36}\cdot 3^{\frac{2}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{18}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{6}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{12}\log\left(x^{2}-x+1\right)-\frac{1}{6}\log\left(\left|x+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/36*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/18*3^(2/3)*log(abs(x + 3^(
1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*3^(1/6)*arctan(1/3
*3^(1/6)*(2*x - 3^(1/3))) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x + 1))
```

Mupad [B]
time = 1.36, size = 113, normalized size = 1.01

$$\frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{18}-\frac{\ln\left(x+1\right)}{6}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{36}-\frac{3^{1/6}\,1\mathrm{i}}{12}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{36}+\frac{3^{1/6}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4*x^3 + x^6 + 3),x)
```

```
[Out] (3^(2/3)*log(x + 3^(1/3)))/18 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 + 1/12) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(2/3)/36 - (3^(1/6)*1i)/1
2) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/3)/36 + (3^(1/6)*1i)/12)
```

3.165 $\int \frac{1}{3+4x^3+x^6}\,dx$

Optimal. Leaf size=112

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}\,x\right.}{12\ 3^{2/3}}$$

```
[Out] 1/18*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/18*3^(1/3)*ln(
3^(1/3)+x)-1/12*ln(x^2-x+1)+1/36*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/6*arct
an(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1361, 206, 31, 648, 632, 210, 642, 631}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{12\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{6\ 3^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x^3 + x^6)^(-1),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(3^(1/3) - 2*x)/3^(5/6)]/(6
*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(6*3^(2/3)) - Log[1 - x + x^2]/
12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(12*3^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{3+4x^3+x^6}\,dx = \frac{1}{2}\int \frac{1}{1+x^3}\,dx - \frac{1}{2}\int \frac{1}{3+x^3}\,dx$$

$$= \frac{1}{6}\int \frac{1}{1+x}\,dx + \frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx - \frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{6\ 3^{2/3}} - \frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}\,x+x^2}\,dx}{6\ 3^{2/3}}$$

$$= \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}} - \frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx + \frac{\int \frac{-\sqrt[3]{\ }}{3^{2/3}-}}{1}$$

$$= \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}} - \frac{1}{12}\log\left(1-x+x^2\right) + \frac{\log\left(3^{2/3}-\sqrt[3]{3}\,x+x^2\right)}{12\ 3^{2/3}} - \frac{1}{2}$$

$$= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\sqrt[6]{3}} + \frac{1}{6}\log(1+x) - \frac{\log\left(\sqrt[3]{3}+x\right)}{6\ 3^{2/3}} - \frac{1}{12}\log(1-$$

Mathematica [A]
time = 0.02, size = 107, normalized size = 0.96

$$\frac{1}{36}\left(2\ 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+6\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+6\log(1+x)-2\sqrt[3]{3}\log\left(3+3^{2/3}x\right)-3\log\left(1-x+x^2\right)+\sqrt[3]{3}\log\left(3-3^{2/3}x+\sqrt[3]{3}\,x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x^3 + x^6)^(-1),x]
[Out] (2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 6*Sqrt[3]*ArcTan[(-1 + 2*x)/Sq
rt[3]] + 6*Log[1 + x] - 2*3^(1/3)*Log[3 + 3^(2/3)*x] - 3*Log[1 - x + x^2] +
 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/36
```

Maple [A]
time = 0.03, size = 84, normalized size = 0.75

method	result
risch	$-\frac{\ln(4x^2-4x+4)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}+\frac{\ln(1+x)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(9\_Z^3+1)}\_R\ln\left(x-3\_R\right)\right)}{6}$
default	$-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{18}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{36}-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{18}+\frac{\ln(1+x)}{6}-\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2\right.}{}\right.}{6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/18*3^(1/3)*ln(3^(1/3)+x)+1/36*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/18*3^(
5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)-1/12*ln(x^2-x+1)+1/6
*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.52, size = 84, normalized size = 0.75

$$-\frac{1}{18}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{36}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{18}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3^(
1/3)*log(x + 3^(1/3)) - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Fricas [A]
time = 0.37, size = 124, normalized size = 1.11

$$\frac{1}{18}\cdot 9^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{108}\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(9^{\frac{2}{3}}(-1)^{\frac{1}{3}}x+3x^2+3\cdot 9^{\frac{1}{3}}(-1)^{\frac{2}{3}}\right)+\frac{1}{54}\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-9^{\frac{2}{3}}(-1)^{\frac{1}{3}}+3x\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/18*9^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*(-1)
^(2/3)*x - 3*9^(1/3)*sqrt(3))) - 1/108*9^(2/3)*(-1)^(1/3)*log(9^(2/3)*(-1)^
(1/3)*x + 3*x^2 + 3*9^(1/3)*(-1)^(2/3)) + 1/54*9^(2/3)*(-1)^(1/3)*log(-9^(2
/3)*(-1)^(1/3) + 3*x) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/12*lo
g(x^2 - x + 1) + 1/6*log(x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.12, size = 124, normalized size = 1.11

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{13}{10}-\frac{13\sqrt{3}i}{10}+\frac{23328\left(-\frac{1}{12}+\frac{\sqrt{3}i}{12}\right)^4}{5}\right)+\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)\log\left(x+\frac{13}{10}+\frac{23328\left(-\frac{1}{12}-\frac{\sqrt{3}i}{12}\right)^4}{5}+\frac{13\sqrt{3}i}{10}\right)+\operatorname{RootSum}\left(1944t^3+1,\left(t\mapsto t\log\left(\frac{23328t^4}{5}-\frac{78t}{5}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 13/10 - 13*sqrt(3)*I/10 + 233
28*(-1/12 + sqrt(3)*I/12)**4/5) + (-1/12 - sqrt(3)*I/12)*log(x + 13/10 + 23
328*(-1/12 - sqrt(3)*I/12)**4/5 + 13*sqrt(3)*I/10) + RootSum(1944*_t**3 + 1
, Lambda(_t, _t*log(23328*_t**4/5 - 78*_t/5 + x)))
```

Giac [A]
time = 3.76, size = 86, normalized size = 0.77

$$-\frac{1}{18}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{36}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{18}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/18*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3*
sqrt(3)*(2*x - 1)) + 1/36*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/18*3^(
1/3)*log(abs(x + 3^(1/3))) - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x + 1))
```

Mupad [B]
time = 0.23, size = 110, normalized size = 0.98

$$\frac{\ln(x+1)}{6}-\frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{18}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{(-1)^{1/3}\,3^{1/3}\ln\left(x-(-1)^{1/3}\,3^{1/3}\right)}{18}-\frac{(-1)^{1/3}\ln\left(x+\frac{(-1)^{1/3}\,3^{1/3}}{2}+\frac{(-1)^{1/3}\,3^{5/6}\,1\mathrm{i}}{2}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^3 + x^6 + 3),x)
```

```
[Out] log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/18 - log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 - 1/12) + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)*3^(1/3)))/18 - ((-1)^(1/3
)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/3)*3^(5/6)*1i)/2)*(3^(1/3) + 3^
(5/6)*1i))/36
```

3.166 $\int \frac{1}{x^2(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{3x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\ 3^{5/6}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}\right.}{36\sqrt[3]{3}}$$

```
[Out] -1/3/x-1/18*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+x)-1/54*3^(2
/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/108*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+
1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1382, 1524, 298, 31, 648, 632, 210, 642, 631}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\ 3^{5/6}}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{36\sqrt[3]{3}}-\frac{1}{3x}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{18\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/3*1/x + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)/3
^(5/6)]/(6*3^(5/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(18*3^(1/3)) - Log[1
- x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2(3+4x^3+x^6)}\,dx = -\frac{1}{3x}+\frac{1}{3}\int \frac{x(-4-x^3)}{3+4x^3+x^6}\,dx$$
$$= -\frac{1}{3x}+\frac{1}{6}\int \frac{x}{3+x^3}\,dx-\frac{1}{2}\int \frac{x}{1+x^3}\,dx$$
$$= -\frac{1}{3x}+\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{18\sqrt[3]{3}}+\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}\,x+x^2}\,dx}{18\sqrt[3]{3}}$$
$$= -\frac{1}{3x}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{12}\int \frac{1}{3^{2/3}-\sqrt[3]{3}\,x}$$
$$= -\frac{1}{3x}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/3}-\sqrt[3]{3}\,x\right.}{36\sqrt[3]{3}}$$
$$= -\frac{1}{3x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{6\ 3^{5/6}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{18\sqrt[3]{3}}-\frac{1}{12}$$

Mathematica [A]
time = 0.03, size = 118, normalized size = 0.99

$$-\frac{36+6\sqrt[6]{3}\,x\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+18\sqrt{3}\,x\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-18x\log(1+x)+2\ 3^{2/3}x\log\left(3+3^{2/3}x\right)+9x\log\left(1-x+x^2\right)-3^{2/3}x\log\left(3-3^{2/3}x+\sqrt[3]{3}\,x^2\right)}{108x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/108*(36 + 6*3^(1/6)*x*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 18*Sqrt[3]*x*Arc
Tan[(-1 + 2*x)/Sqrt[3]] - 18*x*Log[1 + x] + 2*3^(2/3)*x*Log[3 + 3^(2/3)*x]
+ 9*x*Log[1 - x + x^2] - 3^(2/3)*x*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/x
```

Maple [A]
time = 0.04, size = 89, normalized size = 0.75

method	result
risch	$-\frac{1}{3x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(3\_Z^3+1\right)}\_R\ln\left(3\_R^2+x\right)\right)}{18}+\frac{\ln(1+x)}{6}-\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\ \arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}$

default	$-\frac{1}{3x}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{54}+\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^{2}\right)}{108}+\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\,\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{18}+\frac{\ln(1+x)}{6}-\frac{\ln(x^{2}-x+1)}{12}-\frac{\sqrt{3}\,\mathrm{arc}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x-1/54*3^(2/3)*ln(3^(1/3)+x)+1/108*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)+1
/18*3^(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))+1/6*ln(1+x)-1/12*ln(x^2-x
+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.52, size = 89, normalized size = 0.75

$$\frac{1}{108}\cdot 3^{\frac{2}{3}}\log\left(x^{2}-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{18}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-\frac{1}{3x}-\frac{1}{12}\log\left(x^{2}-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(x + 3^(1/3)
) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(1/3*3^(
1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(x + 1)
```

Fricas [A]
time = 0.36, size = 117, normalized size = 0.98

$$-\frac{3^{\frac{2}{3}}(-1)^{\frac{1}{3}}x\log\left(-3^{\frac{1}{3}}(-1)^{\frac{2}{3}}x+x^{2}-3^{\frac{2}{3}}(-1)^{\frac{1}{3}}\right)-2\cdot 3^{\frac{2}{3}}(-1)^{\frac{1}{3}}x\log\left(3^{\frac{1}{3}}(-1)^{\frac{2}{3}}+x\right)+18\sqrt{3}\,x\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-6\cdot 3^{\frac{1}{6}}(-1)^{\frac{1}{3}}x\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2(-1)^{\frac{1}{3}}x+3^{\frac{1}{3}}\right)\right)+9x\log\left(x^{2}-x+1\right)-18x\log\left(x+1\right)+36}{108x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/108*(3^(2/3)*(-1)^(1/3)*x*log(-3^(1/3)*(-1)^(2/3)*x + x^2 - 3^(2/3)*(-1)
^(1/3)) - 2*3^(2/3)*(-1)^(1/3)*x*log(3^(1/3)*(-1)^(2/3) + x) + 18*sqrt(3)*x
*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*(-1)^(1/3)*x*arctan(1/3*3^(1/6)*
(2*(-1)^(1/3)*x + 3^(1/3))) + 9*x*log(x^2 - x + 1) - 18*x*log(x + 1) + 36)/
x
```

Sympy [C] Result contains complex when optimal does not.
time = 1.09, size = 139, normalized size = 1.17

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x-\frac{8188128\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{5}}{41}+\frac{39384\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^{2}}{41}\right)+\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{39384\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{2}}{41}-\frac{8188128\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^{5}}{41}\right)+\operatorname{RootSum}\left(17496t^{3}+1,\left(t\mapsto t\log\left(-\frac{8188128t^{5}}{41}+\frac{39384t^{2}}{41}+x\right)\right)\right)-\frac{1}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 - sqrt(3)*I/12)*log(x - 8188128*(-1/12 - sqrt(3)*I/12
)**5/41 + 39384*(-1/12 - sqrt(3)*I/12)**2/41) + (-1/12 + sqrt(3)*I/12)*log(
x + 39384*(-1/12 + sqrt(3)*I/12)**2/41 - 8188128*(-1/12 + sqrt(3)*I/12)**5/
41) + RootSum(17496*_t**3 + 1, Lambda(_t, _t*log(-8188128*_t**5/41 + 39384*
_t**2/41 + x))) - 1/(3*x)
```

Giac [A]
time = 3.55, size = 91, normalized size = 0.76

$$\frac{1}{108} \cdot 3^{\frac{2}{3}} \log\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) - \frac{1}{54} \cdot 3^{\frac{2}{3}} \log\left(\left|x + 3^{\frac{1}{3}}\right|\right) - \frac{1}{6}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x - 1)\right) + \frac{1}{18} \cdot 3^{\frac{1}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}}\left(2\,x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{3\,x} - \frac{1}{12}\log\left(x^2 - x + 1\right) + \frac{1}{6}\log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/108*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/54*3^(2/3)*log(abs(x + 3^(
1/3))) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/18*3^(1/6)*arctan(1/
3*3^(1/6)*(2*x - 3^(1/3))) - 1/3/x - 1/12*log(x^2 - x + 1) + 1/6*log(abs(x
+ 1))
```

Mupad [B]
time = 1.38, size = 119, normalized size = 1.00

$$\frac{\ln(x+1)}{6} - \frac{3^{2/3}\ln\left(x + 3^{1/3}\right)}{54} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \frac{1}{3\,x} - \frac{(-1)^{1/3}\ln\left(x - \frac{(-1)^{1/3}\,3^{1/3}}{2} - \frac{(-1)^{1/6}\,3^{5/6}}{2} + \frac{3^{1/3}}{2}\right)\left(3^{2/3} + 3^{1/6}\,3\mathrm{i}\right)}{108} + \frac{(-1)^{1/3}\,3^{2/3}\ln\left(x + (-1)^{2/3}\,3^{1/3}\right)}{54}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x + 1)/6 - (3^(2/3)*log(x + 3^(1/3)))/54 + log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 - 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 + 1/12) - 1/(3*x) - ((-1)^(1/3)*log(x - ((-1)^(1/3)*3^(1/3))/2 - ((-1)^(1/
6)*3^(5/6))/2 + 3^(1/3)/2)*(3^(2/3) + 3^(1/6)*3i))/108 + ((-1)^(1/3)*3^(2/3
)*log(x + (-1)^(2/3)*3^(1/3)))/54
```

3.167 $\int \frac{1}{x^3(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=119

$$-\frac{1}{6x^2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\right.}{36}$$

```
[Out] -1/6/x^2-1/54*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+1/54*3^
(1/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/108*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2
)+1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {1382, 1436, 206, 31, 648, 632, 210, 642, 631}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}-\frac{1}{6x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}\,x+3^{2/3}\right)}{36\ 3^{2/3}}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{18\ 3^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/6*1/x^2 + ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(3^(1/3) - 2*x)
/3^(5/6)]/(18*3^(1/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(18*3^(2/3)) + Log
[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(36*3^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^3(3+4x^3+x^6)}\,dx = -\frac{1}{6x^2}+\frac{1}{6}\int \frac{-8-2x^3}{3+4x^3+x^6}\,dx$$
$$= -\frac{1}{6x^2}+\frac{1}{6}\int \frac{1}{3+x^3}\,dx-\frac{1}{2}\int \frac{1}{1+x^3}\,dx$$
$$= -\frac{1}{6x^2}-\frac{1}{6}\int \frac{1}{1+x}\,dx-\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx+\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{18\ 3^{2/3}}+\frac{\int \frac{2\sqrt[3]{3}-x}{3^{2/3}-\sqrt[3]{3}x+x^2}\,dx}{18\ 3^{2/3}}$$
$$= -\frac{1}{6x^2}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}$$
$$= -\frac{1}{6x^2}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\sqrt[3]{3}\right.}{36\ 3^{2/3}}$$
$$= -\frac{1}{6x^2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\sqrt[6]{3}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{18\ 3^{2/3}}+$$

Mathematica [A]
time = 0.03, size = 113, normalized size = 0.95

$$\frac{1}{108}\left(-\frac{18}{x^2}-2\,3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)-18\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-18\log(1+x)+2\sqrt[3]{3}\log\left(3+3^{2/3}x\right)+9\log\left(1-x+x^2\right)-\sqrt[3]{3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-18/x^2 - 2*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] - 18*Sqrt[3]*ArcTan[(-
1 + 2*x)/Sqrt[3]] - 18*Log[1 + x] + 2*3^(1/3)*Log[3 + 3^(2/3)*x] + 9*Log[1
- x + x^2] - 3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/108
```

Maple [A]
time = 0.05, size = 89, normalized size = 0.75

method	result
risch	$-\frac{1}{6x^2}-\frac{\ln(1+x)}{6}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\ \arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^3-1\right)}\_R\ln\left(x+3\_R\right)\right)}{18}$
default	$-\frac{1}{6x^2}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{54}-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{108}+\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{54}-\frac{\ln(1+x)}{6}+\frac{\ln(x^2-x+1)}{12}-\frac{\sqrt{3}\ \mathrm{ar}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x^2+1/54*3^(1/3)*ln(3^(1/3)+x)-1/108*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)
+1/54*3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/6*ln(1+x)+1/12*ln(x^2
-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.65, size = 89, normalized size = 0.75

$$\frac{1}{54}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)-\frac{1}{108}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{54}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)-\frac{1}{6\,x^2}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] 1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3^(
1/3)*log(x + 3^(1/3)) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1)
```

Fricas [A]
time = 0.37, size = 126, normalized size = 1.06

$$\frac{6\cdot 9^{\frac{1}{6}}\sqrt{3}\,x^2\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}\,x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-9^{\frac{2}{3}}x^2\log\left(3\,x^2-9^{\frac{2}{3}}x+3\cdot 9^{\frac{1}{3}}\right)+2\cdot 9^{\frac{2}{3}}x^2\log\left(3\,x+9^{\frac{2}{3}}\right)-54\,\sqrt{3}\,x^2\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+27\,x^2\log\left(x^2-x+1\right)-54\,x^2\log\left(x+1\right)-54}{324\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/324*(6*9^(1/6)*sqrt(3)*x^2*arctan(1/27*9^(1/6)*(2*9^(2/3)*sqrt(3)*x - 3*9
^(1/3)*sqrt(3))) - 9^(2/3)*x^2*log(3*x^2 - 9^(2/3)*x + 3*9^(1/3)) + 2*9^(2/
3)*x^2*log(3*x + 9^(2/3)) - 54*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x - 1)) +
27*x^2*log(x^2 - x + 1) - 54*x^2*log(x + 1) - 54)/x^2
```

Sympy [C] Result contains complex when optimal does not.
time = 1.02, size = 128, normalized size = 1.08

$$-\frac{\log\left(x+1\right)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{1093}{244}-\frac{1093\sqrt{3}\,i}{244}+\frac{787320\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^4}{61}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{1093}{244}+\frac{787320\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^4}{61}+\frac{1093\sqrt{3}\,i}{244}\right)+\operatorname{RootSum}\left(52488t^3-1,\left(t\mapsto t\log\left(\frac{787320t^4}{61}+\frac{3279t}{61}+x\right)\right)\right)-\frac{1}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 1093/244 - 1093*sqrt(3)*I/244
 + 787320*(1/12 - sqrt(3)*I/12)**4/61) + (1/12 + sqrt(3)*I/12)*log(x + 1093
/244 + 787320*(1/12 + sqrt(3)*I/12)**4/61 + 1093*sqrt(3)*I/244) + RootSum(5
```

```
2488*_t**3 - 1, Lambda(_t, _t*log(787320*_t**4/61 + 3279*_t/61 + x))) - 1/(
6*x**2)
```

Giac [A]
time = 4.29, size = 91, normalized size = 0.76

$$\frac{1}{54} \cdot 3^{\frac{5}{6}} \arctan\left(\frac{1}{3} \cdot 3^{\frac{1}{6}} \left(2 x - 3^{\frac{1}{3}}\right)\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2 x - 1)\right) - \frac{1}{108} \cdot 3^{\frac{1}{3}} \log\left(x^2 - 3^{\frac{1}{3}} x + 3^{\frac{2}{3}}\right) + \frac{1}{54} \cdot 3^{\frac{1}{3}} \log\left(\left|x + 3^{\frac{1}{3}}\right|\right) - \frac{1}{6 x^2} + \frac{1}{12} \log\left(x^2 - x + 1\right) - \frac{1}{6} \log\left(|x + 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] 1/54*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) - 1/6*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x - 1)) - 1/108*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/54*3^(
1/3)*log(abs(x + 3^(1/3))) - 1/6/x^2 + 1/12*log(x^2 - x + 1) - 1/6*log(abs(
x + 1))
```

Mupad [B]
time = 1.36, size = 118, normalized size = 0.99

$$\frac{3^{1/3} \ln\left(x + 3^{1/3}\right)}{54} - \frac{\ln(x+1)}{6} + \ln\left(x - \frac{1}{2} - \frac{\sqrt{3}\,1\mathrm{i}}{2}\right) \left(\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \ln\left(x - \frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right) \left(-\frac{1}{12} + \frac{\sqrt{3}\,1\mathrm{i}}{12}\right) - \frac{1}{6 x^2} - \ln\left(x - \frac{3^{1/3}}{2} - \frac{3^{5/6}\,1\mathrm{i}}{2}\right) \left(\frac{3^{1/3}}{108} + \frac{3^{5/6}\,1\mathrm{i}}{108}\right) - \ln\left(x - \frac{3^{1/3}}{2} + \frac{3^{5/6}\,1\mathrm{i}}{2}\right) \left(\frac{3^{1/3}}{108} - \frac{3^{5/6}\,1\mathrm{i}}{108}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(4*x^3 + x^6 + 3)),x)
```

```
[Out] (3^(1/3)*log(x + 3^(1/3)))/54 - log(x + 1)/6 + log(x - (3^(1/2)*1i)/2 - 1/2
)*((3^(1/2)*1i)/12 + 1/12) - log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/12
 - 1/12) - 1/(6*x^2) - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3^(1/3)/108 + (
3^(5/6)*1i)/108) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(1/3)/108 - (3^(5
/6)*1i)/108)
```

3.168 $\int \frac{1}{x^5(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{12x^4}+\frac{4}{9x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\ 3^{5/6}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}\right.}{}$$

```
[Out] -1/12/x^4+4/9/x+1/54*3^(1/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))-1/6*ln(1+x)+
1/162*3^(2/3)*ln(3^(1/3)+x)+1/12*ln(x^2-x+1)-1/324*3^(2/3)*ln(3^(2/3)-3^(1/
3)*x+x^2)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1382, 1518, 1524, 298, 31, 648, 632, 210, 642, 631}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\ 3^{5/6}}-\frac{1}{12x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\sqrt[3]{3}}+\frac{4}{9x}-\frac{1}{6}\log(x+1)+\frac{\log\left(x+\sqrt[3]{3}\right)}{54\sqrt[3]{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/12*1/x^4 + 4/(9*x) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3^(
1/3) - 2*x)/3^(5/6)]/(18*3^(5/6)) - Log[1 + x]/6 + Log[3^(1/3) + x]/(54*3^(
1/3)) + Log[1 - x + x^2]/12 - Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
```

```
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(3+4x^3+x^6\right)}\,dx = -\frac{1}{12x^4}+\frac{1}{12}\int \frac{-16-4x^3}{x^2\left(3+4x^3+x^6\right)}\,dx$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{36}\int \frac{x(-52-16x^3)}{3+4x^3+x^6}\,dx$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{18}\int \frac{x}{3+x^3}\,dx+\frac{1}{2}\int \frac{x}{1+x^3}\,dx$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{1+x}{1-x+x^2}\,dx+\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{54\sqrt[3]{3}}-\frac{\int \frac{\sqrt[3]{3}+x}{3^{2/3}-\sqrt[3]{3}\,x-}}{54\sqrt[3]{3}}$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}-\frac{1}{36}\int \frac{1}{3^{2/3}-\sqrt[3]{3}\,x+x^2}\,dx+\frac{1}{12}$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+x\right)}{54\sqrt[3]{3}}+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{\log\left(3^{2/3}-\right.}{10}$$
$$= -\frac{1}{12x^4}+\frac{4}{9x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{18\ 3^{5/6}}-\frac{1}{6}\log(1+x)+\frac{\log\left(\sqrt[3]{3}+\right.}{54\sqrt[3]{3}}$$

Mathematica [A]
time = 0.03, size = 118, normalized size = 0.94

$$\frac{1}{324}\left(-\frac{27}{x^4}+\frac{144}{x}+6\sqrt[6]{3}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+54\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-54\log(1+x)+2\ 3^{2/3}\log\left(3+3^{2/3}x\right)+27\log\left(1-x+x^2\right)-3^{2/3}\log\left(3-3^{2/3}x+\sqrt[3]{3}\,x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-27/x^4 + 144/x + 6*3^(1/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 54*Sqrt[3]*A
rcTan[(-1 + 2*x)/Sqrt[3]] - 54*Log[1 + x] + 2*3^(2/3)*Log[3 + 3^(2/3)*x] +
27*Log[1 - x + x^2] - 3^(2/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/324
```

Maple [A]
time = 0.04, size = 94, normalized size = 0.75

method	result
risch	$\frac{\frac{4x^3}{9}-\frac{1}{12}}{x^4}-\frac{\ln(1+x)}{6}+\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}(3\_Z^3-1)}\_R\ln\left(3\_R^2+x\right)\right)}{54}$
default	$\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{162}-\frac{3^{\frac{2}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{324}-\frac{3^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{54}-\frac{1}{12x^4}+\frac{4}{9x}-\frac{\ln(1+x)}{6}+\frac{\ln(x^2-x+1)}{12}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/162*3^(2/3)*ln(3^(1/3)+x)-1/324*3^(2/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/54*3^
(1/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/12/x^4+4/9/x-1/6*ln(1+x)+1/12
*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.57, size = 96, normalized size = 0.76

$$-\frac{1}{324}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{162}\cdot 3^{\frac{2}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{1}{54}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{16x^3-3}{36x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(x + 3^(1/
3)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arctan(1/3*3
^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x + 1) - 1
/6*log(x + 1)
```

Fricas [A]
time = 0.39, size = 112, normalized size = 0.89

$$-\frac{3^{\frac{2}{3}}x^4\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-2\cdot 3^{\frac{2}{3}}x^4\log\left(x+3^{\frac{1}{3}}\right)-54\sqrt{3}\,x^4\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-6\cdot 3^{\frac{1}{6}}x^4\arctan\left(-\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)-27x^4\log\left(x^2-x+1\right)+54x^4\log\left(x+1\right)-144x^3+27}{324x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] -1/324*(3^(2/3)*x^4*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 2*3^(2/3)*x^4*log(x +
3^(1/3)) - 54*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*3^(1/6)*x^4*arc
tan(-1/3*3^(1/6)*(2*x - 3^(1/3))) - 27*x^4*log(x^2 - x + 1) + 54*x^4*log(x
+ 1) - 144*x^3 + 27)/x^4
```

Sympy [C] Result contains complex when optimal does not.
time = 1.18, size = 141, normalized size = 1.12

$$-\frac{\log\left(x+1\right)}{6}+\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{4782978\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^2}{547}+\frac{1028869776\left(\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^5}{547}\right)+\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{1028869776\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^5}{547}+\frac{4782978\left(\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^2}{547}\right)+\mathrm{RootSum}\left(472392t^3-1,\left(t\mapsto t\log\left(\frac{1028869776t^5}{547}+\frac{4782978t^2}{547}+x\right)\right)\right)+\frac{16x^3-3}{36x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**6+4*x**3+3),x)
```

```
[Out] -log(x + 1)/6 + (1/12 - sqrt(3)*I/12)*log(x + 4782978*(1/12 - sqrt(3)*I/12)
**2/547 + 1028869776*(1/12 - sqrt(3)*I/12)**5/547) + (1/12 + sqrt(3)*I/12)*
log(x + 1028869776*(1/12 + sqrt(3)*I/12)**5/547 + 4782978*(1/12 + sqrt(3)*I
/12)**2/547) + RootSum(472392*_t**3 - 1, Lambda(_t, _t*log(1028869776*_t**5
/547 + 4782978*_t**2/547 + x))) + (16*x**3 - 3)/(36*x**4)
```

Giac [A]
time = 4.25, size = 98, normalized size = 0.78

$$-\frac{1}{324}\cdot 3^{\frac{2}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)+\frac{1}{162}\cdot 3^{\frac{2}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{54}\cdot 3^{\frac{1}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{16\,x^3-3}{36\,x^4}+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/324*3^(2/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) + 1/162*3^(2/3)*log(abs(x + 3
^(1/3))) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/54*3^(1/6)*arctan(
1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/36*(16*x^3 - 3)/x^4 + 1/12*log(x^2 - x + 1
) - 1/6*log(abs(x + 1))
```

Mupad [B]
time = 0.19, size = 124, normalized size = 0.98

$$\frac{3^{2/3}\ln\left(x+3^{1/3}\right)}{162}-\frac{\ln\left(x+1\right)}{6}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{\frac{4\,x^3}{9}-\frac{1}{12}}{x^4}-\ln\left(x-\frac{3^{1/3}}{2}-\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{324}-\frac{3^{1/6}\,1\mathrm{i}}{108}\right)-\ln\left(x-\frac{3^{1/3}}{2}+\frac{3^{5/6}\,1\mathrm{i}}{2}\right)\left(\frac{3^{2/3}}{324}+\frac{3^{1/6}\,1\mathrm{i}}{108}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(4*x^3 + x^6 + 3)),x)
```

```
[Out] (3^(2/3)*log(x + 3^(1/3)))/162 - log(x + 1)/6 - log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 - 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 + 1/12) + ((4*x^3)/9 - 1/12)/x^4 - log(x - 3^(1/3)/2 - (3^(5/6)*1i)/2)*(3
^(2/3)/324 - (3^(1/6)*1i)/108) - log(x - 3^(1/3)/2 + (3^(5/6)*1i)/2)*(3^(2/
3)/324 + (3^(1/6)*1i)/108)
```

3.169 $\int \frac{1}{x^6(3+4x^3+x^6)}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(\right.}{}$$

```
[Out] -1/15/x^5+2/9/x^2+1/162*3^(5/6)*arctan(1/3*(3^(1/3)-2*x)*3^(1/6))+1/6*ln(1+
x)-1/162*3^(1/3)*ln(3^(1/3)+x)-1/12*ln(x^2-x+1)+1/324*3^(1/3)*ln(3^(2/3)-3^
(1/3)*x+x^2)-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1382, 1518, 1436, 206, 31, 648, 632, 210, 642, 631}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}-\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{\log\left(x^2-\sqrt[3]{3}x+3^{2/3}\right)}{108\ 3^{2/3}}+\frac{1}{6}\log(x+1)-\frac{\log\left(x+\sqrt[3]{3}\right)}{54\ 3^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] -1/15*1/x^5 + 2/(9*x^2) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(3
^(1/3) - 2*x)/3^(5/6)]/(54*3^(1/6)) + Log[1 + x]/6 - Log[3^(1/3) + x]/(54*3
^(2/3)) - Log[1 - x + x^2]/12 + Log[3^(2/3) - 3^(1/3)*x + x^2]/(108*3^(2/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6(3+4x^3+x^6)}\,dx &= -\frac{1}{15x^5}+\frac{1}{15}\int \frac{-20-5x^3}{x^3(3+4x^3+x^6)}\,dx\\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{90}\int \frac{-130-40x^3}{3+4x^3+x^6}\,dx\\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{1}{18}\int \frac{1}{3+x^3}\,dx+\frac{1}{2}\int \frac{1}{1+x^3}\,dx\\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\int \frac{1}{1+x}\,dx+\frac{1}{6}\int \frac{2-x}{1-x+x^2}\,dx-\frac{\int \frac{1}{\sqrt[3]{3}+x}\,dx}{54\ 3^{2/3}}-\frac{\int \frac{2\sqrt[3]{3}}{3^{2/3}-\sqrt[3]{3}}}{54\ 3}\\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}}-\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{}{1}\\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}+x\right)}{54\ 3^{2/3}}-\frac{1}{12}\log\left(1-x+x^2\right)+\frac{\log\left(3^{2/}\right.}{}\\
&= -\frac{1}{15x^5}+\frac{2}{9x^2}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)}{54\sqrt[6]{3}}+\frac{1}{6}\log(1+x)-\frac{\log\left(\sqrt[3]{3}\right.}{54\ 3^{2/}}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 118, normalized size = 0.94

$$\frac{-\frac{108}{x^5}+\frac{360}{x^2}+10\ 3^{5/6}\tan^{-1}\left(\frac{\sqrt[3]{3}-2x}{3^{5/6}}\right)+270\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+270\log(1+x)-10\sqrt[3]{3}\log\left(3+3^{2/3}x\right)-135\log\left(1-x+x^2\right)+5\sqrt[3]{3}\log\left(3-3^{2/3}x+\sqrt[3]{3}x^2\right)}{1620}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(3 + 4*x^3 + x^6)),x]
```

```
[Out] (-108/x^5 + 360/x^2 + 10*3^(5/6)*ArcTan[(3^(1/3) - 2*x)/3^(5/6)] + 270*Sqrt
[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 270*Log[1 + x] - 10*3^(1/3)*Log[3 + 3^(2/3
)*x] - 135*Log[1 - x + x^2] + 5*3^(1/3)*Log[3 - 3^(2/3)*x + 3^(1/3)*x^2])/1
620
```

Maple [A]
time = 0.04, size = 94, normalized size = 0.75

method	result
risch	$\frac{\frac{2x^3}{9}-\frac{1}{15}}{x^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^3+1\right)}\_R\ln\left(x-3\_R\right)\right)}{54}-\frac{\ln(x^2-x+1)}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{6}+\frac{\ln(1+x)}{6}$
default	$-\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{1}{3}}+x\right)}{162}+\frac{3^{\frac{1}{3}}\ln\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}x+x^2\right)}{324}-\frac{3^{\frac{5}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}x}{3}-1\right)}{3}\right)}{162}-\frac{1}{15x^5}+\frac{2}{9x^2}+\frac{\ln(1+x)}{6}-\frac{\ln(x^2-x+1)}{12}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^6+4*x^3+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/162*3^(1/3)*ln(3^(1/3)+x)+1/324*3^(1/3)*ln(3^(2/3)-3^(1/3)*x+x^2)-1/162*
3^(5/6)*arctan(1/3*3^(1/2)*(2/3*3^(2/3)*x-1))-1/15/x^5+2/9/x^2+1/6*ln(1+x)-
1/12*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 96, normalized size = 0.76

$$-\frac{1}{162}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{324}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{162}\cdot 3^{\frac{1}{3}}\log\left(x+3^{\frac{1}{3}}\right)+\frac{10x^3-3}{45x^5}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="maxima")
```

```
[Out] -1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/162*
3^(1/3)*log(x + 3^(1/3)) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x + 1) +
1/6*log(x + 1)
```

Fricas [A]
time = 0.38, size = 153, normalized size = 1.21

$$\frac{30\cdot 9^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}x^5\arctan\left(\frac{1}{27}\cdot 9^{\frac{1}{6}}\left(2\cdot 9^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}x-3\cdot 9^{\frac{1}{3}}\sqrt{3}\right)\right)-5\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(9^{\frac{2}{3}}(-1)^{\frac{1}{3}}x+3x^2+3\cdot 9^{\frac{1}{3}}(-1)^{\frac{2}{3}}\right)+10\cdot 9^{\frac{2}{3}}(-1)^{\frac{1}{3}}x^5\log\left(-9^{\frac{2}{3}}(-1)^{\frac{1}{3}}+3x\right)+810\sqrt{3}\,x^5\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-405x^5\log\left(x^2-x+1\right)+810x^5\log\left(x+1\right)+1080x^3-324}{4860x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="fricas")
```

```
[Out] 1/4860*(30*9^(1/6)*sqrt(3)*(-1)^(1/3)*x^5*arctan(1/27*9^(1/6)*(2*9^(2/3)*sq
rt(3)*(-1)^(2/3)*x - 3*9^(1/3)*sqrt(3))) - 5*9^(2/3)*(-1)^(1/3)*x^5*log(9^(
2/3)*(-1)^(1/3)*x + 3*x^2 + 3*9^(1/3)*(-1)^(2/3)) + 10*9^(2/3)*(-1)^(1/3)*x
^5*log(-9^(2/3)*(-1)^(1/3) + 3*x) + 810*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x
```

```
 - 1)) - 405*x^5*log(x^2 - x + 1) + 810*x^5*log(x + 1) + 1080*x^3 - 324)/x^
5
```

Sympy [C] Result contains complex when optimal does not.
time = 1.00, size = 136, normalized size = 1.08

$$\frac{\log(x+1)}{6}+\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{88573}{6562}-\frac{88573\sqrt{3}\,i}{6562}+\frac{119042784\left(-\frac{1}{12}+\frac{\sqrt{3}\,i}{12}\right)^4}{3281}\right)+\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)\log\left(x+\frac{88573}{6562}+\frac{119042784\left(-\frac{1}{12}-\frac{\sqrt{3}\,i}{12}\right)^4}{3281}+\frac{88573\sqrt{3}\,i}{6562}\right)+\operatorname{RootSum}\left(1417176t^3+1,\left(t\mapsto t\log\left(\frac{119042784t^4}{3281}-\frac{531438t}{3281}+x\right)\right)\right)+\frac{10x^3-3}{45x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**6+4*x**3+3),x)
```

```
[Out] log(x + 1)/6 + (-1/12 + sqrt(3)*I/12)*log(x + 88573/6562 - 88573*sqrt(3)*I/
6562 + 119042784*(-1/12 + sqrt(3)*I/12)**4/3281) + (-1/12 - sqrt(3)*I/12)*l
og(x + 88573/6562 + 119042784*(-1/12 - sqrt(3)*I/12)**4/3281 + 88573*sqrt(3
)*I/6562) + RootSum(1417176*_t**3 + 1, Lambda(_t, _t*log(119042784*_t**4/32
81 - 531438*_t/3281 + x))) + (10*x**3 - 3)/(45*x**5)
```

Giac [A]
time = 5.60, size = 98, normalized size = 0.78

$$-\frac{1}{162}\cdot 3^{\frac{5}{6}}\arctan\left(\frac{1}{3}\cdot 3^{\frac{1}{6}}\left(2\,x-3^{\frac{1}{3}}\right)\right)+\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,(2\,x-1)\right)+\frac{1}{324}\cdot 3^{\frac{1}{3}}\log\left(x^2-3^{\frac{1}{3}}x+3^{\frac{2}{3}}\right)-\frac{1}{162}\cdot 3^{\frac{1}{3}}\log\left(\left|x+3^{\frac{1}{3}}\right|\right)+\frac{10\,x^3-3}{45\,x^5}-\frac{1}{12}\log\left(x^2-x+1\right)+\frac{1}{6}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^6+4*x^3+3),x, algorithm="giac")
```

```
[Out] -1/162*3^(5/6)*arctan(1/3*3^(1/6)*(2*x - 3^(1/3))) + 1/6*sqrt(3)*arctan(1/3
*sqrt(3)*(2*x - 1)) + 1/324*3^(1/3)*log(x^2 - 3^(1/3)*x + 3^(2/3)) - 1/162*
3^(1/3)*log(abs(x + 3^(1/3))) + 1/45*(10*x^3 - 3)/x^5 - 1/12*log(x^2 - x +
1) + 1/6*log(abs(x + 1))
```

Mupad [B]
time = 1.40, size = 121, normalized size = 0.96

$$\frac{\ln(x+1)}{6}-\frac{3^{1/3}\ln\left(x+3^{1/3}\right)}{162}-\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1i}{2}\right)\left(\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)+\ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)\left(-\frac{1}{12}+\frac{\sqrt{3}\,1i}{12}\right)+\frac{\frac{2x^3}{9}-\frac{1}{15}}{x^5}+\frac{(-1)^{1/3}\,3^{1/3}\ln\left(x-(-1)^{1/3}\,3^{1/3}\right)}{162}-\frac{(-1)^{1/3}\ln\left(x+\frac{(-1)^{1/3}\,3^{1/3}}{2}+\frac{(-1)^{1/3}\,3^{5/6}\,1i}{2}\right)\left(3^{1/3}+3^{5/6}\,1i\right)}{324}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(4*x^3 + x^6 + 3)),x)
```

```
[Out] log(x + 1)/6 - (3^(1/3)*log(x + 3^(1/3)))/162 - log(x - (3^(1/2)*1i)/2 - 1/
2)*((3^(1/2)*1i)/12 + 1/12) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/1
2 - 1/12) + ((2*x^3)/9 - 1/15)/x^5 + ((-1)^(1/3)*3^(1/3)*log(x - (-1)^(1/3)
*3^(1/3)))/162 - ((-1)^(1/3)*log(x + ((-1)^(1/3)*3^(1/3))/2 + ((-1)^(1/3)*3
^(5/6)*1i)/2)*(3^(1/3) + 3^(5/6)*1i))/324
```

3.170 $\int \frac{x^6}{1-x^3+x^6}\,dx$

Optimal. Leaf size=412

$$x+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\cdots\right.}{9\sqrt[3]{2}\left(\cdots\right.}$$

```
[Out] x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2
^(2/3)/(1-I*3^(1/2))^(2/3)+1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*3^(
1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2
))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/1
8*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2
/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*
(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+
I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Rubi [A]
time = 0.28, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1381, 1436, 206, 31, 648, 631, 210, 642}

$$\frac{\left(-\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+x+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - x^3 + x^6),x]
```

```
[Out] x + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(
3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I
*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*S
qrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^
(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3
)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*
(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3))
 - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x
 + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
```

```
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^6}{1-x^3+x^6}\,dx = x-\int \frac{1-x^3}{1-x^3+x^6}\,dx$$

$$= x-\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= x+\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-\right.}}}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$= x+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$= x+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$= x+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.01, size = 59, normalized size = 0.14

$$x+\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1^2+2\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - x^3 + x^6),x]
```

```
[Out] x + RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 +
2*#1^5) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 44, normalized size = 0.11

method	result	size
default	$x+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(\_R^3-1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44
risch	$x+\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(\_R^3-1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/3*sum((_R^3-1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] x + integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 768 vs. 2(268) = 536.
time = 0.42, size = 768, normalized size = 1.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(72*18^(2/3)*12^(1/6
)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 216*18^(2/3)*12^(1/6)*x*cos(2/3*
arctan(sqrt(3) - 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3)) - 2/27*18^(2/3)*12
^(1/6)*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
 2)) - sqrt(2)*sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)
) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 18*x^2 + 3*18^(1/3
)*12^(1/3))*(18^(1/3)*12^(5/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^
(1/3)*12^(5/6)*sin(2/3*arctan(sqrt(3) - 2))) - 18*(18^(1/3)*12^(5/6)*x - 24
*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqrt(3) - 2)) - 108*sqrt(3))/
(4*cos(2/3*arctan(sqrt(3) - 2))^2 - 3))*sin(2/3*arctan(sqrt(3) - 2)) + 1/27
*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6
)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*
x*cos(2/3*arctan(sqrt(3) - 2)) - sqrt(2)*sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*s
in(2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3)
- 2)) + 18*x^2 + 3*18^(1/3)*12^(1/3))*(18^(1/3)*12^(5/6)*sqrt(3)*cos(2/3*ar
ctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sin(2/3*arctan(sqrt(3) - 2))) + 18
*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqr
t(3) - 2)) + 108*sqrt(3))/(4*cos(2/3*arctan(sqrt(3) - 2))^2 - 3)) + 1/27*(1
8^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*s
in(2/3*arctan(sqrt(3) - 2)))*arctan(-1/2592*(72*18^(1/3)*12^(5/6)*sqrt(3)*x
 - 18^(1/3)*12^(5/6)*sqrt(3)*sqrt(-576*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*
arctan(sqrt(3) - 2)) + 5184*x^2 + 864*18^(1/3)*12^(1/3)) - 2592*sin(2/3*arc
tan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2))) - 1/108*(18^(2/3)*12^(1/6)
*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sq
rt(3) - 2)))*log(288*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2
)) + 864*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 5184*x^2 + 864*
18^(1/3)*12^(1/3)) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3
) - 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(-576*18^(2/3)
*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 5184*x^2 + 864*18^(1/3)*
12^(1/3)) + x
```

Sympy [**A**]
time = 0.07, size = 26, normalized size = 0.06

$$x + \operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 - 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**6-x**3+1),x)
```

```
[Out] x + RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t
 + x)))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(268) = 536.
time = 4.86, size = 641, normalized size = 1.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos
(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9
*pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*
cos(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9
*pi) + 2*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2
*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*co
s(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9
*pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))
*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*si
n(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9
*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/
9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) - 2*cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi
) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) -
 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(
2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log((-I*
sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*p
i)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*
cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos
(1/9*pi))*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) + x
```

Mupad [B]
time = 1.82, size = 320, normalized size = 0.78

$$x+\frac{\ln\left(x+\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9i}{2}\right)\left(36+\sqrt{3}\,12i\right)^{1/3}}{54}\right)\left(36+\sqrt{3}\,12i\right)^{1/3}}{18}+\frac{\ln\left(x-\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9i}{2}\right)\left(36-\sqrt{3}\,12i\right)^{1/3}}{54}\right)\left(36-\sqrt{3}\,12i\right)^{1/3}}{18}-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(3-\sqrt{3}\,1i\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1i\right)\left(\frac{3\left(-3+\sqrt{3}\,1i\right)\left(3^{1/3}-3^{5/6}\,1i\right)^3}{16}-27\right)}{108}\right)\left(3-\sqrt{3}\,1i\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1i\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(3+\sqrt{3}\,1i\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1i\right)\left(\frac{3\left(3+\sqrt{3}\,1i\right)\left(3^{1/3}+3^{5/6}\,1i\right)^3}{16}+27\right)}{108}\right)\left(3+\sqrt{3}\,1i\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1i\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(3-\sqrt{3}\,1i\right)^{1/3}\,1i}{6}\right)\left(3-\sqrt{3}\,1i\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1i\right)}{36}-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(3+\sqrt{3}\,1i\right)^{1/3}\,1i}{6}\right)\left(3+\sqrt{3}\,1i\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1i\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^6 - x^3 + 1),x)
```

```
[Out] x + (log(x + (((3^(1/2)*9i)/2 - 27/2)*(3^(1/2)*12i + 36)^(1/3))/54)*(3^(1/2
)*12i + 36)^(1/3))/18 + (log(x - (((3^(1/2)*9i)/2 + 27/2)*(36 - 3^(1/2)*12i
)^(1/3))/54)*(36 - 3^(1/2)*12i)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*(3 -
3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^
(5/6)*1i)^3)/16 - 27))/108)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/
36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i
)*((3*(3^(1/2)*1i + 3)*(3^(1/3) + 3^(5/6)*1i)^3)/16 + 27))/108)*(3^(1/2)*1i
 + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(5/6)*
(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i)
)/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(
1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

3.171 $\int \frac{x^5}{1-x^3+x^6}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] 1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1371, 648, 632, 210, 642}

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\text{ArcTan}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{1-x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)+\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - x^3 + x^6),x]
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Maple [**A**]
time = 0.02, size = 33, normalized size = 0.85

method	result	size
default	$\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	33

risch	$\frac{\ln(4x^6-4x^3+4)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A]
time = 0.59, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="maxima")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Fricas [A]
time = 0.35, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Sympy [A]
time = 0.04, size = 37, normalized size = 0.95

$$\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**6-x**3+1),x)
[Out] log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Giac [A]
time = 4.32, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^3-1)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Mupad [B]
time = 1.21, size = 34, normalized size = 0.87

$$\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^6 - x^3 + 1),x)
```

```
[Out] log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

3.172 $\int \frac{x^4}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{\vphantom{x}}\right.}{9\ 2^{2/3}\sqrt[3]{1}}$$

[Out] -1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1-I*3^(1/2))^(1/3)

Rubi [A]

time = 0.19, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1388, 298, 31, 648, 631, 210, 642}

$$\frac{\left(\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(-\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(1 - x^3 + x^6),x]

[Out] ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{1-x^3+x^6}\,dx = -\left(\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\left(-\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}\right)+\frac{\left(3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{}}{}}{9\ 2^{2/3}\sqrt[3]{}}$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-$$

$$= \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 41, normalized size = 0.10

$$\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^3 + x^6),x]
```

[Out] RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) &]/3

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 40, normalized size = 0.10

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	40

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(x^6-x^3+1),x,method=_RETURNVERBOSE)

[Out] 1/3*sum(_R^4/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6-x^3+1),x, algorithm="maxima")

[Out] integrate(x^4/(x^6 - x^3 + 1), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 994 vs. 2(267) = 534.
time = 0.45, size = 994, normalized size = 2.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(x^6-x^3+1),x, algorithm="fricas")

[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(-48*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 48*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 144*x^2 - 24*18^(1/3)*12^(1/3)*x + 4*18^(2/3)*12^(2/3)) + 2/27*18^(2/3)*12^(1/6)*arctan(1/216*(24*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 - 12*18^(2/3)*12^(2/3)*sqrt(3)*x - 24*(144*cos(2/3*arctan(sqrt(3) + 2))^3 + (18^(2/3)*12^(2/3)*x - 72)*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-48*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*a

```
rctan(sqrt(3) + 2)) + 48*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2
 + 144*x^2 - 24*18^(1/3)*12^(1/3)*x + 4*18^(2/3)*12^(2/3))*(2*18^(2/3)*12^(
2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2/3*a
rctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(2/3)*sqrt(3
)) + 216*sqrt(3))/(16*cos(2/3*arctan(sqrt(3) + 2))^4 - 16*cos(2/3*arctan(sq
rt(3) + 2))^2 + 3))*sin(2/3*arctan(sqrt(3) + 2)) + 1/27*(18^(2/3)*12^(1/6)*
sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqr
t(3) + 2)))*arctan(-1/108*(12*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sq
rt(3) + 2))^2 - 6*18^(2/3)*12^(2/3)*sqrt(3)*x + 12*(144*cos(2/3*arctan(sqrt
(3) + 2))^3 + (18^(2/3)*12^(2/3)*x - 72)*cos(2/3*arctan(sqrt(3) + 2)))*sin(
2/3*arctan(sqrt(3) + 2)) - sqrt(12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arct
an(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 12*18^(1/3)*12^(1/3)*x*cos(
2/3*arctan(sqrt(3) + 2))^2 + 36*x^2 - 6*18^(1/3)*12^(1/3)*x + 18^(2/3)*12^(
2/3))*(2*18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*18^(2
/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) - 18
^(2/3)*12^(2/3)*sqrt(3)) + 108*sqrt(3))/(16*cos(2/3*arctan(sqrt(3) + 2))^4
- 16*cos(2/3*arctan(sqrt(3) + 2))^2 + 3)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)
*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) +
2)))*arctan(-1/1728*(24*18^(2/3)*12^(2/3)*x - 1728*cos(2/3*arctan(sqrt(3) +
 2))^2 - 18^(2/3)*12^(2/3)*sqrt(-384*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqr
t(3) + 2))^2 + 576*x^2 + 192*18^(1/3)*12^(1/3)*x + 16*18^(2/3)*12^(2/3)) +
864)/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)))) - 1/108*(
18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*
cos(2/3*arctan(sqrt(3) + 2)))*log(192*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*a
rctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 192*18^(1/3)*12^(1/3)*x*
cos(2/3*arctan(sqrt(3) + 2))^2 + 576*x^2 - 96*18^(1/3)*12^(1/3)*x + 16*18^(
2/3)*12^(2/3)) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) +
2)) - 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-384*18^(1/3)*12^
(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 576*x^2 + 192*18^(1/3)*12^(1/3)*x
+ 16*18^(2/3)*12^(2/3))
```

Sympy [A]
time = 0.07, size = 26, normalized size = 0.06

$$\text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(6561t^5 + 54t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t**
2 + x)))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 827 vs. 2(267) = 534.
time = 3.76, size = 827, normalized size = 2.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10
*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(
4/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)
*sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*co
s(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(
2/9*pi)^5 - 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)
*sin(2/9*pi)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi
)^3 - 2*sin(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*c
os(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1
/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(
3)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*
cos(1/9*pi)^4*sin(1/9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^
5 - sqrt(3)*cos(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*p
i))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)
*sin(1/9*pi))) - 1/18*(10*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*co
s(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqrt(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*
cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(
4/9*pi)*sin(4/9*pi) + cos(4/9*pi)^2 - sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/
9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*
pi) - 20*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*
cos(2/9*pi)^5 - 20*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)
^4 + 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log
((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(10*sqrt(3)*cos
(1/9*pi)^4*sin(1/9*pi) - 20*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)
*sin(1/9*pi)^5 - 2*cos(1/9*pi)^5 + 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(
1/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 -
 sin(1/9*pi)^2)*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B]
time = 1.72, size = 304, normalized size = 0.74

$$\frac{\ln\left(x+\left(162\,x+\frac{27\,{\left(-36+\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(-36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x-\left(162\,x+\frac{27\,{\left(-36-\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(-36-\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right)\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right)\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(-3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(-3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (162*x + (27*(3^(1/2)*12i - 36)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/16
2))*(3^(1/2)*12i - 36)^(1/3))/18 + (log(x - (162*x + (27*(- 3^(1/2)*12i - 3
6)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(- 3^(1/2)*12i - 36)^(1/3))/18 - (
2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(- 3^(1/2)*1i - 3)^(2/3))/12 + (2^(1/3)*3^
(1/6)*(- 3^(1/2)*1i - 3)^(2/3)*1i)/4)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3
^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/
```

```
12 - (2^(1/3)*3^(1/6)*(3^(1/2)*1i - 3)^(2/3)*1i)/4)*(3^(1/2)*1i - 3)^(1/3)*
(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(- 3^(1/2)*1
i - 3)^(2/3))/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(
2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*1i - 3)^(
1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.173 $\int \frac{x^3}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\right.}{9\sqrt[3]{2}\left(\right.}$$

```
[Out] 1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^(
2/3)/(1+I*3^(1/2))^(2/3)+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3-I*3^(1/
2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))
^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/18*
ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3
)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3
+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*
3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1-I*3^(1/2))^(2/3)
```

Rubi [A]

time = 0.18, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1388, 206, 31, 648, 631, 210, 642}

$$-\frac{\left(\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(-\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/
(2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*
Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sq
rt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(
2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)
*(1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(
1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3))
- ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x
+ 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^3}{1-x^3+x^6}\,dx = -\left(\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx\right)+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= \frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{\right.}\right.}}}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-$$

$$= \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-$$

$$= -\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right.}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 39, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{-1+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - x^3 + x^6),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 40, normalized size = 0.10

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}}{3}$	40
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}}{3}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(267) = 534$.
time = 0.42, size = 765, normalized size = 1.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(36*18^(2/3)*12^(1/6
)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 324*x^2 + 54*18^(1/3)*12^(1/3))
+ 2/27*18^(2/3)*12^(1/6)*arctan(1/216*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*sq
rt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 18*x^2 + 3*
18^(1/3)*12^(1/3)) - 6*18^(1/3)*12^(5/6)*sqrt(3)*x - 216*sin(2/3*arctan(sqr
t(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) + 1/
27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1
/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/648*(36*18^(1/3)*12^(5/6)*sqrt(3
)*x*cos(2/3*arctan(sqrt(3) + 2)) - 108*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*ar
ctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-72*18^(2/3)*12^(1/
6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 216*18^(2/3)*12^(1/6)*x*cos(2/3
```

```
*arctan(sqrt(3) + 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3))*(18^(1/3)*12^(5/6
)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sin(2/3*arctan
(sqrt(3) + 2))) + 648*sqrt(3))/(4*cos(2/3*arctan(sqrt(3) + 2))^2 - 3)) + 1/
27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1
/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/648*(36*18^(1/3)*12^(5/6)*sqrt(
3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 108*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*a
rctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-72*18^(2/3)*12^(1
/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 216*18^(2/3)*12^(1/6)*x*cos(2/
3*arctan(sqrt(3) + 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3))*(18^(1/3)*12^(5/
6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/6)*sin(2/3*arcta
n(sqrt(3) + 2))) - 648*sqrt(3))/(4*cos(2/3*arctan(sqrt(3) + 2))^2 - 3)) - 1
/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^
(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-72*18^(2/3)*12^(1/6)*sqrt(3)*x*sin
(2/3*arctan(sqrt(3) + 2)) + 216*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3)
+ 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3)) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3
)*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) +
 2)))*log(-72*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 21
6*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 1296*x^2 + 216*18^(1/3
)*12^(1/3))
```

Sympy [**A**]
time = 0.07, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-1458t^4 - 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t +
 x)))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(267) = 534$.
time = 3.67, size = 640, normalized size = 1.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2*
sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4/9
*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*co
s(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(
2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^
4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos
```

```
(2/9*pi) + sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1
/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(
3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*
sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*p
i))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)
*sin(1/9*pi))) - 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(
4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 -
2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9*pi) - cos(4/9*pi))*log((-I*sqrt(3)*cos(4/
9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*p
i) - 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)
^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log
((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(8*sqrt(3)*cos(
1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)
^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi)
 - cos(1/9*pi))*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B]

time = 1.84, size = 327, normalized size = 0.80

$$\frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(-3-\sqrt{3}\,1i\right)}^{1/3}\,1i}{6}\right)\,{\left(-36-\sqrt{3}\,12i\right)}^{1/3}}{18}+\frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(-3+\sqrt{3}\,1i\right)}^{1/3}\,1i}{6}\right)\,{\left(-36+\sqrt{3}\,12i\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(-3-\sqrt{3}\,1i\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(-3-\sqrt{3}\,1i\right)}^{4/3}}{12}\right)\,{\left(-3-\sqrt{3}\,1i\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1i\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(-3+\sqrt{3}\,1i\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(-3+\sqrt{3}\,1i\right)}^{4/3}}{12}\right)\,{\left(-3+\sqrt{3}\,1i\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1i\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(-3-\sqrt{3}\,1i\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(-3-\sqrt{3}\,1i\right)}^{1/3}\,1i}{12}\right)\,{\left(-3-\sqrt{3}\,1i\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1i\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(-3+\sqrt{3}\,1i\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(-3+\sqrt{3}\,1i\right)}^{1/3}\,1i}{12}\right)\,{\left(-3+\sqrt{3}\,1i\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1i\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i -
36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(3^
(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i
 - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)*(- 3^(1/2)*
1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3
)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(4/3))/12)*
(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/
3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3
)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2
/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(
3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)
)/36
```

3.174 $\int \frac{x^2}{1-x^3+x^6}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1366, 632, 210}

$$-\frac{2\text{ArcTan}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^3 + x^6),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^2}{1-x^3+x^6}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)$$
$$= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\right)$$
$$= -\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - x^3 + x^6),x]
[Out] (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.83

method	result	size
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	19
risch	$\frac{2\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] 2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A]
time = 0.55, size = 18, normalized size = 0.78

$$\frac{2}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2x^3-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="maxima")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Fricas [A]
time = 0.36, size = 18, normalized size = 0.78

$$\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="fricas")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Sympy [A]
time = 0.04, size = 27, normalized size = 1.17

$$\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6-x**3+1),x)
[Out] 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Giac [A]
time = 3.59, size = 18, normalized size = 0.78

$$\frac{2}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-x^3+1),x, algorithm="giac")
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Mupad [B]
time = 1.22, size = 20, normalized size = 0.87

$$-\frac{2\,\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6 - x^3 + 1),x)
[Out] -(2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

3.175 $\int \frac{x}{1-x^3+x^6} \, dx$

Optimal. Leaf size=375

$$\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(\sqrt[3]{\right.}}{3\sqrt{3}}$$

```
[Out] 1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))/(1-I*
3^(1/2))^(1/3)-1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))
*3^(1/2))/(1+I*3^(1/2))^(1/3)+1/9*I*2^(1/3)*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/
3))/(1-I*3^(1/2))^(1/3)*3^(1/2)-1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2
))^(1/3)+(1-I*3^(1/2))^(2/3))*2^(1/3)/(1-I*3^(1/2))^(1/3)*3^(1/2)-1/9*I*2^(
1/3)*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))/(1+I*3^(1/2))^(1/3)*3^(1/2)+1/18*I*
ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*2^(1/3)/(
1+I*3^(1/2))^(1/3)*3^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1389, 298, 31, 648, 631, 210, 642}

$$\frac{i\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^3 + x^6),x]
```

```
[Out] ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sqrt[
3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]]
)/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x]
)/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3) -
2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 - I*Sqrt[3]
)^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 -
 I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))
^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x}{1-x^3+x^6}\,dx = -\frac{i\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}} + \frac{i\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$= \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\int \cdots}{3\sqrt{3}\cdots}$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\cdots}}\cdots}{2\sqrt{\cdots}}$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} - \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}\cdots\right)}{3\;2^{2/3}\sqrt{\cdots}}$$

$$= \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}\cdots\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-\cdots\right)}}$$

<u>**Mathematica**</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 40, normalized size = 0.11

$$\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^3 + x^6),x]
[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 38, normalized size = 0.10

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	38
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sum(_R/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 994 vs. 2(241) = 482.
time = 0.45, size = 994, normalized size = 2.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(48*18^(1/3)*12^(1/3
)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 48*
18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 144*x^2 - 24*18^(1/3)*
12^(1/3)*x + 4*18^(2/3)*12^(2/3)) + 2/27*18^(2/3)*12^(1/6)*arctan(-1/108*(1
2*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 - 6*18^(2/3)*1
2^(2/3)*sqrt(3)*x + 12*(144*cos(2/3*arctan(sqrt(3) - 2))^3 + (18^(2/3)*12^(
2/3)*x - 72)*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqrt(3) - 2)) - s
qrt(12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arc
tan(sqrt(3) - 2)) + 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 +
```

```
36*x^2 - 6*18^(1/3)*12^(1/3)*x + 18^(2/3)*12^(2/3))*(2*18^(2/3)*12^(2/3)*s
qrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(
sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(2/3)*sqrt(3)) + 1
08*sqrt(3))/(16*cos(2/3*arctan(sqrt(3) - 2))^4 - 16*cos(2/3*arctan(sqrt(3)
- 2))^2 + 3))*sin(2/3*arctan(sqrt(3) - 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3
)*cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) -
 2)))*arctan(1/432*(48*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
 2))^2 - 24*18^(2/3)*12^(2/3)*sqrt(3)*x - 48*(144*cos(2/3*arctan(sqrt(3) -
2))^3 + (18^(2/3)*12^(2/3)*x - 72)*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*ar
ctan(sqrt(3) - 2)) - sqrt(-192*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(s
qrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 192*18^(1/3)*12^(1/3)*x*cos(2/3
*arctan(sqrt(3) - 2))^2 + 576*x^2 - 96*18^(1/3)*12^(1/3)*x + 16*18^(2/3)*12
^(2/3))*(2*18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 2*18^
(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) -
18^(2/3)*12^(2/3)*sqrt(3)) + 432*sqrt(3))/(16*cos(2/3*arctan(sqrt(3) - 2))^
4 - 16*cos(2/3*arctan(sqrt(3) - 2))^2 + 3)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(
3)*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3)
- 2)))*arctan(-1/1728*(24*18^(2/3)*12^(2/3)*x - 1728*cos(2/3*arctan(sqrt(3)
 - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(-384*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(s
qrt(3) - 2))^2 + 576*x^2 + 192*18^(1/3)*12^(1/3)*x + 16*18^(2/3)*12^(2/3))
+ 864)/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)))) + 1/108
*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6
)*cos(2/3*arctan(sqrt(3) - 2)))*log(-192*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/
3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 192*18^(1/3)*12^(1/3)
*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 576*x^2 - 96*18^(1/3)*12^(1/3)*x + 16*1
8^(2/3)*12^(2/3)) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3)
 - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(-384*18^(1/3)*
12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 576*x^2 + 192*18^(1/3)*12^(1/3)
*x + 16*18^(2/3)*12^(2/3))
```

Sympy [**A**]
time = 0.07, size = 26, normalized size = 0.07

$$\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(6561t^5 - 27t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 - 27*_t**
2 + x)))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(241) = 482.
time = 4.18, size = 815, normalized size = 2.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sq
rt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*
pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 + sqrt(3)*sin(4
/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(4/9*
pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^
5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*
pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(
2/9*pi)^5 - sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2 + 2*cos(2/9*pi)*s
in(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*I*sqrt(3)
 + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)
^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*si
n(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 + sqrt(3)*cos(1/
9*pi)^2 - sqrt(3)*sin(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi))*arctan(-1/2*((
-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(1/9*pi))) - 1
/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9
*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*p
i)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 - 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) - co
s(4/9*pi)^2 + sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))*x +
 x^2 + 1) - 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*
pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi
)^3*sin(2/9*pi)^2 + 5*cos(2/9*pi)*sin(2/9*pi)^4 - 2*sqrt(3)*cos(2/9*pi)*sin
(2/9*pi) - cos(2/9*pi)^2 + sin(2/9*pi)^2)*log((-I*sqrt(3)*cos(2/9*pi) - cos
(2/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqr
t(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 +
10*cos(1/9*pi)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 + 2*sqrt(3)*co
s(1/9*pi)*sin(1/9*pi) - cos(1/9*pi)^2 + sin(1/9*pi)^2)*log((I*sqrt(3)*cos(1
/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B]
time = 0.45, size = 304, normalized size = 0.81

$$\frac{\ln\left(x+\left(81\,x-\frac{27\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x-\left(81\,x-\frac{27\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}\,1\mathrm{i}}{4}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162
))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)^(
2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3
)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3)*3^(1/6)*(3
 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/
36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12 - (2^(1/3
```

```
)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3
^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/
6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^
(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) +
3^(5/6)*1i))/36
```

3.176 $\int \frac{1}{1-x^3+x^6}\,dx$

Optimal. Leaf size=186

$$-\frac{1}{3}(-1)^{13/18}\tan^{-1}\left(\frac{1+2\sqrt[9]{-1}\,x}{\sqrt{3}}\right)+\frac{1}{3}(-1)^{5/18}\tan^{-1}\left(\frac{1-2(-1)^{8/9}x}{\sqrt{3}}\right)-\frac{(-1)^{5/18}\left(\log(2)+3\log\left(\sqrt[9]{-1}\right.\right.}{9\sqrt{3}}$$

```
[Out] -1/3*(-1)^(13/18)*arctan(1/3*(1+2*(-1)^(1/9)*x)*3^(1/2))+1/3*(-1)^(5/18)*ar
ctan(1/3*(1-2*(-1)^(8/9)*x)*3^(1/2))-1/27*(-1)^(5/18)*(ln(2)+3*ln((-1)^(1/9
)-x))*3^(1/2)+1/9*(-1)^(13/18)*ln(-2^(1/3)*((-1)^(8/9)+x))*3^(1/2)-1/18*(-1
)^(13/18)*ln(-2^(2/3)*((-1)^(7/9)+((-1)^(8/9)-x)*x))*3^(1/2)+1/18*(-1)^(5/1
8)*ln(2^(2/3)*((-1)^(2/9)+x*((-1)^(1/9)+x)))*3^(1/2)
```

Rubi [C] Result contains complex when optimal does not.
time = 0.15, antiderivative size = 375, normalized size of antiderivative = 2.02, number of steps used = 13, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$,
Rules used = {1361, 206, 31, 648, 631, 210, 642}

$$-\frac{i\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3 + x^6)^(-1),x]
```

```
[Out] ((-1/3*I)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*Sq
rt[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)
*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/3)
 - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt
[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(
1 - I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3
]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{1-x^3+x^6}\,dx = -\frac{i\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}} + \frac{i\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$= \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} + \frac{i\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i\int \cdots}{3\sqrt{3}\cdots}$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}} - \frac{i\int \frac{\sqrt[3]{\frac{1}{2}}\left(1-\cdots\right.}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\cdots}}{3\sqrt[3]{2}\sqrt{3}\left(\cdots\right.}$$

$$= \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}\cdots\right.}{3\sqrt[3]{2}\sqrt{\cdots}}$$

$$= -\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} + \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1-i\cdots}\right.}{3\sqrt{3}\left(\frac{1}{2}\left(1\cdots\right.\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 42, normalized size = 0.23

$$\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-\#1^2+2\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3 + x^6)^(-1),x]
```

```
[Out] RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.01, size = 37, normalized size = 0.20

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	37
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sum(1/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 767 vs. $2(131) = 262$.
time = 0.43, size = 767, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(72*18^(2/3)*12^(1/6
)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 216*18^(2/3)*12^(1/6)*x*cos(2/3*
arctan(sqrt(3) - 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3)) + 2/27*18^(2/3)*12
^(1/6)*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3)
- 2)) - sqrt(2)*sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2
)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 18*x^2 + 3*18^(1/
3)*12^(1/3))*(18^(1/3)*12^(5/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 3*18
^(1/3)*12^(5/6)*sin(2/3*arctan(sqrt(3) - 2))) + 18*(18^(1/3)*12^(5/6)*x + 2
4*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqrt(3) - 2)) + 108*sqrt(3))
/(4*cos(2/3*arctan(sqrt(3) - 2))^2 - 3))*sin(2/3*arctan(sqrt(3) - 2)) + 1/2
7*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/
```

```
6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*
x*cos(2/3*arctan(sqrt(3) - 2)) - sqrt(2)*sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*s
in(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3)
- 2)) + 18*x^2 + 3*18^(1/3)*12^(1/3))*(18^(1/3)*12^(5/6)*sqrt(3)*cos(2/3*ar
ctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sin(2/3*arctan(sqrt(3) - 2))) - 18
*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqr
t(3) - 2)) - 108*sqrt(3))/(4*cos(2/3*arctan(sqrt(3) - 2))^2 - 3)) - 1/27*(1
8^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*s
in(2/3*arctan(sqrt(3) - 2)))*arctan(-1/2592*(72*18^(1/3)*12^(5/6)*sqrt(3)*x
 - 18^(1/3)*12^(5/6)*sqrt(3)*sqrt(-576*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*
arctan(sqrt(3) - 2)) + 5184*x^2 + 864*18^(1/3)*12^(1/3)) - 2592*sin(2/3*arc
tan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2))) + 1/108*(18^(2/3)*12^(1/6)
*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arctan(sq
rt(3) - 2)))*log(288*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2
)) - 864*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 5184*x^2 + 864*
18^(1/3)*12^(1/3)) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3
) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(-576*18^(2/3)
*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 5184*x^2 + 864*18^(1/3)*
12^(1/3))
```

Sympy [**A**]
time = 0.07, size = 20, normalized size = 0.11

$$\operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(729t^4+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-x**3+1),x)
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x)))
```

Giac [**C**] Result contains complex when optimal does not.
time = 3.95, size = 632, normalized size = 3.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(4/9
*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)
^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(
2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9*pi) -
 sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*I*sqrt(
3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi
```

```
)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4
*cos(1/9*pi)*sin(1/9*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*arctan(-1/2
*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(1/9*pi)))
- 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9
*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 - sq
rt(3)*sin(4/9*pi) + cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))
*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2
/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(
2/9*pi)^4 - sqrt(3)*sin(2/9*pi) + cos(2/9*pi))*log((-I*sqrt(3)*cos(2/9*pi)
- cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4
*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/
9*pi)^2 + sin(1/9*pi)^4 + sqrt(3)*sin(1/9*pi) + cos(1/9*pi))*log((I*sqrt(3)
*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [**B**]
time = 1.79, size = 327, normalized size = 1.76

$$\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 -
 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x + (2^(2/3)
*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3
)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*
(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12)*(3
 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)
*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(4/3
))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x +
 (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(
1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)
^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.177 $\int \frac{1}{x(1-x^3+x^6)}\,dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1371, 719, 29, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - x^3 + x^6)),x]

[Out] -1/3*ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[1 - x^3 + x^6]/6

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1-x^3+x^6)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(1-x+x^2)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^3\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)-\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 55, normalized size = 1.34

$$\log(x)-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3 + x^6)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-1
+ 2*#1^3) & ]/3
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.85

method	result	size
risch	$\ln(x)-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$	33
default	$\ln(x)-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A]
time = 0.52, size = 38, normalized size = 0.93

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3*
log(x^3)
```

Fricas [A]
time = 0.39, size = 34, normalized size = 0.83

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

Sympy [A]
time = 0.05, size = 41, normalized size = 1.00

$$\log\left(x\right) - \frac{\log\left(x^{6} - x^{3} + 1\right)}{6} + \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^{3}}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9
```

Giac [A]
time = 3.79, size = 35, normalized size = 0.85

$$\frac{1}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^{3} - 1\right)\right) - \frac{1}{6}\,\log\left(x^{6} - x^{3} + 1\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

Mupad [B]
time = 1.23, size = 36, normalized size = 0.88

$$\ln\left(x\right) - \frac{\ln\left(x^{6} - x^{3} + 1\right)}{6} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\,\sqrt{3}\,x^{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/9
```

3.178 $\int \frac{1}{x^2(1-x^3+x^6)}\,dx$

Optimal. Leaf size=416

$$-\frac{1}{x}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\mathrm{l}}{9}$$

[Out] -1/x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2)
)*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*
3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(
1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-
1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))
^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3
))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/
(1+I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1+I*3^(1/2))^(1/3)

Rubi [A]

time = 0.18, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1382, 1524, 298, 31, 648, 631, 210, 642}

$$\frac{\left(-\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(1 - x^3 + x^6)),x]

[Out] -x^(-1) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/(
(1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3
 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqr
t[3])^(1/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*
2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
 + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^
(1/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(
1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
```

```
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2(1-x^3+x^6)}\,dx = -\frac{1}{x}+\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{x}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}$$

$$= -\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}}\left(1-\right.}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{}}}{9\ 2^{2/3}\sqrt[3]{1-i}}$$

$$= -\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\right.}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -\frac{1}{x}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\right.}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -\frac{1}{x}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{}{\sqrt[3]{\frac{1}{2}\left(1\right.}}}{\sqrt{}}\right.}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 61, normalized size = 0.15

$$-\frac{1}{x}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#
1 + 2*#1^4) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 50, normalized size = 0.12

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^{6}+9\_Z^{3}+1\right)}\_R\ln\left(-3\_R^{2}+x\right)\right)}{3}$	35
default	$-\frac{1}{x}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(\_R^{4}-\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x-1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^4 - x)/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1007 vs. 2(272) = 544.
time = 0.48, size = 1007, normalized size = 2.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2))*log(-24*18^(1/3)*
12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2 + 12*18^(1/3)*12^(1/3)*x
 + 18^(2/3)*12^(2/3)) + 8*18^(2/3)*12^(1/6)*x*arctan(-1/432*(6*18^(2/3)*12^
(2/3)*x - 432*cos(2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(-24*1
```

```
8^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2 + 12*18^(1/3)*12
^(1/3)*x + 18^(2/3)*12^(2/3)) + 216)/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*
arctan(sqrt(3) + 2))))*sin(2/3*arctan(sqrt(3) + 2)) + 4*(18^(2/3)*12^(1/6)*
sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*x*sin(2/3*arctan
(sqrt(3) + 2)))*arctan(1/216*(24*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan
(sqrt(3) + 2))^2 - 12*18^(2/3)*12^(2/3)*sqrt(3)*x - 24*(144*cos(2/3*arctan(
sqrt(3) + 2))^3 + (18^(2/3)*12^(2/3)*x - 72)*cos(2/3*arctan(sqrt(3) + 2)))*
sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-48*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3
*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 48*18^(1/3)*12^(1/3)*x
*cos(2/3*arctan(sqrt(3) + 2))^2 + 144*x^2 - 24*18^(1/3)*12^(1/3)*x + 4*18^(
2/3)*12^(2/3))*(2*18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2
- 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) +
 2)) - 18^(2/3)*12^(2/3)*sqrt(3)) + 216*sqrt(3))/(16*cos(2/3*arctan(sqrt(3)
 + 2))^4 - 16*cos(2/3*arctan(sqrt(3) + 2))^2 + 3)) - 4*(18^(2/3)*12^(1/6)*s
qrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(
sqrt(3) + 2)))*arctan(-1/108*(12*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan
(sqrt(3) + 2))^2 - 6*18^(2/3)*12^(2/3)*sqrt(3)*x + 12*(144*cos(2/3*arctan(s
qrt(3) + 2))^3 + (18^(2/3)*12^(2/3)*x - 72)*cos(2/3*arctan(sqrt(3) + 2)))*s
in(2/3*arctan(sqrt(3) + 2)) - sqrt(12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*a
rctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 12*18^(1/3)*12^(1/3)*x*c
os(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2 - 6*18^(1/3)*12^(1/3)*x + 18^(2/3)*1
2^(2/3))*(2*18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*18
^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) -
 18^(2/3)*12^(2/3)*sqrt(3)) + 108*sqrt(3))/(16*cos(2/3*arctan(sqrt(3) + 2))
^4 - 16*cos(2/3*arctan(sqrt(3) + 2))^2 + 3)) + (18^(2/3)*12^(1/6)*sqrt(3)*x
*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3)
+ 2)))*log(48*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(
2/3*arctan(sqrt(3) + 2)) + 48*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) +
2))^2 + 144*x^2 - 24*18^(1/3)*12^(1/3)*x + 4*18^(2/3)*12^(2/3)) - (18^(2/3)
*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*x*cos(
2/3*arctan(sqrt(3) + 2)))*log(-48*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arcta
n(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 48*18^(1/3)*12^(1/3)*x*cos(2
/3*arctan(sqrt(3) + 2))^2 + 144*x^2 - 24*18^(1/3)*12^(1/3)*x + 4*18^(2/3)*1
2^(2/3)) - 108)/x
```

Sympy [**A**]
time = 0.08, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(-27t^2+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x))) - 1
/x
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 829 vs. $2(272) = 544$.
time = 4.20, size = 829, normalized size = 1.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sqr
t(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*p
i)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)*si
n(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(4
/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*p
i)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2
/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - s
in(2/9*pi)^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 - 4*cos(2/
9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*I*
sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(
1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*p
i)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 - 2*sqrt(
3)*cos(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin(1/9*pi))*arc
tan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(1/
9*pi))) + 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi
)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^
3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 + 4*sqrt(3)*cos(4/9*pi)*sin(4
/9*pi) + 2*cos(4/9*pi)^2 - 2*sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9*pi) - c
os(4/9*pi))*x + x^2 + 1) + 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*s
qrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5
 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*cos(2/9*pi)*sin(2/9*pi)^4 + 4*sqrt(3)*
cos(2/9*pi)*sin(2/9*pi) + 2*cos(2/9*pi)^2 - 2*sin(2/9*pi)^2)*log((-I*sqrt(3
)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(5*sqrt(3)*cos(1/9*pi)^4*s
in(1/9*pi) - 10*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5
 - cos(1/9*pi)^5 + 10*cos(1/9*pi)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*p
i)^4 - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) + 2*cos(1/9*pi)^2 - 2*sin(1/9*pi)^
2)*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/x
```

Mupad [B]
time = 1.66, size = 286, normalized size = 0.69

$$\frac{\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(-36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{1}{x}+\frac{\ln\left(x-\frac{{\left(-36-\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{12}\right)\,{\left(-36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,{\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}^2}{24}\right)\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,{\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}^2}{24}\right)\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,{\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}^2}{24}\right)\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,{\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}^2}{24}\right)\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^6 - x^3 + 1)),x)
```

```
[Out] (log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1/
3))/18 - 1/x + (log(x - (- 3^(1/2)*12i - 36)^(2/3)/12)*(- 3^(1/2)*12i - 36)
^(1/3))/18 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) -
3^(5/6)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2
^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/
24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x -
(2^(1/3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i -
 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)*1
i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3)
+ 3^(5/6)*1i))/36
```

3.179 $\int \frac{1}{x^3(1-x^3+x^6)} \, dx$

Optimal. Leaf size=418

$$-\frac{1}{2x^2}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)}{9\sqrt[3]{}}$$

```
[Out] -1/2/x^2-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(
1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(
3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I
*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2
/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1
/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^
(2/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/6*arctan(1/3*(1+2*2^(1/3
)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Rubi [A]
time = 0.21, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1382, 1436, 206, 31, 648, 631, 210, 642}

$$-\frac{\left(-\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{1}{2x^2}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/2*1/x^2 - ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(1 + (2*x
)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) -
((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*
Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/
(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2
/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3
])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
```

```
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^3(1-x^3+x^6)}\,dx = -\frac{1}{2x^2}+\frac{1}{2}\int \frac{2-2x^3}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{2x^2}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1}}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+}}{9\sqrt[3]{2}\left(1-i\sqrt{}\right.}$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}\right.}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}\right.}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$= -\frac{1}{2x^2}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+}{\sqrt[3]{\frac{1}{2}\left(1\right.}}\right.}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 65, normalized size = 0.16

$$-\frac{1}{2x^2}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1^2+2\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/
(-#1^2 + 2*#1^5) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 50, normalized size = 0.12

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^6+9\_Z^3+1\right)}\_R\ln\left(9\_R^4+3\_R+x\right)\right)}{3}$	38
default	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(-\_R^3+1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2+1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 800 vs. 2(272) = 544.
time = 0.41, size = 800, normalized size = 1.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) + 2))*log(-72*18^(2/3
)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 216*18^(2/3)*12^(1/6)*x
*cos(2/3*arctan(sqrt(3) + 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3)) + 8*18^(2
/3)*12^(1/6)*x^2*arctan(1/648*(36*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arcta
```

```
n(sqrt(3) + 2)) - 108*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) + 2)
))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-72*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) + 2)) + 216*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) +
 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3))*(18^(1/3)*12^(5/6)*sqrt(3)*cos(2/3
*arctan(sqrt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sin(2/3*arctan(sqrt(3) + 2))) +
 648*sqrt(3))/(4*cos(2/3*arctan(sqrt(3) + 2))^2 - 3))*sin(2/3*arctan(sqrt(3
) + 2)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arctan(sqrt(3) + 2)) - 1
8^(2/3)*12^(1/6)*x^2*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/1296*(72*18^(1
/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 216*(18^(1/3)*12^(5/6
)*x - 24*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(
-288*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 864*18^(2/3
)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 5184*x^2 + 864*18^(1/3)*12^(1/3
))*(18^(1/3)*12^(5/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^
(5/6)*sin(2/3*arctan(sqrt(3) + 2))) - 1296*sqrt(3))/(4*cos(2/3*arctan(sqrt(
3) + 2))^2 - 3)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arctan(sqrt(3)
+ 2)) + 18^(2/3)*12^(1/6)*x^2*sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/216*(1
8^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3
*arctan(sqrt(3) + 2)) + 18*x^2 + 3*18^(1/3)*12^(1/3)) - 6*18^(1/3)*12^(5/6)
*sqrt(3)*x - 216*sin(2/3*arctan(sqrt(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2))
) + (18^(2/3)*12^(1/6)*sqrt(3)*x^2*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*
12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) + 2)))*log(576*18^(2/3)*12^(1/6)*sqrt(3
)*x*sin(2/3*arctan(sqrt(3) + 2)) + 5184*x^2 + 864*18^(1/3)*12^(1/3)) - (18^
(2/3)*12^(1/6)*sqrt(3)*x^2*sin(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)
*x^2*cos(2/3*arctan(sqrt(3) + 2)))*log(-288*18^(2/3)*12^(1/6)*sqrt(3)*x*sin
(2/3*arctan(sqrt(3) + 2)) - 864*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3)
+ 2)) + 5184*x^2 + 864*18^(1/3)*12^(1/3)) - 54)/x^2
```

Sympy [A]
time = 0.08, size = 31, normalized size = 0.07

$$\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 + 9t + x\right)\right)\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + 9*_t + x
))) - 1/(2*x**2)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(272) = 544.
time = 4.34, size = 645, normalized size = 1.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(3
)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^3
 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(
4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*
pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*c
os(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*
pi) + 2*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*
I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos
(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*
pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*
arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin
(1/9*pi))) + 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*
pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9
*pi)^4 + 2*sqrt(3)*sin(4/9*pi) - 2*cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi)
 - cos(4/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) -
4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2
/9*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log((-I*s
qrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(1/9*pi
)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*c
os(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(
1/9*pi))*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/2/x^2
```

Mupad [B]
time = 1.72, size = 324, normalized size = 0.78

$$\frac{\ln\left(x-\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9i}{2}\right)\left(-36-\sqrt{3}\,12i\right)^{1/3}}{54}\right)\left(-36-\sqrt{3}\,12i\right)^{1/3}}{18}+\frac{\ln\left(x+\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9i}{2}\right)\left(-36+\sqrt{3}\,12i\right)^{1/3}}{54}\right)\left(-36+\sqrt{3}\,12i\right)^{1/3}}{18}-\frac{1}{2\,x^2}-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(-3-\sqrt{3}\,1i\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1i\right)\left(\frac{3\left(3+\sqrt{3}\,1i\right)\left(3^{1/3}+3^{5/6}\,1i\right)^3}{16}+27\right)}{108}\right)\left(-3-\sqrt{3}\,1i\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1i\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(-3+\sqrt{3}\,1i\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1i\right)\left(\frac{3\left(-3+\sqrt{3}\,1i\right)\left(3^{1/3}-3^{5/6}\,1i\right)^3}{16}-27\right)}{108}\right)\left(-3+\sqrt{3}\,1i\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1i\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(-3-\sqrt{3}\,1i\right)^{1/3}\,1i}{6}\right)\left(-3-\sqrt{3}\,1i\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1i\right)}{36}-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(-3+\sqrt{3}\,1i\right)^{1/3}\,1i}{6}\right)\left(-3+\sqrt{3}\,1i\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1i\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^6 - x^3 + 1)),x)
```

```
[Out] (log(x - (((3^(1/2)*9i)/2 - 27/2)*(- 3^(1/2)*12i - 36)^(1/3))/54)*(- 3^(1/2
)*12i - 36)^(1/3))/18 + (log(x + (((3^(1/2)*9i)/2 + 27/2)*(3^(1/2)*12i - 36
)^(1/3))/54)*(3^(1/2)*12i - 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x - (2
^(2/3)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)*((3*(3^(1/2)*1i + 3)
*(3^(1/3) + 3^(5/6)*1i)^3)/16 + 27))/108)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3)
 + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i - 3)^(1/3)*(3^(1
/3) - 3^(5/6)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^(5/6)*1i)^3)/16 - 27))/
108)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (
2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3
^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i -
3)^(1/3)*1i)/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.180 $\int \frac{1}{x^4(1-x^3+x^6)} dx$

Optimal. Leaf size=48

$$-\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] -1/3/x^3+ln(x)-1/6*ln(x^6-x^3+1)+1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1371, 723, 814, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/3*1/x^3 + ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3
 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(1-x^3+x^6)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(1-x+x^2)}\,dx,x,x^3\right)\\ &= -\frac{1}{3x^3}+\frac{1}{3}\text{Subst}\left(\int \frac{1-x}{x(1-x+x^2)}\,dx,x,x^3\right)\\ &= -\frac{1}{3x^3}+\frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)\,dx,x,x^3\right)\\ &= -\frac{1}{3x^3}+\log(x)-\frac{1}{3}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^3\right)\\ &= -\frac{1}{3x^3}+\log(x)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)-\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x\right.\\ &= -\frac{1}{3x^3}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\ &= -\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 51, normalized size = 1.06

$$-\frac{1}{3x^3}+\log(x)-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/3*1/x^3 + Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 +
2*#1^3) & ]/3
```

Maple [A]
time = 0.02, size = 40, normalized size = 0.83

method	result	size
risch	$-\frac{1}{3x^3}+\ln(x)-\frac{\ln(x^6-x^3+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$	38
default	$-\frac{1}{3x^3}+\ln(x)-\frac{\ln(x^6-x^3+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3+ln(x)-1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A]
time = 0.53, size = 43, normalized size = 0.90

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3 - 1/6*log(x^6 - x^3
+ 1) + 1/3*log(x^3)
```

Fricas [A]
time = 0.35, size = 51, normalized size = 1.06

$$-\frac{2\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+3x^3\log\left(x^6-x^3+1\right)-18x^3\log\left(x\right)+6}{18x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3*x^3*log(x^6 - x^3
+ 1) - 18*x^3*log(x) + 6)/x^3
```

Sympy [A]
time = 0.06, size = 48, normalized size = 1.00

$$\log\left(x\right)-\frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9 - 1/(3*x**3)
```

Giac [A]
time = 4.38, size = 45, normalized size = 0.94

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{x^3+1}{3x^3}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3*(x^3 + 1)/x^3 - 1/6*log(
x^6 - x^3 + 1) + log(abs(x))
```

Mupad [B]
time = 0.06, size = 41, normalized size = 0.85

$$\ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\,x^3}{3}\right)}{9} - \frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/9 - 1/(3*x^3)
```

3.181 $\int \frac{1}{x^5(1-x^3+x^6)}\,dx$

Optimal. Leaf size=423

$$-\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\ldots\right.}{}$$

```
[Out] -1/4/x^4-1/x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I
-3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3
))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*
(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2)
)^(1/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*
3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/
2))^(2/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^
(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1-I*3^(1/2))^(1/
3)
```

Rubi [A]

time = 0.25, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1382, 1518, 12, 1388, 298, 31, 648, 631, 210, 642}

$$-\frac{\left(\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{4x^4}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^3 + x^6)),x]
```

```
[Out] -1/4*1/x^4 - x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^
(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])*ArcTan[
(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^
(1/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3
)*(1 - I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(
1/3)*x])/(9*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sq
rt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 -
 I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I
*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1}{x^5(1-x^3+x^6)}\,dx = -\frac{1}{4x^4}+\frac{1}{4}\int \frac{4-4x^3}{x^2(1-x^3+x^6)}\,dx$$
$$= -\frac{1}{4x^4}-\frac{1}{x}-\frac{1}{4}\int \frac{4x^4}{1-x^3+x^6}\,dx$$
$$= -\frac{1}{4x^4}-\frac{1}{x}-\int \frac{x^4}{1-x^3+x^6}\,dx$$
$$= -\frac{1}{4x^4}-\frac{1}{x}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}$$
$$= -\frac{1}{4x^4}-\frac{1}{x}+-\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\right.}{}$$
$$= -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\right.}{9\ 2^{2/3}\sqrt[3]{1+i}}$$
$$= -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\right.}{9\ 2^{2/3}\sqrt[3]{1+i}}$$
$$= -\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(1+\right.}{3\ 2^{2/3}\sqrt[3]{1+}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 54, normalized size = 0.13

$$-\frac{1}{4x^4}-\frac{1}{x}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^3 + x^6)),x]
```

```
[Out] -1/4*1/x^4 - x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 +
2*#1^3) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 51, normalized size = 0.12

method	result	size
risch	$\frac{-x^3-\frac{1}{4}}{x^4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^6-9\_Z^3+1\right)}\_R\ln\left(-27\_R^5+6\_R^2+x\right)\right)}{3}$	46
default	$-\frac{1}{4x^4}-\frac{1}{x}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4-1/x-1/3*sum(_R^4/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/4*(4*x^3 + 1)/x^4 - integrate(x^4/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(277) = 554.
time = 0.44, size = 1034, normalized size = 2.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x^4*cos(2/3*arctan(sqrt(3) - 2))*log(-48*18^(1/3
)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) -
2)) + 48*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 144*x^2 - 24*
18^(1/3)*12^(1/3)*x + 4*18^(2/3)*12^(2/3)) + 8*18^(2/3)*12^(1/6)*x^4*arctan
```

```
(1/216*(24*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 - 12*
18^(2/3)*12^(2/3)*sqrt(3)*x - 24*(144*cos(2/3*arctan(sqrt(3) - 2))^3 + (18^
(2/3)*12^(2/3)*x - 72)*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqrt(3)
 - 2)) - sqrt(-48*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*
sin(2/3*arctan(sqrt(3) - 2)) + 48*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3
) - 2))^2 + 144*x^2 - 24*18^(1/3)*12^(1/3)*x + 4*18^(2/3)*12^(2/3))*(2*18^(
2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2/3)*
cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(2/
3)*sqrt(3)) + 216*sqrt(3))/(16*cos(2/3*arctan(sqrt(3) - 2))^4 - 16*cos(2/3*
arctan(sqrt(3) - 2))^2 + 3))*sin(2/3*arctan(sqrt(3) - 2)) - 108*x^3 + 4*(18
^(2/3)*12^(1/6)*sqrt(3)*x^4*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6
)*x^4*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/108*(12*18^(2/3)*12^(2/3)*sqr
t(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 - 6*18^(2/3)*12^(2/3)*sqrt(3)*x + 12*
(144*cos(2/3*arctan(sqrt(3) - 2))^3 + (18^(2/3)*12^(2/3)*x - 72)*cos(2/3*ar
ctan(sqrt(3) - 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(12*18^(1/3)*12^(1/3
)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 12*
18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2 - 6*18^(1/3)*12
^(1/3)*x + 18^(2/3)*12^(2/3))*(2*18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(s
qrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*a
rctan(sqrt(3) - 2)) - 18^(2/3)*12^(2/3)*sqrt(3)) + 108*sqrt(3))/(16*cos(2/3
*arctan(sqrt(3) - 2))^4 - 16*cos(2/3*arctan(sqrt(3) - 2))^2 + 3)) - 4*(18^(
2/3)*12^(1/6)*sqrt(3)*x^4*cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*
x^4*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/1728*(24*18^(2/3)*12^(2/3)*x -
1728*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(-384*18^(1/3)*
12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 576*x^2 + 192*18^(1/3)*12^(1/3)
*x + 16*18^(2/3)*12^(2/3)) + 864)/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arc
tan(sqrt(3) - 2)))) - (18^(2/3)*12^(1/6)*sqrt(3)*x^4*sin(2/3*arctan(sqrt(3)
 - 2)) + 18^(2/3)*12^(1/6)*x^4*cos(2/3*arctan(sqrt(3) - 2)))*log(192*18^(1/
3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) -
 2)) + 192*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 576*x^2 - 9
6*18^(1/3)*12^(1/3)*x + 16*18^(2/3)*12^(2/3)) + (18^(2/3)*12^(1/6)*sqrt(3)*
x^4*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*x^4*cos(2/3*arctan(sqr
t(3) - 2)))*log(-384*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 5
76*x^2 + 192*18^(1/3)*12^(1/3)*x + 16*18^(2/3)*12^(2/3)) - 27)/x^4
```

Sympy [A]
time = 0.08, size = 39, normalized size = 0.09

$$\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(-6561t^5 + 54t^2 + x\right)\right)\right) + \frac{-4x^3 - 1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**6-x**3+1),x)
```

```
[Out] RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**5 + 54*_t*
*2 + x))) + (-4*x**3 - 1)/(4*x**4)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 839 vs. $2(277) = 554$.
time = 4.28, size = 839, normalized size = 1.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(4
/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)*
sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos
(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2
/9*pi)^5 - 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*
sin(2/9*pi)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)
^3 - 2*sin(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*co
s(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/
2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3
)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*c
os(1/9*pi)^4*sin(1/9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5
 - sqrt(3)*cos(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi
))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*
sin(1/9*pi))) + 1/18*(10*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos
(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqrt(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*c
os(4/9*pi)^3*sin(4/9*pi)^2 + 10*cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4
/9*pi)*sin(4/9*pi) + cos(4/9*pi)^2 - sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9
*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*p
i) - 20*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*c
os(2/9*pi)^5 - 20*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^
4 + 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(
(-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(
1/9*pi)^4*sin(1/9*pi) - 20*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*
sin(1/9*pi)^5 - 2*cos(1/9*pi)^5 + 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1
/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 -
sin(1/9*pi)^2)*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/4
*(4*x^3 + 1)/x^4
```

Mupad [B]
time = 1.59, size = 318, normalized size = 0.75

$$\frac{\ln\left(-x+\left(162\,x+\frac{27\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(-x-\left(162\,x+\frac{27\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{x^3+\frac{1}{4}}{x^4}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^6 - x^3 + 1)),x)
```

```
[Out] (log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162) -
 x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(- x - (162*x + (27*(36 - 3^(1/2)*12
i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 - (x^
3 + 1/4)/x^4 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12
 - (2^(1/3)*3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3
^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i +
3)^(2/3))/12 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i +
 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3
 - 3^(1/2)*1i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
- (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i
+ 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

3.182 $\int \frac{1}{2+x^3+x^6}\,dx$

Optimal. Leaf size=381

$$\frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}-\frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{i\log\left(\cdots\right.}{3\sqrt{7}\cdots}$$

```
[Out] -1/21*I*2^(2/3)*ln(2^(1/3)*x+(1-I*7^(1/2))^(1/3))/(1-I*7^(1/2))^(2/3)*7^(1/
2)+1/42*I*ln(2^(2/3)*x^2-2^(1/3)*x*(1-I*7^(1/2))^(1/3)+(1-I*7^(1/2))^(2/3))
*2^(2/3)/(1-I*7^(1/2))^(2/3)*7^(1/2)+1/21*I*2^(2/3)*ln(2^(1/3)*x+(1+I*7^(1/
2))^(1/3))/(1+I*7^(1/2))^(2/3)*7^(1/2)-1/42*I*ln(2^(2/3)*x^2-2^(1/3)*x*(1+I
*7^(1/2))^(1/3)+(1+I*7^(1/2))^(2/3))*2^(2/3)/(1+I*7^(1/2))^(2/3)*7^(1/2)+1/
21*I*2^(2/3)*arctan(1/3*(1-2*2^(1/3)*x/(1-I*7^(1/2))^(1/3))*3^(1/2))/(1-I*7
^(1/2))^(2/3)*21^(1/2)-1/21*I*2^(2/3)*arctan(1/3*(1-2*2^(1/3)*x/(1+I*7^(1/2
))^(1/3))*3^(1/2))/(1+I*7^(1/2))^(2/3)*21^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {1361, 206, 31, 648, 631, 210, 642}

$$\frac{i\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}-\frac{i\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}\,x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{i\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}\,x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{7}\left(1+i\sqrt{7}\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{7}}\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}+\frac{i\log\left(\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{7}}\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^3 + x^6)^(-1),x]
```

```
[Out] (I*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/(Sqrt[21]*((1 - I
*Sqrt[7])/2)^(2/3)) - (I*ArcTan[(1 - (2*x)/((1 + I*Sqrt[7])/2)^(1/3))/Sqrt[
3]])/(Sqrt[21]*((1 + I*Sqrt[7])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sqrt[7])^(1/3
) + 2^(1/3)*x])/(Sqrt[7]*((1 - I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[(1 + I*Sqr
t[7])^(1/3) + 2^(1/3)*x])/(Sqrt[7]*((1 + I*Sqrt[7])/2)^(2/3)) + ((I/3)*Log[
(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3
)*Sqrt[7]*(1 - I*Sqrt[7])^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[7])^(2/3) - (2*(1
 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[7]*(1 + I*Sqrt[7])^(2/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{2+x^3+x^6}\,dx = -\frac{i\int \frac{1}{\frac{1}{2}-\frac{i\sqrt{7}}{2}+x^3}\,dx}{\sqrt{7}} + \frac{i\int \frac{1}{\frac{1}{2}+\frac{i\sqrt{7}}{2}+x^3}\,dx}{\sqrt{7}}$$

$$= -\frac{i\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}+x}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\int \frac{2^{2/3}\sqrt[3]{1-i\sqrt{7}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\,x+x^2}\,dx}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\int}{3\sqrt{7}}$$

$$= -\frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}(}}{\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}}}{3\sqrt[3]{2}\sqrt{7}}$$

$$= -\frac{i\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{3\sqrt{7}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} + \frac{i\log\left(\left(1-i\sqrt{7}\right)^{2}\right.}{3\sqrt[3]{2}}$$

$$= \frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1-i\sqrt{7}\right)\right)^{2/3}} - \frac{i\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1-i\sqrt{}}\right.}{3\sqrt{7}\left(\frac{1}{2}\left(1-\right.\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 38, normalized size = 0.10

$$\frac{1}{3}\text{RootSum}\left[2+\#1^3+\#1^6\&,\frac{\log(x-\#1)}{\#1^2+2\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^3 + x^6)^(-1),x]
```

```
[Out] RootSum[2 + #1^3 + #1^6 & , Log[x - #1]/(#1^2 + 2*#1^5) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 33, normalized size = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+\_Z^{3}+2\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}+\_R^{2}}\right)}{3}$	33
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+\_Z^{3}+2\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}+\_R^{2}}\right)}{3}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+x^3+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sum(1/(2*_R^5+_R^2)*ln(x-_R),_R=RootOf(_Z^6+_Z^3+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^6 + x^3 + 2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1491 vs. 2(247) = 494.
time = 0.80, size = 1491, normalized size = 3.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="fricas")
```

```
[Out] 1/294*112^(1/6)*49^(2/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))*log(392*112^(1/
6)*49^(2/3)*sqrt(7)*x*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 2744*112^(1/6)*4
9^(2/3)*x*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + 38416*x^2 + 5488*49^(1/3)*14
^(1/3)) + 2/147*112^(1/6)*49^(2/3)*arctan(-1/2744*(14*112^(5/6)*49^(1/3)*sq
rt(7)*x*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - sqrt(2)*sqrt(112^(1/6)*49^(2/3
)*sqrt(7)*x*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 7*112^(1/6)*49^(2/3)*x*cos
(2/3*arctan(1/3*sqrt(7) + 4/3)) + 98*x^2 + 14*49^(1/3)*14^(1/3))*(112^(5/6)
*49^(1/3)*sqrt(7)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + 7*112^(5/6)*49^(1/3)
*sin(2/3*arctan(1/3*sqrt(7) + 4/3))) + 98*(112^(5/6)*49^(1/3)*x + 224*cos(2
/3*arctan(1/3*sqrt(7) + 4/3)))*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 2744*sq
rt(7))/(8*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 - 7))*sin(2/3*arctan(1/3*sqr
```

```
t(7) + 4/3)) + 1/147*(112^(1/6)*49^(2/3)*sqrt(3)*cos(2/3*arctan(1/3*sqrt(7)
 + 4/3)) - 112^(1/6)*49^(2/3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)))*arctan(1/
5488*(112*112^(5/6)*49^(1/3)*(sqrt(7)*x + 7*sqrt(3)*x)*cos(2/3*arctan(1/3*s
qrt(7) + 4/3))^3 - 14*112^(5/6)*49^(1/3)*(13*sqrt(7)*x + 21*sqrt(3)*x)*cos(
2/3*arctan(1/3*sqrt(7) + 4/3)) + 21952*(sqrt(7) - 3*sqrt(3))*cos(2/3*arctan
(1/3*sqrt(7) + 4/3))^2 - 14*(8*112^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3)*x - 7*x)
*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 25088*cos(2/3*arctan(1/3*sqrt(7) +
4/3))^3 + 112^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3)*x + 7*x) - 1568*(sqrt(7)*sqrt
(3) + 5)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)))*sin(2/3*arctan(1/3*sqrt(7) + 4
/3)) - (8*112^(5/6)*49^(1/3)*(sqrt(7) + 7*sqrt(3))*cos(2/3*arctan(1/3*sqrt(
7) + 4/3))^3 - 112^(5/6)*49^(1/3)*(13*sqrt(7) + 21*sqrt(3))*cos(2/3*arctan(
1/3*sqrt(7) + 4/3)) - (8*112^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3) - 7)*cos(2/3*a
rctan(1/3*sqrt(7) + 4/3))^2 + 112^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3) + 7))*sin
(2/3*arctan(1/3*sqrt(7) + 4/3)))*sqrt(112^(1/6)*49^(2/3)*(sqrt(7)*sqrt(3)*x
 - 7*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^(1/6)*49^(2/3)*(sqrt(7)*x
+ 7*sqrt(3)*x)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 196*x^2 + 28*49^(1/3)*1
4^(1/3)) + 5488*sqrt(7) + 10976*sqrt(3))/(64*cos(2/3*arctan(1/3*sqrt(7) + 4
/3))^4 - 40*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 1)) + 1/147*(112^(1/6)*4
9^(2/3)*sqrt(3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + 112^(1/6)*49^(2/3)*sin
(2/3*arctan(1/3*sqrt(7) + 4/3)))*arctan(-1/153664*(3136*112^(5/6)*49^(1/3)*
(sqrt(7)*x - 7*sqrt(3)*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^3 - 392*112^(5
/6)*49^(1/3)*(13*sqrt(7)*x - 21*sqrt(3)*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3
)) + 614656*(sqrt(7) + 3*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 39
2*(8*112^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3)*x + 7*x)*cos(2/3*arctan(1/3*sqrt(7
) + 4/3))^2 - 25088*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^3 + 112^(5/6)*49^(1/
3)*(sqrt(7)*sqrt(3)*x - 7*x) - 1568*(sqrt(7)*sqrt(3) - 5)*cos(2/3*arctan(1/
3*sqrt(7) + 4/3)))*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) - (8*112^(5/6)*49^(1/
3)*(sqrt(7) - 7*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^3 - 112^(5/6)*4
9^(1/3)*(13*sqrt(7) - 21*sqrt(3))*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) + (8*1
12^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3) + 7)*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^
2 + 112^(5/6)*49^(1/3)*(sqrt(7)*sqrt(3) - 7))*sin(2/3*arctan(1/3*sqrt(7) +
4/3)))*sqrt(-784*112^(1/6)*49^(2/3)*(sqrt(7)*sqrt(3)*x + 7*x)*cos(2/3*arcta
n(1/3*sqrt(7) + 4/3)) - 784*112^(1/6)*49^(2/3)*(sqrt(7)*x - 7*sqrt(3)*x)*si
n(2/3*arctan(1/3*sqrt(7) + 4/3)) + 153664*x^2 + 21952*49^(1/3)*14^(1/3)) +
153664*sqrt(7) - 307328*sqrt(3))/(64*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^4 -
 40*cos(2/3*arctan(1/3*sqrt(7) + 4/3))^2 + 1)) + 1/588*(112^(1/6)*49^(2/3)*
sqrt(3)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) - 112^(1/6)*49^(2/3)*cos(2/3*arc
tan(1/3*sqrt(7) + 4/3)))*log(-784*112^(1/6)*49^(2/3)*(sqrt(7)*sqrt(3)*x + 7
*x)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)) - 784*112^(1/6)*49^(2/3)*(sqrt(7)*x
- 7*sqrt(3)*x)*sin(2/3*arctan(1/3*sqrt(7) + 4/3)) + 153664*x^2 + 21952*49^(
1/3)*14^(1/3)) - 1/588*(112^(1/6)*49^(2/3)*sqrt(3)*sin(2/3*arctan(1/3*sqrt(
7) + 4/3)) + 112^(1/6)*49^(2/3)*cos(2/3*arctan(1/3*sqrt(7) + 4/3)))*log(784
*112^(1/6)*49^(2/3)*(sqrt(7)*sqrt(3)*x - 7*x)*cos(2/3*arctan(1/3*sqrt(7) +
4/3)) - 784*112^(1/6)*49^(2/3)*(sqrt(7)*x + 7*sqrt(3)*x)*sin(2/3*arctan(1/3
*sqrt(7) + 4/3)) + 153664*x^2 + 21952*49^(1/3)*14^(1/3))
```

Sympy [A]
time = 0.05, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(1000188t^6 + 1323t^3 + 1, \left(t \mapsto t\log\left(-5292t^4 + 7t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+x**3+2),x)
```

```
[Out] RootSum(1000188*_t**6 + 1323*_t**3 + 1, Lambda(_t, _t*log(-5292*_t**4 + 7*_
t + x)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+x^3+2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^6 + x^3 + 2), x)
```

Mupad [B]
time = 2.61, size = 513, normalized size = 1.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3 + x^6 + 2),x)
```

```
[Out] (log(x + (7^(1/3)*(- 7^(1/2)*3i - 7)^(1/3))/4 + (7^(5/6)*(- 7^(1/2)*3i - 7)
^(1/3)*1i)/28)*(- 7^(1/2)*21i - 49)^(1/3))/42 + (log(x + (7^(1/3)*(7^(1/2)*
3i - 7)^(1/3))/4 - (7^(5/6)*(7^(1/2)*3i - 7)^(1/3)*1i)/28)*(7^(1/2)*21i - 4
9)^(1/3))/42 + (7^(1/3)*log(6*x + (7^(1/3)*(3^(1/2)*1i - 1)*(- 7^(1/2)*3i -
 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i - 1)^2*(- 7^(1/2)*3i - 7)^(2/3)*(3969*x + (
567*7^(1/3)*(3^(1/2)*1i - 1)*(- 7^(1/2)*3i - 7)^(1/3))/2))/7056 + 63))/84)*
(3^(1/2)*1i - 1)*(- 7^(1/2)*3i - 7)^(1/3))/84 + (7^(1/3)*log(6*x + (7^(1/3)
*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i - 1)^2*(7^(1
/2)*3i - 7)^(2/3)*(3969*x + (567*7^(1/3)*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^
(1/3))/2))/7056 + 63))/84)*(3^(1/2)*1i - 1)*(7^(1/2)*3i - 7)^(1/3))/84 - (7
^(1/3)*log(6*x - (7^(1/3)*(3^(1/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1/3)*((7^(2/
3)*(3^(1/2)*1i + 1)^2*(- 7^(1/2)*3i - 7)^(2/3)*(3969*x - (567*7^(1/3)*(3^(1
/2)*1i + 1)*(- 7^(1/2)*3i - 7)^(1/3))/2))/7056 + 63))/84)*(3^(1/2)*1i + 1)*
(- 7^(1/2)*3i - 7)^(1/3))/84 - (7^(1/3)*log(6*x - (7^(1/3)*(3^(1/2)*1i + 1)
*(7^(1/2)*3i - 7)^(1/3)*((7^(2/3)*(3^(1/2)*1i + 1)^2*(7^(1/2)*3i - 7)^(2/3)
*(3969*x - (567*7^(1/3)*(3^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(1/3))/2))/7056 +
 63))/84)*(3^(1/2)*1i + 1)*(7^(1/2)*3i - 7)^(1/3))/84
```

3.183 $\int \frac{x^2}{2+x^3+x^6}\,dx$

Optimal. Leaf size=23

$$\frac{2\tan^{-1}\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}$$

```
[Out] 2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1366, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{2x^3+1}{\sqrt{7}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + x^3 + x^6),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^2}{2+x^3+x^6}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{2+x+x^2}\,dx, x, x^3\right)$$
$$= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, 1+2x^3\right)\right)$$
$$= \frac{2\tan^{-1}\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{1+2x^3}{\sqrt{7}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + x^3 + x^6),x]
[Out] (2*ArcTan[(1 + 2*x^3)/Sqrt[7]])/(3*Sqrt[7])
```

Maple [A]
time = 0.03, size = 19, normalized size = 0.83

method	result	size
default	$\frac{2\arctan\left(\frac{(2x^3+1)\sqrt{7}}{7}\right)\sqrt{7}}{21}$	19
risch	$\frac{2\arctan\left(\frac{(2x^3+1)\sqrt{7}}{7}\right)\sqrt{7}}{21}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6+x^3+2),x,method=_RETURNVERBOSE)
[Out] 2/21*arctan(1/7*(2*x^3+1)*7^(1/2))*7^(1/2)
```

Maxima [A]
time = 0.54, size = 18, normalized size = 0.78

$$\frac{2}{21}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^3+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="maxima")
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Fricas [A]
time = 0.36, size = 18, normalized size = 0.78

$$\frac{2}{21}\,\sqrt{7}\,\arctan\left(\frac{1}{7}\,\sqrt{7}\,\left(2\,x^3+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="fricas")
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Sympy [A]
time = 0.04, size = 27, normalized size = 1.17

$$\frac{2\sqrt{7}\,\operatorname{atan}\left(\frac{2\sqrt{7}\,x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6+x**3+2),x)
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**3/7 + sqrt(7)/7)/21
```

Giac [A]
time = 3.37, size = 18, normalized size = 0.78

$$\frac{2}{21}\,\sqrt{7}\,\arctan\left(\frac{1}{7}\,\sqrt{7}\,\left(2\,x^3+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6+x^3+2),x, algorithm="giac")
[Out] 2/21*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^3 + 1))
```

Mupad [B]
time = 0.05, size = 20, normalized size = 0.87

$$\frac{2\,\sqrt{7}\,\operatorname{atan}\left(\frac{2\,\sqrt{7}\,x^3}{7}+\frac{\sqrt{7}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^3 + x^6 + 2),x)
[Out] (2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^3)/7))/21
```

3.184 $\int \frac{x^3}{2+x^3+x^6}\,dx$

Optimal. Leaf size=399

$$-\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}+\frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}+\frac{(7 \cdots}{}$$

```
[Out] 1/42*ln(2^(1/3)*x+(1+I*7^(1/2))^(1/3))*(7-I*7^(1/2))*2^(2/3)/(1+I*7^(1/2))^
(2/3)-1/84*ln(2^(2/3)*x^2-2^(1/3)*x*(1+I*7^(1/2))^(1/3)+(1+I*7^(1/2))^(2/3)
)*(7-I*7^(1/2))*2^(2/3)/(1+I*7^(1/2))^(2/3)+1/42*ln(2^(1/3)*x+(1-I*7^(1/2))
^(1/3))*(7+I*7^(1/2))*2^(2/3)/(1-I*7^(1/2))^(2/3)-1/84*ln(2^(2/3)*x^2-2^(1/
3)*x*(1-I*7^(1/2))^(1/3)+(1-I*7^(1/2))^(2/3))*(7+I*7^(1/2))*2^(2/3)/(1-I*7^
(1/2))^(2/3)-1/42*I*arctan(1/3*(1-2*2^(1/3)*x/(1-I*7^(1/2))^(1/3))*3^(1/2))
*(1-I*7^(1/2))^(1/3)*2^(2/3)*21^(1/2)+1/42*I*arctan(1/3*(1-2*2^(1/3)*x/(1+I
*7^(1/2))^(1/3))*3^(1/2))*(1+I*7^(1/2))^(1/3)*2^(2/3)*21^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1388, 206, 31, 648, 631, 210, 642}

$$-\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\,\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}+\frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\,\text{ArcTan}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}}-\frac{\left(7+i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1-i\sqrt{7}\right)}\,x+\left(1-i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}-\frac{\left(7-i\sqrt{7}\right)\log\left(2^{2/3}x^2-\sqrt[3]{2\left(1+i\sqrt{7}\right)}\,x+\left(1+i\sqrt{7}\right)^{2/3}\right)}{42\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}+\frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{7}}\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}}+\frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{7}}\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 + x^3 + x^6),x]
```

```
[Out] ((-I)*((1 - I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 - I*Sqrt[7])/2)^(1/3)
)/Sqrt[3]])/Sqrt[21] + (I*((1 + I*Sqrt[7])/2)^(1/3)*ArcTan[(1 - (2*x)/((1 +
 I*Sqrt[7])/2)^(1/3))/Sqrt[3]])/Sqrt[21] + ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt
[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) + ((7 - I*Sqrt[
7])*Log[(1 + I*Sqrt[7])^(1/3) + 2^(1/3)*x])/(21*2^(1/3)*(1 + I*Sqrt[7])^(2/
3)) - ((7 + I*Sqrt[7])*Log[(1 - I*Sqrt[7])^(2/3) - (2*(1 - I*Sqrt[7]))^(1/3
)*x + 2^(2/3)*x^2])/(42*2^(1/3)*(1 - I*Sqrt[7])^(2/3)) - ((7 - I*Sqrt[7])*L
og[(1 + I*Sqrt[7])^(2/3) - (2*(1 + I*Sqrt[7]))^(1/3)*x + 2^(2/3)*x^2])/(42*
2^(1/3)*(1 + I*Sqrt[7])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^3}{2+x^3+x^6}\,dx = \frac{1}{14}\left(7-i\sqrt{7}\right)\int \frac{1}{\frac{1}{2}+\frac{i\sqrt{7}}{2}+x^3}\,dx + \frac{1}{14}\left(7+i\sqrt{7}\right)\int \frac{1}{\frac{1}{2}-\frac{i\sqrt{7}}{2}+x^3}\,dx$$

$$= \frac{\left(7-i\sqrt{7}\right)\int \frac{1}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}+x}\,dx}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} + \frac{\left(7-i\sqrt{7}\right)\int \frac{2^{2/3}\sqrt[3]{1+i\sqrt{7}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{7}\right)\right)^{2/3}-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)} \cdots}}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} \cdots$$

$$= \frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} + \frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} - \cdots$$

$$= \frac{\left(7+i\sqrt{7}\right)\log\left(\sqrt[3]{1-i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{21\sqrt[3]{2}\left(1-i\sqrt{7}\right)^{2/3}} + \frac{\left(7-i\sqrt{7}\right)\log\left(\sqrt[3]{1+i\sqrt{7}}+\sqrt[3]{2}\,x\right)}{21\sqrt[3]{2}\left(1+i\sqrt{7}\right)^{2/3}} - \cdots$$

$$= -\frac{i\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{7}\right)}}}{\sqrt{3}}\right)}{\sqrt{21}} + \frac{i\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{7}\right)}\tan^{-1}\left(\cdots\right)}{\sqrt{21}} \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 37, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[2+\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{1+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 + x^3 + x^6),x]
```

```
[Out] RootSum[2 + #1^3 + #1^6 & , (Log[x - #1]*#1)/(1 + 2*#1^3) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 36, normalized size = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+\_Z^{3}+2\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}+\_R^{2}}\right)}{3}$	36
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}+\_Z^{3}+2\right)}\frac{\_R^{3}\ln\left(x-\_R\right)}{2\_R^{5}+\_R^{2}}\right)}{3}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^6+x^3+2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sum(_R^3/(2*_R^5+_R^2)*ln(x-_R),_R=RootOf(_Z^6+_Z^3+2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(x^6 + x^3 + 2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. $2(263) = 526$.
time = 0.43, size = 1048, normalized size = 2.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="fricas")
```

```
[Out] 1/294*98^(2/3)*56^(1/6)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))*log
(-196*98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqr
t(7))) + 9604*x^2 + 1372*98^(1/3)*7^(1/3)) - 2/147*98^(2/3)*56^(1/6)*arctan
(-1/38416*(98*98^(1/3)*56^(5/6)*sqrt(7)*x - 98^(1/3)*56^(5/6)*sqrt(7)*sqrt(
-196*98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt
(7))) + 9604*x^2 + 1372*98^(1/3)*7^(1/3)) - 38416*sin(2/3*arctan(2/7*sqrt(1
4)*sqrt(7) + sqrt(7))))/cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*si
n(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) - 1/147*(98^(2/3)*56^(1/6)*sq
rt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) - 98^(2/3)*56^(1/6)*s
in(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*arctan(1/38416*(196*98^(1/3
)*56^(5/6)*sqrt(7)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) - 196*
```

```
(98^(1/3)*56^(5/6)*sqrt(7)*sqrt(3)*x - 784*cos(2/3*arctan(2/7*sqrt(14)*sqrt
(7) + sqrt(7))))*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + sqrt(-39
2*98^(2/3)*56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) +
 sqrt(7))) + 392*98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sq
rt(7) + sqrt(7))) + 38416*x^2 + 5488*98^(1/3)*7^(1/3))*(98^(1/3)*56^(5/6)*s
qrt(7)*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) - 98^(1/3)*5
6^(5/6)*sqrt(7)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))) - 38416*sq
rt(3))/(4*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 - 3)) - 1/147*(
98^(2/3)*56^(1/6)*sqrt(3)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) +
 98^(2/3)*56^(1/6)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*arctan(
-1/2744*(14*98^(1/3)*56^(5/6)*sqrt(7)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7)
 + sqrt(7))) - sqrt(2)*sqrt(98^(2/3)*56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arc
tan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + 98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*
arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) + 98*x^2 + 14*98^(1/3)*7^(1/3))*(98
^(1/3)*56^(5/6)*sqrt(7)*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(
7))) + 98^(1/3)*56^(5/6)*sqrt(7)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt
(7)))) + 14*(98^(1/3)*56^(5/6)*sqrt(7)*sqrt(3)*x + 784*cos(2/3*arctan(2/7*s
qrt(14)*sqrt(7) + sqrt(7))))*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))
) + 2744*sqrt(3))/(4*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)))^2 - 3)
) + 1/588*(98^(2/3)*56^(1/6)*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) +
sqrt(7))) - 98^(2/3)*56^(1/6)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7)
)))*log(392*98^(2/3)*56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*sqrt(14)
*sqrt(7) + sqrt(7))) + 392*98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*s
qrt(14)*sqrt(7) + sqrt(7))) + 38416*x^2 + 5488*98^(1/3)*7^(1/3)) - 1/588*(9
8^(2/3)*56^(1/6)*sqrt(3)*sin(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))) +
98^(2/3)*56^(1/6)*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) + sqrt(7))))*log(-392
*98^(2/3)*56^(1/6)*sqrt(7)*sqrt(3)*x*cos(2/3*arctan(2/7*sqrt(14)*sqrt(7) +
sqrt(7))) + 392*98^(2/3)*56^(1/6)*sqrt(7)*x*sin(2/3*arctan(2/7*sqrt(14)*sqr
t(7) + sqrt(7))) + 38416*x^2 + 5488*98^(1/3)*7^(1/3))
```

Sympy [A]
time = 0.05, size = 24, normalized size = 0.06

$$\operatorname{RootSum}\left(250047t^6 + 1323t^3 + 2, \left(t \mapsto t\log\left(7938t^4 + 21t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**6+x**3+2),x)
```

```
[Out] RootSum(250047*_t**6 + 1323*_t**3 + 2, Lambda(_t, _t*log(7938*_t**4 + 21*_t
 + x)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^6+x^3+2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(x^6 + x^3 + 2), x)
```

Mupad [B]
time = 2.61, size = 351, normalized size = 0.88

$$\frac{\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,{\left(-7-\sqrt{7}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{14}\right)\,{\left(-196-\sqrt{7}\,28\mathrm{i}\right)}^{1/3}}{42}+\frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,{\left(-7+\sqrt{7}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{14}\right)\,{\left(-7+\sqrt{7}\,1\mathrm{i}\right)}^{1/3}}{42}-\frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,{\left(-7-\sqrt{7}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{28}-\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,{\left(-7-\sqrt{7}\,1\mathrm{i}\right)}^{1/3}}{28}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,{\left(-7-\sqrt{7}\,1\mathrm{i}\right)}^{1/3}}{84}+\frac{2^{2/3}\,7^{1/3}\,\ln\left(x+\frac{2^{2/3}\,7^{5/6}\,{\left(-7-\sqrt{7}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{28}+\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,{\left(-7-\sqrt{7}\,1\mathrm{i}\right)}^{1/3}}{28}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,{\left(-7-\sqrt{7}\,1\mathrm{i}\right)}^{1/3}}{84}+\frac{2^{2/3}\,7^{1/3}\,\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,{\left(-7+\sqrt{7}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{28}-\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,{\left(-7+\sqrt{7}\,1\mathrm{i}\right)}^{1/3}}{28}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,{\left(-7+\sqrt{7}\,1\mathrm{i}\right)}^{1/3}}{84}-\frac{2^{2/3}\,7^{1/3}\,\ln\left(x-\frac{2^{2/3}\,7^{5/6}\,{\left(-7+\sqrt{7}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{28}+\frac{2^{2/3}\,\sqrt{3}\,7^{5/6}\,{\left(-7+\sqrt{7}\,1\mathrm{i}\right)}^{1/3}}{28}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\,{\left(-7+\sqrt{7}\,1\mathrm{i}\right)}^{1/3}}{84}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^3 + x^6 + 2),x)
```

```
[Out] (log(x - (2^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/14)*(- 7^(1/2)*28i -
 196)^(1/3))/42 + (2^(2/3)*7^(1/3)*log(x + (2^(2/3)*7^(5/6)*(7^(1/2)*1i - 7
)^(1/3)*1i)/14)*(7^(1/2)*1i - 7)^(1/3))/42 - (2^(2/3)*7^(1/3)*log(x + (2^(2
/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/28 - (2^(2/3)*3^(1/2)*7^(5/6)*(- 7
^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i + 1)*(- 7^(1/2)*1i - 7)^(1/3))/84 + (
2^(2/3)*7^(1/3)*log(x + (2^(2/3)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3)*1i)/28 +
(2^(2/3)*3^(1/2)*7^(5/6)*(- 7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i - 1)*(-
7^(1/2)*1i - 7)^(1/3))/84 + (2^(2/3)*7^(1/3)*log(x - (2^(2/3)*7^(5/6)*(7^(1
/2)*1i - 7)^(1/3)*1i)/28 - (2^(2/3)*3^(1/2)*7^(5/6)*(7^(1/2)*1i - 7)^(1/3))
/28)*(3^(1/2)*1i - 1)*(7^(1/2)*1i - 7)^(1/3))/84 - (2^(2/3)*7^(1/3)*log(x -
 (2^(2/3)*7^(5/6)*(7^(1/2)*1i - 7)^(1/3)*1i)/28 + (2^(2/3)*3^(1/2)*7^(5/6)*
(7^(1/2)*1i - 7)^(1/3))/28)*(3^(1/2)*1i + 1)*(7^(1/2)*1i - 7)^(1/3))/84
```

3.185 $\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=231

$$\frac{(21b^4-56ab^2c+16a^2c^2)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{1536c^5}-\frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2}+\frac{x^9(a+bx^3+cx^6)^{3/2}}{18c}-\frac{(7b(15b\ldots}{}$$

```
[Out] -1/20*b*x^6*(c*x^6+b*x^3+a)^(3/2)/c^2+1/18*x^9*(c*x^6+b*x^3+a)^(3/2)/c-1/28
80*(7*b*(-28*a*c+15*b^2)-6*c*(-20*a*c+21*b^2)*x^3)*(c*x^6+b*x^3+a)^(3/2)/c^
4-1/3072*(-4*a*c+b^2)*(16*a^2*c^2-56*a*b^2*c+21*b^4)*arctanh(1/2*(2*c*x^3+b
)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(11/2)+1/1536*(16*a^2*c^2-56*a*b^2*c+21*
b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^5
```

Rubi [A]

time = 0.20, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1371, 756, 846, 793, 626, 635, 212}

$$-\frac{(b^2-4ac)\,(16a^2c^2-56ab^2c+21b^4)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\,\sqrt{a+bx^3+cx^6}}\right)}{3072c^{11/2}}+\frac{(16a^2c^2-56ab^2c+21b^4)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{1536c^5}-\frac{(7b(15b^2-28ac)-6cx^3(21b^2-20ac))\,(a+bx^3+cx^6)^{3/2}}{2880c^4}-\frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2}+\frac{x^9(a+bx^3+cx^6)^{3/2}}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((21*b^4 - 56*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/
(1536*c^5) - (b*x^6*(a + b*x^3 + c*x^6)^(3/2))/(20*c^2) + (x^9*(a + b*x^3 +
 c*x^6)^(3/2))/(18*c) - ((7*b*(15*b^2 - 28*a*c) - 6*c*(21*b^2 - 20*a*c)*x^3
)*(a + b*x^3 + c*x^6)^(3/2))/(2880*c^4) - ((b^2 - 4*a*c)*(21*b^4 - 56*a*b^2
*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]
)/(3072*c^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx=\frac{1}{3}\text{Subst}\left(\int x^4\sqrt{a+bx+cx^2}\,dx,x,x^3\right)$$

$$=\frac{x^9(a+bx^3+cx^6)^{3/2}}{18c}+\frac{\text{Subst}\left(\int x^2\left(-3a-\frac{9bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{18c}$$

$$=-\frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2}+\frac{x^9(a+bx^3+cx^6)^{3/2}}{18c}+\frac{\text{Subst}\left(\int x\left(9ab+\frac{3}{4}(21b^2-20\right.\right.}{9}$$

$$=-\frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2}+\frac{x^9(a+bx^3+cx^6)^{3/2}}{18c}-\frac{(7b(15b^2-28ac)-6c(21b^2-}{288}$$

$$=\frac{(21b^4-56ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^5}-\frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2}+$$

$$=\frac{(21b^4-56ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^5}-\frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2}+$$

$$=\frac{(21b^4-56ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1536c^5}-\frac{bx^6(a+bx^3+cx^6)^{3/2}}{20c^2}+$$

Mathematica [A]
time = 0.42, size = 206, normalized size = 0.89

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(315b^5-210b^4cx^3+16b^2c^2x^3(56a-9cx^6)+168b^3c(-10a+cx^6)+16bc^2(113a^2-34acx^6+8c^2x^{12})+160c^3x^3(-3a^2+2acx^6+8c^2x^{12})\right)+15(21b^6-140ab^4c+240a^2b^2c^2-64a^3c^3)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{46080c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(315*b^5 - 210*b^4*c*x^3 + 16*b^2*c^2*x^
3*(56*a - 9*c*x^6) + 168*b^3*c*(-10*a + c*x^6) + 16*b*c^2*(113*a^2 - 34*a*c
*x^6 + 8*c^2*x^12) + 160*c^3*x^3*(-3*a^2 + 2*a*c*x^6 + 8*c^2*x^12)) + 15*(2
1*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*Log[b + 2*c*x^3 - 2*Sqr
t[c]*Sqrt[a + b*x^3 + c*x^6]])/(46080*c^(11/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{14}\sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)
```

[Out] int(x^14*(c*x^6+b*x^3+a)^(1/2),x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 451, normalized size = 1.95

$$\left[-\frac{15\,(21\,b^6-140\,ab^4c+240\,a^2b^2c^2-64\,a^3c^3)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,(1280\,c^6x^{15}+128\,bc^5x^{12}-16\,(9\,b^2c^4-20\,ac^5)x^9+8\,(21\,b^3c^3-68\,abc^4)x^6+315\,b^5c-1680\,ab^3c^2+1808\,a^2bc^3-2\,(105\,b^4c^2-448\,ab^2c^3+240\,a^2c^4)x^3)\sqrt{cx^6+bx^3+a}}{92160\,c^6},\ \frac{15\,(21\,b^6-140\,ab^4c+240\,a^2b^2c^2-64\,a^3c^3)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)+2\,(1280\,c^6x^{15}+128\,bc^5x^{12}-16\,(9\,b^2c^4-20\,ac^5)x^9+8\,(21\,b^3c^3-68\,abc^4)x^6+315\,b^5c-1680\,ab^3c^2+1808\,a^2bc^3-2\,(105\,b^4c^2-448\,ab^2c^3+240\,a^2c^4)x^3)\sqrt{cx^6+bx^3+a}}{46080\,c^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)
*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)
*sqrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*(9*b^2*c^4 - 20*
a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c - 1680*a*b^3*c^2 +
 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c
*x^6 + b*x^3 + a))/c^6, 1/46080*(15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2
 - 64*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sq
rt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(1280*c^6*x^15 + 128*b*c^5*x^12 - 16*
(9*b^2*c^4 - 20*a*c^5)*x^9 + 8*(21*b^3*c^3 - 68*a*b*c^4)*x^6 + 315*b^5*c -
1680*a*b^3*c^2 + 1808*a^2*b*c^3 - 2*(105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*
c^4)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14}\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**14*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^14, x)
```

Mupad [B]
time = 2.94, size = 543, normalized size = 2.35

$$\frac{x^9\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{18\,c}-\frac{b\,\left(\frac{x^6\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{5\,c}+\frac{7\,b\,\left(\frac{a\,\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{4\,c}+\frac{5\,b\,\left(\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{8\,c}\right)}{10\,c}-\frac{2\,a\,\left(\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{5\,c}\right)}{4\,c}+\frac{a\,\left(\frac{a\,\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{4\,c}+\frac{5\,b\,\left(\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{8\,c}\right)}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (x^9*(a + b*x^3 + c*x^6)^(3/2))/(18*c) - (b*((x^6*(a + b*x^3 + c*x^6)^(3/2)
)/(5*c) + (7*b*((a*((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a +
 b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))
/(4*c) - (x^3*(a + b*x^3 + c*x^6)^(3/2))/(4*c) + (5*b*(((8*c*(a + c*x^6) -
3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3
+ c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*
c)))/(10*c) - (2*a*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x
^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1
/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(5*c)))/(4*c) + (a*((a*((b/(4*c) + x^3
/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x
^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^3*(a + b*x^3 + c*x^6)^
(3/2))/(4*c) + (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*
x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(
1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(6*c)
```

3.186 $\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=171

$$-\frac{b(7b^2-12ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c}+\frac{\left(35b^2-32ac-42bcx^3\right)\left(a+bx^3+cx^6\right)}{720c^3}$$

```
[Out] 1/15*x^6*(c*x^6+b*x^3+a)^(3/2)/c+1/720*(-42*b*c*x^3-32*a*c+35*b^2)*(c*x^6+b
*x^3+a)^(3/2)/c^3+1/768*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3
+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(9/2)-1/384*b*(-12*a*c+7*b^2)*(2*c*x^3
+b)*(c*x^6+b*x^3+a)^(1/2)/c^4
```

Rubi [A]

time = 0.11, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 756, 793, 626, 635, 212}

$$\frac{b(7b^2-12ac)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{768c^{9/2}}-\frac{b(7b^2-12ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{\left(-32ac+35b^2-42bcx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{720c^3}+\frac{x^6\left(a+bx^3+cx^6\right)^{3/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -1/384*(b*(7*b^2 - 12*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/c^4 + (x^
6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + ((35*b^2 - 32*a*c - 42*b*c*x^3)*(a +
b*x^3 + c*x^6)^(3/2))/(720*c^3) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(768*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx=\frac{1}{3}\text{Subst}\left(\int x^3\sqrt{a+bx+cx^2}\,dx,x,x^3\right)$$

$$=\frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}+\frac{\text{Subst}\left(\int x\left(-2a-\frac{7bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{15c}$$

$$=\frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}+\frac{(35b^2-32ac-42bcx^3)\,(a+bx^3+cx^6)^{3/2}}{720c^3}-\frac{(b(7b^2-}{}$$

$$=-\frac{b(7b^2-12ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}+\frac{(35b^2-}{}$$

$$=-\frac{b(7b^2-12ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}+\frac{(35b^2-}{}$$

$$=-\frac{b(7b^2-12ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{384c^4}+\frac{x^6(a+bx^3+cx^6)^{3/2}}{15c}+\frac{(35b^2-}{}$$

Mathematica [A]
time = 0.29, size = 166, normalized size = 0.97

$$\frac{\sqrt{a+bx^3+cx^6}\,(-105b^4+70b^3cx^3+4b^2c(115a-14cx^6)+8bc^2x^3(-29a+6cx^6)+128c^2(-2a^2+acx^6+3c^2x^{12}))}{5760c^4}-\frac{(7b^5-40ab^3c+48a^2bc^2)\log\left(c^4\left(b+2cx^3-2\sqrt{c}\,\sqrt{a+bx^3+cx^6}\right)\right)}{768c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-105*b^4 + 70*b^3*c*x^3 + 4*b^2*c*(115*a - 14*c*x
^6) + 8*b*c^2*x^3*(-29*a + 6*c*x^6) + 128*c^2*(-2*a^2 + a*c*x^6 + 3*c^2*x^1
2)))/(5760*c^4) - ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*Log[c^4*(b + 2*c*x^3
 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(768*c^(9/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}\sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^11*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 367, normalized size = 2.15

$$\left[\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{c}-4\,ac\right)+4\,(384\,c^5x^{12}+48\,bc^4x^9-8\,(7\,b^2c^3-16\,ac^4)x^6-105\,b^4c+460\,ab^2c^2-256\,a^2c^3+2\,(35\,b^3c^2-116\,abc^3)x^3)\sqrt{cx^6+bx^3+a}}{23040\,c^5},-\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,(384\,c^5x^{12}+48\,bc^4x^9-8\,(7\,b^2c^3-16\,ac^4)x^6-105\,b^4c+460\,ab^2c^2-256\,a^2c^3+2\,(35\,b^3c^2-116\,abc^3)x^3)\sqrt{cx^6+bx^3+a}}{11520\,c^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8
*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) +
 4*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a*c^4)*x^6 - 105*b^4*c
+ 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x^3)*sqrt(c*x^
6 + b*x^3 + a))/c^5, -1/11520*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(
-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*
c*x^3 + a*c)) - 2*(384*c^5*x^12 + 48*b*c^4*x^9 - 8*(7*b^2*c^3 - 16*a*c^4)*x
^6 - 105*b^4*c + 460*a*b^2*c^2 - 256*a^2*c^3 + 2*(35*b^3*c^2 - 116*a*b*c^3)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**11*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^11, x)
```

Mupad [B]
time = 1.87, size = 315, normalized size = 1.84

$$\frac{x^6\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{15\,c}+\frac{7\,b\,\left(\frac{a\,\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c}-\frac{x^3\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{4\,c}+\frac{5\,b\,\left(\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{8\,c}\right)}{30\,c}-\frac{2\,a\,\left(\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{15\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^(3/2))/(15*c) + (7*b*((a*((b/(4*c) + x^3/2)*(a + b
*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2
))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (x^3*(a + b*x^3 + c*x^6)^(3/2))/(4*
c) + (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2)
)/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3
 - 4*a*b*c))/(16*c^(5/2))))/(8*c)))/(30*c) - (2*a*(((8*c*(a + c*x^6) - 3*b^
2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*
x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(15*c)
```

3.187 $\int x^8 \sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=153

$$\frac{(5b^2-4ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c}-\frac{(b^2-4ac)\,(5b^2-4ac)}{}$$

```
[Out] -5/72*b*(c*x^6+b*x^3+a)^(3/2)/c^2+1/12*x^3*(c*x^6+b*x^3+a)^(3/2)/c-1/384*(-
4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(
1/2))/c^(7/2)+1/192*(-4*a*c+5*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^3
```

Rubi [A]
time = 0.09, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 756, 654, 626, 635, 212}

$$-\frac{(b^2-4ac)\,(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\,\sqrt{a+bx^3+cx^6}}\right)}{384c^{7/2}}+\frac{(5b^2-4ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b\left(a+bx^3+cx^6\right)^{3/2}}{72c^2}+\frac{x^3\left(a+bx^3+cx^6\right)^{3/2}}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(192*c^3) - (5*b*(a
 + b*x^3 + c*x^6)^(3/2))/(72*c^2) + (x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c)
- ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6])])/(384*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^8\sqrt{a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int x^2\sqrt{a+bx+cx^2}\,dx,x,x^3\right)\\
&= \frac{x^3(a+bx^3+cx^6)^{3/2}}{12c}+\frac{\text{Subst}\left(\int\left(-a-\frac{5bx}{2}\right)\sqrt{a+bx+cx^2}\,dx,x,x^3\right)}{12c}\\
&= -\frac{5b(a+bx^3+cx^6)^{3/2}}{72c^2}+\frac{x^3(a+bx^3+cx^6)^{3/2}}{12c}+\frac{(5b^2-4ac)\,\text{Subst}\left(\int\sqrt{a+bx\right.}{48c^2}\\
&= \frac{(5b^2-4ac)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b(a+bx^3+cx^6)^{3/2}}{72c^2}+\frac{x^3(a+bx^3+}{12c}\\
&= \frac{(5b^2-4ac)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b(a+bx^3+cx^6)^{3/2}}{72c^2}+\frac{x^3(a+bx^3+}{12c}\\
&= \frac{(5b^2-4ac)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{192c^3}-\frac{5b(a+bx^3+cx^6)^{3/2}}{72c^2}+\frac{x^3(a+bx^3+}{12c}
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 132, normalized size = 0.86

$$\frac{\sqrt{a+bx^3+cx^6}\,(15b^3-52abc-10b^2cx^3+24ac^2x^3+8bc^2x^6+48c^3x^9)}{576c^3}+\frac{(5b^4-24ab^2c+16a^2c^2)\log\left(b+2cx^3-2\sqrt{c}\,\sqrt{a+bx^3+cx^6}\right)}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(15*b^3 - 52*a*b*c - 10*b^2*c*x^3 + 24*a*c^2*x^3 +
 8*b*c^2*x^6 + 48*c^3*x^9))/(576*c^3) + ((5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*
Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(384*c^(7/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^8\sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 303, normalized size = 1.98

$$\left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{c}-4\,ac\right)+4\,(48\,c^4x^9+8\,bc^3x^6+15\,b^3c-52\,abc^2-2\,(5\,b^2c^2-12\,ac^3)x^3)\sqrt{cx^6+bx^3+a}}{2304\,c^4},\ \frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)+2\,(48\,c^4x^9+8\,bc^3x^6+15\,b^3c-52\,abc^2-2\,(5\,b^2c^2-12\,ac^3)x^3)\sqrt{cx^6+bx^3+a}}{1152\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c
*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(
```

```
48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/1152*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*
c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^
2*x^6 + b*c*x^3 + a*c)) + 2*(48*c^4*x^9 + 8*b*c^3*x^6 + 15*b^3*c - 52*a*b*c
^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**8*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^8, x)
```

Mupad [**B**]
time = 1.59, size = 193, normalized size = 1.26

$$\frac{x^3\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{12\,c}-\frac{a\,\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{12\,c}-\frac{5\,b\,\left(\frac{\left(8\,c\,\left(c\,x^6+a\right)-3\,b^2+2\,b\,c\,x^3\right)\,\sqrt{c\,x^6+b\,x^3+a}}{24\,c^2}+\frac{\ln\left(2\,\sqrt{c\,x^6+b\,x^3+a}+\frac{2\,c\,x^3+b}{\sqrt{c}}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}\right)}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (x^3*(a + b*x^3 + c*x^6)^(3/2))/(12*c) - (a*((b/(4*c) + x^3/2)*(a + b*x^3 +
 c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*
c - b^2/4))/(2*c^(3/2))))/(12*c) - (5*b*(((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*
x^3)*(a + b*x^3 + c*x^6)^(1/2))/(24*c^2) + (log(2*(a + b*x^3 + c*x^6)^(1/2)
 + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))))/(24*c)
```

3.188 $\int x^5 \sqrt{a+bx^3+cx^6}\, dx$

Optimal. Leaf size=108

$$-\frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{(a+bx^3+cx^6)^{3/2}}{9c}+\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}$$

```
[Out] 1/9*(c*x^6+b*x^3+a)^(3/2)/c+1/48*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(
1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)-1/24*b*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2
)/c^2
```

Rubi [A]
time = 0.05, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 654, 626, 635, 212}

$$\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{5/2}}-\frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2}+\frac{(a+bx^3+cx^6)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -1/24*(b*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/c^2 + (a + b*x^3 + c*x^6)^(
3/2)/(9*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x
^3 + c*x^6])])/(48*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\sqrt{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{Subst}\left(\int x\sqrt{a+bx+cx^2}\,dx, x, x^3\right)$$

$$= \frac{(a+bx^3+cx^6)^{3/2}}{9c} - \frac{b\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^3\right)}{6c}$$

$$= -\frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2} + \frac{(a+bx^3+cx^6)^{3/2}}{9c} + \frac{(b(b^2-4ac))\,\text{Subst}\left(\int \ldots\right.}{48\ldots}$$

$$= -\frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2} + \frac{(a+bx^3+cx^6)^{3/2}}{9c} + \frac{(b(b^2-4ac))\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= -\frac{b(b+2cx^3)\sqrt{a+bx^3+cx^6}}{24c^2} + \frac{(a+bx^3+cx^6)^{3/2}}{9c} + \frac{b(b^2-4ac)\tanh^{-1}\left(\ldots\right.}{48c^{5/\ldots}}$$

Mathematica [A]
time = 0.20, size = 101, normalized size = 0.94

$$\frac{\sqrt{a+bx^3+cx^6}\,(-3b^2+2bcx^3+8c(a+cx^6))}{72c^2} - \frac{(b^3-4abc)\log\left(c^2\left(b+2cx^3-2\sqrt{c}\,\sqrt{a+bx^3+cx^6}\right)\right)}{48c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^3 + c*x^6],x]
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 2*b*c*x^3 + 8*c*(a + c*x^6)))/(72*c^2) -
 ((b^3 - 4*a*b*c)*Log[c^2*(b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])
])/(48*c^(5/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \sqrt{c\,x^6 + b\,x^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 237, normalized size = 2.19

$$\left[-\frac{3\,(b^3 - 4\,abc)\sqrt{c}\,\log\left(-8\,c^2x^6 - 8\,bcx^3 - b^2 + 4\,\sqrt{cx^6 + bx^3 + a}\,(2\,cx^3 + b)\sqrt{c} - 4\,ac\right) - 4\,(8\,c^3x^6 + 2\,bc^2x^3 - 3\,b^2c + 8\,ac^2)\sqrt{cx^6 + bx^3 + a}}{288\,c^3}, -\frac{3\,(b^3 - 4\,abc)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^6 + bx^3 + a}\,(2\,cx^3 + b)\sqrt{-c}}{2\,(c^2x^6 + bcx^3 + ac)}\right) - 2\,(8\,c^3x^6 + 2\,bc^2x^3 - 3\,b^2c + 8\,ac^2)\sqrt{cx^6 + bx^3 + a}}{144\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(b^3 - 4*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqr
t(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 + 2*b*c^
2*x^3 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^3, -1/144*(3*(b^3 - 4
*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/
(c^2*x^6 + b*c*x^3 + a*c)) - 2*(8*c^3*x^6 + 2*b*c^2*x^3 - 3*b^2*c + 8*a*c^2
)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \sqrt{a + bx^3 + cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [A]
time = 3.93, size = 98, normalized size = 0.91

$$\frac{1}{72}\sqrt{cx^6+bx^3+a}\left(2\left(4x^3+\frac{b}{c}\right)x^3-\frac{3b^2-8ac}{c^2}\right)-\frac{(b^3-4abc)\log\left(\left|-2\left(\sqrt{c}x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{48c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*sqrt(c*x^6 + b*x^3 + a)*(2*(4*x^3 + b/c)*x^3 - (3*b^2 - 8*a*c)/c^2) -
1/48*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqr
t(c) - b))/c^(5/2)
```

Mupad [B]
time = 1.39, size = 87, normalized size = 0.81

$$\frac{(8c(cx^6+a)-3b^2+2bcx^3)\sqrt{cx^6+bx^3+a}}{72c^2}+\frac{\ln\left(2\sqrt{cx^6+bx^3+a}+\frac{2cx^3+b}{\sqrt{c}}\right)(b^3-4abc)}{48c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] ((8*c*(a + c*x^6) - 3*b^2 + 2*b*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(72*c^2)
+ (log(2*(a + b*x^3 + c*x^6)^(1/2) + (b + 2*c*x^3)/c^(1/2))*(b^3 - 4*a*b*c)
)/(48*c^(5/2))
```

3.189 $\int x^2\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=83

$$\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

```
[Out] -1/24*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c
^(3/2)+1/12*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1366, 626, 635, 212}

$$\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) - ((b^2 - 4*a*c)*ArcTanh[(b
+ 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(24*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int x^2\sqrt{a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \sqrt{a+bx+cx^2}\,dx, x, x^3\right)\\
&= \frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24c}\\
&= \frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12c}\\
&= \frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c} - \frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{24c^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 85, normalized size = 1.02

$$\frac{(b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c}+\frac{(b^2-4ac)\log\left(bc+2c^2x^3-2c^{3/2}\sqrt{a+bx^3+cx^6}\right)}{24c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^3 + c*x^6],x]
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c) + ((b^2 - 4*a*c)*Log[b*c + 2
*c^2*x^3 - 2*c^(3/2)*Sqrt[a + b*x^3 + c*x^6]])/(24*c^(3/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(x^2*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 197, normalized size = 2.37

$$\left[-\frac{\left(b^{2}-4\,ac\right)\sqrt{c}\,\log\left(-8\,c^{2}x^{6}-8\,bcx^{3}-b^{2}-4\,\sqrt{cx^{6}+bx^{3}+a}\,\left(2\,cx^{3}+b\right)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^{6}+bx^{3}+a}\,\left(2\,c^{2}x^{3}+bc\right)}{48\,c^{2}}, \frac{\left(b^{2}-4\,ac\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,\left(2\,cx^{3}+b\right)\sqrt{-c}}{2\,\left(c^{2}x^{6}+bcx^{3}+ac\right)}\right)+2\,\sqrt{cx^{6}+bx^{3}+a}\,\left(2\,c^{2}x^{3}+bc\right)}{24\,c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x
^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*
(2*c^2*x^3 + b*c))/c^2, 1/24*((b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x
^6 + b*x^3 + a)*(2*c^2*x^3 + b*c))/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\sqrt{a+bx^{3}+cx^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [A]
time = 4.53, size = 76, normalized size = 0.92

$$\frac{1}{12}\,\sqrt{cx^{6}+bx^{3}+a}\,\left(2\,x^{3}+\frac{b}{c}\right)+\frac{\left(b^{2}-4\,ac\right)\log\left(\left|-2\,\left(\sqrt{c}\,x^{3}-\sqrt{cx^{6}+bx^{3}+a}\right)\sqrt{c}-b\right|\right)}{24\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*log(abs(-2*
(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(3/2)
```

Mupad [B]
time = 1.39, size = 72, normalized size = 0.87

$$\frac{\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}}{3}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{6\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] ((b/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2))/3 + (log((a + b*x^3 + c*x^6)^
(1/2) + (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(6*c^(3/2))
```

3.190 $\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx$

Optimal. Leaf size=109

$$\frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\,\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\,\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

```
[Out] -1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))*a^(1/2)+1/6*b*a
rctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(1/2)+1/3*(c*x^6+b*
x^3+a)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 748, 857, 635, 212, 738}

$$\frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\,\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\,\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/3 - (Sqrt[a]*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])/3 + (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^
3 + c*x^6])])/(6*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^3+cx^6}}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx,x,x^3\right)\\
&= \frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{6}\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\sqrt{a+bx^3+cx^6}+\frac{1}{3}a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)+\frac{1}{6}b\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.\\
&= \frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}(2a)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)+\frac{1}{3}b\text{Subs}\\
&= \frac{1}{3}\sqrt{a+bx^3+cx^6}-\frac{1}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\,\sqrt{a+bx^3+cx^6}}\right)+\frac{b\tanh^{-1}\left(\frac{}{2\sqrt{c}\,\sqrt{}}\right.}{6\sqrt{}}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 105, normalized size = 0.96

$$\frac{1}{6}\left(2\sqrt{a+bx^3+cx^6}+4\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)-\frac{b\log\left(b+2cx^3-2\sqrt{c}\,\sqrt{a+bx^3+cx^6}\right)}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^3 + c*x^6] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^
3 + c*x^6])/Sqrt[a]] - (b*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^
6]])/Sqrt[c])/6
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.41, size = 566, normalized size = 5.19

$$\left[\frac{b\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{c}-4\,ac\right)+2\,\sqrt{a}\,c\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,\sqrt{cx^6+bx^3+a}\,c}{12\,c}, -\frac{b\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-\sqrt{a}\,c\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-2\,\sqrt{cx^6+bx^3+a}\,c}{6\,c}, \frac{4\,\sqrt{-a}\,c\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+b\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{c}-4\,ac\right)+4\,\sqrt{cx^6+bx^3+a}\,c}{12\,c}, \frac{2\,\sqrt{-a}\,c\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)-b\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)+2\,\sqrt{cx^6+bx^3+a}\,c}{6\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 +
a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 2*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^6 + 8
*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) +
4*sqrt(c*x^6 + b*x^3 + a)*c)/c, -1/6*(b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*
x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - sqrt(a)*c*log(
-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(a) + 8*a^2)/x^6) - 2*sqrt(c*x^6 + b*x^3 + a)*c)/c, 1/12*(4*sqrt(-a)*c*a
rctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3
 + a^2)) + b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^
3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c, 1/6
*(2*sqrt(-a)*c*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a
*c*x^6 + a*b*x^3 + a^2)) - b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2
*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)
*c)/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x,x)
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x, x)
```

Mupad [B]
time = 1.36, size = 88, normalized size = 0.81

$$\frac{\sqrt{c\,x^6+b\,x^3+a}}{3}-\frac{\sqrt{a}\,\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{3}+\frac{b\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{6\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x,x)
[Out] (a + b*x^3 + c*x^6)^(1/2)/3 - (a^(1/2)*log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^
3 + c*x^6)^(1/2))/x^3))/3 + (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3
)/c^(1/2)))/(6*c^(1/2))
```

3.191 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^4} \, dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

```
[Out] -1/6*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)+1/3*a
rctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))*c^(1/2)-1/3*(c*x^6+b*
x^3+a)^(1/2)/x^3
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 746, 857, 635, 212, 738}

$$-\frac{\sqrt{a+bx^3+cx^6}}{3x^3}-\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}}+\frac{1}{3}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/x^3 - (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt
[a + b*x^3 + c*x^6])])/(6*Sqrt[a]) + (Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt
[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^4}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} + \frac{1}{6}\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} + \frac{1}{6}b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right) + \frac{1}{3}c\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{1}{3}b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right) + \frac{1}{3}(2c)\text{Subs}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{3x^3} - \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6\sqrt{a}} + \frac{1}{3}\sqrt{c}\,\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.$$

Mathematica [A]
time = 0.17, size = 107, normalized size = 0.96

$$\frac{1}{3}\left(-\frac{\sqrt{a+bx^3+cx^6}}{x^3}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{\sqrt{a}}-\sqrt{c}\,\log\left(b+2cx^3-2\sqrt{c}\,\sqrt{a+bx^3+cx^6}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^4,x]
```

```
[Out] (-(Sqrt[a + b*x^3 + c*x^6]/x^3) + (b*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3
+ c*x^6])/Sqrt[a]])/Sqrt[a] - Sqrt[c]*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a +
b*x^3 + c*x^6]])/3
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^4,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 601, normalized size = 5.37

$$\left[\frac{2\,a\sqrt{c}\,x^3\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{c}-4\,ac\right)+\sqrt{a}\,bx^3\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,\sqrt{cx^6+bx^3+a}\,a}{12\,ax^3},-\frac{4\,a\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-\sqrt{a}\,bx^3\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,\sqrt{cx^6+bx^3+a}\,a}{12\,ax^3},\frac{\sqrt{-a}\,bx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+a\sqrt{c}\,x^3\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{c}-4\,ac\right)-2\,\sqrt{cx^6+bx^3+a}\,a}{6\,ax^3},\frac{\sqrt{-a}\,bx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)-2\,a\sqrt{-c}\,x^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)-2\,\sqrt{cx^6+bx^3+a}\,a}{6\,ax^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/12*(2*a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*
x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)
*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)
/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), -1/12*(4*a*sqrt(-c)*x^3*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a
*c)) - sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b
*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*a
)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + a*sqrt(c)*x^3*log(-8*c^2*x^6 - 8
*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) -
 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3), 1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(
c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*a*
sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2
*x^6 + b*c*x^3 + a*c)) - 2*sqrt(c*x^6 + b*x^3 + a)*a)/(a*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**4,x)
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^4, x)
```

Mupad [B]
time = 1.55, size = 91, normalized size = 0.81

$$\frac{\sqrt{c}\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3}-\frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,x^3}-\frac{b\,\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{6\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^4,x)
[Out] (c^(1/2)*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2)))/3 - (a + b
*x^3 + c*x^6)^(1/2)/(3*x^3) - (b*log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 + c*
x^6)^(1/2))/x^3))/(6*a^(1/2))
```

3.192 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx$

Optimal. Leaf size=88

$$-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}+\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}$$

```
[Out] 1/24*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^
(3/2)-1/12*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a/x^6
```

Rubi [A]
time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1371, 734, 738, 212}

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] -1/12*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) + ((b^2 - 4*a*c)*ArcT
anh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(24*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)\\ &= -\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{24a}\\ &= -\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6} + \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12a}\\ &= -\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6} + \frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 91, normalized size = 1.03

$$\frac{(-2a-bx^3)\sqrt{a+bx^3+cx^6}}{12ax^6} + \frac{(-b^2+4ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{12a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^7,x]
```

```
[Out] ((-2*a - b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a*x^6) + ((-b^2 + 4*a*c)*ArcTa
nh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(12*a^(3/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^7,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 215, normalized size = 2.44

$$\left[-\frac{(b^2-4\,ac)\sqrt{a}\,x^6\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,\sqrt{cx^6+bx^3+a}\,(abx^3+2\,a^2)}{48\,a^2x^6},-\frac{(b^2-4\,ac)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,\sqrt{cx^6+bx^3+a}\,(abx^3+2\,a^2)}{24\,a^2x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/48*((b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*s
qrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 +
 b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6), -1/24*((b^2 - 4*a*c)*sqrt(-a)*x^6
*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x
^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*x^3 + 2*a^2))/(a^2*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a+bx^3+cx^6}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**7, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^7,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^7, x)
```

3.193 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx$

Optimal. Leaf size=116

$$\frac{b(2a+bx^3)\sqrt{a+bx^3+cx^6}}{24a^2x^6}-\frac{(a+bx^3+cx^6)^{3/2}}{9ax^9}-\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}$$

```
[Out] -1/9*(c*x^6+b*x^3+a)^(3/2)/a/x^9-1/48*b*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a
)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)+1/24*b*(b*x^3+2*a)*(c*x^6+b*x^3+a)
^(1/2)/a^2/x^6
```

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 744, 734, 738, 212}

$$-\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{5/2}}+\frac{b(2a+bx^3)\sqrt{a+bx^3+cx^6}}{24a^2x^6}-\frac{(a+bx^3+cx^6)^{3/2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] (b*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*a^2*x^6) - (a + b*x^3 + c*x^6
)^(3/2)/(9*a*x^9) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[
a + b*x^3 + c*x^6])])/(48*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{10}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} - \frac{b\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)}{6a}$$

$$= \frac{b(2a+bx^3)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} + \frac{(b(b^2-4ac))\,\text{Subst}\left(\int \frac{}{x\sqrt{a}}\right.}{48a^2}$$

$$= \frac{b(2a+bx^3)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} - \frac{(b(b^2-4ac))\,\text{Subst}\left(\int \frac{1}{4a-x}\right.}{24a}$$

$$= \frac{b(2a+bx^3)\sqrt{a+bx^3+cx^6}}{24a^2x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{9ax^9} - \frac{b(b^2-4ac)\tanh^{-1}\left(\frac{}{2\sqrt{a}\,\sqrt{}}\right.}{48a^{5/2}}$$

Mathematica [A]

time = 0.41, size = 108, normalized size = 0.93

$$\frac{\sqrt{a+bx^3+cx^6}\,(-8a^2-2abx^3+3b^2x^6-8acx^6)}{72a^2x^9}+\frac{(b^3-4abc)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^10,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 2*a*b*x^3 + 3*b^2*x^6 - 8*a*c*x^6))/(72*
a^2*x^9) + ((b^3 - 4*a*b*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])
/Sqrt[a]])/(24*a^(5/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^10,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 259, normalized size = 2.23

$$\left[-\frac{3\,(b^3-4\,abc)\sqrt{a}\,x^9\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,((3\,ab^2-8\,a^2c)x^6-2\,a^2bx^3-8\,a^3)\sqrt{cx^6+bx^3+a}}{288\,a^3x^9},\ \frac{3\,(b^3-4\,abc)\sqrt{-a}\,x^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((3\,ab^2-8\,a^2c)x^6-2\,a^2bx^3-8\,a^3)\sqrt{cx^6+bx^3+a}}{144\,a^3x^9}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3
+ 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b
```

```
^2 - 8*a^2*c)*x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^9)
, 1/144*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*
(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 - 8*a^2*c)*
x^6 - 2*a^2*b*x^3 - 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^9)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**10,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**10, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^10, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^10,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^10, x)
```

3.194 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$

Optimal. Leaf size=161

$$-\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6}-\frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}}+\frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9}+\frac{(b^2-4ac)(5b^2-4ac)}{}$$

```
[Out] -1/12*(c*x^6+b*x^3+a)^(3/2)/a/x^12+5/72*b*(c*x^6+b*x^3+a)^(3/2)/a^2/x^9+1/3
84*(-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3
+a)^(1/2))/a^(7/2)-1/192*(-4*a*c+5*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a
^3/x^6
```

Rubi [A]
time = 0.10, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 758, 820, 734, 738, 212}

$$\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{7/2}}-\frac{(5b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6}+\frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9}-\frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

```
[Out] -1/192*((5*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a^3*x^6) -
(a + b*x^3 + c*x^6)^(3/2)/(12*a*x^12) + (5*b*(a + b*x^3 + c*x^6)^(3/2))/(72
*a^2*x^9) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]
*Sqrt[a + b*x^3 + c*x^6])])/(384*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{5b}{2}+cx\right)\sqrt{a+bx+cx^2}}{x^4}\,dx, x, x^3\right)}{12a}$$

$$= -\frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6)^{3/2}}{72a^2x^9} + \frac{(5b^2-4ac)\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx}}{x^3}\right.}{48a^2}$$

$$= -\frac{(5b^2-4ac)\,(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6}{72a^2x^9}$$

$$= -\frac{(5b^2-4ac)\,(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6}{72a^2x^9}$$

$$= -\frac{(5b^2-4ac)\,(2a+bx^3)\sqrt{a+bx^3+cx^6}}{192a^3x^6} - \frac{(a+bx^3+cx^6)^{3/2}}{12ax^{12}} + \frac{5b(a+bx^3+cx^6}{72a^2x^9}$$

Mathematica [A]
time = 0.50, size = 141, normalized size = 0.88

$$\frac{\sqrt{a+bx^3+cx^6}\,(-48a^3-8a^2bx^3+10ab^2x^6-24a^2cx^6-15b^3x^9+52abcx^9)}{576a^3x^{12}} + \frac{(-5b^4+24ab^2c-16a^2c^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{192a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^13,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 - 8*a^2*b*x^3 + 10*a*b^2*x^6 - 24*a^2*c*x
^6 - 15*b^3*x^9 + 52*a*b*c*x^9))/(576*a^3*x^12) + ((-5*b^4 + 24*a*b^2*c - 1
6*a^2*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(192*a
^(7/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)
```

[Out] int((c*x^6+b*x^3+a)^(1/2)/x^13,x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.42, size = 325, normalized size = 2.02

$$\left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{a}\,x^{12}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,((15\,ab^3-52\,a^2bc)x^9+8\,a^3bx^3-2\,(5\,a^2b^2-12\,a^3c)x^6+48\,a^4)\sqrt{cx^6+bx^3+a}}{2304\,a^4x^{12}},\ -\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{-a}\,x^{12}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((15\,ab^3-52\,a^2bc)x^9+8\,a^3bx^3-2\,(5\,a^2b^2-12\,a^3c)x^6+48\,a^4)\sqrt{cx^6+bx^3+a}}{1152\,a^4x^{12}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*
c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^
2)/x^6) - 4*((15*a*b^3 - 52*a^2*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*
a^3*c)*x^6 + 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^12), -1/1152*(3*(5*b^4
 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a
)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3 - 52*a^2
*b*c)*x^9 + 8*a^3*b*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^6 + 48*a^4)*sqrt(c*x^6
 + b*x^3 + a))/(a^4*x^12)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a+bx^3+cx^6}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**13, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^13, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^13,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^13, x)
```

3.195 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$

Optimal. Leaf size=199

$$\frac{b(7b^2-12ac)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}}+\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}}-\frac{(35b^2-32ac)\left(a+b\right.}{720a^3x^9}$$

```
[Out] -1/15*(c*x^6+b*x^3+a)^(3/2)/a/x^15+7/120*b*(c*x^6+b*x^3+a)^(3/2)/a^2/x^12-1
/720*(-32*a*c+35*b^2)*(c*x^6+b*x^3+a)^(3/2)/a^3/x^9-1/768*b*(-12*a*c+7*b^2)
*(-4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(9/2
)+1/384*b*(-12*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^4/x^6
```

Rubi [A]
time = 0.15, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1371, 758, 848, 820, 734, 738, 212}

$$-\frac{b(7b^2-12ac)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{768a^{9/2}}+\frac{b(7b^2-12ac)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6}-\frac{(35b^2-32ac)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9}+\frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}}-\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(384*a^4*x^6) -
(a + b*x^3 + c*x^6)^(3/2)/(15*a*x^15) + (7*b*(a + b*x^3 + c*x^6)^(3/2))/(12
0*a^2*x^12) - ((35*b^2 - 32*a*c)*(a + b*x^3 + c*x^6)^(3/2))/(720*a^3*x^9) -
 (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a
+ b*x^3 + c*x^6])])/(768*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^6}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+2cx\right)\sqrt{a+bx+cx^2}}{x^5}\,dx, x, x^3\right)}{15a}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{4}(35b^2-32ac)+\frac{7bcx}{2}\right)\sqrt{a+\cdots}}{x^4}\cdots\right)}{60a^2}$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b\left(a+bx^3+cx^6\right)^{3/2}}{120a^2x^{12}} - \frac{(35b^2-32ac)\left(a+bx^3+cx^6\right)^{3/2}}{720a^3x^9} - \cdots$$

$$= \frac{b(7b^2-12ac)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b(a+bx^3+c\cdots}{120a^2x^{12}}$$

$$= \frac{b(7b^2-12ac)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b(a+bx^3+c\cdots}{120a^2x^{12}}$$

$$= \frac{b(7b^2-12ac)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{384a^4x^6} - \frac{\left(a+bx^3+cx^6\right)^{3/2}}{15ax^{15}} + \frac{7b(a+bx^3+c\cdots}{120a^2x^{12}}$$

Mathematica [A]
time = 0.67, size = 176, normalized size = 0.88

$$\frac{\sqrt{a+bx^3+cx^6}\left(-384a^4-48a^3bx^3+56a^2b^2x^6-128a^3cx^6-70ab^3x^9+232a^2bcx^9+105b^4x^{12}-460ab^2cx^{12}+256a^2c^2x^{12}\right)}{5760a^4x^{15}} + \frac{\left(7b^5-40ab^3c+48a^2bc^2\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{384a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^16,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-384*a^4 - 48*a^3*b*x^3 + 56*a^2*b^2*x^6 - 128*a^
3*c*x^6 - 70*a*b^3*x^9 + 232*a^2*b*c*x^9 + 105*b^4*x^12 - 460*a*b^2*c*x^12
+ 256*a^2*c^2*x^12))/(5760*a^4*x^15) + ((7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)
*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(384*a^(9/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^16,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.47, size = 389, normalized size = 1.95

$$\left[\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{a}\,x^{15}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,((105\,ab^4-460\,a^2b^2c+256\,a^3c^2)x^{12}-2\,(35\,a^2b^3-116\,a^3bc)x^9-48\,a^4bx^3+8\,(7\,a^3b^2-16\,a^4c)x^6-384\,a^5)\sqrt{cx^6+bx^3+a}}{23040\,a^5x^{15}},\ \frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\sqrt{-a}\,x^{15}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((105\,ab^4-460\,a^2b^2c+256\,a^3c^2)x^{12}-2\,(35\,a^2b^3-116\,a^3bc)x^9-48\,a^4bx^3+8\,(7\,a^3b^2-16\,a^4c)x^6-384\,a^5)\sqrt{cx^6+bx^3+a}}{11520\,a^5x^{15}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="fricas")
```

```
[Out] [1/23040*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 +
4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) +
8*a^2)/x^6) + 4*((105*a*b^4 - 460*a^2*b^2*c + 256*a^3*c^2)*x^12 - 2*(35*a^2
*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a^3*b^2 - 16*a^4*c)*x^6 - 384
*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15), 1/11520*(15*(7*b^5 - 40*a*b^3*c
+ 48*a^2*b*c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2
*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^4 - 460*a^2*b^2*c + 2
56*a^3*c^2)*x^12 - 2*(35*a^2*b^3 - 116*a^3*b*c)*x^9 - 48*a^4*b*x^3 + 8*(7*a
^3*b^2 - 16*a^4*c)*x^6 - 384*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^15)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**16,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**16, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^16,x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^16, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^16,x)
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^16, x)
```

3.196 $\int x^3\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=140

$$\frac{x^4\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{4}{3};-\frac{1}{2},-\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,-1/2,-1/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3
/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$\frac{x^4\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{4}{3};-\frac{1}{2},-\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -1/2, -1/2, 7/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx = \frac{\sqrt{a+bx^3+cx^6}\int x^3\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{x^4\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{4}{3};-\frac{1}{2},-\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 358 vs. $2(140) = 280$.
time = 9.19, size = 358, normalized size = 2.56

$$\frac{x\left(8(3b+8cx^3)(a+bx^3+cx^6)-24ab\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+3(-5b^2+16ac)x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)}{448c\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(8*(3*b + 8*c*x^3)*(a + b*x^3 + c*x^6) - 24*a*b*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*(-5*b^2 + 16*a*c
)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/
2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
 4*a*c])]))/(448*c*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^3*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(x**3*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int(x^3*(a + b*x^3 + c*x^6)^(1/2), x)
```

3.197 $\int x\sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=140

$$\frac{x^2\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,-1/2,-1/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3
/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1399, 524}

$$\frac{x^2\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -1/2, -1/2, 5/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int x\sqrt{a+bx^3+cx^6}\,dx = \frac{\sqrt{a+bx^3+cx^6}\int x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{x^2\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 337 vs. 2(140) = 280.
time = 10.28, size = 337, normalized size = 2.41

$$\frac{x^2\left(10(a+bx^3+cx^6)+15a\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+3bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)}{50\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*(10*(a + b*x^3 + c*x^6) + 15*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(50*Sqrt[a + b*x^3
+ c*x^6])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+bx^3+cx^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(x*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int(x*(a + b*x^3 + c*x^6)^(1/2), x)
```

3.198 $\int \sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=135

$$\frac{x\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] x*AppellF1(1/3,-1/2,-1/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-
4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1362, 440}

$$\frac{x\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -1/2, -1/2, 4/3, (-2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^3)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
```

```
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+bx^3+cx^6}\,dx &= \frac{\sqrt{a+bx^3+cx^6}\int \sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\
&= \frac{x\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 335 vs. 2(135) = 270.

time = 10.24, size = 335, normalized size = 2.48

$$
\frac{x\left(8(a+bx^3+cx^6)+24a\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+3bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)}{32\sqrt{a+bx^3+cx^6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(8*(a + b*x^3 + c*x^6) + 24*a*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(32*Sqrt[a + b*x^3 + c
*x^6])
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$
\int \sqrt{c\,x^{6}+b\,x^{3}+a}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx^3 + cx^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x^6 + b\,x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2),x)
[Out] int((a + b*x^3 + c*x^6)^(1/2), x)
```

3.199 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -AppellF1(-1/3,-1/2,-1/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-
4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$-\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -1/2, -1/2, 2/3, (-2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c*x
^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
```

```
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3+cx^6}}{x^2}\,dx = \frac{\sqrt{a+bx^3+cx^6}\int \frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}{x^2}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{1}{2},-\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 340 vs. 2(138) = 276.
time = 10.25, size = 340, normalized size = 2.46

$$\frac{-20(a+bx^3+cx^6)+15bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+12cx^6\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{20x\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^2,x]
```

```
[Out] (-20*(a + b*x^3 + c*x^6) + 15*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 12*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*x*Sqrt[a + b*x^
3 + c*x^6])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6 + b\,x^3 + a}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^2,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^2, x)
```

3.200 $\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -1/2*AppellF1(-2/3,-1/2,-1/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x^2/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$-\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

```
[Out] -1/2*(Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -1/2, -1/2, 1/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[1 + (
2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
```

```
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx^3+cx^6}}{x^3}\,dx &= \frac{\sqrt{a+bx^3+cx^6}\int \frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}{x^3}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\\ &= -\frac{\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{2}{3};-\frac{1}{2},-\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 340 vs. 2(140) = 280.
time = 10.21, size = 340, normalized size = 2.43

$$\frac{-4(a+bx^3+cx^6)+6bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+3cx^6\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{8x^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3 + c*x^6]/x^3,x]
```

```
[Out] (-4*(a + b*x^3 + c*x^6) + 6*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 3*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(8*x^2*Sqrt[a + b*x^3
+ c*x^6])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(1/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^3 + cx^6}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**3 + c*x**6)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^6+b\,x^3+a}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(1/2)/x^3,x)
[Out] int((a + b*x^3 + c*x^6)^(1/2)/x^3, x)
```

3.201 $\int x^{14}(a+bx^3+cx^6)^{3/2}\,dx$

Optimal. Leaf size=293

$$-\frac{(b^2-4ac)(33b^4-72ab^2c+16a^2c^2)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{(33b^4-72ab^2c+16a^2c^2)(b+2cx^3)(a-}{6144c^5}$$

```
[Out] 1/6144*(16*a^2*c^2-72*a*b^2*c+33*b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^5
-11/336*b*x^6*(c*x^6+b*x^3+a)^(5/2)/c^2+1/24*x^9*(c*x^6+b*x^3+a)^(5/2)/c-1/
13440*(3*b*(-124*a*c+77*b^2)-10*c*(-28*a*c+33*b^2)*x^3)*(c*x^6+b*x^3+a)^(5/
2)/c^4+1/32768*(-4*a*c+b^2)^2*(16*a^2*c^2-72*a*b^2*c+33*b^4)*arctanh(1/2*(2
*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(13/2)-1/16384*(-4*a*c+b^2)*(16*
a^2*c^2-72*a*b^2*c+33*b^4)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^6
```

Rubi [A]
time = 0.26, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1371, 756, 846, 793, 626, 635, 212}

$$\frac{(b^2-4ac)^2(16a^2c^2-72ab^2c+33b^4)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{32768c^{13/2}}-\frac{(b^2-4ac)(16a^2c^2-72ab^2c+33b^4)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{(16a^2c^2-72ab^2c+33b^4)(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{6144c^5}-\frac{(3b(77b^2-124ac)-10cx^3(33b^2-28ac))(a+bx^3+cx^6)^{5/2}}{13440c^4}-\frac{11bx^6(a+bx^3+cx^6)^{5/2}}{336c^2}+\frac{x^9(a+bx^3+cx^6)^{5/2}}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -1/16384*((b^2 - 4*a*c)*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x^3)*Sq
rt[a + b*x^3 + c*x^6])/c^6 + ((33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(b + 2*c*x
^3)*(a + b*x^3 + c*x^6)^(3/2))/(6144*c^5) - (11*b*x^6*(a + b*x^3 + c*x^6)^(
5/2))/(336*c^2) + (x^9*(a + b*x^3 + c*x^6)^(5/2))/(24*c) - ((3*b*(77*b^2 -
124*a*c) - 10*c*(33*b^2 - 28*a*c)*x^3)*(a + b*x^3 + c*x^6)^(5/2))/(13440*c^
4) + ((b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*ArcTanh[(b + 2*c*x
^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(32768*c^(13/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{14}\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{1}{3}\text{Subst}\left(\int x^4\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)$$

$$=\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}+\frac{\text{Subst}\left(\int x^2\left(-3a-\frac{11bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)}{24c}$$

$$=-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}+\frac{\text{Subst}\left(\int x\left(11ab+\frac{3}{4}(33\right.\right.}{}$$

$$=-\frac{11bx^6\left(a+bx^3+cx^6\right)^{5/2}}{336c^2}+\frac{x^9\left(a+bx^3+cx^6\right)^{5/2}}{24c}-\frac{(3b(77b^2-124ac)-10}{}$$

$$=\frac{\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{6144c^5}-\frac{11bx^6\left(a+bx^3+c\right.}{336c^2}$$

$$=-\frac{\left(b^2-4ac\right)\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{(33b^4-}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{(33b^4-}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(33b^4-72ab^2c+16a^2c^2\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{16384c^6}+\frac{(33b^4-}{}$$

Mathematica [A]
time = 0.85, size = 289, normalized size = 0.99

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(-3465b^7+2310b^6cx^3+84b^5c(365a-22cx^6)+24b^4c^2x^3(-749a+66cx^6)+32b^2c^3x^3(1181a^2-284acx^6+40c^2x^{12})-16b^3c^2(5103a^2-780acx^6+88c^2x^{12})+4480c^4x^3(-3a^3+2a^2cx^6+24ac^2x^{12}+16c^3x^{18})+64bc^3(919a^3-302a^2cx^6+104ac^2x^{12}+1360c^3x^{18})\right)-105(b^2-4ac)^2(33b^4-72ab^2c+16a^2c^2)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{3440640c^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-3465*b^7 + 2310*b^6*c*x^3 + 84*b^5*c*(
365*a - 22*c*x^6) + 24*b^4*c^2*x^3*(-749*a + 66*c*x^6) + 32*b^2*c^3*x^3*(11
81*a^2 - 284*a*c*x^6 + 40*c^2*x^12) - 16*b^3*c^2*(5103*a^2 - 780*a*c*x^6 +
88*c^2*x^12) + 4480*c^4*x^3*(-3*a^3 + 2*a^2*c*x^6 + 24*a*c^2*x^12 + 16*c^3*
x^18) + 64*b*c^3*(919*a^3 - 302*a^2*c*x^6 + 104*a*c^2*x^12 + 1360*c^3*x^18)
) - 105*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*Log[b + 2*c*x^3
- 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(3440640*c^(13/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14}\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^14*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.41, size = 641, normalized size = 2.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6881280*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3
 + 256*a^4*c^4)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b
*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(71680*c^8*x^21 + 87040*b*c^7*
x^18 + 1280*(b^2*c^6 + 84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12
+ 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*
b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3
*c^4 + 2416*a^2*b*c^5)*x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b
^2*c^4 - 6720*a^3*c^5)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7, -1/3440640*(105*(
33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*s
qrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6
+ b*c*x^3 + a*c)) - 2*(71680*c^8*x^21 + 87040*b*c^7*x^18 + 1280*(b^2*c^6 +
84*a*c^7)*x^15 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^12 + 16*(99*b^4*c^4 - 568*
a*b^2*c^5 + 560*a^2*c^6)*x^9 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3
*c^3 + 58816*a^3*b*c^4 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*
x^6 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*
x^3)*sqrt(c*x^6 + b*x^3 + a))/c^7]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{14}\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**14*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^14, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{14}\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^14*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.202 $\int x^{11}(a+bx^3+cx^6)^{3/2}\,dx$

Optimal. Leaf size=223

$$\frac{b(b^2-4ac)(3b^2-4ac)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b(3b^2-4ac)(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{384c^4}+\frac{x^6(a+bx^3+\cdots}{21c\cdots}$$

```
[Out] -1/384*b*(-4*a*c+3*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^4+1/21*x^6*(c*x
^6+b*x^3+a)^(5/2)/c+1/840*(-30*b*c*x^3-16*a*c+21*b^2)*(c*x^6+b*x^3+a)^(5/2)
/c^3-1/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)
/(c*x^6+b*x^3+a)^(1/2))/c^(11/2)+1/1024*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(2*c*
x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^5
```

Rubi [A]
time = 0.14, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 756, 793, 626, 635, 212}

$$-\frac{b(b^2-4ac)^2(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2048c^{11/2}}+\frac{b(b^2-4ac)(3b^2-4ac)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b(3b^2-4ac)(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{384c^4}+\frac{(-16ac+21b^2-30bcx^3)(a+bx^3+cx^6)^{5/2}}{840c^3}+\frac{x^6(a+bx^3+cx^6)^{5/2}}{21c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(10
24*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(384*
c^4) + (x^6*(a + b*x^3 + c*x^6)^(5/2))/(21*c) + ((21*b^2 - 16*a*c - 30*b*c*
x^3)*(a + b*x^3 + c*x^6)^(5/2))/(840*c^3) - (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a
*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2048*c^(11
/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^{11}\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{1}{3}\text{Subst}\left(\int x^3\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)$$

$$=\frac{x^6(a+bx^3+cx^6)^{5/2}}{21c}+\frac{\text{Subst}\left(\int x\left(-2a-\frac{9bx}{2}\right)\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)}{21c}$$

$$=\frac{x^6(a+bx^3+cx^6)^{5/2}}{21c}+\frac{(21b^2-16ac-30bcx^3)\left(a+bx^3+cx^6\right)^{5/2}}{840c^3}-\frac{(b(3b^2-}{}$$

$$=-\frac{b(3b^2-4ac)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384c^4}+\frac{x^6(a+bx^3+cx^6)^{5/2}}{21c}+\frac{(21b^2-}{}$$

$$=\frac{b(b^2-4ac)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b(3b^2-4ac)\left(b+2cx^3\right)}{384c^4}$$

$$=\frac{b(b^2-4ac)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b(3b^2-4ac)\left(b+2cx^3\right)}{384c^4}$$

$$=\frac{b(b^2-4ac)\left(3b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1024c^5}-\frac{b(3b^2-4ac)\left(b+2cx^3\right)}{384c^4}$$

Mathematica [A]
time = 0.64, size = 220, normalized size = 0.99

$$\frac{\sqrt{a+bx^3+cx^6}\left(315b^6-210b^5cx^3+16b^3c^2x^3(91a-9cx^6)+168b^4c(-15a+cx^6)+1024c^3(a+cx^6)^2(-2a+5cx^6)+16b^2c^2(343a^2-62acx^6+8c^2x^{12})+32bc^3x^3(-73a^2+22acx^6+200c^2x^{12})\right)}{107520c^5}+\frac{b(b^2-4ac)^2(3b^2-4ac)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{2048c^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(315*b^6 - 210*b^5*c*x^3 + 16*b^3*c^2*x^3*(91*a -
9*c*x^6) + 168*b^4*c*(-15*a + c*x^6) + 1024*c^3*(a + c*x^6)^2*(-2*a + 5*c*x
^6) + 16*b^2*c^2*(343*a^2 - 62*a*c*x^6 + 8*c^2*x^12) + 32*b*c^3*x^3*(-73*a^
2 + 22*a*c*x^6 + 200*c^2*x^12)))/(107520*c^5) + (b*(b^2 - 4*a*c)^2*(3*b^2 -
 4*a*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(2048*c^(11/2
))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{11}\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^11*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.43, size = 535, normalized size = 2.40

$$\left[-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,(5120\,c^7x^{18}+6400\,bc^6x^{15}+128\,(b^2c^5+64\,ac^6)x^{12}-16\,(9\,b^3c^4-44\,abc^5)x^9+315\,b^6c-2520\,ab^4c^2+5488\,a^2b^2c^3-2048\,a^3c^4+8\,(21\,b^4c^3-124\,ab^2c^4+128\,a^2c^5)x^6-2\,(105\,b^5c^2-728\,ab^3c^3+1168\,a^2bc^4)x^3)\sqrt{cx^6+bx^3+a}}{430080\,c^6},\ \frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)+2\,(5120\,c^7x^{18}+6400\,bc^6x^{15}+128\,(b^2c^5+64\,ac^6)x^{12}-16\,(9\,b^3c^4-44\,abc^5)x^9+315\,b^6c-2520\,ab^4c^2+5488\,a^2b^2c^3-2048\,a^3c^4+8\,(21\,b^4c^3-124\,ab^2c^4+128\,a^2c^5)x^6-2\,(105\,b^5c^2-728\,ab^3c^3+1168\,a^2bc^4)x^3)\sqrt{cx^6+bx^3+a}}{215040\,c^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(c) - 4*a*c) - 4*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128*(b^2*c^5 + 64
*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c - 2520*a*b^4*c^2
 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^
2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x^3)*sqrt(c*x
^6 + b*x^3 + a))/c^6, 1/215040*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 -
64*a^3*b*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(5120*c^7*x^18 + 6400*b*c^6*x^15 + 128
*(b^2*c^5 + 64*a*c^6)*x^12 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^9 + 315*b^6*c -
2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 8*(21*b^4*c^3 - 124*a*b^
2*c^4 + 128*a^2*c^5)*x^6 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)
*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^6]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11} \left(a + b x^{3} + c x^{6}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**11*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^11, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{11}\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x^11*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.203 $\int x^8(a+bx^3+cx^6)^{3/2}\,dx$

Optimal. Leaf size=204

$$-\frac{(b^2-4ac)\,(7b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{(7b^2-4ac)\,(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{576c^3}-\frac{7b(a+bx^3}{180}$$

```
[Out] 1/576*(-4*a*c+7*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^3-7/180*b*(c*x^6+b
*x^3+a)^(5/2)/c^2+1/18*x^3*(c*x^6+b*x^3+a)^(5/2)/c+1/3072*(-4*a*c+b^2)^2*(-
4*a*c+7*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(9/2)
-1/1536*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^4
```

Rubi [A]
time = 0.12, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 756, 654, 626, 635, 212}

$$\frac{(b^2-4ac)^2\,(7b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\,\sqrt{a+bx^3+cx^6}}\right)}{3072c^{9/2}}-\frac{(b^2-4ac)\,(7b^2-4ac)\,(b+2cx^3)\,\sqrt{a+bx^3+cx^6}}{1536c^4}+\frac{(7b^2-4ac)\,(b+2cx^3)\,(a+bx^3+cx^6)^{3/2}}{576c^3}-\frac{7b(a+bx^3+cx^6)^{5/2}}{180c^2}+\frac{x^3(a+bx^3+cx^6)^{5/2}}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -1/1536*((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6
])/c^4 + ((7*b^2 - 4*a*c)*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*c^3
) - (7*b*(a + b*x^3 + c*x^6)^(5/2))/(180*c^2) + (x^3*(a + b*x^3 + c*x^6)^(5
/2))/(18*c) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqr
t[c]*Sqrt[a + b*x^3 + c*x^6])])/(3072*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^8 \left(a+bx^3+cx^6\right)^{3/2} \, dx = \frac{1}{3}\text{Subst}\left(\int x^2 \left(a+bx+cx^2\right)^{3/2} \, dx, x, x^3\right)$$

$$= \frac{x^3(a+bx^3+cx^6)^{5/2}}{18c} + \frac{\text{Subst}\left(\int \left(-a-\frac{7bx}{2}\right)\left(a+bx+cx^2\right)^{3/2} \, dx, x, x^3\right)}{18c}$$

$$= -\frac{7b(a+bx^3+cx^6)^{5/2}}{180c^2} + \frac{x^3(a+bx^3+cx^6)^{5/2}}{18c} + \frac{(7b^2-4ac)\,\text{Subst}\left(\int (a+bx\right.}{72c^2}$$

$$= \frac{(7b^2-4ac)\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{576c^3} - \frac{7b(a+bx^3+cx^6)^{5/2}}{180c^2} + \frac{x^3(a+bx}{1}$$

$$= -\frac{(b^2-4ac)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4} + \frac{(7b^2-4ac)\left(b+2cx^3\right)}{576c}$$

$$= -\frac{(b^2-4ac)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4} + \frac{(7b^2-4ac)\left(b+2cx^3\right)}{576c}$$

$$= -\frac{(b^2-4ac)\left(7b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{1536c^4} + \frac{(7b^2-4ac)\left(b+2cx^3\right)}{576c}$$

Mathematica [A]
time = 0.53, size = 194, normalized size = 0.95

$$\frac{2\sqrt{c}\sqrt{a+bx^3+cx^6}\left(-105b^5+70b^4cx^3+8b^3c(95a-7cx^6)+48b^2c^2x^3(-9a+cx^6)+160c^3x^3(3a^2+14acx^6+8c^2x^{12})+16bc^2(-81a^2+18acx^6+104c^2x^{12})\right)-15(b^2-4ac)^2(7b^2-4ac)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{46080c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]*(-105*b^5 + 70*b^4*c*x^3 + 8*b^3*c*(95*a
 - 7*c*x^6) + 48*b^2*c^2*x^3*(-9*a + c*x^6) + 160*c^3*x^3*(3*a^2 + 14*a*c*x
^6 + 8*c^2*x^12) + 16*b*c^2*(-81*a^2 + 18*a*c*x^6 + 104*c^2*x^12)) - 15*(b^
2 - 4*a*c)^2*(7*b^2 - 4*a*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c
*x^6]])/(46080*c^(9/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{8} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^8*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 451, normalized size = 2.21

$$\left[-\frac{15\,{\left(7\,b^{6} - 60\,a b^{4} c + 144\,a^{2} b^{2} c^{2} - 64\,a^{3} c^{3}\right)} \sqrt{c} \log\left(-8\,c^{2} x^{6} - 8\,b c x^{3} - b^{2} + 4\,\sqrt{c x^{6} + b x^{3} + a} {\left(2\,c x^{3} + b\right)} \sqrt{c} - 4\,a c\right) - 4\,{\left(1280\,c^{6} x^{15} + 1664\,b c^{5} x^{12} + 16\,{\left(3\,b^{2} c^{4} + 140\,a c^{5}\right)} x^{9} - 8\,{\left(7\,b^{3} c^{3} - 36\,a b c^{4}\right)} x^{6} - 105\,b^{5} c + 760\,a b^{3} c^{2} - 1296\,a^{2} b c^{3} + 2\,{\left(35\,b^{4} c^{2} - 216\,a b^{2} c^{3} + 240\,a^{2} c^{4}\right)} x^{3}\right)} \sqrt{c x^{6} + b x^{3} + a}}{92160\,c^{5}}, -\frac{15\,{\left(7\,b^{6} - 60\,a b^{4} c + 144\,a^{2} b^{2} c^{2} - 64\,a^{3} c^{3}\right)} \sqrt{-c} \arctan\left(\frac{\sqrt{c x^{6} + b x^{3} + a} {\left(2\,c x^{3} + b\right)} \sqrt{-c}}{2\,{\left(c^{2} x^{6} + b c x^{3} + a c\right)}}\right) - 2\,{\left(1280\,c^{6} x^{15} + 1664\,b c^{5} x^{12} + 16\,{\left(3\,b^{2} c^{4} + 140\,a c^{5}\right)} x^{9} - 8\,{\left(7\,b^{3} c^{3} - 36\,a b c^{4}\right)} x^{6} - 105\,b^{5} c + 760\,a b^{3} c^{2} - 1296\,a^{2} b c^{3} + 2\,{\left(35\,b^{4} c^{2} - 216\,a b^{2} c^{3} + 240\,a^{2} c^{4}\right)} x^{3}\right)} \sqrt{c x^{6} + b x^{3} + a}}{46080\,c^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*l
og(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*s
qrt(c) - 4*a*c) - 4*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*b^2*c^4 + 140*
a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760*a*b^3*c^2 - 1
296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x^3)*sqrt(c*x^
6 + b*x^3 + a))/c^5, -1/46080*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 6
4*a^3*c^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-
c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(1280*c^6*x^15 + 1664*b*c^5*x^12 + 16*(3*
b^2*c^4 + 140*a*c^5)*x^9 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^6 - 105*b^5*c + 760
*a*b^3*c^2 - 1296*a^2*b*c^3 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*
x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{8} \left(a + b x^{3} + c x^{6}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**8*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^8, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^8\,\left(c\,x^6+b\,x^3+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x^8*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.204 $\int x^5 (a+bx^3+cx^6)^{3/2}\, dx$

Optimal. Leaf size=150

$$\frac{b(b^2-4ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}-\frac{b(b^2-4ac)^2\ \mathrm{ta}}{}$$

```
[Out] -1/48*b*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c^2+1/15*(c*x^6+b*x^3+a)^(5/2)/c-
1/256*b*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2
))/c^(7/2)+1/128*b*(-4*a*c+b^2)*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/c^3
```

Rubi [A]

time = 0.08, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 654, 626, 635, 212}

$$-\frac{b(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{256c^{7/2}}+\frac{b(b^2-4ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (b*(b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(128*c^3) - (b*(b +
 2*c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*c^2) + (a + b*x^3 + c*x^6)^(5/2)/(
15*c) - (b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3
+ c*x^6])])/(256*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{1}{3}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)$$

$$=\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}-\frac{b\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)}{6c}$$

$$=-\frac{b(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15c}+\frac{(b(b^2-4ac))\,\text{Subst}\left(\right.}{}$$

$$=\frac{b(b^2-4ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{(a}{}$$

$$=\frac{b(b^2-4ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{(a}{}$$

$$=\frac{b(b^2-4ac)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{128c^3}-\frac{b(b+2cx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48c^2}+\frac{(a}{}$$

Mathematica [A]
time = 0.38, size = 140, normalized size = 0.93

$$\frac{\sqrt{a+bx^3+cx^6}\left(15b^4-10b^3cx^3+128c^2(a+cx^6)^2+4b^2c(-25a+2cx^6)+8bc^2x^3(7a+22cx^6)\right)}{1920c^3}+\frac{b(b^2-4ac)^2\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{256c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(15*b^4 - 10*b^3*c*x^3 + 128*c^2*(a + c*x^6)^2 + 4
*b^2*c*(-25*a + 2*c*x^6) + 8*b*c^2*x^3*(7*a + 22*c*x^6)))/(1920*c^3) + (b*(
b^2 - 4*a*c)^2*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(256*c
^(7/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5} \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.41, size = 361, normalized size = 2.41

$$\left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{c}-4\,ac\right)+4\,(128\,c^5x^{12}+176\,bc^4x^9+8\,(b^2c^3+32\,ac^4)x^6+15\,b^4c-100\,ab^2c^2+128\,a^2c^3-2\,(5\,b^3c^2-28\,abc^3)x^3)\sqrt{cx^6+bx^3+a}}{7680\,c^4},\ \frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)+2\,(128\,c^5x^{12}+176\,bc^4x^9+8\,(b^2c^3+32\,ac^4)x^6+15\,b^4c-100\,ab^2c^2+128\,a^2c^3-2\,(5\,b^3c^2-28\,abc^3)x^3)\sqrt{cx^6+bx^3+a}}{3840\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c
*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(
128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c - 100*
a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6 + b*x^
3 + a))/c^4, 1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*arctan(1/
2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c))
 + 2*(128*c^5*x^12 + 176*b*c^4*x^9 + 8*(b^2*c^3 + 32*a*c^4)*x^6 + 15*b^4*c
- 100*a*b^2*c^2 + 128*a^2*c^3 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^3)*sqrt(c*x^6
+ b*x^3 + a))/c^4]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [A]

time = 3.65, size = 172, normalized size = 1.15

$$\frac{1}{1920}\sqrt{cx^6+bx^3+a}\left(2\left(4\left(2\left(8cx^3+11b\right)x^3+\frac{b^2c^3+32ac^4}{c^4}\right)x^3-\frac{5b^3c^2-28abc^3}{c^4}\right)x^3+\frac{15b^4c-100ab^2c^2+128a^2c^3}{c^4}\right)+\frac{\left(b^5-8ab^3c+16a^2bc^2\right)\log\left(\left|-2\left(\sqrt{c}\,x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{256c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*(8*c*x^3 + 11*b)*x^3 + (b^2*c^3 + 3
2*a*c^4)/c^4)*x^3 - (5*b^3*c^2 - 28*a*b*c^3)/c^4)*x^3 + (15*b^4*c - 100*a*b
^2*c^2 + 128*a^2*c^3)/c^4) + 1/256*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*log(abs
(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(7/2)
```

Mupad [B]

time = 1.58, size = 223, normalized size = 1.49

$$\frac{\left(cx^6+bx^3+a\right)^{5/2}}{15c}-\frac{b\left(\frac{3a\left(\ln\left(\sqrt{cx^6+bx^3+a}+\frac{cx^3+\frac{b}{2}}{\sqrt{c}}\right)\left(\frac{a}{2\sqrt{c}}-\frac{b^2}{8c^{3/2}}\right)+\frac{\left(2cx^3+b\right)\sqrt{cx^6+bx^3+a}}{4c}\right)}{4}+\frac{x^3\left(cx^6+bx^3+a\right)^{3/2}}{4}-\frac{3b^2\left(\ln\left(\sqrt{cx^6+bx^3+a}+\frac{cx^3+\frac{b}{2}}{\sqrt{c}}\right)\left(\frac{a}{2\sqrt{c}}-\frac{b^2}{8c^{3/2}}\right)+\frac{\left(2cx^3+b\right)\sqrt{cx^6+bx^3+a}}{4c}\right)}{16c}+\frac{b\left(cx^6+bx^3+a\right)^{3/2}}{8c}\right)}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] (a + b*x^3 + c*x^6)^(5/2)/(15*c) - (b*((3*a*(log((a + b*x^3 + c*x^6)^(1/2)
+ (b/2 + c*x^3)/c^(1/2))*(a/(2*c^(1/2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^3)
*(a + b*x^3 + c*x^6)^(1/2))/(4*c)))/4 + (x^3*(a + b*x^3 + c*x^6)^(3/2))/4 -
 (3*b^2*(log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))*(a/(2*c^(1/
2)) - b^2/(8*c^(3/2))) + ((b + 2*c*x^3)*(a + b*x^3 + c*x^6)^(1/2))/(4*c)))/
(16*c) + (b*(a + b*x^3 + c*x^6)^(3/2))/(8*c)))/(6*c)
```

3.205 $\int x^2(a+bx^3+cx^6)^{3/2}\,dx$

Optimal. Leaf size=124

$$-\frac{(b^2-4ac)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{24c}+\frac{(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3}}\right.}{128c^{5/2}}$$

```
[Out] 1/24*(2*c*x^3+b)*(c*x^6+b*x^3+a)^(3/2)/c+1/128*(-4*a*c+b^2)^2*arctanh(1/2*(
2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)-1/64*(-4*a*c+b^2)*(2*c*x^
3+b)*(c*x^6+b*x^3+a)^(1/2)/c^2
```

Rubi [A]
time = 0.06, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1366, 626, 635, 212}

$$\frac{(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{128c^{5/2}}-\frac{(b^2-4ac)(b+2cx^3)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{(b+2cx^3)(a+bx^3+cx^6)^{3/2}}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] -1/64*((b^2 - 4*a*c)*(b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/c^2 + ((b + 2*c
*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*c) + ((b^2 - 4*a*c)^2*ArcTanh[(b + 2*c
*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(128*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int x^2\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{1}{3}\text{Subst}\left(\int\left(a+bx+cx^2\right)^{3/2}\,dx,x,x^3\right)$$

$$=\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}-\frac{\left(b^2-4ac\right)\text{Subst}\left(\int\sqrt{a+bx+cx^2}\,dx,x,x^3\right.}{16c}$$

$$=-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}+\frac{(b}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}+\frac{(b}{}$$

$$=-\frac{\left(b^2-4ac\right)\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}}{64c^2}+\frac{\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{24c}+\frac{(b}{}$$

Mathematica [A]
time = 0.31, size = 110, normalized size = 0.89

$$\frac{\left(b+2cx^3\right)\sqrt{a+bx^3+cx^6}\left(-3b^2+20ac+8bcx^3+8c^2x^6\right)}{192c^2}-\frac{\left(-b^2+4ac\right)^2\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{128c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6]*(-3*b^2 + 20*a*c + 8*b*c*x^3 + 8*c^2
*x^6))/(192*c^2) - ((-b^2 + 4*a*c)^2*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b
*x^3 + c*x^6]])/(128*c^(5/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.39, size = 297, normalized size = 2.40

$$\left[\frac{3\,{\left(b^{4} - 8\,a b^{2} c + 16\,a^{2} c^{2}\right)} \sqrt{c} \log\left(-8\,c^{2} x^{6} - 8\,b c x^{3} - b^{2} - 4\,\sqrt{c x^{6} + b x^{3} + a} {\left(2\,c x^{3} + b\right)} \sqrt{c} - 4\,a c\right) + 4\,{\left(16\,c^{4} x^{9} + 24\,b c^{3} x^{6} - 3\,b^{3} c + 20\,a b c^{2} + 2\,{\left(b^{2} c^{2} + 20\,a c^{3}\right)} x^{3}\right)} \sqrt{c x^{6} + b x^{3} + a}}{768\,c^{3}}, -\frac{3\,{\left(b^{4} - 8\,a b^{2} c + 16\,a^{2} c^{2}\right)} \sqrt{-c} \arctan\left(\frac{\sqrt{c x^{6} + b x^{3} + a} {\left(2\,c x^{3} + b\right)} \sqrt{-c}}{2\,{\left(c^{2} x^{6} + b c x^{3} + a c\right)}}\right) - 2\,{\left(16\,c^{4} x^{9} + 24\,b c^{3} x^{6} - 3\,b^{3} c + 20\,a b c^{2} + 2\,{\left(b^{2} c^{2} + 20\,a c^{3}\right)} x^{3}\right)} \sqrt{c x^{6} + b x^{3} + a}}{384\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3
 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(16*c
^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^3)*
sqrt(c*x^6 + b*x^3 + a))/c^3, -1/384*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt
(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b
*c*x^3 + a*c)) - 2*(16*c^4*x^9 + 24*b*c^3*x^6 - 3*b^3*c + 20*a*b*c^2 + 2*(b
^2*c^2 + 20*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^3]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(a + b x^{3} + c x^{6}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [**A**]
time = 3.46, size = 135, normalized size = 1.09

$$\frac{1}{192} \, \sqrt{c x^{6} + b x^{3} + a} {\left(2 \, {\left(4 \, {\left(2 \, c x^{3} + 3 \, b\right)} x^{3} + \frac{b^{2} c^{2} + 20 \, a c^{3}}{c^{3}}\right)} x^{3} - \frac{3 \, b^{3} c - 20 \, a b c^{2}}{c^{3}}\right)} - \frac{{\left(b^{4} - 8 \, a b^{2} c + 16 \, a^{2} c^{2}\right)} \log\left({\left| -2 \, {\left(\sqrt{c} x^{3} - \sqrt{c x^{6} + b x^{3} + a}\right)} \sqrt{c} - b \right|}\right)}{128 \, c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^6 + b*x^3 + a)*(2*(4*(2*c*x^3 + 3*b)*x^3 + (b^2*c^2 + 20*a*c
^3)/c^3)*x^3 - (3*b^3*c - 20*a*b*c^2)/c^3) - 1/128*(b^4 - 8*a*b^2*c + 16*a^
2*c^2)*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(
5/2)
```

Mupad [B]
time = 1.44, size = 115, normalized size = 0.93

$$\frac{\left(c\,x^3+\frac{b}{2}\right)\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{12\,c}+\frac{\left(3\,a\,c-\frac{3\,b^2}{4}\right)\,\left(\left(\frac{b}{4\,c}+\frac{x^3}{2}\right)\,\sqrt{c\,x^6+b\,x^3+a}+\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] ((b/2 + c*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(12*c) + ((3*a*c - (3*b^2)/4)*((b
/(4*c) + x^3/2)*(a + b*x^3 + c*x^6)^(1/2) + (log((a + b*x^3 + c*x^6)^(1/2)
+ (b/2 + c*x^3)/c^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(12*c)
```

3.206 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx$

Optimal. Leaf size=155

$$\frac{(b^2+8ac+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c}+\frac{1}{9}(a+bx^3+cx^6)^{3/2}-\frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b(b^2-\cdots}{\cdots}$$

```
[Out] 1/9*(c*x^6+b*x^3+a)^(3/2)-1/3*a^(3/2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^
6+b*x^3+a)^(1/2))-1/48*b*(-12*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x
^6+b*x^3+a)^(1/2))/c^(3/2)+1/24*(2*b*c*x^3+8*a*c+b^2)*(c*x^6+b*x^3+a)^(1/2)
/c
```

Rubi [A]
time = 0.12, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1371, 748, 828, 857, 635, 212, 738}

$$-\frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)-\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{3/2}}+\frac{(8ac+b^2+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c}+\frac{1}{9}(a+bx^3+cx^6)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(24*c) + (a + b*x^3 + c
*x^6)^(3/2)/9 - (a^(3/2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 +
c*x^6])])/3 - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b
*x^3 + c*x^6])])/(48*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x}\,dx, x, x^3\right)$$

$$= \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} - \frac{1}{6}\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^3\right)$$

$$= \frac{(b^2+8ac+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} + \frac{\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b(b^2}{x\sqrt{a+bx}}\right.}{24}$$

$$= \frac{(b^2+8ac+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} + \frac{1}{3}a^2\text{Subst}\left(\int \frac{}{x\sqrt{a}}\right.$$

$$= \frac{(b^2+8ac+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} - \frac{1}{3}\left(2a^2\right)\text{Subst}\left(\int \frac{}{4}\right.$$

$$= \frac{(b^2+8ac+2bcx^3)\sqrt{a+bx^3+cx^6}}{24c} + \frac{1}{9}\left(a+bx^3+cx^6\right)^{3/2} - \frac{1}{3}a^{3/2}\tanh^{-1}\left(\frac{}{2\sqrt{a}}\right.$$

Mathematica [A]
time = 0.49, size = 143, normalized size = 0.92

$$\frac{1}{144}\left(\frac{2\sqrt{a+bx^3+cx^6}\,(3b^2+14bcx^3+8c(4a+cx^6))}{c} + 96a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right) + \frac{3(b^3-12abc)\log\left(c\left(b+2cx^3-2\sqrt{c}\,\sqrt{a+bx^3+cx^6}\right)\right)}{c^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x,x]
```

```
[Out] ((2*Sqrt[a + b*x^3 + c*x^6]*(3*b^2 + 14*b*c*x^3 + 8*c*(4*a + c*x^6)))/c + 9
6*a^(3/2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]] + (3*(b^
3 - 12*a*b*c)*Log[c*(b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/c^(
3/2))/144
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.48, size = 727, normalized size = 4.69

$$\left[\frac{48\,a^{\frac{3}{2}}c^{2}\log\left(-\frac{(b^{2}+4\,ac)x^{6}+8\,abx^{3}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(bx^{3}+2\,a)\sqrt{a}+8\,a^{2}}{x^{6}}\right)-3\,(b^{3}-12\,abc)\sqrt{c}\,\log\left(-8\,c^{2}x^{6}-8\,bcx^{3}-b^{2}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{c}-4\,ac\right)+4\,(8\,c^{3}x^{6}+14\,bc^{2}x^{3}+3\,b^{2}c+32\,ac^{2})\sqrt{cx^{6}+bx^{3}+a}}{288\,c^{2}},\ \frac{24\,a^{\frac{3}{2}}c^{2}\log\left(-\frac{(b^{2}+4\,ac)x^{6}+8\,abx^{3}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(bx^{3}+2\,a)\sqrt{a}+8\,a^{2}}{x^{6}}\right)+3\,(b^{3}-12\,abc)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{-c}}{2\,(c^{2}x^{6}+bcx^{3}+ac)}\right)+2\,(8\,c^{3}x^{6}+14\,bc^{2}x^{3}+3\,b^{2}c+32\,ac^{2})\sqrt{cx^{6}+bx^{3}+a}}{144\,c^{2}},\ \frac{96\,\sqrt{-a}\,ac^{2}\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(bx^{3}+2\,a)\sqrt{-a}}{2\,(acx^{6}+abx^{3}+a^{2})}\right)-3\,(b^{3}-12\,abc)\sqrt{c}\,\log\left(-8\,c^{2}x^{6}-8\,bcx^{3}-b^{2}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{c}-4\,ac\right)+4\,(8\,c^{3}x^{6}+14\,bc^{2}x^{3}+3\,b^{2}c+32\,ac^{2})\sqrt{cx^{6}+bx^{3}+a}}{288\,c^{2}},\ \frac{48\,\sqrt{-a}\,ac^{2}\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(bx^{3}+2\,a)\sqrt{-a}}{2\,(acx^{6}+abx^{3}+a^{2})}\right)+3\,(b^{3}-12\,abc)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{-c}}{2\,(c^{2}x^{6}+bcx^{3}+ac)}\right)+2\,(8\,c^{3}x^{6}+14\,bc^{2}x^{3}+3\,b^{2}c+32\,ac^{2})\sqrt{cx^{6}+bx^{3}+a}}{144\,c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/288*(48*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 +
 b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^3 - 12*a*b*c)*sqrt(c
)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt
(c*x^6 + b*x^3 + a))/c^2, 1/144*(24*a^(3/2)*c^2*log(-((b^2 + 4*a*c)*x^6 + 8
*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) +
3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b
)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 + 14*b*c^2*x^3 + 3*b^2
*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/288*(96*sqrt(-a)*a*c^2*arcta
n(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a
^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt
(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*x^6 + 14*b*c^
2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2, 1/144*(48*sqrt(-a
)*a*c^2*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6
+ a*b*x^3 + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x
^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 +
14*b*c^2*x^3 + 3*b^2*c + 32*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x, x)
```

3.207 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4} dx$

Optimal. Leaf size=150

$$\frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6}-\frac{(a+bx^3+cx^6)^{3/2}}{3x^3}-\frac{1}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)+\frac{(b^2+4ac)\,\mathrm{t}}{}$$

[Out] -1/3*(c*x^6+b*x^3+a)^(3/2)/x^3-1/2*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))*a^(1/2)+1/8*(4*a*c+b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(1/2)+1/4*(2*c*x^3+3*b)*(c*x^6+b*x^3+a)^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1371, 746, 828, 857, 635, 212, 738}

$$\frac{(4ac+b^2)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{c}}-\frac{(a+bx^3+cx^6)^{3/2}}{3x^3}+\frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6}-\frac{1}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]

[Out] ((3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/4 - (a + b*x^3 + c*x^6)^(3/2)/(3*x^3) - (Sqrt[a]*b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/2 + ((b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(8*Sqrt[c])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^4}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^2}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3+cx^6)^{3/2}}{3x^3} + \frac{1}{2}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^3\right)$$

$$= \frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6} - \frac{(a+bx^3+cx^6)^{3/2}}{3x^3} - \frac{\text{Subst}\left(\int \frac{-4abc-c(b^2+4ac)}{x\sqrt{a+bx+cx}}\right.}{8c}$$

$$= \frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6} - \frac{(a+bx^3+cx^6)^{3/2}}{3x^3} + \frac{1}{2}(ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+b}}\right.$$

$$= \frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6} - \frac{(a+bx^3+cx^6)^{3/2}}{3x^3} - (ab)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx\right.$$

$$= \frac{1}{4}(3b+2cx^3)\sqrt{a+bx^3+cx^6} - \frac{(a+bx^3+cx^6)^{3/2}}{3x^3} - \frac{1}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2}{2\sqrt{a}\sqrt{}}\right.$$

Mathematica [A]
time = 0.49, size = 131, normalized size = 0.87

$$\frac{\sqrt{a+bx^3+cx^6}\,(-4a+5bx^3+2cx^6)}{12x^3} + \sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right) - \frac{(b^2+4ac)\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{8\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^4,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-4*a + 5*b*x^3 + 2*c*x^6))/(12*x^3) + Sqrt[a]*b*A
rcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]] - ((b^2 + 4*a*c)*Lo
g[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/(8*Sqrt[c])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^4,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.45, size = 713, normalized size = 4.75

$$\left[\frac{12\sqrt{a}bcx^3\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right)+3(b^2+4ac)\sqrt{c}x^3\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{c}-4ac\right)+4(2c^2x^6+5bcx^3-4ac)\sqrt{cx^6+bx^3+a}}{48cx^3}, \frac{6\sqrt{a}bcx^3\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right)-3(b^2+4ac)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{-c}}{2(c^2x^6+bcx^3+ac)}\right)+2(2c^2x^6+5bcx^3-4ac)\sqrt{cx^6+bx^3+a}}{24cx^3}, \frac{24\sqrt{-a}bcx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right)+3(b^2+4ac)\sqrt{c}x^3\log\left(-8c^2x^6-8bcx^3-b^2-4\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{c}-4ac\right)+4(2c^2x^6+5bcx^3-4ac)\sqrt{cx^6+bx^3+a}}{48cx^3}, \frac{12\sqrt{-a}bcx^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right)-3(b^2+4ac)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{-c}}{2(c^2x^6+bcx^3+ac)}\right)+2(2c^2x^6+5bcx^3-4ac)\sqrt{cx^6+bx^3+a}}{24cx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(12*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^
6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 3*(b^2 + 4*a*c)*sqrt(c
)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c*x^6 + b*x
^3 + a))/(c*x^3), 1/24*(6*sqrt(a)*b*c*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x
^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c
*x^6 + b*x^3 + a))/(c*x^3), 1/48*(24*sqrt(-a)*b*c*x^3*arctan(1/2*sqrt(c*x^6
 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 3*(b^2 +
4*a*c)*sqrt(c)*x^3*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqr
t(c*x^6 + b*x^3 + a))/(c*x^3), 1/24*(12*sqrt(-a)*b*c*x^3*arctan(1/2*sqrt(c*
x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 3*(b^2
 + 4*a*c)*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(2*c^2*x^6 + 5*b*c*x^3 - 4*a*c)*sqrt(c
*x^6 + b*x^3 + a))/(c*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**4,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^4,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^4, x)
```

3.208 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=151

$$-\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3}-\frac{(a+bx^3+cx^6)^{3/2}}{6x^6}-\frac{(b^2+4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}}+\frac{1}{2}b\sqrt{c}\ \mathrm{tanl}$$

```
[Out] -1/6*(c*x^6+b*x^3+a)^(3/2)/x^6-1/8*(4*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1
/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)+1/2*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c
*x^6+b*x^3+a)^(1/2))*c^(1/2)-1/4*(-2*c*x^3+b)*(c*x^6+b*x^3+a)^(1/2)/x^3
```

Rubi [A]
time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1371, 746, 826, 857, 635, 212, 738}

$$-\frac{(4ac+b^2)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8\sqrt{a}}-\frac{(a+bx^3+cx^6)^{3/2}}{6x^6}-\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3}+\frac{1}{2}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] -1/4*((b - 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/x^3 - (a + b*x^3 + c*x^6)^(3/2
)/(6*x^6) - ((b^2 + 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3
+ c*x^6])])/(8*Sqrt[a]) + (b*Sqrt[c]*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[
a + b*x^3 + c*x^6])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^7}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^3}\,dx, x, x^3\right)$$
$$= -\frac{(a+bx^3+cx^6)^{3/2}}{6x^6} + \frac{1}{4}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2}\,dx, x, x^3\right)$$
$$= -\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{(a+bx^3+cx^6)^{3/2}}{6x^6} - \frac{1}{8}\text{Subst}\left(\int \frac{-b^2-4ac-4b}{x\sqrt{a+bx+cx}}\right.$$
$$= -\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{(a+bx^3+cx^6)^{3/2}}{6x^6} + \frac{1}{2}(bc)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+}}\right.$$
$$= -\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{(a+bx^3+cx^6)^{3/2}}{6x^6} + (bc)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x\right.$$
$$= -\frac{(b-2cx^3)\sqrt{a+bx^3+cx^6}}{4x^3} - \frac{(a+bx^3+cx^6)^{3/2}}{6x^6} - \frac{(b^2+4ac)\tanh^{-1}\left(\frac{}{2\sqrt{a}\sqrt{}}\right.}{8\sqrt{a}}$$

Mathematica [A]
time = 0.51, size = 131, normalized size = 0.87

$$\frac{1}{12}\left(\frac{\sqrt{a+bx^3+cx^6}\,(-2a-5bx^3+4cx^6)}{x^6} + \frac{3(b^2+4ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{\sqrt{a}} - 6b\sqrt{c}\,\log\left(b+2cx^3-2\sqrt{c}\,\sqrt{a+bx^3+cx^6}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^7,x]
```

```
[Out] ((Sqrt[a + b*x^3 + c*x^6]*(-2*a - 5*b*x^3 + 4*c*x^6))/x^6 + (3*(b^2 + 4*a*c
)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/Sqrt[a] - 6*b*S
qrt[c]*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]])/12
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^7,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.44, size = 713, normalized size = 4.72

$$\left[\frac{12\,ab\sqrt{c}\,x^{6}\log\left(-8\,c^{2}x^{6}-8\,bcx^{3}-b^{2}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{c}-4\,ac\right)+3\,(b^{2}+4\,ac)\sqrt{a}\,x^{6}\log\left(-\frac{(b^{2}+4\,ac)x^{6}+8\,abx^{3}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(bx^{3}+2\,a)\sqrt{a}+8\,a^{2}}{x^{6}}\right)+4\,(4\,acx^{6}-5\,abx^{3}-2\,a^{2})\sqrt{cx^{6}+bx^{3}+a}}{48\,ax^{6}},\ -\frac{24\,ab\sqrt{-c}\,x^{6}\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{-c}}{2\,(c^{2}x^{6}+bcx^{3}+ac)}\right)-3\,(b^{2}+4\,ac)\sqrt{a}\,x^{6}\log\left(-\frac{(b^{2}+4\,ac)x^{6}+8\,abx^{3}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(bx^{3}+2\,a)\sqrt{a}+8\,a^{2}}{x^{6}}\right)-4\,(4\,acx^{6}-5\,abx^{3}-2\,a^{2})\sqrt{cx^{6}+bx^{3}+a}}{48\,ax^{6}},\ \frac{6\,ab\sqrt{c}\,x^{6}\log\left(-8\,c^{2}x^{6}-8\,bcx^{3}-b^{2}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{c}-4\,ac\right)+3\,(b^{2}+4\,ac)\sqrt{-a}\,x^{6}\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(bx^{3}+2\,a)\sqrt{-a}}{2\,(acx^{6}+abx^{3}+a^{2})}\right)+2\,(4\,acx^{6}-5\,abx^{3}-2\,a^{2})\sqrt{cx^{6}+bx^{3}+a}}{24\,ax^{6}},\ -\frac{12\,ab\sqrt{-c}\,x^{6}\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{-c}}{2\,(c^{2}x^{6}+bcx^{3}+ac)}\right)-3\,(b^{2}+4\,ac)\sqrt{-a}\,x^{6}\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(bx^{3}+2\,a)\sqrt{-a}}{2\,(acx^{6}+abx^{3}+a^{2})}\right)-2\,(4\,acx^{6}-5\,abx^{3}-2\,a^{2})\sqrt{cx^{6}+bx^{3}+a}}{24\,ax^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/48*(12*a*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 +
 b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*(b^2 + 4*a*c)*sqrt(a)*x^6*lo
g(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)
*sqrt(a) + 8*a^2)/x^6) + 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt(c*x^6 + b*x
^3 + a))/(a*x^6), -1/48*(24*a*b*sqrt(-c)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 3*(b^2 + 4*a*c)*sq
rt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(
b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt
(c*x^6 + b*x^3 + a))/(a*x^6), 1/24*(6*a*b*sqrt(c)*x^6*log(-8*c^2*x^6 - 8*b*
c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 3*
(b^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)
*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sq
rt(c*x^6 + b*x^3 + a))/(a*x^6), -1/24*(12*a*b*sqrt(-c)*x^6*arctan(1/2*sqrt(
c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 3*(b
^2 + 4*a*c)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*s
qrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) - 2*(4*a*c*x^6 - 5*a*b*x^3 - 2*a^2)*sqrt
(c*x^6 + b*x^3 + a))/(a*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}{x^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**7,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**7, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^7, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^7,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^7, x)
```

3.209 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}} dx$

Optimal. Leaf size=163

$$-\frac{(2ab+(b^2+8ac)x^3)\sqrt{a+bx^3+cx^6}}{24ax^6}-\frac{(a+bx^3+cx^6)^{3/2}}{9x^9}+\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}}$$

```
[Out] -1/9*(c*x^6+b*x^3+a)^(3/2)/x^9+1/48*b*(-12*a*c+b^2)*arctanh(1/2*(b*x^3+2*a)
/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)+1/3*c^(3/2)*arctanh(1/2*(2*c*x^3+b)
/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))-1/24*(2*a*b+(8*a*c+b^2)*x^3)*(c*x^6+b*x^3+a
)^(1/2)/a/x^6
```

Rubi [A]
time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1371, 746, 824, 857, 635, 212, 738}

$$\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{3/2}}-\frac{(x^3(8ac+b^2)+2ab)\sqrt{a+bx^3+cx^6}}{24ax^6}+\frac{1}{3}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)-\frac{(a+bx^3+cx^6)^{3/2}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] -1/24*((2*a*b + (b^2 + 8*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a*x^6) - (a +
b*x^3 + c*x^6)^(3/2)/(9*x^9) + (b*(b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*S
qrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(48*a^(3/2)) + (c^(3/2)*ArcTanh[(b + 2*c*
x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{10}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^4}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3+cx^6)^{3/2}}{9x^9} + \frac{1}{6}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right)$$

$$= -\frac{(2ab+(b^2+8ac)\,x^3)\,\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9x^9} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}b(b^2}{x\sqrt{a}}\right.}{}$$

$$= -\frac{(2ab+(b^2+8ac)\,x^3)\,\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9x^9} + \frac{1}{3}c^2\text{Subst}\left(\int\right.$$

$$= -\frac{(2ab+(b^2+8ac)\,x^3)\,\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9x^9} + \frac{1}{3}(2c^2)\,\text{Subst}\left(\right.$$

$$= -\frac{(2ab+(b^2+8ac)\,x^3)\,\sqrt{a+bx^3+cx^6}}{24ax^6} - \frac{(a+bx^3+cx^6)^{3/2}}{9x^9} + \frac{b(b^2-12ac)\,\text{ta}}{}$$

Mathematica [A]
time = 0.70, size = 148, normalized size = 0.91

$$\frac{\sqrt{a+bx^3+cx^6}\,(-8a^2-14abx^3-3b^2x^6-32acx^6)}{72ax^9} + \frac{(b^3-12abc)\tanh^{-1}\left(\frac{-\sqrt{c}\,x^3+\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{3/2}} - \frac{1}{3}c^{3/2}\log\left(b+2cx^3-2\sqrt{c}\,\sqrt{a+bx^3+cx^6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^10,x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 - 14*a*b*x^3 - 3*b^2*x^6 - 32*a*c*x^6))/(7
2*a*x^9) + ((b^3 - 12*a*b*c)*ArcTanh[(-(Sqrt[c]*x^3) + Sqrt[a + b*x^3 + c*x
^6])/Sqrt[a]])/(24*a^(3/2)) - (c^(3/2)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a +
 b*x^3 + c*x^6]])/3
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)
```

[Out] int((c*x^6+b*x^3+a)^(3/2)/x^10,x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.49, size = 771, normalized size = 4.73

$$\left[\frac{48\,a^{2}c^{\frac{3}{2}}x^{9}\log\left(-8\,c^{2}x^{6}-8\,bcx^{3}-b^{2}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{c}-4\,ac\right)-3\,(b^{3}-12\,abc)\sqrt{a}\,x^{9}\log\left(-\frac{(b^{2}+4\,ac)x^{6}+8\,abx^{3}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(bx^{3}+2\,a)\sqrt{a}+8\,a^{2}}{x^{6}}\right)-4\,\left((3\,ab^{2}+32\,a^{2}c)x^{6}+14\,a^{2}bx^{3}+8\,a^{3}\right)\sqrt{cx^{6}+bx^{3}+a}}{288\,a^{2}x^{9}},\ -\frac{96\,a^{2}\sqrt{-c}\,cx^{9}\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{-c}}{2\,(c^{2}x^{6}+bcx^{3}+ac)}\right)+3\,(b^{3}-12\,abc)\sqrt{a}\,x^{9}\log\left(-\frac{(b^{2}+4\,ac)x^{6}+8\,abx^{3}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(bx^{3}+2\,a)\sqrt{a}+8\,a^{2}}{x^{6}}\right)+4\,\left((3\,ab^{2}+32\,a^{2}c)x^{6}+14\,a^{2}bx^{3}+8\,a^{3}\right)\sqrt{cx^{6}+bx^{3}+a}}{288\,a^{2}x^{9}},\ \frac{24\,a^{2}c^{\frac{3}{2}}x^{9}\log\left(-8\,c^{2}x^{6}-8\,bcx^{3}-b^{2}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{c}-4\,ac\right)-3\,(b^{3}-12\,abc)\sqrt{-a}\,x^{9}\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(bx^{3}+2\,a)\sqrt{-a}}{2\,(acx^{6}+abx^{3}+a^{2})}\right)-2\,\left((3\,ab^{2}+32\,a^{2}c)x^{6}+14\,a^{2}bx^{3}+8\,a^{3}\right)\sqrt{cx^{6}+bx^{3}+a}}{144\,a^{2}x^{9}},\ -\frac{48\,a^{2}\sqrt{-c}\,cx^{9}\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{-c}}{2\,(c^{2}x^{6}+bcx^{3}+ac)}\right)+3\,(b^{3}-12\,abc)\sqrt{-a}\,x^{9}\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(bx^{3}+2\,a)\sqrt{-a}}{2\,(acx^{6}+abx^{3}+a^{2})}\right)+2\,\left((3\,ab^{2}+32\,a^{2}c)x^{6}+14\,a^{2}bx^{3}+8\,a^{3}\right)\sqrt{cx^{6}+bx^{3}+a}}{144\,a^{2}x^{9}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/288*(48*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6
+ b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(a)*x^
9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8
*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/288*(96*a^2*sqrt(-c)*c*x^9*arc
tan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 +
 a*c)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3
 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((3*a*
b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x
^9), 1/144*(24*a^2*c^(3/2)*x^9*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*
x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 3*(b^3 - 12*a*b*c)*sqrt(-
a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 +
 a*b*x^3 + a^2)) - 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14*a^2*b*x^3 + 8*a^3)*sqrt
(c*x^6 + b*x^3 + a))/(a^2*x^9), -1/144*(48*a^2*sqrt(-c)*c*x^9*arctan(1/2*sq
rt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 3
*(b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2 + 32*a^2*c)*x^6 + 14
*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^2*x^9)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**10,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**10, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^10, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^10,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^10, x)
```

3.210 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=133

$$\frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}}-\frac{(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}$$

```
[Out] -1/24*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a/x^12-1/128*(-4*a*c+b^2)^2*arctanh
(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)+1/64*(-4*a*c+b^2)*(
b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^2/x^6
```

Rubi [A]
time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1371, 734, 738, 212}

$$-\frac{(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{128a^{5/2}}+\frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6}-\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] ((b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(64*a^2*x^6) - ((2*a
+ b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(24*a*x^12) - ((b^2 - 4*a*c)^2*ArcTanh[
(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(128*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{13}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^5}\,dx, x, x^3\right)$$

$$= -\frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx, x, x^3\right.}{16a}$$

$$= \frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} + \frac{(b^2-}{}$$

$$= \frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2-}{}$$

$$= \frac{(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{64a^2x^6} - \frac{(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{24ax^{12}} - \frac{(b^2-}{}$$

Mathematica [A]
time = 0.64, size = 118, normalized size = 0.89

$$-\frac{(2a+bx^3)\sqrt{a+bx^3+cx^6}\,(8a^2+8abx^3-3b^2x^6+20acx^6)}{192a^2x^{12}} + \frac{(b^2-4ac)^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{64a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^13,x]
```

```
[Out] -1/192*((2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6]*(8*a^2 + 8*a*b*x^3 - 3*b^2*x^
6 + 20*a*c*x^6))/(a^2*x^12) + ((b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]*x^3 - Sqrt[
a + b*x^3 + c*x^6])/Sqrt[a]])/(64*a^(5/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^13,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.42, size = 319, normalized size = 2.40

$$\left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{a}\,x^{12}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,((3\,ab^3-20\,a^2bc)x^9-24\,a^3bx^3-2\,(a^2b^2+20\,a^3c)x^6-16\,a^4)\sqrt{cx^6+bx^3+a}}{768\,a^3x^{12}}, \frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-a}\,x^{12}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((3\,ab^3-20\,a^2bc)x^9-24\,a^3bx^3-2\,(a^2b^2+20\,a^3c)x^6-16\,a^4)\sqrt{cx^6+bx^3+a}}{384\,a^3x^{12}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*a*c)*x
^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x
^6) + 4*((3*a*b^3 - 20*a^2*b*c)*x^9 - 24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)
*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^3*x^12), 1/384*(3*(b^4 - 8*a*b^2
*c + 16*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^3 - 20*a^2*b*c)*x^9 -
24*a^3*b*x^3 - 2*(a^2*b^2 + 20*a^3*c)*x^6 - 16*a^4)*sqrt(c*x^6 + b*x^3 + a)
)/(a^3*x^12)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}+cx^{6}\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**13,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**13, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^13, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^13,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^13, x)
```

3.211 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=162

$$-\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}}-\frac{(a+bx^3+cx^6)^{5/2}}{15ax^{15}}+\frac{b(b^2-4ac)^2}{}$$

```
[Out] 1/48*b*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^2/x^12-1/15*(c*x^6+b*x^3+a)^(5/2
)/a/x^15+1/256*b*(-4*a*c+b^2)^2*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^
3+a)^(1/2))/a^(7/2)-1/128*b*(-4*a*c+b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/
a^3/x^6
```

Rubi [A]
time = 0.10, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 744, 734, 738, 212}

$$\frac{b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{256a^{7/2}}-\frac{b(b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{128a^3x^6}+\frac{b(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{48a^2x^{12}}-\frac{(a+bx^3+cx^6)^{5/2}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] -1/128*(b*(b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(a^3*x^6) +
(b*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(48*a^2*x^12) - (a + b*x^3 + c*
x^6)^(5/2)/(15*a*x^15) + (b*(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a
]*Sqrt[a + b*x^3 + c*x^6])])/(256*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{16}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^6}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3+cx^6\right)^{5/2}}{15ax^{15}} - \frac{b\text{Subst}\left(\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^5}\,dx, x, x^3\right)}{6a}$$

$$= \frac{b(2a+bx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}} - \frac{\left(a+bx^3+cx^6\right)^{5/2}}{15ax^{15}} + \frac{\left(b(b^2-4ac)\right)\text{Subst}\left(\int \right.}{32}$$

$$= -\frac{b(b^2-4ac)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b(2a+bx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}} - (a$$

$$= -\frac{b(b^2-4ac)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b(2a+bx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}} - (a$$

$$= -\frac{b(b^2-4ac)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{128a^3x^6} + \frac{b(2a+bx^3)\left(a+bx^3+cx^6\right)^{3/2}}{48a^2x^{12}} - (a$$

Mathematica [A]

time = 0.95, size = 160, normalized size = 0.99

$$-\frac{\frac{\sqrt{a}\sqrt{a+bx^3+cx^6}\,(128a^4+15b^4x^{12}-10ab^2x^9(b+10cx^3)+16a^3(11bx^3+16cx^6)+8a^2x^6(b^2+7bcx^3+16c^2x^6))}{x^{15}}-15b(b^2-4ac)^2\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{1920a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^16,x]
```

```
[Out] (-((Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(128*a^4 + 15*b^4*x^12 - 10*a*b^2*x^9*(
b + 10*c*x^3) + 16*a^3*(11*b*x^3 + 16*c*x^6) + 8*a^2*x^6*(b^2 + 7*b*c*x^3 +
 16*c^2*x^6)))/x^15) - 15*b*(b^2 - 4*a*c)^2*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a +
 b*x^3 + c*x^6])/Sqrt[a]])/(1920*a^(7/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^16,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.47, size = 383, normalized size = 2.36

$$\left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{a}\,x^{15}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,((15\,ab^4-100\,a^2b^2c+128\,a^3c^2)x^{12}-2\,(5\,a^2b^3-28\,a^3bc)x^9+176\,a^4bx^3+8\,(a^3b^2+32\,a^4c)x^6+128\,a^5)\sqrt{cx^6+bx^3+a}}{7680\,a^4x^{15}},\ -\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{-a}\,x^{15}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((15\,ab^4-100\,a^2b^2c+128\,a^3c^2)x^{12}-2\,(5\,a^2b^3-28\,a^3bc)x^9+176\,a^4bx^3+8\,(a^3b^2+32\,a^4c)x^6+128\,a^5)\sqrt{cx^6+bx^3+a}}{3840\,a^4x^{15}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^15*log(-((b^2 + 4*a*
c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^
2)/x^6) - 4*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*x^12 - 2*(5*a^2*b^3 -
 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^6 + 128*a^5)*sq
rt(c*x^6 + b*x^3 + a))/(a^4*x^15), -1/3840*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*sqrt(-a)*x^15*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a
)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^4 - 100*a^2*b^2*c + 128*a^3*c^2)*
x^12 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^9 + 176*a^4*b*x^3 + 8*(a^3*b^2 + 32*a^4
*c)*x^6 + 128*a^5)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^15)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3 + cx^6)^{\frac{3}{2}}}{x^{16}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**16,x)
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**16, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^16,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^16, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{x^{16}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^16,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^16, x)
```

3.212 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}} \, dx$

Optimal. Leaf size=216

$$\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6}-\frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}}-\frac{(a+bx^3+cx}{18ax^{18}}$$

```
[Out] -1/576*(-4*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^3/x^12-1/18*(c*x^
6+b*x^3+a)^(5/2)/a/x^18+7/180*b*(c*x^6+b*x^3+a)^(5/2)/a^2/x^15-1/3072*(-4*a
*c+b^2)^2*(-4*a*c+7*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1
/2))/a^(9/2)+1/1536*(-4*a*c+b^2)*(-4*a*c+7*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)
^(1/2)/a^4/x^6
```

Rubi [A]
time = 0.14, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 758, 820, 734, 738, 212}

$$-\frac{(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3072a^{9/2}}+\frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6}-\frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}}+\frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}}-\frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(1536
*a^4*x^6) - ((7*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2))/(576*
a^3*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(18*a*x^18) + (7*b*(a + b*x^3 + c*x^6
)^(5/2))/(180*a^2*x^15) - ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b
*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(3072*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{19}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^7}\,dx, x, x^3\right)$$

$$= -\frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{7b}{2}+cx\right)(a+bx+cx^2)^{3/2}}{x^6}\,dx, x, x^3\right)}{18a}$$

$$= -\frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}} + \frac{7b(a+bx^3+cx^6)^{5/2}}{180a^2x^{15}} + \frac{(7b^2-4ac)\,\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^5}\right.}{72a^2}$$

$$= -\frac{(7b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{576a^3x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{18ax^{18}} + \frac{7b(a+bx^3+}{180a^2x^1}$$

$$= \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{(7b^2-4ac)(2a+bx^3)(a+}{576a^3x^{12}}$$

$$= \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{(7b^2-4ac)(2a+bx^3)(a+}{576a^3x^{12}}$$

$$= \frac{(b^2-4ac)(7b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1536a^4x^6} - \frac{(7b^2-4ac)(2a+bx^3)(a+}{576a^3x^{12}}$$

Mathematica [A]
time = 1.32, size = 201, normalized size = 0.93

$$\frac{-\frac{\sqrt{a}\sqrt{a+bx^3+cx^6}\,(1280a^5-105b^5x^{15}+10ab^3x^{12}(7b+76cx^3)+64a^4(26bx^3+35cx^6)+48a^3x^6(b^2+6bcx^3+10c^2x^6)-8a^2bx^9(7b^2+54bcx^3+162c^2x^6))}{x^{18}}+15(b^2-4ac)^2(7b^2-4ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{23040a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^19,x]
```

```
[Out] (-((Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(1280*a^5 - 105*b^5*x^15 + 10*a*b^3*x^1
2*(7*b + 76*c*x^3) + 64*a^4*(26*b*x^3 + 35*c*x^6) + 48*a^3*x^6*(b^2 + 6*b*c
*x^3 + 10*c^2*x^6) - 8*a^2*b*x^9*(7*b^2 + 54*b*c*x^3 + 162*c^2*x^6)))/x^18)
 + 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3
 + c*x^6])/Sqrt[a]])/(23040*a^(9/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^19,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.55, size = 473, normalized size = 2.19

$$\left[-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{a}\,x^{18}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,((105\,ab^5-760\,a^2b^3c+1296\,a^3bc^2)x^{15}-2\,(35\,a^2b^4-216\,a^3b^2c+240\,a^4c^2)x^{12}+8\,(7\,a^3b^3-36\,a^4bc)x^9-1664\,a^5bx^3-16\,(3\,a^4b^2+140\,a^5c)x^6-1280\,a^6)\sqrt{cx^6+bx^3+a}}{92160\,a^5x^{18}},\ \frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{-a}\,x^{18}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((105\,ab^5-760\,a^2b^3c+1296\,a^3bc^2)x^{15}-2\,(35\,a^2b^4-216\,a^3b^2c+240\,a^4c^2)x^{12}+8\,(7\,a^3b^3-36\,a^4bc)x^9-1664\,a^5bx^3-16\,(3\,a^4b^2+140\,a^5c)x^6-1280\,a^6)\sqrt{cx^6+bx^3+a}}{46080\,a^5x^{18}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="fricas")
```

```
[Out] [-1/92160*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(a)*x
^18*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
+ 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((105*a*b^5 - 760*a^2*b^3*c + 1296*a^3*b*c
^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a^4*c^2)*x^12 + 8*(7*a^3*b^3
 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*b^2 + 140*a^5*c)*x^6 - 1280
*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18), 1/46080*(15*(7*b^6 - 60*a*b^4*c
+ 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-a)*x^18*arctan(1/2*sqrt(c*x^6 + b*x^3
 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((105*a*b^5 - 7
60*a^2*b^3*c + 1296*a^3*b*c^2)*x^15 - 2*(35*a^2*b^4 - 216*a^3*b^2*c + 240*a
^4*c^2)*x^12 + 8*(7*a^3*b^3 - 36*a^4*b*c)*x^9 - 1664*a^5*b*x^3 - 16*(3*a^4*
b^2 + 140*a^5*c)*x^6 - 1280*a^6)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^18)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}{x^{19}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**19,x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**19, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^19,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^19, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^{3/2}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^19,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^19, x)
```

3.213 $\int \frac{\left(a+bx^3+cx^6\right)^{3/2}}{x^{22}}\,dx$

Optimal. Leaf size=255

$$-\frac{b(b^2-4ac)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6}+\frac{b(3b^2-4ac)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384a^4x^{12}}-\frac{(a+bx^3}{21}$$

```
[Out] 1/384*b*(-4*a*c+3*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(3/2)/a^4/x^12-1/21*(c*x
^6+b*x^3+a)^(5/2)/a/x^21+1/28*b*(c*x^6+b*x^3+a)^(5/2)/a^2/x^18-1/840*(-16*a
*c+21*b^2)*(c*x^6+b*x^3+a)^(5/2)/a^3/x^15+1/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3
*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(11/2)-1/102
4*b*(-4*a*c+b^2)*(-4*a*c+3*b^2)*(b*x^3+2*a)*(c*x^6+b*x^3+a)^(1/2)/a^5/x^6
```

Rubi [A]
time = 0.21, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1371, 758, 848, 820, 734, 738, 212}

$$\frac{b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2048a^{11/2}}-\frac{b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(2a+bx^3\right)\sqrt{a+bx^3+cx^6}}{1024a^5x^6}+\frac{b\left(3b^2-4ac\right)\left(2a+bx^3\right)\left(a+bx^3+cx^6\right)^{3/2}}{384a^4x^{12}}-\frac{\left(21b^2-16ac\right)\left(a+bx^3+cx^6\right)^{5/2}}{840a^3x^{15}}+\frac{b\left(a+bx^3+cx^6\right)^{5/2}}{28a^2x^{18}}-\frac{\left(a+bx^3+cx^6\right)^{5/2}}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

```
[Out] -1/1024*(b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(2*a + b*x^3)*Sqrt[a + b*x^3 + c*x
^6])/(a^5*x^6) + (b*(3*b^2 - 4*a*c)*(2*a + b*x^3)*(a + b*x^3 + c*x^6)^(3/2)
)/(384*a^4*x^12) - (a + b*x^3 + c*x^6)^(5/2)/(21*a*x^21) + (b*(a + b*x^3 +
c*x^6)^(5/2))/(28*a^2*x^18) - ((21*b^2 - 16*a*c)*(a + b*x^3 + c*x^6)^(5/2))
/(840*a^3*x^15) + (b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/
(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2048*a^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
```

```
0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
```

```
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^{22}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^8}\,dx,x,x^3\right)$$

$$= -\frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} - \frac{\text{Subst}\left(\int \frac{\left(\frac{9b}{2}+2cx\right)(a+bx+cx^2)^{3/2}}{x^7}\,dx,x,x^3\right)}{21a}$$

$$= -\frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} + \frac{b(a+bx^3+cx^6)^{5/2}}{28a^2x^{18}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{3}{4}(21b^2-16ac)+\frac{9bcx}{2}\right)(a+bx+c}{x^6}\right.}{126a^2}$$

$$= -\frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} + \frac{b(a+bx^3+cx^6)^{5/2}}{28a^2x^{18}} - \frac{(21b^2-16ac)(a+bx^3+cx^6)^{5/2}}{840a^3x^{15}}$$

$$= \frac{b(3b^2-4ac)(2a+bx^3)(a+bx^3+cx^6)^{3/2}}{384a^4x^{12}} - \frac{(a+bx^3+cx^6)^{5/2}}{21ax^{21}} + \frac{b(a+bx^3+c}{28a^2x^{18}}$$

$$= -\frac{b(b^2-4ac)(3b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b(3b^2-4ac)(2a+bx^3)}{384a^4x}$$

$$= -\frac{b(b^2-4ac)(3b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b(3b^2-4ac)(2a+bx^3)}{384a^4x}$$

$$= -\frac{b(b^2-4ac)(3b^2-4ac)(2a+bx^3)\sqrt{a+bx^3+cx^6}}{1024a^5x^6} + \frac{b(3b^2-4ac)(2a+bx^3)}{384a^4x}$$

Mathematica [A]
time = 1.54, size = 244, normalized size = 0.96

$$\frac{-\frac{\sqrt{a}\sqrt{a+bx^3+cx^6}\,(5120a^6+315b^6x^{18}-210ab^4x^{15}(b+12cx^3)+256a^5(25bx^3+32cx^6)+64a^4x^6(2b^2+11bcx^3+16c^2x^6)+56a^2b^2x^{12}(3b^2+26bcx^3+98c^2x^6)-16a^3x^9(9b^3+62b^2cx^3+146bc^2x^6+128c^3x^9))}{x^{21}}-105b(b^2-4ac)^2(3b^2-4ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{107520a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^22,x]
```

```
[Out] (-((Sqrt[a]*Sqrt[a + b*x^3 + c*x^6]*(5120*a^6 + 315*b^6*x^18 - 210*a*b^4*x^
15*(b + 12*c*x^3) + 256*a^5*(25*b*x^3 + 32*c*x^6) + 64*a^4*x^6*(2*b^2 + 11*
b*c*x^3 + 16*c^2*x^6) + 56*a^2*b^2*x^12*(3*b^2 + 26*b*c*x^3 + 98*c^2*x^6) -
 16*a^3*x^9*(9*b^3 + 62*b^2*c*x^3 + 146*b*c^2*x^6 + 128*c^3*x^9)))/x^21) -
```

```
105*b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3
 + c*x^6])/Sqrt[a]])/(107520*a^(11/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^{6}+b\,x^{3}+a)^{\frac{3}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

```
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^22,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.65, size = 557, normalized size = 2.18

$$\left[-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{a}\,x^{21}\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,((315\,ab^6-2520\,a^2b^4c+5488\,a^3b^2c^2-2048\,a^4c^3)x^{18}-2\,(105\,a^2b^5-728\,a^3b^3c+1168\,a^4bc^2)x^{15}+8\,(21\,a^3b^4-124\,a^4b^2c+128\,a^5c^2)x^{12}+6400\,a^6bx^3-16\,(9\,a^4b^3-44\,a^5bc)x^9+5120\,a^7+128\,(a^5b^2+64\,a^6c)x^6)\sqrt{cx^6+bx^3+a}}{430080\,a^6x^{21}},\ -\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{-a}\,x^{21}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((315\,ab^6-2520\,a^2b^4c+5488\,a^3b^2c^2-2048\,a^4c^3)x^{18}-2\,(105\,a^2b^5-728\,a^3b^3c+1168\,a^4bc^2)x^{15}+8\,(21\,a^3b^4-124\,a^4b^2c+128\,a^5c^2)x^{12}+6400\,a^6bx^3-16\,(9\,a^4b^3-44\,a^5bc)x^9+5120\,a^7+128\,(a^5b^2+64\,a^6c)x^6)\sqrt{cx^6+bx^3+a}}{215040\,a^6x^{21}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="fricas")
```

```
[Out] [-1/430080*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(a
)*x^21*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x
^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((315*a*b^6 - 2520*a^2*b^4*c + 5488*a^3
*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5 - 728*a^3*b^3*c + 1168*a^4*b
*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 128*a^5*c^2)*x^12 + 6400*a^6*b
*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*a^7 + 128*(a^5*b^2 + 64*a^6*c
)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(a^6*x^21), -1/215040*(105*(3*b^7 - 28*a*b^
5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-a)*x^21*arctan(1/2*sqrt(c*x^6 +
b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((315*a*b^
6 - 2520*a^2*b^4*c + 5488*a^3*b^2*c^2 - 2048*a^4*c^3)*x^18 - 2*(105*a^2*b^5
 - 728*a^3*b^3*c + 1168*a^4*b*c^2)*x^15 + 8*(21*a^3*b^4 - 124*a^4*b^2*c + 1
```

```
28*a^5*c^2)*x^12 + 6400*a^6*b*x^3 - 16*(9*a^4*b^3 - 44*a^5*b*c)*x^9 + 5120*
a^7 + 128*(a^5*b^2 + 64*a^6*c)*x^6)*sqrt(c*x^6 + b*x^3 + a))/(a^6*x^21)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**22,x)
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**22, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^22,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^22, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{x^{22}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^22,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^22, x)
```

3.214 $\int x^3(a+bx^3+cx^6)^{3/2}\,dx$

Optimal. Leaf size=141

$$\frac{ax^4\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{4}{3};-\frac{3}{2},-\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/4*a*x^4*AppellF1(4/3,-3/2,-3/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$\frac{ax^4\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{4}{3};-\frac{3}{2},-\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x^4*Sqrt[a + b*x^3 + c*x^6]*AppellF1[4/3, -3/2, -3/2, 7/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
```

```
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int x^3\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int x^3\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$=\frac{ax^4\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{4}{3};-\frac{3}{2},-\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 453 vs. 2(141) = 282.
time = 10.56, size = 453, normalized size = 3.21

$$\frac{x\left(8\left(-297b^4x^3-81b^3cx^6+3464b^2c^2x^9+5488bc^3x^{12}+2240c^4x^{15}+4a^2c(459b+1280cx^3)+a(-297b^3+2052b^2cx^3+10204bc^2x^6+7360c^3x^9)\right)+216ab(11b^2-68ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+27(55b^4-404ab^2c+640a^2c^2)x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)}{232960c^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(8*(-297*b^4*x^3 - 81*b^3*c*x^6 + 3464*b^2*c^2*x^9 + 5488*b*c^3*x^12 + 2
240*c^4*x^15 + 4*a^2*c*(459*b + 1280*c*x^3) + a*(-297*b^3 + 2052*b^2*c*x^3
+ 10204*b*c^2*x^6 + 7360*c^3*x^9)) + 216*a*b*(11*b^2 - 68*a*c)*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 27*(55
*b^4 - 404*a*b^2*c + 640*a^2*c^2)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(232960*c^2*Sqrt[a + b*x^3 + c*
x^6])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(x^3*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] integral((c*x^9 + b*x^6 + a*x^3)*sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+bx^{3}+cx^{6}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x**3*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x^3*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.215 $\int x(a+bx^3+cx^6)^{3/2}\,dx$

Optimal. Leaf size=141

$$\frac{ax^2\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/2*a*x^2*AppellF1(2/3,-3/2,-3/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1399, 524}

$$\frac{ax^2\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^3 + c*x^6]*AppellF1[2/3, -3/2, -3/2, 5/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[1 + (2*
c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]
)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
```

```
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int x\left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$=\frac{ax^2\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 410 vs. 2(141) = 282.
time = 10.46, size = 410, normalized size = 2.91

$$\frac{x^2\left(10(27ab^2+448a^2c+27b^3x^3+698abcx^3+277b^2cx^6+608ac^2x^6+410bc^2x^9+160c^3x^{12})-270a(b^2-16ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)-27b(7b^2-52ac)x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)}{17600c\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*(10*(27*a*b^2 + 448*a^2*c + 27*b^3*x^3 + 698*a*b*c*x^3 + 277*b^2*c*x^6
 + 608*a*c^2*x^6 + 410*b*c^2*x^9 + 160*c^3*x^12) - 270*a*(b^2 - 16*a*c)*Sqr
t[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5
/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]
 - 27*b*(7*b^2 - 52*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(17600*c*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(x*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] integral((c*x^7 + b*x^4 + a*x)*sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+bx^3+cx^6\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.216 $\int (a+bx^3+cx^6)^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{ax\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] a*x*AppellF1(1/3,-3/2,-3/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+
(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1362, 440}

$$\frac{ax\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^3 + c*x^6]*AppellF1[1/3, -3/2, -3/2, 4/3, (-2*c*x^3)/(b -
 Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[1 + (2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
```

```
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+bx^3+cx^6\right)^{3/2}\,dx=\frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int \left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$
$$=\frac{ax\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 408 vs. 2(136) = 272.
time = 10.43, size = 408, normalized size = 3.00

$$\frac{x\left(8(27ab^2+364a^2c+27b^3x^3+548abcx^3+211b^2cx^6+476ac^2x^6+296bc^2x^9+112c^3x^{12})-216a(b^2-28ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)-27b(5b^2-44ac)x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)}{8960c\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(8*(27*a*b^2 + 364*a^2*c + 27*b^3*x^3 + 548*a*b*c*x^3 + 211*b^2*c*x^6 +
476*a*c^2*x^6 + 296*b*c^2*x^9 + 112*c^3*x^12) - 216*a*(b^2 - 28*a*c)*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] -
27*b*(5*b^2 - 44*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(8960*c*Sqrt[a + b*x^3 + c*x^6])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2),x)
```

[Out] int((c*x^6+b*x^3+a)^(3/2),x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2), x)

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")

[Out] integral((c*x^6 + b*x^3 + a)^(3/2), x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**6+b*x**3+a)**(3/2),x)

[Out] Integral((a + b*x**3 + c*x**6)**(3/2), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")

[Out] integrate((c*x^6 + b*x^3 + a)^(3/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,x^6+b\,x^3+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3 + c*x^6)^(3/2),x)

[Out] int((a + b*x^3 + c*x^6)^(3/2), x)

3.217 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=139

$$-\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -a*AppellF1(-1/3,-3/2,-3/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+
(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x/(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$-\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-1/3, -3/2, -3/2, 2/3, (-2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[1 + (2*c
*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
```

```
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^2}\,dx = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^2}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{1}{3};-\frac{3}{2},-\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 379 vs. 2(139) = 278.
time = 10.33, size = 379, normalized size = 2.73

$$\frac{10(-80a^2-61abx^3+19b^2x^6-70acx^6+29bcx^9+10c^2x^{12})+810abx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+27(b^2+20ac)x^6\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{800x\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^2,x]
```

```
[Out] (10*(-80*a^2 - 61*a*b*x^3 + 19*b^2*x^6 - 70*a*c*x^6 + 29*b*c*x^9 + 10*c^2*x
^12) + 810*a*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*App
ellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b
 + Sqrt[b^2 - 4*a*c])] + 27*(b^2 + 20*a*c)*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(800*x*Sqrt[a + b*x^3 +
 c*x^6])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3 + cx^6)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**2,x)
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^2,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^2, x)
```

3.218 $\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3} dx$

Optimal. Leaf size=141

$$-\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -1/2*a*AppellF1(-2/3,-3/2,-3/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3
/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/x^2/(1+2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$-\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[a + b*x^3 + c*x^6]*AppellF1[-2/3, -3/2, -3/2, 1/3, (-2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[1 +
 (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
```

```
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^{3/2}}{x^3}\,dx = \frac{\left(a\sqrt{a+bx^3+cx^6}\right)\int \frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^3}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{a\sqrt{a+bx^3+cx^6}\,F_1\left(-\frac{2}{3};-\frac{3}{2},-\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 379 vs. 2(141) = 282.
time = 10.33, size = 379, normalized size = 2.69

$$\frac{8(-28a^2-11abx^3+17b^2x^6-20acx^6+25bcx^9+8c^2x^{12})+648abx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+27(b^2+8ac)x^6\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{448x^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(3/2)/x^3,x]
```

```
[Out] (8*(-28*a^2 - 11*a*b*x^3 + 17*b^2*x^6 - 20*a*c*x^6 + 25*b*c*x^9 + 8*c^2*x^1
2) + 648*a*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
 Sqrt[b^2 - 4*a*c])] + 27*(b^2 + 8*a*c)*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
+ Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(448*x^2*Sqrt[a + b*x^3 +
c*x^6])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
[Out] int((c*x^6+b*x^3+a)^(3/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^3+cx^6)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**(3/2)/x**3,x)
[Out] Integral((a + b*x**3 + c*x**6)**(3/2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^(3/2)/x^3,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^(3/2)/x^3,x)
[Out] int((a + b*x^3 + c*x^6)^(3/2)/x^3, x)
```

3.219 $\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=171

$$-\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2}+\frac{x^9\sqrt{a+bx^3+cx^6}}{12c}-\frac{(5b(21b^2-44ac)-2c(35b^2-36ac)\,x^3)\sqrt{a+bx^3+cx^6}}{576c^4}+\frac{(3$$

```
[Out] 1/384*(48*a^2*c^2-120*a*b^2*c+35*b^4)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^
6+b*x^3+a)^(1/2))/c^(9/2)-7/72*b*x^6*(c*x^6+b*x^3+a)^(1/2)/c^2+1/12*x^9*(c*
x^6+b*x^3+a)^(1/2)/c-1/576*(5*b*(-44*a*c+21*b^2)-2*c*(-36*a*c+35*b^2)*x^3)*
(c*x^6+b*x^3+a)^(1/2)/c^4
```

Rubi [A]
time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 756, 846, 793, 635, 212}

$$\frac{(48a^2c^2-120ab^2c+35b^4)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{384c^{9/2}}-\frac{(5b(21b^2-44ac)-2cx^3(35b^2-36ac))\sqrt{a+bx^3+cx^6}}{576c^4}-\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2}+\frac{x^9\sqrt{a+bx^3+cx^6}}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (-7*b*x^6*Sqrt[a + b*x^3 + c*x^6])/(72*c^2) + (x^9*Sqrt[a + b*x^3 + c*x^6])
/(12*c) - ((5*b*(21*b^2 - 44*a*c) - 2*c*(35*b^2 - 36*a*c)*x^3)*Sqrt[a + b*x
^3 + c*x^6])/(576*c^4) + ((35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*ArcTanh[(b +
2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(384*c^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
```

```
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{14}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} + \frac{\text{Subst}\left(\int \frac{x^2\left(-3a-\frac{7bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{12c}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} + \frac{\text{Subst}\left(\int \frac{x\left(7ab+\frac{1}{4}(35b^2-36ac)x\right)}{\sqrt{a+bx+cx^2}}\,dx, x\right.}{36c^2}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{(5b(21b^2-44ac)-2c(35b^2-36}{576c^4}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{(5b(21b^2-44ac)-2c(35b^2-36}{576c^4}$$

$$= -\frac{7bx^6\sqrt{a+bx^3+cx^6}}{72c^2} + \frac{x^9\sqrt{a+bx^3+cx^6}}{12c} - \frac{(5b(21b^2-44ac)-2c(35b^2-36}{576c^4}$$

Mathematica [A]
time = 0.26, size = 138, normalized size = 0.81

$$\frac{\sqrt{a+bx^3+cx^6}\,(-105b^3+220abc+70b^2cx^3-72ac^2x^3-56bc^2x^6+48c^3x^9)}{576c^4}+\frac{(-35b^4+120ab^2c-48a^2c^2)\log\left(bc^4+2c^5x^3-2c^{9/2}\sqrt{a+bx^3+cx^6}\right)}{384c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-105*b^3 + 220*a*b*c + 70*b^2*c*x^3 - 72*a*c^2*x^
3 - 56*b*c^2*x^6 + 48*c^3*x^9))/(576*c^4) + ((-35*b^4 + 120*a*b^2*c - 48*a^
2*c^2)*Log[b*c^4 + 2*c^5*x^3 - 2*c^(9/2)*Sqrt[a + b*x^3 + c*x^6]])/(384*c^(
9/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^14/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 303, normalized size = 1.77

$$\left[\frac{3\,{\left(35\,b^{4} - 120\,a b^{2} c + 48\,a^{2} c^{2}\right)} \sqrt{c} \log\left(-8\,c^{2} x^{6} - 8\,b c x^{3} - b^{2} - 4\,\sqrt{c x^{6} + b x^{3} + a} {\left(2\,c x^{3} + b\right)} \sqrt{c} - 4\,a c\right) + 4\,{\left(48\,c^{4} x^{9} - 56\,b c^{3} x^{6} - 105\,b^{3} c + 220\,a b c^{2} + 2\,{\left(35\,b^{2} c^{2} - 36\,a c^{3}\right)} x^{3}\right)} \sqrt{c x^{6} + b x^{3} + a}}{2304\,c^{5}}, -\frac{3\,{\left(35\,b^{4} - 120\,a b^{2} c + 48\,a^{2} c^{2}\right)} \sqrt{-c} \arctan\left(\frac{\sqrt{c x^{6} + b x^{3} + a} {\left(2\,c x^{3} + b\right)} \sqrt{-c}}{2\,{\left(c^{2} x^{6} + b c x^{3} + a c\right)}}\right) - 2\,{\left(48\,c^{4} x^{9} - 56\,b c^{3} x^{6} - 105\,b^{3} c + 220\,a b c^{2} + 2\,{\left(35\,b^{2} c^{2} - 36\,a c^{3}\right)} x^{3}\right)} \sqrt{c x^{6} + b x^{3} + a}}{1152\,c^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(c)*log(-8*c^2*x^6 - 8*b
*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4
*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*
a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^5, -1/1152*(3*(35*b^4 - 120*a*b^2*c
+ 48*a^2*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqr
t(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*(48*c^4*x^9 - 56*b*c^3*x^6 - 105*b^3*c
 + 220*a*b*c^2 + 2*(35*b^2*c^2 - 36*a*c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/c^
5]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt{a + b x^{3} + c x^{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**14/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(x^14/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.220 $\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=121

$$\frac{x^6\sqrt{a+bx^3+cx^6}}{9c}+\frac{(15b^2-16ac-10bcx^3)\sqrt{a+bx^3+cx^6}}{72c^3}-\frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}$$

```
[Out] -1/48*b*(-12*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/
2))/c^(7/2)+1/9*x^6*(c*x^6+b*x^3+a)^(1/2)/c+1/72*(-10*b*c*x^3-16*a*c+15*b^2
)*(c*x^6+b*x^3+a)^(1/2)/c^3
```

Rubi [A]

time = 0.07, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 756, 793, 635, 212}

$$-\frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{48c^{7/2}}+\frac{(-16ac+15b^2-10bcx^3)\sqrt{a+bx^3+cx^6}}{72c^3}+\frac{x^6\sqrt{a+bx^3+cx^6}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^6*Sqrt[a + b*x^3 + c*x^6])/(9*c) + ((15*b^2 - 16*a*c - 10*b*c*x^3)*Sqrt[
a + b*x^3 + c*x^6])/(72*c^3) - (b*(5*b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^3)/(2
*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(48*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
```

```
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{\text{Subst}\left(\int \frac{x\left(-2a-\frac{5bx}{2}\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{9c}$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{(15b^2-16ac-10bcx^3)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{(b(5b^2-12ac))}{}$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{(15b^2-16ac-10bcx^3)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{(b(5b^2-12ac))}{}$$

$$= \frac{x^6\sqrt{a+bx^3+cx^6}}{9c} + \frac{(15b^2-16ac-10bcx^3)\sqrt{a+bx^3+cx^6}}{72c^3} - \frac{b(5b^2-12ac)\,\mathrm{t}}{}$$

Mathematica [A]
time = 0.22, size = 101, normalized size = 0.83

$$\frac{\sqrt{a+bx^3+cx^6}\,(15b^2-16ac-10bcx^3+8c^2x^6)}{72c^3} + \frac{(5b^3-12abc)\log\left(b+2cx^3-2\sqrt{c}\,\sqrt{a+bx^3+cx^6}\right)}{48c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(15*b^2 - 16*a*c - 10*b*c*x^3 + 8*c^2*x^6))/(72*c^
3) + ((5*b^3 - 12*a*b*c)*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6
]])/(48*c^(7/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt{c\,x^{6}+b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^11/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 241, normalized size = 1.99

$$\left[-\frac{3\,(5\,b^{3}-12\,abc)\sqrt{c}\,\log\left(-8\,c^{2}x^{6}-8\,bcx^{3}-b^{2}-4\,\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{c}-4\,ac\right)-4\,(8\,c^{3}x^{6}-10\,bc^{2}x^{3}+15\,b^{2}c-16\,ac^{2})\sqrt{cx^{6}+bx^{3}+a}}{288\,c^{4}},\ \frac{3\,(5\,b^{3}-12\,abc)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,(2\,cx^{3}+b)\sqrt{-c}}{2\,(c^{2}x^{6}+bcx^{3}+ac)}\right)+2\,(8\,c^{3}x^{6}-10\,bc^{2}x^{3}+15\,b^{2}c-16\,ac^{2})\sqrt{cx^{6}+bx^{3}+a}}{144\,c^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*
sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*(8*c^3*x^6 - 10*
b*c^2*x^3 + 15*b^2*c - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4, 1/144*(3*(5*
b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*s
qrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) + 2*(8*c^3*x^6 - 10*b*c^2*x^3 + 15*b^2*c
 - 16*a*c^2)*sqrt(c*x^6 + b*x^3 + a))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(x**11/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(x^11/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int(x^11/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.221 $\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=104

$$-\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}+\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\,\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}$$

```
[Out] 1/24*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/
c^(5/2)-1/4*b*(c*x^6+b*x^3+a)^(1/2)/c^2+1/6*x^3*(c*x^6+b*x^3+a)^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 756, 654, 635, 212}

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\,\sqrt{a+bx^3+cx^6}}\right)}{24c^{5/2}}-\frac{b\sqrt{a+bx^3+cx^6}}{4c^2}+\frac{x^3\sqrt{a+bx^3+cx^6}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] -1/4*(b*Sqrt[a + b*x^3 + c*x^6])/c^2 + (x^3*Sqrt[a + b*x^3 + c*x^6])/(6*c)
+ ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]
)])/(24*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{\text{Subst}\left(\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6c}\\
&= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\right.}{24c^2}\\
&= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \right.}{12c^2}\\
&= -\frac{b\sqrt{a+bx^3+cx^6}}{4c^2} + \frac{x^3\sqrt{a+bx^3+cx^6}}{6c} + \frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3}}\right.}{24c^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.18, size = 91, normalized size = 0.88

$$
\frac{(-3b+2cx^3)\sqrt{a+bx^3+cx^6}}{12c^2} + \frac{(-3b^2+4ac)\log\left(bc^2+2c^3x^3-2c^{5/2}\sqrt{a+bx^3+cx^6}\right)}{24c^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((-3*b + 2*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c^2) + ((-3*b^2 + 4*a*c)*Log
[b*c^2 + 2*c^3*x^3 - 2*c^(5/2)*Sqrt[a + b*x^3 + c*x^6]])/(24*c^(5/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\sqrt{c\,x^{6}+b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^8/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 203, normalized size = 1.95

$$\left[-\frac{\left(3\,b^{2}-4\,ac\right)\sqrt{c}\,\log\left(-8\,c^{2}x^{6}-8\,bcx^{3}-b^{2}+4\,\sqrt{cx^{6}+bx^{3}+a}\,\left(2\,cx^{3}+b\right)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^{6}+bx^{3}+a}\,\left(2\,c^{2}x^{3}-3\,bc\right)}{48\,c^{3}},\,-\frac{\left(3\,b^{2}-4\,ac\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,\left(2\,cx^{3}+b\right)\sqrt{-c}}{2\,\left(c^{2}x^{6}+bcx^{3}+ac\right)}\right)-2\,\sqrt{cx^{6}+bx^{3}+a}\,\left(2\,c^{2}x^{3}-3\,bc\right)}{24\,c^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*b^2 - 4*a*c)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c
*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a
)*(2*c^2*x^3 - 3*b*c))/c^3, -1/24*((3*b^2 - 4*a*c)*sqrt(-c)*arctan(1/2*sqrt
(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c)) - 2*s
qrt(c*x^6 + b*x^3 + a)*(2*c^2*x^3 - 3*b*c))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\sqrt{a + bx^{3} + cx^{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**8/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(x^8/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.222 $\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a+bx^3+cx^6}}{3c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

```
[Out] -1/6*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(3/2)+1/3*(
c*x^6+b*x^3+a)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1371, 654, 635, 212}

$$\frac{\sqrt{a+bx^3+cx^6}}{3c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) - (b*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a
+ b*x^3 + c*x^6])])/(6*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\ &= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6c}\\ &= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3c}\\ &= \frac{\sqrt{a+bx^3+cx^6}}{3c} - \frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{6c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 70, normalized size = 1.03

$$\frac{\sqrt{a+bx^3+cx^6}}{3c} + \frac{b\log\left(bc+2c^2x^3-2c^{3/2}\sqrt{a+bx^3+cx^6}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] Sqrt[a + b*x^3 + c*x^6]/(3*c) + (b*Log[b*c + 2*c^2*x^3 - 2*c^(3/2)*Sqrt[a +
 b*x^3 + c*x^6]])/(6*c^(3/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x^5/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.43, size = 161, normalized size = 2.37

$$\left[\frac{b\sqrt{c}\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{c}-4\,ac\right)+4\,\sqrt{cx^6+bx^3+a}\,c}{12\,c^2},\frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)+2\,\sqrt{cx^6+bx^3+a}\,c}{6\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(b*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 +
a)*(2*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^6 + b*x^3 + a)*c)/c^2, 1/6*(
b*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x
^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*c)/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [A]
time = 3.87, size = 61, normalized size = 0.90

$$\frac{b\log\left(\left|-2\left(\sqrt{c}\,x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c}-b\right|\right)}{6\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^6+bx^3+a}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*b*log(abs(-2*(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(3
/2) + 1/3*sqrt(c*x^6 + b*x^3 + a)/c
```

Mupad [B]
time = 1.49, size = 55, normalized size = 0.81

$$\frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,c}-\frac{b\,\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{6\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] (a + b*x^3 + c*x^6)^(1/2)/(3*c) - (b*log((a + b*x^3 + c*x^6)^(1/2) + (b/2 +
 c*x^3)/c^(1/2)))/(6*c^(3/2))
```

3.223 $\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

```
[Out] 1/3*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1366, 635, 212}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= \frac{2}{3}\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^3}{\sqrt{a+bx^3+cx^6}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{c}}$$

Mathematica [A]
time = 0.08, size = 41, normalized size = 0.95

$$-\frac{\log\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)}{3\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^3 + c*x^6],x]
[Out] -1/3*Log[b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6]]/Sqrt[c]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.36, size = 118, normalized size = 2.74

$$\left[\frac{\log\left(-8\,c^2x^6 - 8\,bcx^3 - b^2 - 4\,\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{c} - 4\,ac\right)}{6\,\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)}{3\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3
+ b)*sqrt(c) - 4*a*c)/sqrt(c), -1/3*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3
+ a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*x^3 + a*c))/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(33) = 66$.
time = 3.66, size = 76, normalized size = 1.77

$$\frac{1}{12}\,\sqrt{cx^6+bx^3+a}\,\left(2\,x^3+\frac{b}{c}\right) + \frac{(b^2-4\,ac)\log\left(\left|-2\left(\sqrt{c}\,x^3-\sqrt{cx^6+bx^3+a}\right)\sqrt{c} - b\right|\right)}{24\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c*x^6 + b*x^3 + a)*(2*x^3 + b/c) + 1/24*(b^2 - 4*a*c)*log(abs(-2*
(sqrt(c)*x^3 - sqrt(c*x^6 + b*x^3 + a))*sqrt(c) - b))/c^(3/2)
```

Mupad [B]
time = 1.58, size = 34, normalized size = 0.79

$$\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(1/2))
```

3.224 $\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

```
[Out] -1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1371, 738, 212}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/3*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$
$$= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3\sqrt{a}}$$

Mathematica [**A**]
time = 0.07, size = 44, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^3 + c*x^6]),x]
[Out] (2*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(3*Sqrt[a])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(1/x/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.36, size = 124, normalized size = 2.82

$$\left[\frac{\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\,(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right)}{6\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right)}{3a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] [1/6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(a) + 8*a^2)/x^6)/sqrt(a), 1/3*sqrt(-a)*arctan(1/2*sqrt(c*x^6 +
 b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x), x)
```

Mupad [B]
time = 1.57, size = 36, normalized size = 0.82

$$-\frac{\ln\left(\frac{b}{2}+\frac{a}{x^3}+\frac{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}{x^3}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3 + c*x^6)^(1/2)),x)
[Out] -log(b/2 + a/x^3 + (a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))/x^3)/(3*a^(1/2))
```

3.225 $\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}+\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}$$

```
[Out] 1/6*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)-1/3*(c
*x^6+b*x^3+a)^(1/2)/a/x^3
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1371, 744, 738, 212}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/(a*x^3) + (b*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*
Sqrt[a + b*x^3 + c*x^6])])/(6*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
```

```
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{6a}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} + \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3a}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} + \frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{6a^{3/2}}
\end{aligned}$$

Mathematica [**A**]
time = 0.13, size = 72, normalized size = 1.00

$$-\frac{\sqrt{a+bx^3+cx^6}}{3ax^3} - \frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/3*Sqrt[a + b*x^3 + c*x^6]/(a*x^3) - (b*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b
*x^3 + c*x^6])/Sqrt[a]])/(3*a^(3/2))
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^4/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 179, normalized size = 2.49

$$\left[\frac{\sqrt{a} b x^3 \log\left(-\frac{(b^2+4ac)x^6+8abx^3+4\sqrt{cx^6+bx^3+a}\,(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right) - 4\sqrt{cx^6+bx^3+a}\,a}{12a^2x^3}, -\frac{\sqrt{-a}\,bx^3 \arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2a)\sqrt{-a}}{2(acx^6+abx^3+a^2)}\right) + 2\sqrt{cx^6+bx^3+a}\,a}{6a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(a)*b*x^3*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b
*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6 + b*x^3 + a)*a
)/(a^2*x^3), -1/6*(sqrt(-a)*b*x^3*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3
 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*a)/
(a^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^4), x)
```

Mupad [B]
time = 1.56, size = 56, normalized size = 0.78

$$\frac{b\,\mathrm{atanh}\left(\frac{\frac{b\,x^3}{2}+a}{\sqrt{a}\,\sqrt{c\,x^6+b\,x^3+a}}\right)}{6\,a^{3/2}}-\frac{\sqrt{c\,x^6+b\,x^3+a}}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3 + c*x^6)^(1/2)),x)
```

```
[Out] (b*atanh((a + (b*x^3)/2)/(a^(1/2)*(a + b*x^3 + c*x^6)^(1/2))))/(6*a^(3/2))
- (a + b*x^3 + c*x^6)^(1/2)/(3*a*x^3)
```

3.226 $\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}$$

```
[Out] -1/24*(-4*a*c+3*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))
/a^(5/2)-1/6*(c*x^6+b*x^3+a)^(1/2)/a/x^6+1/4*b*(c*x^6+b*x^3+a)^(1/2)/a^2/x^
3
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 758, 820, 738, 212}

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/6*Sqrt[a + b*x^3 + c*x^6]/(a*x^6) + (b*Sqrt[a + b*x^3 + c*x^6])/(4*a^2*x
^3) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*
x^6])])/(24*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
```

```
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}-\frac{\text{Subst}\left(\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{6a}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}+\frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{24a^2}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{12a^2}\\
&= -\frac{\sqrt{a+bx^3+cx^6}}{6ax^6}+\frac{b\sqrt{a+bx^3+cx^6}}{4a^2x^3}-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{24a^{5/2}}
\end{aligned}$$

Mathematica [A]

time = 0.22, size = 91, normalized size = 0.84

$$\frac{(-2a+3bx^3)\sqrt{a+bx^3+cx^6}}{12a^2x^6}+\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{12a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] ((-2*a + 3*b*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*a^2*x^6) + ((3*b^2 - 4*a*c)*
ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(12*a^(5/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^7/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 221, normalized size = 2.05

$$\left[-\frac{(3\,b^2-4\,ac)\sqrt{a}\,x^6\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\,\sqrt{cx^6+bx^3+a}\,(3\,abx^3-2\,a^2)}{48\,a^3x^6},\ \frac{(3\,b^2-4\,ac)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,\sqrt{cx^6+bx^3+a}\,(3\,abx^3-2\,a^2)}{24\,a^3x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*b^2 - 4*a*c)*sqrt(a)*x^6*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4
*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*sqrt(c*x^6
```

```
 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6), 1/24*((3*b^2 - 4*a*c)*sqrt(-a
)*x^6*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 +
a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(3*a*b*x^3 - 2*a^2))/(a^3*x^6)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/(x**7*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^7), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)),x)
[Out] int(1/(x^7*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.227 $\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=145

$$-\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{2a\cdots}{2\sqrt{a}\sqrt{a+\cdots}}\right)}{48a^{7/2}}$$

```
[Out] 1/48*b*(-12*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2
))/a^(7/2)-1/9*(c*x^6+b*x^3+a)^(1/2)/a/x^9+5/36*b*(c*x^6+b*x^3+a)^(1/2)/a^2
/x^6-1/72*(-16*a*c+15*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/x^3
```

Rubi [A]
time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 758, 848, 820, 738, 212}

$$\frac{b(5b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{7/2}}-\frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3}+\frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6}-\frac{\sqrt{a+bx^3+cx^6}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/9*Sqrt[a + b*x^3 + c*x^6]/(a*x^9) + (5*b*Sqrt[a + b*x^3 + c*x^6])/(36*a^
2*x^6) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(72*a^3*x^3) + (b*(5*b
^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(4
8*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
```

```
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} - \frac{\text{Subst}\left(\int \frac{\frac{5b}{2}+2cx}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{9a}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(15b^2-16ac)+\frac{5bcx}{2}}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x\right.}{18a^2}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3} - \frac{(}{}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3} + \frac{(}{}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{9ax^9} + \frac{5b\sqrt{a+bx^3+cx^6}}{36a^2x^6} - \frac{(15b^2-16ac)\sqrt{a+bx^3+cx^6}}{72a^3x^3} + \frac{b}{}$$

Mathematica [A]
time = 0.35, size = 110, normalized size = 0.76

$$\frac{\sqrt{a+bx^3+cx^6}\,(-8a^2+10abx^3-15b^2x^6+16acx^6)}{72a^3x^9} + \frac{(-5b^3+12abc)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-8*a^2 + 10*a*b*x^3 - 15*b^2*x^6 + 16*a*c*x^6))/(
72*a^3*x^9) + ((-5*b^3 + 12*a*b*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 +
c*x^6])/Sqrt[a]])/(24*a^(7/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^10/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.41, size = 263, normalized size = 1.81

$$\left[-\frac{3\,(5\,b^3-12\,abc)\sqrt{a}\,x^9\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,((15\,ab^2-16\,a^2c)x^6-10\,a^2bx^3+8\,a^3)\sqrt{cx^6+bx^3+a}}{288\,a^4x^9},-\frac{3\,(5\,b^3-12\,abc)\sqrt{-a}\,x^9\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,((15\,ab^2-16\,a^2c)x^6-10\,a^2bx^3+8\,a^3)\sqrt{cx^6+bx^3+a}}{144\,a^4x^9}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(5*b^3 - 12*a*b*c)*sqrt(a)*x^9*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x
^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((15
*a*b^2 - 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^
4*x^9), -1/144*(3*(5*b^3 - 12*a*b*c)*sqrt(-a)*x^9*arctan(1/2*sqrt(c*x^6 + b
*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^2
- 16*a^2*c)*x^6 - 10*a^2*b*x^3 + 8*a^3)*sqrt(c*x^6 + b*x^3 + a))/(a^4*x^9)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**10*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^10), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{10}\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^10*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.228 $\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=192

$$-\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}}+\frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9}-\frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6}+\frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{576a^4x^3}$$

```
[Out] -1/384*(48*a^2*c^2-120*a*b^2*c+35*b^4)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x
^6+b*x^3+a)^(1/2))/a^(9/2)-1/12*(c*x^6+b*x^3+a)^(1/2)/a/x^12+7/72*b*(c*x^6+
b*x^3+a)^(1/2)/a^2/x^9-1/288*(-36*a*c+35*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/x^6
+5/576*b*(-44*a*c+21*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^4/x^3
```

Rubi [A]
time = 0.16, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 758, 848, 820, 738, 212}

$$\frac{5b(21b^2-44ac)\sqrt{a+bx^3+cx^6}}{576a^4x^3}-\frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6}+\frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9}-\frac{(48a^2c^2-120ab^2c+35b^4)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{384a^{9/2}}-\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/12*Sqrt[a + b*x^3 + c*x^6]/(a*x^12) + (7*b*Sqrt[a + b*x^3 + c*x^6])/(72*
a^2*x^9) - ((35*b^2 - 36*a*c)*Sqrt[a + b*x^3 + c*x^6])/(288*a^3*x^6) + (5*b
*(21*b^2 - 44*a*c)*Sqrt[a + b*x^3 + c*x^6])/(576*a^4*x^3) - ((35*b^4 - 120*
a*b^2*c + 48*a^2*c^2)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x
^6])])/(384*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
```

```
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^5\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} - \frac{\text{Subst}\left(\int \frac{\frac{7b}{2}+3cx}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{12a}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(35b^2-36ac)+7bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x,\right.}{36a^2}$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} -$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} +$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} +$$

$$= -\frac{\sqrt{a+bx^3+cx^6}}{12ax^{12}} + \frac{7b\sqrt{a+bx^3+cx^6}}{72a^2x^9} - \frac{(35b^2-36ac)\sqrt{a+bx^3+cx^6}}{288a^3x^6} +$$

Mathematica [A]
time = 0.47, size = 141, normalized size = 0.73

$$\frac{\sqrt{a+bx^3+cx^6}\,(-48a^3+56a^2bx^3-70ab^2x^6+72a^2cx^6+105b^3x^9-220abcx^9)}{576a^4x^{12}} + \frac{(35b^4-120ab^2c+48a^2c^2)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{192a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^13*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (Sqrt[a + b*x^3 + c*x^6]*(-48*a^3 + 56*a^2*b*x^3 - 70*a*b^2*x^6 + 72*a^2*c*
x^6 + 105*b^3*x^9 - 220*a*b*c*x^9))/(576*a^4*x^12) + ((35*b^4 - 120*a*b^2*c
 + 48*a^2*c^2)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]])/(1
92*a^(9/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{13}\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^13/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.48, size = 327, normalized size = 1.70

$$\left[\frac{3\,(35\,b^4-120\,ab^2c+48\,a^2c^2)\sqrt{a}\,x^{12}\log\left(-\frac{(b^2+4ac)x^6+8abx^3-4\sqrt{cx^6+bx^3+a}\,(bx^3+2a)\sqrt{a}+8a^2}{x^6}\right)+4\,(5\,(21\,ab^3-44\,a^2bc)x^9+56\,a^3bx^3-2\,(35\,a^2b^2-36\,a^3c)x^6-48\,a^4)\sqrt{cx^6+bx^3+a}}{2304\,a^5x^{12}},\ \frac{3\,(35\,b^4-120\,ab^2c+48\,a^2c^2)\sqrt{-a}\,x^{12}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,(5\,(21\,ab^3-44\,a^2bc)x^9+56\,a^3bx^3-2\,(35\,a^2b^2-36\,a^3c)x^6-48\,a^4)\sqrt{cx^6+bx^3+a}}{1152\,a^5x^{12}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2304*(3*(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(a)*x^12*log(-((b^2 + 4*
a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*
a^2)/x^6) + 4*(5*(21*a*b^3 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2
 - 36*a^3*c)*x^6 - 48*a^4)*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12), 1/1152*(3*(
35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*sqrt(-a)*x^12*arctan(1/2*sqrt(c*x^6 + b*
x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*(5*(21*a*b^3
 - 44*a^2*b*c)*x^9 + 56*a^3*b*x^3 - 2*(35*a^2*b^2 - 36*a^3*c)*x^6 - 48*a^4)
*sqrt(c*x^6 + b*x^3 + a))/(a^5*x^12)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{13}\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**13*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^13), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{13}\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^13*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.229 $\int \frac{x^3}{\sqrt{a+bx^3+cx^6}} \, dx$

Optimal. Leaf size=140

$$\frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1/2,1/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\,\sqrt{1-}}}{\sqrt{a+bx^3+cx^6}}$$

$$= \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\right.}{4\sqrt{a+bx^3+cx^6}}$$

Mathematica [A]
time = 10.07, size = 168, normalized size = 1.20

$$\frac{x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\,\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/Sqrt[a + b*x^3 + c*x^6],x]
[Out] (x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2
, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
4*a*c])])/(4*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
[Out] int(x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{c\,x^6 + b\,x^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(x^3/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.230 $\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=140

$$\frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1/2,1/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1399, 524}

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\cdots}}}{\sqrt{a+bx^3+cx^6}}$$

$$= \frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\cdots\right.}{2\sqrt{a+bx^3+cx^6}}$$

Mathematica [A]
time = 10.07, size = 168, normalized size = 1.20

$$\frac{x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2
, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 -
4*a*c])])/(2*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(x/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{c\,x^6 + b\,x^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(x/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.231 $\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=135

$$\frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

```
[Out] x*AppellF1(1/3,1/2,1/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1362, 440}

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\cdots}}}{\sqrt{a+bx^3+cx^6}}$$

$$= \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\cdots\right.}{\sqrt{a+bx^3+cx^6}}$$

Mathematica [A]
time = 10.05, size = 163, normalized size = 1.21

$$\frac{x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^3+cx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2,
1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*
a*c])])/Sqrt[a + b*x^3 + c*x^6]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,x^6+b\,x^3+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int(1/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.232 $\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

```
[Out] -AppellF1(-1/3,1/2,1/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/x/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-1/3, 1/2, 1/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*
a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^3 + c*x^6]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\cdots}}}{\sqrt{a+bx^3+cx^6}}$$

$$= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},\cdots\right.}{x\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 343 vs. 2(138) = 276.

time = 10.23, size = 343, normalized size = 2.49

$$\frac{-20(a+bx^3+cx^6)+5bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+8cx^6\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{20ax\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (-20*(a + b*x^3 + c*x^6) + 5*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 8*c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(20*a*x*Sqrt[a + b*x^
3 + c*x^6])
```

Maple [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^2/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c*x^8 + b*x^5 + a*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3 + c*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.233 $\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=140

$$-\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

```
[Out] -1/2*AppellF1(-2/3,1/2,1/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+
(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/x^2/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] -1/2*(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-2/3, 1/2, 1/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[a + b*x^3 + c*x^6]
)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\ \cdots}}{\sqrt{a+bx^3+cx^6}}$$

$$= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\ \cdots\right.}{2x^2\sqrt{a+bx^3+cx^6}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(140) = 280.
time = 10.20, size = 342, normalized size = 2.44

$$\frac{-4(a+bx^3+cx^6)-2bx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+cx^6\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{8ax^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^3 + c*x^6]),x]
```

```
[Out] (-4*(a + b*x^3 + c*x^6) - 2*b*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*x^6*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(8*a*x^2*Sqrt[a + b*x^3
+ c*x^6])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int(1/x^3/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c*x^9 + b*x^6 + a*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+bx^3+cx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + b*x**3 + c*x**6)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^6 + b*x^3 + a)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3 + c*x^6)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^3 + c*x^6)^(1/2)), x)
```

3.234 $\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)}-\frac{(b(15b^2-52ac)-2c(5b^2-12ac)x^3)\sqrt{a+bx^3+cx^6}}{12c^3(b^2-4ac)}$$

```
[Out] 1/8*(-4*a*c+5*b^2)*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c
^(7/2)+2/3*x^9*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)-2/3*b*x^6*(c*
x^6+b*x^3+a)^(1/2)/c/(-4*a*c+b^2)-1/12*(b*(-52*a*c+15*b^2)-2*c*(-12*a*c+5*b
^2)*x^3)*(c*x^6+b*x^3+a)^(1/2)/c^3/(-4*a*c+b^2)
```

Rubi [A]

time = 0.16, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 752, 846, 793, 635, 212}

$$\frac{(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{8c^{7/2}}-\frac{(b(15b^2-52ac)-2cx^3(5b^2-12ac))\sqrt{a+bx^3+cx^6}}{12c^3(b^2-4ac)}-\frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)}+\frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^9*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*x^6*
Sqrt[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52*a*c) - 2*c*
(5*b^2 - 12*a*c)*x^3)*Sqrt[a + b*x^3 + c*x^6])/(12*c^3*(b^2 - 4*a*c)) + ((5
*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(
8*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
```

```
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{14}}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{(a+bx+cx^2)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{x^2(6a+3bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3(b^2-4ac)}$$

$$= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{2\text{Subst}\left(\int \frac{x(-6ab-\frac{3}{2}(5b^2-}{\sqrt{a+bx+}}\right.}{9c(b^2-4}$$

$$= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5}{12c}$$

$$= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5}{12c}$$

$$= \frac{2x^9(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2bx^6\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} - \frac{(b(15b^2-52ac)-2c(5}{12c}$$

Mathematica [A]
time = 0.67, size = 170, normalized size = 0.87

$$\frac{4a^2c(-13b+6cx^3)+b^2x^3(15b^2+5bcx^3-2c^2x^6)+a(15b^3-62b^2cx^3-20bc^2x^6+8c^3x^9)}{12c^3(-b^2+4ac)\sqrt{a+bx^3+cx^6}} + \frac{(-5b^2+4ac)\log\left(c^3\left(b+2cx^3-2\sqrt{c}\sqrt{a+bx^3+cx^6}\right)\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (4*a^2*c*(-13*b + 6*c*x^3) + b^2*x^3*(15*b^2 + 5*b*c*x^3 - 2*c^2*x^6) + a*(
15*b^3 - 62*b^2*c*x^3 - 20*b*c^2*x^6 + 8*c^3*x^9))/(12*c^3*(-b^2 + 4*a*c)*S
qrt[a + b*x^3 + c*x^6]) + ((-5*b^2 + 4*a*c)*Log[c^3*(b + 2*c*x^3 - 2*Sqrt[c
]*Sqrt[a + b*x^3 + c*x^6])])/(8*c^(7/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^14/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.42, size = 591, normalized size = 3.03

$$\left[-\frac{3\left((5b^4c-24ab^2c^2+16a^2c^3)x^6+5ab^4-24a^2b^2c+16a^3c^2+(5b^5-24ab^3c+16a^2bc^2)x^3\right)\sqrt{c}\log\left(-8c^2x^6-8bcx^3-b^2+4\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{c}-4ac\right)-4\left(2(b^2c^3-4ac^4)x^9-5(b^3c^2-4abc^3)x^6-15ab^3c+52a^2bc^2-(15b^4c-62ab^2c^2+24a^2c^3)x^3\right)\sqrt{cx^6+bx^3+a}}{48\left(ab^2c^4-4a^2c^5+(b^2c^5-4ac^6)x^6+(b^3c^4-4abc^5)x^3\right)}, -\frac{3\left((5b^4c-24ab^2c^2+16a^2c^3)x^6+5ab^4-24a^2b^2c+16a^3c^2+(5b^5-24ab^3c+16a^2bc^2)x^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^6+bx^3+a}(2cx^3+b)\sqrt{-c}}{2(c^2x^6+bcx^3+ac)}\right)-2\left(2(b^2c^3-4ac^4)x^9-5(b^3c^2-4abc^3)x^6-15ab^3c+52a^2bc^2-(15b^4c-62ab^2c^2+24a^2c^3)x^3\right)\sqrt{cx^6+bx^3+a}}{24\left(ab^2c^4-4a^2c^5+(b^2c^5-4ac^6)x^6+(b^3c^4-4abc^5)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6 + 5*a*b^4 - 24*a^2*b^2
*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^3)*sqrt(c)*log(-8*c
^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(c)
- 4*a*c) - 4*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*b*c^3)*x^6 - 15*
a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x^3)*sqrt(c
*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^6 + (b^3*
c^4 - 4*a*b*c^5)*x^3), -1/24*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^6
+ 5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)
*x^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c
^2*x^6 + b*c*x^3 + a*c)) - 2*(2*(b^2*c^3 - 4*a*c^4)*x^9 - 5*(b^3*c^2 - 4*a*
b*c^3)*x^6 - 15*a*b^3*c + 52*a^2*b*c^2 - (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*
c^3)*x^3)*sqrt(c*x^6 + b*x^3 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*
c^6)*x^6 + (b^3*c^4 - 4*a*b*c^5)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**14/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^14/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.235 $\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{2x^6(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}+\frac{(3b^2-8ac-2bcx^3)\sqrt{a+bx^3+cx^6}}{3c^2(b^2-4ac)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

```
[Out] -1/2*b*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(5/2)+2/3*x
^6*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)+1/3*(-2*b*c*x^3-8*a*c+3*b
^2)*(c*x^6+b*x^3+a)^(1/2)/c^2/(-4*a*c+b^2)
```

Rubi [A]

time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 752, 793, 635, 212}

$$\frac{(-8ac+3b^2-2bcx^3)\sqrt{a+bx^3+cx^6}}{3c^2(b^2-4ac)}+\frac{2x^6(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}-\frac{b\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*x^6*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + ((3*b^2 -
 8*a*c - 2*b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(3*c^2*(b^2 - 4*a*c)) - (b*Arc
Tanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])])/(2*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
```

```
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(a+bx+cx^2)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2x^6(2a+bx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{x(4a+2bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3\,(b^2-4ac)}$$

$$= \frac{2x^6(2a+bx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}} + \frac{(3b^2-8ac-2bcx^3)\,\sqrt{a+bx^3+cx^6}}{3c^2\,(b^2-4ac)} - \frac{b\text{Subst}}{}$$

$$= \frac{2x^6(2a+bx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}} + \frac{(3b^2-8ac-2bcx^3)\,\sqrt{a+bx^3+cx^6}}{3c^2\,(b^2-4ac)} - \frac{b\text{Subst}}{}$$

$$= \frac{2x^6(2a+bx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}} + \frac{(3b^2-8ac-2bcx^3)\,\sqrt{a+bx^3+cx^6}}{3c^2\,(b^2-4ac)} - \frac{b\tanh^{-}}{}$$

Mathematica [A]

time = 0.45, size = 131, normalized size = 0.96

$$\frac{-3ab^2+8a^2c-3b^3x^3+10abcx^3-b^2cx^6+4ac^2x^6}{3c^2\left(-b^2+4ac\right)\sqrt{a+bx^3+cx^6}}+\frac{b\log\left(bc^2+2c^3x^3-2c^{5/2}\sqrt{a+bx^3+cx^6}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-3*a*b^2 + 8*a^2*c - 3*b^3*x^3 + 10*a*b*c*x^3 - b^2*c*x^6 + 4*a*c^2*x^6)/(
3*c^2*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) + (b*Log[b*c^2 + 2*c^3*x^3 -
2*c^(5/2)*Sqrt[a + b*x^3 + c*x^6]])/(2*c^(5/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^11/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.46, size = 459, normalized size = 3.35

$$\left[\frac{3\left(\left(b^3c-4\,abc^2\right)x^6+ab^3-4\,a^2bc+\left(b^4-4\,ab^2c\right)x^3\right)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2+4\,\sqrt{cx^6+bx^3+a}\,\left(2\,cx^3+b\right)\sqrt{c}-4\,ac\right)+4\left(\left(b^2c^2-4\,ac^3\right)x^6+3\,ab^2c-8\,a^2c^2+\left(3\,b^3c-10\,abc^2\right)x^3\right)\sqrt{cx^6+bx^3+a}}{12\left(\left(b^2c^4-4\,ac^5\right)x^6+ab^2c^3-4\,a^2c^4+\left(b^3c^3-4\,abc^4\right)x^3\right)},\ \frac{3\left(\left(b^3c-4\,abc^2\right)x^6+ab^3-4\,a^2bc+\left(b^4-4\,ab^2c\right)x^3\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,\left(2\,cx^3+b\right)\sqrt{-c}}{2\left(c^2x^6+bcx^3+ac\right)}\right)+2\left(\left(b^2c^2-4\,ac^3\right)x^6+3\,ab^2c-8\,a^2c^2+\left(3\,b^3c-10\,abc^2\right)x^3\right)\sqrt{cx^6+bx^3+a}}{6\left(\left(b^2c^4-4\,ac^5\right)x^6+ab^2c^3-4\,a^2c^4+\left(b^3c^3-4\,abc^4\right)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x
^3)*sqrt(c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 + 4*sqrt(c*x^6 + b*x^3 + a)*(2
```

```
*c*x^3 + b)*sqrt(c) - 4*a*c) + 4*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a
^2*c^2 + (3*b^3*c - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4
*a*c^5)*x^6 + a*b^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3), 1/6*(3*((
b^3*c - 4*a*b*c^2)*x^6 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^3)*sqrt(-c
)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^2*x^6 + b*c*
x^3 + a*c)) + 2*((b^2*c^2 - 4*a*c^3)*x^6 + 3*a*b^2*c - 8*a^2*c^2 + (3*b^3*c
 - 10*a*b*c^2)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((b^2*c^4 - 4*a*c^5)*x^6 + a*b
^2*c^3 - 4*a^2*c^4 + (b^3*c^3 - 4*a*b*c^4)*x^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a + bx^3 + cx^6)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x**11/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate(x^11/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int(x^11/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.236 $\int \frac{x^8}{\left(a+bx^3+cx^6\right)^{3/2}} dx$

Optimal. Leaf size=120

$$\frac{2x^3(2a+bx^3)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

[Out] 1/3*arctanh(1/2*(2*c*x^3+b)/c^(1/2)/(c*x^6+b*x^3+a)^(1/2))/c^(3/2)+2/3*x^3*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)-2/3*b*(c*x^6+b*x^3+a)^(1/2)/c/(-4*a*c+b^2)

Rubi [A]
time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 752, 654, 635, 212}

$$\frac{2x^3(2a+bx^3)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2b\sqrt{a+bx^3+cx^6}}{3c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3+cx^6}}\right)}{3c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a + b*x^3 + c*x^6)^(3/2),x]

[Out] (2*x^3*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - (2*b*Sqrt[a + b*x^3 + c*x^6])/(3*c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^3)/(2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*c^(3/2))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2x^3(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{2a+bx}{\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3(b^2-4ac)}$$

$$= \frac{2x^3(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}} \cdots\right.}{3c}$$

$$= \frac{2x^3(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} + \frac{2\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \cdots\right.}{3c}$$

$$= \frac{2x^3(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}} - \frac{2b\sqrt{a+bx^3+cx^6}}{3c(b^2-4ac)} + \frac{\tanh^{-1}\left(\frac{b+2cx^3}{2\sqrt{c}\sqrt{a+bx^3 \cdots}}\right.}{3c^{3/2}}$$

Mathematica [A]

time = 0.38, size = 97, normalized size = 0.81

$$\frac{2(b^2x^3+a(b-2cx^3))}{3c\left(-b^2+4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\log\left(c\left(b+2cx^3-2\sqrt{c}\,\sqrt{a+bx^3+cx^6}\right)\right)}{3c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*(b^2*x^3 + a*(b - 2*c*x^3)))/(3*c*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6]
) - Log[c*(b + 2*c*x^3 - 2*Sqrt[c]*Sqrt[a + b*x^3 + c*x^6])]/(3*c^(3/2))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^8/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.39, size = 387, normalized size = 3.22

$$\left[\frac{((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)\sqrt{c}\,\log\left(-8\,c^2x^6-8\,bcx^3-b^2-4\,\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{c}-4\,ac\right)-4\,\sqrt{cx^6+bx^3+a}\,((b^2c-2\,ac^2)x^3+abc)}{6\left((b^2c^3-4\,ac^4)x^6+ab^2c^2-4\,a^2c^3+(b^3c^2-4\,abc^3)x^3\right)},\ -\frac{((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(2\,cx^3+b)\sqrt{-c}}{2\,(c^2x^6+bcx^3+ac)}\right)+2\,\sqrt{cx^6+bx^3+a}\,((b^2c-2\,ac^2)x^3+abc)}{3\left((b^2c^3-4\,ac^4)x^6+ab^2c^2-4\,a^2c^3+(b^3c^2-4\,abc^3)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(
c)*log(-8*c^2*x^6 - 8*b*c*x^3 - b^2 - 4*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 +
```

```
b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3 + a*
b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a*b*c
^3)*x^3), -1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^
2*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)*sqrt(-c)/(c^
2*x^6 + b*c*x^3 + a*c)) + 2*sqrt(c*x^6 + b*x^3 + a)*((b^2*c - 2*a*c^2)*x^3
+ a*b*c))/((b^2*c^3 - 4*a*c^4)*x^6 + a*b^2*c^2 - 4*a^2*c^3 + (b^3*c^2 - 4*a
*b*c^3)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(x**8/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate(x^8/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [B]
time = 1.66, size = 84, normalized size = 0.70

$$\frac{\ln\left(\sqrt{c\,x^6+b\,x^3+a}+\frac{c\,x^3+\frac{b}{2}}{\sqrt{c}}\right)}{3\,c^{3/2}}+\frac{\frac{a\,b}{2}-x^3\,\left(a\,c-\frac{b^2}{2}\right)}{3\,c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^6+b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^3 + c*x^6)^(3/2),x)
[Out] log((a + b*x^3 + c*x^6)^(1/2) + (b/2 + c*x^3)/c^(1/2))/(3*c^(3/2)) + ((a*b)
/2 - x^3*(a*c - b^2/2))/(3*c*(a*c - b^2/4)*(a + b*x^3 + c*x^6)^(1/2))
```

3.237 $\int \frac{x^5}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2(2a+bx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}$$

```
[Out] 2/3*(b*x^3+2*a)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1371, 650}

$$\frac{2(2a+bx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)\\ &= \frac{2(2a+bx^3)}{3\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [**A**]
time = 0.25, size = 39, normalized size = 1.00

$$\frac{2(2a+bx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (2*(2*a + b*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [**A**]
time = 0.03, size = 38, normalized size = 0.97

method	result	size
gosper	$-\frac{2(b\,x^3+2a)}{3\sqrt{c\,x^6+b\,x^3+a}\,(4ac-b^2)}$	38
trager	$-\frac{2(b\,x^3+2a)}{3\sqrt{c\,x^6+b\,x^3+a}\,(4ac-b^2)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^6+b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/(c*x^6+b*x^3+a)^(1/2)*(b*x^3+2*a)/(4*a*c-b^2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.36, size = 68, normalized size = 1.74

$$\frac{2\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)}{3\,((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)/((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4
*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**5/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [A]
time = 2.98, size = 45, normalized size = 1.15

$$\frac{2\left(\frac{bx^3}{b^2-4\,ac}+\frac{2\,a}{b^2-4\,ac}\right)}{3\,\sqrt{cx^6+bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*x^3/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

Mupad [B]
time = 1.43, size = 38, normalized size = 0.97

$$-\frac{2\,b\,x^3+4\,a}{\left(12\,a\,c-3\,b^2\right)\,\sqrt{c\,x^6+b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] -(4*a + 2*b*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))
```

3.238 $\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

```
[Out] -2/3*(2*c*x^3+b)/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1366, 627}

$$-\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^3+cx^6)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^{3/2}}\,dx, x, x^3\right)\\ &= -\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 38, normalized size = 1.00

$$-\frac{2(b+2cx^3)}{3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (-2*(b + 2*c*x^3))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [A]
time = 0.03, size = 37, normalized size = 0.97

method	result	size
gosper	$\frac{\frac{4c\,x^{3}}{3}+\frac{2b}{3}}{\sqrt{c\,x^{6}+b\,x^{3}+a}\,\left(4ac-b^{2}\right)}$	37
trager	$\frac{\frac{4c\,x^{3}}{3}+\frac{2b}{3}}{\sqrt{c\,x^{6}+b\,x^{3}+a}\,\left(4ac-b^{2}\right)}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^6+b*x^3+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/(c*x^6+b*x^3+a)^(1/2)*(2*c*x^3+b)/(4*a*c-b^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 67, normalized size = 1.76

$$-\frac{2\,\sqrt{cx^{6}+bx^{3}+a}\,\left(2\,cx^{3}+b\right)}{3\,\left(\left(b^{2}c-4\,ac^{2}\right)x^{6}+\left(b^{3}-4\,abc\right)x^{3}+ab^{2}-4\,a^{2}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(c*x^6 + b*x^3 + a)*(2*c*x^3 + b)/((b^2*c - 4*a*c^2)*x^6 + (b^3 -
4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^3+cx^6)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [**A**]
time = 2.96, size = 45, normalized size = 1.18

$$-\frac{2\left(\frac{2\,cx^3}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}\right)}{3\,\sqrt{cx^6+bx^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(2*c*x^3/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^6 + b*x^3 + a)
```

Mupad [**B**]
time = 1.37, size = 37, normalized size = 0.97

$$\frac{4\,c\,x^3+2\,b}{(12\,a\,c-3\,b^2)\,\sqrt{c\,x^6+b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] (2*b + 4*c*x^3)/((12*a*c - 3*b^2)*(a + b*x^3 + c*x^6)^(1/2))
```

3.239 $\int \frac{1}{x(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=92

$$\frac{2(b^2-2ac+bcx^3)}{3a\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\,\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

```
[Out] -1/3*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(3/2)+2/3*(b*
c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 754, 12, 738, 212}

$$\frac{2(-2ac+b^2+bcx^3)}{3a\,(b^2-4ac)\,\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\,\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*Sqrt[a + b*x^3 + c*x^6]) - A
rcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])]/(3*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^3+cx^6\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^3\right)\\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{3a\left(b^2-4ac\right)}\\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{3a}\\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^3}{\sqrt{a+bx^3+cx^6}}\right)}{3a}\\
&= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}-\frac{\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{3a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.42, size = 131, normalized size = 1.42

$$\frac{2\left(\frac{\sqrt{a}\left(-b^2+2ac-bcx^3\right)\sqrt{a+bx^3+cx^6}}{4a^2c-b^2x^3\left(b+cx^3\right)+a\left(-b^2+4bcx^3+4c^2x^6\right)}+\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*((Sqrt[a]*(-b^2 + 2*a*c - b*c*x^3)*Sqrt[a + b*x^3 + c*x^6])/(4*a^2*c - b
^2*x^3*(b + c*x^3) + a*(-b^2 + 4*b*c*x^3 + 4*c^2*x^6)) + ArcTanh[(Sqrt[c]*x
^3 - Sqrt[a + b*x^3 + c*x^6])/Sqrt[a]]))/(3*a^(3/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(c\,x^{6}+b\,x^{3}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(78) = 156$.
time = 0.43, size = 389, normalized size = 4.23

$$\left[\frac{\left((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c\right)\sqrt{a}\,\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3-4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)+4\,\sqrt{cx^6+bx^3+a}\,(abcx^3+ab^2-2\,a^2c)}{6\left((a^2b^2c-4\,a^3c^2)x^6+a^3b^2-4\,a^4c+(a^2b^3-4\,a^3bc)x^3\right)},\ \frac{\left((b^2c-4\,ac^2)x^6+(b^3-4\,abc)x^3+ab^2-4\,a^2c\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\,\sqrt{cx^6+bx^3+a}\,(abcx^3+ab^2-2\,a^2c)}{3\left((a^2b^2c-4\,a^3c^2)x^6+a^3b^2-4\,a^4c+(a^2b^3-4\,a^3bc)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 - 4*a^2*c)*sqrt(
a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 +
 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b^2
- 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3 - 4
*a^3*b*c)*x^3), 1/3*(((b^2*c - 4*a*c^2)*x^6 + (b^3 - 4*a*b*c)*x^3 + a*b^2 -
```

```
 4*a^2*c)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a
)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*sqrt(c*x^6 + b*x^3 + a)*(a*b*c*x^3 + a*b^2
 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^6 + a^3*b^2 - 4*a^4*c + (a^2*b^3 -
4*a^3*b*c)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3+cx^6\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.240 $\int \frac{1}{x^4 (a+bx^3+cx^6)^{3/2}} \, dx$

Optimal. Leaf size=142

$$\frac{2(b^2-2ac+bcx^3)}{3a(b^2-4ac)x^3\sqrt{a+bx^3+cx^6}}-\frac{(3b^2-8ac)\sqrt{a+bx^3+cx^6}}{3a^2(b^2-4ac)x^3}+\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}$$

```
[Out] 1/2*b*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/a^(5/2)+2/3*(b
*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x^6+b*x^3+a)^(1/2)-1/3*(-8*a*c+3*b^
2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^3
```

Rubi [A]
time = 0.09, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 754, 820, 738, 212}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{2a^{5/2}}-\frac{(3b^2-8ac)\sqrt{a+bx^3+cx^6}}{3a^2x^3(b^2-4ac)}+\frac{2(-2ac+b^2+bcx^3)}{3ax^3(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6])
 - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^3 + c*x^6])/(3*a^2*(b^2 - 4*a*c)*x^3) + (b
*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])/(2*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
```

```
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^3\right)$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{3a\left(b^2-4ac\right)}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} - \frac{b\text{Subst}\left(\int \right.}{}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\text{Subst}\left(\int \right.}{}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^3\sqrt{a+bx^3+cx^6}} - \frac{\left(3b^2-8ac\right)\sqrt{a+bx^3+cx^6}}{3a^2\left(b^2-4ac\right)x^3} + \frac{b\tanh^{-1}}{}$$

Mathematica [A]
time = 0.53, size = 125, normalized size = 0.88

$$\frac{-4a^2c+3b^2x^3(b+cx^3)+a(b^2-10bcx^3-8c^2x^6)}{3a^2\left(-b^2+4ac\right)x^3\sqrt{a+bx^3+cx^6}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (-4*a^2*c + 3*b^2*x^3*(b + c*x^3) + a*(b^2 - 10*b*c*x^3 - 8*c^2*x^6))/(3*a^
2*(-b^2 + 4*a*c)*x^3*Sqrt[a + b*x^3 + c*x^6]) - (b*ArcTanh[(Sqrt[c]*x^3 - S
qrt[a + b*x^3 + c*x^6])/Sqrt[a]])/a^(5/2)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^4/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 485, normalized size = 3.42

$$\left[\frac{3\left((b^3c-4\,abc^2)x^9+(b^4-4\,ab^2c)x^6+(ab^3-4\,a^2bc)x^3\right)\sqrt{a}\,\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^3+4\,\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{a}+8\,a^2}{x^6}\right)-4\left((3\,ab^2c-8\,a^2c^2)x^6+a^2b^2-4\,a^3c+(3\,ab^3-10\,a^2bc)x^3\right)\sqrt{cx^6+bx^3+a}}{12\left((a^3b^2c-4\,a^4c^2)x^9+(a^3b^3-4\,a^4bc)x^6+(a^4b^2-4\,a^5c)x^3\right)},-\frac{3\left((b^3c-4\,abc^2)x^9+(b^4-4\,ab^2c)x^6+(ab^3-4\,a^2bc)x^3\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^6+bx^3+a}\,(bx^3+2\,a)\sqrt{-a}}{2\,(acx^6+abx^3+a^2)}\right)+2\left((3\,ab^2c-8\,a^2c^2)x^6+a^2b^2-4\,a^3c+(3\,ab^3-10\,a^2bc)x^3\right)\sqrt{cx^6+bx^3+a}}{6\left((a^3b^2c-4\,a^4c^2)x^9+(a^3b^3-4\,a^4bc)x^6+(a^4b^2-4\,a^5c)x^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^3 - 4*a^2*
b*c)*x^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^
3 + a)*(b*x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) - 4*((3*a*b^2*c - 8*a^2*c^2)*x^6
 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))
/((a^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^
5*c)*x^3), -1/6*(3*((b^3*c - 4*a*b*c^2)*x^9 + (b^4 - 4*a*b^2*c)*x^6 + (a*b^
3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*
a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((3*a*b^2*c - 8*a^2*c^2)*x^6 + a
^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a^2*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a
^3*b^2*c - 4*a^4*c^2)*x^9 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)
*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.241 $\int \frac{1}{x^7(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=198

$$\frac{2(b^2-2ac+bcx^3)}{3a\,(b^2-4ac)\,x^6\sqrt{a+bx^3+cx^6}}-\frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{6a^2\,(b^2-4ac)\,x^6}+\frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3\,(b^2-4ac)\,x^3}-\frac{(5b^2-4}{}$$

```
[Out] -1/8*(-4*a*c+5*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2))/
a^(7/2)+2/3*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^6/(c*x^6+b*x^3+a)^(1/2)-1/
6*(-12*a*c+5*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^6+1/12*b*(-52*a*
c+15*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/(-4*a*c+b^2)/x^3
```

Rubi [A]

time = 0.15, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 754, 848, 820, 738, 212}

$$-\frac{(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{8a^{7/2}}+\frac{b(15b^2-52ac)\sqrt{a+bx^3+cx^6}}{12a^3x^3\,(b^2-4ac)}-\frac{(5b^2-12ac)\sqrt{a+bx^3+cx^6}}{6a^2x^6\,(b^2-4ac)}+\frac{2(-2ac+b^2+bcx^3)}{3ax^6\,(b^2-4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^6*Sqrt[a + b*x^3 + c*x^6])
 - ((5*b^2 - 12*a*c)*Sqrt[a + b*x^3 + c*x^6])/(6*a^2*(b^2 - 4*a*c)*x^6) + (
b*(15*b^2 - 52*a*c)*Sqrt[a + b*x^3 + c*x^6])/(12*a^3*(b^2 - 4*a*c)*x^3) - (
(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sqrt[a + b*x^3 + c*x^6])])
/(8*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
```

```
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7 \left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3 \left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5b^2+12ac\right)-2bcx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^3\right)}{3a\left(b^2-4ac\right)}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b(15b^2-}{12}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b(15b^2-}{12}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^6\sqrt{a+bx^3+cx^6}} - \frac{\left(5b^2-12ac\right)\sqrt{a+bx^3+cx^6}}{6a^2\left(b^2-4ac\right)x^6} + \frac{b(15b^2-}{12}$$

Mathematica [A]
time = 0.72, size = 166, normalized size = 0.84

$$\frac{-8a^3c-15b^3x^6(b+cx^3)+2a^2(b^2+10bcx^3-12c^2x^6)+abx^3(-5b^2+62bcx^3+52c^2x^6)}{12a^3\left(-b^2+4ac\right)x^6\sqrt{a+bx^3+cx^6}} + \frac{\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (-8*a^3*c - 15*b^3*x^6*(b + c*x^3) + 2*a^2*(b^2 + 10*b*c*x^3 - 12*c^2*x^6)
+ a*b*x^3*(-5*b^2 + 62*b*c*x^3 + 52*c^2*x^6))/(12*a^3*(-b^2 + 4*a*c)*x^6*Sq
rt[a + b*x^3 + c*x^6]) + ((5*b^2 - 4*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a + b
*x^3 + c*x^6])/Sqrt[a]])/(4*a^(7/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^7/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.51, size = 615, normalized size = 3.11

$$\left[-\frac{3\,\left(\left(5\,b^{4}c-24\,ab^{2}c^{2}+16\,a^{2}c^{3}\right)x^{12}+\left(5\,b^{5}-24\,ab^{3}c+16\,a^{2}bc^{2}\right)x^{9}+\left(5\,ab^{4}-24\,a^{2}b^{2}c+16\,a^{3}c^{2}\right)x^{6}\right)\sqrt{a}\log\left(-\frac{\left(b^{2}+4\,ac\right)x^{6}+8\,abx^{3}+4\,\sqrt{cx^{6}+bx^{3}+a}\,\left(bx^{3}+2\,a\right)\sqrt{a}+8\,a^{2}}{x^{6}}\right)-4\,\left(\left(15\,ab^{3}c-52\,a^{2}bc^{2}\right)x^{9}+\left(15\,ab^{4}-62\,a^{2}b^{2}c+24\,a^{3}c^{2}\right)x^{6}-2\,a^{3}b^{2}+8\,a^{4}c+5\,\left(a^{2}b^{3}-4\,a^{3}bc\right)x^{3}\right)\sqrt{cx^{6}+bx^{3}+a}}{48\,\left(\left(a^{4}b^{2}c-4\,a^{5}c^{2}\right)x^{12}+\left(a^{4}b^{3}-4\,a^{5}bc\right)x^{9}+\left(a^{5}b^{2}-4\,a^{6}c\right)x^{6}\right)},\ \frac{3\,\left(\left(5\,b^{4}c-24\,ab^{2}c^{2}+16\,a^{2}c^{3}\right)x^{12}+\left(5\,b^{5}-24\,ab^{3}c+16\,a^{2}bc^{2}\right)x^{9}+\left(5\,ab^{4}-24\,a^{2}b^{2}c+16\,a^{3}c^{2}\right)x^{6}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^{6}+bx^{3}+a}\,\left(bx^{3}+2\,a\right)\sqrt{-a}}{2\,\left(acx^{6}+abx^{3}+a^{2}\right)}\right)+2\,\left(\left(15\,ab^{3}c-52\,a^{2}bc^{2}\right)x^{9}+\left(15\,ab^{4}-62\,a^{2}b^{2}c+24\,a^{3}c^{2}\right)x^{6}-2\,a^{3}b^{2}+8\,a^{4}c+5\,\left(a^{2}b^{3}-4\,a^{3}bc\right)x^{3}\right)\sqrt{cx^{6}+bx^{3}+a}}{24\,\left(\left(a^{4}b^{2}c-4\,a^{5}c^{2}\right)x^{12}+\left(a^{4}b^{3}-4\,a^{5}bc\right)x^{9}+\left(a^{5}b^{2}-4\,a^{6}c\right)x^{6}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c
 + 16*a^2*b*c^2)*x^9 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(a)*l
og(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 + 4*sqrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a
)*sqrt(a) + 8*a^2)/x^6) - 4*((15*a*b^3*c - 52*a^2*b*c^2)*x^9 + (15*a*b^4 -
62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*a^3*b^2 + 8*a^4*c + 5*(a^2*b^3 - 4*a^3*b
*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^4*b^2*c - 4*a^5*c^2)*x^12 + (a^4*b^3
- 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a^6*c)*x^6), 1/24*(3*((5*b^4*c - 24*a*b^2*c
^2 + 16*a^2*c^3)*x^12 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^9 + (5*a*b^4
- 24*a^2*b^2*c + 16*a^3*c^2)*x^6)*sqrt(-a)*arctan(1/2*sqrt(c*x^6 + b*x^3 +
a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) + 2*((15*a*b^3*c - 52*
a^2*b*c^2)*x^9 + (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^6 - 2*a^3*b^2 + 8
*a^4*c + 5*(a^2*b^3 - 4*a^3*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^4*b^2*c
- 4*a^5*c^2)*x^12 + (a^4*b^3 - 4*a^5*b*c)*x^9 + (a^5*b^2 - 4*a^6*c)*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7}\left(a+bx^{3}+cx^{6}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^7), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.242 $\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=256

$$\frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}}-\frac{(7b^2-16ac)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9}+\frac{b(35b^2-116ac)\sqrt{a+bx^3+cx^6}}{36a^3\left(b^2-4ac\right)x^6}-\frac{(105b$$

```
[Out] 5/48*b*(-12*a*c+7*b^2)*arctanh(1/2*(b*x^3+2*a)/a^(1/2)/(c*x^6+b*x^3+a)^(1/2
))/a^(9/2)+2/3*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/x^9/(c*x^6+b*x^3+a)^(1/2)
-1/9*(-16*a*c+7*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^2/(-4*a*c+b^2)/x^9+1/36*b*(-11
6*a*c+35*b^2)*(c*x^6+b*x^3+a)^(1/2)/a^3/(-4*a*c+b^2)/x^6-1/72*(256*a^2*c^2-
460*a*b^2*c+105*b^4)*(c*x^6+b*x^3+a)^(1/2)/a^4/(-4*a*c+b^2)/x^3
```

Rubi [A]
time = 0.20, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 754, 848, 820, 738, 212}

$$\frac{5b(7b^2-12ac)\tanh^{-1}\left(\frac{2a+bx^3}{2\sqrt{a}\sqrt{a+bx^3+cx^6}}\right)}{48a^{9/2}}+\frac{b(35b^2-116ac)\sqrt{a+bx^3+cx^6}}{36a^3x^6\left(b^2-4ac\right)}-\frac{(7b^2-16ac)\sqrt{a+bx^3+cx^6}}{9a^2x^9\left(b^2-4ac\right)}-\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{a+bx^3+cx^6}}{72a^4x^3\left(b^2-4ac\right)}+\frac{2(-2ac+b^2+bcx^3)}{3ax^9\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*x^9*Sqrt[a + b*x^3 + c*x^6])
 - ((7*b^2 - 16*a*c)*Sqrt[a + b*x^3 + c*x^6])/(9*a^2*(b^2 - 4*a*c)*x^9) + (
b*(35*b^2 - 116*a*c)*Sqrt[a + b*x^3 + c*x^6])/(36*a^3*(b^2 - 4*a*c)*x^6) -
((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a + b*x^3 + c*x^6])/(72*a^4*(b^
2 - 4*a*c)*x^3) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^3)/(2*Sqrt[a]*Sq
rt[a + b*x^3 + c*x^6])])/(48*a^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^{10}\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx+cx^2\right)^{3/2}}\,dx,x,x^3\right)$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{2\text{Subst}\left(\int \frac{\frac{1}{2}\left(-7b^2+16ac\right)-3bcx}{x^4\sqrt{a+bx+cx^2}}\,dx,x,x^3\right)}{3a\left(b^2-4ac\right)}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{2\text{Subst}}{}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b(35b^2}{}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b(35b^2}{}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b(35b^2}{}$$

$$= \frac{2(b^2-2ac+bcx^3)}{3a\left(b^2-4ac\right)x^9\sqrt{a+bx^3+cx^6}} - \frac{\left(7b^2-16ac\right)\sqrt{a+bx^3+cx^6}}{9a^2\left(b^2-4ac\right)x^9} + \frac{b(35b^2}{}$$

Mathematica [A]
time = 0.95, size = 210, normalized size = 0.82

$$\frac{-32a^4c+105b^4x^9(b+cx^3)+5ab^2x^6(7b^2-106bcx^3-92c^2x^6)+8a^3(b^2+7bcx^3+16c^2x^6)+2a^2x^3(-7b^3-86b^2cx^3+244bc^2x^6+128c^3x^9)}{72a^4\left(-b^2+4ac\right)x^9\sqrt{a+bx^3+cx^6}} + \frac{5b(-7b^2+12ac)\tanh^{-1}\left(\frac{\sqrt{c}\,x^3-\sqrt{a+bx^3+cx^6}}{\sqrt{a}}\right)}{24a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] (-32*a^4*c + 105*b^4*x^9*(b + c*x^3) + 5*a*b^2*x^6*(7*b^2 - 106*b*c*x^3 - 9
2*c^2*x^6) + 8*a^3*(b^2 + 7*b*c*x^3 + 16*c^2*x^6) + 2*a^2*x^3*(-7*b^3 - 86*
b^2*c*x^3 + 244*b*c^2*x^6 + 128*c^3*x^9))/(72*a^4*(-b^2 + 4*a*c)*x^9*Sqrt[a
 + b*x^3 + c*x^6]) + (5*b*(-7*b^2 + 12*a*c)*ArcTanh[(Sqrt[c]*x^3 - Sqrt[a +
 b*x^3 + c*x^6])/Sqrt[a]])/(24*a^(9/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^10/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.55, size = 705, normalized size = 2.75

$$\left[-\frac{15\,{\left({\left(7\,b^{5} c - 40\,a b^{3} c^{2} + 48\,a^{2} b c^{3}\right)} x^{15} + {\left(7\,b^{6} - 40\,a b^{4} c + 48\,a^{2} b^{2} c^{2}\right)} x^{12} + {\left(7\,a b^{5} - 40\,a^{2} b^{3} c + 48\,a^{3} b c^{2}\right)} x^{9}\right)} \sqrt{a} \log\left(-\frac{{\left(b^{2} + 4\,a c\right)} x^{6} + 8\,a b x^{3} - 4\,\sqrt{c x^{6} + b x^{3} + a} {\left(b x^{3} + 2\,a\right)} \sqrt{a} + 8\,a^{2}}{x^{6}}\right) + 4\,{\left({\left(105\,a b^{4} c - 460\,a^{2} b^{2} c^{2} + 256\,a^{3} c^{3}\right)} x^{12} + {\left(105\,a b^{5} - 530\,a^{2} b^{3} c + 488\,a^{3} b c^{2}\right)} x^{9} + {\left(35\,a^{2} b^{4} - 172\,a^{3} b^{2} c + 128\,a^{4} c^{2}\right)} x^{6} + 8\,a^{4} b^{2} - 32\,a^{5} c - 14\,{\left(a^{3} b^{3} - 4\,a^{4} b c\right)} x^{3}\right)} \sqrt{c x^{6} + b x^{3} + a}}{288\,{\left({\left(a^{5} b^{2} c - 4\,a^{6} c^{2}\right)} x^{15} + {\left(a^{5} b^{3} - 4\,a^{6} b c\right)} x^{12} + {\left(a^{6} b^{2} - 4\,a^{7} c\right)} x^{9}\right)}}, -\frac{15\,{\left({\left(7\,b^{5} c - 40\,a b^{3} c^{2} + 48\,a^{2} b c^{3}\right)} x^{15} + {\left(7\,b^{6} - 40\,a b^{4} c + 48\,a^{2} b^{2} c^{2}\right)} x^{12} + {\left(7\,a b^{5} - 40\,a^{2} b^{3} c + 48\,a^{3} b c^{2}\right)} x^{9}\right)} \sqrt{-a} \arctan\left(\frac{\sqrt{c x^{6} + b x^{3} + a} {\left(b x^{3} + 2\,a\right)} \sqrt{-a}}{2\,{\left(a c x^{6} + a b x^{3} + a^{2}\right)}}\right) + 2\,{\left({\left(105\,a b^{4} c - 460\,a^{2} b^{2} c^{2} + 256\,a^{3} c^{3}\right)} x^{12} + {\left(105\,a b^{5} - 530\,a^{2} b^{3} c + 488\,a^{3} b c^{2}\right)} x^{9} + {\left(35\,a^{2} b^{4} - 172\,a^{3} b^{2} c + 128\,a^{4} c^{2}\right)} x^{6} + 8\,a^{4} b^{2} - 32\,a^{5} c - 14\,{\left(a^{3} b^{3} - 4\,a^{4} b c\right)} x^{3}\right)} \sqrt{c x^{6} + b x^{3} + a}}{144\,{\left({\left(a^{5} b^{2} c - 4\,a^{6} c^{2}\right)} x^{15} + {\left(a^{5} b^{3} - 4\,a^{6} b c\right)} x^{12} + {\left(a^{6} b^{2} - 4\,a^{7} c\right)} x^{9}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b
^4*c + 48*a^2*b^2*c^2)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*
sqrt(a)*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^3 - 4*sqrt(c*x^6 + b*x^3 + a)*(b*
x^3 + 2*a)*sqrt(a) + 8*a^2)/x^6) + 4*((105*a*b^4*c - 460*a^2*b^2*c^2 + 256*
a^3*c^3)*x^12 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*a^2*b
^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^3*b^3
- 4*a^4*b*c)*x^3)*sqrt(c*x^6 + b*x^3 + a))/((a^5*b^2*c - 4*a^6*c^2)*x^15 +
(a^5*b^3 - 4*a^6*b*c)*x^12 + (a^6*b^2 - 4*a^7*c)*x^9), -1/144*(15*((7*b^5*c
 - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^15 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2
)*x^12 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^9)*sqrt(-a)*arctan(1/2*s
qrt(c*x^6 + b*x^3 + a)*(b*x^3 + 2*a)*sqrt(-a)/(a*c*x^6 + a*b*x^3 + a^2)) +
2*((105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^12 + (105*a*b^5 - 530*a^
2*b^3*c + 488*a^3*b*c^2)*x^9 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x
^6 + 8*a^4*b^2 - 32*a^5*c - 14*(a^3*b^3 - 4*a^4*b*c)*x^3)*sqrt(c*x^6 + b*x^
3 + a))/((a^5*b^2*c - 4*a^6*c^2)*x^15 + (a^5*b^3 - 4*a^6*b*c)*x^12 + (a^6*b
^2 - 4*a^7*c)*x^9)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10} \left(a + b x^{3} + c x^{6}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**10*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^10), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{10}\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^10*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.243 $\int \frac{x^3}{\left(a+bx^3+cx^6\right)^{3/2}} \, dx$

Optimal. Leaf size=143

$$\frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{3};\frac{3}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,3/2,3/2,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$\frac{x^4\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{4}{3};\frac{3}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/3, 3/2, 3/2, 7/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^3 + c*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{\cdots}{b+\cdots}\right.}}{a\sqrt{a+bx^3+cx^6}}$$

$$= \frac{x^4\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{3};\frac{3}{2},\frac{3}{2};\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},\cdots\right.}{4a\sqrt{a+bx^3+cx^6}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 340 vs. 2(143) = 286.
time = 10.21, size = 340, normalized size = 2.38

$$\frac{x\left(-2(b+2cx^3)+2b\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+cx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)}{3\left(b^2-4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(-2*(b + 2*c*x^3) + 2*b*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt
[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*
c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^
3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(3*(b^2 - 4*a*c)*Sqrt[a + b*x^
3 + c*x^6])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x^3/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*
x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx^3+cx^6)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x**3/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x^3/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.244 $\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,3/2,3/2,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1399, 524}

$$\frac{x^2\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/3, 3/2, 3/2, 5/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^3 + c*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{x}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{}{b+\sqrt{}}\right.}}{a\sqrt{a+bx^3+cx^6}}$$

$$= \frac{x^2\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\right.}{2a\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 362 vs. 2(143) = 286.
time = 10.34, size = 362, normalized size = 2.53

$$\frac{x^2\left(-20(b^2-2ac+bcx^3)+5(b^2+4ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+8bcx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)}{30a\left(-b^2+4ac\right)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x^2*(-20*(b^2 - 2*a*c + b*c*x^3) + 5*(b^2 + 4*a*c)*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + 8*b*c*x^3*Sqrt[(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3,
(-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(
30*a*(-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(x/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*x/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^
6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + bx^3 + cx^6)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(x/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(x/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.245 $\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

```
[Out] x*AppellF1(1/3,3/2,3/2,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/a/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1362, 440}

$$\frac{x\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^(-3/2),x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[1/3, 3/2, 3/2, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a
*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^3 + c*x^6])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{}{b+}\right.}}{a\sqrt{a+bx^3+cx^6}}$$

$$= \frac{x\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\right.}{a\sqrt{a+bx^3+cx^6}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 359 vs. 2(138) = 276.
time = 10.32, size = 359, normalized size = 2.60

$$\frac{x\left(-4(b^2-2ac+bcx^3)-2(b^2-8ac)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+bcx^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)}{6a(-b^2+4ac)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^(-3/2),x]
```

```
[Out] (x*(-4*(b^2 - 2*a*c + b*c*x^3) - 2*(b^2 - 8*a*c)*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + b*c*x^3*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/(6*a*(
-b^2 + 4*a*c)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6
+ 2*a*b*x^3 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{3} + cx^{6}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**3 + c*x**6)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,x^6+b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int(1/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.246 $\int \frac{1}{x^2(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

```
[Out] -AppellF1(-1/3,3/2,3/2,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*
a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/a/x/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-1/3, 3/2, 3/2, 2/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*
a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*x^3 + c*x^6]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\cdots\right.}}{a\sqrt{a+bx^3+cx^6}}$$

$$= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{3};\frac{3}{2},\frac{3}{2};\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\cdots\right.}{ax\sqrt{a+bx^3+cx^6}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 407 vs. 2(141) = 282.

time = 10.50, size = 407, normalized size = 2.89

$$-\frac{5b(-5b^2+12ac)x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)-4\left(60a^2c-25b^2x^3(b+cx^3)+5a(-3b^2+18bcx^3+16c^2x^6)+2c(5b^2-16ac)x^6\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)}{60a^2(b^2-4ac)x\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -1/60*(5*b*(-5*b^2 + 12*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])] - 4*(60*a^2*c - 25*b^2*x^3*(b + c*x^3)
 + 5*a*(-3*b^2 + 18*b*c*x^3 + 16*c^2*x^6) + 2*c*(5*b^2 - 16*a*c)*x^6*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/3, 1/2, 1/2, 8/3,
 (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])]))/
(a^2*(b^2 - 4*a*c)*x*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int(1/x^2/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^14 + 2*b*c*x^11 + (b^2 + 2*a*c)*x^8
 + 2*a*b*x^5 + a^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3 + c*x^6)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.247 $\int \frac{1}{x^3(a+bx^3+cx^6)^{3/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

```
[Out] -1/2*AppellF1(-2/3,3/2,3/2,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+
(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^3/(b
+(-4*a*c+b^2)^(1/2)))^(1/2)/a/x^2/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$-\frac{\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^3)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-2/3, 3/2, 3/2, 1/3, (-2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*x^2*Sqrt[a + b*x^3 + c*x^
6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{1}{x^3 \left(a+bx^3+cx^6\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(\right.}}{a\sqrt{a+bx^3+cx^6}}$$

$$= -\frac{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{3};\frac{3}{2},\frac{3}{2};\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4a}}\right.}{2ax^2\sqrt{a+bx^3+cx^6}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 405 vs. 2(143) = 286.
time = 10.44, size = 405, normalized size = 2.83

$$\frac{-48a^2c+28b^2x^3(b+cx^3)+4a(3b^2-24bcx^3-20c^2x^6)+2b(7b^2-36ac)x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+c(-7b^2+20ac)x^6\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{24a^2(-b^2+4ac)x^2\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x]
[Out] (-48*a^2*c + 28*b^2*x^3*(b + c*x^3) + 4*a*(3*b^2 - 24*b*c*x^3 - 20*c^2*x^6)
 + 2*b*(7*b^2 - 36*a*c)*x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^3)/(-b + Sqrt[b^2 - 4*a*c])] + c*(-7*b^2 + 20*a*c)*x^6*Sqrt[(b - Sqrt[b^
2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/3, 1/2, 1/2, 7/3, (-2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(24*a^2*(-b^
2 + 4*a*c)*x^2*Sqrt[a + b*x^3 + c*x^6])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
[Out] int(1/x^3/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)/(c^2*x^15 + 2*b*c*x^12 + (b^2 + 2*a*c)*x^9
 + 2*a*b*x^6 + a^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**6+b*x**3+a)**(3/2),x)
[Out] Integral(1/(x**3*(a + b*x**3 + c*x**6)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^6 + b*x^3 + a)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(c\,x^6+b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3 + c*x^6)^(3/2)),x)
[Out] int(1/(x^3*(a + b*x^3 + c*x^6)^(3/2)), x)
```

3.248 $\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx$

Optimal. Leaf size=101

$$\frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{4+m}}{d^4(4+m)}+\frac{\left(b^2+2ac\right)(dx)^{7+m}}{d^7(7+m)}+\frac{2bc(dx)^{10+m}}{d^{10}(10+m)}+\frac{c^2(dx)^{13+m}}{d^{13}(13+m)}$$

```
[Out] a^2*(d*x)^(1+m)/d/(1+m)+2*a*b*(d*x)^(4+m)/d^4/(4+m)+(2*a*c+b^2)*(d*x)^(7+m)
/d^7/(7+m)+2*b*c*(d*x)^(10+m)/d^10/(10+m)+c^2*(d*x)^(13+m)/d^13/(13+m)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1367}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{2ab(dx)^{m+4}}{d^4(m+4)}+\frac{2bc(dx)^{m+10}}{d^{10}(m+10)}+\frac{c^2(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(4 + m))/(d^4*(4 + m)) + ((b
^2 + 2*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*c*(d*x)^(10 + m))/(d^10*(10
 + m)) + (c^2*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^3+cx^6\right)^2 \, dx &= \int \left(a^2(dx)^m+\frac{2ab(dx)^{3+m}}{d^3}+\frac{\left(b^2+2ac\right)(dx)^{6+m}}{d^6}+\frac{2bc(dx)^{9+m}}{d^9}+\frac{c^2(dx)^{1}}{d^{12}}\right.\\ &= \frac{a^2(dx)^{1+m}}{d(1+m)}+\frac{2ab(dx)^{4+m}}{d^4(4+m)}+\frac{\left(b^2+2ac\right)(dx)^{7+m}}{d^7(7+m)}+\frac{2bc(dx)^{10+m}}{d^{10}(10+m)}+\frac{c^2(dx)^{13}}{d^{13}(13+}\end{aligned}$$

Mathematica [A]
time = 0.54, size = 70, normalized size = 0.69

$$(dx)^m\left(\frac{a^2x}{1+m}+\frac{2abx^4}{4+m}+\frac{\left(b^2+2ac\right)x^7}{7+m}+\frac{2bcx^{10}}{10+m}+\frac{c^2x^{13}}{13+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] (d*x)^m*((a^2*x)/(1 + m) + (2*a*b*x^4)/(4 + m) + ((b^2 + 2*a*c)*x^7)/(7 + m
) + (2*b*c*x^10)/(10 + m) + (c^2*x^13)/(13 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(101) = 202$.
time = 0.01, size = 301, normalized size = 2.98

method	result
gosper	$\frac{x\left(c^2m^4x^{12}+22c^2m^3x^{12}+159c^2m^2x^{12}+2bc\,m^4x^9+418m\,x^{12}c^2+50bc\,m^3x^9+280c^2x^{12}+390bc\,m^2x^9+2ac\,m^4x^6+b^2m^4x^6+1070m\,x^9b\right.}{}$
risch	$\frac{x\left(c^2m^4x^{12}+22c^2m^3x^{12}+159c^2m^2x^{12}+2bc\,m^4x^9+418m\,x^{12}c^2+50bc\,m^3x^9+280c^2x^{12}+390bc\,m^2x^9+2ac\,m^4x^6+b^2m^4x^6+1070m\,x^9b\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(c^2*m^4*x^12+22*c^2*m^3*x^12+159*c^2*m^2*x^12+2*b*c*m^4*x^9+418*c^2*m*x^
12+50*b*c*m^3*x^9+280*c^2*x^12+390*b*c*m^2*x^9+2*a*c*m^4*x^6+b^2*m^4*x^6+10
70*b*c*m*x^9+56*a*c*m^3*x^6+28*b^2*m^3*x^6+728*b*c*x^9+498*a*c*m^2*x^6+249*
b^2*m^2*x^6+2*a*b*m^4*x^3+1484*a*c*m*x^6+742*b^2*m*x^6+62*a*b*m^3*x^3+1040*
a*c*x^6+520*b^2*x^6+642*a*b*m^2*x^3+a^2*m^4+2402*a*b*m*x^3+34*a^2*m^3+1820*
a*b*x^3+411*a^2*m^2+2074*a^2*m+3640*a^2)*(d*x)^m/(13+m)/(10+m)/(7+m)/(4+m)/
(1+m)
```

Maxima [A]
time = 0.34, size = 110, normalized size = 1.09

$$\frac{c^2d^mx^{13}x^m}{m+13}+\frac{2\,bcd^mx^{10}x^m}{m+10}+\frac{b^2d^mx^7x^m}{m+7}+\frac{2\,acd^mx^7x^m}{m+7}+\frac{2\,abd^mx^4x^m}{m+4}+\frac{(dx)^{m+1}\,a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] c^2*d^m*x^13*x^m/(m + 13) + 2*b*c*d^m*x^10*x^m/(m + 10) + b^2*d^m*x^7*x^m/(
m + 7) + 2*a*c*d^m*x^7*x^m/(m + 7) + 2*a*b*d^m*x^4*x^m/(m + 4) + (d*x)^(m +
 1)*a^2/(d*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(101) = 202$.
time = 0.41, size = 241, normalized size = 2.39

$$\frac{\left((c^2m^4+22\,c^2m^3+159\,c^2m^2+418\,c^2m+280\,c^2)x^{13}+2\,(bcm^4+25\,bcm^3+195\,bcm^2+535\,bcm+364\,bc)x^{10}+((b^2+2\,ac)m^4+28\,(b^2+2\,ac)m^3+249\,(b^2+2\,ac)m^2+520\,b^2+1040\,ac+742\,(b^2+2\,ac)m)x^7+2\,(abm^4+31\,abm^3+321\,abm^2+1201\,abm+910\,ab)x^4+(a^2m^4+34\,a^2m^3+411\,a^2m^2+2074\,a^2m+3640\,a^2)x\right)(dx)^m}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 22*c^2*m^3 + 159*c^2*m^2 + 418*c^2*m + 280*c^2)*x^13 + 2*(b*c*m
^4 + 25*b*c*m^3 + 195*b*c*m^2 + 535*b*c*m + 364*b*c)*x^10 + ((b^2 + 2*a*c)*
m^4 + 28*(b^2 + 2*a*c)*m^3 + 249*(b^2 + 2*a*c)*m^2 + 520*b^2 + 1040*a*c + 7
42*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 31*a*b*m^3 + 321*a*b*m^2 + 1201*a*b*
m + 910*a*b)*x^4 + (a^2*m^4 + 34*a^2*m^3 + 411*a^2*m^2 + 2074*a^2*m + 3640*
a^2)*x)*(d*x)^m/(m^5 + 35*m^4 + 445*m^3 + 2485*m^2 + 5714*m + 3640)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1459 vs. $2(90) = 180$.
time = 0.81, size = 1459, normalized size = 14.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(12*x**12) - 2*a*b/(9*x**9) - a*c/(3*x**6) - b**2/(6*x**6
) - 2*b*c/(3*x**3) + c**2*log(x))/d**13, Eq(m, -13)), ((-a**2/(9*x**9) - a*
b/(3*x**6) - 2*a*c/(3*x**3) - b**2/(3*x**3) + 2*b*c*log(x) + c**2*x**3/3)/d
**10, Eq(m, -10)), ((-a**2/(6*x**6) - 2*a*b/(3*x**3) + 2*a*c*log(x) + b**2*
log(x) + 2*b*c*x**3/3 + c**2*x**6/6)/d**7, Eq(m, -7)), ((-a**2/(3*x**3) + 2
*a*b*log(x) + 2*a*c*x**3/3 + b**2*x**3/3 + b*c*x**6/3 + c**2*x**9/9)/d**4,
Eq(m, -4)), ((a**2*log(x) + 2*a*b*x**3/3 + a*c*x**6/3 + b**2*x**6/6 + 2*b*c
*x**9/9 + c**2*x**12/12)/d, Eq(m, -1)), (a**2*m**4*x*(d*x)**m/(m**5 + 35*m*
*4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 34*a**2*m**3*x*(d*x)**m/(m**5
+ 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 411*a**2*m**2*x*(d*x)**
m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2074*a**2*m*x*(
d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 3640*a**2
*x*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*a*b
*m**4*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640)
 + 62*a*b*m**3*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*
m + 3640) + 642*a*b*m**2*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m*
*2 + 5714*m + 3640) + 2402*a*b*m*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 +
 2485*m**2 + 5714*m + 3640) + 1820*a*b*x**4*(d*x)**m/(m**5 + 35*m**4 + 445*
m**3 + 2485*m**2 + 5714*m + 3640) + 2*a*c*m**4*x**7*(d*x)**m/(m**5 + 35*m**
4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 56*a*c*m**3*x**7*(d*x)**m/(m**5
 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 498*a*c*m**2*x**7*(d*x
)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 1484*a*c*m*x
**7*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 1040
*a*c*x**7*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640)
+ b**2*m**4*x**7*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
 3640) + 28*b**2*m**3*x**7*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2
+ 5714*m + 3640) + 249*b**2*m**2*x**7*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 +
 2485*m**2 + 5714*m + 3640) + 742*b**2*m*x**7*(d*x)**m/(m**5 + 35*m**4 + 44
```

```
5*m**3 + 2485*m**2 + 5714*m + 3640) + 520*b**2*x**7*(d*x)**m/(m**5 + 35*m**
4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 2*b*c*m**4*x**10*(d*x)**m/(m**5
 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 50*b*c*m**3*x**10*(d*x
)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 390*b*c*m**2
*x**10*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 1
070*b*c*m*x**10*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m +
3640) + 728*b*c*x**10*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 571
4*m + 3640) + c**2*m**4*x**13*(d*x)**m/(m**5 + 35*m**4 + 445*m**3 + 2485*m*
*2 + 5714*m + 3640) + 22*c**2*m**3*x**13*(d*x)**m/(m**5 + 35*m**4 + 445*m**
3 + 2485*m**2 + 5714*m + 3640) + 159*c**2*m**2*x**13*(d*x)**m/(m**5 + 35*m*
*4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 418*c**2*m*x**13*(d*x)**m/(m**
5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640) + 280*c**2*x**13*(d*x)*
*m/(m**5 + 35*m**4 + 445*m**3 + 2485*m**2 + 5714*m + 3640), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 449 vs. $2(101) = 202$.
time = 4.34, size = 449, normalized size = 4.45

$$\frac{(dx)^m c^2m^4x^{13} + 22\,(dx)^m c^2m^3x^{13} + 159\,(dx)^m c^2m^2x^{13} + 2\,(dx)^m bcm^4x^{10} + 418\,(dx)^m c^2mx^{13} + 50\,(dx)^m bcm^3x^{10} + 280\,(dx)^m c^2x^{13} + 390\,(dx)^m bcm^2x^{10} + (dx)^m b^2m^4x^7 + 2\,(dx)^m acm^4x^7 + 1070\,(dx)^m bcmx^{10} + 28\,(dx)^m b^2m^3x^7 + 56\,(dx)^m acm^3x^7 + 728\,(dx)^m bcx^{10} + 249\,(dx)^m b^2m^2x^7 + 498\,(dx)^m acm^2x^7 + 2\,(dx)^m abm^4x^4 + 742\,(dx)^m b^2mx^7 + 1484\,(dx)^m acmx^7 + 62\,(dx)^m abm^3x^4 + 520\,(dx)^m b^2x^7 + 1040\,(dx)^m acx^7 + 642\,(dx)^m abm^2x^4 + (dx)^m a^2m^4x + 2402\,(dx)^m abmx^4 + 34\,(dx)^m a^2m^3x + 1820\,(dx)^m abx^4 + 411\,(dx)^m a^2m^2x + 2074\,(dx)^m a^2mx + 3640\,(dx)^m a^2x}{m^5 + 35\,m^4 + 445\,m^3 + 2485\,m^2 + 5714\,m + 3640}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*c^2*m^4*x^13 + 22*(d*x)^m*c^2*m^3*x^13 + 159*(d*x)^m*c^2*m^2*x^13
+ 2*(d*x)^m*b*c*m^4*x^10 + 418*(d*x)^m*c^2*m*x^13 + 50*(d*x)^m*b*c*m^3*x^10
 + 280*(d*x)^m*c^2*x^13 + 390*(d*x)^m*b*c*m^2*x^10 + (d*x)^m*b^2*m^4*x^7 +
2*(d*x)^m*a*c*m^4*x^7 + 1070*(d*x)^m*b*c*m*x^10 + 28*(d*x)^m*b^2*m^3*x^7 +
56*(d*x)^m*a*c*m^3*x^7 + 728*(d*x)^m*b*c*x^10 + 249*(d*x)^m*b^2*m^2*x^7 + 4
98*(d*x)^m*a*c*m^2*x^7 + 2*(d*x)^m*a*b*m^4*x^4 + 742*(d*x)^m*b^2*m*x^7 + 14
84*(d*x)^m*a*c*m*x^7 + 62*(d*x)^m*a*b*m^3*x^4 + 520*(d*x)^m*b^2*x^7 + 1040*
(d*x)^m*a*c*x^7 + 642*(d*x)^m*a*b*m^2*x^4 + (d*x)^m*a^2*m^4*x + 2402*(d*x)^
m*a*b*m*x^4 + 34*(d*x)^m*a^2*m^3*x + 1820*(d*x)^m*a*b*x^4 + 411*(d*x)^m*a^2
*m^2*x + 2074*(d*x)^m*a^2*m*x + 3640*(d*x)^m*a^2*x)/(m^5 + 35*m^4 + 445*m^3
 + 2485*m^2 + 5714*m + 3640)
```

Mupad [B]
time = 1.52, size = 260, normalized size = 2.57

$$(d\,x)^m\,\left(\frac{c^2\,x^{13}\,\left(m^4+22\,m^3+159\,m^2+418\,m+280\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}+\frac{x^7\,\left(b^2+2\,a\,c\right)\,\left(m^4+28\,m^3+249\,m^2+742\,m+520\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}+\frac{a^2\,x\,\left(m^4+34\,m^3+411\,m^2+2074\,m+3640\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}+\frac{2\,a\,b\,x^4\,\left(m^4+31\,m^3+321\,m^2+1201\,m+910\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}+\frac{2\,b\,c\,x^{10}\,\left(m^4+25\,m^3+195\,m^2+535\,m+364\right)}{m^5+35\,m^4+445\,m^3+2485\,m^2+5714\,m+3640}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^2,x)
```

```
[Out] (d*x)^m*((c^2*x^13*(418*m + 159*m^2 + 22*m^3 + m^4 + 280))/(5714*m + 2485*m
^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (x^7*(2*a*c + b^2)*(742*m + 249*m^2 +
 28*m^3 + m^4 + 520))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) +
```

```
 (a^2*x*(2074*m + 411*m^2 + 34*m^3 + m^4 + 3640))/(5714*m + 2485*m^2 + 445*
m^3 + 35*m^4 + m^5 + 3640) + (2*a*b*x^4*(1201*m + 321*m^2 + 31*m^3 + m^4 +
910))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4 + m^5 + 3640) + (2*b*c*x^10*(53
5*m + 195*m^2 + 25*m^3 + m^4 + 364))/(5714*m + 2485*m^2 + 445*m^3 + 35*m^4
+ m^5 + 3640))
```

3.249 $\int (dx)^m \left(a+bx^3+cx^6\right) \, dx$

Optimal. Leaf size=52

$$\frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{4+m}}{d^4(4+m)}+\frac{c(dx)^{7+m}}{d^7(7+m)}$$

```
[Out] a*(d*x)^(1+m)/d/(1+m)+b*(d*x)^(4+m)/d^4/(4+m)+c*(d*x)^(7+m)/d^7/(7+m)
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{14\}$

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+4}}{d^4(m+4)}+\frac{c(dx)^{m+7}}{d^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6),x]
```

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(4 + m))/(d^4*(4 + m)) + (c*(d*x)^
(7 + m))/(d^7*(7 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^3+cx^6\right) \, dx &= \int \left(a(dx)^m+\frac{b(dx)^{3+m}}{d^3}+\frac{c(dx)^{6+m}}{d^6}\right) \, dx\\ &= \frac{a(dx)^{1+m}}{d(1+m)}+\frac{b(dx)^{4+m}}{d^4(4+m)}+\frac{c(dx)^{7+m}}{d^7(7+m)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 35, normalized size = 0.67

$$x(dx)^m \left(\frac{a}{1+m}+\frac{bx^3}{4+m}+\frac{cx^6}{7+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6),x]
[Out] x*(d*x)^m*(a/(1 + m) + (b*x^3)/(4 + m) + (c*x^6)/(7 + m))
```

Maple [A]
time = 0.02, size = 51, normalized size = 0.98

method	result	size
norman	$\frac{a x\,\mathrm{e}^{m\ln(dx)}}{1+m}+\frac{b\,x^{4}\mathrm{e}^{m\ln(dx)}}{4+m}+\frac{c\,x^{7}\mathrm{e}^{m\ln(dx)}}{7+m}$	51
gosper	$\frac{x\left(c\,m^{2}x^{6}+5cm\,x^{6}+4c\,x^{6}+b\,m^{2}x^{3}+8bm\,x^{3}+7b\,x^{3}+a\,m^{2}+11am+28a\right)(dx)^{m}}{(7+m)(4+m)(1+m)}$	78
risch	$\frac{x\left(c\,m^{2}x^{6}+5cm\,x^{6}+4c\,x^{6}+b\,m^{2}x^{3}+8bm\,x^{3}+7b\,x^{3}+a\,m^{2}+11am+28a\right)(dx)^{m}}{(7+m)(4+m)(1+m)}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] a/(1+m)*x*exp(m*ln(d*x))+b/(4+m)*x^4*exp(m*ln(d*x))+c/(7+m)*x^7*exp(m*ln(d*
x))
```

Maxima [A]
time = 0.29, size = 50, normalized size = 0.96

$$\frac{cd^{m}x^{7}x^{m}}{m+7}+\frac{bd^{m}x^{4}x^{m}}{m+4}+\frac{(dx)^{m+1}\,a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] c*d^m*x^7*x^m/(m + 7) + b*d^m*x^4*x^m/(m + 4) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Fricas [A]
time = 0.36, size = 71, normalized size = 1.37

$$\frac{\left((cm^{2}+5\,cm+4\,c)x^{7}+(bm^{2}+8\,bm+7\,b)x^{4}+(am^{2}+11\,am+28\,a)x\right)(dx)^{m}}{m^{3}+12\,m^{2}+39\,m+28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] ((c*m^2 + 5*c*m + 4*c)*x^7 + (b*m^2 + 8*b*m + 7*b)*x^4 + (a*m^2 + 11*a*m +
28*a)*x)*(d*x)^m/(m^3 + 12*m^2 + 39*m + 28)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(42) = 84.
time = 0.31, size = 299, normalized size = 5.75

$$\begin{cases} \frac{-\frac{a}{6x^{6}}-\frac{b}{3x^{3}}+c\log(x)}{d^{7}} & \text{for}\ m=-7 \\ \frac{-\frac{a}{3x^{3}}+b\log(x)+\frac{cx^{3}}{3}}{d^{4}} & \text{for}\ m=-4 \\ \frac{a\log(x)+\frac{bx^{3}}{3}+\frac{cx^{6}}{6}}{d} & \text{for}\ m=-1 \\ \frac{am^{2}x(dx)^{m}}{m^{3}+12m^{2}+39m+28}+\frac{11amx(dx)^{m}}{m^{3}+12m^{2}+39m+28}+\frac{28ax(dx)^{m}}{m^{3}+12m^{2}+39m+28}+\frac{bm^{2}x^{4}(dx)^{m}}{m^{3}+12m^{2}+39m+28}+\frac{8bmx^{4}(dx)^{m}}{m^{3}+12m^{2}+39m+28}+\frac{7bx^{4}(dx)^{m}}{m^{3}+12m^{2}+39m+28}+\frac{cm^{2}x^{7}(dx)^{m}}{m^{3}+12m^{2}+39m+28}+\frac{5cmx^{7}(dx)^{m}}{m^{3}+12m^{2}+39m+28}+\frac{4cx^{7}(dx)^{m}}{m^{3}+12m^{2}+39m+28} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a),x)
```

```
[Out] Piecewise(((-a/(6*x**6) - b/(3*x**3) + c*log(x))/d**7, Eq(m, -7)), ((-a/(3*
x**3) + b*log(x) + c*x**3/3)/d**4, Eq(m, -4)), ((a*log(x) + b*x**3/3 + c*x*
*6/6)/d, Eq(m, -1)), (a*m**2*x*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 11*a
*m*x*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 28*a*x*(d*x)**m/(m**3 + 12*m**
2 + 39*m + 28) + b*m**2*x**4*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 8*b*m*
x**4*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 7*b*x**4*(d*x)**m/(m**3 + 12*m
**2 + 39*m + 28) + c*m**2*x**7*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 5*c*
m*x**7*(d*x)**m/(m**3 + 12*m**2 + 39*m + 28) + 4*c*x**7*(d*x)**m/(m**3 + 12
*m**2 + 39*m + 28), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(52) = 104.
time = 3.26, size = 119, normalized size = 2.29

$$\frac{(dx)^m cm^2x^7 + 5\,(dx)^m cmx^7 + 4\,(dx)^m cx^7 + (dx)^m bm^2x^4 + 8\,(dx)^m bmx^4 + 7\,(dx)^m bx^4 + (dx)^m am^2x + 11\,(dx)^m amx + 28\,(dx)^m ax}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*c*m^2*x^7 + 5*(d*x)^m*c*m*x^7 + 4*(d*x)^m*c*x^7 + (d*x)^m*b*m^2*x^
4 + 8*(d*x)^m*b*m*x^4 + 7*(d*x)^m*b*x^4 + (d*x)^m*a*m^2*x + 11*(d*x)^m*a*m*
x + 28*(d*x)^m*a*x)/(m^3 + 12*m^2 + 39*m + 28)
```

Mupad [B]
time = 1.36, size = 89, normalized size = 1.71

$$(d\,x)^m\,\left(\frac{b\,x^4\,\left(m^2+8\,m+7\right)}{m^3+12\,m^2+39\,m+28}+\frac{c\,x^7\,\left(m^2+5\,m+4\right)}{m^3+12\,m^2+39\,m+28}+\frac{a\,x\,\left(m^2+11\,m+28\right)}{m^3+12\,m^2+39\,m+28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6),x)
```

```
[Out] (d*x)^m*((b*x^4*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (c*x^7*(5*m +
 m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (a*x*(11*m + m^2 + 28))/(39*m + 12*
m^2 + m^3 + 28))
```

3.250 $\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=173

$$\frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)}-\frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

```
[Out] 2*c*(d*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*(d*x)^(1+m
)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/d/(
1+m)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.16, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1389, 371}

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b
 - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m
)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(
1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(dx)^m}{a+bx^3+cx^6}\,dx = \frac{c\int \frac{(dx)^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{}{b+\sqrt{b}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.10, size = 84, normalized size = 0.49

$$\frac{(dx)^m \text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1^2+2c\#1^5}\&\right]}{3m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((d*x)^m*RootSum[a + b*#1^3 + c*#1^6 & , Hypergeometric2F1[-m, -m, 1 - m, -
(#1/(x - #1))]/((x/(x - #1))^m*(b*#1^2 + 2*c*#1^5)) & ])/(3*m)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^6+b*x^3+a),x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] integral((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^3 + c*x^6),x)
[Out] int((d*x)^m/(a + b*x^3 + c*x^6), x)
```

3.251 $\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^2}\,dx$

Optimal. Leaf size=315

$$\frac{(dx)^{1+m}\left(b^2-2ac+bcx^3\right)}{3a\left(b^2-4ac\right)d\left(a+bx^3+cx^6\right)}+\frac{c\left(b^2(2-m)+b\sqrt{b^2-4ac}\,(2-m)-4ac(5-m)\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+}{3}\right.}{3a\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)d(1+m)}$$

```
[Out] 1/3*(d*x)^(1+m)*(b*c*x^3-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^6+b*x^3+a)-1/3*c*
(d*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-2*c*x^3/(b+(-4*a*c+b^2)^(
1/2)))*(b^2*(2-m)-4*a*c*(5-m)-b*(2-m)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3
/2)/d/(1+m)/(b+(-4*a*c+b^2)^(1/2))+1/3*c*(d*x)^(1+m)*hypergeom([1, 1/3+1/3*
m],[4/3+1/3*m],-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))*(b^2*(2-m)-4*a*c*(5-m)+b*(2
-m)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.48, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1380, 1524, 371}

$$\frac{c(dx)^{m+1}\left(b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)}{3ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(2-m)\sqrt{b^2-4ac}-4ac(5-m)+b^2(2-m)\right)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{3ad(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^3\right)}{3ad\left(b^2-4ac\right)\left(a+bx^3+cx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^3))/(3*a*(b^2 - 4*a*c)*d*(a + b*x^3 + c
*x^6)) + (c*(b^2*(2 - m) + b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*
x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c])])/(3*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m))
 - (c*(b^2*(2 - m) - b*Sqrt[b^2 - 4*a*c]*(2 - m) - 4*a*c*(5 - m))*(d*x)^(1
+ m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c])])/(3*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1380

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
```

```
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b
^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(m + n*(
p + 1) + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(m + n*(2*p + 3) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] && ILtQ[p, -1]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^2}\,dx = \frac{(dx)^{1+m}(b^2-2ac+bcx^3)}{3a(b^2-4ac)d(a+bx^3+cx^6)} - \frac{\int \frac{(dx)^m(-b^2(2-m)+2ac(5-m)-bc(2-m)x^3)}{a+bx^3+cx^6}\,dx}{3a(b^2-4ac)}$$

$$= \frac{(dx)^{1+m}(b^2-2ac+bcx^3)}{3a(b^2-4ac)d(a+bx^3+cx^6)} - \frac{\left(c\left(b^2(2-m)-b\sqrt{b^2-4ac}\,(2-m)-4ac(5-\right.\right.}{6a(b^2-4ac)^{3/2}}$$

$$= \frac{(dx)^{1+m}(b^2-2ac+bcx^3)}{3a(b^2-4ac)d(a+bx^3+cx^6)} + \frac{c\left(b^2(2-m)+b\sqrt{b^2-4ac}\,(2-m)-4ac(5-\right.}{3a(b^2-4ac)^{3/2}\left(b-\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.15, size = 78, normalized size = 0.25

$$\frac{x(dx)^m F_1\left(\frac{1+m}{3};2,2;\frac{4+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{a^2(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^2,x]
```

```
[Out] (x*(d*x)^m*AppellF1[(1 + m)/3, 2, 2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(a^2*(1 + m))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(c\,x^6+b\,x^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^6+b*x^3+a)^2,x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a*c)*x^6 + 2*a*b*x^3 + a^
2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^m}{{\left(c\,x^6+b\,x^3+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^3 + c*x^6)^2,x)
```

```
[Out] int((d*x)^m/(a + b*x^3 + c*x^6)^2, x)
```

3.252 $\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx$

Optimal. Leaf size=158

$$\frac{a(dx)^{1+m}\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1+m}{3};-\frac{3}{2},-\frac{3}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] a*(d*x)^(1+m)*AppellF1(1/3+1/3*m,-3/2,-3/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^
2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/d/(1+m)/(1
+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{a(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{m+1}{3};-\frac{3}{2},-\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -3/2, -3/2, (4
 + m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
```

```
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int (dx)^m \left(a+bx^3+cx^6\right)^{3/2} dx = \frac{\left(a\sqrt{a+bx^3+cx^6}\right) \int (dx)^m \left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2} \left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right.}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{a(dx)^{1+m}\sqrt{a+bx^3+cx^6}\, F_1\left(\frac{1+m}{3};-\frac{3}{2},-\frac{3}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{}{b+\sqrt{b}}\right.}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 357 vs. 2(158) = 316.
time = 0.78, size = 357, normalized size = 2.26

$$\frac{x(dx)^m\sqrt{a+bx^3+cx^6}\left(a(28+11m+m^2)F_1\left(\frac{1+m}{3};-\frac{1}{2},-\frac{1}{2};\frac{4+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+(1+m)x^3\left(b(7+m)F_1\left(\frac{4+m}{3};-\frac{1}{2},-\frac{1}{2};\frac{7+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)+c(4+m)x^3F_1\left(\frac{7+m}{3};-\frac{1}{2},-\frac{1}{2};\frac{10+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{(1+m)(4+m)(7+m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^3 + c*x^6]*(a*(28 + 11*m + m^2)*AppellF1[(1 + m)/3,
 -1/2, -1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b +
 Sqrt[b^2 - 4*a*c])] + (1 + m)*x^3*(b*(7 + m)*AppellF1[(4 + m)/3, -1/2, -1/
2, (7 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2
- 4*a*c])] + c*(4 + m)*x^3*AppellF1[(7 + m)/3, -1/2, -1/2, (10 + m)/3, (-2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])))/((1
+ m)*(4 + m)*(7 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (dx)^m \left(c\,x^6+b\,x^3+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

```
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + bx^3 + cx^6\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^(3/2)*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(c\,x^6+b\,x^3+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^(3/2),x)
[Out] int((d*x)^m*(a + b*x^3 + c*x^6)^(3/2), x)
```

3.253 $\int (dx)^m \sqrt{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{1+m}\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1+m}{3};-\frac{1}{2},-\frac{1}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/3+1/3*m,-1/2,-1/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(c*x^6+b*x^3+a)^(1/2)/d/(1+m)/(1+2
*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/
2)
```

Rubi [A]
time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{(dx)^{m+1}\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{m+1}{3};-\frac{1}{2},-\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4 +
 m)/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c
])])/(d*(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
```

```
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int (dx)^m\sqrt{a+bx^3+cx^6}\,dx = \frac{\sqrt{a+bx^3+cx^6}\int (dx)^m\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{(dx)^{1+m}\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1+m}{3};-\frac{1}{2},-\frac{1}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-}}\right.}{d(1+m)\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A]
time = 0.50, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\sqrt{a+bx^3+cx^6}\,F_1\left(\frac{1+m}{3};-\frac{1}{2},-\frac{1}{2};\frac{4+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(d*x)^m*Sqrt[a + b*x^3 + c*x^6]*AppellF1[(1 + m)/3, -1/2, -1/2, (4 + m)/
3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + bx^3 + cx^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\sqrt{c\,x^6+b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^(1/2),x)
[Out] int((d*x)^m*(a + b*x^3 + c*x^6)^(1/2), x)
```

3.254 $\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{3};\frac{1}{2},\frac{1}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/3+1/3*m,1/2,1/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(1+m)/(c*x^6+b*x^3+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{3};\frac{1}{2},\frac{1}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 +
m)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a+bx^3+cx^6}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+}}}{\sqrt{a+bx^3+cx^6}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{3};\frac{1}{2},\frac{1}{2};\frac{4+m}{3};-\frac{2c}{b-\sqrt{b^2}}\right.}{d(1+m)\sqrt{a+bx^3+cx^6}}$$

Mathematica [A]
time = 0.71, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{3};\frac{1}{2},\frac{1}{2};\frac{4+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a + b*x^3 + c*x^6],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1
 + m)/3, 1/2, 1/2, (4 + m)/3, (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)
/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + b*x^3 + c*x^6])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{c\,x^6+b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

```
[Out] int((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a + bx^3 + cx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a)**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt(a + b*x**3 + c*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^6 + b*x^3 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{c\,x^6 + b\,x^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^3 + c*x^6)^(1/2),x)
```

```
[Out] int((d*x)^m/(a + b*x^3 + c*x^6)^(1/2), x)
```

3.255 $\int \frac{(dx)^m}{\left(a+bx^3+cx^6\right)^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{3};\frac{3}{2},\frac{3}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(1+m)\sqrt{a+bx^3+cx^6}}$$

```
[Out] (d*x)^(1+m)*AppellF1(1/3+1/3*m,3/2,3/2,4/3+1/3*m,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^(
1/2)*(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(1+m)/(c*x^6+b*x^3+a)^(1/
2)
```

Rubi [A]
time = 0.10, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{3};\frac{3}{2},\frac{3}{2};\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^3+cx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*c*
x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(a*d*(1
+ m)*Sqrt[a + b*x^3 + c*x^6])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(dx)^m}{(a+bx^3+cx^6)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx}{a\sqrt{a+bx^3+cx^6}}$$
$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{3};\frac{3}{2},\frac{3}{2};\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{ad(1+m)\sqrt{a+bx^3+cx^6}}$$

Mathematica [A]
time = 5.78, size = 221, normalized size = 1.38

$$\frac{x(dx)^m\left(-b+\sqrt{b^2-4ac}-2cx^3\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{3/2}F_1\left(\frac{1+m}{3};\frac{3}{2},\frac{3}{2};\frac{4+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{\left(-b+\sqrt{b^2-4ac}\right)(1+m)\left(a+bx^3+cx^6\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x]
[Out] (x*(d*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b +
Sqrt[b^2 - 4*a*c]))^(3/2)*AppellF1[(1 + m)/3, 3/2, 3/2, (4 + m)/3, (-2*c*x^
3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqr
t[b^2 - 4*a*c])*(1 + m)*(a + b*x^3 + c*x^6)^(3/2))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(c\,x^6+b\,x^3+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
[Out] int((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^6 + b*x^3 + a)*(d*x)^m/(c^2*x^12 + 2*b*c*x^9 + (b^2 + 2*a
*c)*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + bx^3 + cx^6)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(c*x**6+b*x**3+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**3 + c*x**6)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(c*x^6+b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^6 + b*x^3 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{(c\,x^6 + b\,x^3 + a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^3 + c*x^6)^(3/2),x)
```

```
[Out] int((d*x)^m/(a + b*x^3 + c*x^6)^(3/2), x)
```

3.256 $\int (dx)^m \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=155

$$\frac{(dx)^{1+m}\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1+m}{3};-p,-p;\frac{4+m}{3};-\frac{2cx^3}{b-\sqrt{b^2-}}\right.}{d(1+m)}$$

```
[Out] (d*x)^(1+m)*(c*x^6+b*x^3+a)^p*AppellF1(1/3+1/3*m,-p,-p,4/3+1/3*m,-2*c*x^3/(
b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*x^3/
(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.07, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$\frac{(dx)^{m+1}\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{m+1}{3};-p,-p;\frac{m+4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3,
 (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(
d*(1 + m)*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int (dx)^m \left(a+bx^3+cx^6\right)^p dx = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int (\cdots$$

$$= \frac{(dx)^{1+m}\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1 \cdots}{d(1+m)}$$

Mathematica [A]
time = 0.19, size = 179, normalized size = 1.15

$$\frac{x(dx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1+m}{3};-p,-p;\frac{4+m}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{1+m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x*(d*x)^m*(a + b*x^3 + c*x^6)^p*AppellF1[(1 + m)/3, -p, -p, (4 + m)/3, (-2
*c*x^3)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/((1 +
 m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqr
t[b^2 - 4*a*c] + 2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx)^m \left(c\,x^6+b\,x^3+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int((d*x)^m*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*(d*x)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int((d*x)^m*(a + b*x^3 + c*x^6)^p, x)
```

3.257 $\int x^8(a+bx^3+cx^6)^p\,dx$

Optimal. Leaf size=224

$$-\frac{b(2+p)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)}+\frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)}+\frac{2^p\left(2ac-b^2(2+p)\right)\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+}{3c^2\sqrt{b^2-4ac}}$$

```
[Out] -1/6*b*(2+p)*(c*x^6+b*x^3+a)^(1+p)/c^2/(2*p^2+5*p+3)+1/3*x^3*(c*x^6+b*x^3+a
)^(1+p)/c/(3+2*p)+1/3*2^p*(2*a*c-b^2*(2+p))*(c*x^6+b*x^3+a)^(1+p)*hypergeom
([-p, 1+p],[2+p],1/2*(b+2*c*x^3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*((-
b-2*c*x^3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(-1-p)/c^2/(1+p)/(3+2*p)/
(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {1371, 756, 654, 638}

$$\frac{2^p\left(2ac-b^2(p+2)\right)\left(a+bx^3+cx^6\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c^2(p+1)(2p+3)\sqrt{b^2-4ac}}-\frac{b(p+2)\left(a+bx^3+cx^6\right)^{p+1}}{6c^2(p+1)(2p+3)}+\frac{x^3\left(a+bx^3+cx^6\right)^{p+1}}{3c(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] -1/6*(b*(2 + p)*(a + b*x^3 + c*x^6)^(1 + p))/(c^2*(1 + p)*(3 + 2*p)) + (x^3
*(a + b*x^3 + c*x^6)^(1 + p))/(3*c*(3 + 2*p)) + (2^p*(2*a*c - b^2*(2 + p))*
(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^
3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(3*c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 +
 2*p))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a+bx^3+cx^6\right)^p\,dx &= \frac{1}{3}\text{Subst}\left(\int x^2\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)\\ &= \frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)}+\frac{\text{Subst}\left(\int(-a-b(2+p)x)\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)}{3c(3+2p)}\\ &= -\frac{b(2+p)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)}+\frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)}-\frac{\left(2ac-b^2(2+p)\right)\text{Subs}}{6}\\ &= -\frac{b(2+p)\left(a+bx^3+cx^6\right)^{1+p}}{6c^2(1+p)(3+2p)}+\frac{x^3\left(a+bx^3+cx^6\right)^{1+p}}{3c(3+2p)}+\frac{2^p\left(2ac-b^2(2+p)\right)\Big(}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.33, size = 162, normalized size = 0.72

$$\frac{1}{9}x^9\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(3;-p,-p;4;-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^9*(a + b*x^3 + c*x^6)^p*AppellF1[3, -p, -p, 4, (-2*c*x^3)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(9*((b - Sqrt[b^2 - 4*a*c]
```

```
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^{8}\left(c\,x^{6}+b\,x^{3}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(c*x^6+b*x^3+a)^p,x)
[Out] int(x^8*(c*x^6+b*x^3+a)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(c*x**6+b*x**3+a)**p,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^8 \left(c x^6 + b x^3 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int(x^8*(a + b*x^3 + c*x^6)^p, x)
```

3.258 $\int x^5(a+bx^3+cx^6)^p\,dx$

Optimal. Leaf size=161

$$\frac{(a+bx^3+cx^6)^{1+p}}{6c(1+p)}+\frac{2^p b\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+\dots}{2\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}\,(1+p)}$$

```
[Out] 1/6*(c*x^6+b*x^3+a)^(1+p)/c/(1+p)+1/3*2^p*b*(c*x^6+b*x^3+a)^(1+p)*hypergeom
([-p, 1+p],[2+p],1/2*(b+2*c*x^3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*((-
b-2*c*x^3+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(-1-p)/c/(1+p)/(-4*a*c+b^
2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {1371, 654, 638}

$$\frac{b2^p(a+bx^3+cx^6)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3c(p+1)\sqrt{b^2-4ac}}+\frac{(a+bx^3+cx^6)^{p+1}}{6c(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(6*c*(1 + p)) + (2^p*b*(-((b - Sqrt[b^2 - 4*a*c
] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hyper
geometric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^
2 - 4*a*c])])/(3*c*Sqrt[b^2 - 4*a*c]*(1 + p))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5\left(a+bx^3+cx^6\right)^p\,dx=\frac{1}{3}\text{Subst}\left(\int x\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)$$
$$=\frac{\left(a+bx^3+cx^6\right)^{1+p}}{6c(1+p)}-\frac{b\text{Subst}\left(\int\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)}{6c}$$
$$=\frac{\left(a+bx^3+cx^6\right)^{1+p}}{6c(1+p)}+\frac{2^pb\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}\left(a+bx^3+cx^6\right)^{1+p}\,{}_2F_1\left(\right.}{3c\sqrt{b^2-4ac}\,(1+p)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.25, size = 162, normalized size = 1.01

$$\frac{1}{6}x^6\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(2;-p,-p;3;-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^6*(a + b*x^3 + c*x^6)^p*AppellF1[2, -p, -p, 3, (-2*c*x^3)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(6*((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^5\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^5*(c*x^6+b*x^3+a)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**6+b*x**3+a)**p,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,{\left(c\,x^6+b\,x^3+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^3 + c*x^6)^p,x)
[Out] int(x^5*(a + b*x^3 + c*x^6)^p, x)
```

3.259 $\int x^2(a+bx^3+cx^6)^p\,dx$

Optimal. Leaf size=130

$$-\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{b^2-4ac}+2cx^3}{2\sqrt{b^2-4ac}}\right)}{3\sqrt{b^2-4ac}\,(1+p)}$$

```
[Out] -1/3*2^(1+p)*(c*x^6+b*x^3+a)^(1+p)*hypergeom([-p, 1+p],[2+p],1/2*(b+2*c*x^3
+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))*((-b-2*c*x^3+(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2))^(-1-p)/(1+p)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1366, 638}

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p-1}(a+bx^3+cx^6)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{2cx^3+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{3(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] -1/3*(2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a*c]))^(-
1 - p)*(a + b*x^3 + c*x^6)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(2*Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(1
 + p))
```

Rule 638

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int x^2(a+bx^3+cx^6)^p\,dx = \frac{1}{3}\text{Subst}\left(\int (a+bx+cx^2)^p\,dx, x, x^3\right)$$
$$= -\frac{2^{1+p}\left(-\frac{b-\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-1-p}(a+bx^3+cx^6)^{1+p}\,{}_2F_1\left(-p,1+p;2+p;\frac{b+\sqrt{}}{2}\right.}{3\sqrt{b^2-4ac}\,(1+p)}$$

Mathematica [A]
time = 0.15, size = 138, normalized size = 1.06

$$\frac{2^{-1+p}\left(b-\sqrt{b^2-4ac}+2cx^3\right)\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,{}_2F_1\left(-p,1+p;2+p;\frac{-b+\sqrt{b^2-4ac}-2cx^3}{2\sqrt{b^2-4ac}}\right)}{3c(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)*(a + b*x^3 + c*x^6)^p*Hyperge
ometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*x^3)/(2*Sqrt[b^2
 - 4*a*c])])/(3*c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/Sqrt[b^2 - 4*a
*c])^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^2\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^2*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int(x^2*(a + b*x^3 + c*x^6)^p, x)
```

3.260 $\int x^4(a+bx^3+cx^6)^p\,dx$

Optimal. Leaf size=138

$$\frac{1}{5}x^5\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right.$$

```
[Out] 1/5*x^5*(c*x^6+b*x^3+a)^p*AppellF1(5/3,-p,-p,8/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1399, 524}

$$\frac{1}{5}x^5\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p F_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(5*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int x^4\left(a+bx^3+cx^6\right)^p\,dx=\left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int x^4($$

$$=\frac{1}{5}x^5\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\Bigg(\frac{5}{3}$$

Mathematica [A]
time = 0.38, size = 166, normalized size = 1.20

$$\frac{1}{5}x^5\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(\frac{5}{3};-p,-p;\frac{8}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^5*(a + b*x^3 + c*x^6)^p*AppellF1[5/3, -p, -p, 8/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(5*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^4\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^4*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^4, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int(x^4*(a + b*x^3 + c*x^6)^p, x)
```

3.261 $\int x^3(a+bx^3+cx^6)^p\,dx$

Optimal. Leaf size=138

$$\frac{1}{4}x^4\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right.$$

```
[Out] 1/4*x^4*(c*x^6+b*x^3+a)^p*AppellF1(4/3,-p,-p,7/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1399, 524}

$$\frac{1}{4}x^4\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^3+cx^6\right)^pF_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(4*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int x^3 \left(a+bx^3+cx^6\right)^p\,dx = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int x^3 \Bigg($$
$$= \frac{1}{4}x^4\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\Bigg(\frac{4}{3};$$

Mathematica [A]
time = 0.40, size = 166, normalized size = 1.20

$$\frac{1}{4}x^4\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{4}{3};-p,-p;\frac{7}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^4*(a + b*x^3 + c*x^6)^p*AppellF1[4/3, -p, -p, 7/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(4*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x^3 \left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] int(x^3*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int(x^3*(a + b*x^3 + c*x^6)^p, x)
```

3.262 $\int x(a+bx^3+cx^6)^p\,dx$

Optimal. Leaf size=138

$$\frac{1}{2}x^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right.$$

```
[Out] 1/2*x^2*(c*x^6+b*x^3+a)^p*AppellF1(2/3,-p,-p,5/3,-2*c*x^3/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^
p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]

time = 0.05, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1399, 524}

$$\frac{1}{2}x^2\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p F_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b - Sqrt[
b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*(1 + (2*c*x^3)/(b -
Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int x\left(a+bx^3+cx^6\right)^p\,dx=\left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int x\Bigg(1$$
$$=\frac{1}{2}x^2\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\Bigg(\frac{2}{3};$$

Mathematica [A]
time = 0.40, size = 166, normalized size = 1.20

$$\frac{1}{2}x^2\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{2}{3};-p,-p;\frac{5}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(a + b*x^3 + c*x^6)^p,x]
[Out] (x^2*(a + b*x^3 + c*x^6)^p*AppellF1[2/3, -p, -p, 5/3, (-2*c*x^3)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(2*((b - Sqrt[b^2 - 4*a
*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3
)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int x\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^6+b*x^3+a)^p,x)
[Out] int(x*(c*x^6+b*x^3+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p*x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p*x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int(x*(a + b*x^3 + c*x^6)^p, x)
```

3.263 $\int (a+bx^3+cx^6)^p \, dx$

Optimal. Leaf size=133

$$x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},\right.$$

[Out] x*(c*x^6+b*x^3+a)^p*AppellF1(1/3,-p,-p,4/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)), -2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1 +2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)

Rubi [A]

time = 0.05, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1362, 440}

$$x\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)^p,x]

[Out] (x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/((1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+bx^3+cx^6\right)^p\,dx = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p\right)\int \left(1+\right.$$

$$= x\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,\right.$$

Mathematica [A]
time = 0.11, size = 161, normalized size = 1.21

$$x\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^3+cx^6\right)^p F_1\left(\frac{1}{3};-p,-p;\frac{4}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p,x]
[Out] (x*(a + b*x^3 + c*x^6)^p*AppellF1[1/3, -p, -p, 4/3, (-2*c*x^3)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(((b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(b
 + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(c\,x^6+b\,x^3+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p,x)
[Out] int((c*x^6+b*x^3+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x^6+b\,x^3+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^p, x)
```

3.264 $\int \frac{(a+bx^3+cx^6)^p}{x}\,dx$

Optimal. Leaf size=157

$$\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-}}{2cx^3}\right.}{3p}$$

```
[Out] 1/3*2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(-2*p,-p,-p,1-2*p,1/2*(-b-(-4*a*c+
b^2)^(1/2))/c/x^3,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^3)/p/(((b+2*c*x^3-(-4*a*c
+b^2)^(1/2))/c/x^3)^p)/(((b+2*c*x^3+(-4*a*c+b^2)^(1/2))/c/x^3)^p)
```

Rubi [A]

time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1371, 772, 138}

$$\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3p}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b
 - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/(3*p*
((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c
*x^3)/(c*x^3))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p)), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
- q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
+ e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m
, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^p}{x}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^p}{x}\,dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\left(2^{2p}\left(\frac{1}{x^3}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a\right.\right.$$

$$= \frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-2p\right.}{3p}$$

Mathematica [A]
time = 0.21, size = 157, normalized size = 1.00

$$\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{-b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x,x]
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, -1/2*(b
 + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*p*((
b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x
^3)/(c*x^3))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x,x)
[Out] int((c*x^6+b*x^3+a)^p/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p/x, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x,x)
[Out] int((a + b*x^3 + c*x^6)^p/x, x)
```

3.265 $\int \frac{(a+bx^3+cx^6)^p}{x^2} dx$

Optimal. Leaf size=136

$$-\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

```
[Out] -(c*x^6+b*x^3+a)^p*AppellF1(-1/3,-p,-p,2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/(
(1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1399, 524}

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^2,x]
```

```
[Out] -(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x*(1 + (2*c*x^3)/(b - S
qrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^p}{x^2}\,dx = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\right)\int \frac{\left(1+\right.}{b}$$

$$= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,F_1\left(-\frac{1}{3};-p,-p\right.}{x}$$

Mathematica [A]
time = 0.39, size = 164, normalized size = 1.21

$$-\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,F_1\left(-\frac{1}{3};-p,-p;\frac{2}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^2,x]
```

```
[Out] -(((a + b*x^3 + c*x^6)^p*AppellF1[-1/3, -p, -p, 2/3, (-2*c*x^3)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^3)
/(b + Sqrt[b^2 - 4*a*c]))^p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^2,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^2,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x^2,x)
[Out] int((a + b*x^3 + c*x^6)^p/x^2, x)
```

3.266 $\int \frac{(a+bx^3+cx^6)^p}{x^3} dx$

Optimal. Leaf size=138

$$-\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

```
[Out] -1/2*(c*x^6+b*x^3+a)^p*AppellF1(-2/3,-p,-p,1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^2/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2))
)^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1399, 524}

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^3,x]
```

```
[Out] -1/2*((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b - Sqr
t[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*(1 + (2*c*x^3)/(
b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^p}{x^3}\,dx = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\right)\int \frac{\left(1+\right.}{}$$

$$= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,F_1\left(-\frac{2}{3};-p,-\right.}{2x^2}$$

Mathematica [A]
time = 0.40, size = 166, normalized size = 1.20

$$-\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,F_1\left(-\frac{2}{3};-p,-p;\frac{1}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^3,x]
```

```
[Out] -1/2*((a + b*x^3 + c*x^6)^p*AppellF1[-2/3, -p, -p, 1/3, (-2*c*x^3)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^2*((b - Sqrt[b^2 -
 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c
*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^3,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p/x^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^3,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x^3,x)
[Out] int((a + b*x^3 + c*x^6)^p/x^3, x)
```

3.267 $\int \frac{(a+bx^3+cx^6)^p}{x^4} \, dx$

Optimal. Leaf size=164

$$-\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b}}{2}\right.}{3(1-2p)x^3}$$

```
[Out] -1/3*4^p*(c*x^6+b*x^3+a)^p*AppellF1(1-2*p,-p,-p,2-2*p,1/2*(-b-(-4*a*c+b^2)^
(1/2))/c/x^3,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^3)/(1-2*p)/x^3/(((b+2*c*x^3-(-
4*a*c+b^2)^(1/2))/c/x^3)^p)/(((b+2*c*x^3+(-4*a*c+b^2)^(1/2))/c/x^3)^p)
```

Rubi [A]

time = 0.09, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1371, 772, 138}

$$-\frac{4^p\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-2p)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^4,x]
```

```
[Out] -1/3*(4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2*(1 - p), -1/2*(
b - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^3)])/((1
- 2*p)*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4
*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p)), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
- q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
+ e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m
, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^p}{x^4}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^p}{x^2}\,dx, x, x^3\right)$$
$$= -\left(\frac{1}{3}\left(2^{2p}\left(\frac{1}{x^3}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+$$
$$= -\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(1-2p;\right.}{3(1-2p)x^3}$$

Mathematica [A]
time = 0.26, size = 162, normalized size = 0.99

$$\frac{4^p\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(1-2p;-p,-p;2-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{-b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(-1+2p)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^4,x]
```

```
[Out] (4^p*(a + b*x^3 + c*x^6)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -1/2*(b + Sqr
t[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)])/(3*(-1 + 2*p)
*x^3*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^3)/(c*x^3))^p)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^4,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**4,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^4,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^6+b\,x^3+a\right)}^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x^4,x)
[Out] int((a + b*x^3 + c*x^6)^p/x^4, x)
```

3.268 $\int \frac{(a+bx^3+cx^6)^p}{x^5} \, dx$

Optimal. Leaf size=138

$$-\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

```
[Out] -1/4*(c*x^6+b*x^3+a)^p*AppellF1(-4/3,-p,-p,-1/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1
/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^4/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)
))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1399, 524}

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^5,x]
```

```
[Out] -1/4*((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^4*(1 + (2*c*x^3)/
(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^p}{x^5}\,dx = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\right)\int \frac{\left(1+\right.}{}$$

$$= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{4}{3};-p,-\right.}{4x^4}$$

Mathematica [A]
time = 0.41, size = 166, normalized size = 1.20

$$-\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{4}{3};-p,-p;-\frac{1}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{4x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^5,x]
```

```
[Out] -1/4*((a + b*x^3 + c*x^6)^p*AppellF1[-4/3, -p, -p, -1/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^4*((b - Sqrt[b^2
- 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^5,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)^p/x^5, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**5,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x^5,x)
```

```
[Out] int((a + b*x^3 + c*x^6)^p/x^5, x)
```

3.269 $\int \frac{(a+bx^3+cx^6)^p}{x^6} dx$

Optimal. Leaf size=138

$$-\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

[Out] -1/5*(c*x^6+b*x^3+a)^p*AppellF1(-5/3,-p,-p,-2/3,-2*c*x^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))/x^5/((1+2*c*x^3/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^3/(b+(-4*a*c+b^2)^(1/2)))^p)

Rubi [A]
time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1399, 524}

$$-\frac{\left(\frac{2cx^3}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^3}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^3+cx^6)^p F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b-\sqrt{b^2-4ac}},-\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x^3 + c*x^6)^p/x^6,x]

[Out] -1/5*((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^3)/(b + Sqrt[b^2 - 4*a*c])])/(x^5*(1 + (2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^p}{x^6}\,dx = \left(\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\right)\int \frac{\left(1+\right.}{b}$$

$$= -\frac{\left(1+\frac{2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,F_1\left(-\frac{5}{3};-p,-p\right.}{5x^5}$$

Mathematica [A]
time = 0.40, size = 166, normalized size = 1.20

$$-\frac{\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^3+cx^6)^p\,F_1\left(-\frac{5}{3};-p,-p;-\frac{2}{3};-\frac{2cx^3}{b+\sqrt{b^2-4ac}},\frac{2cx^3}{-b+\sqrt{b^2-4ac}}\right)}{5x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^6,x]
```

```
[Out] -1/5*((a + b*x^3 + c*x^6)^p*AppellF1[-5/3, -p, -p, -2/3, (-2*c*x^3)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^3)/(-b + Sqrt[b^2 - 4*a*c])])/(x^5*((b - Sqrt[b^2
- 4*a*c] + 2*c*x^3)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*
c*x^3)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^6,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**6,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^6,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^6+b\,x^3+a\right)^p}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x^6,x)
[Out] int((a + b*x^3 + c*x^6)^p/x^6, x)
```

3.270 $\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx$

Optimal. Leaf size=168

$$-\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\cdots}{\cdots}\right.}{3(1-p)x^6}$$

```
[Out] -1/3*2^(-1+2*p)*(c*x^6+b*x^3+a)^p*AppellF1(2-2*p,-p,-p,3-2*p,1/2*(-b-(-4*a*
c+b^2)^(1/2))/c/x^3,1/2*(-b+(-4*a*c+b^2)^(1/2))/c/x^3)/(1-p)/x^6/(((b+2*c*x
^3-(-4*a*c+b^2)^(1/2))/c/x^3)^p)/(((b+2*c*x^3+(-4*a*c+b^2)^(1/2))/c/x^3)^p)
```

Rubi [A]
time = 0.09, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1371, 772, 138}

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^3},-\frac{b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(1-p)x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)^p/x^7,x]
```

```
[Out] -1/3*(2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2*(1 - p), -p, -p, 3 - 2*
p, -1/2*(b - Sqrt[b^2 - 4*a*c])/(c*x^3), -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*x^
3)])/((1 - p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[
b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p)), Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
- q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
+ e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[p] && ILtQ[m
, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3+cx^6)^p}{x^7}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx+cx^2)^p}{x^3}\,dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\left(2^{2p}\left(\frac{1}{x^3}\right)^{2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a\right.\right.$$

$$= -\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(2(\right.}{3(1-p)x^6}$$

Mathematica [A]
time = 0.29, size = 164, normalized size = 0.98

$$\frac{2^{-1+2p}\left(\frac{b-\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^3}{cx^3}\right)^{-p}(a+bx^3+cx^6)^p F_1\left(2-2p;-p,-p;3-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^3},\frac{-b+\sqrt{b^2-4ac}}{2cx^3}\right)}{3(-1+p)x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)^p/x^7,x]
```

```
[Out] (2^(-1 + 2*p)*(a + b*x^3 + c*x^6)^p*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -1/2
*(b + Sqrt[b^2 - 4*a*c])/(c*x^3), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^3)]])/(3*(
-1 + p)*x^6*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^3)/(c*x^3))^p*((b + Sqrt[b^2 -
4*a*c] + 2*c*x^3)/(c*x^3))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)^p/x^7,x)
```

```
[Out] int((c*x^6+b*x^3+a)^p/x^7,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="maxima")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="fricas")
[Out] integral((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)**p/x**7,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)^p/x^7,x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)^p/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^6+b\,x^3+a)^p}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)^p/x^7,x)
[Out] int((a + b*x^3 + c*x^6)^p/x^7, x)
```

3.271 $\int \frac{x^m}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=32

$$\frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{4};\frac{5+m}{4};-x^4\right)}{1+m}$$

```
[Out] x^(1+m)*hypergeom([2, 1/4+1/4*m],[5/4+1/4*m],-x^4)/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 371}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};-x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 + 2*x^4 + x^8),x]
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, -x^4])/(1 + m)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{1+2x^4+x^8}\,dx &= \int \frac{x^m}{\left(1+x^4\right)^2}\,dx\\ &= \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{4};\frac{5+m}{4};-x^4\right)}{1+m}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 1.06

$$\frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{4};1+\frac{1+m}{4};-x^4\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(1 + 2*x^4 + x^8),x]
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, -x^4])/(1 + m)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 + 2x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8+2*x^4+1),x)
[Out] int(x^m/(x^8+2*x^4+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] integral(x^m/(x^8 + 2*x^4 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(x^4 + 1\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8+2*x**4+1),x)
[Out] Integral(x**m/(x**4 + 1)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+2*x^4+1),x, algorithm="giac")
[Out] integrate(x^m/(x^8 + 2*x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{x^8+2\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(2*x^4 + x^8 + 1),x)
[Out] int(x^m/(2*x^4 + x^8 + 1), x)
```

3.272 $\int \frac{x^9}{1+2x^4+x^8} \, dx$

Optimal. Leaf size=30

$$\frac{3x^2}{4} - \frac{x^6}{4(1+x^4)} - \frac{3}{4}\tan^{-1}\left(x^2\right)$$

```
[Out] 3/4*x^2-1/4*x^6/(x^4+1)-3/4*arctan(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 281, 294, 327, 209}

$$-\frac{3\text{ArcTan}(x^2)}{4} + \frac{3x^2}{4} - \frac{x^6}{4(x^4+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + 2*x^4 + x^8),x]
```

```
[Out] (3*x^2)/4 - x^6/(4*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{1+2x^4+x^8}\,dx &= \int \frac{x^9}{(1+x^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, x^2\right) \\
&= -\frac{x^6}{4(1+x^4)} + \frac{3}{4}\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, x^2\right) \\
&= \frac{3x^2}{4} - \frac{x^6}{4(1+x^4)} - \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\
&= \frac{3x^2}{4} - \frac{x^6}{4(1+x^4)} - \frac{3}{4}\tan^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.80

$$\frac{1}{4}\left(x^2\left(2+\frac{1}{1+x^4}\right)-3\tan^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + 2*x^4 + x^8),x]
```

```
[Out] (x^2*(2 + (1 + x^4)^(-1)) - 3*ArcTan[x^2])/4
```

Maple [A]
time = 0.02, size = 25, normalized size = 0.83

method	result	size
default	$\frac{x^2}{2}+\frac{x^2}{4x^4+4}-\frac{3\arctan(x^2)}{4}$	25
risch	$\frac{x^2}{2}+\frac{x^2}{4x^4+4}-\frac{3\arctan(x^2)}{4}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+1/4*x^2/(x^4+1)-3/4*arctan(x^2)
```

Maxima [A]
time = 0.52, size = 24, normalized size = 0.80

$$\frac{1}{2}\,x^2+\frac{x^2}{4\,(x^4+1)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

Fricas [A]
time = 0.33, size = 31, normalized size = 1.03

$$\frac{2\,x^6+3\,x^2-3\,(x^4+1)\arctan\left(x^2\right)}{4\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x^6 + 3*x^2 - 3*(x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A]
time = 0.04, size = 22, normalized size = 0.73

$$\frac{x^2}{2}+\frac{x^2}{4x^4+4}-\frac{3\,\mathrm{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+2*x**4+1),x)
```

```
[Out] x**2/2 + x**2/(4*x**4 + 4) - 3*atan(x**2)/4
```

Giac [A]
time = 3.17, size = 24, normalized size = 0.80

$$\frac{1}{2}\,x^2+\frac{x^2}{4\,(x^4+1)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*x^2/(x^4 + 1) - 3/4*arctan(x^2)
```

Mupad [B]
time = 0.05, size = 25, normalized size = 0.83

$$\frac{x^2}{4\,(x^4+1)}-\frac{3\,\mathrm{atan}(x^2)}{4}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(2*x^4 + x^8 + 1),x)
[Out] x^2/(4*(x^4 + 1)) - (3*atan(x^2))/4 + x^2/2
```

3.273 $\int \frac{x^7}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=22

$$\frac{1}{4(1+x^4)}+\frac{1}{4}\log(1+x^4)$$

[Out] 1/4/(x^4+1)+1/4*ln(x^4+1)

Rubi [A]

time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {28, 272, 45}

$$\frac{1}{4(x^4+1)}+\frac{1}{4}\log(x^4+1)$$

Antiderivative was successfully verified.

[In] Int[x^7/(1 + 2*x^4 + x^8),x]

[Out] 1/(4*(1 + x^4)) + Log[1 + x^4]/4

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{1+2x^4+x^8}\,dx &= \int \frac{x^7}{(1+x^4)^2}\,dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(1+x)^2}\,dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \left(-\frac{1}{(1+x)^2}+\frac{1}{1+x}\right)\,dx, x, x^4\right) \\
&= \frac{1}{4(1+x^4)}+\frac{1}{4}\log\left(1+x^4\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.82

$$\frac{1}{4}\left(\frac{1}{1+x^4}+\log\left(1+x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + 2*x^4 + x^8),x]
[Out] ((1 + x^4)^(-1) + Log[1 + x^4])/4
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.86

method	result	size
default	$\frac{1}{4x^4+4}+\frac{\ln(x^4+1)}{4}$	19
norman	$\frac{1}{4x^4+4}+\frac{\ln(x^4+1)}{4}$	19
risch	$\frac{1}{4x^4+4}+\frac{\ln(x^4+1)}{4}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4/(x^4+1)+1/4*ln(x^4+1)
```

Maxima [A]
time = 0.29, size = 18, normalized size = 0.82

$$\frac{1}{4(x^4+1)}+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

Fricas [A]
time = 0.33, size = 23, normalized size = 1.05

$$\frac{(x^4+1)\log(x^4+1)+1}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*((x^4 + 1)*log(x^4 + 1) + 1)/(x^4 + 1)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.68

$$\frac{\log(x^4+1)}{4}+\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+2*x**4+1),x)
```

```
[Out] log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

Giac [A]
time = 3.74, size = 18, normalized size = 0.82

$$\frac{1}{4(x^4+1)}+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4/(x^4 + 1) + 1/4*log(x^4 + 1)
```

Mupad [B]
time = 1.32, size = 18, normalized size = 0.82

$$\frac{\ln(x^4+1)}{4}+\frac{1}{4\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(2*x^4 + x^8 + 1),x)
```

```
[Out] log(x^4 + 1)/4 + 1/(4*(x^4 + 1))
```

3.274 $\int \frac{x^5}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=23

$$-\frac{x^2}{4\,(1+x^4)}+\frac{1}{4}\tan^{-1}\left(x^2\right)$$

[Out] -1/4*x^2/(x^4+1)+1/4*arctan(x^2)

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {28, 281, 294, 209}

$$\frac{\text{ArcTan}(x^2)}{4}-\frac{x^2}{4\,(x^4+1)}$$

Antiderivative was successfully verified.

[In] Int[x^5/(1 + 2*x^4 + x^8),x]

[Out] -1/4*x^2/(1 + x^4) + ArcTan[x^2]/4

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{1+2x^4+x^8}\,dx &= \int \frac{x^5}{\left(1+x^4\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx, x, x^2\right) \\
&= -\frac{x^2}{4\left(1+x^4\right)} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right) \\
&= -\frac{x^2}{4\left(1+x^4\right)} + \frac{1}{4}\tan^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 23, normalized size = 1.00

$$-\frac{x^2}{4\left(1+x^4\right)} + \frac{1}{4}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + 2*x^4 + x^8),x]
[Out] -1/4*x^2/(1 + x^4) + ArcTan[x^2]/4
```

Maple [**A**]
time = 0.02, size = 20, normalized size = 0.87

method	result	size
default	$-\frac{x^2}{4(x^4+1)}+\frac{\arctan(x^2)}{4}$	20
risch	$-\frac{x^2}{4(x^4+1)}+\frac{\arctan(x^2)}{4}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Maxima [**A**]
time = 0.51, size = 19, normalized size = 0.83

$$-\frac{x^2}{4\left(x^4+1\right)} + \frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Fricas [A]
time = 0.34, size = 24, normalized size = 1.04

$$-\frac{x^2 - (x^4 + 1)\arctan(x^2)}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*(x^2 - (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.65

$$-\frac{x^2}{4x^4 + 4} + \frac{\operatorname{atan}(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8+2*x**4+1),x)
```

```
[Out] -x**2/(4*x**4 + 4) + atan(x**2)/4
```

Giac [A]
time = 4.72, size = 19, normalized size = 0.83

$$-\frac{x^2}{4(x^4 + 1)} + \frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Mupad [B]
time = 1.37, size = 21, normalized size = 0.91

$$\frac{\operatorname{atan}(x^2)}{4} - \frac{x^2}{4\,(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(2*x^4 + x^8 + 1),x)
```

```
[Out] atan(x^2)/4 - x^2/(4*(x^4 + 1))
```

3.275 $\int \frac{x^3}{1+2x^4+x^8} \, dx$

Optimal. Leaf size=11

$$-\frac{1}{4\left(1+x^4\right)}$$

[Out] -1/4/(x^4+1)

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 267}

$$-\frac{1}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 + 2*x^4 + x^8),x]

[Out] -1/4*1/(1 + x^4)

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{1+2x^4+x^8} \, dx &= \int \frac{x^3}{\left(1+x^4\right)^2} \, dx \\ &= -\frac{1}{4\left(1+x^4\right)} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$-\frac{1}{4\left(1+x^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 2*x^4 + x^8),x]
```

```
[Out] -1/4*1/(1 + x^4)
```

Maple [A]
time = 0.01, size = 10, normalized size = 0.91

method	result	size
gosper	$-\frac{1}{4(x^4+1)}$	10
default	$-\frac{1}{4(x^4+1)}$	10
norman	$-\frac{1}{4(x^4+1)}$	10
risch	$-\frac{1}{4(x^4+1)}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(x^4+1)
```

Maxima [A]
time = 0.28, size = 9, normalized size = 0.82

$$-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 + 1)
```

Fricas [A]
time = 0.35, size = 9, normalized size = 0.82

$$-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4/(x^4 + 1)
```

Sympy [A]
time = 0.03, size = 8, normalized size = 0.73

$$-\frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+2*x**4+1),x)
```

```
[Out] -1/(4*x**4 + 4)
```

Giac [A]
time = 4.34, size = 9, normalized size = 0.82

$$-\frac{1}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4/(x^4 + 1)
```

Mupad [B]
time = 0.02, size = 11, normalized size = 1.00

$$-\frac{1}{4\ \left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(2*x^4 + x^8 + 1),x)
```

```
[Out] -1/(4*(x^4 + 1))
```

3.276 $\int \frac{x}{1+2x^4+x^8} \, dx$

Optimal. Leaf size=23

$$\frac{x^2}{4\left(1+x^4\right)}+\frac{1}{4}\tan^{-1}\left(x^2\right)$$

```
[Out] 1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {28, 281, 205, 209}

$$\frac{\text{ArcTan}(x^2)}{4}+\frac{x^2}{4\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 + x^4)) + ArcTan[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+2x^4+x^8}\,dx &= \int \frac{x}{(1+x^4)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx,x,x^2\right)\\ &= \frac{x^2}{4(1+x^4)}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)\\ &= \frac{x^2}{4(1+x^4)}+\frac{1}{4}\tan^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 0.87

$$\frac{1}{4}\left(\frac{x^2}{1+x^4}+\tan^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + 2*x^4 + x^8),x]
```

```
[Out] (x^2/(1 + x^4) + ArcTan[x^2])/4
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.87

method	result	size
default	$\frac{x^2}{4x^4+4}+\frac{\arctan(x^2)}{4}$	20
risch	$\frac{x^2}{4x^4+4}+\frac{\arctan(x^2)}{4}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^2/(x^4+1)+1/4*arctan(x^2)
```

Maxima [A]
time = 0.50, size = 19, normalized size = 0.83

$$\frac{x^2}{4(x^4+1)}+\frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Fricas [A]
time = 0.33, size = 23, normalized size = 1.00

$$\frac{x^2 + (x^4 + 1)\arctan(x^2)}{4(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*(x^2 + (x^4 + 1)*arctan(x^2))/(x^4 + 1)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.65

$$\frac{x^2}{4x^4 + 4} + \frac{\operatorname{atan}(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+2*x**4+1),x)
```

```
[Out] x**2/(4*x**4 + 4) + atan(x**2)/4
```

Giac [A]
time = 3.60, size = 19, normalized size = 0.83

$$\frac{x^2}{4(x^4 + 1)} + \frac{1}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^2/(x^4 + 1) + 1/4*arctan(x^2)
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.87

$$\frac{\operatorname{atan}(x^2)}{4} + \frac{x^2}{4\,(x^4 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^4 + x^8 + 1),x)
```

```
[Out] atan(x^2)/4 + x^2/(4*(x^4 + 1))
```

3.277 $\int \frac{1}{x(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{4(1+x^4)} + \log(x) - \frac{1}{4}\log\left(1+x^4\right)$$

```
[Out] 1/4/(x^4+1)+ln(x)-1/4*ln(x^4+1)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 272, 46}

$$\frac{1}{4(x^4+1)} - \frac{1}{4}\log\left(x^4+1\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + 2*x^4 + x^8)),x]
```

```
[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x(1+2x^4+x^8)}\,dx = \int \frac{1}{x(1+x^4)^2}\,dx$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1+x)^2}\,dx, x, x^4\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{-1-x}+\frac{1}{x}-\frac{1}{(1+x)^2}\right)\,dx, x, x^4\right)$$
$$= \frac{1}{4(1+x^4)}+\log(x)-\frac{1}{4}\log\left(1+x^4\right)$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{4(1+x^4)}+\log(x)-\frac{1}{4}\log\left(1+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + 2*x^4 + x^8)),x]
[Out] 1/(4*(1 + x^4)) + Log[x] - Log[1 + x^4]/4
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.88

method	result	size
default	$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln(x^4+1)}{4}$	21
norman	$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln(x^4+1)}{4}$	21
risch	$\frac{1}{4x^4+4}+\ln(x)-\frac{\ln(x^4+1)}{4}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4/(x^4+1)+ln(x)-1/4*ln(x^4+1)
```

Maxima [A]
time = 0.31, size = 24, normalized size = 1.00

$$\frac{1}{4(x^4+1)}-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

Fricas [A]
time = 0.33, size = 32, normalized size = 1.33

$$-\frac{(x^4+1)\log(x^4+1) - 4(x^4+1)\log(x) - 1}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*((x^4 + 1)*log(x^4 + 1) - 4*(x^4 + 1)*log(x) - 1)/(x^4 + 1)
```

Sympy [A]
time = 0.05, size = 19, normalized size = 0.79

$$\log(x) - \frac{\log(x^4+1)}{4} + \frac{1}{4x^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+2*x**4+1),x)
```

```
[Out] log(x) - log(x**4 + 1)/4 + 1/(4*x**4 + 4)
```

Giac [A]
time = 3.69, size = 29, normalized size = 1.21

$$\frac{x^4+2}{4(x^4+1)} - \frac{1}{4}\log\left(x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*(x^4 + 2)/(x^4 + 1) - 1/4*log(x^4 + 1) + 1/4*log(x^4)
```

Mupad [B]
time = 0.04, size = 20, normalized size = 0.83

$$\ln(x) - \frac{\ln(x^4+1)}{4} + \frac{1}{4\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(2*x^4 + x^8 + 1)),x)
```

```
[Out] log(x) - log(x^4 + 1)/4 + 1/(4*(x^4 + 1))
```

3.278 $\int \frac{1}{x^3(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=30

$$-\frac{3}{4x^2}+\frac{1}{4x^2\left(1+x^4\right)}-\frac{3}{4}\tan^{-1}\left(x^2\right)$$

```
[Out] -3/4/x^2+1/4/x^2/(x^4+1)-3/4*arctan(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 281, 296, 331, 209}

$$-\frac{3}{4}\text{ArcTan}\left(x^2\right)-\frac{3}{4x^2}+\frac{1}{4x^2\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 + x^4)) - (3*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^3\left(1+x^4\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)^2}\,dx,x,x^2\right) \\
&= \frac{1}{4x^2\left(1+x^4\right)}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx,x,x^2\right) \\
&= -\frac{3}{4x^2}+\frac{1}{4x^2\left(1+x^4\right)}-\frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right) \\
&= -\frac{3}{4x^2}+\frac{1}{4x^2\left(1+x^4\right)}-\frac{3}{4}\tan^{-1}\left(x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 1.00

$$-\frac{1}{2x^2}-\frac{x^2}{4\left(1+x^4\right)}+\frac{3}{4}\tan^{-1}\left(\frac{1}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + 2*x^4 + x^8)),x]
[Out] -1/2*1/x^2 - x^2/(4*(1 + x^4)) + (3*ArcTan[x^(-2)])/4
```

Maple [A]
time = 0.02, size = 25, normalized size = 0.83

method	result	size
default	$-\frac{x^2}{4(x^4+1)}-\frac{3\arctan(x^2)}{4}-\frac{1}{2x^2}$	25

risch	$\frac{-\frac{3x^4}{4}-\frac{1}{2}}{x^2(x^4+1)} - \frac{3\arctan(x^2)}{4}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^2/(x^4+1)-3/4*arctan(x^2)-1/2/x^2
```

Maxima [A]
time = 0.56, size = 25, normalized size = 0.83

$$-\frac{3\,x^4+2}{4\,(x^6+x^2)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

Fricas [A]
time = 0.33, size = 31, normalized size = 1.03

$$-\frac{3\,x^4+3\,(x^6+x^2)\arctan\left(x^2\right)+2}{4\,(x^6+x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*(3*x^4 + 3*(x^6 + x^2)*arctan(x^2) + 2)/(x^6 + x^2)
```

Sympy [A]
time = 0.05, size = 26, normalized size = 0.87

$$\frac{-3x^4-2}{4x^6+4x^2}-\frac{3\,\mathrm{atan}\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+2*x**4+1),x)
```

```
[Out] (-3*x**4 - 2)/(4*x**6 + 4*x**2) - 3*atan(x**2)/4
```

Giac [A]
time = 3.84, size = 25, normalized size = 0.83

$$-\frac{3\,x^4+2}{4\,(x^6+x^2)}-\frac{3}{4}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+2*x^4+1),x, algorithm="giac")
[Out] -1/4*(3*x^4 + 2)/(x^6 + x^2) - 3/4*arctan(x^2)
```

Mupad [B]
time = 0.04, size = 25, normalized size = 0.83

$$-\frac{3\,\mathrm{atan}\left(x^2\right)}{4}-\frac{\frac{3\,x^4}{4}+\frac{1}{2}}{x^6+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(2*x^4 + x^8 + 1)),x)
[Out] - (3*atan(x^2))/4 - ((3*x^4)/4 + 1/2)/(x^2 + x^6)
```

3.279 $\int \frac{1}{x^5(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{4x^4}-\frac{1}{4\left(1+x^4\right)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)$$

```
[Out] -1/4/x^4-1/4/(x^4+1)-2*ln(x)+1/2*ln(x^4+1)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 272, 46}

$$-\frac{1}{4\left(x^4+1\right)}-\frac{1}{4x^4}+\frac{1}{2}\log\left(x^4+1\right)-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^5\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^5\left(1+x^4\right)^2}\,dx \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(1+x)^2}\,dx, x, x^4\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{2}{x}+\frac{1}{(1+x)^2}+\frac{2}{1+x}\right)dx, x, x^4\right) \\
&= -\frac{1}{4x^4}-\frac{1}{4\left(1+x^4\right)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$-\frac{1}{4x^4}-\frac{1}{4\left(1+x^4\right)}-2\log(x)+\frac{1}{2}\log\left(1+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 - 1/(4*(1 + x^4)) - 2*Log[x] + Log[1 + x^4]/2
```

Maple [A]
time = 0.02, size = 28, normalized size = 0.85

method	result	size
default	$-\frac{1}{4x^4}-\frac{1}{4(x^4+1)}-2\ln(x)+\frac{\ln(x^4+1)}{2}$	28
norman	$\frac{-\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4+1)}-2\ln(x)+\frac{\ln(x^4+1)}{2}$	32
risch	$\frac{-\frac{1}{4}-\frac{x^4}{2}}{x^4(x^4+1)}-2\ln(x)+\frac{\ln(x^4+1)}{2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4-1/4/(x^4+1)-2*ln(x)+1/2*ln(x^4+1)
```

Maxima [A]
time = 0.33, size = 33, normalized size = 1.00

$$-\frac{2\,x^4+1}{4\left(x^8+x^4\right)}+\frac{1}{2}\,\log\left(x^4+1\right)-\frac{1}{2}\,\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

Fricas [A]
time = 0.34, size = 44, normalized size = 1.33

$$-\frac{2\,x^4-2\,(x^8+x^4)\log\left(x^4+1\right)+8\,(x^8+x^4)\log\left(x\right)+1}{4\,(x^8+x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^4 - 2*(x^8 + x^4)*log(x^4 + 1) + 8*(x^8 + x^4)*log(x) + 1)/(x^8 +
 x^4)
```

Sympy [A]
time = 0.06, size = 31, normalized size = 0.94

$$\frac{-2x^4-1}{4x^8+4x^4}-2\log\left(x\right)+\frac{\log\left(x^4+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+2*x**4+1),x)
```

```
[Out] (-2*x**4 - 1)/(4*x**8 + 4*x**4) - 2*log(x) + log(x**4 + 1)/2
```

Giac [A]
time = 3.25, size = 33, normalized size = 1.00

$$-\frac{2\,x^4+1}{4\,(x^8+x^4)}+\frac{1}{2}\,\log\left(x^4+1\right)-\frac{1}{2}\,\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^4 + 1)/(x^8 + x^4) + 1/2*log(x^4 + 1) - 1/2*log(x^4)
```

Mupad [B]
time = 0.05, size = 31, normalized size = 0.94

$$\frac{\ln\left(x^4+1\right)}{2}-2\,\ln\left(x\right)-\frac{\frac{x^4}{2}+\frac{1}{4}}{x^8+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(2*x^4 + x^8 + 1)),x)
```

```
[Out] log(x^4 + 1)/2 - 2*log(x) - (x^4/2 + 1/4)/(x^4 + x^8)
```

3.280 $\int \frac{1}{x^7\left(1+2x^4+x^8\right)}\,dx$

Optimal. Leaf size=37

$$-\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{1}{4x^6\left(1+x^4\right)}+\frac{5}{4}\tan^{-1}\left(x^2\right)$$

```
[Out] -5/12/x^6+5/4/x^2+1/4/x^6/(x^4+1)+5/4*arctan(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 281, 296, 331, 209}

$$\frac{5\text{ArcTan}(x^2)}{4}-\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{1}{4x^6\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -5/(12*x^6) + 5/(4*x^2) + 1/(4*x^6*(1 + x^4)) + (5*ArcTan[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(1+2x^4+x^8\right)}\,dx &= \int \frac{1}{x^7\left(1+x^4\right)^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{1}{4x^6\left(1+x^4\right)}+\frac{5}{4}\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{5}{12x^6}+\frac{1}{4x^6\left(1+x^4\right)}-\frac{5}{4}\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)}\,dx,x,x^2\right)\\
&= -\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{1}{4x^6\left(1+x^4\right)}+\frac{5}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)\\
&= -\frac{5}{12x^6}+\frac{5}{4x^2}+\frac{1}{4x^6\left(1+x^4\right)}+\frac{5}{4}\tan^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.89

$$-\frac{1}{6x^6}+\frac{1}{x^2}+\frac{x^2}{4\left(1+x^4\right)}-\frac{5}{4}\tan^{-1}\left(\frac{1}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 + x^(-2) + x^2/(4*(1 + x^4)) - (5*ArcTan[x^(-2)])/4
```

Maple [A]
time = 0.03, size = 28, normalized size = 0.76

method	result	size

default	$\frac{x^2}{4x^4+4}+\frac{5\arctan(x^2)}{4}-\frac{1}{6x^6}+\frac{1}{x^2}$	28
risch	$\frac{\frac{5}{4}x^8+\frac{5}{6}x^4-\frac{1}{6}}{x^6(x^4+1)}+\frac{5\arctan(x^2)}{4}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*x^2/(x^4+1)+5/4*arctan(x^2)-1/6/x^6+1/x^2
```

Maxima [A]
time = 0.52, size = 30, normalized size = 0.81

$$\frac{15\,x^8+10\,x^4-2}{12\,(x^{10}+x^6)}+\frac{5}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="maxima")
[Out] 1/12*(15*x^8 + 10*x^4 - 2)/(x^10 + x^6) + 5/4*arctan(x^2)
```

Fricas [A]
time = 0.33, size = 36, normalized size = 0.97

$$\frac{15\,x^8+10\,x^4+15\,(x^{10}+x^6)\arctan\left(x^2\right)-2}{12\,(x^{10}+x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="fricas")
[Out] 1/12*(15*x^8 + 10*x^4 + 15*(x^10 + x^6)*arctan(x^2) - 2)/(x^10 + x^6)
```

Sympy [A]
time = 0.06, size = 29, normalized size = 0.78

$$\frac{5\operatorname{atan}\left(x^2\right)}{4}+\frac{15x^8+10x^4-2}{12x^{10}+12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+2*x**4+1),x)
[Out] 5*atan(x**2)/4 + (15*x**8 + 10*x**4 - 2)/(12*x**10 + 12*x**6)
```

Giac [A]
time = 4.48, size = 31, normalized size = 0.84

$$\frac{x^2}{4\,(x^4+1)}+\frac{6\,x^4-1}{6\,x^6}+\frac{5}{4}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^2/(x^4 + 1) + 1/6*(6*x^4 - 1)/x^6 + 5/4*arctan(x^2)
```

Mupad [B]
time = 0.05, size = 30, normalized size = 0.81

$$\frac{5\,\mathrm{atan}\left(x^2\right)}{4}+\frac{\frac{5\,x^8}{4}+\frac{5\,x^4}{6}-\frac{1}{6}}{x^6\,\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(2*x^4 + x^8 + 1)),x)
```

```
[Out] (5*atan(x^2))/4 + ((5*x^4)/6 + (5*x^8)/4 - 1/6)/(x^6*(x^4 + 1))
```

3.281 $\int \frac{x^8}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=104

$$\frac{5x}{4}-\frac{x^5}{4(1+x^4)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{5\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{5\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}$$

```
[Out] 5/4*x-1/4*x^5/(x^4+1)-5/16*arctan(-1+x*2^(1/2))*2^(1/2)-5/16*arctan(1+x*2^(
1/2))*2^(1/2)+5/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-5/32*ln(1+x^2+x*2^(1/2))*2^(
1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {28, 294, 327, 217, 1179, 642, 1176, 631, 210}

$$\frac{5\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{5\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{5\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{5\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{x^5}{4(x^4+1)}+\frac{5x}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] (5*x)/4 - x^5/(4*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*Ar
cTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]
) - (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^8}{1+2x^4+x^8}\,dx = \int \frac{x^8}{(1+x^4)^2}\,dx$$
$$= -\frac{x^5}{4(1+x^4)} + \frac{5}{4}\int \frac{x^4}{1+x^4}\,dx$$
$$= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} - \frac{5}{4}\int \frac{1}{1+x^4}\,dx$$
$$= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} - \frac{5}{8}\int \frac{1-x^2}{1+x^4}\,dx - \frac{5}{8}\int \frac{1+x^2}{1+x^4}\,dx$$
$$= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} - \frac{5}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx - \frac{5}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx + \frac{5\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-}}{16\sqrt{2}}$$
$$= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} + \frac{5\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} - \frac{5\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} - \frac{5\mathrm{Subst}\left(\int \frac{1}{-1-x}\right.}{8}$$
$$= \frac{5x}{4} - \frac{x^5}{4(1+x^4)} + \frac{5\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}} - \frac{5\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}} + \frac{5\log\left(1-\sqrt{2}\,x+x\right.}{16\sqrt{2}}$$

Mathematica [A]
time = 0.05, size = 94, normalized size = 0.90

$$\frac{1}{32}\left(32x+\frac{8x}{1+x^4}+10\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\,x\right)-10\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\,x\right)+5\sqrt{2}\log\left(1-\sqrt{2}\,x+x^2\right)-5\sqrt{2}\log\left(1+\sqrt{2}\,x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + 2*x^4 + x^8),x]
```

```
[Out] (32*x + (8*x)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*Arc
Tan[1 + Sqrt[2]*x] + 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 5*Sqrt[2]*Log[1 +
 Sqrt[2]*x + x^2])/32
```

Maple [A]
time = 0.04, size = 64, normalized size = 0.62

method	result	size
risch	$x+\frac{x}{4x^4+4}-\frac{5\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16}$	34
default	$x+\frac{x}{4x^4+4}-\frac{5\sqrt{2}\left(\ln\left(\frac{1+x^2+\sqrt{2}\,x}{1+x^2-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{32}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/4*x/(x^4+1)-5/32*2^(1/2)*(ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))+2*arc
tan(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))
```

Maxima [A]
time = 0.51, size = 83, normalized size = 0.80

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+x+\frac{x}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

Fricas [A]
time = 0.35, size = 137, normalized size = 1.32

$$\frac{32x^5+20\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+20\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-5\sqrt{2}(x^4+1)\log\left(4x^2+4\sqrt{2}x+4\right)+5\sqrt{2}(x^4+1)\log\left(4x^2-4\sqrt{2}x+4\right)+40x}{32(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/32*(32*x^5 + 20*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 +
sqrt(2)*x + 1) - 1) + 20*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt
(x^2 - sqrt(2)*x + 1) + 1) - 5*sqrt(2)*(x^4 + 1)*log(4*x^2 + 4*sqrt(2)*x +
4) + 5*sqrt(2)*(x^4 + 1)*log(4*x^2 - 4*sqrt(2)*x + 4) + 40*x)/(x^4 + 1)
```

Sympy [A]
time = 0.07, size = 90, normalized size = 0.87

$$x+\frac{x}{4x^4+4}+\frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}-\frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+2*x**4+1),x)
```

```
[Out] x + x/(4*x**4 + 4) + 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 5*sqrt(2)*log
(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*sqrt(2)*at
an(sqrt(2)*x + 1)/16
```

Giac [A]
time = 5.04, size = 83, normalized size = 0.80

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+x+\frac{x}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) + x + 1/4*x/(x^4 + 1)
```

Mupad [B]
time = 1.37, size = 45, normalized size = 0.43

$$x+\frac{x}{4\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{16}-\frac{5}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{16}+\frac{5}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(2*x^4 + x^8 + 1),x)
```

```
[Out] x - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(5/16 + 5i/16) - 2^(1/2)*atan(2^(1
/2)*x*(1/2 + 1i/2))*(5/16 - 5i/16) + x/(4*(x^4 + 1))
```

3.282 $\int \frac{x^6}{1+2x^4+x^8} \, dx$

Optimal. Leaf size=99

$$-\frac{x^3}{4(1+x^4)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{3\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{3\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}$$

```
[Out] -1/4*x^3/(x^4+1)+3/16*arctan(-1+x*2^(1/2))*2^(1/2)+3/16*arctan(1+x*2^(1/2))
*2^(1/2)+3/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-3/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {28, 294, 303, 1176, 631, 210, 1179, 642}

$$-\frac{3\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{3\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{3\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{3\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{x^3}{4(x^4+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + 2*x^4 + x^8),x]
```

```
[Out] -1/4*x^3/(1 + x^4) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 +
Sqrt[2]*x])/(8*Sqrt[2]) + (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) - (3*Lo
g[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^6}{1+2x^4+x^8}\,dx = \int \frac{x^6}{(1+x^4)^2}\,dx$$
$$= -\frac{x^3}{4(1+x^4)} + \frac{3}{4}\int \frac{x^2}{1+x^4}\,dx$$
$$= -\frac{x^3}{4(1+x^4)} - \frac{3}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{3}{8}\int \frac{1+x^2}{1+x^4}\,dx$$
$$= -\frac{x^3}{4(1+x^4)} + \frac{3}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{3}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx + \frac{3\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}}{16\sqrt{2}}$$
$$= -\frac{x^3}{4(1+x^4)} + \frac{3\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} - \frac{3\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} + \frac{3\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,d\right.}{8\sqrt{}}$$
$$= -\frac{x^3}{4(1+x^4)} - \frac{3\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}} + \frac{3\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}$$

Mathematica [A]
time = 0.04, size = 93, normalized size = 0.94

$$\frac{1}{32}\left(-\frac{8x^3}{1+x^4}-6\sqrt{2}\,\tan^{-1}\left(1-\sqrt{2}\,x\right)+6\sqrt{2}\,\tan^{-1}\left(1+\sqrt{2}\,x\right)+3\sqrt{2}\,\log\left(1-\sqrt{2}\,x+x^2\right)-3\sqrt{2}\,\log\left(1+\sqrt{2}\,x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((-8*x^3)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1
+ Sqrt[2]*x] + 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 3*Sqrt[2]*Log[1 + Sqrt[
2]*x + x^2])/32
```

Maple [A]
time = 0.02, size = 65, normalized size = 0.66

method	result	size
risch	$-\frac{x^3}{4(x^4+1)}+\frac{3\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{\_R}\right)}{16}$	35
default	$-\frac{x^3}{4(x^4+1)}+\frac{3\sqrt{2}\,\left(\ln\left(\frac{1+x^2-\sqrt{2}\,x}{1+x^2+\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{32}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^3/(x^4+1)+3/32*2^(1/2)*(ln((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))+2*ar
ctan(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))
```

Maxima [A]
time = 0.51, size = 84, normalized size = 0.85

$$-\frac{x^3}{4\,(x^4+1)}+\frac{3}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x+\sqrt{2}\right)\right)+\frac{3}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x-\sqrt{2}\right)\right)-\frac{3}{32}\,\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)+\frac{3}{32}\,\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A]
time = 0.35, size = 134, normalized size = 1.35

$$-\frac{8\,x^3+12\,\sqrt{2}\,(x^4+1)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+\sqrt{2}\,x+1}-1\right)+12\,\sqrt{2}\,(x^4+1)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2-\sqrt{2}\,x+1}+1\right)+3\,\sqrt{2}\,(x^4+1)\log\left(4\,x^2+4\,\sqrt{2}\,x+4\right)-3\,\sqrt{2}\,(x^4+1)\log\left(4\,x^2-4\,\sqrt{2}\,x+4\right)}{32\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(8*x^3 + 12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 +
sqrt(2)*x + 1) - 1) + 12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt
(x^2 - sqrt(2)*x + 1) + 1) + 3*sqrt(2)*(x^4 + 1)*log(4*x^2 + 4*sqrt(2)*x +
4) - 3*sqrt(2)*(x^4 + 1)*log(4*x^2 - 4*sqrt(2)*x + 4))/(x^4 + 1)
```

Sympy [A]
time = 0.07, size = 90, normalized size = 0.91

$$-\frac{x^3}{4x^4+4}+\frac{3\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)}{32}-\frac{3\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)}{32}+\frac{3\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{16}+\frac{3\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 + 4) + 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 3*sqrt(2)*log
(x**2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*at
an(sqrt(2)*x + 1)/16
```

Giac [A]
time = 6.49, size = 84, normalized size = 0.85

$$-\frac{x^3}{4\,(x^4+1)}+\frac{3}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x+\sqrt{2}\right)\right)+\frac{3}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x-\sqrt{2}\right)\right)-\frac{3}{32}\,\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)+\frac{3}{32}\,\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 + 1) + 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/1
6*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 3/32*sqrt(2)*log(x^2 + sqrt
(2)*x + 1) + 3/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B]
time = 1.33, size = 47, normalized size = 0.47

$$-\frac{x^3}{4\,\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(3/16 - 3i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(3/16 + 3i/16) - x^3/(4*(x^4 + 1))
```

3.283 $\int \frac{x^4}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=97

$$-\frac{x}{4\left(1+x^4\right)}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}$$

```
[Out] -1/4*x/(x^4+1)+1/16*arctan(-1+x*2^(1/2))*2^(1/2)+1/16*arctan(1+x*2^(1/2))*2
^(1/2)-1/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+1/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {28, 294, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{x}{4\left(x^4+1\right)}-\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + 2*x^4 + x^8),x]
```

```
[Out] -1/4*x/(1 + x^4) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x
]/(8*Sqrt[2]) - Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) + Log[1 + Sqrt[2]*x +
 x^2]/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^4}{1+2x^4+x^8}\,dx = \int \frac{x^4}{(1+x^4)^2}\,dx$$
$$= -\frac{x}{4(1+x^4)} + \frac{1}{4}\int \frac{1}{1+x^4}\,dx$$
$$= -\frac{x}{4(1+x^4)} + \frac{1}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{8}\int \frac{1+x^2}{1+x^4}\,dx$$
$$= -\frac{x}{4(1+x^4)} + \frac{1}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{1}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx - \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{16\sqrt{2}}$$
$$= -\frac{x}{4(1+x^4)} - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{8\sqrt{2}}$$
$$= -\frac{x}{4(1+x^4)} - \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}} - \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} + \frac{\log}{}$$

Mathematica [A]
time = 0.04, size = 90, normalized size = 0.93

$$\frac{1}{32}\left(-\frac{8x}{1+x^4} - 2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\,x\right) + 2\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\,x\right) - \sqrt{2}\log\left(1-\sqrt{2}\,x+x^2\right) + \sqrt{2}\log\left(1+\sqrt{2}\,x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((-8*x)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + Sqrt[2]*Log[1 + Sqrt[2]*x +
 x^2])/32
```

Maple [A]
time = 0.02, size = 63, normalized size = 0.65

method	result	size
risch	$-\frac{x}{4(x^4+1)} + \frac{\left(\sum_{\_R=\text{RootOf}(\_Z^4+1)} \frac{\ln(x-\_R)}{\_R^3}\right)}{16}$	33
default	$-\frac{x}{4(x^4+1)} + \frac{\sqrt{2}\left(\ln\left(\frac{1+x^2+\sqrt{2}\,x}{1+x^2-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{32}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x/(x^4+1)+1/32*2^(1/2)*(ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))+2*arct
an(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))
```

Maxima [A]
time = 0.50, size = 82, normalized size = 0.85

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{x}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

Fricas [A]
time = 0.37, size = 131, normalized size = 1.35

$$-\frac{4\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+4\sqrt{2}\left(x^4+1\right)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\sqrt{2}\left(x^4+1\right)\log\left(4x^2+4\sqrt{2}x+4\right)+\sqrt{2}\left(x^4+1\right)\log\left(4x^2-4\sqrt{2}x+4\right)+8x}{32\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x
 + 1) - 1) + 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqr
t(2)*x + 1) + 1) - sqrt(2)*(x^4 + 1)*log(4*x^2 + 4*sqrt(2)*x + 4) + sqrt(2)
*(x^4 + 1)*log(4*x^2 - 4*sqrt(2)*x + 4) + 8*x)/(x^4 + 1)
```

Sympy [A]
time = 0.06, size = 82, normalized size = 0.85

$$-\frac{x}{4x^4+4}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+2*x**4+1),x)
```

```
[Out] -x/(4*x**4 + 4) - sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + sqrt(2)*log(x**2 +
 sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt(2)*
x + 1)/16
```

Giac [A]
time = 6.86, size = 82, normalized size = 0.85

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{x}{4\left(x^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 1/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1)
```

Mupad [B]
time = 0.08, size = 45, normalized size = 0.46

$$-\frac{x}{4\,\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}+\frac{1}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}-\frac{1}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/16 + 1i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(1/16 - 1i/16) - x/(4*(x^4 + 1))
```

3.284 $\int \frac{x^2}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=99

$$\frac{x^3}{4(1+x^4)}-\frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}$$

```
[Out] 1/4*x^3/(x^4+1)+1/16*arctan(-1+x*2^(1/2))*2^(1/2)+1/16*arctan(1+x*2^(1/2))*
2^(1/2)+1/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-1/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {28, 296, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{x^3}{4(x^4+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 + x^4)) - ArcTan[1 - Sqrt[2]*x]/(8*Sqrt[2]) + ArcTan[1 + Sqrt[2]*
x]/(8*Sqrt[2]) + Log[1 - Sqrt[2]*x + x^2]/(16*Sqrt[2]) - Log[1 + Sqrt[2]*x
+ x^2]/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^2}{1+2x^4+x^8}\,dx = \int \frac{x^2}{(1+x^4)^2}\,dx$$
$$= \frac{x^3}{4(1+x^4)} + \frac{1}{4}\int \frac{x^2}{1+x^4}\,dx$$
$$= \frac{x^3}{4(1+x^4)} - \frac{1}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{1}{8}\int \frac{1+x^2}{1+x^4}\,dx$$
$$= \frac{x^3}{4(1+x^4)} + \frac{1}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{1}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{16\sqrt{2}}$$
$$= \frac{x^3}{4(1+x^4)} + \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1\right.}{8\sqrt{2}}$$
$$= \frac{x^3}{4(1+x^4)} - \frac{\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} - \frac{\log}{}$$

Mathematica [A]
time = 0.03, size = 92, normalized size = 0.93

$$\frac{1}{32}\left(\frac{8x^3}{1+x^4} - 2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\,x\right) + 2\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\,x\right) + \sqrt{2}\log\left(1-\sqrt{2}\,x+x^2\right) - \sqrt{2}\log\left(1+\sqrt{2}\,x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 2*x^4 + x^8),x]
```

```
[Out] ((8*x^3)/(1 + x^4) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 2*Sqrt[2]*ArcTan[1 +
 Sqrt[2]*x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - Sqrt[2]*Log[1 + Sqrt[2]*x
+ x^2])/32
```

Maple [A]
time = 0.01, size = 65, normalized size = 0.66

method	result	size
risch	$\frac{x^3}{4x^4+4} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+1\right)} \frac{\ln\left(x-\_R\right)}{\_R}\right)}{16}$	35
default	$\frac{x^3}{4x^4+4} + \frac{\sqrt{2}\left(\ln\left(\frac{1+x^2-\sqrt{2}\,x}{1+x^2+\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{32}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^3/(x^4+1)+1/32*2^(1/2)*(ln((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))+2*arc
tan(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))
```

Maxima [A]
time = 0.51, size = 84, normalized size = 0.85

$$\frac{x^3}{4\,(x^4+1)} + \frac{1}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{32}\,\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right) + \frac{1}{32}\,\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A]
time = 0.39, size = 133, normalized size = 1.34

$$\frac{8\,x^3 - 4\,\sqrt{2}\,(x^4+1)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+\sqrt{2}\,x+1}-1\right) - 4\,\sqrt{2}\,(x^4+1)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2-\sqrt{2}\,x+1}+1\right) - \sqrt{2}\,(x^4+1)\log\left(4\,x^2+4\,\sqrt{2}\,x+4\right) + \sqrt{2}\,(x^4+1)\log\left(4\,x^2-4\,\sqrt{2}\,x+4\right)}{32\,(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/32*(8*x^3 - 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sq
rt(2)*x + 1) - 1) - 4*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^
2 - sqrt(2)*x + 1) + 1) - sqrt(2)*(x^4 + 1)*log(4*x^2 + 4*sqrt(2)*x + 4) +
sqrt(2)*(x^4 + 1)*log(4*x^2 - 4*sqrt(2)*x + 4))/(x^4 + 1)
```

Sympy [A]
time = 0.06, size = 83, normalized size = 0.84

$$\frac{x^3}{4x^4+4} + \frac{\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)}{32} - \frac{\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)}{32} + \frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{16} + \frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+2*x**4+1),x)
```

```
[Out] x**3/(4*x**4 + 4) + sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - sqrt(2)*log(x**2
 + sqrt(2)*x + 1)/32 + sqrt(2)*atan(sqrt(2)*x - 1)/16 + sqrt(2)*atan(sqrt(2
)*x + 1)/16
```

Giac [A]
time = 4.34, size = 84, normalized size = 0.85

$$\frac{x^3}{4\,(x^4+1)} + \frac{1}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x+\sqrt{2}\right)\right) + \frac{1}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x-\sqrt{2}\right)\right) - \frac{1}{32}\,\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right) + \frac{1}{32}\,\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 1/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 1/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 1/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Mupad [B]
time = 0.05, size = 46, normalized size = 0.46

$$\frac{x^3}{4\,(x^4+1)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}-\frac{1}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{16}+\frac{1}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(1/16 - 1i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(1/16 + 1i/16) + x^3/(4*(x^4 + 1))
```

3.285 $\int \frac{1}{1+2x^4+x^8}\,dx$

Optimal. Leaf size=97

$$\frac{x}{4(1+x^4)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{3\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{3\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\frac{3\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}$$

```
[Out] 1/4*x/(x^4+1)+3/16*arctan(-1+x*2^(1/2))*2^(1/2)+3/16*arctan(1+x*2^(1/2))*2^
(1/2)-3/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+3/32*ln(1+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$,
Rules used = {28, 205, 217, 1179, 642, 1176, 631, 210}

$$-\frac{3\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{3\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{x}{4(x^4+1)}-\frac{3\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{3\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^4 + x^8)^(-1),x]
```

```
[Out] x/(4*(1 + x^4)) - (3*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) + (3*ArcTan[1 + Sqr
t[2]*x])/(8*Sqrt[2]) - (3*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2]) + (3*Log[1
 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{1+2x^4+x^8}\,dx = \int \frac{1}{(1+x^4)^2}\,dx$$
$$= \frac{x}{4(1+x^4)} + \frac{3}{4}\int \frac{1}{1+x^4}\,dx$$
$$= \frac{x}{4(1+x^4)} + \frac{3}{8}\int \frac{1-x^2}{1+x^4}\,dx + \frac{3}{8}\int \frac{1+x^2}{1+x^4}\,dx$$
$$= \frac{x}{4(1+x^4)} + \frac{3}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx + \frac{3}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx - \frac{3\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}\,x-x^2}\,dx}{16\sqrt{2}}$$
$$= \frac{x}{4(1+x^4)} - \frac{3\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} + \frac{3\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} + \frac{3\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x\right.}{8\sqrt{2}}$$
$$= \frac{x}{4(1+x^4)} - \frac{3\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}} + \frac{3\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}} - \frac{3\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} +$$

Mathematica [A]
time = 0.03, size = 91, normalized size = 0.94

$$\frac{1}{32}\left(\frac{8x}{1+x^4} - 6\sqrt{2}\,\tan^{-1}\left(1-\sqrt{2}\,x\right) + 6\sqrt{2}\,\tan^{-1}\left(1+\sqrt{2}\,x\right) - 3\sqrt{2}\,\log\left(1-\sqrt{2}\,x+x^2\right) + 3\sqrt{2}\,\log\left(1+\sqrt{2}\,x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^4 + x^8)^(-1),x]
```

```
[Out] ((8*x)/(1 + x^4) - 6*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] + 6*Sqrt[2]*ArcTan[1 + S
qrt[2]*x] - 3*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 3*Sqrt[2]*Log[1 + Sqrt[2]*
x + x^2])/32
```

Maple [A]
time = 0.02, size = 63, normalized size = 0.65

method	result	size
risch	$\frac{x}{4x^4+4} + \frac{3\left(\sum_{\_R=\text{RootOf}\left(\_Z^4+1\right)} \frac{\ln\left(x-\_R\right)}{\_R^3}\right)}{16}$	33
default	$\frac{x}{4x^4+4} + \frac{3\sqrt{2}\,\left(\ln\left(\frac{1+x^2+\sqrt{2}\,x}{1+x^2-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{32}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x/(x^4+1)+3/32*2^(1/2)*(ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))+2*arcta
n(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))
```

Maxima [A]
time = 0.50, size = 82, normalized size = 0.85

$$\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

Fricas [A]
time = 0.35, size = 132, normalized size = 1.36

$$-\frac{12\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+12\sqrt{2}(x^4+1)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-3\sqrt{2}(x^4+1)\log\left(4x^2+4\sqrt{2}x+4\right)+3\sqrt{2}(x^4+1)\log\left(4x^2-4\sqrt{2}x+4\right)-8x}{32(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*
x + 1) - 1) + 12*sqrt(2)*(x^4 + 1)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - s
qrt(2)*x + 1) + 1) - 3*sqrt(2)*(x^4 + 1)*log(4*x^2 + 4*sqrt(2)*x + 4) + 3*s
qrt(2)*(x^4 + 1)*log(4*x^2 - 4*sqrt(2)*x + 4) - 8*x)/(x^4 + 1)
```

Sympy [A]
time = 0.07, size = 88, normalized size = 0.91

$$\frac{x}{4x^4+4}-\frac{3\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}+\frac{3\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+2*x**4+1),x)
```

```
[Out] x/(4*x**4 + 4) - 3*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 3*sqrt(2)*log(x**
2 + sqrt(2)*x + 1)/32 + 3*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 3*sqrt(2)*atan(s
qrt(2)*x + 1)/16
```

Giac [A]
time = 4.10, size = 82, normalized size = 0.85

$$\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{3}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{3}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{3}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{4(x^4+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 3/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) + 3/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 3/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1)
```

Mupad [B]
time = 1.31, size = 44, normalized size = 0.45

$$\frac{x}{4\,\left(x^4+1\right)}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}+\frac{3}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{16}-\frac{3}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^4 + x^8 + 1),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(3/16 + 3i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(3/16 - 3i/16) + x/(4*(x^4 + 1))
```

3.286 $\int \frac{1}{x^2(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=106

$$-\frac{5}{4x}+\frac{1}{4x(1+x^4)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{5\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}\,x\right.}{16\sqrt{2}}$$

```
[Out] -5/4/x+1/4/x/(x^4+1)-5/16*arctan(-1+x*2^(1/2))*2^(1/2)-5/16*arctan(1+x*2^(1
/2))*2^(1/2)-5/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+5/32*ln(1+x^2+x*2^(1/2))*2^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {28, 296, 331, 303, 1176, 631, 210, 1179, 642}

$$\frac{5\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{5\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}+\frac{1}{4x(x^4+1)}-\frac{5\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{5\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{5}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -5/(4*x) + 1/(4*x*(1 + x^4)) + (5*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) - (5*A
rcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (5*Log[1 - Sqrt[2]*x + x^2])/(16*Sqrt[2
]) + (5*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(1+2x^4+x^8)}\,dx &= \int \frac{1}{x^2(1+x^4)^2}\,dx\\ &= \frac{1}{4x(1+x^4)}+\frac{5}{4}\int \frac{1}{x^2(1+x^4)}\,dx\\ &= -\frac{5}{4x}+\frac{1}{4x(1+x^4)}-\frac{5}{4}\int \frac{x^2}{1+x^4}\,dx\\ &= -\frac{5}{4x}+\frac{1}{4x(1+x^4)}+\frac{5}{8}\int \frac{1-x^2}{1+x^4}\,dx-\frac{5}{8}\int \frac{1+x^2}{1+x^4}\,dx\\ &= -\frac{5}{4x}+\frac{1}{4x(1+x^4)}-\frac{5}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx-\frac{5}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx-\frac{5\int}{\ }\\ &= -\frac{5}{4x}+\frac{1}{4x(1+x^4)}-\frac{5\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{5\mathrm{Subs}}{\ }\\ &= -\frac{5}{4x}+\frac{1}{4x(1+x^4)}+\frac{5\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{5\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{5\log\left(1-\right.}{16}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 98, normalized size = 0.92

$$\frac{1}{32}\left(-\frac{32}{x}-\frac{8x^3}{1+x^4}+10\sqrt{2}\,\tan^{-1}\left(1-\sqrt{2}\,x\right)-10\sqrt{2}\,\tan^{-1}\left(1+\sqrt{2}\,x\right)-5\sqrt{2}\,\log\left(1-\sqrt{2}\,x+x^2\right)+5\sqrt{2}\,\log\left(1+\sqrt{2}\,x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x - (8*x^3)/(1 + x^4) + 10*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 10*Sqrt[2]*
ArcTan[1 + Sqrt[2]*x] - 5*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] + 5*Sqrt[2]*Log[
1 + Sqrt[2]*x + x^2])/32
```

Maple [A]
time = 0.02, size = 70, normalized size = 0.66

method	result	size
risch	$\frac{-\frac{5x^4}{4}-1}{x(x^4+1)}+\frac{5\left(\sum_{\_R=\mathrm{RootOf}(\_Z^4+1)}\_R\ln\left(-\_R^3+x\right)\right)}{16}$	41
default	$-\frac{x^3}{4(x^4+1)}-\frac{5\sqrt{2}\left(\ln\left(\frac{1+x^2-\sqrt{2}\,x}{1+x^2+\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{32}-\frac{1}{x}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^3/(x^4+1)-5/32*2^(1/2)*(ln((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))+2*ar
ctan(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))-1/x
```

Maxima [A]
time = 0.54, size = 88, normalized size = 0.83

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{5x^4+4}{4(x^5+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

Fricas [A]
time = 0.35, size = 135, normalized size = 1.27

$$-\frac{40x^4-20\sqrt{2}(x^5+x)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-20\sqrt{2}(x^5+x)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-5\sqrt{2}(x^5+x)\log\left(4x^2+4\sqrt{2}x+4\right)+5\sqrt{2}(x^5+x)\log\left(4x^2-4\sqrt{2}x+4\right)+32}{32(x^5+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/32*(40*x^4 - 20*sqrt(2)*(x^5 + x)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 +
 sqrt(2)*x + 1) - 1) - 20*sqrt(2)*(x^5 + x)*arctan(-sqrt(2)*x + sqrt(2)*sqr
t(x^2 - sqrt(2)*x + 1) + 1) - 5*sqrt(2)*(x^5 + x)*log(4*x^2 + 4*sqrt(2)*x +
 4) + 5*sqrt(2)*(x^5 + x)*log(4*x^2 - 4*sqrt(2)*x + 4) + 32)/(x^5 + x)
```

Sympy [A]
time = 0.08, size = 97, normalized size = 0.92

$$\frac{-5x^4-4}{4x^5+4x}-\frac{5\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}+\frac{5\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}-\frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{5\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+2*x**4+1),x)
```

```
[Out] (-5*x**4 - 4)/(4*x**5 + 4*x) - 5*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 5*s
qrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 5*sqrt(2)*atan(sqrt(2)*x - 1)/16 - 5*
sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Giac [A]
time = 2.76, size = 88, normalized size = 0.83

$$-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{5}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{5}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{5}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{5x^4+4}{4(x^5+x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -5/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 5/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) + 5/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 5/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*(5*x^4 + 4)/(x^5 + x)
```

Mupad [B]
time = 1.31, size = 49, normalized size = 0.46

$$-\frac{\frac{5x^4}{4}+1}{x^5+x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{16}+\frac{5}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{5}{16}-\frac{5}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(2*x^4 + x^8 + 1)),x)
```

```
[Out] - ((5*x^4)/4 + 1)/(x + x^5) - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(5/16 -
5i/16) - 2^(1/2)*atan(2^(1/2)*x*(1/2 + 1i/2))*(5/16 + 5i/16)
```

3.287 $\int \frac{1}{x^4(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=106

$$-\frac{7}{12x^3}+\frac{1}{4x^3(1+x^4)}+\frac{7\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{7\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{7\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{7\log\left(1+\sqrt{2}\,x\right.}{16\sqrt{2}}$$

```
[Out] -7/12/x^3+1/4/x^3/(x^4+1)-7/16*arctan(-1+x*2^(1/2))*2^(1/2)-7/16*arctan(1+x
*2^(1/2))*2^(1/2)+7/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-7/32*ln(1+x^2+x*2^(1/2))
*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {28, 296, 331, 217, 1179, 642, 1176, 631, 210}

$$\frac{7\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{7\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{7}{12x^3}+\frac{7\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{7\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{1}{4x^3(x^4+1)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 + x^4)) + (7*ArcTan[1 - Sqrt[2]*x])/(8*Sqrt[2]) -
 (7*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (7*Log[1 - Sqrt[2]*x + x^2])/(16*S
qrt[2]) - (7*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^4(1+2x^4+x^8)}\,dx = \int \frac{1}{x^4(1+x^4)^2}\,dx$$
$$= \frac{1}{4x^3(1+x^4)} + \frac{7}{4}\int \frac{1}{x^4(1+x^4)}\,dx$$
$$= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} - \frac{7}{4}\int \frac{1}{1+x^4}\,dx$$
$$= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} - \frac{7}{8}\int \frac{1-x^2}{1+x^4}\,dx - \frac{7}{8}\int \frac{1+x^2}{1+x^4}\,dx$$
$$= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} - \frac{7}{16}\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx - \frac{7}{16}\int \frac{1}{1+\sqrt{2}\,x+x^2}\,dx + \frac{7}{\ldots}$$
$$= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} + \frac{7\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} - \frac{7\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} - \frac{7\mathrm{Sul}}{\ldots}$$
$$= -\frac{7}{12x^3} + \frac{1}{4x^3(1+x^4)} + \frac{7\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}} - \frac{7\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}} + \frac{7\log\left(1-\ldots\right.}{1\ldots}$$

Mathematica [A]
time = 0.05, size = 96, normalized size = 0.91

$$\frac{1}{96}\left(-\frac{32}{x^3} - \frac{24x}{1+x^4} + 42\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\,x\right) - 42\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\,x\right) + 21\sqrt{2}\log\left(1-\sqrt{2}\,x+x^2\right) - 21\sqrt{2}\log\left(1+\sqrt{2}\,x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + 2*x^4 + x^8)),x]
[Out] (-32/x^3 - (24*x)/(1 + x^4) + 42*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 42*Sqrt[2]
*ArcTan[1 + Sqrt[2]*x] + 21*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 21*Sqrt[2]*L
og[1 + Sqrt[2]*x + x^2])/96
```

Maple [A]
time = 0.02, size = 68, normalized size = 0.64

method	result	size
risch	$\frac{-\frac{7x^4}{12}-\frac{1}{3}}{x^3(x^4+1)} + \frac{7\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)} \_R\ln\left(x-\_R\right)\right)}{16}$	39
default	$-\frac{x}{4(x^4+1)} - \frac{7\sqrt{2}\left(\ln\left(\frac{1+x^2+\sqrt{2}\,x}{1+x^2-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{32} - \frac{1}{3x^3}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x/(x^4+1)-7/32*2^(1/2)*(ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))+2*arct
an(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))-1/3/x^3
```

Maxima [A]
time = 0.53, size = 90, normalized size = 0.85

$$-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{7}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{7}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{7x^4+4}{12(x^7+x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] -7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/12*(7*x^4 + 4)/(x^7 + x^3)
```

Fricas [A]
time = 0.35, size = 145, normalized size = 1.37

$$-\frac{56x^4-84\sqrt{2}(x^7+x^3)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-84\sqrt{2}(x^7+x^3)\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)+21\sqrt{2}(x^7+x^3)\log\left(4x^2+4\sqrt{2}x+4\right)-21\sqrt{2}(x^7+x^3)\log\left(4x^2-4\sqrt{2}x+4\right)+32}{96(x^7+x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/96*(56*x^4 - 84*sqrt(2)*(x^7 + x^3)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2
 + sqrt(2)*x + 1) - 1) - 84*sqrt(2)*(x^7 + x^3)*arctan(-sqrt(2)*x + sqrt(2)
*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 21*sqrt(2)*(x^7 + x^3)*log(4*x^2 + 4*sqrt
(2)*x + 4) - 21*sqrt(2)*(x^7 + x^3)*log(4*x^2 - 4*sqrt(2)*x + 4) + 32)/(x^7
 + x^3)
```

Sympy [A]
time = 0.08, size = 99, normalized size = 0.93

$$\frac{-7x^4-4}{12x^7+12x^3}+\frac{7\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{32}-\frac{7\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{32}-\frac{7\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{16}-\frac{7\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+2*x**4+1),x)
```

```
[Out] (-7*x**4 - 4)/(12*x**7 + 12*x**3) + 7*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32
- 7*sqrt(2)*log(x**2 + sqrt(2)*x + 1)/32 - 7*sqrt(2)*atan(sqrt(2)*x - 1)/16
 - 7*sqrt(2)*atan(sqrt(2)*x + 1)/16
```

Giac [A]
time = 3.31, size = 87, normalized size = 0.82

$$-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{7}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{7}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{7}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\frac{x}{4(x^4+1)}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] -7/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 7/16*sqrt(2)*arctan(1/2
*sqrt(2)*(2*x - sqrt(2))) - 7/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 7/32*sq
rt(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*x/(x^4 + 1) - 1/3/x^3
```

Mupad [B]
time = 1.36, size = 51, normalized size = 0.48

$$-\frac{\frac{7x^4}{12}+\frac{1}{3}}{x^7+x^3}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{16}-\frac{7}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{16}+\frac{7}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(2*x^4 + x^8 + 1)),x)
```

```
[Out] - 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(7/16 + 7i/16) - 2^(1/2)*atan(2^(1/2
)*x*(1/2 + 1i/2))*(7/16 - 7i/16) - ((7*x^4)/12 + 1/3)/(x^3 + x^7)
```

3.288 $\int \frac{1}{x^6(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=113

$$-\frac{9}{20x^5}+\frac{9}{4x}+\frac{1}{4x^5(1+x^4)}-\frac{9\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{9\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{9\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}-\frac{9\log\left(1-\right.}{1}$$

```
[Out] -9/20/x^5+9/4/x+1/4/x^5/(x^4+1)+9/16*arctan(-1+x*2^(1/2))*2^(1/2)+9/16*arct
an(1+x*2^(1/2))*2^(1/2)+9/32*ln(1+x^2-x*2^(1/2))*2^(1/2)-9/32*ln(1+x^2+x*2^
(1/2))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {28, 296, 331, 303, 1176, 631, 210, 1179, 642}

$$-\frac{9\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{9\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{9}{20x^5}+\frac{9\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}-\frac{9\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{1}{4x^5\left(x^4+1\right)}+\frac{9}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -9/(20*x^5) + 9/(4*x) + 1/(4*x^5*(1 + x^4)) - (9*ArcTan[1 - Sqrt[2]*x])/(8*
Sqrt[2]) + (9*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) + (9*Log[1 - Sqrt[2]*x + x
^2])/(16*Sqrt[2]) - (9*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^6 (1+2x^4+x^8)} \, dx = \int \frac{1}{x^6 (1+x^4)^2} \, dx$$
$$= \frac{1}{4x^5 (1+x^4)} + \frac{9}{4} \int \frac{1}{x^6 (1+x^4)} \, dx$$
$$= -\frac{9}{20x^5} + \frac{1}{4x^5 (1+x^4)} - \frac{9}{4} \int \frac{1}{x^2 (1+x^4)} \, dx$$
$$= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5 (1+x^4)} + \frac{9}{4} \int \frac{x^2}{1+x^4} \, dx$$
$$= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5 (1+x^4)} - \frac{9}{8} \int \frac{1-x^2}{1+x^4} \, dx + \frac{9}{8} \int \frac{1+x^2}{1+x^4} \, dx$$
$$= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5 (1+x^4)} + \frac{9}{16} \int \frac{1}{1-\sqrt{2}\,x+x^2} \, dx + \frac{9}{16} \int \frac{1}{1+\sqrt{2}\,x+x^2} \, d$$
$$= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5 (1+x^4)} + \frac{9\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}} - \frac{9\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}$$
$$= -\frac{9}{20x^5} + \frac{9}{4x} + \frac{1}{4x^5 (1+x^4)} - \frac{9\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}} + \frac{9\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}} + \frac{9\,\text{l}}{}$$

Mathematica [A]
time = 0.05, size = 103, normalized size = 0.91

$$\frac{1}{160}\left(-\frac{32}{x^5}+\frac{320}{x}+\frac{40x^3}{1+x^4}-90\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\,x\right)+90\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\,x\right)+45\sqrt{2}\log\left(1-\sqrt{2}\,x+x^2\right)-45\sqrt{2}\log\left(1+\sqrt{2}\,x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-32/x^5 + 320/x + (40*x^3)/(1 + x^4) - 90*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] +
90*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] + 45*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - 45
*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/160
```

Maple [A]
time = 0.02, size = 75, normalized size = 0.66

method	result	size
risch	$\frac{\frac{9}{4}x^8+\frac{9}{5}x^4-\frac{1}{5}}{x^5(x^4+1)}+\frac{9\left(\sum_{\_R=\text{RootOf}(\_Z^4+1)}\_R\ln\left(\_R^3+x\right)\right)}{16}$	44

default	$\frac{x^{3}}{4x^{4}+4}+\frac{9\sqrt{2}\left(\ln\left(\frac{1+x^{2}-\sqrt{2}\,x}{1+x^{2}+\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{32}-\frac{1}{5x^{5}}+\frac{2}{x}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^3/(x^4+1)+9/32*2^(1/2)*(ln((1+x^2-2^(1/2)*x)/(1+x^2+2^(1/2)*x))+2*arc
tan(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))-1/5/x^5+2/x
```

Maxima [A]
time = 0.51, size = 95, normalized size = 0.84

$$\frac{9}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x+\sqrt{2}\right)\right)+\frac{9}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x-\sqrt{2}\right)\right)-\frac{9}{32}\,\sqrt{2}\,\log\left(x^{2}+\sqrt{2}\,x+1\right)+\frac{9}{32}\,\sqrt{2}\,\log\left(x^{2}-\sqrt{2}\,x+1\right)+\frac{45\,x^{8}+36\,x^{4}-4}{20\,(x^{9}+x^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16*sqrt(2)*arctan(1/2*
sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 9/32*sqr
t(2)*log(x^2 - sqrt(2)*x + 1) + 1/20*(45*x^8 + 36*x^4 - 4)/(x^9 + x^5)
```

Fricas [A]
time = 0.34, size = 150, normalized size = 1.33

$$\frac{360\,x^{8}+288\,x^{4}-180\,\sqrt{2}\,(x^{9}+x^{5})\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^{2}+\sqrt{2}\,x+1}-1\right)-180\,\sqrt{2}\,(x^{9}+x^{5})\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^{2}-\sqrt{2}\,x+1}+1\right)-45\,\sqrt{2}\,(x^{9}+x^{5})\log\left(4\,x^{2}+4\,\sqrt{2}\,x+4\right)+45\,\sqrt{2}\,(x^{9}+x^{5})\log\left(4\,x^{2}-4\,\sqrt{2}\,x+4\right)-32}{160\,(x^{9}+x^{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/160*(360*x^8 + 288*x^4 - 180*sqrt(2)*(x^9 + x^5)*arctan(-sqrt(2)*x + sqrt
(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 180*sqrt(2)*(x^9 + x^5)*arctan(-sqrt(2
)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 45*sqrt(2)*(x^9 + x^5)*log(4
*x^2 + 4*sqrt(2)*x + 4) + 45*sqrt(2)*(x^9 + x^5)*log(4*x^2 - 4*sqrt(2)*x +
4) - 32)/(x^9 + x^5)
```

Sympy [A]
time = 0.09, size = 102, normalized size = 0.90

$$\frac{9\sqrt{2}\,\log\left(x^{2}-\sqrt{2}\,x+1\right)}{32}-\frac{9\sqrt{2}\,\log\left(x^{2}+\sqrt{2}\,x+1\right)}{32}+\frac{9\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{16}+\frac{9\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{16}+\frac{45x^{8}+36x^{4}-4}{20x^{9}+20x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8+2*x**4+1),x)
```

```
[Out] 9*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 - 9*sqrt(2)*log(x**2 + sqrt(2)*x + 1
)/32 + 9*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 9*sqrt(2)*atan(sqrt(2)*x + 1)/16
+ (45*x**8 + 36*x**4 - 4)/(20*x**9 + 20*x**5)
```

Giac [A]
time = 4.31, size = 96, normalized size = 0.85

$$\frac{x^3}{4\,(x^4+1)}+\frac{9}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x+\sqrt{2}\right)\right)+\frac{9}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x-\sqrt{2}\right)\right)-\frac{9}{32}\,\sqrt{2}\,\log\left(x^2+\sqrt{2}\,x+1\right)+\frac{9}{32}\,\sqrt{2}\,\log\left(x^2-\sqrt{2}\,x+1\right)+\frac{10\,x^4-1}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^3/(x^4 + 1) + 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 9/16
*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) - 9/32*sqrt(2)*log(x^2 + sqrt(
2)*x + 1) + 9/32*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/5*(10*x^4 - 1)/x^5
```

Mupad [B]
time = 0.09, size = 55, normalized size = 0.49

$$\frac{\frac{9\,x^8}{4}+\frac{9\,x^4}{5}-\frac{1}{5}}{x^9+x^5}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{9}{16}-\frac{9}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{9}{16}+\frac{9}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(2*x^4 + x^8 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(9/16 - 9i/16) + 2^(1/2)*atan(2^(1/2)*
x*(1/2 + 1i/2))*(9/16 + 9i/16) + ((9*x^4)/5 + (9*x^8)/4 - 1/5)/(x^5 + x^9)
```

3.289 $\int \frac{1}{x^8(1+2x^4+x^8)}\,dx$

Optimal. Leaf size=113

$$-\frac{11}{28x^7}+\frac{11}{12x^3}+\frac{1}{4x^7(1+x^4)}-\frac{11\tan^{-1}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{11\tan^{-1}\left(1+\sqrt{2}\,x\right)}{8\sqrt{2}}-\frac{11\log\left(1-\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}+\frac{11\log\left(1+\sqrt{2}\,x+x^2\right)}{16\sqrt{2}}$$

```
[Out] -11/28/x^7+11/12/x^3+1/4/x^7/(x^4+1)+11/16*arctan(-1+x*2^(1/2))*2^(1/2)+11/
16*arctan(1+x*2^(1/2))*2^(1/2)-11/32*ln(1+x^2-x*2^(1/2))*2^(1/2)+11/32*ln(1
+x^2+x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {28, 296, 331, 217, 1179, 642, 1176, 631, 210}

$$-\frac{11\text{ArcTan}\left(1-\sqrt{2}\,x\right)}{8\sqrt{2}}+\frac{11\text{ArcTan}\left(\sqrt{2}\,x+1\right)}{8\sqrt{2}}-\frac{11}{28x^7}+\frac{11}{12x^3}-\frac{11\log\left(x^2-\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{11\log\left(x^2+\sqrt{2}\,x+1\right)}{16\sqrt{2}}+\frac{1}{4x^7\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

```
[Out] -11/(28*x^7) + 11/(12*x^3) + 1/(4*x^7*(1 + x^4)) - (11*ArcTan[1 - Sqrt[2]*x
])/(8*Sqrt[2]) + (11*ArcTan[1 + Sqrt[2]*x])/(8*Sqrt[2]) - (11*Log[1 - Sqrt[
2]*x + x^2])/(16*Sqrt[2]) + (11*Log[1 + Sqrt[2]*x + x^2])/(16*Sqrt[2])
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^8 (1+2x^4+x^8)} \, dx = \int \frac{1}{x^8 (1+x^4)^2} \, dx$$
$$= \frac{1}{4x^7 (1+x^4)} + \frac{11}{4} \int \frac{1}{x^8 (1+x^4)} \, dx$$
$$= -\frac{11}{28x^7} + \frac{1}{4x^7 (1+x^4)} - \frac{11}{4} \int \frac{1}{x^4 (1+x^4)} \, dx$$
$$= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7 (1+x^4)} + \frac{11}{4} \int \frac{1}{1+x^4} \, dx$$
$$= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7 (1+x^4)} + \frac{11}{8} \int \frac{1-x^2}{1+x^4} \, dx + \frac{11}{8} \int \frac{1+x^2}{1+x^4} \, dx$$
$$= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7 (1+x^4)} + \frac{11}{16} \int \frac{1}{1-\sqrt{2}\, x+x^2} \, dx + \frac{11}{16} \int \frac{1}{1+\sqrt{2}\, x+x^2}$$
$$= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7 (1+x^4)} - \frac{11 \log \left(1-\sqrt{2}\, x+x^2\right)}{16\sqrt{2}} + \frac{11 \log \left(1+\sqrt{2}\, x+x\right.}{16\sqrt{2}}$$
$$= -\frac{11}{28x^7} + \frac{11}{12x^3} + \frac{1}{4x^7 (1+x^4)} - \frac{11 \tan^{-1} \left(1-\sqrt{2}\, x\right)}{8\sqrt{2}} + \frac{11 \tan^{-1} \left(1+\sqrt{2}\, x\right)}{8\sqrt{2}} -$$

Mathematica [A]
time = 0.05, size = 101, normalized size = 0.89

$$\frac{1}{672} \left(-\frac{96}{x^7} + \frac{448}{x^3} + \frac{168x}{1+x^4} - 462\sqrt{2}\, \tan^{-1}\left(1-\sqrt{2}\, x\right) + 462\sqrt{2}\, \tan^{-1}\left(1+\sqrt{2}\, x\right) - 231\sqrt{2}\, \log\left(1-\sqrt{2}\, x+x^2\right) + 231\sqrt{2}\, \log\left(1+\sqrt{2}\, x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 + 2*x^4 + x^8)),x]
```

```
[Out] (-96/x^7 + 448/x^3 + (168*x)/(1 + x^4) - 462*Sqrt[2]*ArcTan[1 - Sqrt[2]*x]
+ 462*Sqrt[2]*ArcTan[1 + Sqrt[2]*x] - 231*Sqrt[2]*Log[1 - Sqrt[2]*x + x^2]
+ 231*Sqrt[2]*Log[1 + Sqrt[2]*x + x^2])/672
```

Maple [A]
time = 0.03, size = 73, normalized size = 0.65

method	result	size
risch	$\frac{\frac{11}{12}x^8+\frac{11}{21}x^4-\frac{1}{7}}{x^7(x^4+1)} + \frac{11\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+1\right)} \_R \ln\left(x+\_R\right)\right)}{16}$	42

default	$\frac{x}{4x^4+4}+\frac{11\sqrt{2}\,\left(\ln\left(\frac{1+x^2+\sqrt{2}\,x}{1+x^2-\sqrt{2}\,x}\right)+2\arctan\left(\sqrt{2}\,x+1\right)+2\arctan\left(\sqrt{2}\,x-1\right)\right)}{32}-\frac{1}{7x^7}+\frac{2}{3x^3}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8+2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x/(x^4+1)+11/32*2^(1/2)*(ln((1+x^2+2^(1/2)*x)/(1+x^2-2^(1/2)*x))+2*arct
an(2^(1/2)*x+1)+2*arctan(2^(1/2)*x-1))-1/7/x^7+2/3/x^3
```

Maxima [A]
time = 0.50, size = 95, normalized size = 0.84

$$\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{11}{32}\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)-\frac{11}{32}\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)+\frac{77x^8+44x^4-12}{84\,(x^{11}+x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="maxima")
```

```
[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/32
*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/84*(77*x^8 + 44*x^4 - 12)/(x^11 + x^7
)
```

Fricas [A]
time = 0.36, size = 150, normalized size = 1.33

$$\frac{616x^8+352x^4-924\sqrt{2}\,(x^{11}+x^7)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2+\sqrt{2}\,x+1}-1\right)-924\sqrt{2}\,(x^{11}+x^7)\arctan\left(-\sqrt{2}\,x+\sqrt{2}\,\sqrt{x^2-\sqrt{2}\,x+1}+1\right)+231\sqrt{2}\,(x^{11}+x^7)\log\left(4x^2+4\sqrt{2}\,x+4\right)-231\sqrt{2}\,(x^{11}+x^7)\log\left(4x^2-4\sqrt{2}\,x+4\right)-96}{672\,(x^{11}+x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/672*(616*x^8 + 352*x^4 - 924*sqrt(2)*(x^11 + x^7)*arctan(-sqrt(2)*x + sqr
t(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 924*sqrt(2)*(x^11 + x^7)*arctan(-sqrt
(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 231*sqrt(2)*(x^11 + x^7)*l
og(4*x^2 + 4*sqrt(2)*x + 4) - 231*sqrt(2)*(x^11 + x^7)*log(4*x^2 - 4*sqrt(2
)*x + 4) - 96)/(x^11 + x^7)
```

Sympy [A]
time = 0.11, size = 102, normalized size = 0.90

$$-\frac{11\sqrt{2}\log\left(x^2-\sqrt{2}\,x+1\right)}{32}+\frac{11\sqrt{2}\log\left(x^2+\sqrt{2}\,x+1\right)}{32}+\frac{11\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x-1\right)}{16}+\frac{11\sqrt{2}\operatorname{atan}\left(\sqrt{2}\,x+1\right)}{16}+\frac{77x^8+44x^4-12}{84x^{11}+84x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8+2*x**4+1),x)
```

```
[Out] -11*sqrt(2)*log(x**2 - sqrt(2)*x + 1)/32 + 11*sqrt(2)*log(x**2 + sqrt(2)*x
+ 1)/32 + 11*sqrt(2)*atan(sqrt(2)*x - 1)/16 + 11*sqrt(2)*atan(sqrt(2)*x + 1
)/16 + (77*x**8 + 44*x**4 - 12)/(84*x**11 + 84*x**7)
```

Giac [A]
time = 5.63, size = 94, normalized size = 0.83

$$\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{11}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{11}{32}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{11}{32}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)+\frac{x}{4(x^4+1)}+\frac{14x^4-3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+2*x^4+1),x, algorithm="giac")
```

```
[Out] 11/16*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 11/16*sqrt(2)*arctan(1/
2*sqrt(2)*(2*x - sqrt(2))) + 11/32*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 11/32
*sqrt(2)*log(x^2 - sqrt(2)*x + 1) + 1/4*x/(x^4 + 1) + 1/21*(14*x^4 - 3)/x^7
```

Mupad [B]
time = 0.10, size = 55, normalized size = 0.49

$$\frac{\frac{11x^8}{12}+\frac{11x^4}{21}-\frac{1}{7}}{x^{11}+x^7}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{11}{16}+\frac{11}{16}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{11}{16}-\frac{11}{16}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(2*x^4 + x^8 + 1)),x)
```

```
[Out] 2^(1/2)*atan(2^(1/2)*x*(1/2 - 1i/2))*(11/16 + 11i/16) + 2^(1/2)*atan(2^(1/2
)*x*(1/2 + 1i/2))*(11/16 - 11i/16) + ((11*x^4)/21 + (11*x^8)/12 - 1/7)/(x^7
 + x^11)
```

3.290 $\int \frac{x^m}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=30

$$\frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{4};\frac{5+m}{4};x^4\right)}{1+m}$$

```
[Out] x^(1+m)*hypergeom([2, 1/4+1/4*m],[5/4+1/4*m],x^4)/(1+m)
```

Rubi [A]
time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 371}

$$\frac{x^{m+1}\,{}_2F_1\left(2,\frac{m+1}{4};\frac{m+5}{4};x^4\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 - 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, (5 + m)/4, x^4])/(1 + m)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^m}{1-2x^4+x^8}\,dx &= \int \frac{x^m}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{4};\frac{5+m}{4};x^4\right)}{1+m}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 32, normalized size = 1.07

$$\frac{x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{4};1+\frac{1+m}{4};x^4\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m/(1 - 2*x^4 + x^8),x]
```

```
[Out] (x^(1 + m)*Hypergeometric2F1[2, (1 + m)/4, 1 + (1 + m)/4, x^4])/(1 + m)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 - 2x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8-2*x^4+1),x)
```

```
[Out] int(x^m/(x^8-2*x^4+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] integral(x^m/(x^8 - 2*x^4 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(x-1)^2\,(x+1)^2\,(x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8-2*x**4+1),x)
```

```
[Out] Integral(x**m/((x - 1)**2*(x + 1)**2*(x**2 + 1)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-2*x^4+1),x, algorithm="giac")
[Out] integrate(x^m/(x^8 - 2*x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{x^8-2\,x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8 - 2*x^4 + 1),x)
[Out] int(x^m/(x^8 - 2*x^4 + 1), x)
```

3.291 $\int \frac{x^9}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=32

$$\frac{3x^2}{4} + \frac{x^6}{4(1-x^4)} - \frac{3}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] 3/4*x^2+1/4*x^6/(-x^4+1)-3/4*arctanh(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 281, 294, 327, 213}

$$\frac{3x^2}{4} - \frac{3}{4}\tanh^{-1}\left(x^2\right) + \frac{x^6}{4(1-x^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - 2*x^4 + x^8),x]
```

```
[Out] (3*x^2)/4 + x^6/(4*(1 - x^4)) - (3*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{1-2x^4+x^8}\,dx &= \int \frac{x^9}{(-1+x^4)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(-1+x^2)^2}\,dx,x,x^2\right)\\ &= \frac{x^6}{4(1-x^4)}+\frac{3}{4}\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,x^2\right)\\ &= \frac{3x^2}{4}+\frac{x^6}{4(1-x^4)}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,x^2\right)\\ &= \frac{3x^2}{4}+\frac{x^6}{4(1-x^4)}-\frac{3}{4}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.22

$$\frac{1}{8}\left(2x^2\left(2+\frac{1}{1-x^4}\right)+3\log\left(1-x^2\right)-3\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - 2*x^4 + x^8),x]
[Out] (2*x^2*(2 + (1 - x^4)^(-1)) + 3*Log[1 - x^2] - 3*Log[1 + x^2])/8
```

Maple [A]
time = 0.04, size = 41, normalized size = 1.28

method	result	size
risch	$\frac{x^2}{2}-\frac{x^2}{4(x^4-1)}+\frac{3\ln(x^2-1)}{8}-\frac{3\ln(x^2+1)}{8}$	35
default	$\frac{x^2}{2}-\frac{1}{8(x^2-1)}+\frac{3\ln(x^2-1)}{8}-\frac{1}{8(x^2+1)}-\frac{3\ln(x^2+1)}{8}$	41
norman	$\frac{-\frac{3}{4}x^2+\frac{1}{2}x^6}{x^4-1}+\frac{3\ln(-1+x)}{8}+\frac{3\ln(1+x)}{8}-\frac{3\ln(x^2+1)}{8}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-1/8/(x^2-1)+3/8*ln(x^2-1)-1/8/(x^2+1)-3/8*ln(x^2+1)
```

Maxima [A]
time = 0.29, size = 34, normalized size = 1.06

$$\frac{1}{2}\,x^{2}-\frac{x^{2}}{4\,(x^{4}-1)}-\frac{3}{8}\,\log\left(x^{2}+1\right)+\frac{3}{8}\,\log\left(x^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(x^2 - 1)
```

Fricas [A]
time = 0.34, size = 46, normalized size = 1.44

$$\frac{4\,x^{6}-6\,x^{2}-3\,(x^{4}-1)\log\left(x^{2}+1\right)+3\,(x^{4}-1)\log\left(x^{2}-1\right)}{8\,(x^{4}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/8*(4*x^6 - 6*x^2 - 3*(x^4 - 1)*log(x^2 + 1) + 3*(x^4 - 1)*log(x^2 - 1))/(
x^4 - 1)
```

Sympy [A]
time = 0.04, size = 34, normalized size = 1.06

$$\frac{x^{2}}{2}-\frac{x^{2}}{4x^{4}-4}+\frac{3\log\left(x^{2}-1\right)}{8}-\frac{3\log\left(x^{2}+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8-2*x**4+1),x)
```

```
[Out] x**2/2 - x**2/(4*x**4 - 4) + 3*log(x**2 - 1)/8 - 3*log(x**2 + 1)/8
```

Giac [A]
time = 3.80, size = 35, normalized size = 1.09

$$\frac{1}{2}\,x^{2}-\frac{x^{2}}{4\,(x^{4}-1)}-\frac{3}{8}\,\log\left(x^{2}+1\right)+\frac{3}{8}\,\log\left(\left|x^{2}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/4*x^2/(x^4 - 1) - 3/8*log(x^2 + 1) + 3/8*log(abs(x^2 - 1))
```

Mupad [B]
time = 0.05, size = 26, normalized size = 0.81

$$\frac{x^2}{2} - \frac{x^2}{4\,(x^4-1)} - \frac{3\,\mathrm{atanh}(x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8 - 2*x^4 + 1),x)
```

```
[Out] x^2/2 - x^2/(4*(x^4 - 1)) - (3*atanh(x^2))/4
```

3.292 $\int \frac{x^7}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=26

$$\frac{1}{4(1-x^4)}+\frac{1}{4}\log\left(1-x^4\right)$$

```
[Out] 1/4/(-x^4+1)+1/4*ln(-x^4+1)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 272, 45}

$$\frac{1}{4(1-x^4)}+\frac{1}{4}\log\left(1-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - 2*x^4 + x^8),x]
```

```
[Out] 1/(4*(1 - x^4)) + Log[1 - x^4]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{1-2x^4+x^8}\,dx = \int \frac{x^7}{(-1+x^4)^2}\,dx$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{x}{(-1+x)^2}\,dx, x, x^4\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^2}+\frac{1}{-1+x}\right)\,dx, x, x^4\right)$$
$$= \frac{1}{4(1-x^4)}+\frac{1}{4}\log\left(1-x^4\right)$$

Mathematica [A]
time = 0.01, size = 22, normalized size = 0.85

$$-\frac{1}{4(-1+x^4)}+\frac{1}{4}\log\left(-1+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - 2*x^4 + x^8),x]
```

```
[Out] -1/4*1/(-1 + x^4) + Log[-1 + x^4]/4
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.73

method	result	size
default	$-\frac{1}{4(x^4-1)}+\frac{\ln(x^4-1)}{4}$	19
risch	$-\frac{1}{4(x^4-1)}+\frac{\ln(x^4-1)}{4}$	19
norman	$-\frac{1}{4(x^4-1)}+\frac{\ln(-1+x)}{4}+\frac{\ln(1+x)}{4}+\frac{\ln(x^2+1)}{4}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(x^4-1)+1/4*ln(x^4-1)
```

Maxima [A]
time = 0.28, size = 18, normalized size = 0.69

$$-\frac{1}{4(x^4-1)}+\frac{1}{4}\log\left(x^4-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4/(x^4 - 1) + 1/4*log(x^4 - 1)
```

Fricas [A]
time = 0.35, size = 23, normalized size = 0.88

$$\frac{(x^4-1)\log(x^4-1)-1}{4\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*((x^4 - 1)*log(x^4 - 1) - 1)/(x^4 - 1)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.58

$$\frac{\log(x^4-1)}{4}-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8-2*x**4+1),x)
```

```
[Out] log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

Giac [A]
time = 3.21, size = 19, normalized size = 0.73

$$-\frac{1}{4\,(x^4-1)}+\frac{1}{4}\log\left(\left|x^4-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4/(x^4 - 1) + 1/4*log(abs(x^4 - 1))
```

Mupad [B]
time = 0.05, size = 20, normalized size = 0.77

$$\frac{\ln(x^4-1)}{4}-\frac{1}{4\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8 - 2*x^4 + 1),x)
```

```
[Out] log(x^4 - 1)/4 - 1/(4*(x^4 - 1))
```

3.293 $\int \frac{x^5}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=25

$$\frac{x^2}{4(1-x^4)}-\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] 1/4*x^2/(-x^4+1)-1/4*arctanh(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {28, 281, 294, 213}

$$\frac{x^2}{4(1-x^4)}-\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 - x^4)) - ArcTanh[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{1-2x^4+x^8}\,dx &= \int \frac{x^5}{(-1+x^4)^2}\,dx\\ &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{x^2}{(-1+x^2)^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{4(1-x^4)} + \frac{1}{4}\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{4(1-x^4)} - \frac{1}{4}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.32

$$\frac{1}{8}\left(-\frac{2x^2}{-1+x^4}+\log\left(1-x^2\right)-\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - 2*x^4 + x^8),x]
[Out] ((-2*x^2)/(-1 + x^4) + Log[1 - x^2] - Log[1 + x^2])/8
```

Maple [A]
time = 0.02, size = 36, normalized size = 1.44

method	result	size
risch	$-\frac{x^2}{4(x^4-1)}+\frac{\ln(x^2-1)}{8}-\frac{\ln(x^2+1)}{8}$	30
norman	$-\frac{x^2}{4(x^4-1)}+\frac{\ln(-1+x)}{8}+\frac{\ln(1+x)}{8}-\frac{\ln(x^2+1)}{8}$	34
default	$-\frac{1}{8(x^2-1)}+\frac{\ln(x^2-1)}{8}-\frac{1}{8(x^2+1)}-\frac{\ln(x^2+1)}{8}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/8/(x^2-1)+1/8*ln(x^2-1)-1/8/(x^2+1)-1/8*ln(x^2+1)
```

Maxima [A]
time = 0.28, size = 29, normalized size = 1.16

$$-\frac{x^2}{4(x^4-1)}-\frac{1}{8}\log\left(x^2+1\right)+\frac{1}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(x^2 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.
time = 0.35, size = 40, normalized size = 1.60

$$-\frac{2\,x^2 + (x^4-1)\log(x^2+1) - (x^4-1)\log(x^2-1)}{8\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^2 + (x^4 - 1)*log(x^2 + 1) - (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

Sympy [A]
time = 0.04, size = 26, normalized size = 1.04

$$-\frac{x^2}{4x^4-4} + \frac{\log(x^2-1)}{8} - \frac{\log(x^2+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8-2*x**4+1),x)
```

```
[Out] -x**2/(4*x**4 - 4) + log(x**2 - 1)/8 - log(x**2 + 1)/8
```

Giac [A]
time = 3.11, size = 30, normalized size = 1.20

$$-\frac{x^2}{4\,(x^4-1)} - \frac{1}{8}\log\left(x^2+1\right) + \frac{1}{8}\log\left(|x^2-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^2/(x^4 - 1) - 1/8*log(x^2 + 1) + 1/8*log(abs(x^2 - 1))
```

Mupad [B]
time = 1.27, size = 21, normalized size = 0.84

$$-\frac{\operatorname{atanh}(x^2)}{4} - \frac{x^2}{4\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8 - 2*x^4 + 1),x)
```

```
[Out] - atanh(x^2)/4 - x^2/(4*(x^4 - 1))
```

3.294 $\int \frac{x^3}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=13

$$\frac{1}{4(1-x^4)}$$

[Out] 1/4/(-x^4+1)

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {28, 267}

$$\frac{1}{4(1-x^4)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(1 - 2*x^4 + x^8),x]

[Out] 1/(4*(1 - x^4))

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1-2x^4+x^8}\,dx &= \int \frac{x^3}{(-1+x^4)^2}\,dx\\ &= \frac{1}{4(1-x^4)}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 0.85

$$-\frac{1}{4(-1+x^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - 2*x^4 + x^8),x]
[Out] -1/4*1/(-1 + x^4)
```

Maple [A]
time = 0.01, size = 10, normalized size = 0.77

method	result	size
gosper	$-\frac{1}{4(x^4-1)}$	10
default	$-\frac{1}{4(x^4-1)}$	10
norman	$-\frac{1}{4(x^4-1)}$	10
risch	$-\frac{1}{4(x^4-1)}$	10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/4/(x^4-1)
```

Maxima [A]
time = 0.32, size = 9, normalized size = 0.69

$$-\frac{1}{4\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4/(x^4 - 1)
```

Fricas [A]
time = 0.33, size = 9, normalized size = 0.69

$$-\frac{1}{4\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/4/(x^4 - 1)
```

Sympy [A]
time = 0.04, size = 8, normalized size = 0.62

$$-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-2*x**4+1),x)
```

```
[Out] -1/(4*x**4 - 4)
```

Giac [A]
time = 4.12, size = 9, normalized size = 0.69

$$-\frac{1}{4\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4/(x^4 - 1)
```

Mupad [B]
time = 0.02, size = 11, normalized size = 0.85

$$-\frac{1}{4\,\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8 - 2*x^4 + 1),x)
```

```
[Out] -1/(4*(x^4 - 1))
```

3.295 $\int \frac{x}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=25

$$\frac{x^2}{4(1-x^4)}+\frac{1}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] 1/4*x^2/(-x^4+1)+1/4*arctanh(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {28, 281, 205, 213}

$$\frac{1}{4}\tanh^{-1}\left(x^2\right)+\frac{x^2}{4(1-x^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^2/(4*(1 - x^4)) + ArcTanh[x^2]/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1-2x^4+x^8}\,dx &= \int \frac{x}{\left(-1+x^4\right)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(-1+x^2\right)^2}\,dx,x,x^2\right)\\ &= \frac{x^2}{4\left(1-x^4\right)}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,x^2\right)\\ &= \frac{x^2}{4\left(1-x^4\right)}+\frac{1}{4}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.32

$$\frac{1}{8}\left(-\frac{2x^2}{-1+x^4}-\log\left(1-x^2\right)+\log\left(1+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - 2*x^4 + x^8),x]
```

```
[Out] ((-2*x^2)/(-1 + x^4) - Log[1 - x^2] + Log[1 + x^2])/8
```

Maple [A]
time = 0.02, size = 36, normalized size = 1.44

method	result	size
risch	$-\frac{x^2}{4(x^4-1)}-\frac{\ln(x^2-1)}{8}+\frac{\ln(x^2+1)}{8}$	30
norman	$-\frac{x^2}{4(x^4-1)}-\frac{\ln(-1+x)}{8}-\frac{\ln(1+x)}{8}+\frac{\ln(x^2+1)}{8}$	34
default	$-\frac{1}{8(x^2-1)}-\frac{\ln(x^2-1)}{8}-\frac{1}{8(x^2+1)}+\frac{\ln(x^2+1)}{8}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/(x^2-1)-1/8*ln(x^2-1)-1/8/(x^2+1)+1/8*ln(x^2+1)
```

Maxima [A]
time = 0.28, size = 29, normalized size = 1.16

$$-\frac{x^2}{4\left(x^4-1\right)}+\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{8}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(x^2 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.
time = 0.34, size = 40, normalized size = 1.60

$$-\frac{2x^2 - (x^4-1)\log(x^2+1) + (x^4-1)\log(x^2-1)}{8(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^2 - (x^4 - 1)*log(x^2 + 1) + (x^4 - 1)*log(x^2 - 1))/(x^4 - 1)
```

Sympy [A]
time = 0.04, size = 26, normalized size = 1.04

$$-\frac{x^2}{4x^4-4} - \frac{\log(x^2-1)}{8} + \frac{\log(x^2+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8-2*x**4+1),x)
```

```
[Out] -x**2/(4*x**4 - 4) - log(x**2 - 1)/8 + log(x**2 + 1)/8
```

Giac [A]
time = 4.71, size = 30, normalized size = 1.20

$$-\frac{x^2}{4(x^4-1)} + \frac{1}{8}\log(x^2+1) - \frac{1}{8}\log(|x^2-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^2/(x^4 - 1) + 1/8*log(x^2 + 1) - 1/8*log(abs(x^2 - 1))
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.84

$$\frac{\operatorname{atanh}(x^2)}{4} - \frac{x^2}{4\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8 - 2*x^4 + 1),x)
```

```
[Out] atanh(x^2)/4 - x^2/(4*(x^4 - 1))
```

3.296 $\int \frac{1}{x(1-2x^4+x^8)} \, dx$

Optimal. Leaf size=28

$$\frac{1}{4(1-x^4)} + \log(x) - \frac{1}{4}\log\left(1-x^4\right)$$

[Out] 1/4/(-x^4+1)+ln(x)-1/4*ln(-x^4+1)

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 272, 46}

$$\frac{1}{4(1-x^4)} - \frac{1}{4}\log\left(1-x^4\right) + \log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 - 2*x^4 + x^8)),x]

[Out] 1/(4*(1 - x^4)) + Log[x] - Log[1 - x^4]/4

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x(1-2x^4+x^8)}\,dx = \int \frac{1}{x(-1+x^4)^2}\,dx$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(-1+x)^2x}\,dx, x, x^4\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{(-1+x)^2}+\frac{1}{x}\right)\,dx, x, x^4\right)$$
$$= \frac{1}{4(1-x^4)}+\log(x)-\frac{1}{4}\log\left(1-x^4\right)$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.93

$$-\frac{1}{4(-1+x^4)}+\log(x)-\frac{1}{4}\log\left(1-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/4*1/(-1 + x^4) + Log[x] - Log[1 - x^4]/4
```

Maple [A]
time = 0.03, size = 47, normalized size = 1.68

method	result	size
risch	$-\frac{1}{4(x^4-1)}+\ln(x)-\frac{\ln(x^4-1)}{4}$	21
norman	$-\frac{1}{4(x^4-1)}-\frac{\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}-\frac{\ln(x^2+1)}{4}+\ln(x)$	33
default	$-\frac{1}{16(-1+x)}-\frac{\ln(-1+x)}{4}+\frac{1}{8x^2+8}-\frac{\ln(x^2+1)}{4}+\ln(x)+\frac{1}{16+16x}-\frac{\ln(1+x)}{4}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(-1+x)-1/4*ln(-1+x)+1/8/(x^2+1)-1/4*ln(x^2+1)+ln(x)+1/16/(1+x)-1/4*ln
(1+x)
```

Maxima [A]
time = 0.31, size = 24, normalized size = 0.86

$$-\frac{1}{4(x^4-1)}-\frac{1}{4}\log\left(x^4-1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4/(x^4 - 1) - 1/4*log(x^4 - 1) + 1/4*log(x^4)
```

Fricas [A]
time = 0.34, size = 32, normalized size = 1.14

$$-\frac{(x^4-1)\log(x^4-1)-4(x^4-1)\log(x)+1}{4(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/4*((x^4 - 1)*log(x^4 - 1) - 4*(x^4 - 1)*log(x) + 1)/(x^4 - 1)
```

Sympy [A]
time = 0.05, size = 19, normalized size = 0.68

$$\log(x)-\frac{\log(x^4-1)}{4}-\frac{1}{4x^4-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8-2*x**4+1),x)
[Out] log(x) - log(x**4 - 1)/4 - 1/(4*x**4 - 4)
```

Giac [A]
time = 5.77, size = 30, normalized size = 1.07

$$\frac{x^4-2}{4(x^4-1)}+\frac{1}{4}\log(x^4)-\frac{1}{4}\log(|x^4-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-2*x^4+1),x, algorithm="giac")
[Out] 1/4*(x^4 - 2)/(x^4 - 1) + 1/4*log(x^4) - 1/4*log(abs(x^4 - 1))
```

Mupad [B]
time = 0.06, size = 22, normalized size = 0.79

$$\ln(x)-\frac{\ln(x^4-1)}{4}-\frac{1}{4(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^8 - 2*x^4 + 1)),x)
[Out] log(x) - log(x^4 - 1)/4 - 1/(4*(x^4 - 1))
```

3.297 $\int \frac{1}{x^3(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=32

$$-\frac{3}{4x^2}+\frac{1}{4x^2(1-x^4)}+\frac{3}{4}\tanh^{-1}(x^2)$$

```
[Out] -3/4/x^2+1/4/x^2/(-x^4+1)+3/4*arctanh(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 281, 296, 331, 213}

$$-\frac{3}{4x^2}+\frac{3}{4}\tanh^{-1}(x^2)+\frac{1}{4x^2(1-x^4)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -3/(4*x^2) + 1/(4*x^2*(1 - x^4)) + (3*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(1-2x^4+x^8)}\,dx &= \int \frac{1}{x^3(-1+x^4)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(-1+x^2)^2}\,dx,x,x^2\right)\\ &= \frac{1}{4x^2(1-x^4)}-\frac{3}{4}\text{Subst}\left(\int \frac{1}{x^2(-1+x^2)}\,dx,x,x^2\right)\\ &= -\frac{3}{4x^2}+\frac{1}{4x^2(1-x^4)}-\frac{3}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,x^2\right)\\ &= -\frac{3}{4x^2}+\frac{1}{4x^2(1-x^4)}+\frac{3}{4}\tanh^{-1}(x^2)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.28

$$\frac{1}{8}\left(\frac{4-6x^4}{x^2(-1+x^4)}-3\log(1-x^2)+3\log(1+x^2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - 2*x^4 + x^8)),x]
[Out] ((4 - 6*x^4)/(x^2*(-1 + x^4)) - 3*Log[1 - x^2] + 3*Log[1 + x^2])/8
```

Maple [A]
time = 0.03, size = 50, normalized size = 1.56

method	result	size
risch	$\frac{\frac{1}{2}-\frac{3x^4}{4}}{x^2(x^4-1)}-\frac{3\ln(x^2-1)}{8}+\frac{3\ln(x^2+1)}{8}$	36

norman	$\frac{\frac{1}{2}-\frac{3x^4}{4}}{x^2(x^4-1)}-\frac{3\ln(-1+x)}{8}-\frac{3\ln(1+x)}{8}+\frac{3\ln(x^2+1)}{8}$	40
default	$-\frac{1}{16(-1+x)}-\frac{3\ln(-1+x)}{8}-\frac{1}{8(x^2+1)}+\frac{3\ln(x^2+1)}{8}-\frac{1}{2x^2}+\frac{1}{16+16x}-\frac{3\ln(1+x)}{8}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/16/(-1+x)-3/8*ln(-1+x)-1/8/(x^2+1)+3/8*ln(x^2+1)-1/2/x^2+1/16/(1+x)-3/8*
ln(1+x)
```

Maxima [A]
time = 0.29, size = 37, normalized size = 1.16

$$-\frac{3\,x^4-2}{4\,(x^6-x^2)}+\frac{3}{8}\,\log\left(x^2+1\right)-\frac{3}{8}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="maxima")
[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(x^2 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.
time = 0.34, size = 54, normalized size = 1.69

$$-\frac{6\,x^4-3\,(x^6-x^2)\log\left(x^2+1\right)+3\,(x^6-x^2)\log\left(x^2-1\right)-4}{8\,(x^6-x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="fricas")
[Out] -1/8*(6*x^4 - 3*(x^6 - x^2)*log(x^2 + 1) + 3*(x^6 - x^2)*log(x^2 - 1) - 4)/
(x^6 - x^2)
```

Sympy [A]
time = 0.05, size = 36, normalized size = 1.12

$$\frac{2-3x^4}{4x^6-4x^2}-\frac{3\log\left(x^2-1\right)}{8}+\frac{3\log\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8-2*x**4+1),x)
[Out] (2 - 3*x**4)/(4*x**6 - 4*x**2) - 3*log(x**2 - 1)/8 + 3*log(x**2 + 1)/8
```

Giac [A]
time = 5.85, size = 38, normalized size = 1.19

$$-\frac{3\,x^4-2}{4\,(x^6-x^2)}+\frac{3}{8}\,\log\left(x^2+1\right)-\frac{3}{8}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*(3*x^4 - 2)/(x^6 - x^2) + 3/8*log(x^2 + 1) - 3/8*log(abs(x^2 - 1))
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.81

$$\frac{3\,\mathrm{atanh}(x^2)}{4}+\frac{\frac{3\,x^4}{4}-\frac{1}{2}}{x^2-x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^8 - 2*x^4 + 1)),x)
[Out] (3*atanh(x^2))/4 + ((3*x^4)/4 - 1/2)/(x^2 - x^6)
```

3.298 $\int \frac{1}{x^5(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{4x^4}+\frac{1}{4\left(1-x^4\right)}+2\log(x)-\frac{1}{2}\log\left(1-x^4\right)$$

```
[Out] -1/4/x^4+1/4/(-x^4+1)+2*ln(x)-1/2*ln(-x^4+1)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {28, 272, 46}

$$\frac{1}{4\left(1-x^4\right)}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^4\right)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 + 1/(4*(1 - x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(1-2x^4+x^8\right)}\,dx = \int \frac{1}{x^5\left(-1+x^4\right)^2}\,dx$$
$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(-1+x)^2x^2}\,dx, x, x^4\right)$$
$$= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{(-1+x)^2} - \frac{2}{-1+x} + \frac{1}{x^2} + \frac{2}{x}\right)\,dx, x, x^4\right)$$
$$= -\frac{1}{4x^4} + \frac{1}{4\left(1-x^4\right)} + 2\log(x) - \frac{1}{2}\log\left(1-x^4\right)$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 0.95

$$-\frac{1}{4x^4} - \frac{1}{4\left(-1+x^4\right)} + 2\log(x) - \frac{1}{2}\log\left(1-x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 - 1/(4*(-1 + x^4)) + 2*Log[x] - Log[1 - x^4]/2
```

Maple [A]
time = 0.03, size = 54, normalized size = 1.46

method	result	size
risch	$\frac{\frac{1}{4}-\frac{x^4}{2}}{x^4\left(x^4-1\right)} + 2\ln\left(x\right) - \frac{\ln\left(x^4-1\right)}{2}$	32
norman	$\frac{\frac{1}{4}-\frac{x^4}{2}}{x^4\left(x^4-1\right)} + 2\ln\left(x\right) - \frac{\ln\left(-1+x\right)}{2} - \frac{\ln\left(1+x\right)}{2} - \frac{\ln\left(x^2+1\right)}{2}$	44
default	$-\frac{1}{16(-1+x)} - \frac{\ln(-1+x)}{2} + \frac{1}{8x^2+8} - \frac{\ln\left(x^2+1\right)}{2} - \frac{1}{4x^4} + 2\ln\left(x\right) + \frac{1}{16+16x} - \frac{\ln(1+x)}{2}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(-1+x)-1/2*ln(-1+x)+1/8/(x^2+1)-1/2*ln(x^2+1)-1/4/x^4+2*ln(x)+1/16/(1
+x)-1/2*ln(1+x)
```

Maxima [A]
time = 0.30, size = 35, normalized size = 0.95

$$-\frac{2\,x^4-1}{4\,\left(x^8-x^4\right)} - \frac{1}{2}\,\log\left(x^4-1\right) + \frac{1}{2}\,\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^4 - 1)/(x^8 - x^4) - 1/2*log(x^4 - 1) + 1/2*log(x^4)
```

Fricas [A]
time = 0.36, size = 50, normalized size = 1.35

$$-\frac{2\,x^4+2\,(x^8-x^4)\log\left(x^4-1\right)-8\,(x^8-x^4)\log\left(x\right)-1}{4\,(x^8-x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^4 + 2*(x^8 - x^4)*log(x^4 - 1) - 8*(x^8 - x^4)*log(x) - 1)/(x^8 -
 x^4)
```

Sympy [A]
time = 0.06, size = 29, normalized size = 0.78

$$\frac{1-2x^4}{4x^8-4x^4}+2\log\left(x\right)-\frac{\log\left(x^4-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8-2*x**4+1),x)
```

```
[Out] (1 - 2*x**4)/(4*x**8 - 4*x**4) + 2*log(x) - log(x**4 - 1)/2
```

Giac [A]
time = 5.20, size = 36, normalized size = 0.97

$$-\frac{2\,x^4-1}{4\,(x^8-x^4)}+\frac{1}{2}\,\log\left(x^4\right)-\frac{1}{2}\,\log\left(\left|x^4-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^4 - 1)/(x^8 - x^4) + 1/2*log(x^4) - 1/2*log(abs(x^4 - 1))
```

Mupad [B]
time = 0.05, size = 32, normalized size = 0.86

$$2\,\ln\left(x\right)-\frac{\ln\left(x^4-1\right)}{2}+\frac{\frac{x^4}{2}-\frac{1}{4}}{x^4-x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] 2*log(x) - log(x^4 - 1)/2 + (x^4/2 - 1/4)/(x^4 - x^8)
```

3.299 $\int \frac{1}{x^7(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=39

$$-\frac{5}{12x^6}-\frac{5}{4x^2}+\frac{1}{4x^6(1-x^4)}+\frac{5}{4}\tanh^{-1}\left(x^2\right)$$

```
[Out] -5/12/x^6-5/4/x^2+1/4/x^6/(-x^4+1)+5/4*arctanh(x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 281, 296, 331, 213}

$$-\frac{5}{12x^6}-\frac{5}{4x^2}+\frac{5}{4}\tanh^{-1}\left(x^2\right)+\frac{1}{4x^6(1-x^4)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -5/(12*x^6) - 5/(4*x^2) + 1/(4*x^6*(1 - x^4)) + (5*ArcTanh[x^2])/4
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^7\left(-1+x^4\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(-1+x^2\right)^2}\,dx, x, x^2\right) \\
&= \frac{1}{4x^6\left(1-x^4\right)} - \frac{5}{4}\text{Subst}\left(\int \frac{1}{x^4\left(-1+x^2\right)}\,dx, x, x^2\right) \\
&= -\frac{5}{12x^6} + \frac{1}{4x^6\left(1-x^4\right)} - \frac{5}{4}\text{Subst}\left(\int \frac{1}{x^2\left(-1+x^2\right)}\,dx, x, x^2\right) \\
&= -\frac{5}{12x^6} - \frac{5}{4x^2} + \frac{1}{4x^6\left(1-x^4\right)} - \frac{5}{4}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, x^2\right) \\
&= -\frac{5}{12x^6} - \frac{5}{4x^2} + \frac{1}{4x^6\left(1-x^4\right)} + \frac{5}{4}\tanh^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 1.26

$$
-\frac{1}{6x^6} - \frac{1}{x^2} - \frac{x^2}{4\left(-1+x^4\right)} - \frac{5}{8}\log\left(1-x^2\right) + \frac{5}{8}\log\left(1+x^2\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 - x^(-2) - x^2/(4*(-1 + x^4)) - (5*Log[1 - x^2])/8 + (5*Log[1 +
x^2])/8
```

Maple [A]
time = 0.04, size = 55, normalized size = 1.41

method	result	size

risch	$\frac{\frac{1}{6}+\frac{5}{6}x^4-\frac{5}{4}x^8}{x^6(x^4-1)}+\frac{5\ln(x^2+1)}{8}-\frac{5\ln(x^2-1)}{8}$	41
norman	$\frac{\frac{1}{6}+\frac{5}{6}x^4-\frac{5}{4}x^8}{x^6(x^4-1)}-\frac{5\ln(-1+x)}{8}-\frac{5\ln(1+x)}{8}+\frac{5\ln(x^2+1)}{8}$	45
default	$-\frac{1}{16(-1+x)}-\frac{5\ln(-1+x)}{8}-\frac{1}{8(x^2+1)}+\frac{5\ln(x^2+1)}{8}-\frac{1}{6x^6}-\frac{1}{x^2}+\frac{1}{16+16x}-\frac{5\ln(1+x)}{8}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(-1+x)-5/8*ln(-1+x)-1/8/(x^2+1)+5/8*ln(x^2+1)-1/6/x^6-1/x^2+1/16/(1+x
)-5/8*ln(1+x)
```

Maxima [A]
time = 0.33, size = 42, normalized size = 1.08

$$-\frac{15\,x^8-10\,x^4-2}{12\,(x^{10}-x^6)}+\frac{5}{8}\,\log\left(x^2+1\right)-\frac{5}{8}\,\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*(15*x^8 - 10*x^4 - 2)/(x^10 - x^6) + 5/8*log(x^2 + 1) - 5/8*log(x^2 -
 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(29) = 58.
time = 0.34, size = 59, normalized size = 1.51

$$-\frac{30\,x^8-20\,x^4-15\,(x^{10}-x^6)\log\left(x^2+1\right)+15\,(x^{10}-x^6)\log\left(x^2-1\right)-4}{24\,(x^{10}-x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(30*x^8 - 20*x^4 - 15*(x^10 - x^6)*log(x^2 + 1) + 15*(x^10 - x^6)*log
(x^2 - 1) - 4)/(x^10 - x^6)
```

Sympy [A]
time = 0.07, size = 41, normalized size = 1.05

$$-\frac{5\log\left(x^2-1\right)}{8}+\frac{5\log\left(x^2+1\right)}{8}+\frac{-15x^8+10x^4+2}{12x^{10}-12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8-2*x**4+1),x)
```

```
[Out] -5*log(x**2 - 1)/8 + 5*log(x**2 + 1)/8 + (-15*x**8 + 10*x**4 + 2)/(12*x**10
 - 12*x**6)
```

Giac [A]
time = 4.97, size = 42, normalized size = 1.08

$$-\frac{x^2}{4\,(x^4-1)}-\frac{6\,x^4+1}{6\,x^6}+\frac{5}{8}\,\log\left(x^2+1\right)-\frac{5}{8}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-2*x^4+1),x, algorithm="giac")
[Out] -1/4*x^2/(x^4 - 1) - 1/6*(6*x^4 + 1)/x^6 + 5/8*log(x^2 + 1) - 5/8*log(abs(x
^2 - 1))
```

Mupad [B]
time = 0.05, size = 32, normalized size = 0.82

$$\frac{5\,\mathrm{atanh}(x^2)}{4}-\frac{-\frac{5\,x^8}{4}+\frac{5\,x^4}{6}+\frac{1}{6}}{x^6-x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^8 - 2*x^4 + 1)),x)
[Out] (5*atanh(x^2))/4 - ((5*x^4)/6 - (5*x^8)/4 + 1/6)/(x^6 - x^10)
```

3.300 $\int \frac{x^8}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=34

$$\frac{5x}{4}+\frac{x^5}{4(1-x^4)}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)$$

```
[Out] 5/4*x+1/4*x^5/(-x^4+1)-5/8*arctan(x)-5/8*arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {28, 294, 327, 218, 212, 209}

$$-\frac{5\text{ArcTan}(x)}{8}+\frac{x^5}{4(1-x^4)}+\frac{5x}{4}-\frac{5}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - 2*x^4 + x^8),x]
```

```
[Out] (5*x)/4 + x^5/(4*(1 - x^4)) - (5*ArcTan[x])/8 - (5*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8}{1-2x^4+x^8}\,dx &= \int \frac{x^8}{\left(-1+x^4\right)^2}\,dx\\
&= \frac{x^5}{4\left(1-x^4\right)}+\frac{5}{4}\int \frac{x^4}{-1+x^4}\,dx\\
&= \frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}+\frac{5}{4}\int \frac{1}{-1+x^4}\,dx\\
&= \frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{8}\int \frac{1}{1-x^2}\,dx-\frac{5}{8}\int \frac{1}{1+x^2}\,dx\\
&= \frac{5x}{4}+\frac{x^5}{4\left(1-x^4\right)}-\frac{5}{8}\tan^{-1}(x)-\frac{5}{8}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.12

$$x-\frac{x}{4\left(-1+x^4\right)}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{16}\log(1-x)-\frac{5}{16}\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - 2*x^4 + x^8),x]
[Out] x - x/(4*(-1 + x^4)) - (5*ArcTan[x])/8 + (5*Log[1 - x])/16 - (5*Log[1 + x])
/16
```

Maple [A]
time = 0.03, size = 43, normalized size = 1.26

method	result	size
risch	$x-\frac{x}{4(x^4-1)}+\frac{5\ln(-1+x)}{16}-\frac{5\arctan(x)}{8}-\frac{5\ln(1+x)}{16}$	29
default	$x-\frac{1}{16(-1+x)}+\frac{5\ln(-1+x)}{16}+\frac{x}{8x^2+8}-\frac{5\arctan(x)}{8}-\frac{1}{16(1+x)}-\frac{5\ln(1+x)}{16}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] x-1/16/(-1+x)+5/16*ln(-1+x)+1/8*x/(x^2+1)-5/8*arctan(x)-1/16/(1+x)-5/16*ln(
1+x)
```

Maxima [A]
time = 0.51, size = 28, normalized size = 0.82

$$x-\frac{x}{4\left(x^4-1\right)}-\frac{5}{8}\arctan\left(x\right)-\frac{5}{16}\log\left(x+1\right)+\frac{5}{16}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(x + 1) + 5/16*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(24) = 48$.
time = 0.33, size = 49, normalized size = 1.44

$$\frac{16\,x^5-10\left(x^4-1\right)\arctan\left(x\right)-5\left(x^4-1\right)\log\left(x+1\right)+5\left(x^4-1\right)\log\left(x-1\right)-20\,x}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/16*(16*x^5 - 10*(x^4 - 1)*arctan(x) - 5*(x^4 - 1)*log(x + 1) + 5*(x^4 - 1
)*log(x - 1) - 20*x)/(x^4 - 1)
```

Sympy [A]
time = 0.06, size = 32, normalized size = 0.94

$$x-\frac{x}{4x^4-4}+\frac{5\log\left(x-1\right)}{16}-\frac{5\log\left(x+1\right)}{16}-\frac{5\,\mathrm{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-2*x**4+1),x)
```

```
[Out] x - x/(4*x**4 - 4) + 5*log(x - 1)/16 - 5*log(x + 1)/16 - 5*atan(x)/8
```

Giac [A]
time = 3.65, size = 30, normalized size = 0.88

$$x - \frac{x}{4\,(x^4 - 1)} - \frac{5}{8}\,\arctan\left(x\right) - \frac{5}{16}\,\log\left(|x + 1|\right) + \frac{5}{16}\,\log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-2*x^4+1),x, algorithm="giac")
[Out] x - 1/4*x/(x^4 - 1) - 5/8*arctan(x) - 5/16*log(abs(x + 1)) + 5/16*log(abs(x
 - 1))
```

Mupad [B]
time = 1.29, size = 26, normalized size = 0.76

$$x - \frac{5\,\mathrm{atan}(x)}{8} - \frac{x}{4\,\left(x^4 - 1\right)} + \frac{\mathrm{atan}(x\,1\mathrm{i})\;5\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8 - 2*x^4 + 1),x)
[Out] x + (atan(x*1i)*5i)/8 - (5*atan(x))/8 - x/(4*(x^4 - 1))
```

3.301 $\int \frac{x^6}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=29

$$\frac{x^3}{4(1-x^4)}+\frac{3}{8}\tan^{-1}(x)-\frac{3}{8}\tanh^{-1}(x)$$

```
[Out] 1/4*x^3/(-x^4+1)+3/8*arctan(x)-3/8*arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {28, 294, 304, 209, 212}

$$\frac{3\text{ArcTan}(x)}{8}+\frac{x^3}{4(1-x^4)}-\frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 - x^4)) + (3*ArcTan[x])/8 - (3*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{1-2x^4+x^8}\,dx &= \int \frac{x^6}{(-1+x^4)^2}\,dx\\ &= \frac{x^3}{4(1-x^4)}+\frac{3}{4}\int \frac{x^2}{-1+x^4}\,dx\\ &= \frac{x^3}{4(1-x^4)}-\frac{3}{8}\int \frac{1}{1-x^2}\,dx+\frac{3}{8}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x^3}{4(1-x^4)}+\frac{3}{8}\tan^{-1}(x)-\frac{3}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.21

$$\frac{1}{16}\left(-\frac{4x^3}{-1+x^4}+6\tan^{-1}(x)+3\log(1-x)-3\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - 2*x^4 + x^8),x]
[Out] ((-4*x^3)/(-1 + x^4) + 6*ArcTan[x] + 3*Log[1 - x] - 3*Log[1 + x])/16
```

Maple [A]
time = 0.03, size = 42, normalized size = 1.45

method	result	size
risch	$-\frac{x^3}{4(x^4-1)}-\frac{3\ln(1+x)}{16}+\frac{3\ln(-1+x)}{16}+\frac{3\arctan(x)}{8}$	30
default	$-\frac{1}{16(-1+x)}+\frac{3\ln(-1+x)}{16}-\frac{x}{8(x^2+1)}+\frac{3\arctan(x)}{8}-\frac{1}{16(1+x)}-\frac{3\ln(1+x)}{16}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(-1+x)+3/16*ln(-1+x)-1/8*x/(x^2+1)+3/8*arctan(x)-1/16/(1+x)-3/16*ln(1
+x)
```

Maxima [A]
time = 0.53, size = 29, normalized size = 1.00

$$-\frac{x^3}{4\,(x^4-1)}+\frac{3}{8}\arctan\left(x\right)-\frac{3}{16}\log\left(x+1\right)+\frac{3}{16}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(x + 1) + 3/16*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(21) = 42.
time = 0.33, size = 46, normalized size = 1.59

$$-\frac{4\,x^3-6\,(x^4-1)\arctan\left(x\right)+3\,(x^4-1)\log\left(x+1\right)-3\,(x^4-1)\log\left(x-1\right)}{16\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 - 6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)
*log(x - 1))/(x^4 - 1)
```

Sympy [A]
time = 0.06, size = 32, normalized size = 1.10

$$-\frac{x^3}{4x^4-4}+\frac{3\log\left(x-1\right)}{16}-\frac{3\log\left(x+1\right)}{16}+\frac{3\,\mathrm{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8-2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 - 4) + 3*log(x - 1)/16 - 3*log(x + 1)/16 + 3*atan(x)/8
```

Giac [A]
time = 3.02, size = 31, normalized size = 1.07

$$-\frac{x^3}{4\,(x^4-1)}+\frac{3}{8}\arctan\left(x\right)-\frac{3}{16}\log\left(|x+1|\right)+\frac{3}{16}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) + 3/8*arctan(x) - 3/16*log(abs(x + 1)) + 3/16*log(abs(x
- 1))
```

Mupad [B]
time = 0.03, size = 23, normalized size = 0.79

$$\frac{3\,\mathrm{atan}(x)}{8}-\frac{3\,\mathrm{atanh}(x)}{8}-\frac{x^3}{4\,\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8 - 2*x^4 + 1),x)
```

```
[Out] (3*atan(x))/8 - (3*atanh(x))/8 - x^3/(4*(x^4 - 1))
```

3.302 $\int \frac{x^4}{1-2x^4+x^8}\,dx$

Optimal. Leaf size=27

$$\frac{x}{4(1-x^4)}-\frac{1}{8}\tan^{-1}(x)-\frac{1}{8}\tanh^{-1}(x)$$

```
[Out] 1/4*x/(-x^4+1)-1/8*arctan(x)-1/8*arctanh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.312$, Rules used = {28, 294, 218, 212, 209}

$$-\frac{\text{ArcTan}(x)}{8}+\frac{x}{4(1-x^4)}-\frac{1}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - 2*x^4 + x^8),x]
```

```
[Out] x/(4*(1 - x^4)) - ArcTan[x]/8 - ArcTanh[x]/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{1-2x^4+x^8}\,dx &= \int \frac{x^4}{(-1+x^4)^2}\,dx\\ &= \frac{x}{4(1-x^4)}+\frac{1}{4}\int \frac{1}{-1+x^4}\,dx\\ &= \frac{x}{4(1-x^4)}-\frac{1}{8}\int \frac{1}{1-x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x}{4(1-x^4)}-\frac{1}{8}\tan^{-1}(x)-\frac{1}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.15

$$\frac{1}{16}\left(-\frac{4x}{-1+x^4}-2\tan^{-1}(x)+\log(1-x)-\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - 2*x^4 + x^8),x]
[Out] ((-4*x)/(-1 + x^4) - 2*ArcTan[x] + Log[1 - x] - Log[1 + x])/16
```

Maple [A]
time = 0.03, size = 42, normalized size = 1.56

method	result	size
risch	$-\frac{x}{4(x^4-1)}-\frac{\ln(1+x)}{16}+\frac{\ln(-1+x)}{16}-\frac{\arctan(x)}{8}$	28
default	$-\frac{1}{16(-1+x)}+\frac{\ln(-1+x)}{16}+\frac{x}{8x^2+8}-\frac{\arctan(x)}{8}-\frac{1}{16(1+x)}-\frac{\ln(1+x)}{16}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/16/(-1+x)+1/16*ln(-1+x)+1/8*x/(x^2+1)-1/8*arctan(x)-1/16/(1+x)-1/16*ln(1
+x)
```

Maxima [A]
time = 0.54, size = 27, normalized size = 1.00

$$-\frac{x}{4\,(x^4-1)}-\frac{1}{8}\,\arctan\left(x\right)-\frac{1}{16}\,\log\left(x+1\right)+\frac{1}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(x + 1) + 1/16*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(19) = 38$.
time = 0.34, size = 43, normalized size = 1.59

$$-\frac{2\,(x^4-1)\arctan\left(x\right)+(x^4-1)\log\left(x+1\right)-(x^4-1)\log\left(x-1\right)+4\,x}{16\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(2*(x^4 - 1)*arctan(x) + (x^4 - 1)*log(x + 1) - (x^4 - 1)*log(x - 1)
+ 4*x)/(x^4 - 1)
```

Sympy [A]
time = 0.06, size = 26, normalized size = 0.96

$$-\frac{x}{4x^4-4}+\frac{\log\left(x-1\right)}{16}-\frac{\log\left(x+1\right)}{16}-\frac{\operatorname{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-2*x**4+1),x)
```

```
[Out] -x/(4*x**4 - 4) + log(x - 1)/16 - log(x + 1)/16 - atan(x)/8
```

Giac [A]
time = 3.10, size = 29, normalized size = 1.07

$$-\frac{x}{4\,(x^4-1)}-\frac{1}{8}\,\arctan\left(x\right)-\frac{1}{16}\,\log\left(|x+1|\right)+\frac{1}{16}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/8*arctan(x) - 1/16*log(abs(x + 1)) + 1/16*log(abs(x -
1))
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.78

$$-\frac{\mathrm{atan}(x)}{8}-\frac{\mathrm{atanh}(x)}{8}-\frac{x}{4\,\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8 - 2*x^4 + 1),x)
[Out] - atan(x)/8 - atanh(x)/8 - x/(4*(x^4 - 1))
```

3.303 $\int \frac{x^2}{1-2x^4+x^8} dx$

Optimal. Leaf size=29

$$\frac{x^3}{4(1-x^4)} - \frac{1}{8}\tan^{-1}(x) + \frac{1}{8}\tanh^{-1}(x)$$

```
[Out] 1/4*x^3/(-x^4+1)-1/8*arctan(x)+1/8*arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {28, 296, 304, 209, 212}

$$-\frac{\text{ArcTan}(x)}{8} + \frac{x^3}{4(1-x^4)} + \frac{1}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - 2*x^4 + x^8),x]
```

```
[Out] x^3/(4*(1 - x^4)) - ArcTan[x]/8 + ArcTanh[x]/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{1-2x^4+x^8}\,dx &= \int \frac{x^2}{(-1+x^4)^2}\,dx\\ &= \frac{x^3}{4(1-x^4)}-\frac{1}{4}\int \frac{x^2}{-1+x^4}\,dx\\ &= \frac{x^3}{4(1-x^4)}+\frac{1}{8}\int \frac{1}{1-x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x^3}{4(1-x^4)}-\frac{1}{8}\tan^{-1}(x)+\frac{1}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.14

$$\frac{1}{16}\left(-\frac{4x^3}{-1+x^4}-2\tan^{-1}(x)-\log(1-x)+\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - 2*x^4 + x^8),x]
[Out] ((-4*x^3)/(-1 + x^4) - 2*ArcTan[x] - Log[1 - x] + Log[1 + x])/16
```

Maple [A]
time = 0.03, size = 42, normalized size = 1.45

method	result	size
risch	$-\frac{x^3}{4(x^4-1)}+\frac{\ln(1+x)}{16}-\frac{\ln(-1+x)}{16}-\frac{\arctan(x)}{8}$	30
default	$-\frac{1}{16(-1+x)}-\frac{\ln(-1+x)}{16}-\frac{x}{8(x^2+1)}-\frac{\arctan(x)}{8}-\frac{1}{16(1+x)}+\frac{\ln(1+x)}{16}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(-1+x)-1/16*ln(-1+x)-1/8*x/(x^2+1)-1/8*arctan(x)-1/16/(1+x)+1/16*ln(1
+x)
```

Maxima [A]
time = 0.55, size = 29, normalized size = 1.00

$$-\frac{x^3}{4\,(x^4-1)}-\frac{1}{8}\,\arctan\left(x\right)+\frac{1}{16}\,\log\left(x+1\right)-\frac{1}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(x + 1) - 1/16*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(21) = 42$.
time = 0.36, size = 45, normalized size = 1.55

$$-\frac{4\,x^3+2\,(x^4-1)\arctan\left(x\right)-(x^4-1)\log\left(x+1\right)+(x^4-1)\log\left(x-1\right)}{16\,(x^4-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(4*x^3 + 2*(x^4 - 1)*arctan(x) - (x^4 - 1)*log(x + 1) + (x^4 - 1)*log
(x - 1))/(x^4 - 1)
```

Sympy [A]
time = 0.06, size = 27, normalized size = 0.93

$$-\frac{x^3}{4x^4-4}-\frac{\log\left(x-1\right)}{16}+\frac{\log\left(x+1\right)}{16}-\frac{\operatorname{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-2*x**4+1),x)
```

```
[Out] -x**3/(4*x**4 - 4) - log(x - 1)/16 + log(x + 1)/16 - atan(x)/8
```

Giac [A]
time = 4.18, size = 31, normalized size = 1.07

$$-\frac{x^3}{4\,(x^4-1)}-\frac{1}{8}\,\arctan\left(x\right)+\frac{1}{16}\,\log\left(|x+1|\right)-\frac{1}{16}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/8*arctan(x) + 1/16*log(abs(x + 1)) - 1/16*log(abs(x
- 1))
```

Mupad [B]
time = 0.03, size = 23, normalized size = 0.79

$$\frac{\operatorname{atanh}(x)}{8} - \frac{\operatorname{atan}(x)}{8} - \frac{x^3}{4\,(x^4 - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8 - 2*x^4 + 1),x)
```

```
[Out] atanh(x)/8 - atan(x)/8 - x^3/(4*(x^4 - 1))
```

3.304 $\int \frac{1}{1-2x^4+x^8} \, dx$

Optimal. Leaf size=27

$$\frac{x}{4(1-x^4)} + \frac{3}{8}\tan^{-1}(x) + \frac{3}{8}\tanh^{-1}(x)$$

```
[Out] 1/4*x/(-x^4+1)+3/8*arctan(x)+3/8*arctanh(x)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {28, 205, 218, 212, 209}

$$\frac{3\text{ArcTan}(x)}{8} + \frac{x}{4(1-x^4)} + \frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^4 + x^8)^(-1),x]
```

```
[Out] x/(4*(1 - x^4)) + (3*ArcTan[x])/8 + (3*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-2x^4+x^8}\,dx &= \int \frac{1}{(-1+x^4)^2}\,dx\\ &= \frac{x}{4(1-x^4)}-\frac{3}{4}\int \frac{1}{-1+x^4}\,dx\\ &= \frac{x}{4(1-x^4)}+\frac{3}{8}\int \frac{1}{1-x^2}\,dx+\frac{3}{8}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x}{4(1-x^4)}+\frac{3}{8}\tan^{-1}(x)+\frac{3}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.22

$$\frac{1}{16}\left(-\frac{4x}{-1+x^4}+6\tan^{-1}(x)-3\log(1-x)+3\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^4 + x^8)^(-1),x]
[Out] ((-4*x)/(-1 + x^4) + 6*ArcTan[x] - 3*Log[1 - x] + 3*Log[1 + x])/16
```

Maple [A]
time = 0.03, size = 42, normalized size = 1.56

method	result	size
risch	$-\frac{x}{4(x^4-1)}-\frac{3\ln(-1+x)}{16}+\frac{3\arctan(x)}{8}+\frac{3\ln(1+x)}{16}$	28
default	$-\frac{1}{16(-1+x)}-\frac{3\ln(-1+x)}{16}+\frac{x}{8x^2+8}+\frac{3\arctan(x)}{8}-\frac{1}{16(1+x)}+\frac{3\ln(1+x)}{16}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(-1+x)-3/16*ln(-1+x)+1/8*x/(x^2+1)+3/8*arctan(x)-1/16/(1+x)+3/16*ln(1
+x)
```

Maxima [A]
time = 0.56, size = 27, normalized size = 1.00

$$-\frac{x}{4\left(x^4-1\right)}+\frac{3}{8}\arctan\left(x\right)+\frac{3}{16}\log\left(x+1\right)-\frac{3}{16}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(x + 1) - 3/16*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(19) = 38.
time = 0.38, size = 44, normalized size = 1.63

$$\frac{6\left(x^4-1\right)\arctan\left(x\right)+3\left(x^4-1\right)\log\left(x+1\right)-3\left(x^4-1\right)\log\left(x-1\right)-4\,x}{16\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(x^4 - 1)*arctan(x) + 3*(x^4 - 1)*log(x + 1) - 3*(x^4 - 1)*log(x -
1) - 4*x)/(x^4 - 1)
```

Sympy [A]
time = 0.06, size = 31, normalized size = 1.15

$$-\frac{x}{4x^4-4}-\frac{3\log\left(x-1\right)}{16}+\frac{3\log\left(x+1\right)}{16}+\frac{3\,\mathrm{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-2*x**4+1),x)
```

```
[Out] -x/(4*x**4 - 4) - 3*log(x - 1)/16 + 3*log(x + 1)/16 + 3*atan(x)/8
```

Giac [A]
time = 5.02, size = 29, normalized size = 1.07

$$-\frac{x}{4\left(x^4-1\right)}+\frac{3}{8}\arctan\left(x\right)+\frac{3}{16}\log\left(\left|x+1\right|\right)-\frac{3}{16}\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) + 3/8*arctan(x) + 3/16*log(abs(x + 1)) - 3/16*log(abs(x -
1))
```

Mupad [B]
time = 0.03, size = 21, normalized size = 0.78

$$\frac{3\,\mathrm{atan}(x)}{8}+\frac{3\,\mathrm{atanh}(x)}{8}-\frac{x}{4\,\left(x^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - 2*x^4 + 1),x)
```

```
[Out] (3*atan(x))/8 + (3*atanh(x))/8 - x/(4*(x^4 - 1))
```

3.305 $\int \frac{1}{x^2(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=36

$$-\frac{5}{4x}+\frac{1}{4x(1-x^4)}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{8}\tanh^{-1}(x)$$

```
[Out] -5/4/x+1/4/x/(-x^4+1)-5/8*arctan(x)+5/8*arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {28, 296, 331, 304, 209, 212}

$$-\frac{5\text{ArcTan}(x)}{8}+\frac{1}{4x(1-x^4)}-\frac{5}{4x}+\frac{5}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -5/(4*x) + 1/(4*x*(1 - x^4)) - (5*ArcTan[x])/8 + (5*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^2\left(-1+x^4\right)^2}\,dx\\ &= \frac{1}{4x\left(1-x^4\right)}-\frac{5}{4}\int \frac{1}{x^2\left(-1+x^4\right)}\,dx\\ &= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}-\frac{5}{4}\int \frac{x^2}{-1+x^4}\,dx\\ &= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}+\frac{5}{8}\int \frac{1}{1-x^2}\,dx-\frac{5}{8}\int \frac{1}{1+x^2}\,dx\\ &= -\frac{5}{4x}+\frac{1}{4x\left(1-x^4\right)}-\frac{5}{8}\tan^{-1}(x)+\frac{5}{8}\tanh^{-1}(x)\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 40, normalized size = 1.11

$$\frac{1}{16}\left(-\frac{16}{x}-\frac{4x^3}{-1+x^4}-10\tan^{-1}(x)-5\log(1-x)+5\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-16/x - (4*x^3)/(-1 + x^4) - 10*ArcTan[x] - 5*Log[1 - x] + 5*Log[1 + x])/1
6
```

Maple [A]
time = 0.03, size = 47, normalized size = 1.31

method	result	size
risch	$\frac{-\frac{5x^4}{4}+1}{x(x^4-1)}+\frac{5\ln(1+x)}{16}-\frac{5\arctan(x)}{8}-\frac{5\ln(-1+x)}{16}$	36
default	$-\frac{1}{16(-1+x)}-\frac{5\ln(-1+x)}{16}-\frac{x}{8(x^2+1)}-\frac{5\arctan(x)}{8}-\frac{1}{x}-\frac{1}{16(1+x)}+\frac{5\ln(1+x)}{16}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(-1+x)-5/16*ln(-1+x)-1/8*x/(x^2+1)-5/8*arctan(x)-1/x-1/16/(1+x)+5/16*
ln(1+x)
```

Maxima [A]
time = 0.51, size = 35, normalized size = 0.97

$$-\frac{5\,x^4-4}{4\,(x^5-x)}-\frac{5}{8}\,\arctan\left(x\right)+\frac{5}{16}\,\log\left(x+1\right)-\frac{5}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(x + 1) - 5/16*log(x -
 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(26) = 52$.
time = 0.35, size = 55, normalized size = 1.53

$$-\frac{20\,x^4+10\,(x^5-x)\arctan\left(x\right)-5\,(x^5-x)\log\left(x+1\right)+5\,(x^5-x)\log\left(x-1\right)-16}{16\,(x^5-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/16*(20*x^4 + 10*(x^5 - x)*arctan(x) - 5*(x^5 - x)*log(x + 1) + 5*(x^5 -
x)*log(x - 1) - 16)/(x^5 - x)
```

Sympy [A]
time = 0.07, size = 37, normalized size = 1.03

$$\frac{4-5x^4}{4x^5-4x}-\frac{5\log\left(x-1\right)}{16}+\frac{5\log\left(x+1\right)}{16}-\frac{5\,\mathrm{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8-2*x**4+1),x)
```

```
[Out] (4 - 5*x**4)/(4*x**5 - 4*x) - 5*log(x - 1)/16 + 5*log(x + 1)/16 - 5*atan(x)
/8
```

Giac [A]
time = 4.51, size = 37, normalized size = 1.03

$$-\frac{5\,x^4-4}{4\,(x^5-x)}-\frac{5}{8}\,\arctan\left(x\right)+\frac{5}{16}\,\log\left(|x+1|\right)-\frac{5}{16}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(5*x^4 - 4)/(x^5 - x) - 5/8*arctan(x) + 5/16*log(abs(x + 1)) - 5/16*lo
g(abs(x - 1))
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.72

$$\frac{5\,\mathrm{atanh}(x)}{8}-\frac{5\,\mathrm{atan}(x)}{8}+\frac{\frac{5\,x^4}{4}-1}{x-x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (5*atanh(x))/8 - (5*atan(x))/8 + ((5*x^4)/4 - 1)/(x - x^5)
```

3.306 $\int \frac{1}{x^4(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=36

$$-\frac{7}{12x^3}+\frac{1}{4x^3(1-x^4)}+\frac{7}{8}\tan^{-1}(x)+\frac{7}{8}\tanh^{-1}(x)$$

```
[Out] -7/12/x^3+1/4/x^3/(-x^4+1)+7/8*arctan(x)+7/8*arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {28, 296, 331, 218, 212, 209}

$$\frac{7\text{ArcTan}(x)}{8}-\frac{7}{12x^3}+\frac{1}{4x^3(1-x^4)}+\frac{7}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -7/(12*x^3) + 1/(4*x^3*(1 - x^4)) + (7*ArcTan[x])/8 + (7*ArcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4(1-2x^4+x^8)}\,dx &= \int \frac{1}{x^4(-1+x^4)^2}\,dx \\
&= \frac{1}{4x^3(1-x^4)} - \frac{7}{4}\int \frac{1}{x^4(-1+x^4)}\,dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1-x^4)} - \frac{7}{4}\int \frac{1}{-1+x^4}\,dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1-x^4)} + \frac{7}{8}\int \frac{1}{1-x^2}\,dx + \frac{7}{8}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{7}{12x^3} + \frac{1}{4x^3(1-x^4)} + \frac{7}{8}\tan^{-1}(x) + \frac{7}{8}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.06

$$\frac{1}{48}\left(-\frac{16}{x^3} - \frac{12x}{-1+x^4} + 42\tan^{-1}(x) - 21\log(1-x) + 21\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-16/x^3 - (12*x)/(-1 + x^4) + 42*ArcTan[x] - 21*Log[1 - x] + 21*Log[1 + x]
)/48
```

Maple [A]
time = 0.03, size = 47, normalized size = 1.31

method	result	size
risch	$\frac{-\frac{7x^4}{12}+\frac{1}{3}}{x^3(x^4-1)} - \frac{7\ln(-1+x)}{16} + \frac{7\arctan(x)}{8} + \frac{7\ln(1+x)}{16}$	36
default	$-\frac{1}{16(-1+x)} - \frac{7\ln(-1+x)}{16} + \frac{x}{8x^2+8} + \frac{7\arctan(x)}{8} - \frac{1}{3x^3} - \frac{1}{16(1+x)} + \frac{7\ln(1+x)}{16}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(-1+x)-7/16*ln(-1+x)+1/8*x/(x^2+1)+7/8*arctan(x)-1/3/x^3-1/16/(1+x)+7
/16*ln(1+x)
```

Maxima [A]
time = 0.52, size = 37, normalized size = 1.03

$$-\frac{7\,x^4-4}{12\,(x^7-x^3)}+\frac{7}{8}\,\arctan\left(x\right)+\frac{7}{16}\,\log\left(x+1\right)-\frac{7}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*(7*x^4 - 4)/(x^7 - x^3) + 7/8*arctan(x) + 7/16*log(x + 1) - 7/16*log(
x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.
time = 0.32, size = 63, normalized size = 1.75

$$-\frac{28\,x^4-42\,(x^7-x^3)\arctan\left(x\right)-21\,(x^7-x^3)\log\left(x+1\right)+21\,(x^7-x^3)\log\left(x-1\right)-16}{48\,(x^7-x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/48*(28*x^4 - 42*(x^7 - x^3)*arctan(x) - 21*(x^7 - x^3)*log(x + 1) + 21*(
x^7 - x^3)*log(x - 1) - 16)/(x^7 - x^3)
```

Sympy [A]
time = 0.09, size = 39, normalized size = 1.08

$$\frac{4-7x^4}{12x^7-12x^3}-\frac{7\log\left(x-1\right)}{16}+\frac{7\log\left(x+1\right)}{16}+\frac{7\operatorname{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8-2*x**4+1),x)
```

```
[Out] (4 - 7*x**4)/(12*x**7 - 12*x**3) - 7*log(x - 1)/16 + 7*log(x + 1)/16 + 7*at
an(x)/8
```

Giac [A]
time = 3.28, size = 34, normalized size = 0.94

$$-\frac{x}{4\left(x^{4}-1\right)}-\frac{1}{3\,x^{3}}+\frac{7}{8}\,\arctan\left(x\right)+\frac{7}{16}\,\log\left(\left|x+1\right|\right)-\frac{7}{16}\,\log\left(\left|x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/3/x^3 + 7/8*arctan(x) + 7/16*log(abs(x + 1)) - 7/16*lo
g(abs(x - 1))
```

Mupad [B]
time = 1.29, size = 28, normalized size = 0.78

$$\frac{7\,\mathrm{atan}(x)}{8}+\frac{7\,\mathrm{atanh}(x)}{8}+\frac{\frac{7\,x^4}{12}-\frac{1}{3}}{x^3-x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (7*atan(x))/8 + (7*atanh(x))/8 + ((7*x^4)/12 - 1/3)/(x^3 - x^7)
```

3.307 $\int \frac{1}{x^6(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=43

$$-\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5(1-x^4)}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)$$

```
[Out] -9/20/x^5-9/4/x+1/4/x^5/(-x^4+1)-9/8*arctan(x)+9/8*arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {28, 296, 331, 304, 209, 212}

$$-\frac{9\text{ArcTan}(x)}{8}-\frac{9}{20x^5}+\frac{1}{4x^5(1-x^4)}-\frac{9}{4x}+\frac{9}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -9/(20*x^5) - 9/(4*x) + 1/(4*x^5*(1 - x^4)) - (9*ArcTan[x])/8 + (9*ArcTanh[
x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(1-2x^4+x^8\right)}\,dx &= \int \frac{1}{x^6\left(-1+x^4\right)^2}\,dx\\
&= \frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4}\int \frac{1}{x^6\left(-1+x^4\right)}\,dx\\
&= -\frac{9}{20x^5}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4}\int \frac{1}{x^2\left(-1+x^4\right)}\,dx\\
&= -\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{4}\int \frac{x^2}{-1+x^4}\,dx\\
&= -\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}+\frac{9}{8}\int \frac{1}{1-x^2}\,dx-\frac{9}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{9}{20x^5}-\frac{9}{4x}+\frac{1}{4x^5\left(1-x^4\right)}-\frac{9}{8}\tan^{-1}(x)+\frac{9}{8}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 1.19

$$
-\frac{1}{5x^5}-\frac{2}{x}-\frac{x^3}{4\left(-1+x^4\right)}-\frac{9}{8}\tan^{-1}(x)-\frac{9}{16}\log(1-x)+\frac{9}{16}\log(1+x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - 2/x - x^3/(4*(-1 + x^4)) - (9*ArcTan[x])/8 - (9*Log[1 - x])/16
 + (9*Log[1 + x])/16
```

Maple [A]
time = 0.04, size = 52, normalized size = 1.21

method	result	size
risch	$\frac{-\frac{9}{4}x^8+\frac{9}{5}x^4+\frac{1}{5}}{x^5(x^4-1)}-\frac{9\arctan(x)}{8}+\frac{9\ln(1+x)}{16}-\frac{9\ln(-1+x)}{16}$	41
default	$-\frac{1}{16(-1+x)}-\frac{9\ln(-1+x)}{16}-\frac{x}{8(x^2+1)}-\frac{9\arctan(x)}{8}-\frac{1}{5x^5}-\frac{2}{x}-\frac{1}{16(1+x)}+\frac{9\ln(1+x)}{16}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(-1+x)-9/16*ln(-1+x)-1/8*x/(x^2+1)-9/8*arctan(x)-1/5/x^5-2/x-1/16/(1+
x)+9/16*ln(1+x)
```

Maxima [A]
time = 0.50, size = 42, normalized size = 0.98

$$-\frac{45\,x^8-36\,x^4-4}{20\,(x^9-x^5)}-\frac{9}{8}\,\arctan(x)+\frac{9}{16}\,\log(x+1)-\frac{9}{16}\,\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/20*(45*x^8 - 36*x^4 - 4)/(x^9 - x^5) - 9/8*arctan(x) + 9/16*log(x + 1) -
 9/16*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(31) = 62$.
time = 0.34, size = 68, normalized size = 1.58

$$-\frac{180\,x^8-144\,x^4+90\,(x^9-x^5)\arctan(x)-45\,(x^9-x^5)\log(x+1)+45\,(x^9-x^5)\log(x-1)-16}{80\,(x^9-x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/80*(180*x^8 - 144*x^4 + 90*(x^9 - x^5)*arctan(x) - 45*(x^9 - x^5)*log(x
+ 1) + 45*(x^9 - x^5)*log(x - 1) - 16)/(x^9 - x^5)
```

Sympy [A]
time = 0.08, size = 44, normalized size = 1.02

$$-\frac{9\log(x-1)}{16}+\frac{9\log(x+1)}{16}-\frac{9\,\mathrm{atan}\,(x)}{8}+\frac{-45x^8+36x^4+4}{20x^9-20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8-2*x**4+1),x)
```

```
[Out] -9*log(x - 1)/16 + 9*log(x + 1)/16 - 9*atan(x)/8 + (-45*x**8 + 36*x**4 + 4)
/(20*x**9 - 20*x**5)
```

Giac [A]
time = 3.41, size = 43, normalized size = 1.00

$$-\frac{x^3}{4\,(x^4-1)}-\frac{10\,x^4+1}{5\,x^5}-\frac{9}{8}\,\arctan\left(x\right)+\frac{9}{16}\,\log\left(|x+1|\right)-\frac{9}{16}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x^3/(x^4 - 1) - 1/5*(10*x^4 + 1)/x^5 - 9/8*arctan(x) + 9/16*log(abs(x
+ 1)) - 9/16*log(abs(x - 1))
```

Mupad [B]
time = 0.04, size = 34, normalized size = 0.79

$$\frac{9\,\mathrm{atanh}(x)}{8}-\frac{9\,\mathrm{atan}(x)}{8}-\frac{-\frac{9\,x^8}{4}+\frac{9\,x^4}{5}+\frac{1}{5}}{x^5-x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (9*atanh(x))/8 - (9*atan(x))/8 - ((9*x^4)/5 - (9*x^8)/4 + 1/5)/(x^5 - x^9)
```

3.308 $\int \frac{1}{x^8(1-2x^4+x^8)}\,dx$

Optimal. Leaf size=43

$$-\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}+\frac{11}{8}\tan^{-1}(x)+\frac{11}{8}\tanh^{-1}(x)$$

```
[Out] -11/28/x^7-11/12/x^3+1/4/x^7/(-x^4+1)+11/8*arctan(x)+11/8*arctanh(x)
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {28, 296, 331, 218, 212, 209}

$$\frac{11\text{ArcTan}(x)}{8}-\frac{11}{28x^7}-\frac{11}{12x^3}+\frac{1}{4x^7\left(1-x^4\right)}+\frac{11}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] -11/(28*x^7) - 11/(12*x^3) + 1/(4*x^7*(1 - x^4)) + (11*ArcTan[x])/8 + (11*A
rcTanh[x])/8
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
```

```
, 0]
```

Rule 296

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^8(1-2x^4+x^8)}\,dx &= \int \frac{1}{x^8(-1+x^4)^2}\,dx \\
&= \frac{1}{4x^7(1-x^4)} - \frac{11}{4}\int \frac{1}{x^8(-1+x^4)}\,dx \\
&= -\frac{11}{28x^7} + \frac{1}{4x^7(1-x^4)} - \frac{11}{4}\int \frac{1}{x^4(-1+x^4)}\,dx \\
&= -\frac{11}{28x^7} - \frac{11}{12x^3} + \frac{1}{4x^7(1-x^4)} - \frac{11}{4}\int \frac{1}{-1+x^4}\,dx \\
&= -\frac{11}{28x^7} - \frac{11}{12x^3} + \frac{1}{4x^7(1-x^4)} + \frac{11}{8}\int \frac{1}{1-x^2}\,dx + \frac{11}{8}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{11}{28x^7} - \frac{11}{12x^3} + \frac{1}{4x^7(1-x^4)} + \frac{11}{8}\tan^{-1}(x) + \frac{11}{8}\tanh^{-1}(x)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 1.00

$$
\frac{1}{336}\left(-\frac{48}{x^7} - \frac{224}{x^3} - \frac{84x}{-1+x^4} + 462\tan^{-1}(x) - 231\log(1-x) + 231\log(1+x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - 2*x^4 + x^8)),x]
```

```
[Out] (-48/x^7 - 224/x^3 - (84*x)/(-1 + x^4) + 462*ArcTan[x] - 231*Log[1 - x] + 2
31*Log[1 + x])/336
```

Maple [A]
time = 0.04, size = 52, normalized size = 1.21

method	result	size
risch	$\frac{-\frac{11}{12}x^8+\frac{11}{21}x^4+\frac{1}{7}}{x^7(x^4-1)} - \frac{11\ln(-1+x)}{16} + \frac{11\arctan(x)}{8} + \frac{11\ln(1+x)}{16}$	41
default	$-\frac{1}{16(-1+x)} - \frac{11\ln(-1+x)}{16} + \frac{x}{8x^2+8} + \frac{11\arctan(x)}{8} - \frac{1}{7x^7} - \frac{2}{3x^3} - \frac{1}{16(1+x)} + \frac{11\ln(1+x)}{16}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8-2*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/(-1+x)-11/16*ln(-1+x)+1/8*x/(x^2+1)+11/8*arctan(x)-1/7/x^7-2/3/x^3-1/
16/(1+x)+11/16*ln(1+x)
```

Maxima [A]
time = 0.54, size = 42, normalized size = 0.98

$$-\frac{77\,x^8-44\,x^4-12}{84\,(x^{11}-x^7)}+\frac{11}{8}\,\arctan\left(x\right)+\frac{11}{16}\,\log\left(x+1\right)-\frac{11}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/84*(77*x^8 - 44*x^4 - 12)/(x^11 - x^7) + 11/8*arctan(x) + 11/16*log(x +
1) - 11/16*log(x - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(31) = 62$.
time = 0.36, size = 68, normalized size = 1.58

$$-\frac{308\,x^8-176\,x^4-462\,(x^{11}-x^7)\arctan\left(x\right)-231\,(x^{11}-x^7)\log\left(x+1\right)+231\,(x^{11}-x^7)\log\left(x-1\right)-48}{336\,(x^{11}-x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/336*(308*x^8 - 176*x^4 - 462*(x^11 - x^7)*arctan(x) - 231*(x^11 - x^7)*l
og(x + 1) + 231*(x^11 - x^7)*log(x - 1) - 48)/(x^11 - x^7)
```

Sympy [A]
time = 0.10, size = 44, normalized size = 1.02

$$-\frac{11\log\left(x-1\right)}{16}+\frac{11\log\left(x+1\right)}{16}+\frac{11\,\mathrm{atan}\left(x\right)}{8}+\frac{-77x^8+44x^4+12}{84x^{11}-84x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-2*x**4+1),x)
```

```
[Out] -11*log(x - 1)/16 + 11*log(x + 1)/16 + 11*atan(x)/8 + (-77*x**8 + 44*x**4 +
 12)/(84*x**11 - 84*x**7)
```

Giac [A]
time = 2.94, size = 41, normalized size = 0.95

$$-\frac{x}{4\,(x^4-1)}-\frac{14\,x^4+3}{21\,x^7}+\frac{11}{8}\,\arctan\left(x\right)+\frac{11}{16}\,\log\left(|x+1|\right)-\frac{11}{16}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-2*x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*x/(x^4 - 1) - 1/21*(14*x^4 + 3)/x^7 + 11/8*arctan(x) + 11/16*log(abs(x
 + 1)) - 11/16*log(abs(x - 1))
```

Mupad [B]
time = 0.05, size = 34, normalized size = 0.79

$$\frac{11\,\mathrm{atan}(x)}{8}+\frac{11\,\mathrm{atanh}(x)}{8}-\frac{-\frac{11\,x^8}{12}+\frac{11\,x^4}{21}+\frac{1}{7}}{x^7-x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^8 - 2*x^4 + 1)),x)
```

```
[Out] (11*atan(x))/8 + (11*atanh(x))/8 - ((11*x^4)/21 - (11*x^8)/12 + 1/7)/(x^7 -
 x^11)
```

3.309 $\int \frac{x^m}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=163

$$\frac{2cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)}-\frac{2cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+m)}$$

```
[Out] 2*c*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*c*x^4/(b-(-4*a*c+b^2)^(
1/2)))/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*x^(1+m)*hypergeo
m([1, 1/4+1/4*m],[5/4+1/4*m],-2*c*x^4/(b+(-4*a*c+b^2)^(1/2)))/(1+m)/(-4*a*c
+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.10, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1389, 371}

$$\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2cx^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2cx^4}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b - S
qrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*(1 + m)) - (
2*c*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*c*x^4)/(b + Sq
rt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^m}{a+bx^4+cx^8}\,dx = \frac{c\int \frac{x^m}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{x^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{2cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2cx^4}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)(1+m)} - \frac{2cx^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2cx^4}{b+\sqrt{b^2-4}}\right.}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)(1+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.24, size = 82, normalized size = 0.50

$$\frac{x^m \text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{b\#1^3+2c\#1^7}\&\right]}{4m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(a + b*x^4 + c*x^8),x]
```

```
[Out] (x^m*RootSum[a + b*#1^4 + c*#1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/
(x - #1))]/((x/(x - #1))^m*(b*#1^3 + 2*c*#1^7)) & ])/(4*m)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{c\,x^8+b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(c*x^8+b*x^4+a),x)
```

```
[Out] int(x^m/(c*x^8+b*x^4+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] integral(x^m/(c*x^8 + b*x^4 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] integrate(x^m/(c*x^8 + b*x^4 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{c\,x^8+b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(a + b*x^4 + c*x^8),x)
[Out] int(x^m/(a + b*x^4 + c*x^8), x)
```

3.310 $\int \frac{x^{11}}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=81

$$\frac{x^4}{4c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}$$

```
[Out] 1/4*x^4/c-1/8*b*ln(c*x^8+b*x^4+a)/c^2-1/4*(-2*a*c+b^2)*arctanh((2*c*x^4+b)/
(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1371, 717, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^4/(4*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}}{a+bx^4+cx^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4c}+\frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx,x,x^4\right)}{4c}\\
&= \frac{x^4}{4c}-\frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8c^2}+\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^4\right)}{8c^2}\\
&= \frac{x^4}{4c}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^4\right)}{4c^2}\\
&= \frac{x^4}{4c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^4+cx^8\right)}{8c^2}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 78, normalized size = 0.96

$$
\frac{2cx^4+\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+bx^4+cx^8\right)}{8c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*c*x^4 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^4 + c*x^8])/(8*c^2)
```

Maple [A]
time = 0.04, size = 83, normalized size = 1.02

method	result
default	$\frac{x^{4}}{4c}+\frac{-\frac{b\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{2c}+\frac{2\left(-a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{4c}$
risch	$\frac{x^{4}}{4c}-\frac{\ln\left(\left(\left(-8a^{2}c^{2}+6a\,b^{2}c-b^{4}+\sqrt{-\left(4ac-b^{2}\right)\left(2ac-b^{2}\right)^{2}}\,b\right)x^{4}+2\sqrt{-\left(4ac-b^{2}\right)\left(2ac-b^{2}\right)^{2}}\,a\right)ab}{2c\left(4ac-b^{2}\right)}+$ — ln

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4/c+1/4/c*(-1/2*b/c*ln(c*x^8+b*x^4+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 254, normalized size = 3.14

$$\left[\frac{2\left(b^{2}c-4\,ac^{2}\right)x^{4}-\left(b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{8}+2\,bcx^{4}+b^{2}-2\,ac+\left(2\,cx^{4}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{8}+bx^{4}+a}\right)-\left(b^{3}-4\,abc\right)\log\left(cx^{8}+bx^{4}+a\right)}{8\left(b^{2}c^{2}-4\,ac^{3}\right)},\ \frac{2\left(b^{2}c-4\,ac^{2}\right)x^{4}-2\left(b^{2}-2\,ac\right)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\left(2\,cx^{4}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-\left(b^{3}-4\,abc\right)\log\left(cx^{8}+bx^{4}+a\right)}{8\left(b^{2}c^{2}-4\,ac^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(b^2*c - 4*a*c^2)*x^4 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b
```

```
*x^4 + a)) - (b^3 - 4*a*b*c)*log(c*x^8 + b*x^4 + a))/(b^2*c^2 - 4*a*c^3), 1
/8*(2*(b^2*c - 4*a*c^2)*x^4 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^8 +
 b*x^4 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(73) = 146.
time = 2.76, size = 316, normalized size = 3.90

$$\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)\log\left(x^4+\frac{-ab-16ac^2\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)\log\left(x^4+\frac{-ab-16ac^2\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)+4b^2c\left(-\frac{b}{8c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{8c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right)+\frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))*
log(x**4 + (-a*b - 16*a*c**2*(-b/(8*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b*
*2)/(8*c**2*(4*a*c - b**2))) + 4*b**2*c*(-b/(8*c**2) - sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(8*c**2) + s
qrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a*c - b**2)))*log(x**4 + (-a*b
 - 16*a*c**2*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(8*c**2*(4*a
*c - b**2))) + 4*b**2*c*(-b/(8*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(
8*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**4/(4*c)
```

Giac [A]
time = 7.00, size = 75, normalized size = 0.93

$$\frac{x^4}{4\,c}-\frac{b\log\left(cx^8+bx^4+a\right)}{8\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*x^4/c - 1/8*b*log(c*x^8 + b*x^4 + a)/c^2 + 1/4*(b^2 - 2*a*c)*arctan((2*
c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B]
time = 2.69, size = 2500, normalized size = 30.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x^4 + c*x^8),x)
```

```
[Out] x^4/(4*c) + (log(a + b*x^4 + c*x^8)*(4*b^3 - 16*a*b*c))/(2*(64*a*c^3 - 16*b
^2*c^2)) - (atan((8*c^4*x^4*(((a*c - b^2)*(((((2*a*c - b^2)*((((448*b^4*c^6
```

- 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)) + (4*b^3*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) - ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + ((2*a*c - b^2)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + (((8*a^3*c^5 - 20*b^6*c^2 + 48*a*b^4*c^3 - 36*a^2*b^2*c^4)/c^4 - ((4*b^3 - 16*a*b*c)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (b^3*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(2*c^2*(4*a*c - b^2)^(3/2)*(64*a*c^3 - 16*b^2*c^2))))/(8*a^3*c^2) - ((b^3 - 3*a*b*c)*(((4*b^3 - 16*a*b*c)*((8*a^3*c^5 - 20*b^6*c^2 + 48*a*b^4*c^3 - 36*a^2*b^2*c^4)/c^4 - ((4*b^3 - 16*a*b*c)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) - (b^7 - a^3*b*c^3 + 3*a^2*b^3*c^2 - 3*a*b^5*c)/c^4 + ((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*((((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)) + (4*b^3*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c - b^2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + ((((4*b^3 - 16*a*b*c)*((((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (32*b^3*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + ((2*a*c - b^2)*((144*b^5*c^4 - 240*a*b^3*c^5 + 96*a^2*b*c^6)/c^4 + ((4*b^3 - 16*a*b*c)*((448*b^4*c^6 - 384*a*b^2*c^7)/c^4 + (256*b^3*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b^3*(2*a*c - b^2)^4)/(8*c^4*(4*a*c - b^2)^2)))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^2)/(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3 - 8*a*b^6*c) - (c^2*(a*c - b^2)*(4*a*c - b^2)^2*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*(((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + (((64*a^3*c^6

```
 + 208*a*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c
^6 - 512*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16
*b^2*c^2)))/(2*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2
)^(1/2))))/(2*(64*a*c^3 - 16*b^2*c^2)) + ((2*a*c - b^2)*((24*a*b^5*c^2 + 16
*a^3*b*c^4 - 40*a^2*b^3*c^3)/c^4 + ((4*b^3 - 16*a*b*c)*((64*a^3*c^6 + 208*a
*b^4*c^4 - 256*a^2*b^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c^6 - 512
*a^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2
)))/(2*(64*a*c^3 - 16*b^2*c^2))))/(2*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a
*c - b^2)^(1/2)) - ((2*a*c - b^2)*(((((2*a*c - b^2)*((768*a*b^3*c^6 - 512*a
^2*b*c^7)/c^4 + (512*a*b^2*c^4*(4*b^3 - 16*a*b*c))/(64*a*c^3 - 16*b^2*c^2))
)/(8*c^2*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^2*(4*b^3 - 16*a*b*c)*(2*a*c - b
^2))/((4*a*c - b^2)^(1/2)*(64*a*c^3 - 16*b^2*c^2)))*(2*a*c - b^2))/(8*c^2*(
4*a*c - b^2)^(1/2)) + (8*a*b^2*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/((4*a*c
- b^2)*(64*a*c^3 - 16*b^2*c^2))))/(8*c^2*(4*a*c - b^2)^(1/2)) - (a*b^2*(4*b
^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(c^2*(4*a*c - b^2)^(3/2)*(64*a*c^3 - 16*b^2
*c^2))))/(a^3*(b^8 + 16*a^4*c^4 + 24*a^2*b^4*c^2 - 32*a^3*b^2*c^3 - 8*a*b^6
*c)) + (c^2*(4*a*c - b^2)^(3/2)*(b^3 - 3*a*b*c)*((a*b^6 - 2*a^2*b^4*c + a^3
*b^2*c^2)/c^4 + ((4*b^3 - 16*a*b*c)*((24*a*b^5*c^2 + 16*a^3*b*c^4 - 40*a^2*
b^3*c^3)/c^4 + ((4*b^3 - 16*a*b*c)*((64*a^3*c^6 + 208*a*b^4*c^4 - 256*a^2*b
^2*c^5)/c^4 + ((4*b^3 - 16*a*b*c)*((768*a*b^3*c...
```

3.311 $\int \frac{x^9}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=192

$$\frac{x^2}{2c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x^2/c-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(
2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^
2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1373, 1136, 1180, 211}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^2/(2*c) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^
2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a
*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
```

```
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{a+bx^4+cx^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx,x,x^2\right)\\
&= \frac{x^2}{2c}-\frac{\text{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx,x,x^2\right)}{2c}\\
&= \frac{x^2}{2c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)}{4c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\cdots}{\cdots}\\
&= \frac{x^2}{2c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\cdots\right)}{2\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{\cdots}}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 210, normalized size = 1.09

$$\frac{2\sqrt{c}\,x^2-\frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x^4 + c*x^8),x]
```

```
[Out] (2*Sqrt[c]*x^2 - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

Maple [A]
time = 0.05, size = 176, normalized size = 0.92

method	result
risch	$\frac{x^2}{2c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(\left(16a^2c^3-8a\,b^2c^2+b^4c\right)\_Z^4+\left(12a^2b\,c^2-7a\,b^3c+b^5\right)\_Z^2+a^3c^2\right)} \_R\ln\left(\left(-a^2c^2+a\,b^2c\right)x^2+\left(-4ab\,c^2+b^3c\right)\_R^3+(\right.}{4c}$
default	$\frac{x^2}{2c} - \frac{\left(b^2-2ac-b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}} + \frac{\left(-b\sqrt{-4ac+b^2}+2ac-b^2\right)}{4\sqrt{-4ac+b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c-1/4*(b^2-2*a*c-b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/c*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))+1/4*(-b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/(-4*a*c+b^2)^(1/2)/c
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^4 + a)*x/(c*x^8 + b*x^4 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(150) = 300$.
time = 0.36, size = 1071, normalized size = 5.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 -
 a^2*c)*x^2 + 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b
*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3
*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*
a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^
3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^
3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4
*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
- 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqr
t(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a*b^2 - a^2*c)*x^2 + 1/2*sqr
t(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4
*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(a
*b^2 - a^2*c)*x^2 - 1/2*sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 -
 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b
^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^
6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - 2*x^2)/c
```

Sympy [**A**]
time = 2.88, size = 134, normalized size = 0.70

$$\operatorname{RootSum}\left(t^{4}\cdot\left(4096a^{2}c^{5}-2048ab^{2}c^{4}+256b^{4}c^{3}\right)+t^{2}\cdot\left(192a^{2}bc^{2}-112ab^{3}c+16b^{5}\right)+a^{3},\left(t\mapsto t\log\left(x^{2}+\frac{256t^{3}abc^{4}-64t^{3}b^{3}c^{3}-8ta^{2}c^{2}+16tab^{2}c-4tb^{4}}{a^{2}c-ab^{2}}\right)\right)\right)+\frac{x^{2}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**2*c**5 - 2048*a*b**2*c**4 + 256*b**4*c**3) + _t**2*(
192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + a**3, Lambda(_t, _t*log(x**2 +
(256*_t**3*a*b*c**4 - 64*_t**3*b**3*c**3 - 8*_t*a**2*c**2 + 16*_t*a*b**2*c
- 4*_t*b**4)/(a**2*c - a*b**2)))) + x**2/(2*c)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2041 vs. 2(150) = 300.
time = 6.30, size = 2041, normalized size = 10.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/8*(2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3 + (sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c -
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b
^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 -
 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*x^4*abs(c) + (2*b^4*c^3 -
 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 2*(b^2 -
4*a*c)*b^2*c^3)*x^4 + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^3*c + 2*a*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 16*a^2*b^2*c^2 - 4*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 32*a^3*c^3 - 2*(b^2 - 4*a*c)*a*b^
2*c + 8*(b^2 - 4*a*c)*a^2*c^2)*abs(c))*arctan(2*sqrt(1/2)*x^2/sqrt((b*c + s
qrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*
a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*c^2) + 1/8*(2*a*b^3*c^3 - 8*
a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3
*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 -
 4*a*c)*a*b*c^3 - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 -
 4*a*c)*a*b*c^2)*x^4*abs(c) + (2*b^4*c^3 - 8*a*b^2*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 2*(b^2 - 4*a*c)*b^2*c^3)*x^4 - (sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*a*b^
4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^2 + 16*a^2*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^3 - 32*a^3*c^3 + 2*(b^2 - 4*a*c)*a*b^2*c - 8*(b^2 - 4*a*c)*a^2*c^2)*ab
```

```
s(c))*arctan(2*sqrt(1/2)*x^2/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a
*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3
 - 4*a^2*c^4)*c^2)
```

Mupad [B]
time = 3.07, size = 2500, normalized size = 13.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan(((((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*
c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3
*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^
5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c
- b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(64*a^
4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^
2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*
(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16
*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5
*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c +
 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^
6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^
4*c^7 + 4096*a^3*b^2*c^8)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^
2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3
 - 8*a*b^2*c^4)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*
a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2
*c^4)))^(1/2) + (4*x^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^
2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(
-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4
*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2
*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^
3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*(a*b^
8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16
*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 -
((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)))
```

```
/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 + b^2*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(1
6*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(64*a^4*b*c^5 + 16*a^2*
b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4
*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2
*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(a^2*b^6 - 2*a^5*c^3 - 5*a^3*b^4*c + 6*a^
4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a
*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*
c^4)))^(1/2) + (((16*(a*b^8 + 4*a^5*c^4 - 8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16
*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 - 256*a^4*b*c^6 - 256*a^2*b^5*c^4
+ 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6 - 2048*a^2*b^4*c^7 + 4096*a^3*b^2
*c^8)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*
(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a
*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x
^2*(64*a^4*b*c^5 + 16*a^2*b^5*c^3 - 80*a^3*b^3*c^4))/c^2)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))*(-(b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)
)/(32*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (4*x^2*(a^2*b^6 - 2*a^
5*c^3 - 5*a^3*b^4*c + 6*a^4*b^2*c^2))/c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^
2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i + atan(((((16*(a*b^8 + 4*a^5*c^4 -
8*a^2*b^6*c + 20*a^3*b^4*c^2 - 16*a^4*b^2*c^3))/c^2 + (((16*(32*a*b^7*c^3 -
 256*a^4*b*c^6 - 256*a^2*b^5*c^4 + 576*a^3*b^3*c^5))/c^2 - ((256*a*b^6*c^6
- 2048*a^2*b^4*c^7 + 4096*a^3*b^2*c^8)*(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*c^2*(16*a^2*
c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(16*a^2*c^5 + b^4*
c^3 - 8*a*b^2*c^4)))^(1/2) - (4*x^2*(64*a^4*b*c...
```

3.312 $\int \frac{x^7}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

```
[Out] 1/8*ln(c*x^8+b*x^4+a)/c+1/4*b*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/c/(-4
*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1371, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^4+cx^8\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^4 + c*x^8]/(8*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{a+bx^4+cx^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8c} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8c}\\
&= \frac{\log\left(a+bx^4+cx^8\right)}{8c} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4c}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx^4+cx^8\right)}{8c}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.98

$$\frac{-\frac{2b\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log\left(a+bx^4+cx^8\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^4 + c*x^8])/(8*c)
```

Maple [A]
time = 0.03, size = 60, normalized size = 0.95

method	result

default	$\frac{\ln(c\,x^8+b\,x^4+a)}{8c}-\frac{b\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{4c\sqrt{4ac-b^2}}$
risch	$\frac{\ln\left(\left(-4abc+b^3+\sqrt{-b^2\,(4ac-b^2)}\,b\right)x^4+2\sqrt{-b^2\,(4ac-b^2)}\,a\right)a}{8ac-2b^2}-\frac{\ln\left(\left(-4abc+b^3+\sqrt{-b^2\,(4ac-b^2)}\,b\right)x^4+\right.}{8c(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(c*x^8+b*x^4+a)/c-1/4*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c
-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.36, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)+(b^2-4\,ac)\log\left(cx^8+bx^4+a\right)}{8\,(b^2c-4\,ac^2)},\frac{2\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(b^2-4\,ac)\log\left(cx^8+bx^4+a\right)}{8\,(b^2c-4\,ac^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x
^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) + (b^2 - 4*a*c)*log(c*x^8 +
 b*x^4 + a))/(b^2*c - 4*a*c^2), 1/8*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^8 + b*x^4
+ a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.
time = 1.32, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{8c\,(4ac-b^2)}+\frac{1}{8c}\right)\log\left(x^4+\frac{-16ac\left(-\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)+2a+4b^2\left(-\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{8c\,(4ac-b^2)}+\frac{1}{8c}\right)\log\left(x^4+\frac{-16ac\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)+2a+4b^2\left(\frac{b\sqrt{-4ac+b^2}}{8c(4ac-b^2)}+\frac{1}{8c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*c
*(-b*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(-b
*sqrt(-4*a*c + b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b) + (b*sqrt(-4*a*c +
 b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c))*log(x**4 + (-16*a*c*(b*sqrt(-4*a*c +
 b**2)/(8*c*(4*a*c - b**2)) + 1/(8*c)) + 2*a + 4*b**2*(b*sqrt(-4*a*c + b**2
)/(8*c*(4*a*c - b**2)) + 1/(8*c)))/b)
```

Giac [A]
time = 8.36, size = 59, normalized size = 0.94

$$-\frac{b\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,c}+\frac{\log\left(cx^8+bx^4+a\right)}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
8*log(c*x^8 + b*x^4 + a)/c
```

Mupad [B]
time = 2.61, size = 2654, normalized size = 42.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x^4 + c*x^8),x)
```

```
[Out] (log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - (b*at
an((8*x^4*(((a*c - b^2)*(((((16*a*c - 4*b^2)*((b*(448*b^3*c^3 - (256*b^3*c^
4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32
*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2
*(64*a*c^2 - 16*b^2*c)) - (b*(144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*(1
6*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*
b^2*c))))/(8*c*(4*a*c - b^2)^(1/2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^
2*c)) - (b*((b*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2
- 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*a*c - 4*b^2))/((6
4*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) - (4*b
^5*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(8*c*(4*a*
c - b^2)^(1/2)) + (b*(20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c
^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2
- 16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c))))/(8*c*(4*a*c - b
```

^2)^(1/2)) + (b^6*c*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)*(4*a*c - b^2
)^(3/2))))/(8*a^3*c^2) + ((b^3 - 3*a*b*c)*(b^7/(8*(4*a*c - b^2)^2) + b^3 -
((20*b^3*c - ((144*b^3*c^2 - ((448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4*b^2))
/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*(16*a*
c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*
b^2*c)) + ((16*a*c - 4*b^2)*((b*((b*(448*b^3*c^3 - (256*b^3*c^4*(16*a*c - 4
*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (32*b^4*c^3*(16*
a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b
^2)^(1/2)) - (4*b^5*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b
^2))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((16*a*c - 4*b^2)*((b*(448*b^3*c^3 -
 (256*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^
(1/2)) - (32*b^4*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)
^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) - (b*(144*b^3*c^2 - ((448*b^3*c^3 - (25
6*b^3*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64
*a*c^2 - 16*b^2*c))))/(8*c*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2))
))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^2)/b^4 + ((4*a*c - b^2)^(
3/2)*(b^3 - 3*a*b*c)*(a*b^2 + (((b*((b*(768*a*b^2*c^3 - (512*a*b^2*c^4*(16*
a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (64*a*b^3
c^3(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(
4*a*c - b^2)^(1/2)) - (8*a*b^4*c^2*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)
*(4*a*c - b^2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) + (a*b^6)/(4*(
4*a*c - b^2)^2) - ((16*a*c - 4*b^2)*(((16*a*c - 4*b^2)*(((768*a*b^2*c^3 - (
512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2
*(64*a*c^2 - 16*b^2*c)) - 208*a*b^2*c^2))/(2*(64*a*c^2 - 16*b^2*c)) + 24*a*
b^2*c))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((16*a*c - 4*b^2)*((b*(768*a*b^2*c^
3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c -
b^2)^(1/2)) - (64*a*b^3*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c
 - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((768*a*b^2*c^3 - (512*a*b
^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*
c^2 - 16*b^2*c)) - 208*a*b^2*c^2))/(8*c*(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c
 - b^2)^(1/2))))/(a^3*b^4*c^2) + ((a*c - b^2)*(4*a*c - b^2)^2*(((((16*a*c -
4*b^2)*((b*(768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16
*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) - (64*a*b^3*c^3*(16*a*c - 4*b^2))/((64*
a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) + (b*(((
768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c))*(16
*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - 208*a*b^2*c^2))/(8*c*(4*a*c - b^
2)^(1/2)))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - (b*((b*((b*(768*a*
b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*
a*c - b^2)^(1/2)) - (64*a*b^3*c^3*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*
(4*a*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) - (8*a*b^4*c^2*(16*a*c - 4
*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(8*c*(4*a*c - b^2)^(1/2)) +
(b*(((16*a*c - 4*b^2)*(((768*a*b^2*c^3 - (512*a*b^2*c^4*(16*a*c - 4*b^2))/(
64*a*c^2 - 16*b^2*c))*(16*a*c - 4*b^2))/(2*(64*a*c^2 - 16*b^2*c)) - 208*a*b
^2*c^2))/(2*(64*a*c^2 - 16*b^2*c)) + 24*a*b^2*c))/(8*c*(4*a*c - b^2)^(1/2))
 + (a*b^5*c*(16*a*c - 4*b^2))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(3/2)))

```
/(a^3*b^4*c^2)))/(4*c*(4*a*c - b^2)^(1/2))
```

3.313 $\int \frac{x^5}{a+bx^4+cx^8} dx$

Optimal. Leaf size=159

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^
2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/4*arctan(x^2*2^(1/2)*c
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c
^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1373, 1144, 211}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] -1/2*(Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) + (Sqrt[b + Sqrt[b^2
- 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqr
t[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5}{a+bx^4+cx^8}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx, x, x^2\right)$$

$$= \frac{1}{4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)+\frac{1}{4}\left(1+\frac{b}{\sqrt{b^2-}}\right.$$

$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\right.}{2\sqrt{2}\sqrt{c}\sqrt{b^2-}}$$

Mathematica [A]
time = 0.06, size = 171, normalized size = 1.08

$$\frac{\left(-b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 -
4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(
Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[
b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

Maple [A]
time = 0.04, size = 153, normalized size = 0.96

method	result
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(16a^2c^3-8a\,b^2c^2+b^4c\right)\_Z^4+\left(-4abc+b^3\right)\_Z^2+a\right)}\_R\ln\left(\left(\left(-4c^2a+b^2c\right)\_R^2+b\right)x^2+\left(4ab\,c^2-b^3c\right)\_R^3+\left(2ac-b^2\right)\right.\right.}{4}$

default	$2c\left(-\frac{\left(-b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^{2}\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8c\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\left(b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\arctan\left(\right.}{8c\sqrt{-4ac+b^{2}}\,\sqrt{(b+}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c*(-1/8/c*(-b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))+1/8*(b+(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(119) = 238.
time = 0.37, size = 567, normalized size = 3.57

$$\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}\log\left(x^{2}+\frac{\sqrt{\frac{1}{2}}(b^{2}c-4ac^{2})\sqrt{-\frac{b+\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}}{\sqrt{b^{2}c^{2}-4ac^{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}\log\left(x^{2}-\frac{\sqrt{\frac{1}{2}}(b^{2}c-4ac^{2})\sqrt{-\frac{b+\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}}{\sqrt{b^{2}c^{2}-4ac^{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}\log\left(x^{2}+\frac{\sqrt{\frac{1}{2}}(b^{2}c-4ac^{2})\sqrt{-\frac{b-\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}}{\sqrt{b^{2}c^{2}-4ac^{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}\log\left(x^{2}-\frac{\sqrt{\frac{1}{2}}(b^{2}c-4ac^{2})\sqrt{-\frac{b-\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}}{\sqrt{b^{2}c^{2}-4ac^{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) -
1/4*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) -
1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(x^2 + sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3)) +
1/4*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
```

```
- 4*a*c^2))*log(x^2 - sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3))
```

Sympy [A]
time = 1.50, size = 76, normalized size = 0.48

$$\operatorname{RootSum}\left(t^4\cdot\left(4096a^2c^3-2048ab^2c^2+256b^4c\right)+t^2\left(-64abc+16b^3\right)+a,\left(t\mapsto t\log\left(512t^3ac^2-128t^3b^2c-4tb+x^2\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**2*c**3 - 2048*a*b**2*c**2 + 256*b**4*c) + _t**2*(-64
*a*b*c + 16*b**3) + a, Lambda(_t, _t*log(512*_t**3*a*c**2 - 128*_t**3*b**2*
c - 4*_t*b + x**2)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(119) = 238.
time = 5.98, size = 1034, normalized size = 6.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*
b*c^2)*x^4*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4
- 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^
3)*abs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^
2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(
```

```
b^2 - 4*a*c)*b*c^2)*x^4*arctan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))
/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c
^2 - 4*a^2*c^3)*abs(c))
```

Mupad [B]
time = 2.81, size = 1220, normalized size = 7.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + b^3*x
^2*1i - a*b*c*x^2*4i)/(8*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)
 + 128*b^5*c*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3
 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 64*a^2*c^2*((
(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32
*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2) - 1024*a*b^3*c^2*(((b^6 - 64*a
^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 51
2*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 2048*a^2*b*c^3*(((b^6 - 64*a^3*c^3 + 48
*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 -
 256*a*b^2*c^2))^(3/2) - 48*a*b^2*c*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2)
)^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)*2i - atan((x^2*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - b^3*x^2*1i + a*b*c
*x^2*4i)/(8*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2) + 128*b^5*c
*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)
/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2) + 64*a^2*c^2*(-(b^3 + (b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 5
12*a^2*c^3 - 256*a*b^2*c^2))^(1/2) - 48*a*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 2
56*a*b^2*c^2))^(1/2) - 1024*a*b^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b
^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c
^2))^(3/2) + 2048*a^2*b*c^3*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 1
2*a*b^4*c)^(1/2) - 4*a*b*c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(3/2)
))*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*
c)/(32*b^4*c + 512*a^2*c^3 - 256*a*b^2*c^2))^(1/2)*2i
```

3.314 $\int \frac{x^3}{a+bx^4+cx^8} dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1366, 632, 212}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^4 + c*x^8),x]
```

```
[Out] -1/2*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^3}{a+bx^4+cx^8}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.11

$$\frac{\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{2\sqrt{-b^2+4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^4 + c*x^8),x]
[Out] ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]]/(2*Sqrt[-b^2 + 4*a*c])
```

Maple [A]
time = 0.02, size = 37, normalized size = 0.97

method	result	size
default	$\frac{\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}$	37
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^2}\right)x^4-2a\right)}{4\sqrt{-4ac+b^2}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^2}\right)x^4+2a\right)}{4\sqrt{-4ac+b^2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
[Out] 1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 129, normalized size = 3.39

$$\left[\frac{\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac-(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{4\,\sqrt{b^2-4\,ac}}, -\frac{\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{2\,(b^2-4\,ac)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c - (2*c*x^4 + b)*sqrt(b^2 - 4*
a*c))/(c*x^8 + b*x^4 + a))/sqrt(b^2 - 4*a*c), -1/2*sqrt(-b^2 + 4*a*c)*arcta
n(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(36) = 72.
time = 0.45, size = 131, normalized size = 3.45

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^4+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**4 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4 + sqrt(-1/(4*a*c - b**2))*log(x**4 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/4
```

Giac [A]
time = 7.81, size = 36, normalized size = 0.95

$$\frac{\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

[Out] 1/2*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)

Mupad [B]
time = 1.37, size = 260, normalized size = 6.84

$$-\frac{\operatorname{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^2\,\left(\frac{\left(\frac{4\,a\,c^4}{4\,a\,c-b^2}-\frac{4\,a\,b^2\,c^4}{{\left(4\,a\,c-b^2\right)}^2}\right)\,\left(b^3-3\,a\,b\,c\right)}{8\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}}-x^4\,\left(\frac{\left(\frac{2\,c^4}{\sqrt{4\,a\,c-b^2}}-\frac{6\,b^2\,c^4}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(a\,c-b^2\right)}{8\,a^3\,c^2}-\frac{\left(b^3-3\,a\,b\,c\right)\,\left(\frac{6\,b\,c^4}{4\,a\,c-b^2}-\frac{2\,b^3\,c^4}{{\left(4\,a\,c-b^2\right)}^2}\right)}{8\,a^3\,c^2\,\sqrt{4\,a\,c-b^2}}\right)+\frac{b\,c^2\,\left(a\,c-b^2\right)}{a^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)}{2\,c^4}\right)}{2\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b*x^4 + c*x^8),x)

[Out] -atan(((4*a*c - b^2)^2*((((4*a*c^4)/(4*a*c - b^2) - (4*a*b^2*c^4)/(4*a*c - b^2)^2)*(b^3 - 3*a*b*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)) - x^4*((((2*c^4)/(4*a*c - b^2)^(1/2) - (6*b^2*c^4)/(4*a*c - b^2)^(3/2))*(a*c - b^2))/(8*a^3*c^2) - ((b^3 - 3*a*b*c)*((6*b*c^4)/(4*a*c - b^2) - (2*b^3*c^4)/(4*a*c - b^2)^2))/(8*a^3*c^2*(4*a*c - b^2)^(1/2))) + (b*c^2*(a*c - b^2))/(a^2*(4*a*c - b^2)^(3/2))))/(2*c^4))/(2*(4*a*c - b^2)^(1/2))

3.315 $\int \frac{x}{a+bx^4+cx^8} dx$

Optimal. Leaf size=154

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*2^(1/2
)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x^2*2^(1/2)*c^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1373, 1107, 211}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] (Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*ArcTan[(Sqrt[2]
*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx^4+cx^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx^2+cx^4}\,dx, x, x^2\right)\\ &= \frac{c\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{2\sqrt{b^2-4ac}} - \frac{c\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{2\sqrt{b^2-4ac}}\\ &= \frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 133, normalized size = 0.86

$$\frac{\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^4 + c*x^8),x]
```

```
[Out] (Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/Sqrt[b
- Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(Sqrt[2]*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.04, size = 121, normalized size = 0.79

method	result

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^3c^2-8a^2b^2c+b^4a\right)\_Z^4+\left(-4abc+b^3\right)\_Z^2+c\right)}\_R\ln\left(\left(\left(4abc-b^3\right)\_R^2-c\right)x^2+\left(4a^2bc-a\,b^3\right)\_R^3-2ac\_R\right)\right)}{4}$
default	$2c\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\arctan\left(\frac{c\,x^2\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c*(-1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/4/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(119) = 238.
time = 0.41, size = 619, normalized size = 4.02

$$-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(cx^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(cx^2-\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)-\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(cx^2+\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{4}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(cx^2-\frac{1}{2}\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(c*x^2 + 1/2*sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/s
qrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c
))/(a*b^2 - 4*a^2*c))) + 1/4*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^
2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c
 - (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c
```

```
)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/4*sqrt(1/2)*sqrt(-(b - (
a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(c*x^2 + 1/
2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqr
t(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/
4*sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 -
4*a^2*c))*log(c*x^2 - 1/2*sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt
(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/
(a*b^2 - 4*a^2*c)))
```

Sympy [**A**]
time = 2.51, size = 88, normalized size = 0.57

$$\operatorname{RootSum}\left(t^4\cdot\left(4096a^3c^2-2048a^2b^2c+256ab^4\right)+t^2\left(-64abc+16b^3\right)+c,\left(t\mapsto t\log\left(x^2+\frac{256t^3a^2bc-64t^3ab^3+8tac-4tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**3*c**2 - 2048*a**2*b**2*c + 256*a*b**4) + _t**2*(-64
*a*b*c + 16*b**3) + c, Lambda(_t, _t*log(x**2 + (256*_t**3*a**2*b*c - 64*_t
**3*a*b**3 + 8*_t*a*c - 4*_t*b**2)/c)))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1030 vs. 2(119) = 238.
time = 7.42, size = 1030, normalized size = 6.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*
a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*a
bs(c)) + 1/8*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 +
```

```
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2
- 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a
^2*c^3)*abs(c))
```

Mupad [B]
time = 2.27, size = 1105, normalized size = 7.18

$$\mathrm{atan}\left(\frac{b^4x^2\,1\mathrm{i}+b\,x^2\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}\,1\mathrm{i}+a^2c^2x^2\,8\mathrm{i}-a\,b^2c\,x^2\,6\mathrm{i}}{128a^2b^5\left(-\frac{b^3+\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}-4abc}{512a^3c^2-256a^2b^2c+32ab^4}\right)^{3/2}-64a^3c^2\sqrt{-\frac{b^3+\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}-4abc}{512a^3c^2-256a^2b^2c+32ab^4}}+16a^2b^2c\sqrt{-\frac{b^3+\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}-4abc}{512a^3c^2-256a^2b^2c+32ab^4}}-1024a^3b^3c\left(-\frac{b^3+\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}-4abc}{512a^3c^2-256a^2b^2c+32ab^4}\right)^{3/2}+2048a^4bc^2\left(-\frac{b^3+\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}-4abc}{512a^3c^2-256a^2b^2c+32ab^4}\right)^{3/2}}\right)\sqrt{-\frac{b^3+\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}-4abc}{512a^3c^2-256a^2b^2c+32ab^4}}\,2\mathrm{i}+\mathrm{atan}\left(\frac{b^4x^2\,1\mathrm{i}-b\,x^2\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}\,1\mathrm{i}+a^2c^2x^2\,8\mathrm{i}-a\,b^2c\,x^2\,6\mathrm{i}}{128a^2b^5\left(\frac{\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}-b^3+4abc}{512a^3c^2-256a^2b^2c+32ab^4}\right)^{3/2}-64a^3c^2\sqrt{\frac{\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}-b^3+4abc}{512a^3c^2-256a^2b^2c+32ab^4}}+16a^2b^2c\sqrt{\frac{\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}-b^3+4abc}{512a^3c^2-256a^2b^2c+32ab^4}}-1024a^3b^3c\left(\frac{\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}-b^3+4abc}{512a^3c^2-256a^2b^2c+32ab^4}\right)^{3/2}+2048a^4bc^2\left(\frac{\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}-b^3+4abc}{512a^3c^2-256a^2b^2c+32ab^4}\right)^{3/2}}\right)\sqrt{\frac{\sqrt{-64a^3c^3+48a^2b^2c^2-12ab^4c+b^6}-b^3+4abc}{512a^3c^2-256a^2b^2c+32ab^4}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((b^4*x^2*1i + b*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(
1/2)*1i + a^2*c^2*x^2*8i - a*b^2*c*x^2*6i)/(128*a^2*b^5*(-(b^3 + (b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3
*c^2 - 256*a^2*b^2*c))^(3/2) - 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2
*b^2*c))^(1/2) + 16*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/
2) - 1024*a^3*b^3*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2) + 2048*
a^4*b*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2)))*(-(b^3 + (b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(32*a*b^4 + 512
*a^3*c^2 - 256*a^2*b^2*c))^(1/2)*2i + atan((b^4*x^2*1i - b*x^2*(b^6 - 64*a^
3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x^2*8i - a*b^2*c*x^
2*6i)/(128*a^2*b^5*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)
 - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(3/2) - 64*a^3*
c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*
c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) + 16*a^2*b^2*c*(((b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4
+ 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2) - 1024*a^3*b^3*c*(((b^6 - 64*a^3*c^3
+ 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c
^2 - 256*a^2*b^2*c))^(3/2) + 2048*a^4*b*c^2*(((b^6 - 64*a^3*c^3 + 48*a^2*b^
2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^
2*b^2*c))^(3/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)
- b^3 + 4*a*b*c)/(32*a*b^4 + 512*a^3*c^2 - 256*a^2*b^2*c))^(1/2)*2i
```

3.316 $\int \frac{1}{x(a+bx^4+cx^8)} \, dx$

Optimal. Leaf size=69

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log\left(a+bx^4+cx^8\right)}{8a}$$

```
[Out] ln(x)/a-1/8*ln(c*x^8+b*x^4+a)/a+1/4*b*arctanh((2*c*x^4+b)/(-4*a*c+b^2)^(1/2
))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1371, 719, 29, 648, 632, 212, 642}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}} - \frac{\log\left(a+bx^4+cx^8\right)}{8a} + \frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^4 + c*x^8]/(8*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)}{4a}+\frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^4\right)}{4a}\\
&= \frac{\log(x)}{a}-\frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8a}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}+\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^4+cx^8\right)}{8a}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.02, size = 66, normalized size = 0.96

$$\frac{\log(x)}{a}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b+2c\#1^4}\&\right]}{4a}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*x^4 + c*x^8)),x]

[Out] Log[x]/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]*#1^4)/(b + 2*c*#1^4) &]/(4*a)

Maple [A]
time = 0.04, size = 66, normalized size = 0.96

method	result	size
default	$-\frac{\frac{\ln\left(c\,x^8+b\,x^4+a\right)}{4}+\frac{b\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}}}{2a}+\frac{\ln(x)}{a}$	66
risch	$\frac{\ln(x)}{a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^2c-a\,b^2\right)\_Z^2+\left(4ac-b^2\right)\_Z+c\right)}\_R\ln\left(\left(\left(18ac-5b^2\right)\_R+9c\right)x^4-ab\_R+4b\right)\right)}{4}$	77

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)

[Out] -1/2/a*(1/4*ln(c*x^8+b*x^4+a)+1/2*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2)))+ln(x)/a

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for more deta

Fricas [A]
time = 0.39, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)-(b^2-4\,ac)\log\left(cx^8+bx^4+a\right)+8\,(b^2-4\,ac)\log\left(x\right)}{8\,(ab^2-4\,a^2c)},\ \frac{2\,\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^2-4\,ac)\log\left(cx^8+bx^4+a\right)+8\,(b^2-4\,ac)\log\left(x\right)}{8\,(ab^2-4\,a^2c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x
^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^2 - 4*a*c)*log(c*x^8 +
 b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/8*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^8 + b*x^4 + a) + 8*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.
time = 98.46, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)\log\left(x^4+\frac{-16a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)+4ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)\log\left(x^4+\frac{-16a^2c\left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)+4ab^2\left(\frac{b\sqrt{-4ac+b^2}}{8a(4ac-b^2)}-\frac{1}{8a}\right)-2ac+b^2}{bc}\right)+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**8+b*x**4+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-16*a**
2*c*(-b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(-b*
sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c))
+ (b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a))*log(x**4 + (-16*a*
*2*c*(b*sqrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) + 4*a*b**2*(b*s
qrt(-4*a*c + b**2)/(8*a*(4*a*c - b**2)) - 1/(8*a)) - 2*a*c + b**2)/(b*c)) +
 log(x)/a
```

Giac [A]
time = 7.26, size = 68, normalized size = 0.99

$$-\frac{b\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,a}-\frac{\log\left(cx^8+bx^4+a\right)}{8\,a}+\frac{\log\left(x^4\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
8*log(c*x^8 + b*x^4 + a)/a + 1/4*log(x^4)/a
```

Mupad [B]
time = 2.18, size = 1690, normalized size = 24.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4 + c*x^8)),x)

[Out] log(x)/a + (log(a + b*x^4 + c*x^8)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*
c)) - (b*atan((4*(4*a*c - b^2)^2*(5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 34*
a*b^4*c)*((b^9*c^4)/(128*a^4*(4*a*c - b^2)^(5/2)) + (2*b^5*c^4*(16*a*c - 4*
b^2)^4)/((16*a*b^2 - 64*a^2*c)^4*(4*a*c - b^2)^(1/2)) - (b*(16*a*c - 4*b^2)
^3*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/
(16*a*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)^(1/2)) + (b^3*(16*a*c - 4*b^2)*
(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(25
6*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2)) - (3*b^7*c^4*(16*a*c - 4*b
^2)^2)/(4*a^2*(16*a*b^2 - 64*a^2*c)^2*(4*a*c - b^2)^(3/2))))/(b^4*c^8*(81*a
*c - 20*b^2)) + (128*a^5*x^4*(((5*b^5 + 23*a^2*b*c^2 - 24*a*b^3*c)*(((576*b
^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 -
64*a^2*c)))*(16*a*c - 4*b^2)^4)/(16*(16*a*b^2 - 64*a^2*c)^4) + (b^4*(576*b^
3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 6
4*a^2*c))))/(4096*a^4*(4*a*c - b^2)^2) + (b^2*(1280*b^5*c^4 - 4608*a*b^3*c^
5)*(16*a*c - 4*b^2)^3)/(128*a^2*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)) - (3
*b^2*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(
16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2)^2)/(128*a^2*(16*a*b^2 - 64*a^2*c)^2
*(4*a*c - b^2)) - (b^4*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2
048*a^4*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^2)))/(32*a^5*c^4*(81*a*c - 20*b
^2)) + ((5*b^6 - 18*a^3*c^3 + 61*a^2*b^2*c^2 - 34*a*b^4*c)*((b^5*(1280*b^5*
c^4 - 4608*a*b^3*c^5))/(32768*a^5*(4*a*c - b^2)^(5/2)) - (3*b^3*(1280*b^5*c
^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2)^2)/(1024*a^3*(16*a*b^2 - 64*a^2*c)^2*
(4*a*c - b^2)^(3/2)) + (b*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2)^
4)/(128*a*(16*a*b^2 - 64*a^2*c)^4*(4*a*c - b^2)^(1/2)) - (b*(576*b^3*c^5 -
((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*(16*a*b^2 - 64*a^2*c)
))*(16*a*c - 4*b^2)^3)/(16*a*(16*a*b^2 - 64*a^2*c)^3*(4*a*c - b^2)^(1/2)) +
 (b^3*(576*b^3*c^5 - ((1280*b^5*c^4 - 4608*a*b^3*c^5)*(16*a*c - 4*b^2))/(2*
(16*a*b^2 - 64*a^2*c)))*(16*a*c - 4*b^2))/(256*a^3*(16*a*b^2 - 64*a^2*c)*(4
*a*c - b^2)^(3/2))))/(32*a^5*c^4*(4*a*c - b^2)^(1/2)*(81*a*c - 20*b^2)))*(4
*a*c - b^2)^(5/2))/(b^4*c^4) + (4*(4*a*c - b^2)^(5/2)*(5*b^5 + 23*a^2*b*c^2
 - 24*a*b^3*c)*(((16*a*c - 4*b^2)^4*(256*b^4*c^4 - (128*a*b^4*c^4*(16*a*c -
 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(16*(16*a*b^2 - 64*a^2*c)^4) + (b^4*(256*b
^4*c^4 - (128*a*b^4*c^4*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(4096*a^4
*(4*a*c - b^2)^2) - (b^8*c^4*(16*a*c - 4*b^2))/(8*a^3*(16*a*b^2 - 64*a^2*c)
*(4*a*c - b^2)^2) + (2*b^6*c^4*(16*a*c - 4*b^2)^3)/(a*(16*a*b^2 - 64*a^2*c)
^3*(4*a*c - b^2)) - (3*b^2*(16*a*c - 4*b^2)^2*(256*b^4*c^4 - (128*a*b^4*c^4
*(16*a*c - 4*b^2))/(16*a*b^2 - 64*a^2*c)))/(128*a^2*(16*a*b^2 - 64*a^2*c)^2
*(4*a*c - b^2))))/(b^4*c^8*(81*a*c - 20*b^2))))/(4*a*(4*a*c - b^2)^(1/2))
```

3.317 $\int \frac{1}{x^3(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=184

$$-\frac{1}{2ax^2}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/2/a/x^2-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(
1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*ar
ctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4*a*c
+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1373, 1137, 1180, 211}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/2*1/(a*x^2) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^4+cx^8)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx, x, x^2\right)$$
$$= -\frac{1}{2ax^2} + \frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)}{4a} - \frac{\left(c\left(\right.\right.}{}$$
$$= -\frac{1}{2ax^2} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(1-\frac{}{\sqrt{b^2}}\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 75, normalized size = 0.41

$$-\frac{1}{2ax^2} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b\#1^2+2c\#1^6}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/2*1/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x -
 #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

Maple [A]
time = 0.05, size = 159, normalized size = 0.86

method	result
default	$-\frac{1}{2a\,x^{2}}-\frac{2c\left(-\frac{\left(b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^{2}\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\left(-b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\operatorname{arc}}{8\sqrt{-4ac+b^{2}}\,\sqrt{}}\right.}{a}$
risch	$-\frac{1}{2a\,x^{2}}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^{5}c^{2}-8a^{4}b^{2}c+a^{3}b^{4}\right)\_Z^{4}+\left(12a^{2}b\,c^{2}-7a\,b^{3}c+b^{5}\right)\_Z^{2}+c^{3}\right)}\_R\ln\left(\left(\left(-72a^{5}c^{2}+38a^{4}b^{2}c-5a^{3}b^{4}\right)\_R^{4}+\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/x^2-2/a*c*(-1/8*(b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))+1/8*(-b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2/(a*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(141) = 282$.
time = 0.41, size = 1134, normalized size = 6.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2
```

```
*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4
 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a
^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt
(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^
6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 - 1/2*sq
rt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b
*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*
c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))*log(-(b^2*c^2 - a*c^3)*x^2 + 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c +
 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
 - sqrt(1/2)*a*x^2*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-(b^2*c^
2 - a*c^3)*x^2 - 1/2*sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 -
6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*
c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x^2)
```

Sympy [A]
time = 146.83, size = 153, normalized size = 0.83

$$\operatorname{RootSum}\left(t^4\cdot\left(4096a^5c^2-2048a^4b^2c+256a^3b^4\right)+t^2\cdot\left(192a^2bc^2-112ab^3c+16b^5\right)+c^3,\left(t\mapsto t\log\left(x^2+\frac{-512t^3a^5c^2+384t^3a^4b^2c-64t^3a^3b^4-20ta^2bc^2+20tab^3c-4tb^5}{ac^3-b^2c^2}\right)\right)\right)-\frac{1}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**4*(4096*a**5*c**2 - 2048*a**4*b**2*c + 256*a**3*b**4) + _t**2*(
192*a**2*b*c**2 - 112*a*b**3*c + 16*b**5) + c**3, Lambda(_t, _t*log(x**2 +
(-512*_t**3*a**5*c**2 + 384*_t**3*a**4*b**2*c - 64*_t**3*a**3*b**4 - 20*_t*
a**2*b*c**2 + 20*_t*a*b**3*c - 4*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(2*a*
x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. 2(141) = 282.
time = 5.64, size = 2055, normalized size = 11.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*a*b^4*c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 16*a*b^2*c^
3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 -
 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*c^3)*x^4*abs(a) + (2*a*b^3*c^3 - 8*a^2*
b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a
*c)*a*b*c^3)*x^4 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2
- 4*a*c)*a*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*x^2/sqrt((a*b + sqrt(a^2*b^2 -
 4*a^3*c))/(a*c)))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a
^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*abs(a)*abs(c)) + 1/8*(2*a*b^4*c^2 - 8*a
^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*(b^2 -
4*a*c)*a*b^2*c^2 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^3*c^2 + 2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*c^4 + 32*a^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^
2 - 4*a*c)*a*c^3)*x^4*abs(a) + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*x^4 - (sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5
*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*abs(a
```

```
))*arctan(2*sqrt(1/2)*x^2/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^
2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2
- 4*a^3*c^3)*abs(a)*abs(c)) - 1/2/(a*x^2)
```

Mupad [B]
time = 2.42, size = 2500, normalized size = 13.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4 + c*x^8)),x)
```

```
[Out] - atan((((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*
a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2
*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 245
76*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*
a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b
^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) + x
^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-
(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a
*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a
^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)) - ((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 +
65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11
*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5
*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*c^5 - 3072*a^1
1*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4*c^6 - 896*a^1
0*b^2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*
c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
)^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))*1i)/(32*(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))/(((64*a^10*c^8 + ((-(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(3
2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^4 - 32768*a
```

^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(16384*a^13*b*c^7 - 1024*a^10*b^7*c
^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^
3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 512*a^10*b^5*
c^5 - 3072*a^11*b^3*c^6) + x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 + 448*a^9*b^4
*c^6 - 896*a^10*b^2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
 8*a^4*b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 + b^2*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2)
))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + ((64*a^10*c^8 + ((-(b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5 + b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3
)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6*c^
4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(16384*a^13*b*c^7 - 1024
*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*(-(b^5 + b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/
2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 4096*a^12*b*c^7 + 51
2*a^10*b^5*c^5 - 3072*a^11*b^3*c^6) - x^2*(512*a^11*c^8 - 64*a^8*b^6*c^5 +
448*a^9*b^4*c^6 - 896*a^10*b^2*c^7))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 16*a^8*b^4*c^6 - 64*a^9*b^2*c^7)*(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2)))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))))*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - atan((((64*a^10*
c^8 + ((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a
c(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2)*(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c
*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
*(4096*a^12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(16384
*a^13*b*c^7 - 1024*a^10*b^7*c^4 + 9216*a^11*b^5*c^5 - 24576*a^12*b^3*c^6))*
(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4
*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2...

3.318 $\int \frac{1}{x^5(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{4ax^4}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx^4+cx^8)}{8a^2}$$

```
[Out] -1/4/a/x^4-b*ln(x)/a^2+1/8*b*ln(c*x^8+b*x^4+a)/a^2-1/4*(-2*a*c+b^2)*arctanh
((2*c*x^4+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1371, 723, 814, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}+\frac{b\log(a+bx^4+cx^8)}{8a^2}-\frac{b\log(x)}{a^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/4*1/(a*x^4) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(
4*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^4 + c*x^8])/(8*a
^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^4+cx^8\right)}\,dx=\frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^4\right)$$
$$=-\frac{1}{4ax^4}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx,x,x^4\right)}{4a}$$
$$=-\frac{1}{4ax^4}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx,x,x^4\right)}{4a}$$
$$=-\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,x^4\right)}{4a^2}$$
$$=-\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8a^2}+\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\right.}{8a^2}$$
$$=-\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b\right.}{4a^2}$$
$$=-\frac{1}{4ax^4}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^4+cx^8\right)}{8a^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 92, normalized size = 1.03

$$-\frac{1}{4ax^4}-\frac{b\log(x)}{a^2}+\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b^2\log(x-\#1)-ac\log(x-\#1)+bc\log(x-\#1)\#1^4}{b+2c\#1^4}\&\right]}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/4*1/(a*x^4) - (b*Log[x])/a^2 + RootSum[a + b*#1^4 + c*#1^8 & , (b^2*Log[
x - #1] - a*c*Log[x - #1] + b*c*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a^2
)
```

Maple [A]
time = 0.05, size = 84, normalized size = 0.94

method	result
default	$-\frac{-\frac{b\ln\left(c\,x^8+b\,x^4+a\right)}{4}+\frac{\left(ac-\frac{b^2}{2}\right)\arctan\left(\frac{2c\,x^4+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a^2}-\frac{1}{4a\,x^4}-\frac{b\ln(x)}{a^2}$

risch	$-\frac{1}{4a\,x^4}-\frac{b\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c-a^2b^2\right)\_Z^2+\left(-4abc+b^3\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(18a^3c-5a^2b^2\right)\_R^2-8\_Rabc+4c^2\right)x^4-a\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*(-1/4*b*ln(c*x^8+b*x^4+a)+(a*c-1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^4+b)/(4*a*c-b^2)^(1/2)))-1/4/a/x^4-b*ln(x)/a^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.47, size = 293, normalized size = 3.29

$$\left[-\frac{\left(b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}\,x^{4}\log\left(\frac{2\,c^{2}x^{8}+2\,bcx^{4}+b^{2}-2\,ac+\left(2\,cx^{4}+b\right)\sqrt{b^{2}-4\,ac}}{cx^{8}+bx^{4}+a}\right)-\left(b^{3}-4\,abc\right)x^{4}\log\left(cx^{8}+bx^{4}+a\right)+8\,\left(b^{3}-4\,abc\right)x^{4}\log\left(x\right)+2\,ab^{2}-8\,a^{2}c}{8\,\left(a^{2}b^{2}-4\,a^{3}c\right)x^{4}},\,-\frac{2\,\left(b^{2}-2\,ac\right)\sqrt{-b^{2}+4\,ac}\,x^{4}\arctan\left(-\frac{\left(2\,cx^{4}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-\left(b^{3}-4\,abc\right)x^{4}\log\left(cx^{8}+bx^{4}+a\right)+8\,\left(b^{3}-4\,abc\right)x^{4}\log\left(x\right)+2\,ab^{2}-8\,a^{2}c}{8\,\left(a^{2}b^{2}-4\,a^{3}c\right)x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^4*log((2*c^2*x^8 + 2*b*c*x^4 + b^2
 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)) - (b^3 - 4
*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4), -1/8*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*
a*c)*x^4*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4
*a*b*c)*x^4*log(c*x^8 + b*x^4 + a) + 8*(b^3 - 4*a*b*c)*x^4*log(x) + 2*a*b^2
 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 8.91, size = 94, normalized size = 1.06

$$\frac{b\log\left(cx^8+bx^4+a\right)}{8\,a^2}-\frac{b\log\left(x^4\right)}{4\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,a^2}+\frac{bx^4-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*b*log(c*x^8 + b*x^4 + a)/a^2 - 1/4*b*log(x^4)/a^2 + 1/4*(b^2 - 2*a*c)*a
rctan((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/4*(b*x
^4 - a)/(a^2*x^4)
```

Mupad [B]
time = 2.79, size = 2500, normalized size = 28.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4 + c*x^8)),x)
```

```
[Out] (atan((4*a^5*(4*a*c - b^2)^2*(5*b^7 - 23*a^3*b*c^3 + 66*a^2*b^3*c^2 - 35*a*
b^5*c)*(((4*b^3 - 16*a*b*c)*((((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5
- (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2)
)/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2
))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(
4*a*c - b^2)^(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(
4*a*c - b^2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^
(1/2)) - (b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^3)/(4*a^5*(4*a*c - b^2)^
(3/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - ((4*b^3 - 16
*a*b*c)*(((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c)*((((256*a^4*b^5*c^4 - 256
*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b
^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a
*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(
64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 + ((4*b^
3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*
b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*
a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((
16*a^3*b*c^7 - 96*a^2*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((256*a^3*b^4*c^5
- 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3
*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2
```

*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*
a^2*(4*a*c - b^2)^(1/2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((2*a*c - b^2)*(((
4*b^3 - 16*a*b*c)*((((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4
c^4(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*
a*c - b^2)^(1/2)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c
 - b^2)^(1/2)*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)
^(1/2)) - (2*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2)^2)/(a^3*(4*a*c - b^2)
*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((((4*b^3 - 16*a*
b*c)*((((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 1
6*a*b*c))/(64*a^3*c - 16*a^2*b^2))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2
)) - (16*b^4*c^4*(4*b^3 - 16*a*b*c)*(2*a*c - b^2))/(a*(4*a*c - b^2)^(1/2)*(
64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + (((256*a^3*b^4*c^5
- 96*a^4*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3
*c^5)/a^5 - (128*a*b^4*c^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2
*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)))*(2*a
*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + (((a
^2*c^8 - 16*a*b^2*c^7)/a^5 + ((4*b^3 - 16*a*b*c)*((16*a^3*b*c^7 - 96*a^2*b^
3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((256*a^3*b^4*c^5 - 96*a^4*b^2*c^6)/a^5 +
((4*b^3 - 16*a*b*c)*((256*a^4*b^5*c^4 - 256*a^5*b^3*c^5)/a^5 - (128*a*b^4*c
^4*(4*b^3 - 16*a*b*c))/(64*a^3*c - 16*a^2*b^2)))/(2*(64*a^3*c - 16*a^2*b^2)
)))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c - b^2
))/(8*a^2*(4*a*c - b^2)^(1/2)) + (b^4*c^4*(2*a*c - b^2)^5)/(128*a^9*(4*a*c
- b^2)^(5/2))))/(c^4*(a^2*c^2 - 20*b^4 + 80*a*b^2*c)*(16*a^4*c^8 + b^8*c^4
- 8*a*b^6*c^5 + 24*a^2*b^4*c^6 - 32*a^3*b^2*c^7)) - (128*a^10*x^4*(((5*b^6
- a^3*c^3 + 26*a^2*b^2*c^2 - 25*a*b^4*c)*(c^9/a^5 + ((4*b^3 - 16*a*b*c)*((2
0*b*c^8)/a^4 + ((4*b^3 - 16*a*b*c)*((72*a^3*c^8 + 124*a^2*b^2*c^7)/a^5 + ((
4*b^3 - 16*a*b*c)*((864*a^4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b
c)((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a
^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^
3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^
2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((4*b^3 - 16*a*b*c)*(((4*b^3 - 16*a*b*c
)*(((2*a*c - b^2)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*b^3 - 1
6*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 16*a^2*b
^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*c)*(2*a
c - b^2)(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^2)^(1/2
)*(64*a^3*c - 16*a^2*b^2))))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b
c)(2*a*c - b^2)^2*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(128*a^9*(4*a*c
- b^2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) + ((2*a*c - b
^2)*(((4*b^3 - 16*a*b*c)*((((448*a^4*b^4*c^5 - 3456*a^5*b^2*c^6)/a^5 + ((4*
b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(2*a^5*(64*a^3*c - 1
6*a^2*b^2)))*(2*a*c - b^2))/(8*a^2*(4*a*c - b^2)^(1/2)) + ((4*b^3 - 16*a*b*
c)*(2*a*c - b^2)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*c^5))/(16*a^7*(4*a*c - b^
2)^(1/2)*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)) - (((864*a^
4*b*c^7 + 208*a^3*b^3*c^6)/a^5 - ((4*b^3 - 16*a*b*c)*((448*a^4*b^4*c^5 - 34
56*a^5*b^2*c^6)/a^5 + ((4*b^3 - 16*a*b*c)*(1280*a^5*b^5*c^4 - 4608*a^6*b^3*

```
c^5))/(2*a^5*(64*a^3*c - 16*a^2*b^2))))/(2*(64*a^3*c - 16*a^2*b^2)))*(2*a*c
 - b^2))/(8*a^2*(4*a*c - b^2)^(1/2))))/(8*a^2*(...
```

3.319 $\int \frac{x^{10}}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=381

$$\frac{x^3}{3c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/3*x^3/c-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-
2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4
)+1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b
^2)/(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*a
rctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a
*c+b^2)^(1/2))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*arctanh(2^
(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^
(1/2))*2^(1/4)/c^(7/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A]
time = 0.42, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1381, 1524, 304, 211, 214}

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x^4 + c*x^8),x]
```

```
[Out] x^3/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*
x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2 - 4*a
*c])^(1/4)) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(
1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*
c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(7/4)*(-b + Sqrt[b
^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^{10}}{a+bx^4+cx^8}\,dx = \frac{x^3}{3c} - \frac{\int \frac{x^2(3a+3bx^4)}{a+bx^4+cx^8}\,dx}{3c}$$

$$= \frac{x^3}{3c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-}}}{2c}$$

$$= \frac{x^3}{3c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2\sqrt{2}\,c^{3/2}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)}{}$$

$$= \frac{x^3}{3c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{7/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-}}{2\,2^{3/4}c^{7/4}\sqrt[4]{-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 70, normalized size = 0.18

$$\frac{4x^3 - 3\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{a\log(x-\#1)+b\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{12c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x^4 + c*x^8),x]
```

```
[Out] (4*x^3 - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#
1^4)/(b*#1 + 2*c*#1^5) & ])/(12*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 63, normalized size = 0.17

method	result	size
default	$\frac{x^3}{3c} - \frac{\sum_{\_R=\text{RootOf}(c\_Z^8+\_Z^4b+a)} \frac{(\_R^6b+\_R^2a)\ln(x-\_R)}{2\_R^7c+\_R^3b}}{4c}$	63
risch	$\frac{x^3}{3c} + \frac{\sum_{\_R=\text{RootOf}(c\_Z^8+\_Z^4b+a)} \frac{(-\_R^6b-\_R^2a)\ln(x-\_R)}{2\_R^7c+\_R^3b}}{4c}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3/c-1/4/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z
^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3/c - integrate((b*x^6 + a*x^2)/(c*x^8 + b*x^4 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6646 vs. 2(299) = 598.
time = 2.56, size = 6646, normalized size = 17.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^3 + 12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*
c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6
*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7
 - 8*a*b^2*c^8 + 16*a^2*c^9)))*arctan(-1/2*((a^5*b^12*c^7 - 15*a^6*b^10*c^8
 + 88*a^7*b^8*c^9 - 253*a^8*b^6*c^10 + 362*a^9*b^4*c^11 - 224*a^10*b^2*c^12
 + 32*a^11*c^13)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c
^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 +
 48*a^2*b^2*c^16 - 64*a^3*c^17)) - (a^5*b^15 - 14*a^6*b^13*c + 77*a^7*b^11*
c^2 - 210*a^8*b^9*c^3 + 294*a^9*b^7*c^4 - 196*a^10*b^5*c^5 + 49*a^11*b^3*c^
6 - 4*a^12*b*c^7)*x - (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 +
4*a^4*b*c^4 - (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*sqrt(
(b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^
3*c^17)))*sqrt((a^10*b^12 - 10*a^11*b^10*c + 37*a^12*b^8*c^2 - 62*a^13*b^6*
c^3 + 46*a^14*b^4*c^4 - 12*a^15*b^2*c^5 + a^16*c^6)*x^2 - 1/2*sqrt(1/2)*(a^
7*b^17 - 17*a^8*b^15*c + 119*a^9*b^13*c^2 - 441*a^10*b^11*c^3 + 924*a^11*b^
9*c^4 - 1078*a^12*b^7*c^5 + 637*a^13*b^5*c^6 - 151*a^14*b^3*c^7 + 12*a^15*b
*c^8 - (a^7*b^14*c^7 - 18*a^8*b^12*c^8 + 131*a^9*b^10*c^9 - 491*a^10*b^8*c^
10 + 997*a^11*b^6*c^11 - 1052*a^12*b^4*c^12 + 496*a^13*b^2*c^13 - 64*a^14*c
^14)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^
4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^
```

16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 +
(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^1
4 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8
+ 16*a^2*c^9))))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c
+ 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*
c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7
- 8*a*b^2*c^8 + 16*a^2*c^9)))/(a^7*b^12 - 10*a^8*b^10*c + 37*a^9*b^8*c^2 -
62*a^10*b^6*c^3 + 46*a^11*b^4*c^4 - 12*a^12*b^2*c^5 + a^13*c^6)) - 12*c*sqr
t(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^
7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 6
2*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a
*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^
2*c^9)))*arctan(-1/2*((b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 +
4*a^4*b*c^4 + (b^6*c^7 - 10*a*b^4*c^8 + 32*a^2*b^2*c^9 - 32*a^3*c^10)*sqrt(
(b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^
3*c^17)))*sqrt((a^10*b^12 - 10*a^11*b^10*c + 37*a^12*b^8*c^2 - 62*a^13*b^6*
c^3 + 46*a^14*b^4*c^4 - 12*a^15*b^2*c^5 + a^16*c^6)*x^2 - 1/2*sqrt(1/2)*(a^
7*b^17 - 17*a^8*b^15*c + 119*a^9*b^13*c^2 - 441*a^10*b^11*c^3 + 924*a^11*b^
9*c^4 - 1078*a^12*b^7*c^5 + 637*a^13*b^5*c^6 - 151*a^14*b^3*c^7 + 12*a^15*b
*c^8 + (a^7*b^14*c^7 - 18*a^8*b^12*c^8 + 131*a^9*b^10*c^9 - 491*a^10*b^8*c^
10 + 997*a^11*b^6*c^11 - 1052*a^12*b^4*c^12 + 496*a^13*b^2*c^13 - 64*a^14*c
^14)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^
4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^
16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 -
(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^1
4 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8
+ 16*a^2*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7
*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c
+ 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c
^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 -
8*a*b^2*c^8 + 16*a^2*c^9))) + ((a^5*b^12*c^7 - 15*a^6*b^10*c^8 + 88*a^7*b^
8*c^9 - 253*a^8*b^6*c^10 + 362*a^9*b^4*c^11 - 224*a^10*b^2*c^12 + 32*a^11*c
^13)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*
b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*
c^16 - 64*a^3*c^17)) + (a^5*b^15 - 14*a^6*b^13*c + 77*a^7*b^11*c^2 - 210*a^
8*b^9*c^3 + 294*a^9*b^7*c^4 - 196*a^10*b^5*c^5 + 49*a^11*b^3*c^6 - 4*a^12*b
*c^7)*x)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2
*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6
*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2
*c^8 + 16*a^2*c^9))))/(a^7*b^12 - 10*a^8*b^10*c...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^10/(c*x^8 + b*x^4 + a), x)
```

Mupad [**B**]
time = 3.49, size = 2500, normalized size = 6.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan(((((8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3
*c^5)/c^3 - (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 8
6*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)
^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96
*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 +
2560*a^4*b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b
^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (4*x*(
a^5*b^5 - 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)
^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b
^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i - (((
8192*a^6*b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3
```

+ (4*x*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7
*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
 - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*
c^9 - 256*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*
b^4*c^6 - 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 -
16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (4*x*(a^5*b^5 -
 5*a^6*b^3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) -
112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^
3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 -
16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4)*1i)/((((8192*a^6*
b*c^6 - 256*a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 - (4*x*(
-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 23
1*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b
^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256
*a^3*b^2*c^10)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 -
 8192*a^5*b^2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*
c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) + (4*x*(a^5*b^5 - 5*a^6*b^
3*c + 5*a^7*b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b
*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*
c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5
*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*
c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) + (((8192*a^6*b*c^6 - 256*
a^3*b^7*c^3 + 2560*a^4*b^5*c^4 - 8192*a^5*b^3*c^5)/c^3 + (4*x*(-(b^11 + b^6
*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^
3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2
*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(51
2*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2*b^4*c^9 - 256*a^3*b^2*c^1
0)))^(1/4)*(8192*a^6*c^8 - 256*a^3*b^6*c^5 + 2560*a^4*b^4*c^6 - 8192*a^5*b^
2*c^7))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(3/4) - (4*x*(a^5*b^5 - 5*a^6*b^3*c + 5*a^7*
b*c^2))/c^3)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^
2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^
(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(

```
4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^11 + b^8*c^7 - 16*a*b^6*c^8 + 96*a^2
*b^4*c^9 - 256*a^3*b^2*c^10)))^(1/4) - (2*(a^8*c - a^7*b^2))/c^3))*(-(b^11
+ b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b
^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c +
6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^...
```

3.320 $\int \frac{x^8}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=376

$$\frac{x}{c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] x/c+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+
b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*
arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-
4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctan(
2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2
)^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctanh(2^(1/4)*
c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))
*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A]
time = 0.40, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1381, 1436, 218, 214, 211}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4
*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^8}{a+bx^4+cx^8}\,dx = \frac{x}{c} - \frac{\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-}}}{2c}$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)}{2c}$$

$$= \frac{x}{c} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{2\sqrt[4]{2}\,c^{5/4}\left(-b+\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 70, normalized size = 0.19

$$\frac{x}{c} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x^4 + c*x^8),x]
```

```
[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 59, normalized size = 0.16

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^4b-a\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^4b-a\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*
b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5310 vs. $2(296) = 592$.
time = 0.99, size = 5310, normalized size = 14.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(-1/2*(sqrt(1/2)*(b^11 - 13
*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^
5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b
^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*
sqrt((a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*x^2
 + 1/2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 1
25*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6 + (b^11*c^5 - 15*a*b^9*c^6 + 85
*a^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqrt((b^8
 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b
*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c
^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*
a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*
```

a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))
 - sqrt(1/2)*((a*b^14*c^5 - 19*a^2*b^12*c^6 + 147*a^3*b^10*c^7 - 590*a^4*b^
8*c^8 + 1290*a^5*b^6*c^9 - 1464*a^6*b^4*c^10 + 736*a^7*b^2*c^11 - 128*a^8*c
^12)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (a*b^15 - 16*a^
2*b^13*c + 103*a^3*b^11*c^2 - 340*a^4*b^9*c^3 + 605*a^5*b^7*c^4 - 554*a^6*b
^5*c^5 + 224*a^7*b^3*c^6 - 32*a^8*b*c^7)*x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*
b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(sqrt(1/2)
*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7
)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^
10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7)))/(a^5*b^8 - 6*a^6*b^6*c + 11*a^7*b^4*c^2 - 6*a^8*b^2*c^3 +
 a^9*c^4)) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6
*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/2*(sqrt(1/2)*(b^1
1 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^
5*b*c^5 - (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352
*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))*sqrt((a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^
4)*x^2 + 1/2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c
^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6 - (b^11*c^5 - 15*a*b^9*c^
6 + 85*a^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(sqrt
(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^
2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b
^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*
a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))
/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)) + sqrt(1/2)*((a*b^14*c^5 - 19*a^2*b^
12*c^6 + 147*a^3*b^10*c^7 - 590*a^4*b^8*c^8 + 1290*a^5*b^6*c^9 - 1464*a^6*b
^4*c^10 + 736*a^7*b^2*c^11 - 128*a^8*c^12)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)) - (a*b^15 - 16*a^2*b^13*c + 103*a^3*b^11*c^2 - 340*a^4
*b^9*c^3 + 605*a^5*b^7*c^4 - 554*a^6*b^5*c^5 + 224*a^7*b^3*c^6 - 32*a^8*b*c
^7)*x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a

```
*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2
 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^8/(c*x^8 + b*x^4 + a), x)
```

Mupad [**B**]
time = 3.97, size = 2500, normalized size = 6.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2
*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 204
8*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 6
1*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b
^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*
a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b
*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
- 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*
c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(
```

a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*
(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3
+ a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 25
6*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5
))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
- 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 +
96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a
^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2
*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c
- 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^
6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a
^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 + b^4*(-(4*a*c - b^
2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(
4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(5
12*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8
)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9
+ b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^
3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c
- b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^
7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)
(-(b^9 + b^4(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 12
0*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^
2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b
^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9
+ b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*
a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c -
b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*
(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*
c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-
(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 +
a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5
)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*
a^3*b^2*c^8)))^(1/4))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4
+ 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13
*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5
- 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + atan(((((16
*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^
3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^

```
2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 -
256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c
^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c
^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*
b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4
*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a
^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*
c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(2...
```

3.321 $\int \frac{x^6}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=325

$$-\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4} c^{3/4} \sqrt{b^2-4ac}}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\ 2^{3/4} c^{3/4} \sqrt{b^2-4ac}}$$

```
[Out] -1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^
2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)+1/4*arctanh(2^(1/4)*c^(1
/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2)^(1/2))^(3/4)*2^(1/4)/
c^(3/4)/(-4*a*c+b^2)^(1/2)+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1
/2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2
)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(-b+(-4*a*c+
b^2)^(1/2))^(3/4)*2^(1/4)/c^(3/4)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1388, 304, 211, 214}

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4} \text{ArcTan}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4} c^{3/4} \sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4} \text{ArcTan}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4} c^{3/4} \sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4} c^{3/4} \sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{2} \sqrt[4]{c} x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\ 2^{3/4} c^{3/4} \sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x^4 + c*x^8),x]
```

```
[Out] -1/2*((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[
b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b + Sqrt[b^2
 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]
)/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(3/4)*A
rcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3
/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^(1/4)*c
^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*
a*c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^6}{a+bx^4+cx^8}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int$$

$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2\sqrt{2}\sqrt{c}}+\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int}{}$$

$$= -\frac{\left(-b-\sqrt{b^2-4ac}\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\tan}{2\,2^{3/4}c^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 44, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)\#1^3}{b+2c\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1^3)/(b + 2*c*#1^4) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 43, normalized size = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^6\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^6\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4054 vs. $2(245) = 490$.
time = 0.56, size = 4054, normalized size = 12.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*arctan(1/2*((a^2*b^7
*c^3 - 9*a^3*b^5*c^4 + 24*a^4*b^3*c^5 - 16*a^5*b*c^6)*x*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)) + (a^
2*b^6 - 6*a^3*b^4*c + 9*a^4*b^2*c^2 - 4*a^5*c^3)*x + (b^4 - 5*a*b^2*c + 4*a
^2*c^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt((a^4*b^
4 - 2*a^5*b^2*c + a^6*c^2)*x^2 - 1/2*sqrt(1/2)*(a^3*b^7 - 6*a^4*b^5*c + 9*a
^5*b^3*c^2 - 4*a^6*b*c^3 + (a^3*b^8*c^3 - 13*a^4*b^6*c^4 + 60*a^5*b^4*c^5 -
 112*a^6*b^2*c^6 + 64*a^7*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 -
```

12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c
^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 -
12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a
^2*c^5))))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 1
6*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/(a^3*b^4 -
 2*a^4*b^2*c + a^5*c^2)) + sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a
*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)))*arctan(-1/2*((b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^5*c^3 - 8*a*b^3*c^4 +
16*a^2*b*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48
*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt((a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*x^2 - 1
/2*sqrt(1/2)*(a^3*b^7 - 6*a^4*b^5*c + 9*a^5*b^3*c^2 - 4*a^6*b*c^3 - (a^3*b^
8*c^3 - 13*a^4*b^6*c^4 + 60*a^5*b^4*c^5 - 112*a^6*b^2*c^6 + 64*a^7*c^7)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(sqrt(1/2)*sqrt(-(b^
3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3
- 8*a*b^2*c^4 + 16*a^2*c^5))) - ((a^2*b^7*c^3 - 9*a^3*b^5*c^4 + 24*a^4*b^3*
c^5 - 16*a^5*b*c^6)*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*
c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)) - (a^2*b^6 - 6*a^3*b^4*c + 9*a^4*b^2*c^
2 - 4*a^5*c^3)*x)*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*
c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))))/(a
^3*b^4 - 2*a^4*b^2*c + a^5*c^2)) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
+ (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^
6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^
4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*
a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 12
8*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*
b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^
5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^
9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*x) - 1/4*sqr
t(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*sqrt(1/2)*(b^7
- 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*
a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*
sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a

```
*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*
a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^
4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
) - (a^2*b^2 - a^3*c)*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c
^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 -
12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a
^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^
3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^
7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^
8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] integrate(x^6/(c*x^8 + b*x^4 + a), x)
```

Mupad [**B**]
time = 3.51, size = 2500, normalized size = 7.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x^4 + c*x^8),x)
[Out] atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^
2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3
- 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 +
256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b
```

^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^5)) + x
*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a
^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512
*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6))
)^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2
*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b
^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4096*a^5
*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*a*c - b^2)^
5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)
^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 25
6*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4*b^2*c^
5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2)
 - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^
2*c^6)))^(1/4)*1i)/(((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*
c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4)*(4
096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 + b^2*(-(4*a*c
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384*a^4
*b^2*c^5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^2)^5
)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256
*a^3*b^2*c^6)))^(1/4) + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c
^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*
a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(3/4
)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7 + b^2*(-(4*
a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4
*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b
^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 16384
*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 + b^2*(-(4*a*c - b^
2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b
^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 -
 256*a^3*b^2*c^6)))^(1/4) - 2*a^4*b*c))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/
2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1
/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*
b^2*c^6)))^(1/4)*2i + atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3
*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^
(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 + x*(-(b^7 - b^2*(
-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*
(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a
^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*c^4 - 1
6384*a^4*b^2*c^5)) + x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-(4*a*c

```
- b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c
^5 - 256*a^3*b^2*c^6)))^(1/4)*1i - ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*
c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4*c^3 - 2048*a^4*b^2*c^4 - x*(-(b^7
- b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
 + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4
 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*(32768*a^5*c^6 + 2048*a^3*b^4*
c^4 - 16384*a^4*b^2*c^5)) - x*(4*a^3*b^3*c - 12*a^4*b*c^2))*(-(b^7 - b^2*(-
(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(
-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^
2*b^4*c^5 - 256*a^3*b^2*c^6)))^(1/4)*1i)/(((-(b^7 - b^2*(-(4*a*c - b^2)^5)^
(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)
^(1/2))/(512*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a
^3*b^2*c^6)))^(3/4)*(4096*a^5*c^5 + 256*a^3*b^4...
```

3.322 $\int \frac{x^4}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=325

$$\frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+$$

```
[Out] 1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2
)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)+1/4*arctanh(2^(1/4)*c^(1/
4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(-b-(-4*a*c+b^2)^(1/2))^(1/4)*2^(3/4)/c
^(1/4)/(-4*a*c+b^2)^(1/2)-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/
2))^(1/4))*(-b+(-4*a*c+b^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)
-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(-b+(-4*a*c+b
^2)^(1/2))^(1/4)*2^(3/4)/c^(1/4)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1388, 218, 214, 211}

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 -
 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 -
4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(
2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcT
anh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)
*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*c
])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{a+bx^4+cx^8}\,dx = -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int$$

$$= \frac{\sqrt{-b-\sqrt{b^2-4ac}}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2\sqrt{b^2-4ac}}+\frac{\sqrt{-b-\sqrt{b^2-4ac}}\int}{}$$

$$= \frac{\sqrt[4]{-b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{-b+\sqrt{b^2-4ac}}\tan^{-1}\left(\right.}{2\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 42, normalized size = 0.13

$$\frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)\#1}{b+2c\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x^4 + c*x^8),x]
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[x - #1]*#1)/(b + 2*c*#1^4) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 43, normalized size = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2479 vs. $2(245) = 490$.
time = 0.42, size = 2479, normalized size = 7.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)))*arctan(1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48
*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x^2 + sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(-(b + (b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 6
4*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*((b^4 - 8*a*b^2
*c + 16*a^2*c^2)*x - (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)
*x/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b + (
```

b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*arctan(-1/2*(sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(x^2 + sqrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) - sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*x + (b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/a) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log(x - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))

Sympy [**A**]

time = 3.16, size = 126, normalized size = 0.39

$$\operatorname{RootSum}\left(t^8\cdot\left(16777216a^4c^5-16777216a^3b^2c^4+6291456a^2b^4c^3-1048576ab^6c^2+65536b^8c\right)+t^4\cdot\left(4096a^2bc^2-2048ab^3c+256b^5\right)+a,\left(t\mapsto t\log\left(-32768t^5a^2c^3+16384t^5ab^2c^2-2048t^5b^4c-4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**4*c**5 - 16777216*a**3*b**2*c**4 + 6291456*a**2*
b**4*c**3 - 1048576*a*b**6*c**2 + 65536*b**8*c) + _t**4*(4096*a**2*b*c**2 -
 2048*a*b**3*c + 256*b**5) + a, Lambda(_t, _t*log(-32768*_t**5*a**2*c**3 +
16384*_t**5*a*b**2*c**2 - 2048*_t**5*b**4*c - 4*_t*b + x)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^4/(c*x^8 + b*x^4 + a), x)
```

Mupad [B]
time = 3.63, size = 2500, normalized size = 7.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x^4 + c*x^8),x)
```

```
[Out] - atan((((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512
*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^
(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*
(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(
1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^
2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5
+ (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^
4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b
*c^4 - 16*a^2*b^3*c^3) - x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c -
 b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a
*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*1i - ((-(b^5 + (-(4*a*
c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 1
6*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c
 - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16
*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 40
```

96*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6) - x*(16384*a^4*b*c
^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2
) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96
*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3) +
x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3
 - 256*a^3*b^2*c^4)))^(1/4)*1i)/(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^
2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*
c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2
*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c
^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^
3*b^4*c^5 - 196608*a^4*b^2*c^6) + x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8
192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^
3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^
2*c^4)))^(3/4) + 64*a^3*b*c^4 - 16*a^2*b^3*c^3) - x*(8*a^3*c^4 - 4*a^2*b^2*
c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/
4) + ((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b
^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/
4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^
8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4
)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c
^6) - x*(16384*a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (
-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c
^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4) + 64*a^3*b*c^
4 - 16*a^2*b^3*c^3) + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^
2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^
6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)))*(-(b^5 + (-(4*a*c - b^2
)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6
*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*2i - 2*atan((((-(b^5 + (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5
 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4
*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5
- 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7
- 4096*a^2*b^6*c^4 + 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i + x*(16384*
a^4*b*c^6 + 1024*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^
5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c
^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b
^3*c^3)*1i + x*(8*a^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4) - ((-(b^5 + (-(4*a*c - b^2)^5)^(1/
2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 9
6*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(((-(b^5 + (-(4*a*c - b^2)^5)^(1/2
) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96
*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))^(1/4)*(262144*a^5*c^7 - 4096*a^2*b^6*c^4

```
+ 49152*a^3*b^4*c^5 - 196608*a^4*b^2*c^6)*1i - x*(16384*a^4*b*c^6 + 1024*a^
2*b^5*c^4 - 8192*a^3*b^3*c^5))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b
*c^2 - 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3
 - 256*a^3*b^2*c^4)))^(3/4)*1i - 64*a^3*b*c^4 + 16*a^2*b^3*c^3)*1i - x*(8*a
^3*c^4 - 4*a^2*b^2*c^3))*(-(b^5 + (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 -
 8*a*b^3*c)/(512*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256
*a^3*b^2*c^4)))^(1/4))/(((-(b^5 + (-(4*a*c - b^...
```

3.323 $\int \frac{x^2}{a+bx^4+cx^8} \, dx$

Optimal. Leaf size=315

$$-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}}+\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-}}}\right.}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-}}}$$

```
[Out] -1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4
)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/(-4*a*c+b^2)^(1/2)+1/2*c^(1/4)*arctanh(2^(1
/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(1/4)/(-b-(-4*a*c+b^2)^(1/2)
)^(1/4)/(-4*a*c+b^2)^(1/2)+1/2*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c
+b^2)^(1/2))^(1/4))*2^(1/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4
)-1/2*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(1
/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A]
time = 0.20, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1389, 304, 211, 214}

$$-\frac{\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(
3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(1/4))) + (c^(1/4)*ArcTan[(
2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a
*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2
- 4*a*c])^(1/4)) - (c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2^(3/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2}{a+bx^4+cx^8}\,dx = \frac{c\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{c}\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}} - \frac{\sqrt{c}\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{\sqrt{2}\sqrt{b^2-4ac}}$$

$$= -\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b-\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2^{3/4}\sqrt{b^2-4ac}\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\ \mathrm{t}}{2^{3/4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 43, normalized size = 0.14

$$\frac{1}{4}\mathrm{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)}{b\#1+2c\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1 + 2*c*#1^5) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 43, normalized size = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	43
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2762 vs. $2(245) = 490$.
time = 0.42, size = 2762, normalized size = 8.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2)))*arctan(-((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2)*sqrt(c^2*x^2 - 1/2*sqrt(1/2)*(b^3*c - 4*a*b*c^2 - (a*b^6*c - 12*a^2*b^
4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c +
 16*a^3*c^2)))/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*
sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 -
 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3
```

```
*c^2)))/c) + sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/s
qrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b
^2*c + 16*a^3*c^2)))*arctan(-((a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x*sqrt
(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)))/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3) - (a*b^4 - 8
*a^2*b^2*c + 16*a^3*c^2)*sqrt(c^2*x^2 - 1/2*sqrt(1/2)*(b^3*c - 4*a*b*c^2 +
(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)/sqrt(a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c +
 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a
*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2
*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*
c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48
*a^4*b^2*c^2 - 64*a^5*c^3))/c) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a
^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b
^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/
sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*
sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(
b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4
*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sq
rt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c
^2)))*log(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a
^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4
- 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2
*b^2*c + 16*a^3*c^2)) + c*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2
*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*
c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 - 8*a*b^2
*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sq
rt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sq
rt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b
- (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*x) + 1/4*sqrt
(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)))*log(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*
c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3
*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 -
```

```
8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/s
qrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b
^2*c + 16*a^3*c^2)) + c*x)
```

Sympy [A]
time = 3.08, size = 172, normalized size = 0.55

$$\operatorname{RootSum}\left(t^{8}\cdot\left(16777216a^{5}c^{4}-16777216a^{4}b^{2}c^{3}+6291456a^{3}b^{4}c^{2}-1048576a^{2}b^{6}c+65536ab^{8}\right)+t^{4}\cdot\left(4096a^{2}bc^{2}-2048ab^{3}c+256b^{5}\right)+c,\left(t\mapsto t\log\left(x+\frac{1048576t^{7}a^{4}bc^{3}-786432t^{7}a^{3}b^{3}c^{2}+196608t^{7}a^{2}b^{5}c-16384t^{7}ab^{7}-512t^{3}a^{2}c^{2}+384t^{3}ab^{2}c-64t^{3}b^{4}}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] RootSum(_t**8*(16777216*a**5*c**4 - 16777216*a**4*b**2*c**3 + 6291456*a**3*
b**4*c**2 - 1048576*a**2*b**6*c + 65536*a*b**8) + _t**4*(4096*a**2*b*c**2 -
 2048*a*b**3*c + 256*b**5) + c, Lambda(_t, _t*log(x + (1048576*_t**7*a**4*b
*c**3 - 786432*_t**7*a**3*b**3*c**2 + 196608*_t**7*a**2*b**5*c - 16384*_t**
7*a*b**7 - 512*_t**3*a**2*c**2 + 384*_t**3*a*b**2*c - 64*_t**3*b**4)/c)))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(x^2/(c*x^8 + b*x^4 + a), x)
```

Mupad [B]
time = 2.34, size = 2500, normalized size = 7.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^4 + c*x^8),x)
```

```
[Out] 2*atan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512
*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^
(3/4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2)
 + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*
a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10
240*a^2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i - 4*a*b*c^5*
x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^
8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)
- ((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^
8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*
```

(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*
a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^
4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^
2*b^4*c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(
b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 25
6*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4))/(((-(
b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 25
6*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a
*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2
 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*
c^5 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i - 4*a*b*c^5*x)*(-(b^5 -
(-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*
c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i + ((-(b^5
 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a
^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^
5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2
 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 2
56*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5
 - 32768*a^3*b^2*c^6)*1i - 2048*a^2*b^3*c^5)*1i + 4*a*b*c^5*x)*(-(b^5 - (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4
 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - 2*a*c^5))*
(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 +
 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) - a
tan((((-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a
*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/
4)*(256*a*b^5*c^4 + 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240
*a^2*b^4*c^5 - 32768*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5
 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a
^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i - ((-(
b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 25
6*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a
*b^5*c^4 + 4096*a^3*b*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*
c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2
 - 256*a^4*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*
c^5 - 32768*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) + 4*a*b*c^5*x)*(-(b^5 - (-(4*a
*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 -
16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4)*1i)/(((-(b^5 - (-(
4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4
 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4
+ 4096*a^3*b*c^6 + x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a
*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4
*b^2*c^3)))^(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 327

```
68*a^3*b^2*c^6) - 2048*a^2*b^3*c^5) - 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)
^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^
6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) + ((-(b^5 - (-(4*a*c - b^2)
^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^
6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^(3/4)*(256*a*b^5*c^4 + 4096*a^3*b
*c^6 - x*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512
*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3*b^4*c^2 - 256*a^4*b^2*c^3)))^
(1/4)*(32768*a^4*c^7 - 1024*a*b^6*c^4 + 10240*a^2*b^4*c^5 - 32768*a^3*b^2*c
^6) - 2048*a^2*b^3*c^5) + 4*a*b*c^5*x)*(-(b^5 - (-(4*a*c - b^2)^5)^(1/2) +
16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 + 256*a^5*c^4 - 16*a^2*b^6*c + 96*a^3
*b^4*c^2 - 256*a^4*b^2*c^3)))^(1/4) + 2*a*c^5))*(-(b^5 - (-(4*a*c - b^2)^5)
^(1/2) + 16*a^2*b*c^2 - 8*a*b^3*c)/(512*(a*b^8 ...
```

3.324 $\int \frac{1}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=315

$$\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{}}{\sqrt[4]{-b-\sqrt{b^2}}}\right.}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2}\right.}$$

```
[Out] 1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)
/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)+1/2*c^(3/4)*arctanh(2^(1/
4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/4)/(-b-(-4*a*c+b^2)^(1/2))
^(3/4)/(-4*a*c+b^2)^(1/2)-1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+
b^2)^(1/2))^(1/4))*2^(3/4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
-1/2*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*2^(3/
4)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A]
time = 0.21, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1361, 218, 214, 211}

$$\frac{c^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4 + c*x^8)^(-1),x]
```

```
[Out] (c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/
4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4
*a*c])^(3/4)) - (c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{a+bx^4+cx^8}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{c\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{c\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}\,x^2}\,dx}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}} - \cdots$$

$$= \frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt[4]{2}\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 45, normalized size = 0.14

$$\frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{\log(x-\#1)}{b\#1^3+2c\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4 + c*x^8)^(-1),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[x - #1]/(b*#1^3 + 2*c*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 40, normalized size = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(1/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4041 vs. 2(245) = 490.
time = 0.53, size = 4041, normalized size = 12.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(-1/2*(sqrt(1/
2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (a^3*b^8 - 14*a^4*b^6
*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt((b
^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2 + 1/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 21*a
^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4 - (a^3*b^9 - 13*a^4*b^7*c + 60*a^5*
b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b
*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
```

(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) - sqrt(1/2)*((a^3*b^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5 - 128*a^8*c^6)*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)) - (b^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5)*x)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^4*c^3 - 2*a*b^2*c^4 + a^2*c^5)) + sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(1/2*(sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2 + 1/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 21*a^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4 + (a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)) + sqrt(1/2)*((a^3*b^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5 - 128*a^8*c^6)*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)) + (b^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5)*x)*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(b^4*c^3 - 2*a*b^2*c^4 + a^2*c^5)) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*log(-(b^2*c - a*c^2)*x + 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c

```
+ 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3
*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6
- 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b
^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*lo
g(-(b^2*c - a*c^2)*x - 1/2*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5 - 8*a^4*
b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^
4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c +
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*
b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^8 + b*x^4 + a), x)
```

Mupad [**B**]
time = 3.42, size = 2500, normalized size = 7.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^4 + c*x^8),x)
```

```
[Out] - atan((((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^
4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((
-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a
*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4
*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144
```

*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c^4 - 11
264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c
 - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*
c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 + b^2*(-(4*a
*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*
a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^
4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i - ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b
^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3
*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^
(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b
^3*c^6) - x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b
^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*
c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^
4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6)
 - 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*1i)/(((-(b^7
 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*
c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*
c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 + b^2*(-(
4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5
*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49
152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 -
 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2
) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/
2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b
^2*c^3)))^(3/4) - 16*b^2*c^6) + 8*c^7*x)*(-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1
/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(
1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6
*b^2*c^3)))^(1/4) + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 +
 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8
+ 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(6
4*a*c^7 + ((-(b^7 + b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^
3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*
c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)*(4096*a*b^7*
c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*c^6) - x*(1024*
b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*c^6))*(-(b^7 +
b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c -
 a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c +
 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(3/4) - 16*b^2*c^6) - 8*c^7*x)*(-(b^7

```
+ b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c
 - a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c
 + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/4)))*(-(b^7 + b^2*(-(4*a*c - b^2)
^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c - a*c*(-(4*a*c - b^2
)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 2
56*a^6*b^2*c^3)))^(1/4)*2i - atan((((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) -
 48*a^3*b*c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))
/(512*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*
c^3)))^(1/4)*(64*a*c^7 + ((-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*
c^3 + 40*a^2*b^3*c^2 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3
*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3)))^(1/
4)*(4096*a*b^7*c^4 - 262144*a^4*b*c^7 - 49152*a^2*b^5*c^5 + 196608*a^3*b^3*
c^6) + x*(1024*b^7*c^4 - 11264*a*b^5*c^5 - 49152*a^3*b*c^7 + 40960*a^2*b^3*
c^6))*(-(b^7 - b^2*(-(4*a*c - b^2)^5)^(1/2) - 48*a^3*b*c^3 + 40*a^2*b^3*c^2
 - 11*a*b^5*c + a*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^3*b^8 + 256*a^7*c^4 -
 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^...
```

3.325 $\int \frac{1}{x^2(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=363

$$-\frac{1}{ax}-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\sqrt[4]{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{}}{\sqrt[4]{-b+\sqrt{b}}}\right.}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/a/x-1/4*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(1-b/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4
)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(1-b/(-4*a*c+b^2
)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*c^(1/4)*arctan(2^(1/4)
*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/4)/
a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(
-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b+(-4*a*c+b^
2)^(1/2))^(1/4)
```

Rubi [A]
time = 0.29, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1382, 1524, 304, 211, 214}

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(1/(a*x)) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4
)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sq
rt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^
(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(
1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c
])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^4+cx^8\right)}\,dx = -\frac{1}{ax}+\frac{\int \frac{x^2\left(-b-cx^4\right)}{a+bx^4+cx^8}\,dx}{a}$$

$$= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right.\right.}{}$$

$$= -\frac{1}{ax}+\frac{\left(\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2\sqrt{2}\,a}-\frac{\left(\sqrt{c}\right.}{}$$

$$= -\frac{1}{ax}-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\sqrt[4]{c}\left(1+\frac{}{\sqrt{b^2}}\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 71, normalized size = 0.20

$$-\frac{1}{ax}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(1/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x - #1]
*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 63, normalized size = 0.17

method	result
default	$-\frac{1}{ax}-\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6c+b\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4a}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(256a^9c^4-256b^2c^3a^8+96b^4c^2a^7-16b^6c\,a^6+b^8a^5\right)\_Z^8+\left(80a^4b\,c^4-120a^3b^3c^3+61a^2b^5c^2-13a\,b^7c+b^9\right)\_Z^4+c^5\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/x-1/4/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*
c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^6 + b*x^2)/(c*x^8 + b*x^4 + a), x)/a - 1/(a*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5375 vs. $2(281) = 562$.
time = 1.09, size = 5375, normalized size = 14.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-1/2*((a^5*b^9*c^4 - 11*
a^6*b^7*c^5 + 41*a^7*b^5*c^6 - 56*a^8*b^3*c^7 + 16*a^9*b*c^8)*x*sqrt((b^8 -
 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*
b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) + (b^10*c^4 - 10*a*b^8*c^5 + 35*a^2
*b^6*c^6 - 50*a^3*b^4*c^7 + 25*a^4*b^2*c^8 - 4*a^5*c^9)*x - (b^6 - 7*a*b^4*
c + 13*a^2*b^2*c^2 - 4*a^3*c^3 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((b^8*c^8 - 6*a*b^6*c^
9 + 11*a^2*b^4*c^10 - 6*a^3*b^2*c^11 + a^4*c^12)*x^2 - 1/2*sqrt(1/2)*(b^13*
c^5 - 13*a*b^11*c^6 + 65*a^2*b^9*c^7 - 155*a^3*b^7*c^8 + 175*a^4*b^5*c^9 -
79*a^5*b^3*c^10 + 12*a^6*b*c^11 + (a^5*b^12*c^5 - 16*a^6*b^10*c^6 + 100*a^7
*b^8*c^7 - 305*a^8*b^6*c^8 + 460*a^9*b^4*c^9 - 304*a^10*b^2*c^10 + 64*a^11*
c^11)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^
10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
 + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))))
*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2
*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^4 - 8*a^6*b^2*c + 16*a^7*c^2)))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 -
6*a^3*b^2*c^8 + a^4*c^9)) - 4*a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^1
2*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-1
/2*((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (a^5*b^5 - 8*a^6*b^3*c
+ 16*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^
4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt((b
^8*c^8 - 6*a*b^6*c^9 + 11*a^2*b^4*c^10 - 6*a^3*b^2*c^11 + a^4*c^12)*x^2 - 1
/2*sqrt(1/2)*(b^13*c^5 - 13*a*b^11*c^6 + 65*a^2*b^9*c^7 - 155*a^3*b^7*c^8 +
 175*a^4*b^5*c^9 - 79*a^5*b^3*c^10 + 12*a^6*b*c^11 - (a^5*b^12*c^5 - 16*a^6
*b^10*c^6 + 100*a^7*b^8*c^7 - 305*a^8*b^6*c^8 + 460*a^9*b^4*c^9 - 304*a^10*
b^2*c^10 + 64*a^11*c^11)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2
*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))
)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^
2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*
b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2
*c + 16*a^7*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a
```

```
^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))) + ((a^5*b^9*c^4 - 11*a^6
*b^7*c^5 + 41*a^7*b^5*c^6 - 56*a^8*b^3*c^7 + 16*a^9*b*c^8)*x*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)) - (b^10*c^4 - 10*a*b^8*c^5 + 35*a^2*b^
6*c^6 - 50*a^3*b^4*c^7 + 25*a^4*b^2*c^8 - 4*a^5*c^9)*x)*sqrt(sqrt(1/2)*sqrt
(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2))))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7 - 6*a^3*b^2*c^8 + a^4
*c^9)) - a*x*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*a
*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5
- (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2
*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sq
rt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c
+ 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*
c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b
^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13
*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) +...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^8 + b*x^4 + a)*x^2), x)
```

Mupad [B]
time = 2.81, size = 2500, normalized size = 6.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4 + c*x^8)),x)
```

```
[Out] 2*atan((((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*
c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a
*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
 + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 + b^
4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^
3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^
2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 -
256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b
^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2
816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i + 4*a^11*b*c
^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - ((-(b^9 + b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^
(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^
8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^1
5*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 -
 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4))/(((-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2
 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^
2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c +
96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8 - x*(-(b^9 + b^4*(
-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 +
 a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^
5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256
*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*
c^5 + 51200*a^14*b^4*c^6 - 81920*a^15*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816
*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 - 14336*a^14*b^2*c^7)*1i + 4*a^11*b*c^8*
```

```
x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 -
120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*
a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*1i + ((-(b^9 + b^4*(-(4*a*c - b^2)^5
)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*
c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(
a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^
(3/4)*(4096*a^15*c^8 + x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c
^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^
8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 81920*a^1
5*b^2*c^7)*1i + 256*a^11*b^8*c^4 - 2816*a^12*b^6*c^5 + 10496*a^13*b^4*c^6 -
 14336*a^14*b^2*c^7)*1i - 4*a^11*b*c^8*x)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(
1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c -
 b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5
*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/
4)*1i))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c
^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*
b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c
+ 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4) - atan((((-(b^9 - b^4*(-(4*a*c
- b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2
*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2)
)/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2
*c^3)))^(3/4)*(4096*a^15*c^8 + x*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80
*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^
(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 25
6*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768
*a^16*c^8 + 1024*a^12*b^8*c^4 - 12288*a^13*b^6*c^5 + 51200*a^14*b^4*c^6 - 8
1920*a^15*b^2*c^7) + 256*a^11*b^8*c^4 - 2816*a^^...
```

3.326 $\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=365

$$-\frac{1}{3ax^3}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] -1/3/a/x^3+1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/
4))*(1-b/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^
(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(1-b/(-4*a*c
+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(3/4)*arctan(2^(
1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(3
/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(
-b+(-4*a*c+b^2)^(1/2))^(1/4))*(1+b/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*
c+b^2)^(1/2))^(3/4)
```

Rubi [A]
time = 0.27, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1382, 1436, 218, 214, 211}

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/3*1/(a*x^3) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)
*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^
(3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) +
 (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[
b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/
4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4
*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^4(a+bx^4+cx^8)}\,dx = -\frac{1}{3ax^3} + \frac{\int \frac{-3b-3cx^4}{a+bx^4+cx^8}\,dx}{3a}$$

$$= -\frac{1}{3ax^3} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a} - \frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right.\right.}{}$$

$$= -\frac{1}{3ax^3} + \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{\left(c\left(1\right.\right.}{}$$

$$= -\frac{1}{3ax^3} + \frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(1+\frac{}{\sqrt{b}}\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 75, normalized size = 0.21

$$-\frac{1}{3ax^3} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{b\log(x-\#1)+c\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/3*1/(a*x^3) - RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[x - #1] + c*Log[x -
 #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 62, normalized size = 0.17

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^4c-b\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4a}-\frac{1}{3a\,x^3}$
risch	$-\frac{1}{3a\,x^3}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(256c^4a^{11}-256a^{10}b^2c^3+96a^9b^4c^2-16a^8b^6c+a^7b^8\right)\_Z^8+\left(-112b\,c^5a^5+280b^3c^4a^4-231b^5c^3a^3+86b^7c^2a^2-15b^9ca+\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a)
)-1/3/a/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 + b)/(c*x^8 + b*x^4 + a), x)/a - 1/3/(a*x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6664 vs. $2(281) = 562$.
time = 1.83, size = 6664, normalized size = 18.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/12*(12*a*x^3*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a
^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c +
37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6
)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8
*a^8*b^2*c + 16*a^9*c^2)))*arctan(-1/2*(sqrt(1/2)*(b^14 - 16*a*b^12*c + 102
*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a
^6*b^2*c^6 - 16*a^7*c^7 + (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*
a^10*b^5*c^3 + 560*a^11*b^3*c^4 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c
+ 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c
^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt((b^12
*c^4 - 10*a*b^10*c^5 + 37*a^2*b^8*c^6 - 62*a^3*b^6*c^7 + 46*a^4*b^4*c^8 - 1
2*a^5*b^2*c^9 + a^6*c^10)*x^2 + 1/2*sqrt(1/2)*(b^18 - 18*a*b^16*c + 135*a^2
*b^14*c^2 - 546*a^3*b^12*c^3 + 1288*a^4*b^10*c^4 - 1792*a^5*b^8*c^5 + 1421*
a^6*b^6*c^6 - 592*a^7*b^4*c^7 + 114*a^8*b^2*c^8 - 8*a^9*c^9 + (a^7*b^15 - 1
9*a^8*b^13*c + 148*a^9*b^11*c^2 - 605*a^10*b^9*c^3 + 1374*a^11*b^7*c^4 - 16
72*a^12*b^5*c^5 + 928*a^13*b^3*c^6 - 128*a^14*b*c^7)*sqrt((b^12 - 10*a*b^10
*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^
6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(
```

b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 1
6*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*
a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16
*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(-(b^7
 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a
^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4
*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^
2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)) + sqrt(1/2)*((
a^7*b^17*c^2 - 22*a^8*b^15*c^3 + 204*a^9*b^13*c^4 - 1032*a^10*b^11*c^5 + 30
75*a^11*b^9*c^6 - 5417*a^12*b^7*c^7 + 5324*a^13*b^5*c^8 - 2480*a^14*b^3*c^9
 + 320*a^15*b*c^10)*x*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^
6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*
c + 48*a^16*b^2*c^2 - 64*a^17*c^3)) + (b^20*c^2 - 21*a*b^18*c^3 + 188*a^2*b
^16*c^4 - 935*a^3*b^14*c^5 + 2821*a^4*b^12*c^6 - 5292*a^5*b^10*c^7 + 6083*a
^6*b^8*c^8 - 4071*a^7*b^6*c^9 + 1449*a^8*b^4*c^10 - 248*a^9*b^2*c^11 + 16*a
^10*c^12)*x)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b
^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*
a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a
^9*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*
c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^
2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^
14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*
b^2*c + 16*a^9*c^2)))/(b^12*c^7 - 10*a*b^10*c^8 + 37*a^2*b^8*c^9 - 62*a^3*b
^6*c^10 + 46*a^4*b^4*c^11 - 12*a^5*b^2*c^12 + a^6*c^13)) - 12*a*x^3*sqrt(sq
rt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 -
8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^
3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*
b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^
2)))*arctan(1/2*(sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3
*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7
 - (a^7*b^11 - 17*a^8*b^9*c + 113*a^9*b^7*c^2 - 364*a^10*b^5*c^3 + 560*a^11
*b^3*c^4 - 320*a^12*b*c^5)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15
*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt((b^12*c^4 - 10*a*b^10*c^5 +
37*a^2*b^8*c^6 - 62*a^3*b^6*c^7 + 46*a^4*b^4*c^8 - 12*a^5*b^2*c^9 + a^6*c^1
0)*x^2 + 1/2*sqrt(1/2)*(b^18 - 18*a*b^16*c + 135*a^2*b^14*c^2 - 546*a^3*b^1
2*c^3 + 1288*a^4*b^10*c^4 - 1792*a^5*b^8*c^5 + 1421*a^6*b^6*c^6 - 592*a^7*b
^4*c^7 + 114*a^8*b^2*c^8 - 8*a^9*c^9 - (a^7*b^15 - 19*a^8*b^13*c + 148*a^9*
b^11*c^2 - 605*a^10*b^9*c^3 + 1374*a^11*b^7*c^4 - 1672*a^12*b^5*c^5 + 928*a
^13*b^3*c^6 - 128*a^14*b*c^7)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 6
2*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a
^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^
2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 -
 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^

```
2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)
))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b
^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*
sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^8 + b*x^4 + a)*x^4), x)
```

Mupad [**B**]
time = 5.57, size = 2500, normalized size = 6.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4 + c*x^8)),x)
```

```
[Out] 2*atan(-(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b
^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a
*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^
4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4
 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11
 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*
b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c +
 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2
))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*
b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5
+ 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b
^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b
```

^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4
*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 +
256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i
 - 128*a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 -
 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 8
6*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)
^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c
*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96
*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4) + ((-(b^11 + b^6*(-(4*a*c - b^2)^5
)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^
4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c
 - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a
^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*((x*(81
920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 13312*a^12*b^7*c^5 + 62464*a^13*b^5*c^
6 - 122880*a^14*b^3*c^7) + (-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5
*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*
a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) -
 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*
b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c^8 + 4096*
a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 245760*a^15*b^4*c^6 - 458752*a^16*b^2*c
^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7
*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2)
 - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c
 - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*
c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i + 128*a^11*b*c^9 + 16*a^9*b^5*c^7 - 96*a
^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*b^2*c^9))*(-(b^11 + b^6*(-(4*a*c -
 b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4
*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-
(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8
+ 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4))
/(((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2
- 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15
*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^
2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 -
 256*a^10*b^2*c^3)))^(1/4)*((x*(81920*a^15*b*c^8 + 1024*a^11*b^9*c^4 - 1331
2*a^12*b^7*c^5 + 62464*a^13*b^5*c^6 - 122880*a^14*b^3*c^7) - (-(b^11 + b^6*
(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3
 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*
b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512
*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3
)))^(1/4)*(262144*a^17*c^8 + 4096*a^13*b^8*c^4 - 53248*a^14*b^6*c^5 + 24576
0*a^15*b^4*c^6 - 458752*a^16*b^2*c^7)*1i)*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^
(1/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4
- a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c -
b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^1

1*c^4 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4)*1i - 128*
a^11*b*c^9 - 16*a^9*b^5*c^7 + 96*a^10*b^3*c^8)*1i + x*(8*a^10*c^10 - 4*a^9*
b^2*c^9))*(-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5 + 86*a^2*b
^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 - a^3*c^3*(-(4*a*c - b^2)^5)^(1/
2) - 15*a*b^9*c + 6*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*(-(4*a
c - b^2)^5)^(1/2))/(512(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^
4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*1i - ((-(b^11 + b^6*(-(4*a*c - b^2)^5)^(1
/2) - 112*a^5*b*c^5 + 86*a^2*b^7*c^2 - 231*a^3*b^5*c^3 + 280*a^4*b^3*c^4 -
a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 15*a*b^9*c +...

3.327 $\int \frac{x^m}{1+x^4+x^8}\,dx$

Optimal. Leaf size=127

$$\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}$$

```
[Out] -2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4/(1+I*3^(1/2)))/(1+
m)/(I-3^(1/2))*3^(1/2)+2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*
x^4/(1-I*3^(1/2)))/(1+m)*3^(1/2)/(3^(1/2)+I)
```

Rubi [A]
time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1389, 371}

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 + x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 - I*Sqr
t[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
 (1 + m)/4, (5 + m)/4, (-2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(I - Sqrt[3])*(1
 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^m}{1+x^4+x^8}\,dx = -\frac{i\int \frac{x^m}{\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}} + \frac{i\int \frac{x^m}{\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}}$$
$$= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)} - \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.94, size = 488, normalized size = 3.84

$$\frac{x^m\left(-\frac{i\left(\left(\frac{x}{-\sqrt[3]{-1}+x}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{\sqrt[3]{-1}}{\sqrt[3]{-1}-x}\right)+\left(\frac{x}{-(-1)^{2/3}+x}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{(-1)^{2/3}}{(-1)^{2/3}-x}\right)-\left(\frac{x}{\sqrt[3]{-1}+x}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{\sqrt[3]{-1}}{\sqrt[3]{-1}+x}\right)-\left(\frac{x}{(-1)^{2/3}+x}\right)^{-m}{}_2F_1\left(-m,-m;1-m;\frac{(-1)^{2/3}}{(-1)^{2/3}+x}\right)\right)}{\sqrt{3}}+\text{RootSum}\left[1-\#1^2+\#1^4\&,\frac{{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{-\#1+2\#1^3}\&\right]-\frac{\text{RootSum}\left[1-\#1^2+\#1^4\&,\frac{mx^2+m^2x^2+2mx\#1+m^2x\#1+2\,{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}\#1^2+3m\,{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}\#1^2+m^2\,{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}\#1^2+m\left(\frac{x}{\#1}\right)^{-m}\#1^2}{-\#1+2\#1^3}\&\right]}{2+3m+m^2}\right)}{4m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(1 + x^4 + x^8),x]
```

```
[Out] (x^m*(((-I)*(Hypergeometric2F1[-m, -m, 1 - m, (-1)^(1/3)/((-1)^(1/3) - x)]/
(x/(-(-1)^(1/3) + x))^m + Hypergeometric2F1[-m, -m, 1 - m, (-1)^(2/3)/((-1)
^(2/3) - x)]/(x/(-(-1)^(2/3) + x))^m - Hypergeometric2F1[-m, -m, 1 - m, (-1
)^(1/3)/((-1)^(1/3) + x)]/(x/((-1)^(1/3) + x))^m - Hypergeometric2F1[-m, -m
, 1 - m, (-1)^(2/3)/((-1)^(2/3) + x)]/(x/((-1)^(2/3) + x))^m))/Sqrt[3] + Ro
otSum[1 - #1^2 + #1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]/
((x/(x - #1))^m*(-#1 + 2*#1^3)) & ] - RootSum[1 - #1^2 + #1^4 & , (m*x^2 +
m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x
 - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometric2F1[-m, -m, 1 - m, -(#1/
(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hypergeometric2F1[-m, -m, 1 - m, -(#
1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m)/(-#1 + 2*#1^3) & ]/(
2 + 3*m + m^2)))/(4*m)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8+x^4+1),x)
```

```
[Out] int(x^m/(x^8+x^4+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+x^4+1),x, algorithm="maxima")
[Out] integrate(x^m/(x^8 + x^4 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+x^4+1),x, algorithm="fricas")
[Out] integral(x^m/(x^8 + x^4 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(x^2-x+1)(x^2+x+1)(x^4-x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8+x**4+1),x)
[Out] Integral(x**m/((x**2 - x + 1)*(x**2 + x + 1)*(x**4 - x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+x^4+1),x, algorithm="giac")
[Out] integrate(x^m/(x^8 + x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{x^8+x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^4 + x^8 + 1),x)
[Out] int(x^m/(x^4 + x^8 + 1), x)
```

3.328 $\int \frac{x^{11}}{1+x^4+x^8}\,dx$

Optimal. Leaf size=44

$$\frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1+x^4+x^8\right)$$

[Out] 1/4*x^4-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1371, 717, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{x^4}{4}-\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Antiderivative was successfully verified.

[In] Int[x^11/(1 + x^4 + x^8),x]

[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{1+x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{1+x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{4}\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}-\frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^4\right)\\ &= \frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1+x^4+x^8\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.00

$$\frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1+x^4+x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 + x^4 + x^8),x]
```

```
[Out] x^4/4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4 + x^8]/8
```

Maple [A]
time = 0.02, size = 36, normalized size = 0.82

method	result	size
default	$\frac{x^4}{4}-\frac{\ln(x^8+x^4+1)}{8}-\frac{\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	36
risch	$\frac{x^4}{4}-\frac{\ln(4x^8+4x^4+4)}{8}-\frac{\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.50, size = 35, normalized size = 0.80

$$\frac{1}{4}\,x^4-\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^4+1\right)\right)-\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="maxima")
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

Fricas [A]
time = 0.35, size = 35, normalized size = 0.80

$$\frac{1}{4}\,x^4-\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^4+1\right)\right)-\frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="fricas")
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

Sympy [A]
time = 0.05, size = 42, normalized size = 0.95

$$\frac{x^4}{4}-\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+x**4+1),x)
```

```
[Out] x**4/4 - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3
)/12
```

Giac [A]
time = 5.22, size = 35, normalized size = 0.80

$$\frac{1}{4}\,x^4 - \frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^4+1\right)\right) - \frac{1}{8}\,\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4
+ 1)
```

Mupad [B]
time = 0.05, size = 37, normalized size = 0.84

$$\frac{x^4}{4} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12} - \frac{\ln\left(x^8+x^4+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^4 + x^8 + 1),x)
```

```
[Out] x^4/4 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12 - log(x^4 + x^8 +
1)/8
```

3.329 $\int \frac{x^9}{1+x^4+x^8}\,dx$

Optimal. Leaf size=54

$$\frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/2*x^2+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(2*x^2+1)
*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1373, 1136, 1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + x^4 + x^8),x]
```

```
[Out] x^2/2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/Sqrt[3
]]/(2*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{1+x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{x^2}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{x^2}{2}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^2\right)\\
&= \frac{x^2}{2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.11, size = 98, normalized size = 1.81

$$\frac{x^2}{2}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x^2\right)}{2\sqrt{6+6i\sqrt{3}}}-\frac{\left(-i+\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x^2\right)}{2\sqrt{6-6i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + x^4 + x^8),x]
[Out] x^2/2 - ((I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 + (6*I)*Sq
rt[3]]) - ((-I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(2*Sqrt[6 - (6*I)*
Sqrt[3]])
```

Maple [A]
time = 0.03, size = 43, normalized size = 0.80

method	result	size
default	$\frac{x^{2}}{2}-\frac{\arctan\left(\frac{\left(2x^{2}+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{2}-1\right)\sqrt{3}}{3}\right)}{6}$	43
risch	$\frac{x^{2}}{2}-\frac{\sqrt{3}\,\arctan\left(\frac{x^{6}\sqrt{3}}{3}+\frac{2x^{2}\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{x^{2}\sqrt{3}}{3}\right)}{6}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/2*x^2-1/6*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)-1/6*3^(1/2)*arctan(1/3*(2
*x^2-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 42, normalized size = 0.78

$$\frac{1}{2}\,x^{2}-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^{2}+1\right)\right)-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^{2}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="maxima")
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x^2 - 1))
```

Fricas [A]
time = 0.34, size = 40, normalized size = 0.74

$$\frac{1}{2}\,x^{2}-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,x^{2}\right)-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(x^{6}+2\,x^{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="fricas")
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x^2) - 1/6*sqrt(3)*arctan(1/3*sqrt
(3)*(x^6 + 2*x^2))
```

Sympy [A]
time = 0.05, size = 51, normalized size = 0.94

$$\frac{x^{2}}{2}+\frac{\sqrt{3}\left(-2\,\operatorname{atan}\left(\frac{\sqrt{3}\,x^{2}}{3}\right)-2\,\operatorname{atan}\left(\frac{\sqrt{3}\,x^{6}}{3}+\frac{2\sqrt{3}\,x^{2}}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(
3)*x**2/3))/12
```

Giac [A]
time = 6.34, size = 42, normalized size = 0.78

$$\frac{1}{2}\,x^2-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^2+1\right)\right)-\frac{1}{6}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^2-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(
1/3*sqrt(3)*(2*x^2 - 1))
```

Mupad [B]
time = 0.04, size = 43, normalized size = 0.80

$$\frac{x^2}{2}-\frac{\sqrt{3}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3}+\frac{2\,\sqrt{3}\,x^2}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right)\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^4 + x^8 + 1),x)
```

```
[Out] x^2/2 - (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^(
1/2)*x^2)/3)))/12
```

3.330 $\int \frac{x^7}{1+x^4+x^8} \, dx$

Optimal. Leaf size=37

$$-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1+x^4+x^8\right)$$

```
[Out] 1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1371, 648, 632, 210, 642}

$$\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{\text{ArcTan}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 + x^4 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{1+x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+x+x^2}\,dx, x, x^4\right)\\ &= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^4\right)\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^4\right)\\ &= \frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^4\right)\\ &= -\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1+x^4+x^8\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.00

$$-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1+x^4+x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^4 + x^8),x]
[Out] -1/4*ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 + x^4 + x^8]/8
```

Maple [A]
time = 0.02, size = 31, normalized size = 0.84

method	result	size
default	$\frac{\ln(x^8+x^4+1)}{8}-\frac{\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	31

risch	$\frac{\ln(4x^8+4x^4+4)}{8} - \frac{\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.54, size = 30, normalized size = 0.81

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)+\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="maxima")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

Fricas [A]
time = 0.34, size = 30, normalized size = 0.81

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)+\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="fricas")
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

Sympy [A]
time = 0.05, size = 37, normalized size = 1.00

$$\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+x**4+1),x)
[Out] log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/12
```

Giac [A]
time = 5.36, size = 30, normalized size = 0.81

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)+\frac{1}{8}\log\left(x^8+x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/8*log(x^8 + x^4 + 1)
```

Mupad [B]
time = 0.04, size = 32, normalized size = 0.86

$$\frac{\ln\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^4 + x^8 + 1),x)
```

```
[Out] log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3))/12
```

3.331 $\int \frac{x^5}{1+x^4+x^8}\,dx$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)$$

```
[Out] 1/8*ln(x^4-x^2+1)-1/8*ln(x^4+x^2+1)-1/12*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(
1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1373, 1141, 1175, 632, 210, 1178, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*S
qrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{1+x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\ &= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\ &= \frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{-1-x-x^2}\,dx,x,x^2\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1-2x}{-1+x-x^2}\,dx,x,x^2\right)+\frac{1}{8}\text{Subst}\Bigg(\int\\ &= \frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)-\frac{1}{4}\\ &= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 94, normalized size = 1.25

$$\frac{\sqrt{1-i\sqrt{3}}\left(-i+\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x^2\right)+\sqrt{1+i\sqrt{3}}\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x^2\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + x^4 + x^8),x]
```

```
[Out] (Sqrt[1 - I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((-I + Sqrt[3])*x^2)/2] + Sqrt[1
 + I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((I + Sqrt[3])*x^2)/2])/(4*Sqrt[6])
```

Maple [A]
time = 0.02, size = 62, normalized size = 0.83

method	result	size
default	$-\frac{\ln\left(x^4+x^2+1\right)}{8}+\frac{\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln\left(x^4-x^2+1\right)}{8}+\frac{\sqrt{3}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{12}$	62
risch	$\frac{\ln\left(4x^4-4x^2+4\right)}{8}+\frac{\sqrt{3}\arctan\left(\frac{\left(2x^2-1\right)\sqrt{3}}{3}\right)}{12}-\frac{\ln\left(4x^4+4x^2+4\right)}{8}+\frac{\arctan\left(\frac{\left(2x^2+1\right)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(x^4+x^2+1)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)+1/8*ln(x^4-x^
2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Maxima [A]
time = 0.49, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

Fricas [A]
time = 0.35, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

Sympy [A]
time = 0.08, size = 76, normalized size = 1.01

$$\frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x*
*2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

Giac [A]
time = 4.73, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

Mupad [B]
time = 0.09, size = 51, normalized size = 0.68

$$\operatorname{atanh}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\operatorname{atanh}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4 + x^8 + 1),x)
```

```
[Out] atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3^
(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)
```

3.332 $\int \frac{x^3}{1+x^4+x^8}\,dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] 1/6*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1366, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^3}{1+x^4+x^8}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^4\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^4\right)\right)$$
$$= \frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.83

method	result	size
default	$\frac{\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	19
risch	$\frac{\arctan\left(\frac{(2x^4+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.51, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Fricas [A]
time = 0.34, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^4+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Sympy [A]
time = 0.05, size = 26, normalized size = 1.13

$$\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+x**4+1),x)
[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3)/6
```

Giac [A]
time = 3.02, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^4+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+x^4+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1))
```

Mupad [B]
time = 1.30, size = 17, normalized size = 0.74

$$\frac{\sqrt{3}\,\operatorname{atan}\left(\sqrt{3}\,\left(\frac{2\,x^4}{3}+\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4 + x^8 + 1),x)
[Out] (3^(1/2)*atan(3^(1/2)*((2*x^4)/3 + 1/3)))/6
```

3.333 $\int \frac{x}{1+x^4+x^8}\,dx$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1-x^2+x^4\right)+\frac{1}{8}\log\left(1+x^2+x^4\right)$$

```
[Out] -1/8*ln(x^4-x^2+1)+1/8*ln(x^4+x^2+1)-1/12*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^
(1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {1373, 1108, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^4-x^2+1\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x^2)/Sqrt[3]]/(4*S
qrt[3]) - Log[1 - x^2 + x^4]/8 + Log[1 + x^2 + x^4]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1+x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx,x,x^2\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1+x}{1+x+x^2}\,dx,x,x^2\right)\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)-\frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^2\right)+\frac{1}{8}\text{Subst}\left(\int \frac{}{1}\right.\\
&= -\frac{1}{8}\log\left(1-x^2+x^4\right)+\frac{1}{8}\log\left(1+x^2+x^4\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)-\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1-x^2+x^4\right)+\frac{1}{8}\log\left(1+x^2+x^4\right)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 79, normalized size = 1.05

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(-i+\sqrt{3}\right)x^2\right)-\sqrt{1+i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(i+\sqrt{3}\right)x^2\right)\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^4 + x^8),x]
```

```
[Out] ((I/2)*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x^2)/2] - Sqrt[1 + I*Sqr
t[3]]*ArcTan[((I + Sqrt[3])*x^2)/2]))/Sqrt[6]
```

Maple [A]
time = 0.02, size = 62, normalized size = 0.83

method	result	size
default	$\frac{\ln(x^4+x^2+1)}{8}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\ln(x^4-x^2+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}$	62
risch	$-\frac{\ln(4x^4-4x^2+4)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}+\frac{\ln(4x^4+4x^2+4)}{8}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(x^4+x^2+1)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)-1/8*ln(x^4-x^2
+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Maxima [A]
time = 0.48, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

Fricas [A]
time = 0.35, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

Sympy [A]
time = 0.09, size = 76, normalized size = 1.01

$$-\frac{\log\left(x^4-x^2+1\right)}{8}+\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\ x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\ \operatorname{atan}\left(\frac{2\sqrt{3}\ x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+x**4+1),x)
```

```
[Out] -log(x**4 - x**2 + 1)/8 + log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x
**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12
```

Giac [A]
time = 3.32, size = 61, normalized size = 0.81

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{1}{8}\log\left(x^4+x^2+1\right)-\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/8*log(x^4 + x^2 + 1) - 1/8*log(x^4 - x^2 + 1)
```

Mupad [B]
time = 1.28, size = 51, normalized size = 0.68

$$\operatorname{atan}\left(\frac{\sqrt{3}\ x^2}{2}-\frac{x^2\,1\mathrm{i}}{2}\right)\ \left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)+\operatorname{atan}\left(\frac{\sqrt{3}\ x^2}{2}+\frac{x^2\,1\mathrm{i}}{2}\right)\ \left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4 + x^8 + 1),x)
```

```
[Out] atan((3^(1/2)*x^2)/2 - (x^2*1i)/2)*(3^(1/2)/12 + 1i/4) + atan((3^(1/2)*x^2)
/2 + (x^2*1i)/2)*(3^(1/2)/12 - 1i/4)
```

3.334 $\int \frac{1}{x(1+x^4+x^8)}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1+x^4+x^8\right)$$

```
[Out] ln(x)-1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {1371, 719, 29, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8+x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^4 + x^8)),x]
```

```
[Out] -1/4*ArcTan[(1 + 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[1 + x^4 + x^8]/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(1+x^4+x^8\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(1+x+x^2\right)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)+\frac{1}{4}\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^4\right)-\frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1+x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1+x^4+x^8\right)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 138, normalized size = 3.54

$$\frac{1}{24}\left(2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+24\log(x)-\sqrt{3}\left(-i+\sqrt{3}\right)\log\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^2\right)-\sqrt{3}\left(i+\sqrt{3}\right)\log\left(\frac{1}{2}i\left(i+\sqrt{3}\right)+x^2\right)-3\log\left(1-x+x^2\right)-3\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^4 + x^8)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
 + 24*Log[x] - Sqrt[3]*(-I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^2] - Sqr
t[3]*(I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] - 3*Log[1 - x + x^2] - 3*
Log[1 + x + x^2])/24
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(32) = 64$.
time = 0.03, size = 87, normalized size = 2.23

method	result
risch	$\ln(x) - \frac{\ln(x^8+x^4+1)}{8} - \frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^4+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$
default	$-\frac{\ln(x^2+x+1)}{8} - \frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12} + \ln(x) - \frac{\ln(x^4-x^2+1)}{8} - \frac{\sqrt{3}\,\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12} - \frac{\ln(x^2-x+1)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+ln(x)-1/8*ln(x^4-
x^2+1)-1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/8*ln(x^2-x+1)+1/12*3^(1
/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.52, size = 36, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right) - \frac{1}{8}\log\left(x^8+x^4+1\right) + \frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1/
4*log(x^4)
```

Fricas [A]
time = 0.37, size = 32, normalized size = 0.82

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right) - \frac{1}{8}\log\left(x^8+x^4+1\right) + \log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + lo
g(x)
```

Sympy [A]
time = 0.06, size = 41, normalized size = 1.05

$$\log(x) - \frac{\log(x^8 + x^4 + 1)}{8} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+x**4+1),x)
```

```
[Out] log(x) - log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/3
)/12
```

Giac [A]
time = 2.94, size = 36, normalized size = 0.92

$$-\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^4 + 1\right)\right) - \frac{1}{8}\,\log\left(x^8 + x^4 + 1\right) + \frac{1}{4}\,\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/8*log(x^8 + x^4 + 1) + 1/
4*log(x^4)
```

Mupad [B]
time = 1.30, size = 34, normalized size = 0.87

$$\ln(x) - \frac{\ln(x^8 + x^4 + 1)}{8} - \frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\,\sqrt{3}\,x^4}{3} + \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 + x^8 + 1)),x)
```

```
[Out] log(x) - log(x^4 + x^8 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3
))/12
```

3.335 $\int \frac{1}{x^3(1+x^4+x^8)}\,dx$

Optimal. Leaf size=54

$$-\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/2/x^2+1/6*arctan(1/3*(-2*x^2+1)*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(2*x^2+1
)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1373, 1137, 1175, 632, 210}

$$\frac{\text{ArcTan}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 + ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x^2)/S
qrt[3]]/(2*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(1+x^4+x^8\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{-1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2
\end{aligned}$$

$$= -\frac{1}{2x^2}+\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 100, normalized size = 1.85

$$\frac{1}{12}\left(-\frac{6}{x^2}-2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+i\sqrt{3}\log\left(-1-i\sqrt{3}+2x^2\right)-i\sqrt{3}\log\left(-1+i\sqrt{3}+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x^4 + x^8)),x]
```

```
[Out] (-6/x^2 - 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] + I*Sqrt[3]*Log[-1 - I*Sqrt[3] + 2*x^2] - I*Sqrt[3]*Log[-1 + I*Sq
rt[3] + 2*x^2])/12
```

Maple [A]
time = 0.03, size = 57, normalized size = 1.06

method	result	size
risch	$-\frac{1}{2x^{2}}-\frac{\sqrt{3}\,\arctan\left(\frac{x^{2}\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{x^{6}\sqrt{3}}{3}+\frac{2x^{2}\sqrt{3}}{3}\right)}{6}$	44
default	$\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{1}{2x^{2}}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^{2}-1)\sqrt{3}}{3}\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/2/x^2-1/6*3^(1/2)*arctan(1/3*(2*x
^2-1)*3^(1/2))-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 42, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^{2}+1)\right)-\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^{2}-1)\right)-\frac{1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^2 - 1)) - 1/2/x^2
```

Fricas [A]
time = 0.35, size = 45, normalized size = 0.83

$$-\frac{\sqrt{3}\,x^{2}\arctan\left(\frac{1}{3}\sqrt{3}\,x^{2}\right)+\sqrt{3}\,x^{2}\arctan\left(\frac{1}{3}\sqrt{3}\,(x^{6}+2\,x^{2})\right)+3}{6\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(3)*x^2*arctan(1/3*sqrt(3)*x^2) + sqrt(3)*x^2*arctan(1/3*sqrt(3)*
(x^6 + 2*x^2)) + 3)/x^2
```

Sympy [A]
time = 0.06, size = 53, normalized size = 0.98

$$\frac{\sqrt{3}\left(-2\operatorname{atan}\left(\frac{\sqrt{3}\,x^{2}}{3}\right)-2\operatorname{atan}\left(\frac{\sqrt{3}\,x^{6}}{3}+\frac{2\sqrt{3}\,x^{2}}{3}\right)\right)}{12}-\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(-2*atan(sqrt(3)*x**2/3) - 2*atan(sqrt(3)*x**6/3 + 2*sqrt(3)*x**2/3
))/12 - 1/(2*x**2)
```

Giac [A]
time = 3.05, size = 42, normalized size = 0.78

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^2 - 1)) - 1/2/x^2
```

Mupad [B]
time = 0.04, size = 43, normalized size = 0.80

$$-\frac{\sqrt{3}\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^6}{3}+\frac{2\sqrt{3}\,x^2}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^2}{3}\right)\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^4 + x^8 + 1)),x)
```

```
[Out] - (3^(1/2)*(2*atan((2*3^(1/2)*x^2)/3 + (3^(1/2)*x^6)/3) + 2*atan((3^(1/2)*x
^2)/3)))/12 - 1/(2*x^2)
```

3.336 $\int \frac{1}{x^5(1+x^4+x^8)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{4x^4}-\frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\log(x)+\frac{1}{8}\log\left(1+x^4+x^8\right)$$

```
[Out] -1/4/x^4-ln(x)+1/8*ln(x^8+x^4+1)-1/12*arctan(1/3*(2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1371, 723, 814, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{2x^4+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 - ArcTan[(1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[x] + Log[1 + x^4
 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5(1+x^4+x^8)}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(1+x+x^2)}\,dx, x, x^4\right)$$
$$= -\frac{1}{4x^4} + \frac{1}{4}\text{Subst}\left(\int \frac{-1-x}{x(1+x+x^2)}\,dx, x, x^4\right)$$
$$= -\frac{1}{4x^4} + \frac{1}{4}\text{Subst}\left(\int \left(-\frac{1}{x} + \frac{x}{1+x+x^2}\right)\,dx, x, x^4\right)$$
$$= -\frac{1}{4x^4} - \log(x) + \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+x+x^2}\,dx, x, x^4\right)$$
$$= -\frac{1}{4x^4} - \log(x) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^4\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x,$$
$$= -\frac{1}{4x^4} - \log(x) + \frac{1}{8}\log\left(1+x^4+x^8\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^4\right)$$
$$= -\frac{1}{4x^4} - \frac{\tan^{-1}\left(\frac{1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \log(x) + \frac{1}{8}\log\left(1+x^4+x^8\right)$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 141, normalized size = 2.94

$$\frac{1}{24}\left(-\frac{6}{x^4} + 2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right) - 2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right) - 24\log(x) + \sqrt{3}\left(i+\sqrt{3}\right)\log\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2} + x^2\right) + \sqrt{3}\left(-i+\sqrt{3}\right)\log\left(\frac{1}{2}i\left(i+\sqrt{3}\right) + x^2\right) + 3\log\left(1-x+x^2\right) + 3\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + x^4 + x^8)),x]
```

```
[Out] (-6/x^4 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)
/Sqrt[3]] - 24*Log[x] + Sqrt[3]*(I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^
2] + Sqrt[3]*(-I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] + 3*Log[1 - x +
x^2] + 3*Log[1 + x + x^2])/24
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(39) = 78.
time = 0.03, size = 94, normalized size = 1.96

method	result
risch	$-\frac{1}{4x^4} - \ln(x) + \frac{\ln(x^8+x^4+1)}{8} - \frac{\sqrt{3}\arctan\left(\frac{2\left(x^4+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$
default	$\frac{\ln(x^2+x+1)}{8} - \frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12} - \frac{1}{4x^4} - \ln(x) + \frac{\ln(x^4-x^2+1)}{8} - \frac{\sqrt{3}\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12} + \frac{\ln(x^2-x}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/4/x^4-ln(x)+1/8*
ln(x^4-x^2+1)-1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))+1/8*ln(x^2-x+1)+1/
12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 41, normalized size = 0.85

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-\frac{1}{4x^4}+\frac{1}{8}\log\left(x^8+x^4+1\right)-\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 1/4/x^4 + 1/8*log(x^8 + x^4
 + 1) - 1/4*log(x^4)
```

Fricas [A]
time = 0.37, size = 49, normalized size = 1.02

$$-\frac{2\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4+1\right)\right)-3x^4\log\left(x^8+x^4+1\right)+24x^4\log\left(x\right)+6}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 + 1)) - 3*x^4*log(x^8 + x^4
+ 1) + 24*x^4*log(x) + 6)/x^4
```

Sympy [A]
time = 0.07, size = 48, normalized size = 1.00

$$-\log\left(x\right)+\frac{\log\left(x^8+x^4+1\right)}{8}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}+\frac{\sqrt{3}}{3}\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+x**4+1),x)
```

```
[Out] -log(x) + log(x**8 + x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 + sqrt(3)/
3)/12 - 1/(4*x**4)
```

Giac [A]
time = 3.10, size = 46, normalized size = 0.96

$$-\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^{4}+1\right)\right)+\frac{x^{4}-1}{4\,x^{4}}+\frac{1}{8}\,\log\left(x^{8}+x^{4}+1\right)-\frac{1}{4}\,\log\left(x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 + 1)) + 1/4*(x^4 - 1)/x^4 + 1/8*log
(x^8 + x^4 + 1) - 1/4*log(x^4)
```

Mupad [B]
time = 0.06, size = 41, normalized size = 0.85

$$\frac{\ln\left(x^{8}+x^{4}+1\right)}{8}-\ln\left(x\right)-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^{4}}{3}+\frac{\sqrt{3}}{3}\right)}{12}-\frac{1}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^4 + x^8 + 1)),x)
```

```
[Out] log(x^4 + x^8 + 1)/8 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^4)/3
))/12 - 1/(4*x^4)
```

3.337 $\int \frac{1}{x^7(1+x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)$$

```
[Out] -1/6/x^6+1/2/x^2+1/8*ln(x^4-x^2+1)-1/8*ln(x^4+x^2+1)-1/12*arctan(1/3*(-2*x^
2+1)*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {1373, 1137, 1295, 12, 1141, 1175, 632, 210, 1178, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{8}\log\left(x^4-x^2+1\right)-\frac{1}{8}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 + 1/(2*x^2) - ArcTan[(1 - 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[(
1 + 2*x^2)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^2 + x^4]/8 - Log[1 + x^2 + x^4]
/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(1+x^4+x^8\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(1+x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{-3-3x^2}{x^2\left(1+x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{1}{6}\text{Subst}\left(\int -\frac{3x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1+x^2+x^4}\,dx,x,x^2\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1+x^2+x^4}\,dx,x\right.\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{8}\text{Subst}\left(\int \frac{1+2x}{-1-x-x^2}\,dx,x,x^2\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1-2x}{-1+x-x^2}\,dx\right.\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1+x^2+x^4\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\right.\\
&= -\frac{1}{6x^6}+\frac{1}{2x^2}-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^2+x^4\right)-\frac{1}{8}\log\left(1\right.
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.07, size = 142, normalized size = 1.60

$$\frac{1}{24}\left(-\frac{4}{x^6}+\frac{12}{x^2}+2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+\sqrt{3}\left(-i+\sqrt{3}\right)\log\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^2\right)+\sqrt{3}\left(i+\sqrt{3}\right)\log\left(\frac{1}{2}i\left(i+\sqrt{3}\right)+x^2\right)-3\log\left(1-x+x^2\right)-3\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + x^4 + x^8)),x]
```

```
[Out] (-4/x^6 + 12/x^2 + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[
(1 + 2*x)/Sqrt[3]] + Sqrt[3]*(-I + Sqrt[3])*Log[-1/2 - (I/2)*Sqrt[3] + x^2]
 + Sqrt[3]*(I + Sqrt[3])*Log[(I/2)*(I + Sqrt[3]) + x^2] - 3*Log[1 - x + x^2
] - 3*Log[1 + x + x^2])/24
```

Maple [A]
time = 0.04, size = 95, normalized size = 1.07

method	result
risch	$\frac{\frac{x^4}{2}-\frac{1}{6}}{x^6}-\frac{\ln(x^4+x^2+1)}{8}+\frac{\sqrt{3}\arctan\left(\frac{2\left(x^2+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}+\frac{\ln(x^4-x^2+1)}{8}+\frac{\sqrt{3}\arctan\left(\frac{2\left(x^2-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$
default	$-\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{1}{6x^6}+\frac{1}{2x^2}+\frac{\ln(x^4-x^2+1)}{8}+\frac{\sqrt{3}\arctan\left(\frac{(2x^2-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(x^2-x}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/6/x^6+1/2/x^2+1
/8*ln(x^4-x^2+1)+1/12*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/8*ln(x^2-x+1)
+1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A]
time = 0.48, size = 73, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{3x^4-1}{6x^6}-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt
(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x
^4 - x^2 + 1)
```

Fricas [A]
time = 0.35, size = 84, normalized size = 0.94

$$\frac{2\sqrt{3}x^6\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+2\sqrt{3}x^6\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-3x^6\log\left(x^4+x^2+1\right)+3x^6\log\left(x^4-x^2+1\right)+12x^4-4}{24x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(3)*x^6*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 2*sqrt(3)*x^6*arctan(
1/3*sqrt(3)*(2*x^2 - 1)) - 3*x^6*log(x^4 + x^2 + 1) + 3*x^6*log(x^4 - x^2 +
 1) + 12*x^4 - 4)/x^6
```

Sympy [A]
time = 0.10, size = 88, normalized size = 0.99

$$\frac{\log\left(x^4-x^2+1\right)}{8}-\frac{\log\left(x^4+x^2+1\right)}{8}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{12}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{12}+\frac{3x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+x**4+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/8 - log(x**4 + x**2 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**2/3 - sqrt(3)/3)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/12 + (3*x**4 - 1)/(6*x**6)
```

Giac [A]
time = 3.57, size = 73, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)+\frac{3x^4-1}{6x^6}-\frac{1}{8}\log\left(x^4+x^2+1\right)+\frac{1}{8}\log\left(x^4-x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/6*(3*x^4 - 1)/x^6 - 1/8*log(x^4 + x^2 + 1) + 1/8*log(x^4 - x^2 + 1)
```

Mupad [B]
time = 0.04, size = 62, normalized size = 0.70

$$\mathrm{atanh}\left(\frac{2x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atanh}\left(\frac{2x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\frac{\frac{x^4}{2}-\frac{1}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^4 + x^8 + 1)),x)
```

```
[Out] atanh((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atanh((2*x^2)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) + (x^4/2 - 1/6)/x^6
```

3.338 $\int \frac{x^8}{1+x^4+x^8}\,dx$

Optimal. Leaf size=141

$$x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1\right.$$

```
[Out] x-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/8*ln(x^
2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*3^(
1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2+x*3^(1/2))*3^(
1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1381, 1433, 1108, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\text{ArcTan}\left(\sqrt{3}-2x\right)-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\text{ArcTan}\left(2x+\sqrt{3}\right)+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}+x$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + x^4 + x^8),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 - ArcTa
n[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^
2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 +
Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{1+x^4+x^8}\,dx &= x-\int \frac{1+x^4}{1+x^4+x^8}\,dx\\ &= x-\frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx\\ &= x-\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx-\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}\\ &= x-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1+2x}{1+x+x^2}\,dx\\ &= x+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+\right.}{8\sqrt{3}}\\ &= x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{1}{8}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 139, normalized size = 0.99

$$-\frac{i\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{-6+6i\sqrt{3}}}+\frac{i\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-6-6i\sqrt{3}}}+\frac{1}{24}\left(24x-2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+3\log\left(1-x+x^2\right)-3\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + x^4 + x^8),x]
```

```
[Out] ((-I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[-6 + (6*I)*Sqrt[3]] + (I*ArcTan[(
(1 + I*Sqrt[3])*x)/2])/Sqrt[-6 - (6*I)*Sqrt[3]] + (24*x - 2*Sqrt[3]*ArcTan[
(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*Log[1 - x + x
^2] - 3*Log[1 + x + x^2])/24
```

Maple [A]
time = 0.04, size = 110, normalized size = 0.78

method	result
risch	$x-\frac{\ln(4x^2+4x+4)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(3\_R^3-\_R+x\right)\right)}{4}+\frac{\ln(4x^2-4}{8}$
default	$x-\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] x-1/8*ln(x^2+x+1)-1/12*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/4*arctan(2*x-3
^(1/2))-1/4*arctan(2*x+3^(1/2))+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+
x^2+x*3^(1/2))*3^(1/2)+1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1
/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + x - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x +
 1) + 1/8*log(x^2 - x + 1)
```

Fricas [A]
time = 0.36, size = 216, normalized size = 1.53

$$\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{36}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{-72\sqrt{6}\sqrt{2}x+144x^2+144}+\sqrt{3}\right)+\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(72\sqrt{6}\sqrt{2}x+144x^2+144\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(-72\sqrt{6}\sqrt{2}x+144x^2+144\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/36*s
qrt(6)*sqrt(3)*sqrt(2)*sqrt(-72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) + sqrt(3
)) + 1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1
/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/48*sq
rt(6)*sqrt(2)*log(72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) + 1/48*sqrt(6)*sqrt
(2)*log(-72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) - 1/12*sqrt(3)*arctan(1/3*sq
rt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 1/8*log
(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.36, size = 192, normalized size = 1.36

$$x+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}-9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-9216t^5-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+x**4+1),x)
```

```
[Out] x + (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/24
)**5) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + sqr
t(3)*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 + sq
rt(3)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/
24)**5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*lo
g(-9216*_t**5 - 8*_t + x)))
```

Giac [**A**]
time = 3.51, size = 109, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+x-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqrt(
3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Mupad [**B**]
time = 0.10, size = 100, normalized size = 0.71

$$x-\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^4 + x^8 + 1),x)
```

```
[Out] x - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) - atan((2*x)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/
2)/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

3.339 $\int \frac{x^6}{1+x^4+x^8} dx$

Optimal. Leaf size=88

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}$$

```
[Out] -1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1386, 1178, 642, 1175, 632, 210}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + x^4 + x^8),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sq
rt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1386

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[x^(
m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Dist[1/(2*c*
r), Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6}{1+x^4+x^8}\,dx &= -\left(\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\
&= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}\,x-x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}\,x-x^2}\,dx}{4\sqrt{3}}\\
&= \frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\\
&= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 68, normalized size = 0.77

$$
\frac{2\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+\log\left(-1+\sqrt{3}\,x-x^2\right)-\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + x^4 + x^8),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] + Log[-1 + Sqrt
[3]*x - x^2] - Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

Maple [A]
time = 0.03, size = 67, normalized size = 0.76

method	result	size
default	$\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	67
risch	$\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\sqrt{3}\arctan\left(\frac{x^3\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/
12*ln(1+x^2+x*3^(1/2))*3^(1/2)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Fricas [A]
time = 0.34, size = 70, normalized size = 0.80

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(x^3+2x\right)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{12}\sqrt{3}\log\left(\frac{x^4+5x^2-2\sqrt{3}\left(x^3+x\right)+1}{x^4-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2
+ 1))
```

Sympy [A]
time = 0.07, size = 82, normalized size = 0.93

$$\frac{\sqrt{3}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{3}\,x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\sqrt{3}\,x}{3}\right)\right)}{12}+\frac{\sqrt{3}\,\log\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\,\log\left(x^2+\sqrt{3}\,x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 +
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2
```

Giac [A]
time = 2.86, size = 66, normalized size = 0.75

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)-\frac{1}{12}\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)+\frac{1}{12}\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/12*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1)
```

Mupad [B]
time = 1.31, size = 38, normalized size = 0.43

$$-\frac{\sqrt{3}\,\left(\operatorname{atan}\left(\frac{2\,\sqrt{3}\,x}{3\,\left(\frac{2\,x^2}{3}-\frac{2}{3}\right)}\right)+\operatorname{atanh}\left(\frac{2\,\sqrt{3}\,x}{3\,\left(\frac{2\,x^2}{3}+\frac{2}{3}\right)}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^4 + x^8 + 1),x)
```

```
[Out] -(3^(1/2)*(atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))) + atanh((2*3^(1/2)*x)/
(3*((2*x^2)/3 + 2/3)))))/6
```

3.340 $\int \frac{x^4}{1+x^4+x^8}\,dx$

Optimal. Leaf size=140

$$\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1-\right.$$

```
[Out] 1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/8*ln(x^2+
x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*3^(1/
2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2+x*3^(1/2))*3^(1/
2)
```

Rubi [A]

time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1387, 1141, 1175, 632, 210, 1178, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\text{ArcTan}\left(\sqrt{3}-2x\right)-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\text{ArcTan}\left(2x+\sqrt{3}\right)-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + x^4 + x^8),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1
 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 - Log[1 - x + x^2]/8
 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) - Log[1 + Sqrt
[3]*x + x^2]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{1+x^4+x^8}\,dx = \frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx - \frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx$$
$$= -\left(\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx\right) + \frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx + \frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx - \frac{1}{4}\int \frac{1+x}{1+x^2}$$
$$= -\left(\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx\right) - \frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx - \frac{1}{8}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+x}$$
$$= -\frac{1}{8}\log\left(1-x+x^2\right) + \frac{1}{8}\log\left(1+x+x^2\right) + \frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{8\sqrt{3}} - \frac{\log\left(1+\sqrt{3}\,x+\right.}{8\sqrt{3}}$$
$$= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right) - \frac{1}{8}\mathrm{lo}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.10, size = 135, normalized size = 0.96

$$\frac{1}{24}\left(-2i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)+2i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)-2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-3\log\left(1-x+x^2\right)+3\log\left(1+x+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + x^4 + x^8),x]
[Out] ((-2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] + (2*I)*Sqrt
[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTan[(-1 +
 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x + x^2] +
 3*Log[1 + x + x^2])/24
```

Maple [A]
time = 0.04, size = 121, normalized size = 0.86

method	result
risch	$-\frac{\ln(4x^2-4x+4)}{8} - \frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)} \_R\ln\left(6\_R^3+\_R+x\right)\right)}{4} + \frac{\ln(4x^2+4x}{8}$
default	$\frac{\ln(x^2+x+1)}{8} - \frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12} - \frac{\sqrt{3}\left(-\frac{\ln\left(1+x^2-x\sqrt{3}\right)}{2} - \sqrt{3}\,\arctan\left(2x-\sqrt{3}\right)\right)}{12} - \frac{\sqrt{3}\left(\frac{\ln\left(1+x^2+\right.}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/12*3^(1/2)*(-1/2
*ln(1+x^2-x*3^(1/2))-3^(1/2)*arctan(2*x-3^(1/2)))-1/12*3^(1/2)*(1/2*ln(1+x^
2+x*3^(1/2))-3^(1/2)*arctan(2*x+3^(1/2)))-1/8*ln(x^2-x+1)-1/12*3^(1/2)*arct
an(1/3*(2*x-1)*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 + x + 1
) - 1/8*log(x^2 - x + 1)
```

Fricas [A]
time = 0.36, size = 215, normalized size = 1.54

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{18}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{-18\sqrt{6}\sqrt{2}x+36x^2+36}+\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(18\sqrt{6}\sqrt{2}x+36x^2+36\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(-18\sqrt{6}\sqrt{2}x+36x^2+36\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/18*
sqrt(6)*sqrt(3)*sqrt(2)*sqrt(-18*sqrt(6)*sqrt(2)*x + 36*x^2 + 36) + sqrt(3)
) - 1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/
3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) - 1/48*sqr
t(6)*sqrt(2)*log(18*sqrt(6)*sqrt(2)*x + 36*x^2 + 36) + 1/48*sqrt(6)*sqrt(2)
*log(-18*sqrt(6)*sqrt(2)*x + 36*x^2 + 36) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/8*log(x^2 + x
+ 1) - 1/8*log(x^2 - x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.38, size = 197, normalized size = 1.41

$$\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{3}i}{6}-18432\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+\frac{1}{2}-18432\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5-\frac{\sqrt{3}i}{6}\right)+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-18432t^5-4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+x**4+1),x)
```

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24)
**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 - s
qrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-1/8
 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sq
```

```
rt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_
t, _t*log(-18432*_t**5 - 4*_t + x)))
```

Giac [A]
time = 2.94, size = 108, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3))
+ 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Mupad [B]
time = 0.07, size = 99, normalized size = 0.71

$$-\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4 + x^8 + 1),x)
```

```
[Out] - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4)
```

3.341 $\int \frac{x^2}{1+x^4+x^8} \, dx$

Optimal. Leaf size=140

$$\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+\right.$$

[Out] 1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/8*ln(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x^2+x*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1387, 1108, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\text{ArcTan}\left(\sqrt{3}-2x\right)-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\text{ArcTan}\left(2x+\sqrt{3}\right)+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(1 + x^4 + x^8),x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*x]/4 + Log[1 - x + x^2]/8 - Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{1+x^4+x^8}\,dx &= \frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx\\
&= -\left(\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx\right)-\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}\,x+x^2}\,dx}{4\sqrt{3}}+\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}\,x+x^2}\,dx}{4\sqrt{3}}\\
&= -\left(\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx\right)+\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1+2x}{1+x+x^2
\\
&= \frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)-\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+x^
\\
&= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{1}{8}\text{lo}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.09, size = 135, normalized size = 0.96

$$
\frac{1}{48}\left(4i\sqrt{-6-6i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-4i\sqrt{-6+6i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)-4\sqrt{3}\,\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-4\sqrt{3}\,\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+6\log\left(1-x+x^2\right)-6\log\left(1+x+x^2\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^4 + x^8),x]
```

```
[Out] ((4*I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (4*I)*Sqrt[
-6 + (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 4*Sqrt[3]*ArcTan[(-1 +
2*x)/Sqrt[3]] - 4*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 6*Log[1 - x + x^2] -
6*Log[1 + x + x^2])/48
```

Maple [A]
time = 0.04, size = 109, normalized size = 0.78

method	result
risch	$\frac{\ln(4x^2-4x+4)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}-\frac{\ln(4x^2+4x+4)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\left(\sum_{\_R=\mathrm{RootOf}(9\_Z^4+3\_Z^2$
default	$-\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\arctan\left(2x-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x+\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{24}+$ ln

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/4*arctan(2*x-3^
(1/2))+1/4*arctan(2*x+3^(1/2))-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x
^2+x*3^(1/2))*3^(1/2)+1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/
2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2 + x + 1)
+ 1/8*log(x^2 - x + 1)
```

Fricas [A]
time = 0.40, size = 215, normalized size = 1.54

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{36}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{-72\sqrt{6}\sqrt{2}x+144x^2+144}+\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(72\sqrt{6}\sqrt{2}x+144x^2+144\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(-72\sqrt{6}\sqrt{2}x+144x^2+144\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1/36*
sqrt(6)*sqrt(3)*sqrt(2)*sqrt(-72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) + sqrt(
3)) - 1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x +
1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + 1/48*s
qrt(6)*sqrt(2)*log(72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) - 1/48*sqrt(6)*sqr
t(2)*log(-72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) - 1/12*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/8*log(x^
2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.37, size = 214, normalized size = 1.53

$$\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x+442368\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{7}-192\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x-192\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{3}+442368\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{7}\right)+\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x+442368\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{7}-192\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{3}\right)+\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x-192\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{3}+442368\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{7}\right)+\operatorname{RootSum}\left(2304t^{4}+48t^{2}+1,\left(t\mapsto t\log\left(442368t^{7}-192t^{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x + 442368*(-1/8 - sqrt(3)*I/24)**7 - 192*(-1/8 -
 sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 192*(-1/8 + sqrt(3)*I/24
)**3 + 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x + 4423
68*(1/8 - sqrt(3)*I/24)**7 - 192*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*
I/24)*log(x - 192*(1/8 + sqrt(3)*I/24)**3 + 442368*(1/8 + sqrt(3)*I/24)**7)
 + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(442368*_t**7 - 192*
_t**3 + x)))
```

Giac [A]
time = 3.75, size = 108, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)+\frac{1}{24}\sqrt{3}\log\left(x^{2}+\sqrt{3}\,x+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^{2}-\sqrt{3}\,x+1\right)+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{8}\log\left(x^{2}+x+1\right)+\frac{1}{8}\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + 1/4*arctan(2*x + sqrt(3)) + 1/4*arctan(2*x - sqrt(3))
- 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Mupad [B]
time = 1.31, size = 97, normalized size = 0.69

$$\operatorname{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\operatorname{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\operatorname{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4 + x^8 + 1),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) + atan((2*x)/(3^(1/2)*
1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/1
2 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4)
```

3.342 $\int \frac{1}{1+x^4+x^8} \, dx$

Optimal. Leaf size=88

$$-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}$$

```
[Out] -1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^
(1/2)-1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1360, 1178, 642, 1175, 632, 210}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4 + x^8)^(-1),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sq
rt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1360

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^4+x^8}\,dx &= \frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\ &= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx-\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}\,x-x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}\,x-x^2}\,dx}{4\sqrt{3}}\\ &= -\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 68, normalized size = 0.77

$$\frac{2\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-\log\left(-1+\sqrt{3}\,x-x^2\right)+\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4 + x^8)^(-1),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[3]] + 2*ArcTan[(1 + 2*x)/Sqrt[3]] - Log[-1 + Sqrt
[3]*x - x^2] + Log[1 + Sqrt[3]*x + x^2])/(4*Sqrt[3])
```

Maple [A]
time = 0.03, size = 67, normalized size = 0.76

method	result	size
default	$\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln\left(1+x^{2}-x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^{2}+x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{6}$	67
risch	$\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{x^{3}\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(1+x^{2}+x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^{2}-x\sqrt{3}\right)\sqrt{3}}{12}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/
12*ln(1+x^2+x*3^(1/2))*3^(1/2)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1), x)
```

Fricas [A]
time = 0.38, size = 70, normalized size = 0.80

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(x^{3}+2\,x)\right)+\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,x\right)+\frac{1}{12}\sqrt{3}\,\log\left(\frac{x^{4}+5\,x^{2}+2\,\sqrt{3}\,(x^{3}+x)+1}{x^{4}-x^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3
)*x) + 1/12*sqrt(3)*log((x^4 + 5*x^2 + 2*sqrt(3)*(x^3 + x) + 1)/(x^4 - x^2
+ 1))
```

Sympy [A]
time = 0.07, size = 82, normalized size = 0.93

$$\frac{\sqrt{3}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x}{3}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\sqrt{3}\,x}{3}\right)\right)}{12}-\frac{\sqrt{3}\,\log\left(x^2-\sqrt{3}\,x+1\right)}{12}+\frac{\sqrt{3}\,\log\left(x^2+\sqrt{3}\,x+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 -
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2
```

Giac [A]
time = 4.43, size = 66, normalized size = 0.75

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x+1)\right)+\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x-1)\right)+\frac{1}{12}\sqrt{3}\,\log\left(x^2+\sqrt{3}\,x+1\right)-\frac{1}{12}\sqrt{3}\,\log\left(x^2-\sqrt{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/12*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/12*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1)
```

Mupad [B]
time = 0.04, size = 40, normalized size = 0.45

$$-\frac{\sqrt{3}\,\left(\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3\,\left(\frac{2\,x^2}{3}-\frac{2}{3}\right)}\right)-\mathrm{atanh}\left(\frac{2\,\sqrt{3}\,x}{3\,\left(\frac{2\,x^2}{3}+\frac{2}{3}\right)}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4 + x^8 + 1),x)
```

```
[Out] -(3^(1/2)*(atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))) - atanh((2*3^(1/2)*x)/
(3*((2*x^2)/3 + 2/3)))))/6
```

3.343 $\int \frac{1}{x^2(1+x^4+x^8)}\,dx$

Optimal. Leaf size=145

$$-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\mathrm{lo}$$

```
[Out] -1/x-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2-x+1)+1/8*ln
(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2*x)*
3^(1/2))*3^(1/2)-1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)+1/24*ln(1+x^2+x*3^(1/2))*
3^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1382, 1520, 1141, 1175, 632, 210, 1178, 642}

$$\frac{\mathrm{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\mathrm{ArcTan}\left(\sqrt{3}-2x\right)-\frac{\mathrm{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\mathrm{ArcTan}\left(2x+\sqrt{3}\right)-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/4 -
 ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 - Log[1 -
x + x^2]/8 + Log[1 + x + x^2]/8 - Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) + Lo
g[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1520

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[(f*x)^m*(Simp[d*r - (c*d - e*q)*x^(n/2), x]/
(q - r*x^(n/2) + c*x^n)), x], x] + Dist[c/(2*q*r), Int[(f*x)^m*(Simp[d*r +
(c*d - e*q)*x^(n/2), x]/(q + r*x^(n/2) + c*x^n)), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(1+x^4+x^8\right)}\,dx &= -\frac{1}{x}+\int \frac{x^2(-1-x^4)}{1+x^4+x^8}\,dx \\
&= -\frac{1}{x}-\frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx \\
&= -\frac{1}{x}+\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx-\frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx-\frac{1}{4}\int \frac{}{1} \\
&= -\frac{1}{x}-\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx-\frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{8}\int \\
&= -\frac{1}{x}-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)-\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{8\sqrt{3}}+\frac{\log\left(1+\right.}{} \\
&= -\frac{1}{x}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+\right.
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.13, size = 140, normalized size = 0.97

$$
\frac{1}{24}\left(-\frac{24}{x}+2i\sqrt{-6+6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-2i\sqrt{-6-6i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)-2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-3\log\left(1-x+x^2\right)+3\log\left(1+x+x^2\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x^4 + x^8)),x]
```

```
[Out] (-24/x + (2*I)*Sqrt[-6 + (6*I)*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - (2*
I)*Sqrt[-6 - (6*I)*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2] - 2*Sqrt[3]*ArcTa
n[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 3*Log[1 - x +
 x^2] + 3*Log[1 + x + x^2])/24
```

Maple [A]
time = 0.04, size = 126, normalized size = 0.87

method	result
risch	$-\frac{1}{x}-\frac{\ln\left(4x^2-4x+4\right)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-6\_R^3-\_R+x\right)\right)}{4}+\frac{\ln(}{}$
default	$\frac{\ln\left(x^2+x+1\right)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{1}{x}+\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^2-x\sqrt{3}\right)}{2}-\sqrt{3}\,\arctan\left(2x-\sqrt{3}\right)\right)}{12}+\frac{\sqrt{3}\left(\frac{\ln(}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/x+1/12*3^(1/2)*(
-1/2*ln(1+x^2-x*3^(1/2))-3^(1/2)*arctan(2*x-3^(1/2)))+1/12*3^(1/2)*(1/2*ln(
1+x^2+x*3^(1/2))-3^(1/2)*arctan(2*x+3^(1/2)))-1/8*ln(x^2-x+1)-1/12*3^(1/2)*
arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/x - 1/2*integrate(x^2/(x^4 - x^2 + 1), x) + 1/8*log(x^2 +
 x + 1) - 1/8*log(x^2 - x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(113) = 226.
time = 0.37, size = 228, normalized size = 1.57

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}\,x\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}\,x+\frac{1}{18}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{-18\sqrt{6}\sqrt{2}\,x+36x^2+36}+\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}\,x\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}\,x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}\,x+2x^2+2}-\sqrt{3}\right)+\sqrt{6}\sqrt{2}\,x\log\left(18\sqrt{6}\sqrt{2}\,x+36x^2+36\right)-\sqrt{6}\sqrt{2}\,x\log\left(-18\sqrt{6}\sqrt{2}\,x+36x^2+36\right)-4\sqrt{3}\,x\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-4\sqrt{3}\,x\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+6x\log\left(x^2+x+1\right)-6x\log\left(x^2-x+1\right)-48}{48x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x + 1
/18*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(-18*sqrt(6)*sqrt(2)*x + 36*x^2 + 36) + sqr
t(3)) + 4*sqrt(6)*sqrt(3)*sqrt(2)*x*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x +
 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) + sqrt(
6)*sqrt(2)*x*log(18*sqrt(6)*sqrt(2)*x + 36*x^2 + 36) - sqrt(6)*sqrt(2)*x*lo
g(-18*sqrt(6)*sqrt(2)*x + 36*x^2 + 36) - 4*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*
x + 1)) - 4*sqrt(3)*x*arctan(1/3*sqrt(3)*(2*x - 1)) + 6*x*log(x^2 + x + 1)
- 6*x*log(x^2 - x + 1) - 48)/x
```

Sympy [C] Result contains complex when optimal does not.
time = 0.37, size = 218, normalized size = 1.50

$$\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-384\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3-442368\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-442368\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^7-384\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^3\right)+\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-384\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^3-442368\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^7\right)+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-442368t^7-384t^3+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+x**4+1),x)
```

```
[Out] (-1/8 - sqrt(3)*I/24)*log(x - 442368*(-1/8 - sqrt(3)*I/24)**7 - 384*(-1/8 -
 sqrt(3)*I/24)**3) + (-1/8 + sqrt(3)*I/24)*log(x - 384*(-1/8 + sqrt(3)*I/24
)**3 - 442368*(-1/8 + sqrt(3)*I/24)**7) + (1/8 - sqrt(3)*I/24)*log(x - 4423
68*(1/8 - sqrt(3)*I/24)**7 - 384*(1/8 - sqrt(3)*I/24)**3) + (1/8 + sqrt(3)*
I/24)*log(x - 384*(1/8 + sqrt(3)*I/24)**3 - 442368*(1/8 + sqrt(3)*I/24)**7)
 + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-442368*_t**7 - 384
*_t**3 + x))) - 1/x
```

Giac [**A**]
time = 4.28, size = 113, normalized size = 0.78

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{1}{x}-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{8}\log\left(x^2+x+1\right)-\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) - 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) - 1/x - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x - sqr
t(3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Mupad [**B**]
time = 0.05, size = 102, normalized size = 0.70

$$\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 + x^8 + 1)),x)
```

```
[Out] atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x)/(3^(1/2)*
1i + 1))*((3^(1/2)*1i)/12 - 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/1
2 - 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 + 1i/4) - 1/x
```

3.344 $\int \frac{1}{x^4(1+x^4+x^8)}\,dx$

Optimal. Leaf size=147

$$-\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\,\mathrm{l}$$

```
[Out] -1/3/x^3-1/4*arctan(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))+1/8*ln(x^2-x+1)-1/
8*ln(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2
*x)*3^(1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2+x*3^(1/
2))*3^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1382, 1433, 1108, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\text{ArcTan}\left(\sqrt{3}-2x\right)-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\text{ArcTan}\left(2x+\sqrt{3}\right)-\frac{1}{3x^3}+\frac{1}{8}\log\left(x^2-x+1\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] - 2*x]/
4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqrt[3] + 2*x]/4 + Log[1
 - x + x^2]/8 - Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(8*Sqrt[3]) -
 Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1433

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e +
 q*x^(n/2) + x^n, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x^(n/2) + x
^n, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4
*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[n/2, 0] && (GtQ[2*(d/e) - b/c, 0]
 || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d, e*Rt[a/c, 2]]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4(1+x^4+x^8)}\,dx &= -\frac{1}{3x^3}+\frac{1}{3}\int \frac{-3-3x^4}{1+x^4+x^8}\,dx \\
&= -\frac{1}{3x^3}-\frac{1}{2}\int \frac{1}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{1}{1+x^2+x^4}\,dx \\
&= -\frac{1}{3x^3}-\frac{1}{4}\int \frac{1-x}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1+x}{1+x+x^2}\,dx-\frac{\int \frac{\sqrt{3}-x}{1-\sqrt{3}x+x^2}\,dx}{4\sqrt{3}}-\frac{\int \frac{\sqrt{3}+x}{1+\sqrt{3}x+x^2}}{4\sqrt{3}} \\
&= -\frac{1}{3x^3}-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{8}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+x+x^2}\,dx-\frac{1}{8}\int \frac{1}{1+} \\
&= -\frac{1}{3x^3}+\frac{1}{8}\log\left(1-x+x^2\right)-\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{8\sqrt{3}}-\frac{\log\left(1-\right.}{} \\
&= -\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+\right.
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.17, size = 148, normalized size = 1.01

$$
\frac{1}{24}\left(-\frac{8}{x^3}-\frac{4i\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{\sqrt{\frac{1}{6}i\left(i+\sqrt{3}\right)}}+\frac{4i\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{\sqrt{-\frac{1}{6}i\left(-i+\sqrt{3}\right)}}-2\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+3\log\left(1-x+x^2\right)-3\log\left(1+x+x^2\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + x^4 + x^8)),x]
```

```
[Out] (-8/x^3 - ((4*I)*ArcTan[((1 - I*Sqrt[3])*x)/2])/Sqrt[(I/6)*(I + Sqrt[3])] +
 ((4*I)*ArcTan[((1 + I*Sqrt[3])*x)/2])/Sqrt[(-1/6*I)*(-I + Sqrt[3])] - 2*Sq
rt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 3*
Log[1 - x + x^2] - 3*Log[1 + x + x^2])/24
```

Maple [A]
time = 0.04, size = 114, normalized size = 0.78

method	result
risch	$-\frac{1}{3x^3}-\frac{\ln(4x^2+4x+4)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}+\frac{\ln(4x^2-4x+4)}{8}-\frac{\sqrt{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{12}+\frac{\left(\sum_{\_R=\text{RootOf}(9}\right.}{}$
default	$-\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{1}{3x^3}-\frac{\arctan\left(2x-\sqrt{3}\right)}{4}-\frac{\arctan\left(2x+\sqrt{3}\right)}{4}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{}}{24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(x^2+x+1)-1/12*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/3/x^3-1/4*arcta
n(2*x-3^(1/2))-1/4*arctan(2*x+3^(1/2))+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/2
4*ln(1+x^2+x*3^(1/2))*3^(1/2)+1/8*ln(x^2-x+1)-1/12*3^(1/2)*arctan(1/3*(2*x-
1)*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/3/x^3 - 1/2*integrate(1/(x^4 - x^2 + 1), x) - 1/8*log(x^2
 + x + 1) + 1/8*log(x^2 - x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(113) = 226$.
time = 0.38, size = 244, normalized size = 1.66

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x^3\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{36}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{-72\sqrt{6}\sqrt{2}x+144x^2+144}+\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x^3\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)-\sqrt{6}\sqrt{2}x^3\log\left(72\sqrt{6}\sqrt{2}x+144x^2+144\right)+\sqrt{6}\sqrt{2}x^3\log\left(-72\sqrt{6}\sqrt{2}x+144x^2+144\right)-4\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-4\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-6x^3\log\left(x^2+x+1\right)+6x^3\log\left(x^2-x+1\right)-16}{48x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^3*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x +
 1/36*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(-72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) +
 sqrt(3)) + 4*sqrt(6)*sqrt(3)*sqrt(2)*x^3*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(
2)*x + 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3)) -
 sqrt(6)*sqrt(2)*x^3*log(72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) + sqrt(6)*sq
rt(2)*x^3*log(-72*sqrt(6)*sqrt(2)*x + 144*x^2 + 144) - 4*sqrt(3)*x^3*arctan
(1/3*sqrt(3)*(2*x + 1)) - 4*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*x
^3*log(x^2 + x + 1) + 6*x^3*log(x^2 - x + 1) - 16)/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 0.42, size = 197, normalized size = 1.34

$$\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x-1-\frac{\sqrt{3}i}{3}-9216\left(\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right)+\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x-1-9216\left(\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)\log\left(x+1-\frac{\sqrt{3}i}{3}-9216\left(-\frac{1}{8}+\frac{\sqrt{3}i}{24}\right)^5\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)\log\left(x+1-9216\left(-\frac{1}{8}-\frac{\sqrt{3}i}{24}\right)^5+\frac{\sqrt{3}i}{3}\right)+\operatorname{RootSum}\left(2304t^4+48t^2+1,\left(t\mapsto t\log\left(-9216t^5-8t+x\right)\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+x**4+1),x)
```

```
[Out] (1/8 + sqrt(3)*I/24)*log(x - 1 - sqrt(3)*I/3 - 9216*(1/8 + sqrt(3)*I/24)**5
) + (1/8 - sqrt(3)*I/24)*log(x - 1 - 9216*(1/8 - sqrt(3)*I/24)**5 + sqrt(3)
*I/3) + (-1/8 + sqrt(3)*I/24)*log(x + 1 - sqrt(3)*I/3 - 9216*(-1/8 + sqrt(3
)*I/24)**5) + (-1/8 - sqrt(3)*I/24)*log(x + 1 - 9216*(-1/8 - sqrt(3)*I/24)*
*5 + sqrt(3)*I/3) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_t, _t*log(-9
216*_t**5 - 8*_t + x))) - 1/(3*x**3)
```

Giac [A]
time = 4.13, size = 113, normalized size = 0.77

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)-\frac{1}{3x^3}-\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)-\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)-\frac{1}{8}\log\left(x^2+x+1\right)+\frac{1}{8}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) - 1/3/x^3 - 1/4*arctan(2*x + sqrt(3)) - 1/4*arctan(2*x -
 sqrt(3)) - 1/8*log(x^2 + x + 1) + 1/8*log(x^2 - x + 1)
```

Mupad [B]
time = 0.03, size = 104, normalized size = 0.71

$$-\mathrm{atan}\left(\frac{2x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{2x}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^4 + x^8 + 1)),x)
```

```
[Out] - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 - 1/4) - atan((2*x)/(3^(1/2
)*1i + 1))*((3^(1/2)*1i)/12 + 1/4) - atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)
/12 + 1i/4) - atan((x*2i)/(3^(1/2)*1i + 1))*(3^(1/2)/12 - 1i/4) - 1/(3*x^3)
```

3.345 $\int \frac{1}{x^6\left(1+x^4+x^8\right)} dx$

Optimal. Leaf size=98

$$-\frac{1}{5x^5}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x+x^2\right)}{4\sqrt{3}}$$

```
[Out] -1/5/x^5+1/x-1/6*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2*x)
*3^(1/2))*3^(1/2)+1/12*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))
*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1382, 1518, 12, 1386, 1178, 642, 1175, 632, 210}

$$-\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{5x^5}+\frac{\log\left(x^2-\sqrt{3}\,x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}\,x+1\right)}{4\sqrt{3}}+\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 + x^(-1) - ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2
*x)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 - Sqrt[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + S
qrt[3]*x + x^2]/(4*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1386

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[x^(
m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Dist[1/(2*c*
r), Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
```

```
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(1+x^4+x^8\right)}\,dx &= -\frac{1}{5x^5}+\frac{1}{5}\int \frac{-5-5x^4}{x^2\left(1+x^4+x^8\right)}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}-\frac{1}{5}\int -\frac{5x^6}{1+x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}+\int \frac{x^6}{1+x^4+x^8}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}-\frac{1}{2}\int \frac{1-x^2}{1-x^2+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^2+x^4}\,dx\\
&= -\frac{1}{5x^5}+\frac{1}{x}+\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx+\frac{\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx}{4\sqrt{3}}+\frac{\int -\cdots}{\cdots}\\
&= -\frac{1}{5x^5}+\frac{1}{x}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x\cdots}\right.\\
&= -\frac{1}{5x^5}+\frac{1}{x}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\log\left(1-\sqrt{3}x+x^2\right)}{4\sqrt{3}}-\frac{\log\left(1+\cdots\right.}{4\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 95, normalized size = 0.97

$$
\frac{1}{60}\left(-\frac{12}{x^5}+\frac{60}{x}+10\sqrt{3}\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)+10\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+5\sqrt{3}\log\left(-1+\sqrt{3}x-x^2\right)-5\sqrt{3}\log\left(1+\sqrt{3}x+x^2\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^4 + x^8)),x]
```

```
[Out] (-12/x^5 + 60/x + 10*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 10*Sqrt[3]*ArcTan
[(1 + 2*x)/Sqrt[3]] + 5*Sqrt[3]*Log[-1 + Sqrt[3]*x - x^2] - 5*Sqrt[3]*Log[1
 + Sqrt[3]*x + x^2])/60
```

Maple [A]
time = 0.04, size = 75, normalized size = 0.77

method	result

default	$\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{1}{5x^5}+\frac{1}{x}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right.}{6}$
risch	$\frac{x^4-\frac{1}{5}}{x^5}+\frac{\arctan\left(\frac{x\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{x^3\sqrt{3}}{3}+\frac{2x\sqrt{3}}{3}\right)}{6}+\frac{\ln\left(1+x^2-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^2+x\sqrt{3}\right)\sqrt{3}}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/5/x^5+1/x+1/12*ln(1+x^2-x*3^(1/2)
)*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3
^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/5*(5*x^4 - 1)/x^5 + 1/2*integrate((x^2 - 1)/(x^4 - x^2 + 1),
 x)
```

Fricas [A]
time = 0.35, size = 90, normalized size = 0.92

$$\frac{10\sqrt{3}\,x^5\arctan\left(\frac{1}{3}\sqrt{3}\,(x^3+2x)\right)+10\sqrt{3}\,x^5\arctan\left(\frac{1}{3}\sqrt{3}\,x\right)+5\sqrt{3}\,x^5\log\left(\frac{x^4+5x^2-2\sqrt{3}\,(x^3+x)+1}{x^4-x^2+1}\right)+60x^4-12}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] 1/60*(10*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(x^3 + 2*x)) + 10*sqrt(3)*x^5*arcta
n(1/3*sqrt(3)*x) + 5*sqrt(3)*x^5*log((x^4 + 5*x^2 - 2*sqrt(3)*(x^3 + x) + 1
)/(x^4 - x^2 + 1)) + 60*x^4 - 12)/x^5
```

Sympy [A]
time = 0.09, size = 94, normalized size = 0.96

$$\frac{\sqrt{3}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{3}\,x}{3}\right)+2\operatorname{atan}\left(\frac{\sqrt{3}\,x^3}{3}+\frac{2\sqrt{3}\,x}{3}\right)\right)}{12}+\frac{\sqrt{3}\log\left(x^2-\sqrt{3}\,x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}\,x+1\right)}{12}+\frac{5x^4-1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8+x**4+1),x)
```

```
[Out] sqrt(3)*(2*atan(sqrt(3)*x/3) + 2*atan(sqrt(3)*x**3/3 + 2*sqrt(3)*x/3))/12 +
 sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)*x + 1)/1
2 + (5*x**4 - 1)/(5*x**5)
```

Giac [A]
time = 4.01, size = 100, normalized size = 1.02

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)+\frac{5x^4-1}{5x^5}+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^2 -
 sqrt(3)*x + 1) + 1/5*(5*x^4 - 1)/x^5 + 1/4*arctan(2*x + sqrt(3)) + 1/4*arc
tan(2*x - sqrt(3))
```

Mupad [B]
time = 0.04, size = 52, normalized size = 0.53

$$\frac{x^4-\frac{1}{5}}{x^5}-\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}+\frac{2}{3}\right)}\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3\left(\frac{2x^2}{3}-\frac{2}{3}\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^4 + x^8 + 1)),x)
```

```
[Out] (x^4 - 1/5)/x^5 - (3^(1/2)*atanh((2*3^(1/2)*x)/(3*((2*x^2)/3 + 2/3))))/6 -
(3^(1/2)*atan((2*3^(1/2)*x)/(3*((2*x^2)/3 - 2/3))))/6
```

3.346 $\int \frac{1}{x^8(1+x^4+x^8)}\,dx$

Optimal. Leaf size=154

$$-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x\right)-\frac{1}{8}\log\left(1-x+x^2\right.$$

```
[Out] -1/7/x^7+1/3/x^3+1/4*arctan(2*x-3^(1/2))+1/4*arctan(2*x+3^(1/2))-1/8*ln(x^2
-x+1)+1/8*ln(x^2+x+1)+1/12*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)-1/12*arctan(
1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/24*ln(1+x^2-x*3^(1/2))*3^(1/2)-1/24*ln(1+x^2
+x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {1382, 1518, 12, 1387, 1141, 1175, 632, 210, 1178, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\text{ArcTan}\left(\sqrt{3}-2x\right)-\frac{\text{ArcTan}\left(\frac{2x+1}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\text{ArcTan}\left(2x+\sqrt{3}\right)-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(x^2-x+1\right)+\frac{1}{8}\log\left(x^2+x+1\right)+\frac{\log\left(x^2-\sqrt{3}x+1\right)}{8\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 + x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 + 1/(3*x^3) + ArcTan[(1 - 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcTan[Sqr
t[3] - 2*x]/4 - ArcTan[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) + ArcTan[Sqrt[3] + 2*
x]/4 - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8 + Log[1 - Sqrt[3]*x + x^2]/(
8*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(8*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
```

```
/2)] && NegQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8(1+x^4+x^8)}\,dx &= -\frac{1}{7x^7}+\frac{1}{7}\int \frac{-7-7x^4}{x^4(1+x^4+x^8)}\,dx\\
&= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{21}\int -\frac{21x^4}{1+x^4+x^8}\,dx\\
&= -\frac{1}{7x^7}+\frac{1}{3x^3}+\int \frac{x^4}{1+x^4+x^8}\,dx\\
&= -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{1}{2}\int \frac{x^2}{1-x^2+x^4}\,dx-\frac{1}{2}\int \frac{x^2}{1+x^2+x^4}\,dx\\
&= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{4}\int \frac{1-x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1+x^2}{1-x^2+x^4}\,dx+\frac{1}{4}\int \frac{1-x^2}{1+x^2+x^4}\,dx-\\
&= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\int \frac{1+2x}{-1-x-x^2}\,dx-\frac{1}{8}\int \frac{1-2x}{-1+x-x^2}\,dx-\frac{1}{8}\int \frac{1}{1-x+x^2}\,dx\\
&= -\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)+\frac{\log\left(1-\sqrt{3}\,x+x^2\right)}{8\sqrt{3}}-\\
&= -\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x\right)-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\right.
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.21, size = 171, normalized size = 1.11

$$-\frac{1}{7x^7}+\frac{1}{3x^3}+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)}{2\sqrt{-6+6i\sqrt{3}}}+\frac{\left(-i+\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{2\sqrt{-6-6i\sqrt{3}}}-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1-x+x^2\right)+\frac{1}{8}\log\left(1+x+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 + x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 + 1/(3*x^3) + ((I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2])/(2*S
qrt[-6 + (6*I)*Sqrt[3]]) + ((-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)/2])/(
2*Sqrt[-6 - (6*I)*Sqrt[3]]) - ArcTan[(-1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - ArcT
an[(1 + 2*x)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x + x^2]/8 + Log[1 + x + x^2]/8
```

Maple [A]
time = 0.05, size = 131, normalized size = 0.85

method	result
risch	$\frac{\frac{x^4}{3}-\frac{1}{7}}{x^7}-\frac{\ln(x^2-x+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(6\_R^3+\_R+x\right)\right)}{4}+\ln($
default	$\frac{\ln(x^2+x+1)}{8}-\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{12}-\frac{1}{7x^7}+\frac{1}{3x^3}-\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^2-x\sqrt{3}\right)}{2}-\sqrt{3}\,\arctan\left(2x-\sqrt{3}\right)\right)}{12}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8+x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(x^2+x+1)-1/12*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/7/x^7+1/3/x^3-1/
12*3^(1/2)*(-1/2*ln(1+x^2-x*3^(1/2))-3^(1/2)*arctan(2*x-3^(1/2)))-1/12*3^(1
/2)*(1/2*ln(1+x^2+x*3^(1/2))-3^(1/2)*arctan(2*x+3^(1/2)))-1/8*ln(x^2-x+1)-1
/12*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) + 1/21*(7*x^4 - 3)/x^7 + 1/2*integrate(x^2/(x^4 - x^2 + 1), x
) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(118) = 236$.
time = 0.36, size = 250, normalized size = 1.62

$$-\frac{28\sqrt{6}\sqrt{3}\sqrt{2}x^7\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{18}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{-18\sqrt{6}\sqrt{2}x+36x^2+36}+\sqrt{3}\right)+28\sqrt{6}\sqrt{3}\sqrt{2}x^7\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{\sqrt{6}\sqrt{2}x+2x^2+2}-\sqrt{3}\right)+7\sqrt{6}\sqrt{2}x^7\log\left(18\sqrt{6}\sqrt{2}x+36x^2+36\right)-7\sqrt{6}\sqrt{2}x^7\log\left(-18\sqrt{6}\sqrt{2}x+36x^2+36\right)+28\sqrt{3}x^7\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+28\sqrt{3}x^7\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-42x^7\log(x^2+x+1)+42x^7\log(x^2-x+1)-112x^4+48}{336x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="fricas")
```

```
[Out] -1/336*(28*sqrt(6)*sqrt(3)*sqrt(2)*x^7*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*
x + 1/18*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(-18*sqrt(6)*sqrt(2)*x + 36*x^2 + 36)
+ sqrt(3)) + 28*sqrt(6)*sqrt(3)*sqrt(2)*x^7*arctan(-1/3*sqrt(6)*sqrt(3)*sqr
t(2)*x + 1/3*sqrt(6)*sqrt(3)*sqrt(sqrt(6)*sqrt(2)*x + 2*x^2 + 2) - sqrt(3))
 + 7*sqrt(6)*sqrt(2)*x^7*log(18*sqrt(6)*sqrt(2)*x + 36*x^2 + 36) - 7*sqrt(6
)*sqrt(2)*x^7*log(-18*sqrt(6)*sqrt(2)*x + 36*x^2 + 36) + 28*sqrt(3)*x^7*arc
tan(1/3*sqrt(3)*(2*x + 1)) + 28*sqrt(3)*x^7*arctan(1/3*sqrt(3)*(2*x - 1)) -
 42*x^7*log(x^2 + x + 1) + 42*x^7*log(x^2 - x + 1) - 112*x^4 + 48)/x^7
```

Sympy [C] Result contains complex when optimal does not.
time = 0.43, size = 209, normalized size = 1.36

$$\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x-\frac{1}{2}+\frac{\sqrt{3}\,i}{6}-18432\left(\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{5}\right)+\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x-\frac{1}{2}-18432\left(\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{5}-\frac{\sqrt{3}\,i}{6}\right)+\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{3}\,i}{6}-18432\left(-\frac{1}{8}-\frac{\sqrt{3}\,i}{24}\right)^{5}\right)+\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)\log\left(x+\frac{1}{2}-18432\left(-\frac{1}{8}+\frac{\sqrt{3}\,i}{24}\right)^{5}-\frac{\sqrt{3}\,i}{6}\right)+\operatorname{RootSum}\left(2304t^{4}+48t^{2}+1,\left(t\mapsto t\log\left(-18432t^{5}-4t+x\right)\right)\right)+\frac{7x^{4}-3}{21x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8+x**4+1),x)
```

```
[Out] (1/8 - sqrt(3)*I/24)*log(x - 1/2 + sqrt(3)*I/6 - 18432*(1/8 - sqrt(3)*I/24)
**5) + (1/8 + sqrt(3)*I/24)*log(x - 1/2 - 18432*(1/8 + sqrt(3)*I/24)**5 - s
qrt(3)*I/6) + (-1/8 - sqrt(3)*I/24)*log(x + 1/2 + sqrt(3)*I/6 - 18432*(-1/8
 - sqrt(3)*I/24)**5) + (-1/8 + sqrt(3)*I/24)*log(x + 1/2 - 18432*(-1/8 + sq
rt(3)*I/24)**5 - sqrt(3)*I/6) + RootSum(2304*_t**4 + 48*_t**2 + 1, Lambda(_
t, _t*log(-18432*_t**5 - 4*_t + x))) + (7*x**4 - 3)/(21*x**7)
```

Giac [A]
time = 3.95, size = 120, normalized size = 0.78

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x+1)\right)-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\,(2x-1)\right)-\frac{1}{24}\sqrt{3}\log\left(x^{2}+\sqrt{3}\,x+1\right)+\frac{1}{24}\sqrt{3}\log\left(x^{2}-\sqrt{3}\,x+1\right)+\frac{7x^{4}-3}{21x^{7}}+\frac{1}{4}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2x-\sqrt{3}\right)+\frac{1}{8}\log\left(x^{2}+x+1\right)-\frac{1}{8}\log\left(x^{2}-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8+x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/12*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x - 1)) - 1/24*sqrt(3)*log(x^2 + sqrt(3)*x + 1) + 1/24*sqrt(3)*log(x^
2 - sqrt(3)*x + 1) + 1/21*(7*x^4 - 3)/x^7 + 1/4*arctan(2*x + sqrt(3)) + 1/4
*arctan(2*x - sqrt(3)) + 1/8*log(x^2 + x + 1) - 1/8*log(x^2 - x + 1)
```

Mupad [B]
time = 0.03, size = 110, normalized size = 0.71

$$\frac{\frac{x^4}{3}-\frac{1}{7}}{x^7}-\mathrm{atan}\left(\frac{2\,x}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}-\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{x\,2\mathrm{i}}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{\sqrt{3}}{12}+\frac{1}{4}\mathrm{i}\right)-\mathrm{atan}\left(\frac{2\,x}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^4 + x^8 + 1)),x)
```

```
[Out] (x^4/3 - 1/7)/x^7 - atan((2*x)/(3^(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) -
atan((x*2i)/(3^(1/2)*1i - 1))*(3^(1/2)/12 - 1i/4) - atan((x*2i)/(3^(1/2)*1i
 + 1))*(3^(1/2)/12 + 1i/4) - atan((2*x)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12
+ 1/4)
```

3.347 $\int \frac{x^m}{1-x^4+x^8}\,dx$

Optimal. Leaf size=127

$$\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}$$

```
[Out] -2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(1+I*3^(1/2)))/(1+m
)/(I-3^(1/2))*3^(1/2)+2/3*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^
4/(1-I*3^(1/2)))/(1+m)*3^(1/2)/(3^(1/2)+I)
```

Rubi [A]
time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1389, 371}

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(\sqrt{3}+i\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(-\sqrt{3}+i\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 - x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(1 - I*Sqrt
[3])])/(Sqrt[3]*(I + Sqrt[3])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1,
(1 + m)/4, (5 + m)/4, (2*x^4)/(1 + I*Sqrt[3])])/(Sqrt[3]*(I - Sqrt[3])*(1 +
 m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^m}{1-x^4+x^8}\,dx = -\frac{i\int \frac{x^m}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}} + \frac{i\int \frac{x^m}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^4}\,dx}{\sqrt{3}}$$
$$= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{1-i\sqrt{3}}\right)}{\sqrt{3}\left(i+\sqrt{3}\right)(1+m)} - \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{1+i\sqrt{3}}\right)}{\sqrt{3}\left(i-\sqrt{3}\right)(1+m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.08, size = 79, normalized size = 0.62

$$\frac{x^m\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{-\#1^3+2\#1^7}\&\right]}{4m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(1 - x^4 + x^8),x]
[Out] (x^m*RootSum[1 - #1^4 + #1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x -
 #1))]/((x/(x - #1))^m*(-#1^3 + 2*#1^7)) & ])/(4*m)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8-x^4+1),x)
[Out] int(x^m/(x^8-x^4+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-x^4+1),x, algorithm="maxima")
[Out] integrate(x^m/(x^8 - x^4 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-x^4+1),x, algorithm="fricas")
[Out] integral(x^m/(x^8 - x^4 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 - x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8-x**4+1),x)
[Out] Integral(x**m/(x**8 - x**4 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-x^4+1),x, algorithm="giac")
[Out] integrate(x^m/(x^8 - x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{x^8 - x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8 - x^4 + 1),x)
[Out] int(x^m/(x^8 - x^4 + 1), x)
```

3.348 $\int \frac{x^{11}}{1-x^4+x^8} \, dx$

Optimal. Leaf size=46

$$\frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)$$

[Out] 1/4*x^4+1/8*ln(x^8-x^4+1)+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1371, 717, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{x^4}{4}+\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

[In] Int[x^11/(1 - x^4 + x^8),x]

[Out] x^4/4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}}{1-x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{1-x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\text{Subst}\left(\int \frac{-1+x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)+\frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\
&= \frac{x^4}{4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$$
\frac{x^4}{4}-\frac{\tan^{-1}\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 - x^4 + x^8),x]
```

```
[Out] x^4/4 - ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Maple [A]
time = 0.02, size = 38, normalized size = 0.83

method	result	size
default	$\frac{x^4}{4}+\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	38
risch	$\frac{x^4}{4}+\frac{\ln(4x^8-4x^4+4)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4+1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A]
time = 0.49, size = 37, normalized size = 0.80

$$\frac{1}{4}\,x^4-\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^4-1\right)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="maxima")
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

Fricas [A]
time = 0.36, size = 37, normalized size = 0.80

$$\frac{1}{4}\,x^4-\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^4-1\right)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

Sympy [A]
time = 0.06, size = 42, normalized size = 0.91

$$\frac{x^4}{4}+\frac{\log\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8-x**4+1),x)
```

```
[Out] x**4/4 + log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

Giac [A]
time = 3.97, size = 37, normalized size = 0.80

$$\frac{1}{4}\,x^4-\frac{1}{12}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^4-1\right)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4
+ 1)
```

Mupad [B]
time = 0.05, size = 39, normalized size = 0.85

$$\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8 - x^4 + 1),x)
```

```
[Out] log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12 + x
^4/4
```

3.349 $\int \frac{x^9}{1-x^4+x^8}\,dx$

Optimal. Leaf size=57

$$\frac{x^2}{2}+\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}}$$

```
[Out] 1/2*x^2+1/12*ln(1+x^4-3^(1/2)*x^2)*3^(1/2)-1/12*ln(1+x^4+3^(1/2)*x^2)*3^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1373, 1136, 1178, 642}

$$\frac{x^2}{2}+\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - x^4 + x^8),x]
```

```
[Out] x^2/2 + Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]
/(4*Sqrt[3])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
```

```
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{1-x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1-x^2+x^4}\,dx,x,x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\ &= \frac{x^2}{2}+\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}\,x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}+\frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}\,x-x^2}\,dx,x,x^2\right)}{4\sqrt{3}}\\ &= \frac{x^2}{2}+\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 0.96

$$\frac{1}{12}\left(6x^2+\sqrt{3}\,\log\left(-1+\sqrt{3}\,x^2-x^4\right)-\sqrt{3}\,\log\left(1+\sqrt{3}\,x^2+x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - x^4 + x^8),x]
[Out] (6*x^2 + Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] - Sqrt[3]*Log[1 + Sqrt[3]*x^2
+ x^4])/12
```

Maple [A]
time = 0.02, size = 44, normalized size = 0.77

method	result	size
default	$\frac{x^2}{2}+\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	44

risch	$\frac{x^2}{2}+\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)-1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/
2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 0.35, size = 47, normalized size = 0.82

$$\frac{1}{2}x^2+\frac{1}{12}\sqrt{3}\log\left(\frac{x^8+5x^4-2\sqrt{3}(x^6+x^2)+1}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/12*sqrt(3)*log((x^8 + 5*x^4 - 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 -
 x^4 + 1))
```

Sympy [A]
time = 0.04, size = 48, normalized size = 0.84

$$\frac{x^2}{2}+\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}-\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8-x**4+1),x)
```

```
[Out] x**2/2 + sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 - sqrt(3)*log(x**4 + sqrt(
3)*x**2 + 1)/12
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(43) = 86.
time = 3.93, size = 99, normalized size = 1.74

$$\frac{1}{2}x^2+\frac{1}{4}(x^4-1)\arctan\left(2x^2+\sqrt{3}\right)+\frac{1}{4}(x^4-1)\arctan\left(2x^2-\sqrt{3}\right)+\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4+\sqrt{3}x^2+1\right)-\frac{1}{24}\left(\sqrt{3}x^4-\sqrt{3}\right)\log\left(x^4-\sqrt{3}x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) + 1/4*(x^4 - 1)*arctan(2*x^
2 - sqrt(3)) + 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) - 1/
24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1)
```

Mupad [B]
time = 1.31, size = 29, normalized size = 0.51

$$\frac{x^2}{2}-\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,x^2}{9\left(\frac{2x^4}{9}+\frac{2}{9}\right)}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8 - x^4 + 1),x)
```

```
[Out] x^2/2 - (3^(1/2)*atanh((2*3^(1/2)*x^2)/(9*((2*x^4)/9 + 2/9))))/6
```

3.350 $\int \frac{x^7}{1-x^4+x^8}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[Out] 1/8*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1371, 648, 632, 210, 642}

$$\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\text{ArcTan}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[1 - x^4 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1-x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)+\frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \frac{1}{8}\log\left(1-x^4+x^8\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{8}\log\left(1-x^4+x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - x^4 + x^8),x]
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[1 - x^4 + x^8]/8
```

Maple [A]
time = 0.02, size = 33, normalized size = 0.85

method	result	size
default	$\frac{\ln(x^8-x^4+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	33

risch	$\frac{\ln(4x^8-4x^4+4)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A]
time = 0.52, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^4-1)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="maxima")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

Fricas [A]
time = 0.34, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^4-1)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

Sympy [A]
time = 0.05, size = 37, normalized size = 0.95

$$\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8-x**4+1),x)
[Out] log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

Giac [A]
time = 3.31, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^4-1)\right)+\frac{1}{8}\,\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/8*log(x^8 - x^4 + 1)
```

Mupad [B]
time = 1.28, size = 34, normalized size = 0.87

$$\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8 - x^4 + 1),x)
```

```
[Out] log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12
```

3.351 $\int \frac{x^5}{1-x^4+x^8}\,dx$

Optimal. Leaf size=82

$$-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)+\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}$$

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))+1/24*ln(1+x^4-3^(1/2)*x
^2)*3^(1/2)-1/24*ln(1+x^4+3^(1/2)*x^2)*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1373, 1141, 1175, 632, 210, 1178, 642}

$$-\frac{1}{4}\text{ArcTan}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\text{ArcTan}\left(2x^2+\sqrt{3}\right)+\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{8\sqrt{3}}-\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[Sqrt[3] - 2*x^2] + ArcTan[Sqrt[3] + 2*x^2]/4 + Log[1 - Sqrt[3]*
x^2 + x^4]/(8*Sqrt[3]) - Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
```

```
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{1-x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx,x,x^2\right)\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx,x,x^2\right)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx,x,x^2\right)+\frac{\text{Subst}}{}\\
&= \frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2\right.\\
&= -\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)+\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}-\frac{\log\left(1+\sqrt{}\right.}{8\sqrt{}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.08, size = 98, normalized size = 1.20

$$\frac{\sqrt{-1-i\sqrt{3}}\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)+\sqrt{-1+i\sqrt{3}}\left(-i+\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x^2)/2] + Sqrt[
-1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x^2)/2])/(4*Sqrt[6])
```

Maple [A]
time = 0.03, size = 77, normalized size = 0.94

method	result	s
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(6\_R^3+x^2+\_R\right)\right)}{4}$	3
default	$-\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x^2-\sqrt{3}\right)\right)}{12}-\frac{\sqrt{3}\left(\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x^2+\sqrt{3}\right)\right)}{12}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*3^(1/2)*(-1/2*ln(1+x^4-x^2*3^(1/2))-3^(1/2)*arctan(2*x^2-3^(1/2)))-1/
12*3^(1/2)*(1/2*ln(1+x^4+x^2*3^(1/2))-3^(1/2)*arctan(2*x^2+3^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(64) = 128.
time = 0.36, size = 172, normalized size = 2.10

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4-\sqrt{6}\sqrt{2}x^2+2}+\sqrt{3}\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(36x^4+18\sqrt{6}\sqrt{2}x^2+36\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(36x^4-18\sqrt{6}\sqrt{2}x^2+36\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3
*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) - 1/12*sq
rt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)
*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) - 1/48*sqrt(6)*sq
rt(2)*log(36*x^4 + 18*sqrt(6)*sqrt(2)*x^2 + 36) + 1/48*sqrt(6)*sqrt(2)*log(
36*x^4 - 18*sqrt(6)*sqrt(2)*x^2 + 36)
```

Sympy [A]
time = 0.09, size = 70, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^4-\sqrt{3}\,x^2+1\right)}{24}-\frac{\sqrt{3}\log\left(x^4+\sqrt{3}\,x^2+1\right)}{24}+\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8-x**4+1),x)
```

```
[Out] sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 - sqrt(3)*log(x**4 + sqrt(3)*x**2 +
 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

Giac [A]
time = 3.45, size = 76, normalized size = 0.93

$$\frac{1}{24}\sqrt{3}\,x^4\log\left(x^4+\sqrt{3}\,x^2+1\right)-\frac{1}{24}\sqrt{3}\,x^4\log\left(x^4-\sqrt{3}\,x^2+1\right)+\frac{1}{4}\,x^4\arctan\left(2\,x^2+\sqrt{3}\right)+\frac{1}{4}\,x^4\arctan\left(2\,x^2-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*x^4*log(x^4 - sq
rt(3)*x^2 + 1) + 1/4*x^4*arctan(2*x^2 + sqrt(3)) + 1/4*x^4*arctan(2*x^2 - s
qrt(3))
```

Mupad [B]
time = 0.05, size = 53, normalized size = 0.65

$$-\operatorname{atan}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\operatorname{atan}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8 - x^4 + 1),x)
```

```
[Out] - atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) - atan((2*x^2)/(3^
(1/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4)
```

3.352 $\int \frac{x^3}{1-x^4+x^8}\,dx$

Optimal. Leaf size=23

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1366, 632, 210}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - x^4 + x^8),x]
```

```
[Out] -1/2*ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^3}{1-x^4+x^8}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\right)$$
$$= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [A]
time = 0.00, size = 23, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A]
time = 0.01, size = 19, normalized size = 0.83

method	result	size
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{6}$	19
risch	$\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{6}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^4-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="maxima")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Fricas [A]
time = 0.38, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,\left(2\,x^4-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Sympy [A]
time = 0.05, size = 26, normalized size = 1.13

$$\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-x**4+1),x)
[Out] sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/6
```

Giac [A]
time = 3.75, size = 18, normalized size = 0.78

$$\frac{1}{6}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,\left(2\,x^4-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1))
```

Mupad [B]
time = 1.29, size = 17, normalized size = 0.74

$$\frac{\sqrt{3}\,\operatorname{atan}\left(\sqrt{3}\,\left(\frac{2\,x^4}{3}-\frac{1}{3}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8 - x^4 + 1),x)
[Out] (3^(1/2)*atan(3^(1/2)*((2*x^4)/3 - 1/3)))/6
```

3.353 $\int \frac{x}{1-x^4+x^8} \, dx$

Optimal. Leaf size=82

$$-\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}$$

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))-1/24*ln(1+x^4-3^(1/2)*x
^2)*3^(1/2)+1/24*ln(1+x^4+3^(1/2)*x^2)*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1373, 1108, 648, 632, 210, 642}

$$-\frac{1}{4}\text{ArcTan}\left(\sqrt{3}-2x^2\right)+\frac{1}{4}\text{ArcTan}\left(2x^2+\sqrt{3}\right)-\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[Sqrt[3] - 2*x^2] + ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1 - Sqrt[3]*
x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1-x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2+x^4}\,dx, x, x^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{3}-x}{1-\sqrt{3}\,x+x^2}\,dx, x, x^2\right)}{4\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{\sqrt{3}+x}{1+\sqrt{3}\,x+x^2}\,dx, x, x^2\right)}{4\sqrt{3}}\\ &= \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx, x, x^2\right) + \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx, x, x^2\right) - \frac{\text{Subs}}{}\\ &= -\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}} - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, -\sqrt{3}\right.\\ &= -\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right) + \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right) - \frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\right.}{8}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.03, size = 83, normalized size = 1.01

$$\frac{i\left(\sqrt{-1-i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x^2\right)-\sqrt{-1+i\sqrt{3}}\,\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x^2\right)\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^4 + x^8),x]
```

```
[Out] ((I/2)*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x^2)/2] - Sqrt[-1 + I*
Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x^2)/2]))/Sqrt[6]
```

Maple [A]
time = 0.02, size = 65, normalized size = 0.79

method	result	size
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^{4}+3\_Z^{2}+1\right)}\_R\ln\left(-3\_R^{3}+x^{2}+\_R\right)\right)}{4}$	32
default	$\frac{\arctan\left(2x^{2}-\sqrt{3}\right)}{4}+\frac{\arctan\left(2x^{2}+\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^{4}-x^{2}\sqrt{3}\right)\sqrt{3}}{24}+\frac{\ln\left(1+x^{4}+x^{2}\sqrt{3}\right)\sqrt{3}}{24}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*arctan(2*x^2-3^(1/2))+1/4*arctan(2*x^2+3^(1/2))-1/24*ln(1+x^4-x^2*3^(1/
2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(64) = 128$.
time = 0.37, size = 172, normalized size = 2.10

$$-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^{2}+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^{4}+\sqrt{6}\sqrt{2}x^{2}+2}-\sqrt{3}\right)-\frac{1}{12}\sqrt{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^{2}+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^{4}-\sqrt{6}\sqrt{2}x^{2}+2}+\sqrt{3}\right)+\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(144x^{4}+72\sqrt{6}\sqrt{2}x^{2}+144\right)-\frac{1}{48}\sqrt{6}\sqrt{2}\log\left(144x^{4}-72\sqrt{6}\sqrt{2}x^{2}+144\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3
*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) - 1/12*sq
rt(6)*sqrt(3)*sqrt(2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*sqrt(6)
*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + 1/48*sqrt(6)*sq
rt(2)*log(144*x^4 + 72*sqrt(6)*sqrt(2)*x^2 + 144) - 1/48*sqrt(6)*sqrt(2)*lo
g(144*x^4 - 72*sqrt(6)*sqrt(2)*x^2 + 144)
```

Sympy [A]
time = 0.08, size = 70, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}\,x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}\,x^2+1\right)}{24}+\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}+\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 + atan(2*x**2 - sqrt(3))/4 + atan(2*x**2 + sqrt(3))/4
```

Giac [A]
time = 3.47, size = 64, normalized size = 0.78

$$\frac{1}{24}\sqrt{3}\log\left(x^4+\sqrt{3}\,x^2+1\right)-\frac{1}{24}\sqrt{3}\log\left(x^4-\sqrt{3}\,x^2+1\right)+\frac{1}{4}\arctan\left(2\,x^2+\sqrt{3}\right)+\frac{1}{4}\arctan\left(2\,x^2-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(3)*log(x^4 + sqrt(3)*x^2 + 1) - 1/24*sqrt(3)*log(x^4 - sqrt(3)*x^
2 + 1) + 1/4*arctan(2*x^2 + sqrt(3)) + 1/4*arctan(2*x^2 - sqrt(3))
```

Mupad [B]
time = 0.04, size = 53, normalized size = 0.65

$$-\operatorname{atan}\left(-\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\operatorname{atan}\left(\frac{x^2}{2}+\frac{\sqrt{3}\,x^2\,1\mathrm{i}}{2}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8 - x^4 + 1),x)
```

```
[Out] - atan((3^(1/2)*x^2*1i)/2 - x^2/2)*((3^(1/2)*1i)/12 + 1/4) - atan((3^(1/2)*
x^2*1i)/2 + x^2/2)*((3^(1/2)*1i)/12 - 1/4)
```

3.354 $\int \frac{1}{x(1-x^4+x^8)}\,dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1371, 719, 29, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^4 + x^8)),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[1 - x^4 + x^8]/8
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\left(1-x^4+x^8\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(1-x+x^2\right)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \log(x)+\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)-\frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 55, normalized size = 1.34

$$
\log(x)-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-1+2\#1^4}\&\right]
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^4 + x^8)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-1
+ 2*#1^4) & ]/4
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.85

method	result	size
risch	$\ln\left(x\right)-\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{4}-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$	33
default	$\ln\left(x\right)-\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)}{12}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A]
time = 0.56, size = 38, normalized size = 0.93

$$\frac{1}{12}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{4}-1\right)\right)-\frac{1}{8}\log\left(x^{8}-x^{4}+1\right)+\frac{1}{4}\log\left(x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/4
*log(x^4)
```

Fricas [A]
time = 0.36, size = 34, normalized size = 0.83

$$\frac{1}{12}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{4}-1\right)\right)-\frac{1}{8}\log\left(x^{8}-x^{4}+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + log
(x)
```

Sympy [A]
time = 0.06, size = 41, normalized size = 1.00

$$\log(x) - \frac{\log(x^8 - x^4 + 1)}{8} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

Giac [A]
time = 3.19, size = 38, normalized size = 0.93

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right) - \frac{1}{8}\log\left(x^8 - x^4 + 1\right) + \frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/4
*log(x^4)
```

Mupad [B]
time = 1.29, size = 36, normalized size = 0.88

$$\ln(x) - \frac{\ln(x^8 - x^4 + 1)}{8} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\,x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^8 - x^4 + 1)),x)
```

```
[Out] log(x) - log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3
))/12
```

3.355 $\int \frac{1}{x^3(1-x^4+x^8)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{2x^2}-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}}$$

```
[Out] -1/2/x^2-1/12*ln(1+x^4-3^(1/2)*x^2)*3^(1/2)+1/12*ln(1+x^4+3^(1/2)*x^2)*3^(1
/2)
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1373, 1137, 1178, 642}

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 - Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 +
 x^4]/(4*Sqrt[3])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(1-x^4+x^8)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1-x^2+x^4)}\,dx, x, x^2\right)\\ &= -\frac{1}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right)\\ &= -\frac{1}{2x^2}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx, x, x^2\right)}{4\sqrt{3}}-\frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx, x, x^2\right)}{4\sqrt{3}}\\ &= -\frac{1}{2x^2}-\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 0.96

$$\frac{1}{12}\left(-\frac{6}{x^2}-\sqrt{3}\log\left(-1+\sqrt{3}x^2-x^4\right)+\sqrt{3}\log\left(1+\sqrt{3}x^2+x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] (-6/x^2 - Sqrt[3]*Log[-1 + Sqrt[3]*x^2 - x^4] + Sqrt[3]*Log[1 + Sqrt[3]*x^2
 + x^4])/12
```

Maple [A]
time = 0.03, size = 44, normalized size = 0.77

method	result	size
default	$-\frac{1}{2x^2}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	44
risch	$-\frac{1}{2x^2}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^(1
/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 0.35, size = 50, normalized size = 0.88

$$\frac{\sqrt{3}\,x^2\log\left(\frac{x^8+5\,x^4+2\,\sqrt{3}\,(x^6+x^2)+1}{x^8-x^4+1}\right)-6}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(3)*x^2*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4
+ 1)) - 6)/x^2
```

Sympy [A]
time = 0.08, size = 49, normalized size = 0.86

$$-\frac{\sqrt{3}\,\log\left(x^4-\sqrt{3}\,x^2+1\right)}{12}+\frac{\sqrt{3}\,\log\left(x^4+\sqrt{3}\,x^2+1\right)}{12}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/12 - 1/(2*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(43) = 86$.
time = 3.35, size = 99, normalized size = 1.74

$$-\frac{1}{4}\,(x^4-1)\arctan\left(2\,x^2+\sqrt{3}\right)-\frac{1}{4}\,(x^4-1)\arctan\left(2\,x^2-\sqrt{3}\right)-\frac{1}{24}\left(\sqrt{3}\,x^4-\sqrt{3}\right)\log\left(x^4+\sqrt{3}\,x^2+1\right)+\frac{1}{24}\left(\sqrt{3}\,x^4-\sqrt{3}\right)\log\left(x^4-\sqrt{3}\,x^2+1\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*(x^4 - 1)*arctan(2*x^2 + sqrt(3)) - 1/4*(x^4 - 1)*arctan(2*x^2 - sqrt(
3)) - 1/24*(sqrt(3)*x^4 - sqrt(3))*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*(sqrt(
3)*x^4 - sqrt(3))*log(x^4 - sqrt(3)*x^2 + 1) - 1/2/x^2
```

Mupad [B]
time = 1.27, size = 29, normalized size = 0.51

$$\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{2\sqrt{3}\,x^2}{9\left(\frac{2x^4}{9}+\frac{2}{9}\right)}\right)}{6}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atanh((2*3^(1/2)*x^2)/(9*((2*x^4)/9 + 2/9))))/6 - 1/(2*x^2)
```

3.356 $\int \frac{1}{x^5(1-x^4+x^8)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{4x^4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)$$

[Out] -1/4/x^4+ln(x)-1/8*ln(x^8-x^4+1)+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2
)

Rubi [A]
time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1371, 723, 814, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(1 - x^4 + x^8)),x]

[Out] -1/4*1/x^4 + ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4
 + x^8]/8

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5(1-x^4+x^8)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(1-x+x^2)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{1-x}{x(1-x+x^2)}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\log(x)-\frac{1}{4}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^4\right)\\
&= -\frac{1}{4x^4}+\log(x)-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x\right.\\
&= -\frac{1}{4x^4}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\
&= -\frac{1}{4x^4}+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 51, normalized size = 1.06

$$-\frac{1}{4x^4}+\log(x)-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^4}{-1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 + Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 +
2*#1^4) & ]/4
```

Maple [A]
time = 0.03, size = 40, normalized size = 0.83

method	result	size
risch	$-\frac{1}{4x^4}+\ln(x)-\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^4-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$	38
default	$-\frac{1}{4x^4}+\ln(x)-\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4+ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A]

time = 0.51, size = 43, normalized size = 0.90

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4/x^4 - 1/8*log(x^8 - x^4
 + 1) + 1/4*log(x^4)
```

Fricas [A]

time = 0.35, size = 51, normalized size = 1.06

$$-\frac{2\sqrt{3}x^4\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+3x^4\log\left(x^8-x^4+1\right)-24x^4\log\left(x\right)+6}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(3)*x^4*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 3*x^4*log(x^8 - x^4
+ 1) - 24*x^4*log(x) + 6)/x^4
```

Sympy [A]

time = 0.07, size = 48, normalized size = 1.00

$$\log\left(x\right)-\frac{\log\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12 - 1/(4*x**4)
```

Giac [A]

time = 3.48, size = 48, normalized size = 1.00

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{x^4+1}{4x^4}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/4*(x^4 + 1)/x^4 - 1/8*log
(x^8 - x^4 + 1) + 1/4*log(x^4)
```

Mupad [B]
time = 0.07, size = 41, normalized size = 0.85

$$\ln(x) - \frac{\ln(x^8 - x^4 + 1)}{8} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\,x^4}{3}\right)}{12} - \frac{1}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^8 - x^4 + 1)),x)
```

```
[Out] log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3
))/12 - 1/(4*x^4)
```

3.357 $\int \frac{1}{x^7(1-x^4+x^8)}\,dx$

Optimal. Leaf size=96

$$-\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}$$

```
[Out] -1/6/x^6-1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*l
n(1+x^4-3^(1/2)*x^2)*3^(1/2)+1/24*ln(1+x^4+3^(1/2)*x^2)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1373, 1137, 1295, 12, 1141, 1175, 632, 210, 1178, 642}

$$\frac{1}{4}\text{ArcTan}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\text{ArcTan}\left(2x^2+\sqrt{3}\right)-\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 - 1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2
]/4 - Log[1 - Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(
8*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
```

```
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^7(1-x^4+x^8)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4(1-x^2+x^4)}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{3-3x^2}{x^2(1-x^2+x^4)}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{1}{6}\text{Subst}\left(\int \frac{3x^2}{1-x^2+x^4}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx, x\right.\\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx, x, x^2\right)-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}\,x+x}\right.\\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{1}{4}\text{Subst}\left(\int \frac{}{-1}\right.\\
&= -\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)-\frac{\log\left(1-\sqrt{3}\,x^2\right.}{8\sqrt{3}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 56, normalized size = 0.58

$$-\frac{1}{6x^6}-\frac{1}{2x^2}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 - 1/(2*x^2) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1
 + 2*#1^4) & ]/4
```

Maple [A]
time = 0.03, size = 87, normalized size = 0.91

method	result
risch	$\frac{-\frac{x^4}{2}-\frac{1}{6}}{x^6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+3\_Z^2+1\right)}\_R\ln\left(-6\_R^3+x^2-\_R\right)\right)}{4}$
default	$-\frac{1}{6x^6}-\frac{1}{2x^2}+\frac{\sqrt{3}\left(-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x^2-\sqrt{3}\right)\right)}{12}+\frac{\sqrt{3}\left(\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x^2+\right.\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x^6-1/2/x^2+1/12*3^(1/2)*(-1/2*ln(1+x^4-x^2*3^(1/2))-3^(1/2)*arctan(2*
x^2-3^(1/2)))+1/12*3^(1/2)*(1/2*ln(1+x^4+x^2*3^(1/2))-3^(1/2)*arctan(2*x^2+
3^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/6*(3*x^4 + 1)/x^6 - integrate(x^5/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(74) = 148$.
time = 0.36, size = 194, normalized size = 2.02

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x^6\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x^6\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2x^4-\sqrt{6}\sqrt{2}x^2+2}+\sqrt{3}\right)+\sqrt{6}\sqrt{2}x^6\log\left(36x^4+18\sqrt{6}\sqrt{2}x^2+36\right)-\sqrt{6}\sqrt{2}x^6\log\left(36x^4-18\sqrt{6}\sqrt{2}x^2+36\right)-24x^4-8}{48x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^6*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2
 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) + 4
*sqrt(6)*sqrt(3)*sqrt(2)*x^6*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*
sqrt(6)*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + sqrt(6)*
sqrt(2)*x^6*log(36*x^4 + 18*sqrt(6)*sqrt(2)*x^2 + 36) - sqrt(6)*sqrt(2)*x^6
*log(36*x^4 - 18*sqrt(6)*sqrt(2)*x^2 + 36) - 24*x^4 - 8)/x^6
```

Sympy [A]
time = 0.10, size = 83, normalized size = 0.86

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}-\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}+\frac{-3x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 + (-3*x**4 -
1)/(6*x**6)
```

Giac [A]
time = 3.11, size = 56, normalized size = 0.58

$$-\frac{1}{12}\sqrt{3}\,x^4\log\left(x^4+\sqrt{3}\,x^2+1\right)+\frac{1}{12}\sqrt{3}\,x^4\log\left(x^4-\sqrt{3}\,x^2+1\right)-\frac{3\,x^4+1}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/12*sqrt(3)*x^4*log(x^4 - s
qrt(3)*x^2 + 1) - 1/6*(3*x^4 + 1)/x^6
```

Mupad [B]
time = 0.06, size = 63, normalized size = 0.66

$$\mathrm{atan}\left(\frac{2\,x^2}{-1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)+\mathrm{atan}\left(\frac{2\,x^2}{1+\sqrt{3}\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{12}\right)-\frac{\frac{x^4}{2}+\frac{1}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^8 - x^4 + 1)),x)
```

```
[Out] atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - (x^4/2 + 1/6)/x^6
```

3.358 $\int \frac{x^8}{1-x^4+x^8}\,dx$

Optimal. Leaf size=356

$$x+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] x-1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*l
n(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((
-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6
^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))
/(3/2*2^(1/2)-1/2*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2
^(1/2)+1/6*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+
1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(
1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(
1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A]
time = 0.23, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$,
Rules used = {1381, 1435, 1183, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)+x$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - x^4 + x^8),x]
```

```
[Out] x + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt
[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 +
 Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3
*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*S
qrt[3*(2 + Sqrt[3])]) - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x
+ x^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 +
(Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + S
qrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1435

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8}{1-x^4+x^8}\,dx = x-\int \frac{1-x^4}{1-x^4+x^8}\,dx$$

$$= x+\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}\,x^2-x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}\,x^2-x^4}\,dx}{2\sqrt{3}}$$

$$= x-\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{3(2}}{1-}}{4}$$

$$= x+\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx+\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{}{1+\sqrt{2-}}$$

$$= x-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)+\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{}}\right.$$

$$= x+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+}}{\sqrt{2}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 59, normalized size = 0.17

$$x+\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - x^4 + x^8),x]
```

```
[Out] x + RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 44, normalized size = 0.12

method	result	size
default	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44
risch	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/4*sum((_R^4-1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] x + integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 720 vs. 2(264) = 528.
time = 0.36, size = 720, normalized size = 2.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(144*x^2 +
 24*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 144) +
1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(144*x^2 -
24*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 144) - 1
/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(144*x^2
+ 12*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 144
) + 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(144
*x^2 - 12*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8)
+ 144) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/36*sqrt(6)*sqrt
(3)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(
3) + 8) + 12)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sq
```

```
rt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) -
2) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/36*sqrt(6)*sqrt(3)*
sqrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) +
 8) + 12)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2) +
 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*(2*sqrt(3)*sqrt
(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 1/3*sqrt(6*x^2 + sqrt(6)*(2*sqrt(3
)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 6)*(2*sqrt(3)*sqrt(2) - 3*sq
rt(2))*sqrt(sqrt(3) + 2) + sqrt(3) - 2) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
 + 2)*arctan(-1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3)
+ 2) + 1/3*sqrt(6*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sq
rt(3) + 2) + 6)*(2*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(3) + 2) - sqrt(3)
 + 2) + x
```

Sympy [**A**]
time = 1.59, size = 26, normalized size = 0.07

$$x + \operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(9216t^5 - 8t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-x**4+1),x)
```

```
[Out] x + RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 -
8*_t + x)))
```

Giac [**A**]
time = 2.89, size = 254, normalized size = 0.71

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) + x
```

Mupad [**B**]
time = 0.16, size = 209, normalized size = 0.59

$$x+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}+\frac{\sqrt{3}\,x\,1{}\mathrm{i}}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1{}\mathrm{i}}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}-\frac{\sqrt{3}\,x}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8 - x^4 + 1),x)
```

```
[Out] x + (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i
)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*
8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/12
- (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3
^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (2
^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^
(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.359 $\int \frac{x^6}{1-x^4+x^8} \, dx$

Optimal. Leaf size=275

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

```
[Out] -1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1
/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^
(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))
*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*6^(1/2)+1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2
+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1
/2)))*6^(1/2)-1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1386, 1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - x^4 + x^8),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[6] - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1386

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[x^(
m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Dist[1/(2*c*
r), Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{1-x^4+x^8}\,dx = -\frac{\int \frac{1-\sqrt{3}\,x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{1+\sqrt{3}\,x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{\sqrt{2}}{1+\sqrt{2}}}{}$$

$$= \frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}\,x\right.}{4\sqrt{6}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 41, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^3}{-1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 32, normalized size = 0.12

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)} \_R \ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	32
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)} \_R \ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 0.36, size = 220, normalized size = 0.80

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x+2\right)}{3x^2-2}\right)-\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(36x^4+36\sqrt{3}\sqrt{2}\left(x^3+x\right)+108x^2+36\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(36x^4-36\sqrt{3}\sqrt{2}\left(x^3+x\right)+108x^2+36\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) - 1/24*sqrt(3)*sqrt(2)*log(36*x^4 + 36*sqrt(3)*sqrt(2)*(x^3 + x) + 108
*x^2 + 36) + 1/24*sqrt(3)*sqrt(2)*log(36*x^4 - 36*sqrt(3)*sqrt(2)*(x^3 + x)
 + 108*x^2 + 36)
```

Sympy [A]
time = 0.13, size = 165, normalized size = 0.60

$$\frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\cdot\left(2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24}+\frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24}-\frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
4*x**2 + 2*sqrt(6)*x + 3))/24 + sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
sqrt(6)*x + 1)/24 - sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

Giac [A]
time = 3.10, size = 205, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) - 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) + 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) - 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) + 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B]
time = 0.10, size = 53, normalized size = 0.19

$$\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8 - x^4 + 1),x)
```

```
[Out] - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12)
```

3.360 $\int \frac{x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=347

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}$$

```
[Out] -1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*
2^(1/2)-1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-
1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(
1/2)))/(3/2*2^(1/2)-1/2*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))/
(3/2*2^(1/2)-1/2*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^
(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1
/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/8*ln(1+x
^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/8*ln(1+x^2+x*(1
/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A]
time = 0.13, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1387, 1141, 1175, 632, 210, 1178, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 + Sqrt[3])
]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 - Sqr
t[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
+ Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 - Sqrt[3])]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[
3])]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) + Log[
1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) - Log[1 + Sqrt[2 +
 Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{1-x^4+x^8}\,dx = \frac{\int \frac{x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{1-x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{4\sqrt{3}} + \frac{\int \frac{1+x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{4\sqrt{3}} + \frac{\int \frac{1-x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{4\sqrt{3}} - \frac{\int \frac{1+x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{4\sqrt{3}}$$

$$= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \frac{\int -}{1+}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\log\left(1-\sqrt{2+\sqrt{3}}\right.}{8\sqrt{3\left(2+\sqrt{\right.}}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 39, normalized size = 0.11

$$\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 40, normalized size = 0.12

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	40

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R^4/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(271) = 542.
time = 0.38, size = 581, normalized size = 1.67

$$\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(6\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+36x^2+36\right)-\frac{1}{48}\sqrt{6}\left(\sqrt{3}\sqrt{2}-2\sqrt{2}\right)\sqrt{\sqrt{3}+2}\log\left(-6\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+36x^2+36\right)+\frac{1}{96}\sqrt{6}\left(\sqrt{3}\sqrt{2}+2\sqrt{2}\right)\sqrt{-4\sqrt{3}+8}\log\left(3\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3}+8}+36x^2+36\right)-\frac{1}{96}\sqrt{6}\left(\sqrt{3}\sqrt{2}+2\sqrt{2}\right)\sqrt{-4\sqrt{3}+8}\log\left(-3\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3}+8}+36x^2+36\right)-\frac{1}{12}\sqrt{6}\sqrt{2}\sqrt{\sqrt{3}+2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+\frac{1}{18}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{6\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+36x^2+36}\sqrt{\sqrt{3}+2}-\sqrt{3}-2\right)-\frac{1}{12}\sqrt{6}\sqrt{2}\sqrt{\sqrt{3}+2}\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+\frac{1}{18}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{-6\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+36x^2+36}\sqrt{\sqrt{3}+2}+\sqrt{3}+2\right)+\frac{1}{24}\sqrt{6}\sqrt{2}\sqrt{-4\sqrt{3}+8}\arctan\left(-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3}+8}+\frac{1}{36}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{3\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3}+8}+36x^2+36}\sqrt{-4\sqrt{3}+8}+\sqrt{3}-2\right)+\frac{1}{24}\sqrt{6}\sqrt{2}\sqrt{-4\sqrt{3}+8}\arctan\left(-\frac{1}{6}\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3}+8}+\frac{1}{36}\sqrt{6}\sqrt{3}\sqrt{2}\sqrt{-3\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{-4\sqrt{3}+8}+36x^2+36}\sqrt{-4\sqrt{3}+8}-\sqrt{3}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(6*sqrt(6)*
sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 36*x^2 + 36) - 1/48*sqrt(6)*(sqrt(3)*
sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(-6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqr
t(sqrt(3) + 2) + 36*x^2 + 36) + 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*
sqrt(-4*sqrt(3) + 8)*log(3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) +
 36*x^2 + 36) - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3)
+ 8)*log(-3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 36*x^2 + 36) -
 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)
*x*sqrt(sqrt(3) + 2) + 1/18*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(6*sqrt(6)*sqrt(3)*
sqrt(2)*x*sqrt(sqrt(3) + 2) + 36*x^2 + 36)*sqrt(sqrt(3) + 2) - sqrt(3) - 2)
 - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(
2)*x*sqrt(sqrt(3) + 2) + 1/18*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(-6*sqrt(6)*sqrt(
3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 36*x^2 + 36)*sqrt(sqrt(3) + 2) + sqrt(3) +
 2) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)
*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/36*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(3*sqrt(
6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 36*x^2 + 36)*sqrt(-4*sqrt(3) +
8) + sqrt(3) - 2) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*s
```

```
qrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/36*sqrt(6)*sqrt(3)*sqrt(2
)*sqrt(-3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 36*x^2 + 36)*sqr
t(-4*sqrt(3) + 8) - sqrt(3) + 2)
```

Sympy [A]
time = 1.53, size = 24, normalized size = 0.07

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-18432t^5 + 4t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-x**4+1),x)
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4*
_t + x)))
```

Giac [A]
time = 2.79, size = 253, normalized size = 0.73

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-x^4+1),x, algorithm="giac")
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

Mupad [B]
time = 1.33, size = 474, normalized size = 1.37

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8i\right)^{1/4}}{2\left(\frac{\sqrt{3}\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}+\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8i\right)^{1/4}1i}{2\left(\frac{\sqrt{3}\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}1i}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8-\sqrt{3}\,8i\right)^{1/4}1i}{2\left(\frac{\sqrt{3}\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}-\frac{\sqrt{3}\,x\left(8-\sqrt{3}\,8i\right)^{1/4}}{2\left(\frac{\sqrt{3}\sqrt{8-\sqrt{3}\,8i}\,1i}{4}+\frac{\sqrt{8-\sqrt{3}\,8i}}{4}\right)}\right)\left(8-\sqrt{3}\,8i\right)^{1/4}}{12}+\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(1+\sqrt{3}\,1i\right)^{1/4}}{2\left(\frac{\sqrt{2}\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\sqrt{3}\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}-\frac{2^{3/4}\sqrt{3}\,x\left(1+\sqrt{3}\,1i\right)^{1/4}1i}{2\left(\frac{\sqrt{2}\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\sqrt{3}\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}\right)\left(1+\sqrt{3}\,1i\right)^{1/4}1i}{12}+\frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(1+\sqrt{3}\,1i\right)^{1/4}1i}{2\left(\frac{\sqrt{2}\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\sqrt{3}\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}+\frac{2^{3/4}\sqrt{3}\,x\left(1+\sqrt{3}\,1i\right)^{1/4}}{2\left(\frac{\sqrt{2}\sqrt{1+\sqrt{3}\,1i}}{2}-\frac{\sqrt{2}\sqrt{3}\sqrt{1+\sqrt{3}\,1i}\,1i}{2}\right)}\right)\left(1+\sqrt{3}\,1i\right)^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8 - x^4 + 1),x)
[Out] (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1
/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) - (2
^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(
1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)
^(1/4)*1i)/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8
```

```
 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3
^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*
8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8
i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2
)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)
^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 +
(2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3
^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) +
 (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(
1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)
^(1/4))/12
```

3.361 $\int \frac{x^2}{1-x^4+x^8} dx$

Optimal. Leaf size=355

$$\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\right.}$$

```
[Out] 1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2
^(1/2)-1/6*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1
/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1
/2)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))/(
3/2*2^(1/2)-1/2*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(
1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/
2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/8*ln(1+x^
2-x*(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/8*ln(1+x^2+x*(1/
2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A]

time = 0.14, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1387, 1108, 648, 632, 210, 642}

$$\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)+\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - x^4 + x^8),x]
```

```
[Out] (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]])
/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[
3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 +
Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt
[2 - Sqrt[3]]])/4 + Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[
3])]) - Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 - Sqrt[3])]) - Log[
1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])]) + Log[1 + Sqrt[2 +
 Sqrt[3]]*x + x^2]/(8*Sqrt[3*(2 + Sqrt[3])])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1108

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^2}{1-x^4+x^8}\,dx = \frac{\int \frac{1}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{\sqrt{2-\sqrt{3}}-x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}-x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \text{—} }{\text{1+} \quad 4}$$

$$= -\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} - \frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}} + \frac{\int \text{—}}{\text{1+}}$$

$$= \frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}\right.}{8\sqrt{3\left(2+\sqrt{3}\right.}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 40, normalized size = 0.11

$$\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1+2\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - x^4 + x^8),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1 + 2*#1^5) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 40, normalized size = 0.11

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\_R^{2}\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(_R^2/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 - x^4 + 1), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(263) = 526$.
time = 0.40, size = 573, normalized size = 1.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(96*sqrt(6
)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 576*x^2 + 576) + 1/48*sqrt(6)*(sqrt
(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(-96*sqrt(6)*sqrt(3)*sqrt(2)*
x*sqrt(sqrt(3) + 2) + 576*x^2 + 576) - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sq
rt(2))*sqrt(-4*sqrt(3) + 8)*log(48*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3
) + 8) + 576*x^2 + 576) + 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-
4*sqrt(3) + 8)*log(-48*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 576
*x^2 + 576) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sq
rt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 1/72*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(-96*s
qrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 576*x^2 + 576)*sqrt(sqrt(3) +
2) + sqrt(3) + 2) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt
(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 1/3*sqrt(3)*sqrt(2)*sqrt(sqrt(6)*
sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 6*x^2 + 6)*sqrt(sqrt(3) + 2) - sqrt(3
```

```
) - 2) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt
(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/144*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(-48
*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 576*x^2 + 576)*sqrt(-4*sq
rt(3) + 8) - sqrt(3) + 2) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arcta
n(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2
)*sqrt(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-
4*sqrt(3) + 8) + sqrt(3) - 2)
```

Sympy [**A**]
time = 1.56, size = 26, normalized size = 0.07

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-442368t^7 - 192t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 - 1
92*_t**3 + x)))
```

Giac [**A**]
time = 3.59, size = 253, normalized size = 0.71

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

Mupad [**B**]
time = 0.08, size = 286, normalized size = 0.81

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}+\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}-\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}-\frac{2^{3/4}\,\sqrt{3}\,x\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}+\frac{2^{3/4}\,\sqrt{3}\,x\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8 - x^4 + 1),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)) - (3^(1/2)
*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4))/12
 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i + 1)) + (3^(1/2)
*x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i + 1)))*(8 - 3^(1/2)*8i)^(1/4)*
1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*(3^(1/
2)*1i - 1)) - (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i
- 1)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^
(1/2)*1i + 1)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*
1i + 1)^(1/4))/(2*(3^(1/2)*1i - 1)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.362 $\int \frac{1}{1-x^4+x^8} \, dx$

Optimal. Leaf size=275

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}$$

```
[Out] -1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1
/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^
(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))
*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2
+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1
/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1360, 1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[6] - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1360

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-1+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-1+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\right.}}{1-\sqrt{}}}{4\sqrt{}}$$

$$= -\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{-}{1+}}{}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}\right.}{4\sqrt{6}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 42, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 30, normalized size = 0.11

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	30
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(3\_R^2+3\_Rx+x^2\right)\right)}{4}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="maxima")
[Out] integrate(1/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 0.35, size = 220, normalized size = 0.80

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}\left(x^3+x\right)+3x^2+1}\left(\sqrt{3}\sqrt{2}x+2\right)}{3x^2-2}\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(36x^4+36\sqrt{3}\sqrt{2}\left(x^3+x\right)+108x^2+36\right)-\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(36x^4-36\sqrt{3}\sqrt{2}\left(x^3+x\right)+108x^2+36\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="fricas")
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) + 1/24*sqrt(3)*sqrt(2)*log(36*x^4 + 36*sqrt(3)*sqrt(2)*(x^3 + x) + 108
*x^2 + 36) - 1/24*sqrt(3)*sqrt(2)*log(36*x^4 - 36*sqrt(3)*sqrt(2)*(x^3 + x)
 + 108*x^2 + 36)
```

Sympy [A]
time = 0.11, size = 165, normalized size = 0.60

$$\frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24}-\frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24}+\frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

Giac [A]
time = 5.92, size = 205, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

Mupad [B]
time = 0.04, size = 53, normalized size = 0.19

$$\sqrt{6}\ \mathrm{atan}\left(\frac{\sqrt{6}\ x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\ \mathrm{atan}\left(\frac{\sqrt{6}\ x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - x^4 + 1),x)
```

```
[Out] - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/12)
- 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/12)
```

3.363 $\int \frac{1}{x^2(1-x^4+x^8)}\,dx$

Optimal. Leaf size=360

$$-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right.}{4\sqrt{3\left(2+\sqrt{3}\right.}}$$

```
[Out] -1/x+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/
8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arcta
n((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/
2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2
)))/(3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/
2*2^(1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/
2)+1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*
2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2)
)/(1/2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A]
time = 0.16, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1382, 1520, 1293, 1183, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 + Sqrt[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*
Sqrt[3*(2 - Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]
]/(4*Sqrt[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[
3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]
)/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[
(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1520

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[(f*x)^m*(Simp[d*r - (c*d - e*q)*x^(n/2), x]/
(q - r*x^(n/2) + c*x^n)), x], x] + Dist[c/(2*q*r), Int[(f*x)^m*(Simp[d*r +
(c*d - e*q)*x^(n/2), x]/(q + r*x^(n/2) + c*x^n)), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-x^4+x^8\right)}\,dx = -\frac{1}{x}+\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{x}+\frac{\int \frac{x^2\left(\sqrt{3}-2x^2\right)}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{x^2\left(\sqrt{3}+2x^2\right)}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{x}-\frac{\int \frac{-2+\sqrt{3}\,x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{2+\sqrt{3}\,x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{x}-\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{2\sqrt{2-\sqrt{3}}+\left(2-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{-2\sqrt{2+\cdots}}{1-\sqrt{2\cdots}}}{4\sqrt{3\cdots}}$$

$$= -\frac{1}{x}+\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx+\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{\cdots}{1\cdots}$$

$$= -\frac{1}{x}+\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\,\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(1+\cdots\right.$$

$$= -\frac{1}{x}+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\tan^{-1}\left(\cdots\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.01, size = 61, normalized size = 0.17

$$-\frac{1}{x}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1+2\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#
1 + 2*#1^5) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 52, normalized size = 0.14

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\text{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(-27\_R^7+6\_R^3+x\right)\right)}{4}$	40
default	$-\frac{1}{x}-\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(\_R^6-\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x-1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^6 - x^2)/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 736 vs. 2(268) = 536.
time = 0.40, size = 736, normalized size = 2.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/96*(4*sqrt(6)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8)*arctan(1/36*sqrt(6)*sqrt(3)
*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3)
+ 8) + 12)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(
6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2)
+ 4*sqrt(6)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8)*arctan(1/36*sqrt(6)*sqrt(3)*sqrt
(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8)
+ 12)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2
*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2) - 8*s
qrt(6)*sqrt(2)*x*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x
 - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 1/3*sqrt(6*x^2 + sqrt(6)*(2*sqrt(3)*sqr
t(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 6)*(2*sqrt(3)*sqrt(2) - 3*sqrt(2)
)*sqrt(sqrt(3) + 2) + sqrt(3) - 2) - 8*sqrt(6)*sqrt(2)*x*sqrt(sqrt(3) + 2)*
arctan(-1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) +
 1/3*sqrt(6*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3)
+ 2) + 6)*(2*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(3) + 2) - sqrt(3) + 2)
- 2*sqrt(6)*(sqrt(3)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt(sqrt(3) + 2)*log(576*x^2
 + 96*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 576)
+ 2*sqrt(6)*(sqrt(3)*sqrt(2)*x - 2*sqrt(2)*x)*sqrt(sqrt(3) + 2)*log(576*x^2
 - 96*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 576)
- sqrt(6)*(sqrt(3)*sqrt(2)*x + 2*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8)*log(576*x^
2 + 48*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 5
76) + sqrt(6)*(sqrt(3)*sqrt(2)*x + 2*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8)*log(57
6*x^2 - 48*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8)
 + 576) + 96)/x
```

Sympy [A]
time = 1.49, size = 29, normalized size = 0.08

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-442368t^7 + 384t^3 + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-442368*_t**7 + 3
84*_t**3 + x))) - 1/x
```

Giac [A]
time = 4.72, size = 258, normalized size = 0.72

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
```

```
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/x
```

Mupad [B]
time = 1.29, size = 253, normalized size = 0.70

$$-\frac{1}{x}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}+\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}-\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}-\frac{2^{3/4}\,\sqrt{3}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}+\frac{2^{3/4}\,\sqrt{3}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x*
(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1i)
/12 - 1/x - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)
) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8
i)^(1/4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(3/4))
 - (2^(3/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4
)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4))
+ (2^(3/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4))/1
2
```

3.364 $\int \frac{1}{x^4(1-x^4+x^8)}\,dx$

Optimal. Leaf size=370

$$-\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{}$$

```
[Out] -1/3/x^3+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*
6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)
-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^
(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))
/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/4*arctan((2*x+1/2*6
^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/
8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/8*ln(1+
x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))
```

Rubi [A]
time = 0.16, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1382, 1435, 1183, 648, 632, 210, 642}

$$-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2
 + Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]
] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1435

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4(1-x^4+x^8)}\,dx = -\frac{1}{3x^3}+\frac{1}{3}\int \frac{3-3x^4}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{3x^3}-\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}\,x^2-x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}\,x^2-x^4}\,dx}{2\sqrt{3}}$$
$$= -\frac{1}{3x^3}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+$$
$$= -\frac{1}{3x^3}-\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx-\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int$$
$$= -\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)-\frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\right.$$
$$= -\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 65, normalized size = 0.18

$$-\frac{1}{3x^3}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/
(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 50, normalized size = 0.14

method	result	size

risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(-9\_R^5+2\_R+x\right)\right)}{4}$	38
default	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/3/x^3+1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="maxima")
[Out] -1/3/x^3 - integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(260) = 520$.
time = 0.45, size = 760, normalized size = 2.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/96*(4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arctan(1/36*sqrt(6)*sqrt(3
)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3)
 + 8) + 12)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt
(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2)
 + 4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arctan(1/36*sqrt(6)*sqrt(3)*s
qrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) +
8) + 12)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)
*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2) -
8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*(2*sqrt(3)*sqrt
(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 1/3*sqrt(6*x^2 + sqrt(6)*(2*sqrt(3
)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 6)*(2*sqrt(3)*sqrt(2) - 3*sq
rt(2))*sqrt(sqrt(3) + 2) + sqrt(3) - 2) - 8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3
```

```
) + 2)*arctan(-1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3)
 + 2) + 1/3*sqrt(6*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(s
qrt(3) + 2) + 6)*(2*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(3) + 2) - sqrt(3
) + 2) + 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*
log(144*x^2 + 24*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) +
 2) + 144) - 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) +
 2)*log(144*x^2 - 24*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(
3) + 2) + 144) + sqrt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x^3)*sqrt(-4*sqrt
(3) + 8)*log(144*x^2 + 12*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(
-4*sqrt(3) + 8) + 144) - sqrt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x^3)*sqrt
(-4*sqrt(3) + 8)*log(144*x^2 - 12*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*
x)*sqrt(-4*sqrt(3) + 8) + 144) - 32)/x^3
```

Sympy [A]
time = 1.51, size = 31, normalized size = 0.08

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(-9216t^5 + 8t + x\right)\right)\right) - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-9216*_t**5 + 8*_
t + x))) - 1/(3*x**3)
```

Giac [A]
time = 4.16, size = 258, normalized size = 0.70

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1) - 1/3/x^3
```

Mupad [B]
time = 1.29, size = 213, normalized size = 0.58

$$-\frac{1}{3\,x^3}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}+\frac{\sqrt{3}\,x\,1{}\mathrm{i}}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1{}\mathrm{i}}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}-\frac{\sqrt{3}\,x}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x^8 - x^4 + 1)),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(
1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(
1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))
*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (3^(1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4
) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4))/12 - 1/(3*x
^3) + (2^(3/4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^
(1/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.365 $\int \frac{1}{x^6(1-x^4+x^8)}\,dx$

Optimal. Leaf size=287

$$-\frac{1}{5x^5}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+}}{\sqrt{2}}\right.}{2\sqrt{(}}$$

[Out] -1/5/x^5-1/x+1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)+1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)

Rubi [A]
time = 0.17, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {1382, 1518, 12, 1386, 1183, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{1}{5x^5}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{1}{x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^6*(1 - x^4 + x^8)),x]

[Out] -1/5*1/x^5 - x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6])

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1386

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[x^(
m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Dist[1/(2*c*
r), Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1}{x^6(1-x^4+x^8)}\,dx = -\frac{1}{5x^5}+\frac{1}{5}\int \frac{5-5x^4}{x^2(1-x^4+x^8)}\,dx$$
$$= -\frac{1}{5x^5}-\frac{1}{x}-\frac{1}{5}\int \frac{5x^6}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{5x^5}-\frac{1}{x}-\int \frac{x^6}{1-x^4+x^8}\,dx$$
$$= -\frac{1}{5x^5}-\frac{1}{x}+\frac{\int \frac{1-\sqrt{3}\,x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{1+\sqrt{3}\,x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$
$$= -\frac{1}{5x^5}-\frac{1}{x}-\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2}}{1-}}{4}$$
$$= -\frac{1}{5x^5}-\frac{1}{x}-\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{-\sqrt{2+\sqrt{3}}}{1-\sqrt{2+\sqrt{3}}}}{4\sqrt{6}}$$
$$= -\frac{1}{5x^5}-\frac{1}{x}-\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}}-\frac{\log}{}$$
$$= -\frac{1}{5x^5}-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{}}\right)}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 54, normalized size = 0.19

$$-\frac{1}{5x^5}-\frac{1}{x}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^3}{-1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 43, normalized size = 0.15

method	result	size
default	$-\frac{1}{5x^5}-\frac{1}{x}-\frac{\left(\sum_{\_R=\text{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	43
risch	$\frac{-x^4-\frac{1}{5}}{x^5}+\frac{\left(\sum_{\_R=\text{RootOf}\left(9\_Z^4+1\right)}\_R\ln\left(-9x\_R^3-3\_R^2+x^2\right)\right)}{4}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^5-1/x-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*(5*x^4 + 1)/x^5 - integrate(x^6/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 0.34, size = 243, normalized size = 0.85

$$\frac{20\sqrt{3}\sqrt{2}x^5\arctan\left(-\frac{\sqrt{3}\sqrt{2}(x^3-x)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)+20\sqrt{3}\sqrt{2}x^5\arctan\left(-\frac{\sqrt{3}\sqrt{2}(x^3-x)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}\left(\sqrt{3}\sqrt{2}x+2\right)}{3x^2-2}\right)+5\sqrt{3}\sqrt{2}x^5\log\left(36x^4+36\sqrt{3}\sqrt{2}(x^3+x)+108x^2+36\right)-5\sqrt{3}\sqrt{2}x^5\log\left(36x^4-36\sqrt{3}\sqrt{2}(x^3+x)+108x^2+36\right)-120x^4-24}{120x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(20*sqrt(3)*sqrt(2)*x^5*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sq
rt(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3
*x^2 - 2)) + 20*sqrt(3)*sqrt(2)*x^5*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^
2 - sqrt(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x +
2))/(3*x^2 - 2)) + 5*sqrt(3)*sqrt(2)*x^5*log(36*x^4 + 36*sqrt(3)*sqrt(2)*(x
^3 + x) + 108*x^2 + 36) - 5*sqrt(3)*sqrt(2)*x^5*log(36*x^4 - 36*sqrt(3)*sqr
t(2)*(x^3 + x) + 108*x^2 + 36) - 120*x^4 - 24)/x^5
```

Sympy [A]
time = 0.12, size = 182, normalized size = 0.63

$$\frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}-\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}\,x^3-4x^2+2\sqrt{6}\,x-3\right)\right)}{24}+\frac{\sqrt{6}\left(-2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}+\frac{1}{3}\right)-2\,\mathrm{atan}\left(\sqrt{6}\,x^3+4x^2+2\sqrt{6}\,x+3\right)\right)}{24}-\frac{\sqrt{6}\,\log\left(x^4-\sqrt{6}\,x^3+3x^2-\sqrt{6}\,x+1\right)}{24}+\frac{\sqrt{6}\,\log\left(x^4+\sqrt{6}\,x^3+3x^2+\sqrt{6}\,x+1\right)}{24}+\frac{-5x^4-1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 + sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*x**3
+ 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2
- sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x
+ 1)/24 + (-5*x**4 - 1)/(5*x**5)
```

Giac [A]
time = 3.62, size = 217, normalized size = 0.76

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{5x^4+1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt(6
)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*arct
an((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sq
rt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*
sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/5*(5*x^4 + 1)/x^5
```

Mupad [B]
time = 1.30, size = 63, normalized size = 0.22

$$-\frac{x^4+\frac{1}{5}}{x^5}+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^8 - x^4 + 1)),x)

[Out] 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) +
6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12) -
(x^4 + 1/5)/x^5
```

3.366 $\int \frac{1}{x^8(1-x^4+x^8)}\,dx$

Optimal. Leaf size=377

$$-\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)$$

```
[Out] -1/7/x^7-1/3/x^3-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2
*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2
))/(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2-x*(1/2
*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2
)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*
2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/4*arctan((2
*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(
1/2))+1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1
/8*ln(1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))
```

Rubi [A]
time = 0.19, antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1382, 1518, 12, 1387, 1141, 1175, 632, 210, 1178, 642}

$$-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 - 1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] -
 2*x)/Sqrt[2 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[
3]] - 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt
[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 -
 Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x
^2])/8 + (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^8\left(1-x^4+x^8\right)}\,dx &= -\frac{1}{7x^7}+\frac{1}{7}\int \frac{7-7x^4}{x^4\left(1-x^4+x^8\right)}\,dx \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{1}{21}\int \frac{21x^4}{1-x^4+x^8}\,dx \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}-\int \frac{x^4}{1-x^4+x^8}\,dx \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\int \frac{x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}} \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\int \frac{1-x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1+x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1-x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{4\sqrt{3}}+ \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}+\frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}- \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}+\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}- \\
&= -\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 54, normalized size = 0.14

$$
-\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{1}{4}\mathrm{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+2\#1^4}\&\right]
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 - 1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 +
 2*#1^4) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 51, normalized size = 0.14

method	result	size
risch	$\frac{-\frac{x^4}{3}-\frac{1}{7}}{x^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(18\_R^5-\_R+x\right)\right)}{4}$	44
default	$-\frac{1}{7x^7}-\frac{1}{3x^3}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/x^7-1/3/x^3-1/4*sum(_R^4/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/21*(7*x^4 + 3)/x^7 - integrate(x^4/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 628 vs. 2(265) = 530.
time = 0.38, size = 628, normalized size = 1.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/672*(56*sqrt(6)*sqrt(2)*x^7*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)
*sqrt(2)*x*sqrt(sqrt(3) + 2) + 1/18*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(6*sqrt(6)*
sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 36*x^2 + 36)*sqrt(sqrt(3) + 2) - sqrt
(3) - 2) + 56*sqrt(6)*sqrt(2)*x^7*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqr
t(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 1/18*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(-6*sqr
t(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 36*x^2 + 36)*sqrt(sqrt(3) + 2) +
 sqrt(3) + 2) - 28*sqrt(6)*sqrt(2)*x^7*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqr
t(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/36*sqrt(6)*sqrt(3)*sqrt(2)*
```

```
sqrt(3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 36*x^2 + 36)*sqrt(-
4*sqrt(3) + 8) + sqrt(3) - 2) - 28*sqrt(6)*sqrt(2)*x^7*sqrt(-4*sqrt(3) + 8)
*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/36*sqrt(6)*
sqrt(3)*sqrt(2)*sqrt(-3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 36
*x^2 + 36)*sqrt(-4*sqrt(3) + 8) - sqrt(3) + 2) - 224*x^4 - 14*sqrt(6)*(sqrt
(3)*sqrt(2)*x^7 - 2*sqrt(2)*x^7)*sqrt(sqrt(3) + 2)*log(6*sqrt(6)*sqrt(3)*sq
rt(2)*x*sqrt(sqrt(3) + 2) + 36*x^2 + 36) + 14*sqrt(6)*(sqrt(3)*sqrt(2)*x^7
- 2*sqrt(2)*x^7)*sqrt(sqrt(3) + 2)*log(-6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sq
rt(3) + 2) + 36*x^2 + 36) - 7*sqrt(6)*(sqrt(3)*sqrt(2)*x^7 + 2*sqrt(2)*x^7)
*sqrt(-4*sqrt(3) + 8)*log(3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8)
+ 36*x^2 + 36) + 7*sqrt(6)*(sqrt(3)*sqrt(2)*x^7 + 2*sqrt(2)*x^7)*sqrt(-4*sq
rt(3) + 8)*log(-3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 36*x^2 +
 36) - 96)/x^7
```

Sympy [**A**]
time = 1.55, size = 37, normalized size = 0.10

$$\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(18432t^5 - 4t + x\right)\right)\right) + \frac{-7x^4 - 3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-x**4+1),x)
```

```
[Out] RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(18432*_t**5 - 4*_
t + x))) + (-7*x**4 - 3)/(21*x**7)
```

Giac [**A**]
time = 3.29, size = 265, normalized size = 0.70

$$-\frac{1}{24}\left(\sqrt{6} - 3\sqrt{2}\right)\arctan\left(\frac{4x + \sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6} - 3\sqrt{2}\right)\arctan\left(\frac{4x - \sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6} + 3\sqrt{2}\right)\arctan\left(\frac{4x + \sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) - \frac{1}{24}\left(\sqrt{6} + 3\sqrt{2}\right)\arctan\left(\frac{4x - \sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}\right) - \frac{1}{48}\left(\sqrt{6} - 3\sqrt{2}\right)\log\left(x^2 + \frac{1}{2}x\left(\sqrt{6} + \sqrt{2}\right) + 1\right) + \frac{1}{48}\left(\sqrt{6} - 3\sqrt{2}\right)\log\left(x^2 - \frac{1}{2}x\left(\sqrt{6} + \sqrt{2}\right) + 1\right) - \frac{1}{48}\left(\sqrt{6} + 3\sqrt{2}\right)\log\left(x^2 + \frac{1}{2}x\left(\sqrt{6} - \sqrt{2}\right) + 1\right) + \frac{1}{48}\left(\sqrt{6} + 3\sqrt{2}\right)\log\left(x^2 - \frac{1}{2}x\left(\sqrt{6} - \sqrt{2}\right) + 1\right) - \frac{7x^4 + 3}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/21*(7*x^4 + 3)/x^7
```

Mupad [**B**]
time = 0.06, size = 486, normalized size = 1.29

$$-\frac{\frac{x^4}{3} + \frac{1}{7}}{x^7} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8 - \sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8 - \sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4} + \frac{\sqrt{8 - \sqrt{3}\,8\mathrm{i}}}{4}\right)} + \frac{\sqrt{3}\,x\left(8 - \sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8 - \sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4} + \frac{\sqrt{8 - \sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\left(8 - \sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{12} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\left(8 - \sqrt{3}\,8\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{3}\,\sqrt{8 - \sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4} + \frac{\sqrt{8 - \sqrt{3}\,8\mathrm{i}}}{4}\right)} - \frac{\sqrt{3}\,x\left(8 - \sqrt{3}\,8\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{3}\,\sqrt{8 - \sqrt{3}\,8\mathrm{i}}\,1\mathrm{i}}{4} + \frac{\sqrt{8 - \sqrt{3}\,8\mathrm{i}}}{4}\right)}\right)\left(8 - \sqrt{3}\,8\mathrm{i}\right)^{1/4}}{12} - \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(1 + \sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1 + \sqrt{3}\,1\mathrm{i}}}{2} - \frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1 + \sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)} - \frac{2^{3/4}\sqrt{3}\,x\left(1 + \sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{2}\,\sqrt{1 + \sqrt{3}\,1\mathrm{i}}}{2} - \frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1 + \sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)\left(1 + \sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{12} - \frac{2^{3/4}\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(1 + \sqrt{3}\,1\mathrm{i}\right)^{1/4}1\mathrm{i}}{2\left(\frac{\sqrt{2}\,\sqrt{1 + \sqrt{3}\,1\mathrm{i}}}{2} - \frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1 + \sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)} + \frac{2^{3/4}\sqrt{3}\,x\left(1 + \sqrt{3}\,1\mathrm{i}\right)^{1/4}}{2\left(\frac{\sqrt{2}\,\sqrt{1 + \sqrt{3}\,1\mathrm{i}}}{2} - \frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1 + \sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}\right)\left(1 + \sqrt{3}\,1\mathrm{i}\right)^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2
)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i
)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(
8 - 3^(1/2)*8i)^(1/4)*1i)/12 - (x^4/3 + 1/7)/x^7 + (3^(1/2)*atan((x*(8 - 3^
(1/2)*8i)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2
)*8i)^(1/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1
/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/1
2 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(
3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2))
- (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i +
1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i
+ 1)^(1/4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)
*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i
+ 1)^(1/2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2
)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2
)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.367 $\int \frac{x^m}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=117

$$\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}$$

```
[Out] 2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4/(3-5^(1/2)))/(1+m)/
(3-5^(1/2))*5^(1/2)-2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],-2*x^4
/(3+5^(1/2)))/(1+m)*5^(1/2)/(3+5^(1/2))
```

Rubi [A]

time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1389, 371}

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 + 3*x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (-2*x^4)/(3 - Sqrt[
5])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1, (
1 + m)/4, (5 + m)/4, (-2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1 + m
))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^m}{1+3x^4+x^8}\,dx = \frac{\int \frac{x^m}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{x^m}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$
$$= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)} - \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};-\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.07, size = 79, normalized size = 0.68

$$\frac{x^m\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)\left(\frac{x}{x-\#1}\right)^{-m}}{3\#1^3+2\#1^7}\&\right]}{4m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(1 + 3*x^4 + x^8),x]
```

```
[Out] (x^m*RootSum[1 + 3*#1^4 + #1^8 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(x
 - #1))]/((x/(x - #1))^m*(3*#1^3 + 2*#1^7)) & ])/(4*m)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8+3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8+3*x^4+1),x)
```

```
[Out] int(x^m/(x^8+3*x^4+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+3*x^4+1),x, algorithm="fricas")
[Out] integral(x^m/(x^8 + 3*x^4 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8 + 3x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8+3*x**4+1),x)
[Out] Integral(x**m/(x**8 + 3*x**4 + 1), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8+3*x^4+1),x, algorithm="giac")
[Out] integrate(x^m/(x^8 + 3*x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{x^8 + 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(3*x^4 + x^8 + 1),x)
[Out] int(x^m/(3*x^4 + x^8 + 1), x)
```

3.368 $\int \frac{x^{11}}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=62

$$\frac{x^4}{4}-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

```
[Out] 1/4*x^4-1/40*ln(2*x^4-5^(1/2)+3)*(15-7*5^(1/2))-1/40*ln(2*x^4+5^(1/2)+3)*(1
5+7*5^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 717, 646, 31}

$$\frac{x^4}{4}-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 + 3*x^4 + x^8),x]
```

```
[Out] x^4/4 - ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] + 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{1+3x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{1+3x+x^2}\,dx, x, x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{4}\text{Subst}\left(\int \frac{-1-3x}{1+3x+x^2}\,dx, x, x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{40}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\text{Subst}\\ &= \frac{x^4}{4}-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 0.92

$$\frac{1}{40}\left(10x^4+\left(-15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right)-\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 + 3*x^4 + x^8),x]
```

```
[Out] (10*x^4 + (-15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] - (15 + 7*Sqrt[5])*Lo
g[3 + Sqrt[5] + 2*x^4])/40
```

Maple [A]
time = 0.04, size = 38, normalized size = 0.61

method	result	size
default	$\frac{x^4}{4}-\frac{3\ln(x^8+3x^4+1)}{8}-\frac{7\,\mathrm{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)\sqrt{5}}{20}$	38
risch	$\frac{x^4}{4}-\frac{3\ln\left(2x^4-\sqrt{5}+3\right)}{8}+\frac{7\ln\left(2x^4-\sqrt{5}+3\right)\sqrt{5}}{40}-\frac{3\ln\left(2x^4+\sqrt{5}+3\right)}{8}-\frac{7\ln\left(2x^4+\sqrt{5}+3\right)\sqrt{5}}{40}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4-3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Maxima [A]
time = 0.53, size = 50, normalized size = 0.81

$$\frac{1}{4}\,x^4+\frac{7}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)-\frac{3}{8}\,\log\left(x^8+3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 3
/8*log(x^8 + 3*x^4 + 1)
```

Fricas [A]
time = 0.36, size = 62, normalized size = 1.00

$$\frac{1}{4}\,x^4+\frac{7}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^8+6\,x^4-\sqrt{5}\,(2\,x^4+3)+7}{x^8+3\,x^4+1}\right)-\frac{3}{8}\,\log\left(x^8+3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 +
 3*x^4 + 1)) - 3/8*log(x^8 + 3*x^4 + 1)
```

Sympy [A]
time = 0.05, size = 60, normalized size = 0.97

$$\frac{x^4}{4}+\left(-\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(-\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+3*x**4+1),x)
```

```
[Out] x**4/4 + (-3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (-7*sqrt(5)/40
 - 3/8)*log(x**4 + sqrt(5)/2 + 3/2)
```

Giac [A]
time = 3.61, size = 50, normalized size = 0.81

$$\frac{1}{4}\,x^4+\frac{7}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)-\frac{3}{8}\,\log\left(x^8+3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 3
/8*log(x^8 + 3*x^4 + 1)
```

Mupad [B]
time = 0.13, size = 64, normalized size = 1.03

$$\frac{7\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}-\frac{3\,\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}-\frac{3\,\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}-\frac{7\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(3*x^4 + x^8 + 1),x)
```

```
[Out] (7*5^(1/2)*log(x^4 - 5^(1/2)/2 + 3/2))/40 - (3*log(5^(1/2)/2 + x^4 + 3/2))/
8 - (3*log(x^4 - 5^(1/2)/2 + 3/2))/8 - (7*5^(1/2)*log(5^(1/2)/2 + x^4 + 3/2
))/40 + x^4/4
```

3.369 $\int \frac{x^9}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=90

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

```
[Out] 1/2*x^2+1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(1-2/5*5^(1/2))-1/2*arctan(x^2*2^
(1/2)/(3+5^(1/2))^(1/2))*(1+2/5*5^(1/2))
```

Rubi [A]
time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1373, 1136, 1180, 209}

$$-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 + 3*x^4 + x^8),x]
```

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (Sq
rt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{1+3x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1+3x^2+x^4}\,dx, x, x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{1+3x^2}{1+3x^2+x^4}\,dx, x, x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{20}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)-\frac{1}{20}\left(15+7\sqrt{5}\right)\text{Subst}\left(\right.\\ &= \frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{20}\sqrt{180-80\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{1}{2}}\right.\end{aligned}$$

Mathematica [A]
time = 0.10, size = 97, normalized size = 1.08

$$\frac{1}{40}\left(20x^2-\sqrt{6-2\sqrt{5}}\left(15+7\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\sqrt{2\left(3+\sqrt{5}\right)}\left(-15+7\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 + 3*x^4 + x^8),x]
```

```
[Out] (20*x^2 - Sqrt[6 - 2*Sqrt[5]]*(15 + 7*Sqrt[5])*ArcTan[Sqrt[2/(3 + Sqrt[5])]
*x^2] + Sqrt[2*(3 + Sqrt[5])]*(-15 + 7*Sqrt[5])*ArcTan[Sqrt[(3 + Sqrt[5])/2
]*x^2])/40
```

Maple [A]
time = 0.06, size = 79, normalized size = 0.88

method	result	size
risch	$\frac{x^2}{2}+\frac{\left(\sum_{\_R=\text{RootOf}\left(25\_Z^4+90\_Z^2+1\right)}\_R\ln\left(15\_R^3+8x^2+47\_R\right)\right)}{4}$	42

default	$\frac{x^2}{2}-\frac{\left(7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}-\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2-1/5*(7+3*5^(1/2))*5^(1/2)/(2*5^(1/2)+2)*arctan(4*x^2/(2*5^(1/2)+2))
-1/5*(-7+3*5^(1/2))*5^(1/2)/(2*5^(1/2)-2)*arctan(4*x^2/(2*5^(1/2)-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - integrate((3*x^4 + 1)*x/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(50) = 100$.
time = 0.43, size = 142, normalized size = 1.58

$$\frac{1}{2}x^2-\frac{1}{5}\sqrt{5}\sqrt{-4\sqrt{5}+9}\arctan\left(\frac{1}{4}\sqrt{4x^4-2\sqrt{5}+6}\left(3\sqrt{5}+7\right)\sqrt{-4\sqrt{5}+9}-\frac{1}{2}\left(3\sqrt{5}x^2+7x^2\right)\sqrt{-4\sqrt{5}+9}\right)-\frac{1}{5}\sqrt{5}\sqrt{4\sqrt{5}+9}\arctan\left(-\frac{1}{4}\left(6\sqrt{5}x^2-14x^2-\sqrt{4x^4+2\sqrt{5}+6}\left(3\sqrt{5}-7\right)\right)\sqrt{4\sqrt{5}+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/5*sqrt(5)*sqrt(-4*sqrt(5) + 9)*arctan(1/4*sqrt(4*x^4 - 2*sqrt(5
) + 6)*(3*sqrt(5) + 7)*sqrt(-4*sqrt(5) + 9) - 1/2*(3*sqrt(5)*x^2 + 7*x^2)*s
qrt(-4*sqrt(5) + 9)) - 1/5*sqrt(5)*sqrt(4*sqrt(5) + 9)*arctan(-1/4*(6*sqrt(
5)*x^2 - 14*x^2 - sqrt(4*x^4 + 2*sqrt(5) + 6)*(3*sqrt(5) - 7))*sqrt(4*sqrt(
5) + 9))
```

Sympy [A]
time = 0.12, size = 54, normalized size = 0.60

$$\frac{x^2}{2}+2\cdot\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8+3*x**4+1),x)
```

```
[Out] x**2/2 + 2*(1/4 - sqrt(5)/10)*atan(2*x**2/(-1 + sqrt(5))) - 2*(sqrt(5)/10 +
 1/4)*atan(2*x**2/(1 + sqrt(5)))
```

Giac [A]
time = 3.21, size = 66, normalized size = 0.73

$$\frac{1}{2}x^2 - \frac{1}{20}\left(3x^4\left(\sqrt{5}-5\right)+\sqrt{5}-5\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right) - \frac{1}{20}\left(3x^4\left(\sqrt{5}+5\right)+\sqrt{5}+5\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/20*(3*x^4*(sqrt(5) - 5) + sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) +
1)) - 1/20*(3*x^4*(sqrt(5) + 5) + sqrt(5) + 5)*arctan(2*x^2/(sqrt(5) - 1))
```

Mupad [B]
time = 1.34, size = 130, normalized size = 1.44

$$2\,\mathrm{atanh}\left(\frac{1280\,x^2\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\sqrt{5}-192}+\frac{768\sqrt{5}\,x^2\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\sqrt{5}-192}\right)\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}} - 2\,\mathrm{atanh}\left(\frac{1280\,x^2\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\sqrt{5}+192}-\frac{768\sqrt{5}\,x^2\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{64\sqrt{5}+192}\right)\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(3*x^4 + x^8 + 1),x)
```

```
[Out] 2*atanh((1280*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192) + (768*5^(1
/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) - 192))*(5^(1/2)/20 - 9/80)^
(1/2) - 2*atanh((1280*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192) -
 (768*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(64*5^(1/2) + 192))*(- 5^(1/
2)/20 - 9/80)^(1/2) + x^2/2
```

3.370 $\int \frac{x^7}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=55

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

```
[Out] 1/40*ln(2*x^4-5^(1/2)+3)*(5-3*5^(1/2))+1/40*ln(2*x^4+5^(1/2)+3)*(5+3*5^(1/2
))
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1371, 646, 31}

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] + 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{1+3x^4+x^8}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{x}{1+3x+x^2}\,dx, x, x^4\right)$$
$$= \frac{1}{40}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right) + \frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{}{\frac{3}{2}+}\right.$$
$$= \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right) + \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.96

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right) + \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + S
qrt[5] + 2*x^4])/40
```

Maple [A]
time = 0.02, size = 33, normalized size = 0.60

method	result	size
default	$\frac{\ln\left(x^8+3x^4+1\right)}{8}+\frac{3\,\text{arctanh}\left(\frac{\left(2x^4+3\right)\sqrt{5}}{5}\right)\sqrt{5}}{20}$	33
risch	$\frac{\ln\left(2x^4+\sqrt{5}+3\right)}{8}+\frac{3\ln\left(2x^4+\sqrt{5}+3\right)\sqrt{5}}{40}+\frac{\ln\left(2x^4-\sqrt{5}+3\right)}{8}-\frac{3\ln\left(2x^4-\sqrt{5}+3\right)\sqrt{5}}{40}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Maxima [A]
time = 0.51, size = 45, normalized size = 0.82

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)+\frac{1}{8}\log\left(x^8+3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x^
8 + 3*x^4 + 1)
```

Fricas [A]
time = 0.34, size = 56, normalized size = 1.02

$$\frac{3}{40}\sqrt{5}\,\log\left(\frac{2\,x^8+6\,x^4+\sqrt{5}\,(2\,x^4+3)+7}{x^8+3\,x^4+1}\right)+\frac{1}{8}\,\log\left(x^8+3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
)) + 1/8*log(x^8 + 3*x^4 + 1)
```

Sympy [A]
time = 0.05, size = 53, normalized size = 0.96

$$\left(\frac{1}{8}-\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8+3*x**4+1),x)
```

```
[Out] (1/8 - 3*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (1/8 + 3*sqrt(5)/40)*log
(x**4 + sqrt(5)/2 + 3/2)
```

Giac [A]
time = 3.32, size = 45, normalized size = 0.82

$$-\frac{3}{40}\sqrt{5}\,\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)+\frac{1}{8}\,\log\left(x^8+3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/8*log(x^
8 + 3*x^4 + 1)
```

Mupad [B]
time = 1.36, size = 59, normalized size = 1.07

$$\frac{\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}+\frac{\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{8}-\frac{3\,\sqrt{5}\,\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}+\frac{3\,\sqrt{5}\,\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(3*x^4 + x^8 + 1),x)
```

```
[Out] log(x^4 - 5^(1/2)/2 + 3/2)/8 + log(5^(1/2)/2 + x^4 + 3/2)/8 - (3*5^(1/2)*lo
g(x^4 - 5^(1/2)/2 + 3/2))/40 + (3*5^(1/2)*log(5^(1/2)/2 + x^4 + 3/2))/40
```

3.371 $\int \frac{x^5}{1+3x^4+x^8} dx$

Optimal. Leaf size=81

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] -1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(1/2-1/10*5^(1/2))+1/2*arctan(x^2*2^(1/2
)/(3+5^(1/2))^(1/2))*(1/2+1/10*5^(1/2))
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1373, 1144, 209}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\text{ArcTan}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\text{ArcTan}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + 3*x^4 + x^8),x]
```

```
[Out] (Sqrt[(3 + Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 - (Sqrt[(3 - S
qrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5}{1+3x^4+x^8}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1+3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{1}{20}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right) + \frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{}{\frac{3}{2}+}\right.$$
$$= \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right) - \frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\right.}\right.$$

Mathematica [A]
time = 0.03, size = 75, normalized size = 0.93

$$\frac{2\sqrt{5}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right) + \left(5-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{10\sqrt{6-2\sqrt{5}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + 3*x^4 + x^8),x]
```

```
[Out] (2*Sqrt[5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2] + (5 - 3*Sqrt[5])*ArcTan[Sqrt[
(3 + Sqrt[5])/2]*x^2])/(10*Sqrt[6 - 2*Sqrt[5]])
```

Maple [A]
time = 0.04, size = 70, normalized size = 0.86

method	result	size
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(25\_Z^4+15\_Z^2+1\right)} \_R\ln\left(-10\_R^3+x^2-3\_R\right)\right)}{4}$	34
default	$\frac{\left(3+\sqrt{5}\right)\sqrt{5}\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{10+10\sqrt{5}} + \frac{\sqrt{5}\left(\sqrt{5}-3\right)\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{-10+10\sqrt{5}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*(3+5^(1/2))*5^(1/2)/(2*5^(1/2)+2)*arctan(4*x^2/(2*5^(1/2)+2))+1/5*5^(1/
2)*(5^(1/2)-3)/(2*5^(1/2)-2)*arctan(4*x^2/(2*5^(1/2)-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(41) = 82.
time = 0.37, size = 149, normalized size = 1.84

$$-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+3}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2}\sqrt{2x^4+\sqrt{5}+3}\left(3\sqrt{5}-5\right)\sqrt{\sqrt{5}+3}-\frac{1}{20}\sqrt{10}\left(3\sqrt{5}x^2-5x^2\right)\sqrt{\sqrt{5}+3}\right)+\frac{1}{10}\sqrt{10}\sqrt{-\sqrt{5}+3}\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2}\sqrt{2x^4-\sqrt{5}+3}\left(3\sqrt{5}+5\right)-2\sqrt{10}\left(3\sqrt{5}x^2+5x^2\right)\right)\sqrt{-\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(sqrt(5) + 3)*arctan(1/40*sqrt(10)*sqrt(2)*sqrt(2*x^4 +
sqrt(5) + 3)*(3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 1/20*sqrt(10)*(3*sqrt(5)*x
^2 - 5*x^2)*sqrt(sqrt(5) + 3)) + 1/10*sqrt(10)*sqrt(-sqrt(5) + 3)*arctan(1/
40*(sqrt(10)*sqrt(2)*sqrt(2*x^4 - sqrt(5) + 3)*(3*sqrt(5) + 5) - 2*sqrt(10)
*(3*sqrt(5)*x^2 + 5*x^2))*sqrt(-sqrt(5) + 3))
```

Sympy [A]
time = 0.09, size = 49, normalized size = 0.60

$$-2\cdot\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8+3*x**4+1),x)
```

```
[Out] -2*(1/8 - sqrt(5)/40)*atan(2*x**2/(-1 + sqrt(5))) + 2*(sqrt(5)/40 + 1/8)*at
an(2*x**2/(1 + sqrt(5)))
```

Giac [A]
time = 2.89, size = 47, normalized size = 0.58

$$\frac{1}{20}x^4\left(\sqrt{5}-5\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)+\frac{1}{20}x^4\left(\sqrt{5}+5\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*x^4*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*x^4*(sqrt(5) + 5)
*arctan(2*x^2/(sqrt(5) - 1))
```

Mupad [B]
time = 0.12, size = 117, normalized size = 1.44

$$2\,\mathrm{atanh}\left(\frac{60\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}+\frac{28\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}+3}\right)\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}-2\,\mathrm{atanh}\left(\frac{60\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3}-\frac{28\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{\sqrt{5}-3}\right)\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3*x^4 + x^8 + 1),x)
```

```
[Out] 2*atanh((60*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) + 3) + (28*5^(1/2)*x^
2*(5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) + 3))*(5^(1/2)/160 - 3/160)^(1/2) -
 2*atanh((60*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) - 3) - (28*5^(1/2)
*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(5^(1/2) - 3))*(- 5^(1/2)/160 - 3/160)^
(1/2)
```

3.372 $\int \frac{x^3}{1+3x^4+x^8} dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{3+2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] -1/10*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1366, 632, 212}

$$-\frac{\tanh^{-1}\left(\frac{2x^4+3}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + 3*x^4 + x^8),x]
```

```
[Out] -1/2*ArcTanh[(3 + 2*x^4)/Sqrt[5]]/Sqrt[5]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^3}{1+3x^4+x^8}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+3x+x^2}\,dx, x, x^4\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, 3+2x^4\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{3+2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.65

$$\frac{\log\left(-3+\sqrt{5}-2x^4\right)-\log\left(3+\sqrt{5}+2x^4\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 3*x^4 + x^8),x]
```

```
[Out] (Log[-3 + Sqrt[5] - 2*x^4] - Log[3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.83

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)\sqrt{5}}{10}$	19
risch	$\frac{\ln\left(2x^4-\sqrt{5}+3\right)\sqrt{5}}{20}-\frac{\ln\left(2x^4+\sqrt{5}+3\right)\sqrt{5}}{20}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Maxima [A]
time = 0.56, size = 31, normalized size = 1.35

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(18) = 36$.

time = 0.35, size = 43, normalized size = 1.87

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^8+6x^4-\sqrt{5}\,(2x^4+3)+7}{x^8+3x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
))
```

Sympy [A]

time = 0.04, size = 42, normalized size = 1.83

$$\frac{\sqrt{5}\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20}-\frac{\sqrt{5}\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+3*x**4+1),x)
```

```
[Out] sqrt(5)*log(x**4 - sqrt(5)/2 + 3/2)/20 - sqrt(5)*log(x**4 + sqrt(5)/2 + 3/2
)/20
```

Giac [A]

time = 2.83, size = 31, normalized size = 1.35

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3))
```

Mupad [B]

time = 1.33, size = 30, normalized size = 1.30

$$\frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{8\sqrt{5}\,x^4+3\sqrt{5}}{18x^4+7}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4 + x^8 + 1),x)
```

```
[Out] (5^(1/2)*atanh((3*5^(1/2) + 8*5^(1/2)*x^4)/(18*x^4 + 7)))/10
```

3.373 $\int \frac{x}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=75

$$-\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}+\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

```
[Out] 1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(1/2+1/10*5^(1/2))-arctan(x^2*2^(1/2)/(3+
5^(1/2))^(1/2))/(5+5^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1373, 1107, 209}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)-\frac{\text{ArcTan}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + 3*x^4 + x^8),x]
```

```
[Out] -(ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + Sq
rt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
```

```
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1+3x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+3x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}} - \frac{\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}}\\
&= -\frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}} + \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 74, normalized size = 0.99

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{3-\sqrt{5}}}\,x^2\right)}{\sqrt{10\left(3-\sqrt{5}\right)}} - \frac{\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + 3*x^4 + x^8),x]
```

```
[Out] ArcTan[Sqrt[2/(3 - Sqrt[5])]*x^2]/Sqrt[10*(3 - Sqrt[5])] - ArcTan[Sqrt[2/(3
 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]
```

Maple [A]
time = 0.03, size = 60, normalized size = 0.80

method	result	size
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(25\_Z^4+15\_Z^2+1\right)} \_R\ln\left(-15\_R^3+x^2-7\_R\right)\right)}{4}$	34

default	$-\frac{2\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}+\frac{2\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*5^(1/2)/(2*5^(1/2)+2)*arctan(4*x^2/(2*5^(1/2)+2))+2/5*5^(1/2)/(2*5^(1/
2)-2)*arctan(4*x^2/(2*5^(1/2)-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(43) = 86$.
time = 0.37, size = 128, normalized size = 1.71

$$\frac{1}{10}\sqrt{10}\sqrt{-\sqrt{5}+3}\arctan\left(-\frac{1}{10}\sqrt{10}\sqrt{5}x^2\sqrt{-\sqrt{5}+3}+\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^4+\sqrt{5}+3}\sqrt{-\sqrt{5}+3}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+3}\arctan\left(-\frac{1}{20}\left(2\sqrt{10}\sqrt{5}x^2-\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^4-\sqrt{5}+3}\right)\sqrt{\sqrt{5}+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*sqrt(-sqrt(5) + 3)*arctan(-1/10*sqrt(10)*sqrt(5)*x^2*sqrt(-sq
rt(5) + 3) + 1/20*sqrt(10)*sqrt(5)*sqrt(2)*sqrt(2*x^4 + sqrt(5) + 3)*sqrt(-
sqrt(5) + 3)) - 1/10*sqrt(10)*sqrt(sqrt(5) + 3)*arctan(-1/20*(2*sqrt(10)*sq
rt(5)*x^2 - sqrt(10)*sqrt(5)*sqrt(2)*sqrt(2*x^4 - sqrt(5) + 3))*sqrt(sqrt(5
) + 3))
```

Sympy [A]
time = 0.11, size = 49, normalized size = 0.65

$$2\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)-2\cdot\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8+3*x**4+1),x)
```

```
[Out] 2*(sqrt(5)/40 + 1/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(1/8 - sqrt(5)/40)*ata
n(2*x**2/(1 + sqrt(5)))
```

Giac [A]
time = 3.22, size = 41, normalized size = 0.55

$$\frac{1}{20}\left(\sqrt{5}-5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}+1}\right)+\frac{1}{20}\left(\sqrt{5}+5\right)\arctan\left(\frac{2\,x^2}{\sqrt{5}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 5)*arctan(2*x^2/(sqrt(5) + 1)) + 1/20*(sqrt(5) + 5)*arctan(
2*x^2/(sqrt(5) - 1))
```

Mupad [B]
time = 0.05, size = 125, normalized size = 1.67

$$2\,\mathrm{atanh}\left(\frac{160\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}-18}-\frac{72\,\sqrt{5}\,x^2\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}-18}\right)\,\sqrt{\frac{\sqrt{5}}{160}-\frac{3}{160}}-2\,\mathrm{atanh}\left(\frac{160\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}+18}+\frac{72\,\sqrt{5}\,x^2\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}}{8\,\sqrt{5}+18}\right)\,\sqrt{-\frac{\sqrt{5}}{160}-\frac{3}{160}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3*x^4 + x^8 + 1),x)
```

```
[Out] 2*atanh((160*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) - 18) - (72*5^(1/2
)*x^2*(5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) - 18))*(5^(1/2)/160 - 3/160)^
(1/2) - 2*atanh((160*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) + 18) +
(72*5^(1/2)*x^2*(- 5^(1/2)/160 - 3/160)^(1/2))/(8*5^(1/2) + 18))*(- 5^(1/2)
/160 - 3/160)^(1/2)
```

3.374 $\int \frac{1}{x(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=57

$$\log(x)-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

```
[Out] ln(x)-1/40*ln(2*x^4+5^(1/2)+3)*(5-3*5^(1/2))-1/40*ln(2*x^4-5^(1/2)+3)*(5+3*
5^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1371, 719, 29, 646, 31}

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + 3*x^4 + x^8)),x]
```

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 - ((5 - 3*Sqrt[5])*L
og[3 + Sqrt[5] + 2*x^4])/40
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\,(1+3x^4+x^8)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\,(1+3x+x^2)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)+\frac{1}{4}\text{Subst}\left(\int \frac{-3-x}{1+3x+x^2}\,dx,x,x^4\right)\\
&= \log(x)+\frac{1}{40}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)-\frac{1}{40}\left(5+3\sqrt{5}\right)\text{Sub}\\
&= \log(x)-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x\right.
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.96

$$\log(x)+\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right)+\frac{1}{40}\left(-5+3\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + 3*x^4 + x^8)),x]
```

```
[Out] Log[x] + ((-5 - 3*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4])/40 + ((-5 + 3*Sqrt[5]
)*Log[3 + Sqrt[5] + 2*x^4])/40
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.61

method	result	si
default	$\ln(x)-\frac{\ln(x^8+3x^4+1)}{8}+\frac{3\,\text{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)\sqrt{5}}{20}$	35

risch	$\ln(x)-\frac{\ln\left(3x^4+\frac{9}{2}+\frac{3\sqrt{5}}{2}\right)}{8}+\frac{3\ln\left(3x^4+\frac{9}{2}+\frac{3\sqrt{5}}{2}\right)\sqrt{5}}{40}-\frac{\ln\left(3x^4+\frac{9}{2}-\frac{3\sqrt{5}}{2}\right)}{8}-\frac{3\ln\left(3x^4+\frac{9}{2}-\frac{3\sqrt{5}}{2}\right)\sqrt{5}}{40}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/8*ln(x^8+3*x^4+1)+3/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^(1/2)
```

Maxima [A]
time = 0.50, size = 51, normalized size = 0.89

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)-\frac{1}{8}\log\left(x^8+3x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x^
8 + 3*x^4 + 1) + 1/4*log(x^4)
```

Fricas [A]
time = 0.35, size = 58, normalized size = 1.02

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8+6x^4+\sqrt{5}(2x^4+3)+7}{x^8+3x^4+1}\right)-\frac{1}{8}\log\left(x^8+3x^4+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="fricas")
[Out] 3/40*sqrt(5)*log((2*x^8 + 6*x^4 + sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*x^4 + 1
)) - 1/8*log(x^8 + 3*x^4 + 1) + log(x)
```

Sympy [A]
time = 0.06, size = 58, normalized size = 1.02

$$\log(x)+\left(-\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(-\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8+3*x**4+1),x)
[Out] log(x) + (-3*sqrt(5)/40 - 1/8)*log(x**4 - sqrt(5)/2 + 3/2) + (-1/8 + 3*sqrt
(5)/40)*log(x**4 + sqrt(5)/2 + 3/2)
```

Giac [A]
time = 4.01, size = 51, normalized size = 0.89

$$-\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}+3}{2x^4+\sqrt{5}+3}\right)-\frac{1}{8}\log\left(x^8+3x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -3/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/8*log(x^
8 + 3*x^4 + 1) + 1/4*log(x^4)
```

Mupad [B]
time = 1.41, size = 42, normalized size = 0.74

$$\ln(x)-\ln\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}+\frac{1}{8}\right)+\ln\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x^4 + x^8 + 1)),x)
```

```
[Out] log(x) - log(x^4 - 5^(1/2)/2 + 3/2)*((3*5^(1/2))/40 + 1/8) + log(5^(1/2)/2
+ x^4 + 3/2)*((3*5^(1/2))/40 - 1/8)
```

3.375 $\int \frac{1}{x^3(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}$$

```
[Out] -1/2/x^2-1/40*arctan(x^2*(1/2+1/2*5^(1/2)))*(3+5^(1/2))^(3/2)*10^(1/2)+1/2*
arctan(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(1-2/5*5^(1/2))
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1373, 1137, 1180, 209}

$$\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\text{ArcTan}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)-\frac{\left(3+\sqrt{5}\right)^{3/2}\text{ArcTan}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)}{4\sqrt{10}}-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 + (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTan[Sqrt[2/(3 + Sqrt[5])]*x^2])/2
- ((3 + Sqrt[5])^(3/2)*ArcTan[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^3(1+3x^4+x^8)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(1+3x^2+x^4)}\,dx, x, x^2\right)$$
$$= -\frac{1}{2x^2} + \frac{1}{2}\text{Subst}\left(\int \frac{-3-x^2}{1+3x^2+x^4}\,dx, x, x^2\right)$$
$$= -\frac{1}{2x^2} + \frac{1}{20}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right) - \frac{1}{20}\left(5+3\sqrt{5}\right)\text{S}$$
$$= -\frac{1}{2x^2} + \frac{1}{10}\sqrt{45-20\sqrt{5}}\,\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right) - \frac{\left(3+\sqrt{5}\right)^{3/2}\tan^{-1}\left(\sqrt{\frac{1}{2}}\right.}{4\sqrt{10}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 65, normalized size = 0.73

$$-\frac{1}{2x^2} - \frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&, \frac{3\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^2+2\#1^6}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^
4)/(3*#1^2 + 2*#1^6) & ]/4
```

Maple [A]
time = 0.05, size = 75, normalized size = 0.84

method	result	size

risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+90\_Z^2+1\right)}\_R\ln\left(35\_R^3+8x^2+123\_R\right)\right)}{4}$	42
default	$-\frac{1}{2x^2}-\frac{\sqrt{5}\left(\sqrt{5}-3\right)\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{5\left(2\sqrt{5}+2\right)}-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{5\left(2\sqrt{5}-2\right)}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-1/5*5^(1/2)*(5^(1/2)-3)/(2*5^(1/2)+2)*arctan(4*x^2/(2*5^(1/2)+2))-
1/5*(3+5^(1/2))*5^(1/2)/(2*5^(1/2)-2)*arctan(4*x^2/(2*5^(1/2)-2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate((x^4 + 3)*x/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(53) = 106.
time = 0.35, size = 144, normalized size = 1.62

$$-\frac{2\sqrt{5}\,x^2\sqrt{-4\sqrt{5}+9}\arctan\left(\frac{1}{4}\sqrt{4x^4+2\sqrt{5}+6}\left(\sqrt{5}+3\right)\sqrt{-4\sqrt{5}+9}-\frac{1}{2}\left(\sqrt{5}\,x^2+3x^2\right)\sqrt{-4\sqrt{5}+9}\right)+2\sqrt{5}\,x^2\sqrt{4\sqrt{5}+9}\arctan\left(-\frac{1}{4}\left(2\sqrt{5}\,x^2-6x^2-\sqrt{4x^4-2\sqrt{5}+6}\left(\sqrt{5}-3\right)\right)\sqrt{4\sqrt{5}+9}\right)+5}{10x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*(2*sqrt(5)*x^2*sqrt(-4*sqrt(5) + 9)*arctan(1/4*sqrt(4*x^4 + 2*sqrt(5)
 + 6)*(sqrt(5) + 3)*sqrt(-4*sqrt(5) + 9) - 1/2*(sqrt(5)*x^2 + 3*x^2)*sqrt(-
4*sqrt(5) + 9)) + 2*sqrt(5)*x^2*sqrt(4*sqrt(5) + 9)*arctan(-1/4*(2*sqrt(5)*
x^2 - 6*x^2 - sqrt(4*x^4 - 2*sqrt(5) + 6)*(sqrt(5) - 3))*sqrt(4*sqrt(5) + 9
)) + 5)/x^2
```

Sympy [A]
time = 0.10, size = 56, normalized size = 0.63

$$-2\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\operatorname{atan}\left(\frac{2x^2}{-1+\sqrt{5}}\right)+2\cdot\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\operatorname{atan}\left(\frac{2x^2}{1+\sqrt{5}}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8+3*x**4+1),x)
```

```
[Out] -2*(sqrt(5)/10 + 1/4)*atan(2*x**2/(-1 + sqrt(5))) + 2*(1/4 - sqrt(5)/10)*at
an(2*x**2/(1 + sqrt(5))) - 1/(2*x**2)
```

Giac [A]
time = 4.32, size = 68, normalized size = 0.76

$$-\frac{1}{20}\left(x^4\left(\sqrt{5}-5\right)+3\sqrt{5}-15\right)\arctan\left(\frac{2x^2}{\sqrt{5}+1}\right)-\frac{1}{20}\left(x^4\left(\sqrt{5}+5\right)+3\sqrt{5}+15\right)\arctan\left(\frac{2x^2}{\sqrt{5}-1}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*(x^4*(sqrt(5) - 5) + 3*sqrt(5) - 15)*arctan(2*x^2/(sqrt(5) + 1)) - 1/
20*(x^4*(sqrt(5) + 5) + 3*sqrt(5) + 15)*arctan(2*x^2/(sqrt(5) - 1)) - 1/2/x
^2
```

Mupad [B]
time = 1.30, size = 130, normalized size = 1.46

$$2\,\mathrm{atanh}\left(\frac{26880\,x^2\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\sqrt{5}+7872}+\frac{12032\sqrt{5}\,x^2\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\sqrt{5}+7872}\right)\sqrt{-\frac{\sqrt{5}}{20}-\frac{9}{80}}-2\,\mathrm{atanh}\left(\frac{26880\,x^2\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\sqrt{5}-7872}-\frac{12032\sqrt{5}\,x^2\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}}{3520\sqrt{5}-7872}\right)\sqrt{\frac{\sqrt{5}}{20}-\frac{9}{80}}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(3*x^4 + x^8 + 1)),x)
```

```
[Out] 2*atanh((26880*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872) + (12
032*5^(1/2)*x^2*(- 5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/2) + 7872))*(- 5^(1
/2)/20 - 9/80)^(1/2) - 2*atanh((26880*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(3520*
5^(1/2) - 7872) - (12032*5^(1/2)*x^2*(5^(1/2)/20 - 9/80)^(1/2))/(3520*5^(1/
2) - 7872))*(5^(1/2)/20 - 9/80)^(1/2) - 1/(2*x^2)
```

3.376 $\int \frac{1}{x^5(1+3x^4+x^8)} dx$

Optimal. Leaf size=66

$$-\frac{1}{4x^4}-3\log(x)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)$$

```
[Out] -1/4/x^4-3*ln(x)+1/40*ln(2*x^4+5^(1/2)+3)*(15-7*5^(1/2))+1/40*ln(2*x^4-5^(1
/2)+3)*(15+7*5^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1371, 723, 814, 646, 31}

$$-\frac{1}{4x^4}+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(2x^4+\sqrt{5}+3\right)-3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 - 3*Log[x] + ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] + 2*x^4])/40 + ((
15 - 7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(1+3x^4+x^8)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(1+3x+x^2)}\,dx,x,x^4\right)\\ &= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{-3-x}{x(1+3x+x^2)}\,dx,x,x^4\right)\\ &= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \left(-\frac{3}{x}+\frac{8+3x}{1+3x+x^2}\right)\,dx,x,x^4\right)\\ &= -\frac{1}{4x^4}-3\log(x)+\frac{1}{4}\text{Subst}\left(\int \frac{8+3x}{1+3x+x^2}\,dx,x,x^4\right)\\ &= -\frac{1}{4x^4}-3\log(x)+\frac{1}{40}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)+\frac{1}{40}\left(15-\right.\\ &= -\frac{1}{4x^4}-3\log(x)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}+2x^4\right)+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 0.91

$$\frac{1}{40}\left(-\frac{10}{x^4}-120\log(x)+\left(15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}-2x^4\right)+\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}+2x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 + 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^4 - 120*Log[x] + (15 + 7*Sqrt[5])*Log[-3 + Sqrt[5] - 2*x^4] + (15 -
7*Sqrt[5])*Log[3 + Sqrt[5] + 2*x^4])/40
```

Maple [A]
time = 0.03, size = 42, normalized size = 0.64

method	result
default	$-\frac{1}{4x^4}-3\ln(x)+\frac{3\ln(x^8+3x^4+1)}{8}-\frac{7\,\mathrm{arctanh}\left(\frac{(2x^4+3)\sqrt{5}}{5}\right)\sqrt{5}}{20}$
risch	$-\frac{1}{4x^4}-3\ln(x)+\frac{3\ln\left(7x^4+\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)}{8}+\frac{7\ln\left(7x^4+\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}+\frac{3\ln\left(7x^4+\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)}{8}-\frac{7\ln\left(7x^4+\frac{21}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^4-3*ln(x)+3/8*ln(x^8+3*x^4+1)-7/20*arctanh(1/5*(2*x^4+3)*5^(1/2))*5^
(1/2)
```

Maxima [A]
time = 0.64, size = 56, normalized size = 0.85

$$\frac{7}{40}\sqrt{5}\,\log\left(\frac{2\,x^4-\sqrt{5}+3}{2\,x^4+\sqrt{5}+3}\right)-\frac{1}{4\,x^4}+\frac{3}{8}\,\log\left(x^8+3\,x^4+1\right)-\frac{3}{4}\,\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) - 1/4/x^4 + 3
/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

Fricas [A]
time = 0.40, size = 76, normalized size = 1.15

$$\frac{7\sqrt{5}\,x^4\log\left(\frac{2\,x^8+6\,x^4-\sqrt{5}\,(2\,x^4+3)+7}{x^8+3\,x^4+1}\right)+15\,x^4\log\left(x^8+3\,x^4+1\right)-120\,x^4\log\left(x\right)-10}{40\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(7*sqrt(5)*x^4*log((2*x^8 + 6*x^4 - sqrt(5)*(2*x^4 + 3) + 7)/(x^8 + 3*
x^4 + 1)) + 15*x^4*log(x^8 + 3*x^4 + 1) - 120*x^4*log(x) - 10)/x^4
```

Sympy [A]
time = 0.07, size = 65, normalized size = 0.98

$$-3\log\left(x\right)+\left(\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)+\left(\frac{3}{8}-\frac{7\sqrt{5}}{40}\right)\log\left(x^4+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8+3*x**4+1),x)
```

```
[Out] -3*log(x) + (3/8 + 7*sqrt(5)/40)*log(x**4 - sqrt(5)/2 + 3/2) + (3/8 - 7*sqr
t(5)/40)*log(x**4 + sqrt(5)/2 + 3/2) - 1/(4*x**4)
```

Giac [A]
time = 4.00, size = 63, normalized size = 0.95

$$\frac{7}{40}\sqrt{5}\,\log\left(\frac{2\,x^{4}-\sqrt{5}+3}{2\,x^{4}+\sqrt{5}+3}\right)+\frac{3\,x^{4}-1}{4\,x^{4}}+\frac{3}{8}\,\log\left(x^{8}+3\,x^{4}+1\right)-\frac{3}{4}\,\log\left(x^{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) + 3)/(2*x^4 + sqrt(5) + 3)) + 1/4*(3*x^4
- 1)/x^4 + 3/8*log(x^8 + 3*x^4 + 1) - 3/4*log(x^4)
```

Mupad [B]
time = 1.36, size = 49, normalized size = 0.74

$$\ln\left(x^{4}-\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\,\left(\frac{7\,\sqrt{5}}{40}+\frac{3}{8}\right)-\frac{1}{4\,x^{4}}-3\,\ln\left(x\right)-\ln\left(x^{4}+\frac{\sqrt{5}}{2}+\frac{3}{2}\right)\,\left(\frac{7\,\sqrt{5}}{40}-\frac{3}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(3*x^4 + x^8 + 1)),x)
```

```
[Out] log(x^4 - 5^(1/2)/2 + 3/2)*((7*5^(1/2))/40 + 3/8) - 1/(4*x^4) - 3*log(x) -
log(5^(1/2)/2 + x^4 + 3/2)*((7*5^(1/2))/40 - 3/8)
```

3.377 $\int \frac{1}{x^7(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{1}{2}\left(3+\right.}\right.$$

```
[Out] -1/6/x^6+3/2/x^2-1/2*arctan(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(5/2-11/10*5^(1/
2))+1/2*arctan(x^2*(1/2+1/2*5^(1/2)))*(5/2+11/10*5^(1/2))
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1373, 1137, 1295, 1180, 209}

$$-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)-\frac{1}{6x^6}+\frac{3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 + 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTan[Sqrt[2/(3 + Sq
rt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTan[Sqrt[(3 + Sqrt[5])/2]
*x^2])/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(1+3x^4+x^8\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(1+3x^2+x^4\right)}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{-9-3x^2}{x^2\left(1+3x^2+x^4\right)}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{6}\text{Subst}\left(\int \frac{-24-9x^2}{1+3x^2+x^4}\,dx, x, x^2\right)\\
&= -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{20}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)+\frac{1}{20}\left(15+\right.\\
&= -\frac{1}{6x^6}+\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{20}\sqrt{1230+55
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 73, normalized size = 0.75

$$-\frac{1}{6x^6}+\frac{3}{2x^2}+\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{8\log(x-\#1)+3\log(x-\#1)\#1^4}{3\#1^2+2\#1^6}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 + 3/(2*x^2) + RootSum[1 + 3*#1^4 + #1^8 & , (8*Log[x - #1] + 3*L
og[x - #1]*#1^4)/(3*#1^2 + 2*#1^6) & ]/4
```

Maple [A]
time = 0.04, size = 84, normalized size = 0.87

method	result	size
risch	$\frac{\frac{3x^4}{2}-\frac{1}{6}}{x^6}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+615\_Z^2+1\right)}\_R\ln\left(-90\_R^3+55x^2-2207\_R\right)\right)}{4}$	48
default	$-\frac{1}{6x^6}+\frac{3}{2x^2}+\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}+2}\right)}{10+10\sqrt{5}}+\frac{\left(7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{4x^2}{2\sqrt{5}-2}\right)}{-10+10\sqrt{5}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x^6+3/2/x^2+1/5*(-7+3*5^(1/2))*5^(1/2)/(2*5^(1/2)+2)*arctan(4*x^2/(2*5
^(1/2)+2))+1/5*(7+3*5^(1/2))*5^(1/2)/(2*5^(1/2)-2)*arctan(4*x^2/(2*5^(1/2)-
2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/6*(9*x^4 - 1)/x^6 + integrate((3*x^4 + 8)*x/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(55) = 110$.
time = 0.36, size = 172, normalized size = 1.77

$$\frac{3\sqrt{10}\,x^6\sqrt{-55\sqrt{5}+123}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2}\sqrt{2x^4+\sqrt{5}+3}\left(7\sqrt{5}+15\right)\sqrt{-55\sqrt{5}+123}-\frac{1}{20}\sqrt{10}\left(7\sqrt{5}x^2+15x^2\right)\sqrt{-55\sqrt{5}+123}\right)-3\sqrt{10}\,x^6\sqrt{55\sqrt{5}+123}\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2}\sqrt{2x^4-\sqrt{5}+3}\left(7\sqrt{5}-15\right)-2\sqrt{10}\left(7\sqrt{5}x^2-15x^2\right)\right)\sqrt{55\sqrt{5}+123}\right)+45x^4-5}{30x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/30*(3*sqrt(10)*x^6*sqrt(-55*sqrt(5) + 123)*arctan(1/40*sqrt(10)*sqrt(2)*s
qrt(2*x^4 + sqrt(5) + 3)*(7*sqrt(5) + 15)*sqrt(-55*sqrt(5) + 123) - 1/20*sq
rt(10)*(7*sqrt(5)*x^2 + 15*x^2)*sqrt(-55*sqrt(5) + 123)) - 3*sqrt(10)*x^6*s
qrt(55*sqrt(5) + 123)*arctan(1/40*(sqrt(10)*sqrt(2)*sqrt(2*x^4 - sqrt(5) +
3)*(7*sqrt(5) - 15) - 2*sqrt(10)*(7*sqrt(5)*x^2 - 15*x^2))*sqrt(55*sqrt(5)
+ 123)) + 45*x^4 - 5)/x^6
```

Sympy [A]
time = 0.15, size = 65, normalized size = 0.67

$$2\cdot\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\operatorname{atan}\left(\frac{2x^{2}}{-1+\sqrt{5}}\right)-2\cdot\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\operatorname{atan}\left(\frac{2x^{2}}{1+\sqrt{5}}\right)+\frac{9x^{4}-1}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8+3*x**4+1),x)
```

```
[Out] 2*(11*sqrt(5)/40 + 5/8)*atan(2*x**2/(-1 + sqrt(5))) - 2*(5/8 - 11*sqrt(5)/4
0)*atan(2*x**2/(1 + sqrt(5))) + (9*x**4 - 1)/(6*x**6)
```

Giac [A]
time = 4.02, size = 77, normalized size = 0.79

$$\frac{1}{20}\left(3\,x^{4}\left(\sqrt{5}-5\right)+8\,\sqrt{5}-40\right)\arctan\left(\frac{2\,x^{2}}{\sqrt{5}+1}\right)+\frac{1}{20}\left(3\,x^{4}\left(\sqrt{5}+5\right)+8\,\sqrt{5}+40\right)\arctan\left(\frac{2\,x^{2}}{\sqrt{5}-1}\right)+\frac{9\,x^{4}-1}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*(3*x^4*(sqrt(5) - 5) + 8*sqrt(5) - 40)*arctan(2*x^2/(sqrt(5) + 1)) + 1
/20*(3*x^4*(sqrt(5) + 5) + 8*sqrt(5) + 40)*arctan(2*x^2/(sqrt(5) - 1)) + 1/
6*(9*x^4 - 1)/x^6
```

Mupad [B]
time = 0.12, size = 136, normalized size = 1.40

$$2\,\mathrm{atanh}\left(\frac{3327500\,x^{2}\,\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}-2550075}-\frac{1488300\,\sqrt{5}\,x^{2}\,\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}-2550075}\right)\sqrt{\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}-2\,\mathrm{atanh}\left(\frac{3327500\,x^{2}\,\sqrt{-\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}+2550075}+\frac{1488300\,\sqrt{5}\,x^{2}\,\sqrt{-\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}}{1140425\,\sqrt{5}+2550075}\right)\sqrt{-\frac{11\,\sqrt{5}}{32}-\frac{123}{160}}+\frac{\frac{3\,x^{4}}{2}-\frac{1}{6}}{x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(3*x^4 + x^8 + 1)),x)
```

```
[Out] 2*atanh((3327500*x^2*((11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) -
2550075) - (1488300*5^(1/2)*x^2*((11*5^(1/2))/32 - 123/160)^(1/2))/(1140425
*5^(1/2) - 2550075))*((11*5^(1/2))/32 - 123/160)^(1/2) - 2*atanh((3327500*x
^2*(- (11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) + 2550075) + (1488
300*5^(1/2)*x^2*(- (11*5^(1/2))/32 - 123/160)^(1/2))/(1140425*5^(1/2) + 255
0075))*(- (11*5^(1/2))/32 - 123/160)^(1/2) + ((3*x^4)/2 - 1/6)/x^6
```

3.378 $\int \frac{x^8}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=460

$$x-\frac{\sqrt[4]{123-55\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123+55\sqrt{}}}{}$$

```
[Out] x+1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(123-55*5^(1/2))^(1/4)*2^(1/4
)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(123-55*5^(1/2))^(1/4)
*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(12
3-55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(
1/4)+5^(1/2)-1)*(123-55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/20*arctan(-1+2^(3/
4)*x/(3+5^(1/2))^(1/4))*(123+55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/20*arctan(
1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(123+55*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*
ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(123+55*5^(1/2))^(1/4)*2^
(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(123+5
5*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 440, normalized size of antiderivative = 0.96, number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1381, 1436, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt[4]{984-440\sqrt{5}}\,\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{4\sqrt{10}}+\frac{\sqrt[4]{123+55\sqrt{5}}\,\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\,\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{984-440\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\sqrt{10}}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\sqrt{10}}+\frac{\sqrt[4]{123+55\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{123+55\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+x$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 + 3*x^4 + x^8),x]
```

```
[Out] x - ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(4*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt
[5])^(1/4)])/(4*Sqrt[10]) + ((123 + 55*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x
)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*Arc
Tan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((984 - 440
*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2
*x^2])/(8*Sqrt[10]) + ((984 - 440*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]
 + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(8*Sqrt[10]) + ((123 + 55*Sqrt[5])^
(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) - ((123 + 55*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*
(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^8}{1+3x^4+x^8}\,dx = x-\int \frac{1+3x^4}{1+3x^4+x^8}\,dx$$

$$= x-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= x+\frac{1}{2}\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{2}\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{\sqrt{3-}}{\frac{3}{2}}$$

$$= x+\frac{1}{4}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+x^2}\,dx+\frac{1}{4}\sqrt{\frac{1}{5}\left(9\right.}$$

$$= x-\frac{1}{8}\sqrt[4]{\frac{246}{25}-\frac{22}{\sqrt{5}}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+2x^2\right)+\frac{1}{8}\sqrt[4]{\frac{246}{25}-}$$

$$= x-\frac{\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{123-55\sqrt{5}}\tan^{-1}\left(1+\frac{2}{\sqrt[4]{3-}}\right.}{2\,2^{3/4}\sqrt{5}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 58, normalized size = 0.13

$$x-\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)+3\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 + 3*x^4 + x^8),x]
```

```
[Out] x - RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1] + 3*Log[x - #1]*#1^4)/(3*#1^
3 + 2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 46, normalized size = 0.10

method	result	size
default	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(-3\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	46
risch	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(-3\_R^{4}-1\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/4*sum((-3*_R^4-1)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] x - integrate((3*x^4 + 1)/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. 2(302) = 604.
time = 0.38, size = 1026, normalized size = 2.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/80*sqrt(10)*(110*sqrt(5) + 246)^(3/4)*sqrt(55*sqrt(5) + 123)*(55*sqrt(5)
 - 123)*arctan(1/400*sqrt(10)*sqrt(5)*sqrt(20*x^2 + sqrt(10)*(3*sqrt(5)*sqr
t(2)*x - 5*sqrt(2)*x)*(110*sqrt(5) + 246)^(1/4) - 5*sqrt(110*sqrt(5) + 246)
*(3*sqrt(5) - 7))*(2889*sqrt(5) - 6460)*(110*sqrt(5) + 246)^(5/4)*sqrt(55*s
```

```
qrt(5) + 123) - 1/40*sqrt(10)*(2889*sqrt(5)*x - 6460*x)*(110*sqrt(5) + 246)
^(5/4)*sqrt(55*sqrt(5) + 123) + 1/8*(55*sqrt(5)*sqrt(2) - 123*sqrt(2))*sqrt
(110*sqrt(5) + 246)*sqrt(55*sqrt(5) + 123)) - 1/80*sqrt(10)*(110*sqrt(5) +
246)^(3/4)*sqrt(55*sqrt(5) + 123)*(55*sqrt(5) - 123)*arctan(1/400*sqrt(10)*
sqrt(5)*sqrt(20*x^2 - sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(110*sqr
t(5) + 246)^(1/4) - 5*sqrt(110*sqrt(5) + 246)*(3*sqrt(5) - 7))*(2889*sqrt(5
) - 6460)*(110*sqrt(5) + 246)^(5/4)*sqrt(55*sqrt(5) + 123) - 1/40*sqrt(10)*
(2889*sqrt(5)*x - 6460*x)*(110*sqrt(5) + 246)^(5/4)*sqrt(55*sqrt(5) + 123)
- 1/8*(55*sqrt(5)*sqrt(2) - 123*sqrt(2))*sqrt(110*sqrt(5) + 246)*sqrt(55*sq
rt(5) + 123)) - 1/80*sqrt(10)*(55*sqrt(5) + 123)*sqrt(-55*sqrt(5) + 123)*(-
110*sqrt(5) + 246)^(3/4)*arctan(1/400*sqrt(10)*sqrt(5)*sqrt(20*x^2 + sqrt(1
0)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-110*sqrt(5) + 246)^(1/4) + 5*(3*sq
rt(5) + 7)*sqrt(-110*sqrt(5) + 246))*(2889*sqrt(5) + 6460)*sqrt(-55*sqrt(5)
 + 123)*(-110*sqrt(5) + 246)^(5/4) - 1/40*(sqrt(10)*(2889*sqrt(5)*x + 6460*
x)*(-110*sqrt(5) + 246)^(5/4) + 5*(55*sqrt(5)*sqrt(2) + 123*sqrt(2))*sqrt(-
110*sqrt(5) + 246))*sqrt(-55*sqrt(5) + 123)) - 1/80*sqrt(10)*(55*sqrt(5) +
123)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(3/4)*arctan(1/400*sqrt(1
0)*sqrt(5)*sqrt(20*x^2 - sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-110
*sqrt(5) + 246)^(1/4) + 5*(3*sqrt(5) + 7)*sqrt(-110*sqrt(5) + 246))*(2889*s
qrt(5) + 6460)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(5/4) - 1/40*(s
qrt(10)*(2889*sqrt(5)*x + 6460*x)*(-110*sqrt(5) + 246)^(5/4) - 5*(55*sqrt(5
)*sqrt(2) + 123*sqrt(2))*sqrt(-110*sqrt(5) + 246))*sqrt(-55*sqrt(5) + 123))
 - 1/80*sqrt(10)*sqrt(2)*(110*sqrt(5) + 246)^(1/4)*log(400*x^2 + 20*sqrt(10
)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(110*sqrt(5) + 246)^(1/4) - 100*sqrt(
110*sqrt(5) + 246)*(3*sqrt(5) - 7)) + 1/80*sqrt(10)*sqrt(2)*(110*sqrt(5) +
246)^(1/4)*log(400*x^2 - 20*sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(1
10*sqrt(5) + 246)^(1/4) - 100*sqrt(110*sqrt(5) + 246)*(3*sqrt(5) - 7)) + 1/
80*sqrt(10)*sqrt(2)*(-110*sqrt(5) + 246)^(1/4)*log(400*x^2 + 20*sqrt(10)*(3
*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-110*sqrt(5) + 246)^(1/4) + 100*(3*sqrt(
5) + 7)*sqrt(-110*sqrt(5) + 246)) - 1/80*sqrt(10)*sqrt(2)*(-110*sqrt(5) + 2
46)^(1/4)*log(400*x^2 - 20*sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-1
10*sqrt(5) + 246)^(1/4) + 100*(3*sqrt(5) + 7)*sqrt(-110*sqrt(5) + 246)) + x
```

Sympy [A]
time = 1.03, size = 29, normalized size = 0.06

$$x + \operatorname{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t \log\left(\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8+3*x**4+1),x)
```

```
[Out] x + RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(15360*_t**
5/11 + 1288*_t/55 + x)))
```

Giac [A]

time = 3.83, size = 240, normalized size = 0.52

$$-\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}+55}+\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}+55}+\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}-55}-\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}-55}-\frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(722500\left(x+\sqrt{\sqrt{5}+1}\right)^{2}+722500x^{2}\right)+\frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(722500\left(x-\sqrt{\sqrt{5}+1}\right)^{2}+722500x^{2}\right)+\frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(2992900\left(x+\sqrt{\sqrt{5}-1}\right)^{2}+2992900x^{2}\right)-\frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(2992900\left(x-\sqrt{\sqrt{5}-1}\right)^{2}+2992900x^{2}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/80
*(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) + 55) + 1/80*(pi
 + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi + 4*
arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*sqrt
(5) + 55)*log(722500*(x + sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1/40*sqrt(25
*sqrt(5) + 55)*log(722500*(x - sqrt(sqrt(5) + 1))^2 + 722500*x^2) + 1/40*sq
rt(25*sqrt(5) - 55)*log(2992900*(x + sqrt(sqrt(5) - 1))^2 + 2992900*x^2) -
1/40*sqrt(25*sqrt(5) - 55)*log(2992900*(x - sqrt(sqrt(5) - 1))^2 + 2992900*
x^2) + x
```

Mupad [B]
time = 1.44, size = 216, normalized size = 0.47

$$x-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{1/4}\,x}{2\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}+\frac{2^{1/4}\,\sqrt{5}\,x}{2\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}\right)\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{1/4}\,x}{2\,{\left(55\,\sqrt{5}-123\right)}^{1/4}}-\frac{2^{1/4}\,\sqrt{5}\,x}{2\,{\left(55\,\sqrt{5}-123\right)}^{1/4}}\right)\,{\left(55\,\sqrt{5}-123\right)}^{1/4}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,3{}\mathrm{i}}{2\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}+\frac{2^{1/4}\,\sqrt{5}\,x\,1{}\mathrm{i}}{2\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}}\right)\,{\left(-55\,\sqrt{5}-123\right)}^{1/4}\,1{}\mathrm{i}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,3{}\mathrm{i}}{2\,{\left(55\,\sqrt{5}-123\right)}^{1/4}}-\frac{2^{1/4}\,\sqrt{5}\,x\,1{}\mathrm{i}}{2\,{\left(55\,\sqrt{5}-123\right)}^{1/4}}\right)\,{\left(55\,\sqrt{5}-123\right)}^{1/4}\,1{}\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(3*x^4 + x^8 + 1),x)
```

```
[Out] x - (2^(3/4)*5^(1/2)*atan((3*2^(1/4)*x)/(2*(- 55*5^(1/2) - 123)^(1/4)) + (2
^(1/4)*5^(1/2)*x)/(2*(- 55*5^(1/2) - 123)^(1/4)))*(- 55*5^(1/2) - 123)^(1/4
))/20 + (2^(3/4)*5^(1/2)*atan((3*2^(1/4)*x)/(2*(55*5^(1/2) - 123)^(1/4)) -
(2^(1/4)*5^(1/2)*x)/(2*(55*5^(1/2) - 123)^(1/4)))*(55*5^(1/2) - 123)^(1/4))
/20 + (2^(3/4)*5^(1/2)*atan((2^(1/4)*x*3i)/(2*(- 55*5^(1/2) - 123)^(1/4)) +
 (2^(1/4)*5^(1/2)*x*1i)/(2*(- 55*5^(1/2) - 123)^(1/4)))*(- 55*5^(1/2) - 123
)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(1/4)*x*3i)/(2*(55*5^(1/2) - 123)
^(1/4)) - (2^(1/4)*5^(1/2)*x*1i)/(2*(55*5^(1/2) - 123)^(1/4)))*(55*5^(1/2)
- 123)^(1/4)*1i)/20
```

3.379 $\int \frac{x^6}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=431

$$\frac{\sqrt[4]{9-4\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}}-\frac{\sqrt[4]{9-4\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(1\right.}{4\sqrt[4]{2}\,\sqrt{}}$$

```
[Out] -1/40*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(3/4)*2^(3/4)*5^(1
/2)-1/40*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(3/4)*2^(3/4)*5^
(1/2)-1/80*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3-5^(1/2))^(3
/4)*2^(3/4)*5^(1/2)+1/80*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*
(3-5^(1/2))^(3/4)*2^(3/4)*5^(1/2)+1/40*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4
))*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)+1/40*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1
/4))*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)+1/80*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2)
)^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)-1/80*ln(2*x^2+2*2^(1/4
)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(3/4)*2^(3/4)*5^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1388, 303, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{9-4\sqrt{5}}\,\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}}-\frac{\sqrt[4]{9-4\sqrt{5}}\,\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\sqrt{10}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{4\sqrt[4]{2}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{4\sqrt[4]{2}\sqrt{5}}-\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9-4\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{8\sqrt[4]{2}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{8\sqrt[4]{2}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*Sqrt
[10]) - ((9 - 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(2*Sqrt[10]) - ((3 + Sqrt[5])^(3/4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(
1/4)])/(4*2^(1/4)*Sqrt[5]) + ((3 + Sqrt[5])^(3/4)*ArcTan[1 + (2^(3/4)*x)/(3
 + Sqrt[5])^(1/4)])/(4*2^(1/4)*Sqrt[5]) - ((9 - 4*Sqrt[5])^(1/4)*Log[Sqrt[2
*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9
- 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x
+ 2*x^2])/(4*Sqrt[10]) + ((3 + Sqrt[5])^(3/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2
*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(1/4)*Sqrt[5]) - ((3 + Sqrt[5])^(
3/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2
^(1/4)*Sqrt[5])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^6}{1+3x^4+x^8}\,dx = -\left(\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} - \frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}\,x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}} - \frac{\left(3+\right.}{}$$

$$= -\frac{\sqrt[4]{9-4\sqrt{5}}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x-x^2}\,dx}{4\sqrt{10}} - \frac{\sqrt[4]{9-4\sqrt{5}}\int \frac{}{-\sqrt{\frac{1}{2}}}}{}$$

$$= -\frac{\sqrt[4]{9-4\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9-4\sqrt{5}}\,\mathrm{lo}}{}$$

$$= \frac{\left(3-\sqrt{5}\right)^{3/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt[4]{2}\,\sqrt{5}} - \frac{\sqrt[4]{36-16\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right.}{4\sqrt{5}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 41, normalized size = 0.10

$$\frac{1}{4}\mathrm{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^3}{3+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1^3)/(3 + 2*#1^4) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 40, normalized size = 0.09

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^6\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^6\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R^6/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] integrate(x^6/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(293) = 586$.
time = 0.40, size = 761, normalized size = 1.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="fricas")
[Out] 1/10*sqrt(5)*sqrt(2)*(4*sqrt(5) + 9)^(1/4)*arctan(1/4*sqrt(4*x^2 + 2*(3*sqr
t(5)*sqrt(2)*x - 7*sqrt(2)*x)*(4*sqrt(5) + 9)^(3/4) - 2*sqrt(4*sqrt(5) + 9)
*(sqrt(5) - 3))*(21*sqrt(5)*sqrt(2) - 47*sqrt(2))*(4*sqrt(5) + 9)^(5/4) - 1
/2*(21*sqrt(5)*sqrt(2)*x - 47*sqrt(2)*x)*(4*sqrt(5) + 9)^(5/4) - 1) + 1/10*
sqrt(5)*sqrt(2)*(4*sqrt(5) + 9)^(1/4)*arctan(1/4*sqrt(4*x^2 - 2*(3*sqrt(5)*
sqrt(2)*x - 7*sqrt(2)*x)*(4*sqrt(5) + 9)^(3/4) - 2*sqrt(4*sqrt(5) + 9)*(sqr
t(5) - 3))*(21*sqrt(5)*sqrt(2) - 47*sqrt(2))*(4*sqrt(5) + 9)^(5/4) - 1/2*(2
1*sqrt(5)*sqrt(2)*x - 47*sqrt(2)*x)*(4*sqrt(5) + 9)^(5/4) + 1) + 1/10*sqrt(
5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan(1/4*sqrt(4*x^2 + 2*(3*sqrt(5)*sqrt
(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + 2*(sqrt(5) + 3)*sqrt(-4*sqrt(
5) + 9))*(21*sqrt(5)*sqrt(2) + 47*sqrt(2))*(-4*sqrt(5) + 9)^(5/4) - 1/2*(21
*sqrt(5)*sqrt(2)*x + 47*sqrt(2)*x)*(-4*sqrt(5) + 9)^(5/4) - 1) + 1/10*sqrt(
5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*arctan(1/4*sqrt(4*x^2 - 2*(3*sqrt(5)*sqrt
```

```
(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + 2*(sqrt(5) + 3)*sqrt(-4*sqrt(
5) + 9))*(21*sqrt(5)*sqrt(2) + 47*sqrt(2))*(-4*sqrt(5) + 9)^(5/4) - 1/2*(21
*sqrt(5)*sqrt(2)*x + 47*sqrt(2)*x)*(-4*sqrt(5) + 9)^(5/4) + 1) + 1/40*sqrt(
5)*sqrt(2)*(4*sqrt(5) + 9)^(1/4)*log(4*x^2 + 2*(3*sqrt(5)*sqrt(2)*x - 7*sqr
t(2)*x)*(4*sqrt(5) + 9)^(3/4) - 2*sqrt(4*sqrt(5) + 9)*(sqrt(5) - 3)) - 1/40
*sqrt(5)*sqrt(2)*(4*sqrt(5) + 9)^(1/4)*log(4*x^2 - 2*(3*sqrt(5)*sqrt(2)*x -
 7*sqrt(2)*x)*(4*sqrt(5) + 9)^(3/4) - 2*sqrt(4*sqrt(5) + 9)*(sqrt(5) - 3))
+ 1/40*sqrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*log(4*x^2 + 2*(3*sqrt(5)*sqrt
(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + 2*(sqrt(5) + 3)*sqrt(-4*sqrt(
5) + 9)) - 1/40*sqrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*log(4*x^2 - 2*(3*sqr
t(5)*sqrt(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + 2*(sqrt(5) + 3)*sqrt
(-4*sqrt(5) + 9))
```

Sympy [**A**]
time = 1.07, size = 26, normalized size = 0.06

$$\operatorname{RootSum}\left(40960000t^{8}+115200t^{4}+1,\left(t\mapsto t\log\left(-1792000t^{7}-4920t^{3}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-1792000*_t**7
 - 4920*_t**3 + x)))
```

Giac [**A**]
time = 3.95, size = 239, normalized size = 0.55

$$\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}+20}-\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}+20}-\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}-20}+\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}-20}-\frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x+\sqrt{\sqrt{5}+1}\right)^{2}+400x^{2}\right)+\frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x-\sqrt{\sqrt{5}+1}\right)^{2}+400x^{2}\right)+\frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x+\sqrt{\sqrt{5}-1}\right)^{2}+10000x^{2}\right)-\frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x-\sqrt{\sqrt{5}-1}\right)^{2}+10000x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) + 20) - 1/80*
(pi + 4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) + 20) - 1/80*(pi
+ 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) - 20) + 1/80*(pi + 4*a
rctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) - 20) - 1/40*sqrt(10*sqrt(
5) + 20)*log(400*(x + sqrt(sqrt(5) + 1))^2 + 400*x^2) + 1/40*sqrt(10*sqrt(5
) + 20)*log(400*(x - sqrt(sqrt(5) + 1))^2 + 400*x^2) + 1/40*sqrt(10*sqrt(5)
 - 20)*log(10000*(x + sqrt(sqrt(5) - 1))^2 + 10000*x^2) - 1/40*sqrt(10*sqrt
(5) - 20)*log(10000*(x - sqrt(sqrt(5) - 1))^2 + 10000*x^2)
```

Mupad [**B**]
time = 1.46, size = 149, normalized size = 0.35

$$\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}}{8\,\sqrt{5}+24}\right)\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16\,x\,{\left(4\,\sqrt{5}-9\right)}^{1/4}}{8\,\sqrt{5}-24}\right)\,{\left(4\,\sqrt{5}-9\right)}^{1/4}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}\,16{}\mathrm{i}}{8\,\sqrt{5}+24}\right)\,{\left(-4\,\sqrt{5}-9\right)}^{1/4}\,1{}\mathrm{i}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\,{\left(4\,\sqrt{5}-9\right)}^{1/4}\,16{}\mathrm{i}}{8\,\sqrt{5}-24}\right)\,{\left(4\,\sqrt{5}-9\right)}^{1/4}\,1{}\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^4 + x^8 + 1),x)
```

```
[Out] (5^(1/2)*atan((16*x*(- 4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) + 24))*(- 4*5^(1/2)
 - 9)^(1/4))/10 + (5^(1/2)*atan((16*x*(4*5^(1/2) - 9)^(1/4))/(8*5^(1/2) - 2
4))*(4*5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((x*(- 4*5^(1/2) - 9)^(1/4)*16
i)/(8*5^(1/2) + 24))*(- 4*5^(1/2) - 9)^(1/4)*1i)/10 + (5^(1/2)*atan((x*(4*5
^(1/2) - 9)^(1/4)*16i)/(8*5^(1/2) - 24))*(4*5^(1/2) - 9)^(1/4)*1i)/10
```

3.380 $\int \frac{x^4}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=451

$$\frac{\sqrt[4]{3-\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\,\tan^{-1}\left(1-\cdots\right.}{2\,2^{3/4}\sqrt{5}}$$

```
[Out] -1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5^(1
/2)-1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(3-5^(1/2))^(1/4)*2^(1/4)*5^
(1/2)+1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(3-5^(1/2))^(1
/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*
(3-5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4
))*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1
/4))*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2)
)^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4
)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(3+5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1388, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{3-\sqrt{5}}\,\mathrm{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\mathrm{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\,\mathrm{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\mathrm{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 + 3*x^4 + x^8),x]
```

```
[Out] ((3 - Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4
)*Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4
)])/(2*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 +
Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(
3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((3 - Sqrt[5])^(1/4)*Lo
g[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*
Sqrt[5]) - ((3 - Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[
5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]
) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1
/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{1+3x^4+x^8}\,dx = -\left(\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= -\left(\frac{1}{4}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx\right)-\frac{1}{4}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{\sqrt{3-\sqrt{}}}{\frac{3}{2}-}$$

$$= -\left(\frac{1}{4}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+x^2}\,dx\right)-\frac{1}{4}\sqrt{\frac{1}{10}}$$

$$= \frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+2x^2\right)}{4\;2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\log\left(\right.}{}$$

$$= \frac{\sqrt[4]{3-\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\;2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3-\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\;2^{3/4}\sqrt{5}}-$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 39, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{3+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , (Log[x - #1]*#1)/(3 + 2*#1^4) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 40, normalized size = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	40

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R^4/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] integrate(x^4/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 859 vs. 2(293) = 586.
time = 0.40, size = 859, normalized size = 1.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="fricas")
[Out] 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(
1/800*sqrt(10)*sqrt(10*sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) + 6)^(1/4) + 1
00*x^2 + 50*sqrt(2*sqrt(5) + 6))*(7*sqrt(5) - 15)*(2*sqrt(5) + 6)^(5/4)*sqr
t(sqrt(5) + 3) - 1/80*sqrt(10)*(7*sqrt(5)*x - 15*x)*(2*sqrt(5) + 6)^(5/4)*s
qrt(sqrt(5) + 3) + 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sq
rt(sqrt(5) + 3)) + 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(s
qrt(5) - 3)*arctan(1/800*sqrt(10)*sqrt(-10*sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sq
rt(5) + 6)^(1/4) + 100*x^2 + 50*sqrt(2*sqrt(5) + 6))*(7*sqrt(5) - 15)*(2*sq
rt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3) - 1/80*sqrt(10)*(7*sqrt(5)*x - 15*x)*(2*
sqrt(5) + 6)^(5/4)*sqrt(sqrt(5) + 3) - 1/8*(sqrt(5)*sqrt(2) - 3*sqrt(2))*sq
rt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/80*sqrt(10)*(sqrt(5) + 3)*sqrt(-sq
rt(5) + 3)*(-2*sqrt(5) + 6)^(3/4)*arctan(1/800*sqrt(10)*sqrt(10*sqrt(10)*sq
rt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 100*x^2 + 50*sqrt(-2*sqrt(5) + 6))
*(7*sqrt(5) + 15)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(5/4) - 1/80*(sqrt(10
)*(7*sqrt(5)*x + 15*x)*(-2*sqrt(5) + 6)^(5/4) + 10*(sqrt(5)*sqrt(2) + 3*sqr
t(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)) + 1/80*sqrt(10)*(sqrt(5) +
3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4)*arctan(1/800*sqrt(10)*sqrt(-10
```

```
*sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 100*x^2 + 50*sqrt(-2*s
qrt(5) + 6))*(7*sqrt(5) + 15)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(5/4) - 1
/80*(sqrt(10)*(7*sqrt(5)*x + 15*x)*(-2*sqrt(5) + 6)^(5/4) - 10*(sqrt(5)*sqr
t(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5) + 3)) + 1/80*sqrt(10)
*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(10*sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5)
 + 6)^(1/4) + 100*x^2 + 50*sqrt(2*sqrt(5) + 6)) - 1/80*sqrt(10)*sqrt(2)*(2*
sqrt(5) + 6)^(1/4)*log(-10*sqrt(10)*sqrt(5)*sqrt(2)*x*(2*sqrt(5) + 6)^(1/4)
 + 100*x^2 + 50*sqrt(2*sqrt(5) + 6)) - 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) +
6)^(1/4)*log(10*sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 100*x^2
 + 50*sqrt(-2*sqrt(5) + 6)) + 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*
log(-10*sqrt(10)*sqrt(5)*sqrt(2)*x*(-2*sqrt(5) + 6)^(1/4) + 100*x^2 + 50*sq
rt(-2*sqrt(5) + 6))
```

Sympy [A]
time = 0.95, size = 24, normalized size = 0.05

$$\operatorname{RootSum}\left(40960000t^{8} + 19200t^{4} + 1, \left(t \mapsto t \log\left(-51200t^{5} - 12t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-51200*_t**5 -
12*_t + x)))
```

Giac [A]
time = 3.40, size = 239, normalized size = 0.53

$$\frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(625\left(x+\sqrt{\sqrt{5}+1}\right)^{2}+625x^{2}\right) - \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(625\left(x-\sqrt{\sqrt{5}+1}\right)^{2}+625x^{2}\right) - \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(4225\left(x+\sqrt{\sqrt{5}-1}\right)^{2}+4225x^{2}\right) + \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(4225\left(x-\sqrt{\sqrt{5}-1}\right)^{2}+4225x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi + 4*
arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arctan(
-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) + 5)*l
og(625*(x + sqrt(sqrt(5) + 1))^2 + 625*x^2) - 1/40*sqrt(5*sqrt(5) + 5)*log(
625*(x - sqrt(sqrt(5) + 1))^2 + 625*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*log(422
5*(x + sqrt(sqrt(5) - 1))^2 + 4225*x^2) + 1/40*sqrt(5*sqrt(5) - 5)*log(4225
*(x - sqrt(sqrt(5) - 1))^2 + 4225*x^2)
```

Mupad [B]
time = 0.20, size = 454, normalized size = 1.01

$$\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{3/4}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}}{2\,\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}}{2}\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}}{2\,\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}}{2}\right)}\right)\,{\left(-\sqrt{5}-3\right)}^{1/4}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}\,3{}\mathrm{i}}{2\,\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}}{2}\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(\frac{3\,\sqrt{2}\,\sqrt{-\sqrt{5}-3}}{2}-\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{-\sqrt{5}-3}}{2}\right)}\right)\,{\left(-\sqrt{5}-3\right)}^{1/4}\,1{}\mathrm{i}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{3\,2^{3/4}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}}{2\,\left(\frac{3\,\sqrt{2}\,\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}}{2}\right)}+\frac{2^{3/4}\,\sqrt{5}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}}{2\,\left(\frac{3\,\sqrt{2}\,\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}}{2}\right)}\right)\,{\left(\sqrt{5}-3\right)}^{1/4}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}\,3{}\mathrm{i}}{2\,\left(\frac{3\,\sqrt{2}\,\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}}{2}\right)}+\frac{2^{3/4}\,\sqrt{5}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(\frac{3\,\sqrt{2}\,\sqrt{\sqrt{5}-3}}{2}+\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\sqrt{5}-3}}{2}\right)}\right)\,{\left(\sqrt{5}-3\right)}^{1/4}\,1{}\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(3*x^4 + x^8 + 1),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((3*2^(3/4)*x*(- 5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(-
 5^(1/2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)) - (2^(
3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(- 5^(1/2) - 3)^(1/2))
/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)))*(- 5^(1/2) - 3)^(1/4))/20
 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*((3*2^(1/2
)*(- 5^(1/2) - 3)^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)) -
(2^(3/4)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*1i)/(2*((3*2^(1/2)*(- 5^(1/2) - 3)
^(1/2))/2 - (2^(1/2)*5^(1/2)*(- 5^(1/2) - 3)^(1/2))/2)))*(- 5^(1/2) - 3)^(1
/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((3*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*((3
*2^(1/2)*(5^(1/2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2))
 + (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*((3*2^(1/2)*(5^(1/2) - 3)^(1/
2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)))*(5^(1/2) - 3)^(1/4))/20
+ (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*((3*2^(1/2)*(
5^(1/2) - 3)^(1/2))/2 + (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)) + (2^(3/4
)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*1i)/(2*((3*2^(1/2)*(5^(1/2) - 3)^(1/2))/2 +
 (2^(1/2)*5^(1/2)*(5^(1/2) - 3)^(1/2))/2)))*(5^(1/2) - 3)^(1/4)*1i)/20
```

3.381 $\int \frac{x^2}{1+3x^4+x^8} \, dx$

Optimal. Leaf size=427

$$-\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}+\frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}}+\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}-\frac{\tan^{-1}\left(1+\frac{2^{3/4}}{\sqrt[4]{3+}}\right.}{2\sqrt{5}\sqrt[4]{2\left(3+\right.}}$$

```
[Out] 1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(3/4)/(3-5^(1/2))^(1/4)*5^(1/
2)+1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*2^(3/4)/(3-5^(1/2))^(1/4)*5^(
1/2)+1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*2^(3/4)/(3-5^(1
/2))^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*2
^(3/4)/(3-5^(1/2))^(1/4)*5^(1/2)-1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4)
)*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)-1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/
4))*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))
^(1/4)+5^(1/2)+1)*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)+1/40*ln(2*x^2+2*2^(1/4)
*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*2^(3/4)*5^(1/2)/(3+5^(1/2))^(1/4)
```

Rubi [A]
time = 0.18, antiderivative size = 431, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1389, 303, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt[4]{3+\sqrt{5}}\,\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{3+\sqrt{5}}\,\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}-\frac{\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}+\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{3+\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}+\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 3*x^4 + x^8),x]
```

```
[Out] -1/2*((3 + Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/(2^(
3/4)*Sqrt[5]) + ((3 + Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(
1/4)])/(2*2^(3/4)*Sqrt[5]) + ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(2
*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1
/4)]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) + ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*
(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5])
- ((3 + Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4
)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + S
qrt[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sqrt[5]))^(1/4)) + Log[Sqrt[2*
(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/(4*Sqrt[5]*(2*(3 + Sq
rt[5]))^(1/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2}{1+3x^4+x^8}\,dx = \frac{\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= -\frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} + \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{2\sqrt{10}} -$$

$$= \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}\,dx}{4\sqrt{5}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+}}{4\sqrt{5}}$$

$$= \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\,2^{3/4}\sqrt{5}} - \frac{\sqrt[4]{3+\sqrt{5}}\log\left(\right.}{}$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{5}\sqrt[4]{2\left(3-\sqrt{5}\right)}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right.}{2\sqrt{5}\sqrt[4]{2\left(3+\sqrt{5}\right)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 40, normalized size = 0.09

$$\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)}{3\#1+2\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 3*x^4 + x^8),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1 + 2*#1^5) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 40, normalized size = 0.09

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	40
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/4*sum(_R^2/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="maxima")
[Out] integrate(x^2/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 963 vs. 2(293) = 586.
time = 0.40, size = 963, normalized size = 2.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="fricas")
[Out] 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3)*arctan(
1/1600*sqrt(10)*sqrt(-40*sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sq
rt(5) + 6)^(3/4) + 1600*x^2 - 400*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3))*(3*sqr
t(5) - 5)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3) - 1/40*sqrt(10)*(3*sqrt(5
)*x - 5*x)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3) - 1/8*(sqrt(5)*sqrt(2) -
 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/80*sqrt(10)*(sqrt(5)
 + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4)*arctan(1/1600*sqrt(10)*sqrt
(-40*sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(3/4) +
1600*x^2 + 400*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))*(3*sqrt(5) + 5)*sqrt(-sq
rt(5) + 3)*(-2*sqrt(5) + 6)^(3/4) - 1/40*(sqrt(10)*(3*sqrt(5)*x + 5*x)*(-2*
sqrt(5) + 6)^(3/4) - 5*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*
sqrt(-sqrt(5) + 3)) + 1/80*sqrt(10)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3)
*(sqrt(5) - 3)*arctan(-1/40*sqrt(10)*(3*sqrt(5)*x - 5*x)*(2*sqrt(5) + 6)^(3
```

```
/4)*sqrt(sqrt(5) + 3) + 1/80*sqrt(sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)
*x)*(2*sqrt(5) + 6)^(3/4) + 40*x^2 - 10*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3))*
(3*sqrt(5) - 5)*(2*sqrt(5) + 6)^(3/4)*sqrt(sqrt(5) + 3) + 1/8*(sqrt(5)*sqrt
(2) - 3*sqrt(2))*sqrt(2*sqrt(5) + 6)*sqrt(sqrt(5) + 3)) + 1/80*sqrt(10)*(sq
rt(5) + 3)*sqrt(-sqrt(5) + 3)*(-2*sqrt(5) + 6)^(3/4)*arctan(1/80*sqrt(sqrt(
10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(3/4) + 40*x^2 + 1
0*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6))*(3*sqrt(5) + 5)*sqrt(-sqrt(5) + 3)*(-
2*sqrt(5) + 6)^(3/4) - 1/40*(sqrt(10)*(3*sqrt(5)*x + 5*x)*(-2*sqrt(5) + 6)^
(3/4) + 5*(sqrt(5)*sqrt(2) + 3*sqrt(2))*sqrt(-2*sqrt(5) + 6))*sqrt(-sqrt(5)
 + 3)) - 1/80*sqrt(10)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*log(40*sqrt(10)*(3*sqr
t(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(3/4) + 1600*x^2 - 400*sqrt(2
*sqrt(5) + 6)*(sqrt(5) - 3)) + 1/80*sqrt(10)*sqrt(2)*(2*sqrt(5) + 6)^(1/4)*
log(-40*sqrt(10)*(3*sqrt(5)*sqrt(2)*x - 5*sqrt(2)*x)*(2*sqrt(5) + 6)^(3/4)
+ 1600*x^2 - 400*sqrt(2*sqrt(5) + 6)*(sqrt(5) - 3)) + 1/80*sqrt(10)*sqrt(2)
*(-2*sqrt(5) + 6)^(1/4)*log(40*sqrt(10)*(3*sqrt(5)*sqrt(2)*x + 5*sqrt(2)*x)
*(-2*sqrt(5) + 6)^(3/4) + 1600*x^2 + 400*(sqrt(5) + 3)*sqrt(-2*sqrt(5) + 6)
) - 1/80*sqrt(10)*sqrt(2)*(-2*sqrt(5) + 6)^(1/4)*log(-40*sqrt(10)*(3*sqrt(5
)*sqrt(2)*x + 5*sqrt(2)*x)*(-2*sqrt(5) + 6)^(3/4) + 1600*x^2 + 400*(sqrt(5)
 + 3)*sqrt(-2*sqrt(5) + 6))
```

Sympy [**A**]
time = 0.95, size = 26, normalized size = 0.06

$$\operatorname{RootSum}\left(40960000t^8 + 19200t^4 + 1, \left(t \mapsto t\log\left(-6144000t^7 - 2240t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 19200*_t**4 + 1, Lambda(_t, _t*log(-6144000*_t**7
- 2240*_t**3 + x)))
```

Giac [**A**]
time = 3.55, size = 239, normalized size = 0.56

$$\frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}+1} - 1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}+1} - 1\right)\right)\sqrt{5\sqrt{5}+5} - \frac{1}{80}\left(\pi + 4\arctan\left(x\sqrt{\sqrt{5}-1} + 1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{80}\left(\pi + 4\arctan\left(-x\sqrt{\sqrt{5}-1} + 1\right)\right)\sqrt{5\sqrt{5}-5} + \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x+\sqrt{\sqrt{5}+1}\right)^2 + 16900x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}-5}\log\left(16900\left(x-\sqrt{\sqrt{5}+1}\right)^2 + 16900x^2\right) - \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x+\sqrt{\sqrt{5}-1}\right)^2 + 2500x^2\right) + \frac{1}{40}\sqrt{5\sqrt{5}+5}\log\left(2500\left(x-\sqrt{\sqrt{5}-1}\right)^2 + 2500x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(p
i + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(5*sqrt(5) + 5) - 1/80*(pi + 4*
arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/80*(pi + 4*arctan(
-x*sqrt(sqrt(5) - 1) + 1))*sqrt(5*sqrt(5) - 5) + 1/40*sqrt(5*sqrt(5) - 5)*l
og(16900*(x + sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) - 5)*
log(16900*(x - sqrt(sqrt(5) + 1))^2 + 16900*x^2) - 1/40*sqrt(5*sqrt(5) + 5)
```

```
*log(2500*(x + sqrt(sqrt(5) - 1))^2 + 2500*x^2) + 1/40*sqrt(5*sqrt(5) + 5)*
log(2500*(x - sqrt(sqrt(5) - 1))^2 + 2500*x^2)
```

Mupad [B]
time = 0.09, size = 275, normalized size = 0.64

$$\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{3\,2^{3/4}\,\sqrt{5}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}}{2\,\left(3\,\sqrt{5}-7\right)}\right)\,{\left(\sqrt{5}-3\right)}^{1/4}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}\,7{}\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{2^{3/4}\,\sqrt{5}\,x\,{\left(\sqrt{5}-3\right)}^{1/4}\,3{}\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}\right)\,{\left(\sqrt{5}-3\right)}^{1/4}\,1{}\mathrm{i}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{7\,2^{3/4}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{3\,2^{3/4}\,\sqrt{5}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}}{2\,\left(3\,\sqrt{5}+7\right)}\right)\,{\left(-\sqrt{5}-3\right)}^{1/4}}{20}+\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}\,7{}\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{2^{3/4}\,\sqrt{5}\,x\,{\left(-\sqrt{5}-3\right)}^{1/4}\,3{}\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}\right)\,{\left(-\sqrt{5}-3\right)}^{1/4}\,1{}\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(3*x^4 + x^8 + 1),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7))
 - (3*2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) - 7)))*(5^(1/2)
- 3)^(1/4))/20 + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(5^(1/2) - 3)^(1/4)*7i)/(
2*(3*5^(1/2) - 7)) - (2^(3/4)*5^(1/2)*x*(5^(1/2) - 3)^(1/4)*3i)/(2*(3*5^(1/
2) - 7)))*(5^(1/2) - 3)^(1/4)*1i)/20 + (2^(3/4)*5^(1/2)*atan((7*2^(3/4)*x*(
- 5^(1/2) - 3)^(1/4))/(2*(3*5^(1/2) + 7)) + (3*2^(3/4)*5^(1/2)*x*(- 5^(1/2)
 - 3)^(1/4))/(2*(3*5^(1/2) + 7)))*(- 5^(1/2) - 3)^(1/4))/20 + (2^(3/4)*5^(1
/2)*atan((2^(3/4)*x*(- 5^(1/2) - 3)^(1/4)*7i)/(2*(3*5^(1/2) + 7)) + (2^(3/4
)*5^(1/2)*x*(- 5^(1/2) - 3)^(1/4)*3i)/(2*(3*5^(1/2) + 7)))*(- 5^(1/2) - 3)^
(1/4)*1i)/20
```

3.382 $\int \frac{1}{1+3x^4+x^8}\,dx$

Optimal. Leaf size=414

$$-\frac{\sqrt[4]{9+4\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}}+\frac{\sqrt[4]{9+4\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}}+\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right.}$$

```
[Out] -1/20*arctan(-1+x*(5^(1/2)-1)^(1/2))*(-20+10*5^(1/2))^(1/2)-1/20*arctan(1+x
*(5^(1/2)-1)^(1/2))*(-20+10*5^(1/2))^(1/2)+1/40*ln(1+2*x^2+5^(1/2)-2*x*(5^(
1/2)+1)^(1/2))*(-20+10*5^(1/2))^(1/2)-1/40*ln(1+2*x^2+5^(1/2)+2*x*(5^(1/2)+
1)^(1/2))*(-20+10*5^(1/2))^(1/2)+1/20*arctan(-1+x*(5^(1/2)+1)^(1/2))*(20+10
*5^(1/2))^(1/2)+1/20*arctan(1+x*(5^(1/2)+1)^(1/2))*(20+10*5^(1/2))^(1/2)-1/
40*ln(-1+2*x^2+5^(1/2)-2*x*(5^(1/2)-1)^(1/2))*(20+10*5^(1/2))^(1/2)+1/40*ln
(-1+2*x^2+5^(1/2)+2*x*(5^(1/2)-1)^(1/2))*(20+10*5^(1/2))^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {1361, 217, 1179, 642, 1176, 631, 210}

$$-\frac{\sqrt[4]{9+4\sqrt{5}}\,\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\sqrt{10}}+\frac{\sqrt[4]{9+4\sqrt{5}}\,\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\sqrt{10}}+\frac{\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}-\frac{\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}-\frac{\sqrt[4]{9+4\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\sqrt[4]{9+4\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\sqrt{10}}+\frac{\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}-\frac{\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{2\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x^4 + x^8)^(-1),x]
```

```
[Out] -1/2*((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])/Sq
rt[10] + ((9 + 4*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)]
)/(2*Sqrt[10]) + ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*(2*(3
 + Sqrt[5]))^(3/4)) - ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)]/(Sqrt[5]*
(2*(3 + Sqrt[5]))^(3/4)) - ((9 + 4*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])]
 - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sqrt[10]) + ((9 + 4*Sqrt[5])^(1
/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*Sq
rt[10]) + Log[Sqrt[2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]/
(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4)) - Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 +
 Sqrt[5]))^(1/4)*x + 2*x^2]/(2*Sqrt[5]*(2*(3 + Sqrt[5]))^(3/4))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{1+3x^4+x^8}\,dx = \frac{\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}dx}{\sqrt{5}} - \frac{\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}dx}{\sqrt{5}}$$

$$= \frac{\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}dx}{2\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\int \frac{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}dx}{2\sqrt{5\left(3-\sqrt{5}\right)}} - \frac{\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}dx}{2\sqrt{5\left(3+\sqrt{5}\right)}} - \frac{\int}{}$$

$$= \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x+x^2}dx}{2\sqrt{10\left(3-\sqrt{5}\right)}} + \frac{\int \frac{1}{\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}+\sqrt[4]{2\left(3-\sqrt{5}\right)}x+}}{2\sqrt{10\left(3-\sqrt{5}\right)}}$$

$$= -\frac{\sqrt[4]{9+4\sqrt{5}}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{4\sqrt{10}} + \frac{\sqrt[4]{9+4\sqrt{5}}\,\text{lo}}{}$$

$$= -\frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3-\sqrt{5}\right)\right)^{3/4}} + \frac{\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3-\sqrt{5}\right)\right)^{3/4}} + \frac{\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{\sqrt{5}\left(2\left(3+\sqrt{5}\right)\right)^{3/}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 42, normalized size = 0.10

$$\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{\log(x-\#1)}{3\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x^4 + x^8)^(-1),x]
```

```
[Out] RootSum[1 + 3*#1^4 + #1^8 & , Log[x - #1]/(3*#1^3 + 2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 37, normalized size = 0.09

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	37
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum(1/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 769 vs. $2(221) = 442$.
time = 0.39, size = 769, normalized size = 1.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*sqrt(2)*(4*sqrt(5) + 9)^(1/4)*arctan(1/4*sqrt(4*x^2 - 2*sqrt(4
*sqrt(5) + 9)*(3*sqrt(5) - 7) + 2*(sqrt(5)*sqrt(2)*x - 3*sqrt(2)*x)*(4*sqrt
(5) + 9)^(1/4))*(3*sqrt(5)*sqrt(2) - 7*sqrt(2))*(4*sqrt(5) + 9)^(3/4) - 1/2
*(3*sqrt(5)*sqrt(2)*x - 7*sqrt(2)*x)*(4*sqrt(5) + 9)^(3/4) - 1) + 1/10*sqrt
(5)*sqrt(2)*(4*sqrt(5) + 9)^(1/4)*arctan(1/4*sqrt(4*x^2 - 2*sqrt(4*sqrt(5)
+ 9)*(3*sqrt(5) - 7) - 2*(sqrt(5)*sqrt(2)*x - 3*sqrt(2)*x)*(4*sqrt(5) + 9)^
(1/4))*(3*sqrt(5)*sqrt(2) - 7*sqrt(2))*(4*sqrt(5) + 9)^(3/4) - 1/2*(3*sqrt(
5)*sqrt(2)*x - 7*sqrt(2)*x)*(4*sqrt(5) + 9)^(3/4) + 1) + 1/10*sqrt(5)*sqrt(
2)*(-4*sqrt(5) + 9)^(1/4)*arctan(1/4*sqrt(4*x^2 + 2*(3*sqrt(5) + 7)*sqrt(-4
*sqrt(5) + 9) + 2*(sqrt(5)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)^(1/4))
*(3*sqrt(5)*sqrt(2) + 7*sqrt(2))*(-4*sqrt(5) + 9)^(3/4) - 1/2*(3*sqrt(5)*sq
```

```
rt(2)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) - 1) + 1/10*sqrt(5)*sqrt(2)*(
-4*sqrt(5) + 9)^(1/4)*arctan(1/4*sqrt(4*x^2 + 2*(3*sqrt(5) + 7)*sqrt(-4*sqr
t(5) + 9) - 2*(sqrt(5)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)^(1/4))*(3*
sqrt(5)*sqrt(2) + 7*sqrt(2))*(-4*sqrt(5) + 9)^(3/4) - 1/2*(3*sqrt(5)*sqrt(2
)*x + 7*sqrt(2)*x)*(-4*sqrt(5) + 9)^(3/4) + 1) - 1/40*sqrt(5)*sqrt(2)*(4*sq
rt(5) + 9)^(1/4)*log(4*x^2 - 2*sqrt(4*sqrt(5) + 9)*(3*sqrt(5) - 7) + 2*(sqr
t(5)*sqrt(2)*x - 3*sqrt(2)*x)*(4*sqrt(5) + 9)^(1/4)) + 1/40*sqrt(5)*sqrt(2)
*(4*sqrt(5) + 9)^(1/4)*log(4*x^2 - 2*sqrt(4*sqrt(5) + 9)*(3*sqrt(5) - 7) -
2*(sqrt(5)*sqrt(2)*x - 3*sqrt(2)*x)*(4*sqrt(5) + 9)^(1/4)) - 1/40*sqrt(5)*s
qrt(2)*(-4*sqrt(5) + 9)^(1/4)*log(4*x^2 + 2*(3*sqrt(5) + 7)*sqrt(-4*sqrt(5)
 + 9) + 2*(sqrt(5)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)^(1/4)) + 1/40*
sqrt(5)*sqrt(2)*(-4*sqrt(5) + 9)^(1/4)*log(4*x^2 + 2*(3*sqrt(5) + 7)*sqrt(-
4*sqrt(5) + 9) - 2*(sqrt(5)*sqrt(2)*x + 3*sqrt(2)*x)*(-4*sqrt(5) + 9)^(1/4)
)
```

Sympy [**A**]
time = 1.20, size = 26, normalized size = 0.06

$$\operatorname{RootSum}\left(40960000t^8+115200t^4+1,\left(t\mapsto t\log\left(-9600t^5-\frac{47t}{2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+3*x**4+1),x)
[Out] RootSum(40960000*_t**8 + 115200*_t**4 + 1, Lambda(_t, _t*log(-9600*_t**5 -
47*_t/2 + x)))
```

Giac [**A**]
time = 4.06, size = 239, normalized size = 0.58

$$\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}+20}-\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{10\sqrt{5}+20}-\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}-20}+\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{10\sqrt{5}-20}-\frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x+\sqrt{\sqrt{5}+1}\right)^2+10000x^2\right)+\frac{1}{40}\sqrt{10\sqrt{5}-20}\log\left(10000\left(x-\sqrt{\sqrt{5}+1}\right)^2+10000x^2\right)+\frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x+\sqrt{\sqrt{5}-1}\right)^2+400x^2\right)-\frac{1}{40}\sqrt{10\sqrt{5}+20}\log\left(400\left(x-\sqrt{\sqrt{5}-1}\right)^2+400x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+3*x^4+1),x, algorithm="giac")
[Out] 1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) + 20) - 1/80*
(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(10*sqrt(5) + 20) - 1/80*(pi
+ 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) - 20) + 1/80*(pi + 4*a
rctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(10*sqrt(5) - 20) - 1/40*sqrt(10*sqrt(
5) - 20)*log(10000*(x + sqrt(sqrt(5) + 1))^2 + 10000*x^2) + 1/40*sqrt(10*sq
rt(5) - 20)*log(10000*(x - sqrt(sqrt(5) + 1))^2 + 10000*x^2) + 1/40*sqrt(10
*sqrt(5) + 20)*log(400*(x + sqrt(sqrt(5) - 1))^2 + 400*x^2) - 1/40*sqrt(10*
sqrt(5) + 20)*log(400*(x - sqrt(sqrt(5) - 1))^2 + 400*x^2)
```

Mupad [**B**]
time = 0.08, size = 403, normalized size = 0.97

$$\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{144x\left(-4\sqrt{5}-9\right)^{1/4}}{24\sqrt{5}\sqrt{-4\sqrt{5}-9}+56\sqrt{-4\sqrt{5}-9}}+\frac{64\sqrt{5}\,x\left(-4\sqrt{5}-9\right)^{1/4}}{24\sqrt{5}\sqrt{-4\sqrt{5}-9}+56\sqrt{-4\sqrt{5}-9}}\right)\left(-4\sqrt{5}-9\right)^{1/4}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{144x\left(4\sqrt{5}-9\right)^{1/4}}{24\sqrt{5}\sqrt{4\sqrt{5}-9}-56\sqrt{4\sqrt{5}-9}}-\frac{64\sqrt{5}\,x\left(4\sqrt{5}-9\right)^{1/4}}{24\sqrt{5}\sqrt{4\sqrt{5}-9}-56\sqrt{4\sqrt{5}-9}}\right)\left(4\sqrt{5}-9\right)^{1/4}}{10}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\left(-4\sqrt{5}-9\right)^{1/4}144\mathrm{i}}{24\sqrt{5}\sqrt{-4\sqrt{5}-9}+56\sqrt{-4\sqrt{5}-9}}+\frac{\sqrt{5}\,x\left(-4\sqrt{5}-9\right)^{1/4}64\mathrm{i}}{24\sqrt{5}\sqrt{-4\sqrt{5}-9}+56\sqrt{-4\sqrt{5}-9}}\right)\left(-4\sqrt{5}-9\right)^{1/4}1\mathrm{i}}{10}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\left(4\sqrt{5}-9\right)^{1/4}144\mathrm{i}}{24\sqrt{5}\sqrt{4\sqrt{5}-9}-56\sqrt{4\sqrt{5}-9}}-\frac{\sqrt{5}\,x\left(4\sqrt{5}-9\right)^{1/4}64\mathrm{i}}{24\sqrt{5}\sqrt{4\sqrt{5}-9}-56\sqrt{4\sqrt{5}-9}}\right)\left(4\sqrt{5}-9\right)^{1/4}1\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4 + x^8 + 1),x)
```

```
[Out] (5^(1/2)*atan((144*x*(- 4*5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(- 4*5^(1/2) - 9)
^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)) + (64*5^(1/2)*x*(- 4*5^(1/2) - 9)^(1/4
))/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)))*(- 4*
5^(1/2) - 9)^(1/4))/10 + (5^(1/2)*atan((144*x*(4*5^(1/2) - 9)^(1/4))/(24*5^
(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)) - (64*5^(1/2)*x*(4*
5^(1/2) - 9)^(1/4))/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^
(1/2)))*(4*5^(1/2) - 9)^(1/4))/10 - (5^(1/2)*atan((x*(- 4*5^(1/2) - 9)^(1/4
)*144i)/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2) + 56*(- 4*5^(1/2) - 9)^(1/2)) +
 (5^(1/2)*x*(- 4*5^(1/2) - 9)^(1/4)*64i)/(24*5^(1/2)*(- 4*5^(1/2) - 9)^(1/2
) + 56*(- 4*5^(1/2) - 9)^(1/2)))*(- 4*5^(1/2) - 9)^(1/4)*1i)/10 - (5^(1/2)*
atan((x*(4*5^(1/2) - 9)^(1/4)*144i)/(24*5^(1/2)*(4*5^(1/2) - 9)^(1/2) - 56*
(4*5^(1/2) - 9)^(1/2)) - (5^(1/2)*x*(4*5^(1/2) - 9)^(1/4)*64i)/(24*5^(1/2)*
(4*5^(1/2) - 9)^(1/2) - 56*(4*5^(1/2) - 9)^(1/2)))*(4*5^(1/2) - 9)^(1/4)*1i
)/10
```

3.383 $\int \frac{1}{x^2(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=416

$$-\frac{1}{x}+\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{1}{20}\sqrt[4]{6150-275$$

```
[Out] -1/x+1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(6150-2750*5^(1/2))^(1/4)+
1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(6150-2750*5^(1/2))^(1/4)+1/40*l
n(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(6150-2750*5^(1/2))^(1/4)-
1/40*ln(2*x^2+2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/2)+1)*(6150-2750*5^(1/2))^
(1/4)-1/20*arctan(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(246+110*5^(1/2))^(1/4)*5
^(1/2)-1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(246+110*5^(1/2))^(1/4)*5
^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(246+110*5^(1
/2))^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(
246+110*5^(1/2))^(1/4)*5^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1382, 1524, 303, 1176, 631, 210, 1179, 642}

$$\frac{\left(3+\sqrt{5}\right)^{5/4}\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{1}{20}\sqrt[4]{6150-2750\sqrt{5}}\,\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)+\frac{1}{20}\sqrt[4]{6150-2750\sqrt{5}}\,\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\ 2^{3/4}\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{8\ 2^{3/4}\sqrt{5}}+\frac{1}{40}\sqrt[4]{6150-2750\sqrt{5}}\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)-\frac{1}{40}\sqrt[4]{6150-2750\sqrt{5}}\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}x+\sqrt{2\left(3+\sqrt{5}\right)}\right)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ((3 + Sqrt[5])^(5/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5])^(1/4)])
/(4*2^(3/4)*Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[1 + (2^(3/4)*x)/(3 - Sqr
t[5])^(1/4)])/(4*2^(3/4)*Sqrt[5]) - ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan[1 -
 (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/20 + ((6150 - 2750*Sqrt[5])^(1/4)*ArcTan
[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/20 - ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2
*(3 - Sqrt[5])] - 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5])
 + ((3 + Sqrt[5])^(5/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/
4)*x + 2*x^2])/(8*2^(3/4)*Sqrt[5]) + ((6150 - 2750*Sqrt[5])^(1/4)*Log[Sqrt[
2*(3 + Sqrt[5])] - 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/40 - ((6150 - 2750
*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2
*x^2])/40
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2(1+3x^4+x^8)}\,dx = -\frac{1}{x}+\int \frac{x^2(-3-x^4)}{1+3x^4+x^8}\,dx$$

$$= -\frac{1}{x}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}$$

$$= -\frac{1}{x}-\frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{4\sqrt{10}}+\frac{\left(3-\sqrt{5}\right)\int \frac{\sqrt{3+\sqrt{5}}+\sqrt{2}x^2}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,d}{4\sqrt{10}}$$

$$= -\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)^{5/4}\int \frac{\sqrt[4]{2\left(3-\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3-\sqrt{5}\right)}-\sqrt[4]{2\left(3-\sqrt{5}\right)}x-x^2}\,dx}{8\,2^{3/4}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)}{}$$

$$= -\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)^{5/4}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{8\,2^{3/4}\sqrt{5}}+\frac{\left(3+\right.}{}$$

$$= -\frac{1}{x}+\frac{\sqrt[4]{246+110\sqrt{5}}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{4\sqrt{5}}-\frac{\sqrt[4]{246+110\sqrt{5}}\tan^{-1}\left(\right.}{4\sqrt{5}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 61, normalized size = 0.15

$$-\frac{1}{x}-\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{3\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1+2\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^4)/
(3*#1 + 2*#1^5) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 52, normalized size = 0.12

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(625\_Z^{8}+3075\_Z^{4}+1\right)}\_R\ln\left(1175\_R^{7}+5778\_R^{3}+11x\right)\right)}{4}$	42
default	$-\frac{1}{x}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+3\_Z^{4}+1\right)}\frac{\left(\_R^{6}+3\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}+3\_R^{3}}\right)}{4}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x-1/4*sum((_R^6+3*_R^2)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate((x^6 + 3*x^2)/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1025 vs. $2(270) = 540$.
time = 0.39, size = 1025, normalized size = 2.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/80*(sqrt(10)*(55*sqrt(5)*x - 123*x)*(110*sqrt(5) + 246)^(3/4)*sqrt(55*sq
rt(5) + 123)*arctan(1/800*sqrt(10)*sqrt(-40*sqrt(10)*(47*sqrt(5)*sqrt(2)*x
- 105*sqrt(2)*x)*(110*sqrt(5) + 246)^(3/4) + 1600*x^2 - 800*sqrt(110*sqrt(5
```

```
) + 246)*(4*sqrt(5) - 9))*(161*sqrt(5) - 360)*(110*sqrt(5) + 246)^(3/4)*sqr
t(55*sqrt(5) + 123) - 1/20*sqrt(10)*(161*sqrt(5)*x - 360*x)*(110*sqrt(5) +
246)^(3/4)*sqrt(55*sqrt(5) + 123) - 1/8*(55*sqrt(5)*sqrt(2) - 123*sqrt(2))*
sqrt(110*sqrt(5) + 246)*sqrt(55*sqrt(5) + 123)) + sqrt(10)*(55*sqrt(5)*x +
123*x)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(3/4)*arctan(1/800*sqrt
(10)*sqrt(-40*sqrt(10)*(47*sqrt(5)*sqrt(2)*x + 105*sqrt(2)*x)*(-110*sqrt(5)
 + 246)^(3/4) + 1600*x^2 + 800*(4*sqrt(5) + 9)*sqrt(-110*sqrt(5) + 246))*(1
61*sqrt(5) + 360)*sqrt(-55*sqrt(5) + 123)*(-110*sqrt(5) + 246)^(3/4) - 1/40
*(2*sqrt(10)*(161*sqrt(5)*x + 360*x)*(-110*sqrt(5) + 246)^(3/4) - 5*(55*sqr
t(5)*sqrt(2) + 123*sqrt(2))*sqrt(-110*sqrt(5) + 246))*sqrt(-55*sqrt(5) + 12
3)) + sqrt(10)*(55*sqrt(5)*x - 123*x)*(110*sqrt(5) + 246)^(3/4)*sqrt(55*sqr
t(5) + 123)*arctan(-1/20*sqrt(10)*(161*sqrt(5)*x - 360*x)*(110*sqrt(5) + 24
6)^(3/4)*sqrt(55*sqrt(5) + 123) + 1/40*sqrt(sqrt(10)*(47*sqrt(5)*sqrt(2)*x
- 105*sqrt(2)*x)*(110*sqrt(5) + 246)^(3/4) + 40*x^2 - 20*sqrt(110*sqrt(5) +
 246)*(4*sqrt(5) - 9))*(161*sqrt(5) - 360)*(110*sqrt(5) + 246)^(3/4)*sqrt(5
5*sqrt(5) + 123) + 1/8*(55*sqrt(5)*sqrt(2) - 123*sqrt(2))*sqrt(110*sqrt(5)
+ 246)*sqrt(55*sqrt(5) + 123)) + sqrt(10)*(55*sqrt(5)*x + 123*x)*sqrt(-55*s
qrt(5) + 123)*(-110*sqrt(5) + 246)^(3/4)*arctan(1/40*sqrt(sqrt(10)*(47*sqrt
(5)*sqrt(2)*x + 105*sqrt(2)*x)*(-110*sqrt(5) + 246)^(3/4) + 40*x^2 + 20*(4*
sqrt(5) + 9)*sqrt(-110*sqrt(5) + 246))*(161*sqrt(5) + 360)*sqrt(-55*sqrt(5)
 + 123)*(-110*sqrt(5) + 246)^(3/4) - 1/40*(2*sqrt(10)*(161*sqrt(5)*x + 360*
x)*(-110*sqrt(5) + 246)^(3/4) + 5*(55*sqrt(5)*sqrt(2) + 123*sqrt(2))*sqrt(-
110*sqrt(5) + 246))*sqrt(-55*sqrt(5) + 123)) - sqrt(10)*sqrt(2)*x*(110*sqrt
(5) + 246)^(1/4)*log(40*sqrt(10)*(47*sqrt(5)*sqrt(2)*x - 105*sqrt(2)*x)*(11
0*sqrt(5) + 246)^(3/4) + 1600*x^2 - 800*sqrt(110*sqrt(5) + 246)*(4*sqrt(5)
- 9)) + sqrt(10)*sqrt(2)*x*(110*sqrt(5) + 246)^(1/4)*log(-40*sqrt(10)*(47*s
qrt(5)*sqrt(2)*x - 105*sqrt(2)*x)*(110*sqrt(5) + 246)^(3/4) + 1600*x^2 - 80
0*sqrt(110*sqrt(5) + 246)*(4*sqrt(5) - 9)) + sqrt(10)*sqrt(2)*x*(-110*sqrt(
5) + 246)^(1/4)*log(40*sqrt(10)*(47*sqrt(5)*sqrt(2)*x + 105*sqrt(2)*x)*(-11
0*sqrt(5) + 246)^(3/4) + 1600*x^2 + 800*(4*sqrt(5) + 9)*sqrt(-110*sqrt(5) +
 246)) - sqrt(10)*sqrt(2)*x*(-110*sqrt(5) + 246)^(1/4)*log(-40*sqrt(10)*(47
*sqrt(5)*sqrt(2)*x + 105*sqrt(2)*x)*(-110*sqrt(5) + 246)^(3/4) + 1600*x^2 +
 800*(4*sqrt(5) + 9)*sqrt(-110*sqrt(5) + 246)) + 80)/x
```

Sympy [A]
time = 1.14, size = 32, normalized size = 0.08

$$\operatorname{RootSum}\left(40960000t^8 + 787200t^4 + 1, \left(t \mapsto t \log\left(\frac{19251200t^7}{11} + \frac{369792t^3}{11} + x\right)\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 787200*_t**4 + 1, Lambda(_t, _t*log(19251200*_t**7
/11 + 369792*_t**3/11 + x))) - 1/x
```

Giac [A]
time = 3.68, size = 244, normalized size = 0.59

$$-\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}+55}+\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}-1\right)\right)\sqrt{25\sqrt{5}+55}+\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}-55}-\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}+1\right)\right)\sqrt{25\sqrt{5}-55}-\frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(748225\left(x+\sqrt{\sqrt{5}+1}\right)^2+748225x^2\right)+\frac{1}{40}\sqrt{25\sqrt{5}-55}\log\left(748225\left(x-\sqrt{\sqrt{5}+1}\right)^2+748225x^2\right)+\frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(180625\left(x+\sqrt{\sqrt{5}-1}\right)^2+180625x^2\right)-\frac{1}{40}\sqrt{25\sqrt{5}+55}\log\left(180625\left(x-\sqrt{\sqrt{5}-1}\right)^2+180625x^2\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) + 55) + 1/80
*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) - 1))*sqrt(25*sqrt(5) + 55) + 1/80*(pi
 + 4*arctan(x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) - 55) - 1/80*(pi + 4*
arctan(-x*sqrt(sqrt(5) - 1) + 1))*sqrt(25*sqrt(5) - 55) - 1/40*sqrt(25*sqrt
(5) - 55)*log(748225*(x + sqrt(sqrt(5) + 1))^2 + 748225*x^2) + 1/40*sqrt(25
*sqrt(5) - 55)*log(748225*(x - sqrt(sqrt(5) + 1))^2 + 748225*x^2) + 1/40*sq
rt(25*sqrt(5) + 55)*log(180625*(x + sqrt(sqrt(5) - 1))^2 + 180625*x^2) - 1/
40*sqrt(25*sqrt(5) + 55)*log(180625*(x - sqrt(sqrt(5) - 1))^2 + 180625*x^2)
 - 1/x
```

Mupad [B]
time = 1.29, size = 292, normalized size = 0.70

$$-\frac{1}{x}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2585\,2^{3/4}x\left(-55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}+6765\right)}+\frac{1155\,2^{3/4}\sqrt{5}\,x\left(-55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}+6765\right)}\right)\left(-55\sqrt{5}-123\right)^{1/4}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2585\,2^{3/4}x\left(55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}-6765\right)}-\frac{1155\,2^{3/4}\sqrt{5}\,x\left(55\sqrt{5}-123\right)^{1/4}}{2\left(3025\sqrt{5}-6765\right)}\right)\left(55\sqrt{5}-123\right)^{1/4}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(-55\sqrt{5}-123\right)^{1/4}2585\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}+\frac{2^{3/4}\sqrt{5}\,x\left(-55\sqrt{5}-123\right)^{1/4}1155\mathrm{i}}{2\left(3025\sqrt{5}+6765\right)}\right)\left(-55\sqrt{5}-123\right)^{1/4}1\mathrm{i}}{20}-\frac{2^{3/4}\sqrt{5}\,\mathrm{atan}\left(\frac{2^{3/4}x\left(55\sqrt{5}-123\right)^{1/4}2585\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}-\frac{2^{3/4}\sqrt{5}\,x\left(55\sqrt{5}-123\right)^{1/4}1155\mathrm{i}}{2\left(3025\sqrt{5}-6765\right)}\right)\left(55\sqrt{5}-123\right)^{1/4}1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(3*x^4 + x^8 + 1)),x)
```

```
[Out] - 1/x - (2^(3/4)*5^(1/2)*atan((2585*2^(3/4)*x*(- 55*5^(1/2) - 123)^(1/4))/(
2*(3025*5^(1/2) + 6765)) + (1155*2^(3/4)*5^(1/2)*x*(- 55*5^(1/2) - 123)^(1/
4))/(2*(3025*5^(1/2) + 6765)))*(- 55*5^(1/2) - 123)^(1/4))/20 - (2^(3/4)*5^
(1/2)*atan((2585*2^(3/4)*x*(55*5^(1/2) - 123)^(1/4))/(2*(3025*5^(1/2) - 676
5)) - (1155*2^(3/4)*5^(1/2)*x*(55*5^(1/2) - 123)^(1/4))/(2*(3025*5^(1/2) -
6765)))*(55*5^(1/2) - 123)^(1/4))/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(-
55*5^(1/2) - 123)^(1/4)*2585i)/(2*(3025*5^(1/2) + 6765)) + (2^(3/4)*5^(1/2)
*x*(- 55*5^(1/2) - 123)^(1/4)*1155i)/(2*(3025*5^(1/2) + 6765)))*(- 55*5^(1/
2) - 123)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(55*5^(1/2) - 123
)^(1/4)*2585i)/(2*(3025*5^(1/2) - 6765)) - (2^(3/4)*5^(1/2)*x*(55*5^(1/2) -
 123)^(1/4)*1155i)/(2*(3025*5^(1/2) - 6765)))*(55*5^(1/2) - 123)^(1/4)*1i)/
20
```

3.384 $\int \frac{1}{x^4(1+3x^4+x^8)}\,dx$

Optimal. Leaf size=466

$$-\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\,\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\,\tan^{-1}\left(1+\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843}}{}$$

```
[Out] -1/3/x^3+1/20*arctan(-1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(843-377*5^(1/2))^(1/4
)*2^(1/4)*5^(1/2)+1/20*arctan(1+2^(3/4)*x/(3+5^(1/2))^(1/4))*(843-377*5^(1/
2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2-2*2^(1/4)*x*(3+5^(1/2))^(1/4)+5^(1/
2)+1)*(843-377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)+1/40*ln(2*x^2+2*2^(1/4)*x*(3+
5^(1/2))^(1/4)+5^(1/2)+1)*(843-377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-1/20*arct
an(-1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(843+377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)-
1/20*arctan(1+2^(3/4)*x/(3-5^(1/2))^(1/4))*(843+377*5^(1/2))^(1/4)*2^(1/4)*
5^(1/2)+1/40*ln(2*x^2-2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(1/2)-1)*(843+377*5^(
1/2))^(1/4)*2^(1/4)*5^(1/2)-1/40*ln(2*x^2+2*2^(1/4)*x*(3-5^(1/2))^(1/4)+5^(
1/2)-1)*(843+377*5^(1/2))^(1/4)*2^(1/4)*5^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1382, 1436, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt[4]{843+377\sqrt{5}}\,\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\,\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\,\text{ArcTan}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}\right)}{2\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843-377\sqrt{5}}\,\text{ArcTan}\left(\frac{2^{3/4}x}{\sqrt[4]{3+\sqrt{5}}}+1\right)}{2\,2^{3/4}\sqrt{5}}-\frac{1}{3x^3}+\frac{\sqrt[4]{843+377\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843+377\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3-\sqrt{5}\right)}\,x+\sqrt{2\left(3-\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843-377\sqrt{5}}\,\log\left(2x^2-2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{843-377\sqrt{5}}\,\log\left(2x^2+2\sqrt[4]{2\left(3+\sqrt{5}\right)}\,x+\sqrt{2\left(3+\sqrt{5}\right)}\right)}{4\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 + ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 - (2^(3/4)*x)/(3 - Sqrt[5]
)^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3
/4)*x)/(3 - Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/
4)*ArcTan[1 - (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/4)*Sqrt[5]) + ((843
 - 377*Sqrt[5])^(1/4)*ArcTan[1 + (2^(3/4)*x)/(3 + Sqrt[5])^(1/4)])/(2*2^(3/
4)*Sqrt[5]) + ((843 + 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] - 2*(2*(
3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) - ((843 + 377*Sqrt[5])^
(1/4)*Log[Sqrt[2*(3 - Sqrt[5])] + 2*(2*(3 - Sqrt[5]))^(1/4)*x + 2*x^2])/(4*
2^(3/4)*Sqrt[5]) - ((843 - 377*Sqrt[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] - 2
*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2])/(4*2^(3/4)*Sqrt[5]) + ((843 - 377*Sqrt
[5])^(1/4)*Log[Sqrt[2*(3 + Sqrt[5])] + 2*(2*(3 + Sqrt[5]))^(1/4)*x + 2*x^2]
)/(4*2^(3/4)*Sqrt[5])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^4(1+3x^4+x^8)}\,dx = -\frac{1}{3x^3}+\frac{1}{3}\int \frac{-9-3x^4}{1+3x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}$$

$$= -\frac{1}{3x^3}-\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{8\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/2}\int \frac{\sqrt{3-\sqrt{5}}+}{\frac{3}{2}-\frac{\sqrt{5}}{2}+x}}{8\sqrt{5}}$$

$$= -\frac{1}{3x^3}-\frac{\sqrt[4]{843-377\sqrt{5}}\int \frac{\sqrt[4]{2\left(3+\sqrt{5}\right)}+2x}{-\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}-\sqrt[4]{2\left(3+\sqrt{5}\right)}x-x^2}\,dx}{4\,2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{843}}{}$$

$$= -\frac{1}{3x^3}+\frac{\left(3+\sqrt{5}\right)^{7/4}\log\left(\sqrt{2\left(3-\sqrt{5}\right)}-2\sqrt[4]{2\left(3-\sqrt{5}\right)}x+2x^2\right)}{16\sqrt[4]{2}\sqrt{5}}-\frac{(3}{}$$

$$= -\frac{1}{3x^3}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(1-\frac{2^{3/4}x}{\sqrt[4]{3-\sqrt{5}}}\right)}{8\sqrt[4]{2}\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(1+\right.}{8\sqrt[4]{2}\sqrt{5}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 65, normalized size = 0.14

$$-\frac{1}{3x^3}-\frac{1}{4}\text{RootSum}\left[1+3\#1^4+\#1^8\&,\frac{3\log(x-\#1)+\log(x-\#1)\#1^4}{3\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + 3*x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - RootSum[1 + 3*#1^4 + #1^8 & , (3*Log[x - #1] + Log[x - #1]*#1^
4)/(3*#1^3 + 2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 50, normalized size = 0.11

method	result	size
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(625\_Z^8+21075\_Z^4+1\right)}\_R\ln\left(175\_R^5+5778\_R+377x\right)\right)}{4}$	40
default	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+3\_Z^4+1\right)}\frac{\left(-\_R^4-3\right)\ln\left(x-\_R\right)}{2\_R^7+3\_R^3}\right)}{4}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8+3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^3+1/4*sum((-_R^4-3)/(2*_R^7+3*_R^3)*ln(x-_R),_R=RootOf(_Z^8+3*_Z^4+1
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate((x^4 + 3)/(x^8 + 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(306) = 612$.
time = 0.40, size = 1071, normalized size = 2.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/240*(3*sqrt(10)*sqrt(2)*x^3*(754*sqrt(5) + 1686)^(1/4)*log(400*x^2 + 20*
sqrt(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) -
100*sqrt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47)) - 3*sqrt(10)*sqrt(2)*x^3*(7
```

```
54*sqrt(5) + 1686)^(1/4)*log(400*x^2 - 20*sqrt(10)*(7*sqrt(5)*sqrt(2)*x - 1
5*sqrt(2)*x)*(754*sqrt(5) + 1686)^(1/4) - 100*sqrt(754*sqrt(5) + 1686)*(21*
sqrt(5) - 47)) - 3*sqrt(10)*sqrt(2)*x^3*(-754*sqrt(5) + 1686)^(1/4)*log(400
*x^2 + 20*sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 168
6)^(1/4) + 100*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1686)) + 3*sqrt(10)*sq
rt(2)*x^3*(-754*sqrt(5) + 1686)^(1/4)*log(400*x^2 - 20*sqrt(10)*(7*sqrt(5)*
sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 1686)^(1/4) + 100*(21*sqrt(5) + 4
7)*sqrt(-754*sqrt(5) + 1686)) + 3*sqrt(10)*(377*sqrt(5)*x^3 - 843*x^3)*(754
*sqrt(5) + 1686)^(3/4)*sqrt(377*sqrt(5) + 843)*arctan(1/400*sqrt(10)*sqrt(5
)*sqrt(20*x^2 + sqrt(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5)
+ 1686)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47))*(51841*sqrt(5
) - 115920)*(754*sqrt(5) + 1686)^(5/4)*sqrt(377*sqrt(5) + 843) - 1/40*sqrt(
10)*(51841*sqrt(5)*x - 115920*x)*(754*sqrt(5) + 1686)^(5/4)*sqrt(377*sqrt(5
) + 843) + 1/8*(377*sqrt(5)*sqrt(2) - 843*sqrt(2))*sqrt(754*sqrt(5) + 1686)
*sqrt(377*sqrt(5) + 843)) + 3*sqrt(10)*(377*sqrt(5)*x^3 - 843*x^3)*(754*sqr
t(5) + 1686)^(3/4)*sqrt(377*sqrt(5) + 843)*arctan(1/400*sqrt(10)*sqrt(5)*sq
rt(20*x^2 - sqrt(10)*(7*sqrt(5)*sqrt(2)*x - 15*sqrt(2)*x)*(754*sqrt(5) + 16
86)^(1/4) - 5*sqrt(754*sqrt(5) + 1686)*(21*sqrt(5) - 47))*(51841*sqrt(5) -
115920)*(754*sqrt(5) + 1686)^(5/4)*sqrt(377*sqrt(5) + 843) - 1/40*sqrt(10)*
(51841*sqrt(5)*x - 115920*x)*(754*sqrt(5) + 1686)^(5/4)*sqrt(377*sqrt(5) +
843) - 1/8*(377*sqrt(5)*sqrt(2) - 843*sqrt(2))*sqrt(754*sqrt(5) + 1686)*sqr
t(377*sqrt(5) + 843)) + 3*sqrt(10)*(377*sqrt(5)*x^3 + 843*x^3)*sqrt(-377*sq
rt(5) + 843)*(-754*sqrt(5) + 1686)^(3/4)*arctan(1/400*sqrt(10)*sqrt(5)*sqrt
(20*x^2 + sqrt(10)*(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 168
6)^(1/4) + 5*(21*sqrt(5) + 47)*sqrt(-754*sqrt(5) + 1686))*(51841*sqrt(5) +
115920)*sqrt(-377*sqrt(5) + 843)*(-754*sqrt(5) + 1686)^(5/4) - 1/40*(sqrt(1
0)*(51841*sqrt(5)*x + 115920*x)*(-754*sqrt(5) + 1686)^(5/4) + 5*(377*sqrt(5
)*sqrt(2) + 843*sqrt(2))*sqrt(-754*sqrt(5) + 1686))*sqrt(-377*sqrt(5) + 843
)) + 3*sqrt(10)*(377*sqrt(5)*x^3 + 843*x^3)*sqrt(-377*sqrt(5) + 843)*(-754*
sqrt(5) + 1686)^(3/4)*arctan(1/400*sqrt(10)*sqrt(5)*sqrt(20*x^2 - sqrt(10)*
(7*sqrt(5)*sqrt(2)*x + 15*sqrt(2)*x)*(-754*sqrt(5) + 1686)^(1/4) + 5*(21*sq
rt(5) + 47)*sqrt(-754*sqrt(5) + 1686))*(51841*sqrt(5) + 115920)*sqrt(-377*s
qrt(5) + 843)*(-754*sqrt(5) + 1686)^(5/4) - 1/40*(sqrt(10)*(51841*sqrt(5)*x
 + 115920*x)*(-754*sqrt(5) + 1686)^(5/4) - 5*(377*sqrt(5)*sqrt(2) + 843*sqr
t(2))*sqrt(-754*sqrt(5) + 1686))*sqrt(-377*sqrt(5) + 843)) + 80)/x^3
```

Sympy [A]
time = 0.95, size = 34, normalized size = 0.07

$$\operatorname{RootSum}\left(40960000t^8 + 5395200t^4 + 1, \left(t \mapsto t\log\left(\frac{179200t^5}{377} + \frac{23112t}{377} + x\right)\right)\right) - \frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8+3*x**4+1),x)
```

```
[Out] RootSum(40960000*_t**8 + 5395200*_t**4 + 1, Lambda(_t, _t*log(179200*_t**5/
377 + 23112*_t/377 + x))) - 1/(3*x**3)
```

Giac [A]
time = 4.65, size = 244, normalized size = 0.52

$$-\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{65\sqrt{5}+145}+\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}+1}+1\right)\right)\sqrt{65\sqrt{5}+145}+\frac{1}{80}\left(\pi+4\arctan\left(x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{65\sqrt{5}-145}-\frac{1}{80}\left(\pi+4\arctan\left(-x\sqrt{\sqrt{5}-1}-1\right)\right)\sqrt{65\sqrt{5}-145}+\frac{1}{40}\sqrt{65\sqrt{5}-145}\log\left(93122500\left(x+\sqrt{\sqrt{5}+1}\right)^2+93122500x^2\right)-\frac{1}{40}\sqrt{65\sqrt{5}-145}\log\left(93122500\left(x-\sqrt{\sqrt{5}+1}\right)^2+93122500x^2\right)-\frac{1}{40}\sqrt{65\sqrt{5}+145}\log\left(53728900\left(x+\sqrt{\sqrt{5}-1}\right)^2+53728900x^2\right)+\frac{1}{40}\sqrt{65\sqrt{5}+145}\log\left(53728900\left(x-\sqrt{\sqrt{5}-1}\right)^2+53728900x^2\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8+3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/80*(pi + 4*arctan(x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/8
0*(pi + 4*arctan(-x*sqrt(sqrt(5) + 1) + 1))*sqrt(65*sqrt(5) + 145) + 1/80*(
pi + 4*arctan(x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) - 1/80*(pi +
 4*arctan(-x*sqrt(sqrt(5) - 1) - 1))*sqrt(65*sqrt(5) - 145) + 1/40*sqrt(65*
sqrt(5) - 145)*log(93122500*(x + sqrt(sqrt(5) + 1))^2 + 93122500*x^2) - 1/4
0*sqrt(65*sqrt(5) - 145)*log(93122500*(x - sqrt(sqrt(5) + 1))^2 + 93122500*
x^2) - 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x + sqrt(sqrt(5) - 1))^2 +
 53728900*x^2) + 1/40*sqrt(65*sqrt(5) + 145)*log(53728900*(x - sqrt(sqrt(5)
 - 1))^2 + 53728900*x^2) - 1/3/x^3
```

Mupad [B]
time = 0.18, size = 492, normalized size = 1.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(3*x^4 + x^8 + 1)),x)
```

```
[Out] (2^(3/4)*5^(1/2)*atan((46371*2^(3/4)*x*(377*5^(1/2) - 843)^(1/4))/(2*(3393*
2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*(377*5^(1/2) - 843
)^(1/2))) - (20735*2^(3/4)*5^(1/2)*x*(377*5^(1/2) - 843)^(1/4))/(2*(3393*2^
(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*(377*5^(1/2) - 843)^
(1/2))))*(377*5^(1/2) - 843)^(1/4))/20 - (2^(3/4)*5^(1/2)*atan((46371*2^(3/
4)*x*(- 377*5^(1/2) - 843)^(1/4))/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843)^(1
/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))) + (20735*2^(3/4)*5
^(1/2)*x*(- 377*5^(1/2) - 843)^(1/4))/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843
)^(1/2) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))))*(- 377*5^(1/2
) - 843)^(1/4))/20 - 1/(3*x^3) + (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(- 377*5^
(1/2) - 843)^(1/4)*46371i)/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843)^(1/2) + 1
508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))) + (2^(3/4)*5^(1/2)*x*(- 3
77*5^(1/2) - 843)^(1/4)*20735i)/(2*(3393*2^(1/2)*(- 377*5^(1/2) - 843)^(1/2
) + 1508*2^(1/2)*5^(1/2)*(- 377*5^(1/2) - 843)^(1/2))))*(- 377*5^(1/2) - 84
3)^(1/4)*1i)/20 - (2^(3/4)*5^(1/2)*atan((2^(3/4)*x*(377*5^(1/2) - 843)^(1/4
)*46371i)/(2*(3393*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)
*(377*5^(1/2) - 843)^(1/2))) - (2^(3/4)*5^(1/2)*x*(377*5^(1/2) - 843)^(1/4)
*20735i)/(2*(3393*2^(1/2)*(377*5^(1/2) - 843)^(1/2) - 1508*2^(1/2)*5^(1/2)*
(377*5^(1/2) - 843)^(1/2))))*(377*5^(1/2) - 843)^(1/4)*1i)/20
```

3.385 $\int \frac{x^m}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=117

$$\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)}-\frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}$$

```
[Out] 2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(3-5^(1/2)))/(1+m)/(
3-5^(1/2))*5^(1/2)-2/5*x^(1+m)*hypergeom([1, 1/4+1/4*m],[5/4+1/4*m],2*x^4/(
3+5^(1/2)))/(1+m)*5^(1/2)/(3+5^(1/2))
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1389, 371}

$$\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(m+1)}-\frac{2x^{m+1}\,{}_2F_1\left(1,\frac{m+1}{4};\frac{m+5}{4};\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m/(1 - 3*x^4 + x^8),x]
```

```
[Out] (2*x^(1 + m)*Hypergeometric2F1[1, (1 + m)/4, (5 + m)/4, (2*x^4)/(3 - Sqrt[5
])])/(Sqrt[5]*(3 - Sqrt[5])*(1 + m)) - (2*x^(1 + m)*Hypergeometric2F1[1, (1
 + m)/4, (5 + m)/4, (2*x^4)/(3 + Sqrt[5])])/(Sqrt[5]*(3 + Sqrt[5])*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^m}{1-3x^4+x^8}\,dx = \frac{\int \frac{x^m}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{x^m}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$
$$= \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{3-\sqrt{5}}\right)}{\sqrt{5}\left(3-\sqrt{5}\right)(1+m)} - \frac{2x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{4};\frac{5+m}{4};\frac{2x^4}{3+\sqrt{5}}\right)}{\sqrt{5}\left(3+\sqrt{5}\right)(1+m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.
time = 0.50, size = 575, normalized size = 4.91

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^m/(1 - 3*x^4 + x^8),x]
```

```
[Out] (x^m*(-RootSum[-1 - #1^2 + #1^4 & , Hypergeometric2F1[-m, -m, 1 - m, -(#1/(
x - #1))]/((x/(x - #1))^m*(-#1 + 2*#1^3)) & ] + (RootSum[-1 - #1^2 + #1^4 &
 , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeometric2F1[-m, -m, 1
- m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hypergeometric2F1[-m, -m,
1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hypergeometric2F1[-m, -m
, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2)/(x/#1)^m)/(-#1 + 2
*#1^3) & ] - (2 + 3*m + m^2)*RootSum[-1 + #1^2 + #1^4 & , Hypergeometric2F1
[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(#1 + 2*#1^3)) & ] - RootSu
m[-1 + #1^2 + #1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hypergeo
metric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hyperg
eometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hype
rgeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1^2
)/(x/#1)^m)/(#1 + 2*#1^3) & ])/(2 + 3*m + m^2)))/(4*m)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^m}{x^8-3x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8-3*x^4+1),x)
```

```
[Out] int(x^m/(x^8-3*x^4+1),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-3*x^4+1),x, algorithm="maxima")
[Out] integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-3*x^4+1),x, algorithm="fricas")
[Out] integral(x^m/(x^8 - 3*x^4 + 1), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(x^4 - x^2 - 1)(x^4 + x^2 - 1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(x**8-3*x**4+1),x)
[Out] Integral(x**m/((x**4 - x**2 - 1)*(x**4 + x**2 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(x^8-3*x^4+1),x, algorithm="giac")
[Out] integrate(x^m/(x^8 - 3*x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m}{x^8 - 3\,x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(x^8 - 3*x^4 + 1),x)
[Out] int(x^m/(x^8 - 3*x^4 + 1), x)
```

3.386 $\int \frac{x^{11}}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=62

$$\frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

```
[Out] 1/4*x^4+1/40*ln(-2*x^4-5^(1/2)+3)*(15-7*5^(1/2))+1/40*ln(-2*x^4+5^(1/2)+3)*
(15+7*5^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 717, 646, 31}

$$\frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(1 - 3*x^4 + x^8),x]
```

```
[Out] x^4/4 + ((15 - 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((15 + 7*Sqrt[5])*
Log[3 + Sqrt[5] - 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{1-3x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{1-3x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{4}\text{Subst}\left(\int \frac{-1+3x}{1-3x+x^2}\,dx,x,x^4\right)\\
&= \frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx,x,x^4\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\text{Subst}\\
&= \frac{x^4}{4}+\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 56, normalized size = 0.90

$$\frac{1}{40}\left(10x^4+\left(15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)+\left(15-7\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(1 - 3*x^4 + x^8),x]
```

```
[Out] (10*x^4 + (15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] + (15 - 7*Sqrt[5])*Log[
-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A]
time = 0.02, size = 38, normalized size = 0.61

method	result	size
default	$\frac{x^4}{4}+\frac{3\ln(x^8-3x^4+1)}{8}-\frac{7\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^4-3)\sqrt{5}}{5}\right)}{20}$	38
risch	$\frac{x^4}{4}+\frac{3\ln\left(2x^4-\sqrt{5}-3\right)}{8}+\frac{7\ln\left(2x^4-\sqrt{5}-3\right)\sqrt{5}}{40}+\frac{3\ln\left(2x^4+\sqrt{5}-3\right)}{8}-\frac{7\ln\left(2x^4+\sqrt{5}-3\right)\sqrt{5}}{40}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4+3/8*ln(x^8-3*x^4+1)-7/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Maxima [A]
time = 0.50, size = 50, normalized size = 0.81

$$\frac{1}{4}\,x^4+\frac{7}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^4-\sqrt{5}-3}{2\,x^4+\sqrt{5}-3}\right)+\frac{3}{8}\,\log\left(x^8-3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 3
/8*log(x^8 - 3*x^4 + 1)
```

Fricas [A]
time = 0.34, size = 62, normalized size = 1.00

$$\frac{1}{4}\,x^4+\frac{7}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^8-6\,x^4-\sqrt{5}\,(2\,x^4-3)+7}{x^8-3\,x^4+1}\right)+\frac{3}{8}\,\log\left(x^8-3\,x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 -
 3*x^4 + 1)) + 3/8*log(x^8 - 3*x^4 + 1)
```

Sympy [A]
time = 0.06, size = 58, normalized size = 0.94

$$\frac{x^4}{4}+\left(\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(\frac{3}{8}-\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8-3*x**4+1),x)
```

```
[Out] x**4/4 + (3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (3/8 - 7*sqrt(5
)/40)*log(x**4 - 3/2 + sqrt(5)/2)
```

Giac [A]
time = 4.03, size = 53, normalized size = 0.85

$$\frac{1}{4}\,x^4+\frac{7}{40}\,\sqrt{5}\,\log\left(\frac{\left|2\,x^4-\sqrt{5}-3\right|}{\left|2\,x^4+\sqrt{5}-3\right|}\right)+\frac{3}{8}\,\log\left(\left|x^8-3\,x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3
)) + 3/8*log(abs(x^8 - 3*x^4 + 1))
```

Mupad [B]
time = 0.12, size = 64, normalized size = 1.03

$$\frac{3\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{3\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{7\sqrt{5}\ \ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}-\frac{7\sqrt{5}\ \ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (3*log(x^4 - 5^(1/2)/2 - 3/2))/8 + (3*log(5^(1/2)/2 + x^4 - 3/2))/8 + (7*5^
(1/2)*log(x^4 - 5^(1/2)/2 - 3/2))/40 - (7*5^(1/2)*log(5^(1/2)/2 + x^4 - 3/2
))/40 + x^4/4
```

3.387 $\int \frac{x^9}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=90

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

```
[Out] 1/2*x^2+1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(1-2/5*5^(1/2))-1/2*arctanh(x^2*
2^(1/2)/(3+5^(1/2))^(1/2))*(1+2/5*5^(1/2))
```

Rubi [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1373, 1136, 1180, 213}

$$\frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] x^2/2 - (Sqrt[(9 + 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2 + (S
qrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{1-3x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{1-3x^2+x^4}\,dx, x, x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{2}\text{Subst}\left(\int \frac{1-3x^2}{1-3x^2+x^4}\,dx, x, x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{20}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)+\frac{1}{20}\left(15+7\sqrt{5}\right)\text{Sub}\\ &= \frac{x^2}{2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9+4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{20}\sqrt{180-80\sqrt{5}}\tanh^{-1}\left(\sqrt{\right.\end{aligned}$$

Mathematica [A]
time = 0.04, size = 103, normalized size = 1.14

$$\frac{1}{20}\left(10x^2+\left(-5+2\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+\left(5+2\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)+\left(5-2\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)-\left(5+2\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(1 - 3*x^4 + x^8),x]
```

```
[Out] (10*x^2 + (-5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[
1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + 2*S
qrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

Maple [A]
time = 0.04, size = 67, normalized size = 0.74

method	result
default	$\frac{x^2}{2}+\frac{\ln(x^4-x^2-1)}{4}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{5}-\frac{\ln(x^4+x^2-1)}{4}-\frac{\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{5}$

risch	$\frac{x^2}{2}+\frac{\ln\left(2x^2-\sqrt{5}-1\right)}{4}+\frac{\ln\left(2x^2-\sqrt{5}-1\right)\sqrt{5}}{10}+\frac{\ln\left(2x^2+\sqrt{5}-1\right)}{4}-\frac{\ln\left(2x^2+\sqrt{5}-1\right)\sqrt{5}}{10}-\frac{\ln\left(2x^2-\sqrt{5}+1\right)}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2+1/4*ln(x^4-x^2-1)-1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/4*ln
(x^4+x^2-1)-1/5*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)
```

Maxima [A]
time = 0.95, size = 92, normalized size = 1.02

$$\frac{1}{2}\,x^2+\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{2\,x^2-\sqrt{5}+1}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{2\,x^2-\sqrt{5}-1}{2\,x^2+\sqrt{5}-1}\right)-\frac{1}{4}\,\log\left(x^4+x^2-1\right)+\frac{1}{4}\,\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1
/10*sqrt(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/4*log(x^4
+ x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(50) = 100$.
time = 0.35, size = 114, normalized size = 1.27

$$\frac{1}{2}\,x^2+\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{2\,x^4+2\,x^2-\sqrt{5}\,(2\,x^2+1)+3}{x^4+x^2-1}\right)+\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{2\,x^4-2\,x^2-\sqrt{5}\,(2\,x^2-1)+3}{x^4-x^2-1}\right)-\frac{1}{4}\,\log\left(x^4+x^2-1\right)+\frac{1}{4}\,\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 +
 x^2 - 1)) + 1/10*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^
4 - x^2 - 1)) - 1/4*log(x^4 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(63) = 126$.
time = 0.25, size = 170, normalized size = 1.89

$$\frac{x^2}{2}+\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47}{8}-\frac{47\sqrt{5}}{20}-120\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3\right)+\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47}{8}-120\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)^3+\frac{47\sqrt{5}}{20}\right)+\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{47\sqrt{5}}{20}-120\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{47}{8}\right)+\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-120\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3+\frac{47\sqrt{5}}{20}+\frac{47}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**8-3*x**4+1),x)
```

```
[Out] x**2/2 + (-1/4 - sqrt(5)/10)*log(x**2 - 47/8 - 47*sqrt(5)/20 - 120*(-1/4 -
sqrt(5)/10)**3) + (-1/4 + sqrt(5)/10)*log(x**2 - 47/8 - 120*(-1/4 + sqrt(5)
/10)**3 + 47*sqrt(5)/20) + (1/4 - sqrt(5)/10)*log(x**2 - 47*sqrt(5)/20 - 12
0*(1/4 - sqrt(5)/10)**3 + 47/8) + (sqrt(5)/10 + 1/4)*log(x**2 - 120*(sqrt(5
)/10 + 1/4)**3 + 47*sqrt(5)/20 + 47/8)
```

Giac [A]
time = 3.33, size = 97, normalized size = 1.08

$$\frac{1}{2}\,x^2+\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{10}\,\sqrt{5}\,\log\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right)-\frac{1}{4}\,\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{4}\,\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1))
+ 1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/4
*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

Mupad [B]
time = 1.33, size = 90, normalized size = 1.00

$$\frac{x^2}{2}-\mathrm{atanh}\left(\frac{64\,x^2}{64\,\sqrt{5}+192}+\frac{64\,\sqrt{5}\,x^2}{64\,\sqrt{5}+192}\right)\,\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right)-\mathrm{atanh}\left(\frac{64\,x^2}{64\,\sqrt{5}-192}-\frac{64\,\sqrt{5}\,x^2}{64\,\sqrt{5}-192}\right)\,\left(\frac{\sqrt{5}}{5}-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^8 - 3*x^4 + 1),x)
```

```
[Out] x^2/2 - atanh((64*x^2)/(64*5^(1/2) + 192) + (64*5^(1/2)*x^2)/(64*5^(1/2) +
192))*(5^(1/2)/5 + 1/2) - atanh((64*x^2)/(64*5^(1/2) - 192) - (64*5^(1/2)*x
^2)/(64*5^(1/2) - 192))*(5^(1/2)/5 - 1/2)
```

3.388 $\int \frac{x^7}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=55

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

```
[Out] 1/40*ln(-2*x^4-5^(1/2)+3)*(5-3*5^(1/2))+1/40*ln(-2*x^4+5^(1/2)+3)*(5+3*5^(1
/2))
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1371, 646, 31}

$$\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)+\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((5 - 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 + ((5 + 3*Sqrt[5])*Log[3 + Sq
rt[5] - 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{1-3x^4+x^8}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{x}{1-3x+x^2}\,dx, x, x^4\right)$$
$$= \frac{1}{40}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right) + \frac{1}{40}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{}{-\frac{3}{2}}\right.$$
$$= \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right) + \frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

Mathematica [A]
time = 0.01, size = 53, normalized size = 0.96

$$\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right) + \frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((5 - 3*Sqrt[5])*Log[-3 + S
qrt[5] + 2*x^4])/40
```

Maple [A]
time = 0.02, size = 33, normalized size = 0.60

method	result	size
default	$\frac{\ln(x^8-3x^4+1)}{8}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^4-3)\sqrt{5}}{5}\right)}{20}$	33
risch	$\frac{\ln\left(2x^4-\sqrt{5}-3\right)}{8}+\frac{3\ln\left(2x^4-\sqrt{5}-3\right)\sqrt{5}}{40}+\frac{\ln\left(2x^4+\sqrt{5}-3\right)}{8}-\frac{3\ln\left(2x^4+\sqrt{5}-3\right)\sqrt{5}}{40}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(x^8-3*x^4+1)-3/20*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Maxima [A]
time = 0.51, size = 45, normalized size = 0.82

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)+\frac{1}{8}\log\left(x^8-3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) + 1/8*log(x^8
 - 3*x^4 + 1)
```

Fricas [A]
time = 0.38, size = 57, normalized size = 1.04

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}(2x^4-3)+7}{x^8-3x^4+1}\right)+\frac{1}{8}\log\left(x^8-3x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
)) + 1/8*log(x^8 - 3*x^4 + 1)
```

Sympy [A]
time = 0.05, size = 53, normalized size = 0.96

$$\left(\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(\frac{1}{8}-\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**8-3*x**4+1),x)
```

```
[Out] (1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (1/8 - 3*sqrt(5)/40)*log
(x**4 - 3/2 + sqrt(5)/2)
```

Giac [A]
time = 3.88, size = 48, normalized size = 0.87

$$\frac{3}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)+\frac{1}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/8*l
og(abs(x^8 - 3*x^4 + 1))
```

Mupad [B]
time = 0.10, size = 59, normalized size = 1.07

$$\frac{\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{8}+\frac{3\sqrt{5}\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}-\frac{3\sqrt{5}\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^8 - 3*x^4 + 1),x)
```

```
[Out] log(x^4 - 5^(1/2)/2 - 3/2)/8 + log(5^(1/2)/2 + x^4 - 3/2)/8 + (3*5^(1/2)*lo
g(x^4 - 5^(1/2)/2 - 3/2))/40 - (3*5^(1/2)*log(5^(1/2)/2 + x^4 - 3/2))/40
```

3.389 $\int \frac{x^5}{1-3x^4+x^8} dx$

Optimal. Leaf size=81

$$-\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

```
[Out] 1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(1/2-1/10*5^(1/2))-1/2*arctanh(x^2*2^(1/
2)/(3+5^(1/2))^(1/2))*(1/2+1/10*5^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1373, 1144, 213}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)-\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 - 3*x^4 + x^8),x]
```

```
[Out] -1/2*(Sqrt[(3 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]) + (Sqrt[(3
 - Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^5}{1-3x^4+x^8}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-3x^2+x^4}\,dx, x, x^2\right)$$
$$= \frac{1}{20}\left(5-3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right) + \frac{1}{20}\left(5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{}{-}\right.$$
$$= -\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right) + \frac{1}{2}\sqrt{\frac{1}{10}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}}\right.$$

Mathematica [A]
time = 0.02, size = 91, normalized size = 1.12

$$\frac{1}{40}\left(\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)-\left(-5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)-\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 - 3*x^4 + x^8),x]
[Out] ((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 + Sqrt[5])*Log[1 + Sqrt[5] -
 2*x^2] - (-5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] - (5 + Sqrt[5])*Log[1 +
Sqrt[5] + 2*x^2])/40
```

Maple [A]
time = 0.03, size = 62, normalized size = 0.77

method	result
default	$\frac{\ln(x^4-x^2-1)}{8}-\frac{\sqrt{5}\,\text{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}-\frac{\ln(x^4+x^2-1)}{8}-\frac{\text{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{20}$
risch	$\frac{\ln\left(2x^2-\sqrt{5}-1\right)}{8}+\frac{\ln\left(2x^2-\sqrt{5}-1\right)\sqrt{5}}{40}+\frac{\ln\left(2x^2+\sqrt{5}-1\right)}{8}-\frac{\ln\left(2x^2+\sqrt{5}-1\right)\sqrt{5}}{40}-\frac{\ln\left(2x^2-\sqrt{5}+1\right)}{8}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
[Out] 1/8*ln(x^4-x^2-1)-1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/8*ln(x^4+x^
2-1)-1/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(41) = 82.
time = 0.51, size = 87, normalized size = 1.07

$$\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqrt(5
)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 - 1)
 + 1/8*log(x^4 - x^2 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(41) = 82$.
time = 0.39, size = 109, normalized size = 1.35

$$\frac{1}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^4+2\,x^2-\sqrt{5}\,(2\,x^2+1)+3}{x^4+x^2-1}\right)+\frac{1}{40}\,\sqrt{5}\,\log\left(\frac{2\,x^4-2\,x^2-\sqrt{5}\,(2\,x^2-1)+3}{x^4-x^2-1}\right)-\frac{1}{8}\,\log\left(x^4+x^2-1\right)+\frac{1}{8}\,\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*log((2*x^4 + 2*x^2 - sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1))
 + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 - sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 -
1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(58) = 116$.
time = 0.17, size = 165, normalized size = 2.04

$$\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-\frac{3\sqrt{5}}{10}-640\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3\right)+\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3}{2}-640\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{3\sqrt{5}}{10}\right)+\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{3\sqrt{5}}{10}-640\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{3}{2}\right)+\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-640\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3+\frac{3\sqrt{5}}{10}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**8-3*x**4+1),x)
```

```
[Out] (-1/8 - sqrt(5)/40)*log(x**2 - 3/2 - 3*sqrt(5)/10 - 640*(-1/8 - sqrt(5)/40)
**3) + (-1/8 + sqrt(5)/40)*log(x**2 - 3/2 - 640*(-1/8 + sqrt(5)/40)**3 + 3*
sqrt(5)/10) + (1/8 - sqrt(5)/40)*log(x**2 - 3*sqrt(5)/10 - 640*(1/8 - sqrt(
5)/40)**3 + 3/2) + (sqrt(5)/40 + 1/8)*log(x**2 - 640*(sqrt(5)/40 + 1/8)**3
+ 3*sqrt(5)/10 + 3/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(41) = 82$.
time = 3.59, size = 92, normalized size = 1.14

$$\frac{1}{40}\,\sqrt{5}\,\log\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right)+\frac{1}{40}\,\sqrt{5}\,\log\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right)-\frac{1}{8}\,\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{8}\,\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) + 1/40*sqr
t(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs(x
^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

Mupad [B]
time = 1.38, size = 77, normalized size = 0.95

$$-\operatorname{atanh}\left(\frac{4\,x^2}{\sqrt{5}-3}-\frac{2\,\sqrt{5}\,x^2}{\sqrt{5}-3}\right)\,\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)-\operatorname{atanh}\left(\frac{4\,x^2}{\sqrt{5}+3}+\frac{2\,\sqrt{5}\,x^2}{\sqrt{5}+3}\right)\,\left(\frac{\sqrt{5}}{20}-\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^8 - 3*x^4 + 1),x)
```

```
[Out] - atanh((4*x^2)/(5^(1/2) - 3) - (2*5^(1/2)*x^2)/(5^(1/2) - 3))*(5^(1/2)/20
+ 1/4) - atanh((4*x^2)/(5^(1/2) + 3) + (2*5^(1/2)*x^2)/(5^(1/2) + 3))*(5^(1
/2)/20 - 1/4)
```

3.390 $\int \frac{x^3}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] 1/10*arctanh(1/5*(-2*x^4+3)*5^(1/2))*5^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1366, 632, 212}

$$\frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTanh[(3 - 2*x^4)/Sqrt[5]]/(2*Sqrt[5])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^3}{1-3x^4+x^8}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-3x+x^2}\,dx, x, x^4\right)$$
$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, -3+2x^4\right)\right)$$
$$= \frac{\tanh^{-1}\left(\frac{3-2x^4}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.65

$$\frac{\log\left(3+\sqrt{5}-2x^4\right)-\log\left(-3+\sqrt{5}+2x^4\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - 3*x^4 + x^8),x]
[Out] (Log[3 + Sqrt[5] - 2*x^4] - Log[-3 + Sqrt[5] + 2*x^4])/(4*Sqrt[5])
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.83

method	result	size
default	$-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^4-3)\sqrt{5}}{5}\right)}{10}$	19
risch	$\frac{\ln\left(2x^4-\sqrt{5}-3\right)\sqrt{5}}{20}-\frac{\ln\left(2x^4+\sqrt{5}-3\right)\sqrt{5}}{20}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/10*5^(1/2)*arctanh(1/5*(2*x^4-3)*5^(1/2))
```

Maxima [A]
time = 0.61, size = 31, normalized size = 1.35

$$\frac{1}{20}\,\sqrt{5}\,\log\left(\frac{2\,x^4-\sqrt{5}-3}{2\,x^4+\sqrt{5}-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(18) = 36$.
time = 0.34, size = 43, normalized size = 1.87

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^8 - 6x^4 - \sqrt{5}(2x^4 - 3) + 7}{x^8 - 3x^4 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
))
```

Sympy [A]
time = 0.04, size = 42, normalized size = 1.83

$$\frac{\sqrt{5}\log\left(x^4 - \frac{3}{2} - \frac{\sqrt{5}}{2}\right)}{20} - \frac{\sqrt{5}\log\left(x^4 - \frac{3}{2} + \frac{\sqrt{5}}{2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8-3*x**4+1),x)
```

```
[Out] sqrt(5)*log(x**4 - 3/2 - sqrt(5)/2)/20 - sqrt(5)*log(x**4 - 3/2 + sqrt(5)/2
)/20
```

Giac [A]
time = 3.55, size = 33, normalized size = 1.43

$$\frac{1}{20}\sqrt{5}\log\left(\frac{\left|2x^4 - \sqrt{5} - 3\right|}{\left|2x^4 + \sqrt{5} - 3\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3))
```

Mupad [B]
time = 1.57, size = 30, normalized size = 1.30

$$\frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{3\sqrt{5} - 8\sqrt{5}\,x^4}{18\,x^4 - 7}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8 - 3*x^4 + 1),x)
[Out] (5^(1/2)*atanh((3*5^(1/2) - 8*5^(1/2)*x^4)/(18*x^4 - 7)))/10
```

3.391 $\int \frac{x}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=75

$$-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}+\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)$$

```
[Out] 1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(1/2+1/10*5^(1/2))-arctanh(x^2*2^(1/2)/(
3+5^(1/2))^(1/2))/(5+5^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1373, 1107, 213}

$$\frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2]/Sqrt[10*(3 + Sqrt[5])]) + (Sqrt[(3 + S
qrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
```

```
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1-3x^4+x^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-3x^2+x^4}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)}{2\sqrt{5}}\\
&= -\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)}{\sqrt{10\left(3+\sqrt{5}\right)}} + \frac{1}{2}\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 91, normalized size = 1.21

$$\frac{1}{40}\left(-\left(\left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)\right)-\left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)+\left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)+\left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - 3*x^4 + x^8),x]
```

```
[Out] (-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2]) - (-5 + Sqrt[5])*Log[1 + Sqrt[5
] - 2*x^2] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + Sqrt[5])*Log[1
 + Sqrt[5] + 2*x^2])/40
```

Maple [A]
time = 0.02, size = 62, normalized size = 0.83

method	result
default	$\frac{\ln(x^4-x^2-1)}{8}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}-\frac{\ln(x^4+x^2-1)}{8}+\frac{\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{20}$
risch	$-\frac{\ln\left(2x^2+\sqrt{5}+1\right)}{8}+\frac{\ln\left(2x^2+\sqrt{5}+1\right)\sqrt{5}}{40}-\frac{\ln\left(2x^2-\sqrt{5}+1\right)}{8}-\frac{\ln\left(2x^2-\sqrt{5}+1\right)\sqrt{5}}{40}+\frac{\ln\left(2x^2+\sqrt{5}-1\right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*ln(x^4-x^2-1)+1/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/8*ln(x^4+x^
2-1)+1/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(43) = 86$.
time = 0.50, size = 87, normalized size = 1.16

$$-\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{1}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*sqrt(
5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/8*log(x^4 + x^2 - 1
) + 1/8*log(x^4 - x^2 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(43) = 86$.
time = 0.34, size = 107, normalized size = 1.43

$$\frac{1}{40}\sqrt{5}\log\left(\frac{2x^4+2x^2+\sqrt{5}\,(2x^2+1)+3}{x^4+x^2-1}\right)+\frac{1}{40}\sqrt{5}\log\left(\frac{2x^4-2x^2+\sqrt{5}\,(2x^2-1)+3}{x^4-x^2-1}\right)-\frac{1}{8}\log\left(x^4+x^2-1\right)+\frac{1}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*sqrt(5)*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^2 - 1))
 + 1/40*sqrt(5)*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4 - x^2 -
1)) - 1/8*log(x^4 + x^2 - 1) + 1/8*log(x^4 - x^2 - 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(53) = 106$.
time = 0.18, size = 165, normalized size = 2.20

$$\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)\log\left(x^2-\frac{7}{2}-\frac{7\sqrt{5}}{10}+960\left(\frac{\sqrt{5}}{40}+\frac{1}{8}\right)^3\right)+\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7}{2}+960\left(\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}\right)+\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)\log\left(x^2-\frac{7\sqrt{5}}{10}+960\left(-\frac{1}{8}+\frac{\sqrt{5}}{40}\right)^3+\frac{7}{2}\right)+\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)\log\left(x^2+960\left(-\frac{1}{8}-\frac{\sqrt{5}}{40}\right)^3+\frac{7\sqrt{5}}{10}+\frac{7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**8-3*x**4+1),x)
```

```
[Out] (sqrt(5)/40 + 1/8)*log(x**2 - 7/2 - 7*sqrt(5)/10 + 960*(sqrt(5)/40 + 1/8)**
3) + (1/8 - sqrt(5)/40)*log(x**2 - 7/2 + 960*(1/8 - sqrt(5)/40)**3 + 7*sqrt
(5)/10) + (-1/8 + sqrt(5)/40)*log(x**2 - 7*sqrt(5)/10 + 960*(-1/8 + sqrt(5)
/40)**3 + 7/2) + (-1/8 - sqrt(5)/40)*log(x**2 + 960*(-1/8 - sqrt(5)/40)**3
+ 7*sqrt(5)/10 + 7/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(43) = 86$.
time = 3.13, size = 92, normalized size = 1.23

$$-\frac{1}{40}\,\sqrt{5}\,\log\left(\frac{\left|2\,x^2-\sqrt{5}+1\right|}{2\,x^2+\sqrt{5}+1}\right)-\frac{1}{40}\,\sqrt{5}\,\log\left(\frac{\left|2\,x^2-\sqrt{5}-1\right|}{\left|2\,x^2+\sqrt{5}-1\right|}\right)-\frac{1}{8}\,\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{8}\,\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/40*sq
rt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/8*log(abs(
x^4 + x^2 - 1)) + 1/8*log(abs(x^4 - x^2 - 1))
```

Mupad [B]
time = 1.30, size = 83, normalized size = 1.11

$$\mathrm{atanh}\left(\frac{29\,x^2}{8\,\sqrt{5}-18}-\frac{13\,\sqrt{5}\,x^2}{8\,\sqrt{5}-18}\right)\,\left(\frac{\sqrt{5}}{20}-\frac{1}{4}\right)+\mathrm{atanh}\left(\frac{29\,x^2}{8\,\sqrt{5}+18}+\frac{13\,\sqrt{5}\,x^2}{8\,\sqrt{5}+18}\right)\,\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^8 - 3*x^4 + 1),x)
```

```
[Out] atanh((29*x^2)/(8*5^(1/2) - 18) - (13*5^(1/2)*x^2)/(8*5^(1/2) - 18))*(5^(1/
2)/20 - 1/4) + atanh((29*x^2)/(8*5^(1/2) + 18) + (13*5^(1/2)*x^2)/(8*5^(1/2
) + 18))*(5^(1/2)/20 + 1/4)
```

3.392 $\int \frac{1}{x(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=57

$$\log(x)-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

```
[Out] ln(x)-1/40*ln(-2*x^4+5^(1/2)+3)*(5-3*5^(1/2))-1/40*ln(-2*x^4-5^(1/2)+3)*(5+
3*5^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1371, 719, 29, 646, 31}

$$-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] - ((5 + 3*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((5 - 3*Sqrt[5])*L
og[3 + Sqrt[5] - 2*x^4])/40
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
```

```
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1-3x^4+x^8)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(1-3x+x^2)}\,dx, x, x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^4\right)+\frac{1}{4}\text{Subst}\left(\int \frac{3-x}{1-3x+x^2}\,dx, x, x^4\right)\\ &= \log(x)+\frac{1}{40}\left(-5+3\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)-\frac{1}{40}\left(5+3\sqrt{5}\right)\\ &= \log(x)-\frac{1}{40}\left(5+3\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)-\frac{1}{40}\left(5-3\sqrt{5}\right)\log\left(3+\sqrt{5}-2\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.96

$$\log(x)+\frac{1}{40}\left(-5+3\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)+\frac{1}{40}\left(-5-3\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - 3*x^4 + x^8)),x]
```

```
[Out] Log[x] + ((-5 + 3*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40 + ((-5 - 3*Sqrt[5])
*Log[-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A]
time = 0.03, size = 64, normalized size = 1.12

method	result
default	$-\frac{\ln(x^4+x^2-1)}{8}+\frac{3\,\text{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{20}-\frac{\ln(x^4-x^2-1)}{8}-\frac{3\sqrt{5}\,\text{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}+\ln(x)$

risch	$\ln(x)-\frac{\ln\left(3x^4-\frac{9}{2}-\frac{3\sqrt{5}}{2}\right)}{8}+\frac{3\ln\left(3x^4-\frac{9}{2}-\frac{3\sqrt{5}}{2}\right)\sqrt{5}}{40}-\frac{\ln\left(3x^4-\frac{9}{2}+\frac{3\sqrt{5}}{2}\right)}{8}-\frac{3\ln\left(3x^4-\frac{9}{2}+\frac{3\sqrt{5}}{2}\right)\sqrt{5}}{40}$	7(

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*ln(x^4+x^2-1)+3/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)-1/8*ln(x^4-x
^2-1)-3/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))+ln(x)
```

Maxima [A]
time = 0.52, size = 51, normalized size = 0.89

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)-\frac{1}{8}\log\left(x^8-3x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/8*log(x^8
 - 3*x^4 + 1) + 1/4*log(x^4)
```

Fricas [A]
time = 0.35, size = 59, normalized size = 1.04

$$\frac{3}{40}\sqrt{5}\log\left(\frac{2x^8-6x^4-\sqrt{5}\left(2x^4-3\right)+7}{x^8-3x^4+1}\right)-\frac{1}{8}\log\left(x^8-3x^4+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 3/40*sqrt(5)*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*x^4 + 1
)) - 1/8*log(x^8 - 3*x^4 + 1) + log(x)
```

Sympy [A]
time = 0.06, size = 58, normalized size = 1.02

$$\log\left(x\right)+\left(-\frac{1}{8}+\frac{3\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(-\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**8-3*x**4+1),x)
```

```
[Out] log(x) + (-1/8 + 3*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-3*sqrt(5)/40
 - 1/8)*log(x**4 - 3/2 + sqrt(5)/2)
```

Giac [A]
time = 2.99, size = 54, normalized size = 0.95

$$\frac{3}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)+\frac{1}{4}\log\left(x^4\right)-\frac{1}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) + 1/4*l
og(x^4) - 1/8*log(abs(x^8 - 3*x^4 + 1))
```

Mupad [B]
time = 0.43, size = 42, normalized size = 0.74

$$\ln(x)+\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}-\frac{1}{8}\right)-\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{3\sqrt{5}}{40}+\frac{1}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] log(x) + log(x^4 - 5^(1/2)/2 - 3/2)*((3*5^(1/2))/40 - 1/8) - log(5^(1/2)/2
+ x^4 - 3/2)*((3*5^(1/2))/40 + 1/8)
```

3.393 $\int \frac{1}{x^3(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{4\sqrt{10}}$$

```
[Out] -1/2/x^2+1/40*arctanh(x^2*(1/2+1/2*5^(1/2)))*(3+5^(1/2))^(3/2)*10^(1/2)-1/2
*arctanh(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(1-2/5*5^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1373, 1137, 1180, 213}

$$-\frac{1}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{5}\left(9-4\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x^2\right)}{4\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 - (Sqrt[(9 - 4*Sqrt[5])/5]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x^2])/2
 + ((3 + Sqrt[5])^(3/2)*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x^2])/(4*Sqrt[10])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(1-3x^4+x^8\right)}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{x^2\left(1-3x^2+x^4\right)}\,dx, x, x^2\right)\\ &= -\frac{1}{2x^2}+\frac{1}{2}\mathrm{Subst}\left(\int \frac{3-x^2}{1-3x^2+x^4}\,dx, x, x^2\right)\\ &= -\frac{1}{2x^2}+\frac{1}{20}\left(-5+3\sqrt{5}\right)\mathrm{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx, x, x^2\right)-\frac{1}{20}\left(5+3\sqrt{5}\right.\\ &= -\frac{1}{2x^2}-\frac{1}{10}\sqrt{45-20\sqrt{5}}\,\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{\left(3+\sqrt{5}\right)^{3/2}\tanh^{-1}\left(\right.}{4\sqrt{10}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 103, normalized size = 1.16

$$\frac{1}{20}\left(-\frac{10}{x^2}-\left(5+2\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+\left(5-2\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)+\left(5+2\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)+\left(-5+2\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^2 - (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + (5 - 2*Sqrt[5])*Log[
1 + Sqrt[5] - 2*x^2] + (5 + 2*Sqrt[5])*Log[-1 + Sqrt[5] + 2*x^2] + (-5 + 2*
Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/20
```

Maple [A]
time = 0.03, size = 67, normalized size = 0.75

method	result

default	$-\frac{\ln(x^4+x^2-1)}{4}+\frac{\operatorname{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{5}+\frac{\ln(x^4-x^2-1)}{4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{5}-\frac{1}{2x^2}$
risch	$-\frac{1}{2x^2}+\frac{\ln\left(4x^2-2+2\sqrt{5}\right)}{4}+\frac{\ln\left(4x^2-2+2\sqrt{5}\right)\sqrt{5}}{10}+\frac{\ln\left(4x^2-2-2\sqrt{5}\right)}{4}-\frac{\ln\left(4x^2-2-2\sqrt{5}\right)\sqrt{5}}{10}-\frac{\ln\left(4x^2+2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*ln(x^4+x^2-1)+1/5*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)+1/4*ln(x^4-x^
2-1)+1/5*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/2/x^2
```

Maxima [A]
time = 0.51, size = 92, normalized size = 1.03

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{1}{2x^2}-\frac{1}{4}\log\left(x^4+x^2-1\right)+\frac{1}{4}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sqrt(
5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/4*log(x^4
 + x^2 - 1) + 1/4*log(x^4 - x^2 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(53) = 106.
time = 0.39, size = 125, normalized size = 1.40

$$\frac{2\sqrt{5}\,x^2\log\left(\frac{2x^4+2x^2+\sqrt{5}\,(2x^2+1)+3}{x^4+x^2-1}\right)+2\sqrt{5}\,x^2\log\left(\frac{2x^4-2x^2+\sqrt{5}\,(2x^2-1)+3}{x^4-x^2-1}\right)-5x^2\log\left(x^4+x^2-1\right)+5x^2\log\left(x^4-x^2-1\right)-10}{20x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/20*(2*sqrt(5)*x^2*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 + x^
2 - 1)) + 2*sqrt(5)*x^2*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x^4
- x^2 - 1)) - 5*x^2*log(x^4 + x^2 - 1) + 5*x^2*log(x^4 - x^2 - 1) - 10)/x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(70) = 140.
time = 0.19, size = 172, normalized size = 1.93

$$\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)\log\left(x^2-\frac{123}{8}-\frac{123\sqrt{5}}{20}+280\left(\frac{\sqrt{5}}{10}+\frac{1}{4}\right)^3\right)+\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123}{8}+280\left(\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}\right)+\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)\log\left(x^2-\frac{123\sqrt{5}}{20}+280\left(-\frac{1}{4}+\frac{\sqrt{5}}{10}\right)^3+\frac{123}{8}\right)+\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)\log\left(x^2+280\left(-\frac{1}{4}-\frac{\sqrt{5}}{10}\right)^3+\frac{123\sqrt{5}}{20}+\frac{123}{8}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**8-3*x**4+1),x)
```

```
[Out] (sqrt(5)/10 + 1/4)*log(x**2 - 123/8 - 123*sqrt(5)/20 + 280*(sqrt(5)/10 + 1/
4)**3) + (1/4 - sqrt(5)/10)*log(x**2 - 123/8 + 280*(1/4 - sqrt(5)/10)**3 +
123*sqrt(5)/20) + (-1/4 + sqrt(5)/10)*log(x**2 - 123*sqrt(5)/20 + 280*(-1/4
 + sqrt(5)/10)**3 + 123/8) + (-1/4 - sqrt(5)/10)*log(x**2 + 280*(-1/4 - sqr
t(5)/10)**3 + 123*sqrt(5)/20 + 123/8) - 1/(2*x**2)
```

Giac [A]
time = 3.34, size = 97, normalized size = 1.09

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{1}{2x^2}-\frac{1}{4}\log\left(\left|x^4+x^2-1\right|\right)+\frac{1}{4}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 1/10*sq
rt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/2/x^2 - 1/
4*log(abs(x^4 + x^2 - 1)) + 1/4*log(abs(x^4 - x^2 - 1))
```

Mupad [B]
time = 0.06, size = 88, normalized size = 0.99

$$\operatorname{atanh}\left(\frac{12736x^2}{3520\sqrt{5}-7872}-\frac{5696\sqrt{5}x^2}{3520\sqrt{5}-7872}\right)\left(\frac{\sqrt{5}}{5}-\frac{1}{2}\right)+\operatorname{atanh}\left(\frac{12736x^2}{3520\sqrt{5}+7872}+\frac{5696\sqrt{5}x^2}{3520\sqrt{5}+7872}\right)\left(\frac{\sqrt{5}}{5}+\frac{1}{2}\right)-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] atanh((12736*x^2)/(3520*5^(1/2) - 7872) - (5696*5^(1/2)*x^2)/(3520*5^(1/2)
- 7872))*(5^(1/2)/5 - 1/2) + atanh((12736*x^2)/(3520*5^(1/2) + 7872) + (569
6*5^(1/2)*x^2)/(3520*5^(1/2) + 7872))*(5^(1/2)/5 + 1/2) - 1/(2*x^2)
```

3.394 $\int \frac{1}{x^5(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=66

$$-\frac{1}{4x^4}+3\log(x)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)$$

```
[Out] -1/4/x^4+3*ln(x)-1/40*ln(-2*x^4+5^(1/2)+3)*(15-7*5^(1/2))-1/40*ln(-2*x^4-5^
(1/2)+3)*(15+7*5^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1371, 723, 814, 646, 31}

$$-\frac{1}{4x^4}-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(-2x^4-\sqrt{5}+3\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log\left(-2x^4+\sqrt{5}+3\right)+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/4*1/x^4 + 3*Log[x] - ((15 + 7*Sqrt[5])*Log[3 - Sqrt[5] - 2*x^4])/40 - ((
15 - 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4])/40
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5(1-3x^4+x^8)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(1-3x+x^2)}\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \frac{3-x}{x(1-3x+x^2)}\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+\frac{1}{4}\text{Subst}\left(\int \left(\frac{3}{x}+\frac{8-3x}{1-3x+x^2}\right)dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+3\log(x)+\frac{1}{4}\text{Subst}\left(\int \frac{8-3x}{1-3x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{4x^4}+3\log(x)+\frac{1}{40}\left(-15+7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, x^4\right)-\frac{1}{40}\\
&= -\frac{1}{4x^4}+3\log(x)-\frac{1}{40}\left(15+7\sqrt{5}\right)\log\left(3-\sqrt{5}-2x^4\right)-\frac{1}{40}\left(15-7\sqrt{5}\right)\log
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 0.92

$$\frac{1}{40}\left(-\frac{10}{x^4}+120\log(x)+\left(-15+7\sqrt{5}\right)\log\left(3+\sqrt{5}-2x^4\right)-\left(15+7\sqrt{5}\right)\log\left(-3+\sqrt{5}+2x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-10/x^4 + 120*Log[x] + (-15 + 7*Sqrt[5])*Log[3 + Sqrt[5] - 2*x^4] - (15 +
7*Sqrt[5])*Log[-3 + Sqrt[5] + 2*x^4])/40
```

Maple [A]
time = 0.05, size = 71, normalized size = 1.08

method	result
default	$-\frac{3\ln(x^4+x^2-1)}{8}+\frac{7\,\mathrm{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{20}-\frac{3\ln(x^4-x^2-1)}{8}-\frac{7\sqrt{5}\,\mathrm{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}-\frac{1}{4x^4}+3\ln($
risch	$-\frac{1}{4x^4}+3\ln(x)-\frac{3\ln\left(7x^4-\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)}{8}+\frac{7\ln\left(7x^4-\frac{21}{2}-\frac{7\sqrt{5}}{2}\right)\sqrt{5}}{40}-\frac{3\ln\left(7x^4-\frac{21}{2}+\frac{7\sqrt{5}}{2}\right)}{8}-\frac{7\ln\left(7x^4-\frac{21}{2}+\right.}{40}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*ln(x^4+x^2-1)+7/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)-3/8*ln(x^4-x
^2-1)-7/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/4/x^4+3*ln(x)
```

Maxima [A]
time = 0.51, size = 56, normalized size = 0.85

$$\frac{7}{40}\sqrt{5}\log\left(\frac{2x^4-\sqrt{5}-3}{2x^4+\sqrt{5}-3}\right)-\frac{1}{4x^4}-\frac{3}{8}\log\left(x^8-3x^4+1\right)+\frac{3}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] 7/40*sqrt(5)*log((2*x^4 - sqrt(5) - 3)/(2*x^4 + sqrt(5) - 3)) - 1/4/x^4 - 3
/8*log(x^8 - 3*x^4 + 1) + 3/4*log(x^4)
```

Fricas [A]
time = 0.38, size = 76, normalized size = 1.15

$$\frac{7\sqrt{5}\,x^4\log\left(\frac{2x^8-6x^4-\sqrt{5}(2x^4-3)+7}{x^8-3x^4+1}\right)-15x^4\log\left(x^8-3x^4+1\right)+120x^4\log(x)-10}{40x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/40*(7*sqrt(5)*x^4*log((2*x^8 - 6*x^4 - sqrt(5)*(2*x^4 - 3) + 7)/(x^8 - 3*
x^4 + 1)) - 15*x^4*log(x^8 - 3*x^4 + 1) + 120*x^4*log(x) - 10)/x^4
```

Sympy [A]
time = 0.08, size = 66, normalized size = 1.00

$$3\log(x)+\left(-\frac{3}{8}+\frac{7\sqrt{5}}{40}\right)\log\left(x^4-\frac{3}{2}-\frac{\sqrt{5}}{2}\right)+\left(-\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)\log\left(x^4-\frac{3}{2}+\frac{\sqrt{5}}{2}\right)-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**8-3*x**4+1),x)
```

```
[Out] 3*log(x) + (-3/8 + 7*sqrt(5)/40)*log(x**4 - 3/2 - sqrt(5)/2) + (-7*sqrt(5)/
40 - 3/8)*log(x**4 - 3/2 + sqrt(5)/2) - 1/(4*x**4)
```

Giac [A]
time = 4.00, size = 66, normalized size = 1.00

$$\frac{7}{40}\sqrt{5}\log\left(\frac{\left|2x^4-\sqrt{5}-3\right|}{\left|2x^4+\sqrt{5}-3\right|}\right)-\frac{3x^4+1}{4x^4}+\frac{3}{4}\log\left(x^4\right)-\frac{3}{8}\log\left(\left|x^8-3x^4+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 7/40*sqrt(5)*log(abs(2*x^4 - sqrt(5) - 3)/abs(2*x^4 + sqrt(5) - 3)) - 1/4*(
3*x^4 + 1)/x^4 + 3/4*log(x^4) - 3/8*log(abs(x^8 - 3*x^4 + 1))
```

Mupad [B]
time = 1.35, size = 49, normalized size = 0.74

$$3\ln(x)-\frac{1}{4x^4}+\ln\left(x^4-\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}-\frac{3}{8}\right)-\ln\left(x^4+\frac{\sqrt{5}}{2}-\frac{3}{2}\right)\left(\frac{7\sqrt{5}}{40}+\frac{3}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] 3*log(x) - 1/(4*x^4) + log(x^4 - 5^(1/2)/2 - 3/2)*((7*5^(1/2))/40 - 3/8) -
log(5^(1/2)/2 + x^4 - 3/2)*((7*5^(1/2))/40 + 3/8)
```

3.395 $\int \frac{1}{x^7(1-3x^4+x^8)} \, dx$

Optimal. Leaf size=97

$$-\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\right.}\right.$$

```
[Out] -1/6/x^6-3/2/x^2-1/2*arctanh(x^2*2^(1/2)/(3+5^(1/2))^(1/2))*(5/2-11/10*5^(1
/2))+1/2*arctanh(x^2*(1/2+1/2*5^(1/2)))*(5/2+11/10*5^(1/2))
```

Rubi [A]
time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1373, 1137, 1295, 1180, 213}

$$-\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x^2\right)+\frac{1}{2}\sqrt{\frac{1}{10}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/6*1/x^6 - 3/(2*x^2) - (Sqrt[(123 - 55*Sqrt[5])/10]*ArcTanh[Sqrt[2/(3 + S
qrt[5])]*x^2])/2 + (Sqrt[(123 + 55*Sqrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/
2]*x^2])/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1373

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*
x^(n/k) + c*x^(2*(n/k)))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, p
}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(1-3x^4+x^8\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(1-3x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}+\frac{1}{6}\text{Subst}\left(\int \frac{9-3x^2}{x^2\left(1-3x^2+x^4\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{6}\text{Subst}\left(\int \frac{-24+9x^2}{1-3x^2+x^4}\,dx,x,x^2\right)\\
&= -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{20}\left(15-7\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx,x,x^2\right)-\frac{1}{20}\left(15\right.\\
&= -\frac{1}{6x^6}-\frac{3}{2x^2}-\frac{1}{2}\sqrt{\frac{1}{10}\left(123-55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x^2\right)+\frac{1}{20}\sqrt{1230+}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 111, normalized size = 1.14

$$\frac{1}{120}\left(-\frac{20}{x^6}-\frac{180}{x^2}-3\left(25+11\sqrt{5}\right)\log\left(-1+\sqrt{5}-2x^2\right)+3\left(25-11\sqrt{5}\right)\log\left(1+\sqrt{5}-2x^2\right)+3\left(25+11\sqrt{5}\right)\log\left(-1+\sqrt{5}+2x^2\right)+3\left(-25+11\sqrt{5}\right)\log\left(1+\sqrt{5}+2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 - 3*x^4 + x^8)),x]
```

```
[Out] (-20/x^6 - 180/x^2 - 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[5] - 2*x^2] + 3*(25
- 11*Sqrt[5])*Log[1 + Sqrt[5] - 2*x^2] + 3*(25 + 11*Sqrt[5])*Log[-1 + Sqrt[
5] + 2*x^2] + 3*(-25 + 11*Sqrt[5])*Log[1 + Sqrt[5] + 2*x^2])/120
```

Maple [A]
time = 0.04, size = 72, normalized size = 0.74

method	result
default	$-\frac{5\ln(x^4+x^2-1)}{8}+\frac{11\,\mathrm{arctanh}\left(\frac{(2x^2+1)\sqrt{5}}{5}\right)\sqrt{5}}{20}+\frac{5\ln(x^4-x^2-1)}{8}+\frac{11\sqrt{5}\,\mathrm{arctanh}\left(\frac{(2x^2-1)\sqrt{5}}{5}\right)}{20}-\frac{1}{6x^6}-\frac{3}{2x^2}$
risch	$\frac{-\frac{3x^4}{2}-\frac{1}{6}}{x^6}+\frac{5\ln\left(11x^2-\frac{11}{2}+\frac{11\sqrt{5}}{2}\right)}{8}+\frac{11\ln\left(11x^2-\frac{11}{2}+\frac{11\sqrt{5}}{2}\right)\sqrt{5}}{40}+\frac{5\ln\left(11x^2-\frac{11}{2}-\frac{11\sqrt{5}}{2}\right)}{8}-\frac{11\ln\left(11x^2-\frac{11}{2}-\frac{1}{}\right.}{40}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -5/8*ln(x^4+x^2-1)+11/20*arctanh(1/5*(2*x^2+1)*5^(1/2))*5^(1/2)+5/8*ln(x^4-
x^2-1)+11/20*5^(1/2)*arctanh(1/5*(2*x^2-1)*5^(1/2))-1/6/x^6-3/2/x^2
```

Maxima [A]
time = 0.55, size = 99, normalized size = 1.02

$$-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}+1}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}-1}{2x^2+\sqrt{5}-1}\right)-\frac{9x^4+1}{6x^6}-\frac{5}{8}\log\left(x^4+x^2-1\right)+\frac{5}{8}\log\left(x^4-x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -11/40*sqrt(5)*log((2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40*sqr
t(5)*log((2*x^2 - sqrt(5) - 1)/(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x^4 + 1)/x^6
 - 5/8*log(x^4 + x^2 - 1) + 5/8*log(x^4 - x^2 - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(55) = 110.
time = 0.34, size = 130, normalized size = 1.34

$$\frac{33\sqrt{5}x^6\log\left(\frac{2x^4+2x^2+\sqrt{5}(2x^2+1)+3}{x^4+x^2-1}\right)+33\sqrt{5}x^6\log\left(\frac{2x^4-2x^2+\sqrt{5}(2x^2-1)+3}{x^4-x^2-1}\right)-75x^6\log\left(x^4+x^2-1\right)+75x^6\log\left(x^4-x^2-1\right)-180x^4-20}{120x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(33*sqrt(5)*x^6*log((2*x^4 + 2*x^2 + sqrt(5)*(2*x^2 + 1) + 3)/(x^4 +
x^2 - 1)) + 33*sqrt(5)*x^6*log((2*x^4 - 2*x^2 + sqrt(5)*(2*x^2 - 1) + 3)/(x
```

```
^4 - x^2 - 1)) - 75*x^6*log(x^4 + x^2 - 1) + 75*x^6*log(x^4 - x^2 - 1) - 18
0*x^4 - 20)/x^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(75) = 150$.
time = 0.24, size = 199, normalized size = 2.05

$$\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)\log\left(x^2-\frac{2207}{22}-\frac{2207\sqrt{5}}{50}+\frac{1152\left(\frac{11\sqrt{5}}{40}+\frac{5}{8}\right)^3}{11}\right)+\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207}{22}+\frac{1152\left(\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207\sqrt{5}}{50}\right)+\left(-\frac{5}{8}+\frac{11\sqrt{5}}{40}\right)\log\left(x^2-\frac{2207\sqrt{5}}{50}+\frac{1152\left(-\frac{5}{8}+\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207}{22}\right)+\left(-\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)\log\left(x^2+\frac{1152\left(-\frac{5}{8}-\frac{11\sqrt{5}}{40}\right)^3}{11}+\frac{2207\sqrt{5}}{50}+\frac{2207}{22}\right)+\frac{-9x^4-1}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**8-3*x**4+1),x)
```

```
[Out] (11*sqrt(5)/40 + 5/8)*log(x**2 - 2207/22 - 2207*sqrt(5)/50 + 1152*(11*sqrt(
5)/40 + 5/8)**3/11) + (5/8 - 11*sqrt(5)/40)*log(x**2 - 2207/22 + 1152*(5/8
- 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)/50) + (-5/8 + 11*sqrt(5)/40)*log(x**2
 - 2207*sqrt(5)/50 + 1152*(-5/8 + 11*sqrt(5)/40)**3/11 + 2207/22) + (-5/8 -
 11*sqrt(5)/40)*log(x**2 + 1152*(-5/8 - 11*sqrt(5)/40)**3/11 + 2207*sqrt(5)
/50 + 2207/22) + (-9*x**4 - 1)/(6*x**6)
```

Giac [A]
time = 5.59, size = 104, normalized size = 1.07

$$-\frac{11}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}+1\right|}{2x^2+\sqrt{5}+1}\right)-\frac{11}{40}\sqrt{5}\log\left(\frac{\left|2x^2-\sqrt{5}-1\right|}{\left|2x^2+\sqrt{5}-1\right|}\right)-\frac{9x^4+1}{6x^6}-\frac{5}{8}\log\left(\left|x^4+x^2-1\right|\right)+\frac{5}{8}\log\left(\left|x^4-x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -11/40*sqrt(5)*log(abs(2*x^2 - sqrt(5) + 1)/(2*x^2 + sqrt(5) + 1)) - 11/40*
sqrt(5)*log(abs(2*x^2 - sqrt(5) - 1)/abs(2*x^2 + sqrt(5) - 1)) - 1/6*(9*x^4
 + 1)/x^6 - 5/8*log(abs(x^4 + x^2 - 1)) + 5/8*log(abs(x^4 - x^2 - 1))
```

Mupad [B]
time = 1.38, size = 95, normalized size = 0.98

$$\mathrm{atanh}\left(\frac{4126100\,x^2}{1140425\sqrt{5}-2550075}-\frac{1845250\sqrt{5}\,x^2}{1140425\sqrt{5}-2550075}\right)\left(\frac{11\sqrt{5}}{20}-\frac{5}{4}\right)+\mathrm{atanh}\left(\frac{4126100\,x^2}{1140425\sqrt{5}+2550075}+\frac{1845250\sqrt{5}\,x^2}{1140425\sqrt{5}+2550075}\right)\left(\frac{11\sqrt{5}}{20}+\frac{5}{4}\right)-\frac{\frac{3x^4}{2}+\frac{1}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] atanh((4126100*x^2)/(1140425*5^(1/2) - 2550075) - (1845250*5^(1/2)*x^2)/(11
40425*5^(1/2) - 2550075))*((11*5^(1/2))/20 - 5/4) + atanh((4126100*x^2)/(11
40425*5^(1/2) + 2550075) + (1845250*5^(1/2)*x^2)/(1140425*5^(1/2) + 2550075
))*((11*5^(1/2))/20 + 5/4) - ((3*x^4)/2 + 1/6)/x^6
```

3.396 $\int \frac{x^8}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=170

$$x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(123\right.}}{}$$

```
[Out] x+1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(984-440*5^(1/2))^(1/4)*5^(1
/2)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(984-440*5^(1/2))^(1/4)*5
^(1/2)-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(123+55*5^(1/2))^(1/4)*
2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(123+55*5^(1/
2))^(1/4)*2^(3/4)*5^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1381, 1436, 218, 212, 209}

$$-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\text{ArcTan}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}+x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{984-440\sqrt{5}}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] x - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqr
t[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sq
rt[5]) - (((123 + 55*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/
(2*Sqrt[5]) + ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x]
)/(4*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{1-3x^4+x^8}\,dx &= x-\int \frac{1-3x^4}{1-3x^4+x^8}\,dx\\ &= x-\frac{1}{10}\left(-15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,d\\ &= x+\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}\,dx+\sqrt{\frac{1}{10}\left(9-4\sqrt{5}\right)}\int \frac{}{\sqrt{3-\sqrt{}}}\\ &= x-\frac{\sqrt[4]{\frac{1}{2}\left(123+55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(123-55\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{}\right.}{2\sqrt{5}}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 160, normalized size = 0.94

$$x+\frac{\left(-2+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\left(2+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(-2+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\left(2+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(1 - 3*x^4 + x^8),x]
```

```
[Out] x + ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5]
)] - ((2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5])]
 + ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 + Sqrt[5]
)] - ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 + Sqrt[5])
]
```

Maple [A]
time = 0.08, size = 131, normalized size = 0.77

method	result
risch	$x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+55\_Z^2-1\right)}\_R\ln\left(15\_R^3+29\_R+5x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-55\_Z^2-1\right)}\_R\ln\left(-15\_R^3+29\right.\right.}{4}$
default	$x+\frac{\left(-2+\sqrt{5}\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}-\frac{\sqrt{5}\left(2+\sqrt{5}\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}-\frac{\sqrt{5}\left(2+\sqrt{5}\right)\mathrm{ar}}{5\sqrt{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] x+1/5*(-2+5^(1/2))*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctanh(2*x/(2*5^(1/2)-2)^(1
/2))-1/5*5^(1/2)*(2+5^(1/2))/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(
1/2))-1/5*5^(1/2)*(2+5^(1/2))/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)
^(1/2))+1/5*(-2+5^(1/2))*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctan(2*x/(2*5^(1/2)-
2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] x + 1/2*integrate((2*x^2 + 1)/(x^4 - x^2 - 1), x) - 1/2*integrate((2*x^2 -
1)/(x^4 + x^2 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(118) = 236.
time = 0.36, size = 312, normalized size = 1.84

$$-\frac{1}{10}\sqrt{10}\sqrt{5\sqrt{5}-11}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{5\sqrt{5}-11}\left(2\sqrt{5}+5\right)-\frac{1}{10}\sqrt{10}\left(2\sqrt{5}x+5x\right)\sqrt{5\sqrt{5}-11}\right)-\frac{1}{10}\sqrt{10}\sqrt{5\sqrt{5}+11}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{2}\sqrt{2x^2+\sqrt{5}+1}\sqrt{5\sqrt{5}+11}\left(2\sqrt{5}-5\right)-\frac{1}{10}\sqrt{10}\left(2\sqrt{5}x-5x\right)\sqrt{5\sqrt{5}+11}\right)+\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}-11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}-11}\left(3\sqrt{5}+5\right)+20x\right)-\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}-11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}-11}\left(3\sqrt{5}+5\right)+20x\right)-\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}+11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}+11}\left(3\sqrt{5}-5\right)+20x\right)+\frac{1}{40}\sqrt{10}\sqrt{5\sqrt{5}+11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}+11}\left(3\sqrt{5}-5\right)+20x\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(5*sqrt(5) - 11)*arctan(1/20*sqrt(10)*sqrt(2)*sqrt(2*x^2
 + sqrt(5) - 1)*sqrt(5*sqrt(5) - 11)*(2*sqrt(5) + 5) - 1/10*sqrt(10)*(2*sqr
t(5)*x + 5*x)*sqrt(5*sqrt(5) - 11)) - 1/10*sqrt(10)*sqrt(5*sqrt(5) + 11)*ar
ctan(1/20*sqrt(10)*sqrt(2)*sqrt(2*x^2 + sqrt(5) + 1)*sqrt(5*sqrt(5) + 11)*(
2*sqrt(5) - 5) - 1/10*sqrt(10)*(2*sqrt(5)*x - 5*x)*sqrt(5*sqrt(5) + 11)) +
1/40*sqrt(10)*sqrt(5*sqrt(5) - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(3*sqr
t(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt(5
*sqrt(5) - 11)*(3*sqrt(5) + 5) + 20*x) - 1/40*sqrt(10)*sqrt(5*sqrt(5) + 11)
*log(sqrt(10)*sqrt(5*sqrt(5) + 11)*(3*sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*
sqrt(5*sqrt(5) + 11)*log(-sqrt(10)*sqrt(5*sqrt(5) + 11)*(3*sqrt(5) - 5) + 2
0*x) + x
```

Sympy [A]
time = 0.76, size = 58, normalized size = 0.34

$$x + \operatorname{RootSum}\left(6400t^4 - 880t^2 - 1, \left(t \mapsto t \log\left(-\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right) + \operatorname{RootSum}\left(6400t^4 + 880t^2 - 1, \left(t \mapsto t \log\left(-\frac{15360t^5}{11} + \frac{1288t}{55} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**8-3*x**4+1),x)
```

```
[Out] x + RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(-15360*_t**5/11 +
 1288*_t/55 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*log(
-15360*_t**5/11 + 1288*_t/55 + x)))
```

Giac [A]
time = 4.18, size = 148, normalized size = 0.87

$$-\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) + \frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right) - \frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right) + \frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) - \frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right) + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(50*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(
50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5)
+ 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*
log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*log(abs
(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) - 110)*log(abs(x - sq
rt(1/2*sqrt(5) - 1/2))) + x
```

Mupad [B]
time = 1.44, size = 246, normalized size = 1.45

$$x - \frac{\operatorname{atan}\left(\frac{x\sqrt{-50\sqrt{5}-110}\,55\mathrm{i}}{2\left(275\sqrt{5}+605\right)} + \frac{\sqrt{5}\,x\sqrt{-50\sqrt{5}-110}\,33\mathrm{i}}{2\left(275\sqrt{5}+605\right)}\right)\sqrt{-50\sqrt{5}-110}\,1\mathrm{i}}{20} - \frac{\operatorname{atan}\left(\frac{x\sqrt{110-50\sqrt{5}}\,55\mathrm{i}}{2\left(275\sqrt{5}-605\right)} - \frac{\sqrt{5}\,x\sqrt{110-50\sqrt{5}}\,33\mathrm{i}}{2\left(275\sqrt{5}-605\right)}\right)\sqrt{110-50\sqrt{5}}\,1\mathrm{i}}{20} + \frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}-110}\,55\mathrm{i}}{2\left(275\sqrt{5}-605\right)} - \frac{\sqrt{5}\,x\sqrt{50\sqrt{5}-110}\,33\mathrm{i}}{2\left(275\sqrt{5}-605\right)}\right)\sqrt{50\sqrt{5}-110}\,1\mathrm{i}}{20} + \frac{\operatorname{atan}\left(\frac{x\sqrt{50\sqrt{5}+110}\,55\mathrm{i}}{2\left(275\sqrt{5}+605\right)} + \frac{\sqrt{5}\,x\sqrt{50\sqrt{5}+110}\,33\mathrm{i}}{2\left(275\sqrt{5}+605\right)}\right)\sqrt{50\sqrt{5}+110}\,1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^8 - 3*x^4 + 1),x)
```

```
[Out] x - (atan((x*(- 50*5^(1/2) - 110)^(1/2)*55i)/(2*(275*5^(1/2) + 605)) + (5^(
1/2)*x*(- 50*5^(1/2) - 110)^(1/2)*33i)/(2*(275*5^(1/2) + 605)))*(- 50*5^(1/
2) - 110)^(1/2)*1i)/20 - (atan((x*(110 - 50*5^(1/2))^(1/2)*55i)/(2*(275*5^(
1/2) - 605)) - (5^(1/2)*x*(110 - 50*5^(1/2))^(1/2)*33i)/(2*(275*5^(1/2) - 6
05)))*(110 - 50*5^(1/2))^(1/2)*1i)/20 + (atan((x*(50*5^(1/2) - 110)^(1/2)*5
5i)/(2*(275*5^(1/2) - 605)) - (5^(1/2)*x*(50*5^(1/2) - 110)^(1/2)*33i)/(2*(
275*5^(1/2) - 605)))*(50*5^(1/2) - 110)^(1/2)*1i)/20 + (atan((x*(50*5^(1/2)
 + 110)^(1/2)*55i)/(2*(275*5^(1/2) + 605)) + (5^(1/2)*x*(50*5^(1/2) + 110)^
(1/2)*33i)/(2*(275*5^(1/2) + 605)))*(50*5^(1/2) + 110)^(1/2)*1i)/20
```

3.397 $\int \frac{x^6}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=167

$$\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{144-64\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-}}{2\ 2^{3/4}}$$

```
[Out] -1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(144-64*5^(1/2))^(1/4)*5^(1/2
)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(144-64*5^(1/2))^(1/4)*5^(1
/2)+1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(3/4)*2^(1/4)*
5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(3/4)*2^(
1/4)*5^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1388, 304, 209, 212}

$$\frac{\left(3+\sqrt{5}\right)^{3/4}\text{ArcTan}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{144-64\sqrt{5}}\,\text{ArcTan}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\ 2^{3/4}\sqrt{5}}+\frac{\sqrt[4]{144-64\sqrt{5}}\,\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] ((3 + Sqrt[5])^(3/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt[5])
 - ((144 - 64*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqrt[5])
 - ((3 + Sqrt[5])^(3/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*2^(3/4)*Sqrt
[5]) + ((144 - 64*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4*Sqr
t[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^6}{1-3x^4+x^8}\,dx = \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx+\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$
$$= \frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}\,dx}{2\sqrt{10}} - \frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}\,x^2}\,dx}{2\sqrt{10}} - \frac{\left(3+\sqrt{\right.}}{}$$
$$= \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\left(3-\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}} -$$

Mathematica [A]
time = 0.10, size = 160, normalized size = 0.96

$$\frac{\frac{\left(-3+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{-1+\sqrt{5}}}+\frac{\left(3+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{1+\sqrt{5}}}-\frac{\left(-3+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{-1+\sqrt{5}}}-\frac{\left(3+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 - 3*x^4 + x^8),x]
```

```
[Out] (((-3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] + ((3
 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] - ((-3 + Sqr
t[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((3 + Sqrt[5]
)*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

Maple [A]
time = 0.07, size = 130, normalized size = 0.78

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-20\_Z^2-1\right)}\_R\ln\left(5\_R^3-7\_R+2x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+20\_Z^2-1\right)}\_R\ln\left(5\_R^3+7\_R+2x\right)\right.}{4}$
default	$-\frac{\sqrt{5}\left(\sqrt{5}-3\right)\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}+\frac{\left(3+\sqrt{5}\right)\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\ \mathrm{arcta}}{10\sqrt{2\sqrt{}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*5^(1/2)*(5^(1/2)-3)/(2*5^(1/2)-2)^(1/2)*arctanh(2*x/(2*5^(1/2)-2)^(1/
2))+1/10*(3+5^(1/2))*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(
1/2))-1/10*(3+5^(1/2))*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2
)^(1/2))+1/10*5^(1/2)*(5^(1/2)-3)/(2*5^(1/2)-2)^(1/2)*arctan(2*x/(2*5^(1/2)
-2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(x^8 - 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(113) = 226$.
time = 0.39, size = 243, normalized size = 1.46

$$\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}-2}\arctan\left(\frac{1}{4}\sqrt{4x^2+2\sqrt{5}-2}\left(\sqrt{5}+3\right)\sqrt{\sqrt{5}-2}-\frac{1}{2}\left(\sqrt{5}x+3x\right)\sqrt{\sqrt{5}-2}\right)+\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}+2}\arctan\left(\frac{1}{4}\sqrt{4x^2+2\sqrt{5}+2}\sqrt{\sqrt{5}+2}\left(\sqrt{5}-3\right)-\frac{1}{2}\left(\sqrt{5}x-3x\right)\sqrt{\sqrt{5}+2}\right)-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(\sqrt{\sqrt{5}+2}\left(\sqrt{5}-1\right)+2x\right)+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(-\sqrt{\sqrt{5}+2}\left(\sqrt{5}-1\right)+2x\right)+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(\left(\sqrt{5}+1\right)\sqrt{\sqrt{5}-2}+2x\right)-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\left(\sqrt{5}+1\right)\sqrt{\sqrt{5}-2}+2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*sqrt(sqrt(5) - 2)*arctan(1/4*sqrt(4*x^2 + 2*sqrt(5) - 2)*(sqrt(
5) + 3)*sqrt(sqrt(5) - 2) - 1/2*(sqrt(5)*x + 3*x)*sqrt(sqrt(5) - 2)) + 1/5*
sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*sqrt(4*x^2 + 2*sqrt(5) + 2)*sqrt(sqrt(
5) + 2)*(sqrt(5) - 3) - 1/2*(sqrt(5)*x - 3*x)*sqrt(sqrt(5) + 2)) - 1/20*sqr
t(5)*sqrt(sqrt(5) + 2)*log(sqrt(sqrt(5) + 2)*(sqrt(5) - 1) + 2*x) + 1/20*sq
rt(5)*sqrt(sqrt(5) + 2)*log(-sqrt(sqrt(5) + 2)*(sqrt(5) - 1) + 2*x) + 1/20*
sqrt(5)*sqrt(sqrt(5) - 2)*log((sqrt(5) + 1)*sqrt(sqrt(5) - 2) + 2*x) - 1/20
*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(sqrt(5) + 1)*sqrt(sqrt(5) - 2) + 2*x)
```

Sympy [A]
time = 0.73, size = 53, normalized size = 0.32

$$\operatorname{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(-1792000t^7+4920t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(-1792000*_t**7 + 4920
*_t**3 + x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(-1792
000*_t**7 + 4920*_t**3 + x)))
```

Giac [A]
time = 4.10, size = 147, normalized size = 0.88

$$\frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(5*s
qrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) + 10)*
log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x
 - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x + sqrt(1
/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) - 10)*log(abs(x - sqrt(1/2*sqrt(5
) - 1/2)))
```

Mupad [B]
time = 0.19, size = 147, normalized size = 0.88

$$\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16x\sqrt{2-\sqrt{5}}}{8\sqrt{5}-24}\right)\sqrt{\sqrt{5}-2}\,1\mathrm{i}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{16x\sqrt{-\sqrt{5}-2}}{8\sqrt{5}+24}\right)\sqrt{\sqrt{5}+2}\,1\mathrm{i}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{2-\sqrt{5}}\,16\mathrm{i}}{8\sqrt{5}-24}\right)\sqrt{2-\sqrt{5}}\,1\mathrm{i}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{-\sqrt{5}-2}\,16\mathrm{i}}{8\sqrt{5}+24}\right)\sqrt{-\sqrt{5}-2}\,1\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (5^(1/2)*atan((16*x*(2 - 5^(1/2))^(1/2))/(8*5^(1/2) - 24))*(5^(1/2) - 2)^(1
/2)*1i)/10 + (5^(1/2)*atan((16*x*(- 5^(1/2) - 2)^(1/2))/(8*5^(1/2) + 24))*(
5^(1/2) + 2)^(1/2)*1i)/10 + (5^(1/2)*atan((x*(2 - 5^(1/2))^(1/2)*16i)/(8*5^
(1/2) - 24))*(2 - 5^(1/2))^(1/2)*1i)/10 + (5^(1/2)*atan((x*(- 5^(1/2) - 2)^
(1/2)*16i)/(8*5^(1/2) + 24))*(- 5^(1/2) - 2)^(1/2)*1i)/10
```

3.398 $\int \frac{x^4}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=173

$$-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}}{}$$

```
[Out] 1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3-5^(1/2))^(1/4)*2^(3/4)*5^(1
/2)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3-5^(1/2))^(1/4)*2^(3/4)
*5^(1/2)-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(1/4)*2^(
3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(3+5^(1/2))^(1/4
)*2^(3/4)*5^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1388, 218, 212, 209}

$$-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] -1/2*(((3 + Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/Sqrt[5] +
(((3 - Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (
((3 + Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (
((3 - Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rubi steps

$$\int \frac{x^4}{1-3x^4+x^8}\,dx = \frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx + \frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx$$

$$= \frac{1}{2}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}\,dx + \frac{1}{2}\sqrt{\frac{1}{5}\left(3-\sqrt{5}\right)}\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{\ldots}}$$

$$= -\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}} + \frac{\sqrt[4]{\frac{1}{2}\left(3-\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\ldots\right.}{2\sqrt{5}}$$

Mathematica [**A**]
time = 0.12, size = 132, normalized size = 0.76

$$\frac{\sqrt{-1+\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)-\sqrt{1+\sqrt{5}}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)+\sqrt{-1+\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)-\sqrt{1+\sqrt{5}}\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - 3*x^4 + x^8),x]
```

```
[Out] (Sqrt[-1 + Sqrt[5]]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[1 + Sqrt[5]]*Ar
cTan[Sqrt[2/(1 + Sqrt[5])]*x] + Sqrt[-1 + Sqrt[5]]*ArcTanh[Sqrt[2/(-1 + Sqr
t[5])]*x] - Sqrt[1 + Sqrt[5]]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10]
)
```

Maple [**A**]
time = 0.06, size = 130, normalized size = 0.75

method	result

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-5\_Z^2-1\right)}\_R\ln\left(-10\_R^3+\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+5\_Z^2-1\right)}\_R\ln\left(10\_R^3+\_R+x\right)\right)}{4}$
default	$\frac{\left(\sqrt{5}-1\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\left(\sqrt{5}+1\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}-\frac{\left(\sqrt{5}+1\right)\sqrt{5}\,\mathrm{arctanh}}{10\sqrt{2\sqrt{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(5^(1/2)-1)*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctanh(2*x/(2*5^(1/2)-2)^(1/2
))-1/10*(5^(1/2)+1)*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1
/2))-1/10*(5^(1/2)+1)*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)
^(1/2))+1/10*(5^(1/2)-1)*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctan(2*x/(2*5^(1/2)-
2)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(x^8 - 3*x^4 + 1), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(119) = 238$.
time = 0.38, size = 261, normalized size = 1.51

$$-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}-1}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}-\frac{1}{20}\sqrt{10}\left(\sqrt{5}x+5x\right)\sqrt{\sqrt{5}-1}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2}\sqrt{2x^2+\sqrt{5}+1}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)-\frac{1}{20}\sqrt{10}\left(\sqrt{5}x-5x\right)\sqrt{\sqrt{5}+1}\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}+1}+10x\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}+1}+10x\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}-1}+10x\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\sqrt{10}\sqrt{5}\sqrt{\sqrt{5}-1}+10x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/10*sqrt(10)*sqrt(sqrt(5) - 1)*arctan(1/40*sqrt(10)*sqrt(2)*sqrt(2*x^2 +
sqrt(5) - 1)*(sqrt(5) + 5)*sqrt(sqrt(5) - 1) - 1/20*sqrt(10)*(sqrt(5)*x + 5
*x)*sqrt(sqrt(5) - 1)) - 1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(1/40*sqrt(1
0)*sqrt(2)*sqrt(2*x^2 + sqrt(5) + 1)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) - 1/20
*sqrt(10)*(sqrt(5)*x - 5*x)*sqrt(sqrt(5) + 1)) - 1/40*sqrt(10)*sqrt(sqrt(5)
 + 1)*log(sqrt(10)*sqrt(5)*sqrt(sqrt(5) + 1) + 10*x) + 1/40*sqrt(10)*sqrt(s
qrt(5) + 1)*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(5) + 1) + 10*x) + 1/40*sqrt(10)
*sqrt(sqrt(5) - 1)*log(sqrt(10)*sqrt(5)*sqrt(sqrt(5) - 1) + 10*x) - 1/40*sq
rt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(10)*sqrt(5)*sqrt(sqrt(5) - 1) + 10*x)
```

Sympy [A]
time = 0.71, size = 49, normalized size = 0.28

$$\operatorname{RootSum}\left(6400t^{4}-80t^{2}-1,\ \left(t\mapsto t\log\left(-51200t^{5}+12t+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^{4}+80t^{2}-1,\ \left(t\mapsto t\log\left(-51200t^{5}+12t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 + 12*_t +
 x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(-51200*_t**5 +
 12*_t + x)))
```

Giac [A]
time = 3.88, size = 147, normalized size = 0.85

$$-\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(10*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt(1
0*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5) +
10)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(
abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*log(abs(x +
sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) - 10)*log(abs(x - sqrt(1/2
*sqrt(5) - 1/2)))
```

Mupad [B]
time = 1.47, size = 269, normalized size = 1.55

$$\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,1{}\mathrm{i}}{2\,\left(\sqrt{5}-1\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3{}\mathrm{i}}{10\,\left(\sqrt{5}-1\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1{}\mathrm{i}}{20}+\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,1{}\mathrm{i}}{2\,\left(\sqrt{5}+1\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3{}\mathrm{i}}{10\,\left(\sqrt{5}+1\right)}\right)\,\sqrt{1-\sqrt{5}}\,1{}\mathrm{i}}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,1{}\mathrm{i}}{2\,\left(\sqrt{5}-1\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3{}\mathrm{i}}{10\,\left(\sqrt{5}-1\right)}\right)\,\sqrt{\sqrt{5}+1}\,1{}\mathrm{i}}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,1{}\mathrm{i}}{2\,\left(\sqrt{5}+1\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3{}\mathrm{i}}{10\,\left(\sqrt{5}+1\right)}\right)\,\sqrt{\sqrt{5}-1}\,1{}\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*1i)/(2*(5^(1/2) - 1)) - (5
^(1/2)*10^(1/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(10*(5^(1/2) - 1)))*(- 5^(1/2)
- 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2))^(1/2)*1i)/(2*(
5^(1/2) + 1)) + (5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*3i)/(10*(5^(1/2) +
1)))*(1 - 5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^
(1/2)*1i)/(2*(5^(1/2) - 1)) - (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(
10*(5^(1/2) - 1)))*(5^(1/2) + 1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*
(5^(1/2) - 1)^(1/2)*1i)/(2*(5^(1/2) + 1)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) -
1)^(1/2)*3i)/(10*(5^(1/2) + 1)))*(5^(1/2) - 1)^(1/2)*1i)/20
```

3.399 $\int \frac{x^2}{1-3x^4+x^8} \, dx$

Optimal. Leaf size=145

$$\frac{1}{20}\sqrt{-10+10\sqrt{5}}\ \tan^{-1}\left(\frac{1}{2}\sqrt{-2+2\sqrt{5}}\ x\right)-\frac{1}{20}\sqrt{10+10\sqrt{5}}\ \tan^{-1}\left(\frac{1}{2}\sqrt{2+2\sqrt{5}}\ x\right)-\frac{1}{20}\sqrt{-10+$$

```
[Out] 1/20*arctan(1/2*x*(-2+2*5^(1/2))^(1/2))*(-10+10*5^(1/2))^(1/2)-1/20*arctanh
(1/2*x*(-2+2*5^(1/2))^(1/2))*(-10+10*5^(1/2))^(1/2)-1/20*arctan(1/2*x*(2+2*
5^(1/2))^(1/2))*(10+10*5^(1/2))^(1/2)+1/20*arctanh(1/2*x*(2+2*5^(1/2))^(1/2
))*(10+10*5^(1/2))^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 166, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1389, 304, 209, 212}

$$\frac{\text{ArcTan}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\ x\right)}{2^{3/4}\sqrt{5}\ \sqrt[4]{3+\sqrt{5}}}-\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\ \text{ArcTan}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\ x\right)}{2\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\ x\right)}{2^{3/4}\sqrt{5}\ \sqrt[4]{3+\sqrt{5}}}+\frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\ \tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\ x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^(1/4)) - (
((3 + Sqrt[5])/2)^(1/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - Ar
cTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(3/4)*Sqrt[5]*(3 + Sqrt[5])^(1/4)) + ((
(3 + Sqrt[5])/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
```

```
/b, 0]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2}{1-3x^4+x^8}\,dx = \frac{\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{10}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}x^2}\,dx}{\sqrt{10}} - \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}x^2}\,dx}{\sqrt{10}} + \frac{\int -}{}$$

$$= \frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2^{3/4}\sqrt{5}\sqrt[4]{3+\sqrt{5}}} - \frac{\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{}\right.}{2^{3/4}\sqrt{5}}$$

Mathematica [A]
time = 0.03, size = 131, normalized size = 0.90

$$-\frac{\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 - 3*x^4 + x^8),x]
```

```
[Out] -(ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x]/Sqrt[10*(-1 + Sqrt[5])]) + ArcTan[Sqrt[2
/(1 + Sqrt[5])]*x]/Sqrt[10*(1 + Sqrt[5])] + ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*
x]/Sqrt[10*(-1 + Sqrt[5])] - ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x]/Sqrt[10*(1 +
Sqrt[5])]
```

Maple [A]
time = 0.04, size = 110, normalized size = 0.76

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+5\_Z^2-1\right)}\_R\ln\left(-5\_R^3-3\_R+x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-5\_Z^2-1\right)}\_R\ln\left(-5\_R^3+3\_R+x\right)\right.}{4}$
default	$\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}+\frac{\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}-\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}-\frac{\sqrt{5}\,\mathrm{arc}}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctanh(2*x/(2*5^(1/2)-2)^(1/2))+1/5*5^(1/2
)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/2))-1/5*5^(1/2)/(2*5^(1/2
)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^(1/2))-1/5*5^(1/2)/(2*5^(1/2)-2)^(1/2)
*arctan(2*x/(2*5^(1/2)-2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^8 - 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(97) = 194$.
time = 0.39, size = 255, normalized size = 1.76

$$\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{\sqrt{5}+1}-\frac{1}{10}\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}+1}\right)-\frac{1}{10}\sqrt{10}\sqrt{\sqrt{5}-1}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{5}\sqrt{2}\sqrt{2x^2+\sqrt{5}+1}\sqrt{\sqrt{5}-1}-\frac{1}{10}\sqrt{10}\sqrt{5}x\sqrt{\sqrt{5}-1}\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20x\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}-1}\log\left(-\sqrt{10}\left(\sqrt{5}+5\right)\sqrt{\sqrt{5}-1}+20x\right)-\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20x\right)+\frac{1}{40}\sqrt{10}\sqrt{\sqrt{5}+1}\log\left(-\sqrt{10}\sqrt{\sqrt{5}+1}\left(\sqrt{5}-5\right)+20x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(10)*sqrt(sqrt(5) + 1)*arctan(1/20*sqrt(10)*sqrt(5)*sqrt(2)*sqrt(2
*x^2 + sqrt(5) - 1)*sqrt(sqrt(5) + 1) - 1/10*sqrt(10)*sqrt(5)*x*sqrt(sqrt(5
) + 1)) - 1/10*sqrt(10)*sqrt(sqrt(5) - 1)*arctan(1/20*sqrt(10)*sqrt(5)*sqrt
(2)*sqrt(2*x^2 + sqrt(5) + 1)*sqrt(sqrt(5) - 1) - 1/10*sqrt(10)*sqrt(5)*x*s
qrt(sqrt(5) - 1)) - 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(sqrt(10)*(sqrt(5) +
 5)*sqrt(sqrt(5) - 1) + 20*x) + 1/40*sqrt(10)*sqrt(sqrt(5) - 1)*log(-sqrt(1
0)*(sqrt(5) + 5)*sqrt(sqrt(5) - 1) + 20*x) - 1/40*sqrt(10)*sqrt(sqrt(5) + 1
)*log(sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x) + 1/40*sqrt(10)*sqrt
(sqrt(5) + 1)*log(-sqrt(10)*sqrt(sqrt(5) + 1)*(sqrt(5) - 5) + 20*x)
```

Sympy [A]
time = 0.71, size = 53, normalized size = 0.37

$$\operatorname{RootSum}\left(6400t^4-80t^2-1,\left(t\mapsto t\log\left(6144000t^7-2240t^3+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+80t^2-1,\left(t\mapsto t\log\left(6144000t^7-2240t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 80*_t**2 - 1, Lambda(_t, _t*log(6144000*_t**7 - 2240*_
t**3 + x))) + RootSum(6400*_t**4 + 80*_t**2 - 1, Lambda(_t, _t*log(6144000*
_t**7 - 2240*_t**3 + x)))
```

Giac [A]
time = 3.93, size = 147, normalized size = 1.01

$$\frac{1}{20}\sqrt{10\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{10\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{10\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(10*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(10
*sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(10*sqrt(5) - 1
0)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) - 10)*log(a
bs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x + s
qrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(10*sqrt(5) + 10)*log(abs(x - sqrt(1/2*
sqrt(5) - 1/2)))
```

Mupad [B]
time = 0.08, size = 269, normalized size = 1.86

$$\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,3{}\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}-1}\,7{}\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{\sqrt{5}-1}\,1{}\mathrm{i}}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,3{}\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{\sqrt{5}+1}\,7{}\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{\sqrt{5}+1}\,1{}\mathrm{i}}{20}+\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,3{}\mathrm{i}}{2\,\left(3\,\sqrt{5}-7\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{1-\sqrt{5}}\,7{}\mathrm{i}}{10\,\left(3\,\sqrt{5}-7\right)}\right)\,\sqrt{1-\sqrt{5}}\,1{}\mathrm{i}}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,3{}\mathrm{i}}{2\,\left(3\,\sqrt{5}+7\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-\sqrt{5}-1}\,7{}\mathrm{i}}{10\,\left(3\,\sqrt{5}+7\right)}\right)\,\sqrt{-\sqrt{5}-1}\,1{}\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5
^(1/2)*10^(1/2)*x*(5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) - 7)))*(5^(1/2) -
1)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(5^(1/2) + 1)^(1/2)*3i)/(2*(3*
5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(5^(1/2) + 1)^(1/2)*7i)/(10*(3*5^(1/2)
+ 7)))*(5^(1/2) + 1)^(1/2)*1i)/20 + (10^(1/2)*atan((10^(1/2)*x*(1 - 5^(1/2)
)^(1/2)*3i)/(2*(3*5^(1/2) - 7)) - (5^(1/2)*10^(1/2)*x*(1 - 5^(1/2))^(1/2)*7
i)/(10*(3*5^(1/2) - 7)))*(1 - 5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1
/2)*x*(- 5^(1/2) - 1)^(1/2)*3i)/(2*(3*5^(1/2) + 7)) + (5^(1/2)*10^(1/2)*x*(
- 5^(1/2) - 1)^(1/2)*7i)/(10*(3*5^(1/2) + 7)))*(- 5^(1/2) - 1)^(1/2)*1i)/20
```

3.400 $\int \frac{1}{1-3x^4+x^8}\,dx$

Optimal. Leaf size=169

$$-\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right.}{}$$

```
[Out] -1/10*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*2^(3/4)*5^(1/2)/(3+5^(1/2))^(
3/4)-1/10*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*2^(3/4)*5^(1/2)/(3+5^(1/
2))^(3/4)+1/10*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(9+4*5^(1/2))^(1/4)*
5^(1/2)+1/10*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(9+4*5^(1/2))^(1/4)*5
^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1361, 218, 212, 209}

$$-\frac{\text{ArcTan}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\text{ArcTan}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\sqrt{5}\left(3+\sqrt{5}\right)^{3/4}}+\frac{\left(3+\sqrt{5}\right)^{3/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^4 + x^8)^(-1),x]
```

```
[Out] -(ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4)))
+ ((3 + Sqrt[5])^(3/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sqrt[5
]) - ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]/(2^(1/4)*Sqrt[5]*(3 + Sqrt[5])^(3/4
)) + ((3 + Sqrt[5])^(3/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*2^(3/4)*Sq
rt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{1-3x^4+x^8}\,dx = \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}} - \frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^4}\,dx}{\sqrt{5}}$$

$$= \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}-\sqrt{2}\,x^2}\,dx}{\sqrt{5\left(3-\sqrt{5}\right)}} + \frac{\int \frac{1}{\sqrt{3-\sqrt{5}}+\sqrt{2}\,x^2}\,dx}{\sqrt{5\left(3-\sqrt{5}\right)}} - \frac{\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}\,x^2}\,dx}{\sqrt{5\left(3+\sqrt{5}\right)}} - \cdots$$

$$= -\frac{\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{\sqrt[4]{2}\,\sqrt{5}\,\left(3+\sqrt{5}\right)^{3/4}} + \frac{\left(3+\sqrt{5}\right)^{3/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\,2^{3/4}\sqrt{5}} - \frac{\tanh^{-1}\left(\cdots\right)}{\sqrt[4]{2}\,\sqrt{5}\,\left(3\cdots\right)}$$

Mathematica [A]
time = 0.09, size = 160, normalized size = 0.95

$$\frac{\frac{\left(1+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{-1+\sqrt{5}}} - \frac{\left(-1+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{1+\sqrt{5}}} + \frac{\left(1+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{-1+\sqrt{5}}} - \frac{\left(-1+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{1+\sqrt{5}}}}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x^4 + x^8)^(-1),x]
```

```
[Out] (((1 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-1
 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]] + ((1 + Sqrt
[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[-1 + Sqrt[5]] - ((-1 + Sqrt[5]
)*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[1 + Sqrt[5]])/(2*Sqrt[10])
```

Maple [A]
time = 0.04, size = 130, normalized size = 0.77

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-20\_Z^2-1\right)}\_R\ln\left(15\_R^3-11\_R+2x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+20\_Z^2-1\right)}\_R\ln\left(-15\_R^3-11\_R\right.\right.}{4}$
default	$\frac{\left(\sqrt{5}+1\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\left(\sqrt{5}-1\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}-\frac{\left(\sqrt{5}-1\right)\sqrt{5}\,\operatorname{arctanh}}{10\sqrt{2\sqrt{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(5^(1/2)+1)*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctanh(2*x/(2*5^(1/2)-2)^(1/2
))-1/10*(5^(1/2)-1)*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1
/2))-1/10*(5^(1/2)-1)*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)
^(1/2))+1/10*(5^(1/2)+1)*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctan(2*x/(2*5^(1/2)-
2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - 3*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(113) = 226$.
time = 0.37, size = 241, normalized size = 1.43

$$-\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}+2}\arctan\left(\frac{1}{4}\sqrt{4x^2+2\sqrt{5}-2}\sqrt{\sqrt{5}+2}\left(\sqrt{5}-1\right)-\frac{1}{2}\left(\sqrt{5}x-x\right)\sqrt{\sqrt{5}+2}\right)+\frac{1}{5}\sqrt{5}\sqrt{\sqrt{5}-2}\arctan\left(\frac{1}{4}\sqrt{4x^2+2\sqrt{5}+2}\left(\sqrt{5}+1\right)\sqrt{\sqrt{5}-2}-\frac{1}{2}\left(\sqrt{5}x+x\right)\sqrt{\sqrt{5}-2}\right)-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(\left(\sqrt{5}+3\right)\sqrt{\sqrt{5}-2}+2x\right)+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\left(\sqrt{5}+3\right)\sqrt{\sqrt{5}-2}+2x\right)-\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(\sqrt{\sqrt{5}+2}\left(\sqrt{5}-3\right)+2x\right)+\frac{1}{20}\sqrt{5}\sqrt{\sqrt{5}+2}\log\left(-\sqrt{\sqrt{5}+2}\left(\sqrt{5}-3\right)+2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*sqrt(4*x^2 + 2*sqrt(5) - 2)*sqrt(
sqrt(5) + 2)*(sqrt(5) - 1) - 1/2*(sqrt(5)*x - x)*sqrt(sqrt(5) + 2)) + 1/5*s
qrt(5)*sqrt(sqrt(5) - 2)*arctan(1/4*sqrt(4*x^2 + 2*sqrt(5) + 2)*(sqrt(5) +
1)*sqrt(sqrt(5) - 2) - 1/2*(sqrt(5)*x + x)*sqrt(sqrt(5) - 2)) - 1/20*sqrt(5
)*sqrt(sqrt(5) - 2)*log((sqrt(5) + 3)*sqrt(sqrt(5) - 2) + 2*x) + 1/20*sqrt(
```

```
5)*sqrt(sqrt(5) - 2)*log(-(sqrt(5) + 3)*sqrt(sqrt(5) - 2) + 2*x) - 1/20*sqr
t(5)*sqrt(sqrt(5) + 2)*log(sqrt(sqrt(5) + 2)*(sqrt(5) - 3) + 2*x) + 1/20*sq
rt(5)*sqrt(sqrt(5) + 2)*log(-sqrt(sqrt(5) + 2)*(sqrt(5) - 3) + 2*x)
```

Sympy [A]
time = 0.80, size = 53, normalized size = 0.31

$$\operatorname{RootSum}\left(6400t^4-320t^2-1,\left(t\mapsto t\log\left(9600t^5-\frac{47t}{2}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+320t^2-1,\left(t\mapsto t\log\left(9600t^5-\frac{47t}{2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5 - 47*_t/2
+ x))) + RootSum(6400*_t**4 + 320*_t**2 - 1, Lambda(_t, _t*log(9600*_t**5 -
 47*_t/2 + x)))
```

Giac [A]
time = 3.02, size = 147, normalized size = 0.87

$$-\frac{1}{10}\sqrt{5\sqrt{5}-10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5\sqrt{5}-10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{20}\sqrt{5\sqrt{5}+10}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5*sqrt(5) - 10)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/10*sqrt(5*
sqrt(5) + 10)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(5*sqrt(5) - 10)
*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) - 10)*log(abs(
x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x + sqrt(
1/2*sqrt(5) - 1/2))) - 1/20*sqrt(5*sqrt(5) + 10)*log(abs(x - sqrt(1/2*sqrt(
5) - 1/2)))
```

Mupad [B]
time = 0.08, size = 245, normalized size = 1.45

$$-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{2-\sqrt{5}}\,144{}\mathrm{i}}{104\sqrt{5}-232}-\frac{\sqrt{5}\,x\sqrt{2-\sqrt{5}}\,64{}\mathrm{i}}{104\sqrt{5}-232}\right)\sqrt{2-\sqrt{5}}\,1{}\mathrm{i}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{-\sqrt{5}-2}\,144{}\mathrm{i}}{104\sqrt{5}+232}+\frac{\sqrt{5}\,x\sqrt{-\sqrt{5}-2}\,64{}\mathrm{i}}{104\sqrt{5}+232}\right)\sqrt{-\sqrt{5}-2}\,1{}\mathrm{i}}{10}+\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{\sqrt{5}-2}\,144{}\mathrm{i}}{104\sqrt{5}-232}-\frac{\sqrt{5}\,x\sqrt{\sqrt{5}-2}\,64{}\mathrm{i}}{104\sqrt{5}-232}\right)\sqrt{\sqrt{5}-2}\,1{}\mathrm{i}}{10}-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{x\sqrt{\sqrt{5}+2}\,144{}\mathrm{i}}{104\sqrt{5}+232}+\frac{\sqrt{5}\,x\sqrt{\sqrt{5}+2}\,64{}\mathrm{i}}{104\sqrt{5}+232}\right)\sqrt{\sqrt{5}+2}\,1{}\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8 - 3*x^4 + 1),x)
```

```
[Out] (5^(1/2)*atan((x*(- 5^(1/2) - 2)^(1/2)*144i)/(104*5^(1/2) + 232) + (5^(1/2)
*x*(- 5^(1/2) - 2)^(1/2)*64i)/(104*5^(1/2) + 232))*(- 5^(1/2) - 2)^(1/2)*1i
)/10 - (5^(1/2)*atan((x*(2 - 5^(1/2))^(1/2)*144i)/(104*5^(1/2) - 232) - (5^
(1/2)*x*(2 - 5^(1/2))^(1/2)*64i)/(104*5^(1/2) - 232))*(2 - 5^(1/2))^(1/2)*1
i)/10 + (5^(1/2)*atan((x*(5^(1/2) - 2)^(1/2)*144i)/(104*5^(1/2) - 232) - (5
^(1/2)*x*(5^(1/2) - 2)^(1/2)*64i)/(104*5^(1/2) - 232))*(5^(1/2) - 2)^(1/2)*
1i)/10 - (5^(1/2)*atan((x*(5^(1/2) + 2)^(1/2)*144i)/(104*5^(1/2) + 232) + (
5^(1/2)*x*(5^(1/2) + 2)^(1/2)*64i)/(104*5^(1/2) + 232))*(5^(1/2) + 2)^(1/2)
*1i)/10
```

3.401 $\int \frac{1}{x^2(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=172

$$-\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt[4]{2}\,\sqrt{5}}-\frac{\sqrt[4]{984-440\sqrt{5}}}{}$$

```
[Out] -1/x+1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(984-440*5^(1/2))^(1/4)*5
^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(984-440*5^(1/2))^(1/4
)*5^(1/2)-1/40*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3+5^(1/2))^(5/4)*2^
(3/4)*5^(1/2)+1/40*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(3+5^(1/2))^(5/
4)*2^(3/4)*5^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1382, 1524, 304, 209, 212}

$$\frac{\sqrt[4]{984-440\sqrt{5}}\,\text{ArcTan}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\text{ArcTan}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt[4]{2}\,\sqrt{5}}-\frac{1}{x}-\frac{\sqrt[4]{984-440\sqrt{5}}\,\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{5/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\sqrt[4]{2}\,\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ((984 - 440*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*x])/(4*
Sqrt[5]) - ((3 + Sqrt[5])^(5/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(4*2^(1/
4)*Sqrt[5]) - ((984 - 440*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x]
)/(4*Sqrt[5]) + ((3 + Sqrt[5])^(5/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(4
*2^(1/4)*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(1-3x^4+x^8\right)}\,dx &= -\frac{1}{x}+\int \frac{x^2\left(3-x^4\right)}{1-3x^4+x^8}\,dx\\
&= -\frac{1}{x}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x
\\
&= -\frac{1}{x}-\frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}\,x^2}\,dx}{2\sqrt{10}}+\frac{\left(3-\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}\,x^2}\,dx
}{2\sqrt{10}
\\
&= -\frac{1}{x}+\frac{\sqrt[4]{984-440\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}-\frac{\left(3+\sqrt{5}\right)^{5/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3
\right.}\right.}{4\sqrt[4]{2}\,\sqrt{5}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 174, normalized size = 1.01

$$-\frac{1}{x}-\frac{\left(3+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\left(-3+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(3+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\left(-3+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -x^(-1) - ((3 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[10*(-1 +
 Sqrt[5])]) - ((-3 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(
1 + Sqrt[5])]) + ((3 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*Sqrt[
10*(-1 + Sqrt[5])]) + ((-3 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/(2*
Sqrt[10*(1 + Sqrt[5])])
```

Maple [A]
time = 0.05, size = 135, normalized size = 0.78

method	result
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^{4}+55\_Z^{2}-1\right)}\_R\ln\left(-20\_R^{3}-47\_R+5x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^{4}-55\_Z^{2}-1\right)}\_R\ln\left(-20\_R\right.\right.}{4}$
default	$\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\sqrt{5}\left(\sqrt{5}-3\right)\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\sqrt{5}\left(\sqrt{5}-3\right)\mathrm{arctanh}}{10\sqrt{2\sqrt{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(3+5^(1/2))*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctanh(2*x/(2*5^(1/2)-2)^(1/2
))-1/10*5^(1/2)*(5^(1/2)-3)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1
/2))+1/10*5^(1/2)*(5^(1/2)-3)/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)
^(1/2))-1/10*(3+5^(1/2))*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctan(2*x/(2*5^(1/2)-
2)^(1/2))-1/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - 1/2*integrate((x^2 + 2)/(x^4 + x^2 - 1), x) - 1/2*integrate((x^2 - 2
)/(x^4 - x^2 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(118) = 236.
time = 0.37, size = 321, normalized size = 1.87

$$-\frac{4\sqrt{10}\,x\sqrt{5\sqrt{5}-11}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2}\sqrt{2x^{2}+\sqrt{5}+1}\sqrt{5\sqrt{5}-11}\left(3\sqrt{5}+5\right)-\frac{1}{20}\sqrt{10}\left(3\sqrt{5}x+5x\right)\sqrt{5\sqrt{5}-11}\right)-4\sqrt{10}\,x\sqrt{5\sqrt{5}+11}\arctan\left(\frac{1}{40}\sqrt{10}\sqrt{2}\sqrt{2x^{2}+\sqrt{5}-1}\sqrt{5\sqrt{5}+11}\left(3\sqrt{5}-5\right)-\frac{1}{20}\sqrt{10}\left(3\sqrt{5}x-5x\right)\sqrt{5\sqrt{5}+11}\right)+\sqrt{10}\,x\sqrt{5\sqrt{5}-11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}-11}\left(2\sqrt{5}+5\right)+10x\right)-\sqrt{10}\,x\sqrt{5\sqrt{5}-11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}-11}\left(2\sqrt{5}+5\right)+10x\right)+\sqrt{10}\,x\sqrt{5\sqrt{5}+11}\log\left(\sqrt{10}\sqrt{5\sqrt{5}+11}\left(2\sqrt{5}-5\right)+10x\right)-\sqrt{10}\,x\sqrt{5\sqrt{5}+11}\log\left(-\sqrt{10}\sqrt{5\sqrt{5}+11}\left(2\sqrt{5}-5\right)+10x\right)+40}{40x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/40*(4*sqrt(10)*x*sqrt(5*sqrt(5) - 11)*arctan(1/40*sqrt(10)*sqrt(2)*sqrt(
2*x^2 + sqrt(5) + 1)*sqrt(5*sqrt(5) - 11)*(3*sqrt(5) + 5) - 1/20*sqrt(10)*(
3*sqrt(5)*x + 5*x)*sqrt(5*sqrt(5) - 11)) - 4*sqrt(10)*x*sqrt(5*sqrt(5) + 11
)*arctan(1/40*sqrt(10)*sqrt(2)*sqrt(2*x^2 + sqrt(5) - 1)*sqrt(5*sqrt(5) + 1
1)*(3*sqrt(5) - 5) - 1/20*sqrt(10)*(3*sqrt(5)*x - 5*x)*sqrt(5*sqrt(5) + 11)
) + sqrt(10)*x*sqrt(5*sqrt(5) - 11)*log(sqrt(10)*sqrt(5*sqrt(5) - 11)*(2*sq
rt(5) + 5) + 10*x) - sqrt(10)*x*sqrt(5*sqrt(5) - 11)*log(-sqrt(10)*sqrt(5*s
qrt(5) - 11)*(2*sqrt(5) + 5) + 10*x) + sqrt(10)*x*sqrt(5*sqrt(5) + 11)*log(
sqrt(10)*sqrt(5*sqrt(5) + 11)*(2*sqrt(5) - 5) + 10*x) - sqrt(10)*x*sqrt(5*s
qrt(5) + 11)*log(-sqrt(10)*sqrt(5*sqrt(5) + 11)*(2*sqrt(5) - 5) + 10*x) + 4
0)/x
```

Sympy [A]
time = 0.81, size = 63, normalized size = 0.37

$$\operatorname{RootSum}\left(6400t^4-880t^2-1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+880t^2-1,\left(t\mapsto t\log\left(\frac{19251200t^7}{11}-\frac{369792t^3}{11}+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 880*_t**2 - 1, Lambda(_t, _t*log(19251200*_t**7/11 - 3
69792*_t**3/11 + x))) + RootSum(6400*_t**4 + 880*_t**2 - 1, Lambda(_t, _t*l
og(19251200*_t**7/11 - 369792*_t**3/11 + x))) - 1/x
```

Giac [A]
time = 3.54, size = 152, normalized size = 0.88

$$\frac{1}{20}\sqrt{50\sqrt{5}-110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{50\sqrt{5}+110}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{50\sqrt{5}-110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{50\sqrt{5}+110}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(50*sqrt(5) - 110)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/20*sqrt(5
0*sqrt(5) + 110)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(50*sqrt(5) -
 110)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) - 110)*l
og(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(50*sqrt(5) + 110)*log(abs(
x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(50*sqrt(5) + 110)*log(abs(x - sqr
t(1/2*sqrt(5) - 1/2))) - 1/x
```

Mupad [B]
time = 1.34, size = 250, normalized size = 1.45

$$-\frac{1}{x}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-50\,\sqrt{5}-110}\,1155{}\mathrm{i}}{2\,\left(3025\,\sqrt{5}+6765\right)}+\frac{\sqrt{5}\,x\,\sqrt{-50\,\sqrt{5}-110}\,517{}\mathrm{i}}{2\,\left(3025\,\sqrt{5}+6765\right)}\right)\,\sqrt{-50\,\sqrt{5}-110}\,1{}\mathrm{i}}{20}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{110-50\,\sqrt{5}}\,1155{}\mathrm{i}}{2\,\left(3025\,\sqrt{5}-6765\right)}-\frac{\sqrt{5}\,x\,\sqrt{110-50\,\sqrt{5}}\,517{}\mathrm{i}}{2\,\left(3025\,\sqrt{5}-6765\right)}\right)\,\sqrt{110-50\,\sqrt{5}}\,1{}\mathrm{i}}{20}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{50\,\sqrt{5}-110}\,1155{}\mathrm{i}}{2\,\left(3025\,\sqrt{5}-6765\right)}-\frac{\sqrt{5}\,x\,\sqrt{50\,\sqrt{5}-110}\,517{}\mathrm{i}}{2\,\left(3025\,\sqrt{5}-6765\right)}\right)\,\sqrt{50\,\sqrt{5}-110}\,1{}\mathrm{i}}{20}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{50\,\sqrt{5}+110}\,1155{}\mathrm{i}}{2\,\left(3025\,\sqrt{5}+6765\right)}+\frac{\sqrt{5}\,x\,\sqrt{50\,\sqrt{5}+110}\,517{}\mathrm{i}}{2\,\left(3025\,\sqrt{5}+6765\right)}\right)\,\sqrt{50\,\sqrt{5}+110}\,1{}\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (atan((x*(110 - 50*5^(1/2))^(1/2)*1155i)/(2*(3025*5^(1/2) - 6765)) - (5^(1/
2)*x*(110 - 50*5^(1/2))^(1/2)*517i)/(2*(3025*5^(1/2) - 6765)))*(110 - 50*5^
(1/2))^(1/2)*1i)/20 - (atan((x*(- 50*5^(1/2) - 110)^(1/2)*1155i)/(2*(3025*5
^(1/2) + 6765)) + (5^(1/2)*x*(- 50*5^(1/2) - 110)^(1/2)*517i)/(2*(3025*5^(1
/2) + 6765)))*(- 50*5^(1/2) - 110)^(1/2)*1i)/20 + (atan((x*(50*5^(1/2) - 11
0)^(1/2)*1155i)/(2*(3025*5^(1/2) - 6765)) - (5^(1/2)*x*(50*5^(1/2) - 110)^(
1/2)*517i)/(2*(3025*5^(1/2) - 6765)))*(50*5^(1/2) - 110)^(1/2)*1i)/20 - (at
an((x*(50*5^(1/2) + 110)^(1/2)*1155i)/(2*(3025*5^(1/2) + 6765)) + (5^(1/2)*
x*(50*5^(1/2) + 110)^(1/2)*517i)/(2*(3025*5^(1/2) + 6765)))*(50*5^(1/2) + 1
10)^(1/2)*1i)/20 - 1/x
```

3.402 $\int \frac{1}{x^4(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=182

$$-\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{\sqrt[4]{\frac{1}{2}\left(84\right.}}{}$$

```
[Out] -1/3/x^3-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(843-377*5^(1/2))^(1/
4)*2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(843-377*5
^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*
(843+377*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctanh(1/2*x*(3+5^(1/2))^(1/4
)*2^(3/4))*(843+377*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1382, 1436, 218, 212, 209}

$$-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\text{ArcTan}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\ 2^{3/4}\sqrt{5}}-\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\,\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\left(3+\sqrt{5}\right)^{7/4}\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{4\ 2^{3/4}\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1/4)*
x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTan[((3 + Sqrt[5])/2)^(1/4)*x])/(
4*2^(3/4)*Sqrt[5]) - (((843 - 377*Sqrt[5])/2)^(1/4)*ArcTanh[(2/(3 + Sqrt[5]
))^(1/4)*x])/(2*Sqrt[5]) + ((3 + Sqrt[5])^(7/4)*ArcTanh[((3 + Sqrt[5])/2)^(
1/4)*x])/(4*2^(3/4)*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{3x^3}+\frac{1}{3}\int \frac{9-3x^4}{1-3x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}+\frac{1}{10}\left(-5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}$$

$$= -\frac{1}{3x^3}+\frac{\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}\,x^2}\,dx}{10\sqrt{3+\sqrt{5}}}+\frac{\left(5-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}+\sqrt{2}}{10\sqrt{3+\sqrt{5}}}$$

$$= -\frac{1}{3x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(843-377\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\sqrt[4]{\frac{1}{2}\left(843+377\sqrt{5}\right)}$$

Mathematica [A]
time = 0.16, size = 166, normalized size = 0.91

$$-\frac{1}{3x^3}+\frac{\left(2+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\left(-2+\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}+\frac{\left(2+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\left(-2+\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 + ((2 + Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 +
 Sqrt[5])] - ((-2 + Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 +
Sqrt[5])] + ((2 + Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x])/Sqrt[10*(-1 +
 Sqrt[5])] - ((-2 + Sqrt[5])*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*x])/Sqrt[10*(1 +
 Sqrt[5])]
```

Maple [A]
time = 0.05, size = 135, normalized size = 0.74

method	result
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-145\_Z^2-1\right)}\_R\ln\left(35\_R^3-199\_R+13x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+145\_Z^2-1\right)}\_R\ln\left(-35\_\right.\right.}{4}$
default	$\frac{\sqrt{5}\left(2+\sqrt{5}\right)\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{5\sqrt{2\sqrt{5}-2}}-\frac{\left(-2+\sqrt{5}\right)\sqrt{5}\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{5\sqrt{2\sqrt{5}+2}}-\frac{\left(-2+\sqrt{5}\right)\sqrt{5}\ \mathrm{arctan}}{5\sqrt{2\sqrt{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*5^(1/2)*(2+5^(1/2))/(2*5^(1/2)-2)^(1/2)*arctanh(2*x/(2*5^(1/2)-2)^(1/2)
)-1/5*(-2+5^(1/2))*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+2)^(1/
2))-1/5*(-2+5^(1/2))*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^(1/2)+2)^
(1/2))+1/5*5^(1/2)*(2+5^(1/2))/(2*5^(1/2)-2)^(1/2)*arctan(2*x/(2*5^(1/2)-2)
^(1/2))-1/3/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - 1/2*integrate((2*x^2 + 3)/(x^4 + x^2 - 1), x) + 1/2*integrate((2
*x^2 - 3)/(x^4 - x^2 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(128) = 256.
time = 0.37, size = 335, normalized size = 1.84

$$\frac{12\sqrt{10}x^3\sqrt{13\sqrt{5}-29}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{2}\sqrt{2x^2+\sqrt{5}+1}\sqrt{13\sqrt{5}-29}\left(2\sqrt{5}+5\right)-\frac{1}{10}\sqrt{10}\left(2\sqrt{5}x+5x\right)\sqrt{13\sqrt{5}-29}\right)+12\sqrt{10}x^3\sqrt{13\sqrt{5}+29}\arctan\left(\frac{1}{20}\sqrt{10}\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\sqrt{13\sqrt{5}+29}\left(2\sqrt{5}-5\right)-\frac{1}{10}\sqrt{10}\left(2\sqrt{5}x-5x\right)\sqrt{13\sqrt{5}+29}\right)-3\sqrt{10}x^3\sqrt{13\sqrt{5}-29}\log\left(\sqrt{10}\sqrt{13\sqrt{5}-29}\left(7\sqrt{5}+15\right)+20x\right)+3\sqrt{10}x^3\sqrt{13\sqrt{5}-29}\log\left(-\sqrt{10}\sqrt{13\sqrt{5}-29}\left(7\sqrt{5}+15\right)+20x\right)+3\sqrt{10}x^3\sqrt{13\sqrt{5}+29}\log\left(\sqrt{10}\sqrt{13\sqrt{5}+29}\left(7\sqrt{5}-15\right)+20x\right)-3\sqrt{10}x^3\sqrt{13\sqrt{5}+29}\log\left(-\sqrt{10}\sqrt{13\sqrt{5}+29}\left(7\sqrt{5}-15\right)+20x\right)-40}{120x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/120*(12*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29)*arctan(1/20*sqrt(10)*sqrt(2)*s
qrt(2*x^2 + sqrt(5) + 1)*sqrt(13*sqrt(5) - 29)*(2*sqrt(5) + 5) - 1/10*sqrt(
10)*(2*sqrt(5)*x + 5*x)*sqrt(13*sqrt(5) - 29)) + 12*sqrt(10)*x^3*sqrt(13*sq
rt(5) + 29)*arctan(1/20*sqrt(10)*sqrt(2)*sqrt(2*x^2 + sqrt(5) - 1)*sqrt(13*
sqrt(5) + 29)*(2*sqrt(5) - 5) - 1/10*sqrt(10)*(2*sqrt(5)*x - 5*x)*sqrt(13*s
qrt(5) + 29)) - 3*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29)*log(sqrt(10)*sqrt(13*s
qrt(5) - 29)*(7*sqrt(5) + 15) + 20*x) + 3*sqrt(10)*x^3*sqrt(13*sqrt(5) - 29
)*log(-sqrt(10)*sqrt(13*sqrt(5) - 29)*(7*sqrt(5) + 15) + 20*x) + 3*sqrt(10)
*x^3*sqrt(13*sqrt(5) + 29)*log(sqrt(10)*sqrt(13*sqrt(5) + 29)*(7*sqrt(5) -
15) + 20*x) - 3*sqrt(10)*x^3*sqrt(13*sqrt(5) + 29)*log(-sqrt(10)*sqrt(13*sq
rt(5) + 29)*(7*sqrt(5) - 15) + 20*x) - 40)/x^3
```

Sympy [A]
time = 0.77, size = 63, normalized size = 0.35

$$\operatorname{RootSum}\left(6400t^4-2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+2320t^2-1,\left(t\mapsto t\log\left(\frac{179200t^5}{377}-\frac{23112t}{377}+x\right)\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 2320*_t**2 - 1, Lambda(_t, _t*log(179200*_t**5/377 - 2
3112*_t/377 + x))) + RootSum(6400*_t**4 + 2320*_t**2 - 1, Lambda(_t, _t*log
(179200*_t**5/377 - 23112*_t/377 + x))) - 1/(3*x**3)
```

Giac [A]
time = 3.42, size = 152, normalized size = 0.84

$$-\frac{1}{20}\sqrt{130\sqrt{5}-290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{130\sqrt{5}+290}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{130\sqrt{5}-290}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{130\sqrt{5}-290}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{130\sqrt{5}+290}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{130\sqrt{5}+290}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(130*sqrt(5) - 290)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqrt
(130*sqrt(5) + 290)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(130*sqrt(
5) - 290)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5) - 2
90)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(130*sqrt(5) + 290)*lo
g(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(130*sqrt(5) + 290)*log(abs(
x - sqrt(1/2*sqrt(5) - 1/2))) - 1/3/x^3
```

Mupad [B]
time = 0.20, size = 268, normalized size = 1.47

$$\frac{\mathrm{atan}\left(\frac{x\sqrt{-130\sqrt{5}-290}\,20735\mathrm{i}}{2\left(87841\sqrt{5}+196417\right)}+\frac{\sqrt{5}\,x\sqrt{-130\sqrt{5}-290}\,46371\mathrm{i}}{10\left(87841\sqrt{5}+196417\right)}\right)\sqrt{-130\sqrt{5}-290}\,1\mathrm{i}}{20}+\frac{\mathrm{atan}\left(\frac{x\sqrt{290-130\sqrt{5}}\,20735\mathrm{i}}{2\left(87841\sqrt{5}-196417\right)}-\frac{\sqrt{5}\,x\sqrt{290-130\sqrt{5}}\,46371\mathrm{i}}{10\left(87841\sqrt{5}-196417\right)}\right)\sqrt{290-130\sqrt{5}}\,1\mathrm{i}}{20}-\frac{1}{3\,x^3}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\sqrt{13\sqrt{5}-29}\,20735\mathrm{i}}{2\left(87841\sqrt{5}-196417\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\sqrt{13\sqrt{5}-29}\,46371\mathrm{i}}{10\left(87841\sqrt{5}-196417\right)}\right)\sqrt{13\sqrt{5}-29}\,1\mathrm{i}}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\sqrt{13\sqrt{5}+29}\,20735\mathrm{i}}{2\left(87841\sqrt{5}+196417\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\sqrt{13\sqrt{5}+29}\,46371\mathrm{i}}{10\left(87841\sqrt{5}+196417\right)}\right)\sqrt{13\sqrt{5}+29}\,1\mathrm{i}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(x^8 - 3*x^4 + 1)),x)

[Out] (atan((x*(- 130*5^(1/2) - 290)^(1/2)*20735i)/(2*(87841*5^(1/2) + 196417)) + (5^(1/2)*x*(- 130*5^(1/2) - 290)^(1/2)*46371i)/(10*(87841*5^(1/2) + 196417)))*(- 130*5^(1/2) - 290)^(1/2)*1i)/20 + (atan((x*(290 - 130*5^(1/2))^(1/2) *20735i)/(2*(87841*5^(1/2) - 196417)) - (5^(1/2)*x*(290 - 130*5^(1/2))^(1/2)*46371i)/(10*(87841*5^(1/2) - 196417)))*(290 - 130*5^(1/2))^(1/2)*1i)/20 - 1/(3*x^3) - (10^(1/2)*atan((10^(1/2)*x*(13*5^(1/2) - 29)^(1/2)*20735i)/(2* (87841*5^(1/2) - 196417)) - (5^(1/2)*10^(1/2)*x*(13*5^(1/2) - 29)^(1/2)*463 71i)/(10*(87841*5^(1/2) - 196417)))*(13*5^(1/2) - 29)^(1/2)*1i)/20 - (10^(1 /2)*atan((10^(1/2)*x*(13*5^(1/2) + 29)^(1/2)*20735i)/(2*(87841*5^(1/2) + 19 6417)) + (5^(1/2)*10^(1/2)*x*(13*5^(1/2) + 29)^(1/2)*46371i)/(10*(87841*5^(1/2) + 196417)))*(13*5^(1/2) + 29)^(1/2)*1i)/20

3.403 $\int \frac{1}{x^6(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=173

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{2889-1292\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889+1292\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-$$

```
[Out] -1/5/x^5-3/x+1/10*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(2889-1292*5^(1/2
))^(1/4)*5^(1/2)-1/10*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(2889-1292*5
^(1/2))^(1/4)*5^(1/2)-1/10*arctan(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(2889+12
92*5^(1/2))^(1/4)*5^(1/2)+1/10*arctanh(1/2*x*(3+5^(1/2))^(1/4)*2^(3/4))*(28
89+1292*5^(1/2))^(1/4)*5^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1382, 1518, 1524, 304, 209, 212}

$$\frac{\sqrt[4]{2889-1292\sqrt{5}}\,\text{ArcTan}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}-\frac{\sqrt[4]{2889+1292\sqrt{5}}\,\text{ArcTan}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{1}{5x^5}-\frac{3}{x}-\frac{\sqrt[4]{2889-1292\sqrt{5}}\,\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{2889+1292\sqrt{5}}\,\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - 3/x + ((2889 - 1292*Sqrt[5])^(1/4)*ArcTan[(2/(3 + Sqrt[5]))^(1
/4)*x])/(2*Sqrt[5]) - ((2889 + 1292*Sqrt[5])^(1/4)*ArcTan[((3 + Sqrt[5])/2)
^(1/4)*x])/(2*Sqrt[5]) - ((2889 - 1292*Sqrt[5])^(1/4)*ArcTanh[(2/(3 + Sqrt[
5]))^(1/4)*x])/(2*Sqrt[5]) + ((2889 + 1292*Sqrt[5])^(1/4)*ArcTanh[((3 + Sqr
t[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
```

```
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^6\left(1-3x^4+x^8\right)}\,dx = -\frac{1}{5x^5}+\frac{1}{5}\int \frac{15-5x^4}{x^2\left(1-3x^4+x^8\right)}\,dx$$
$$= -\frac{1}{5x^5}-\frac{3}{x}-\frac{1}{5}\int \frac{x^2(-40+15x^4)}{1-3x^4+x^8}\,dx$$
$$= -\frac{1}{5x^5}-\frac{3}{x}-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{x^2}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{}{-\frac{3}{2}+}$$
$$= -\frac{1}{5x^5}-\frac{3}{x}-\frac{\left(7-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}\,x^2}\,dx}{2\sqrt{10}}+\frac{\left(7-3\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}}}{2\sqrt{10}}$$
$$= -\frac{1}{5x^5}-\frac{3}{x}+\frac{\sqrt[4]{46224-20672\sqrt{5}}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{4\sqrt{5}}-\frac{\sqrt[4]{46224+20672}}{}$$

Mathematica [A]
time = 0.16, size = 189, normalized size = 1.09

$$-\frac{1}{5x^5}-\frac{3}{x}+\frac{\left(-7-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\left(7-3\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\left(-7-3\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\left(7-3\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/5*1/x^5 - 3/x + ((-7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(2*S
qrt[10*(-1 + Sqrt[5])]) + ((7 - 3*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x])
/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(-1 + Sqrt[5
])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((7 - 3*Sqrt[5])*ArcTanh[Sqrt[2/(1 +
Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

Maple [A]
time = 0.06, size = 148, normalized size = 0.86

method	result
risch	$\frac{-3x^4-\frac{1}{5}}{x^5}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-380\_Z^2-1\right)}\_R\ln\left(-55\_R^3+843\_R+34x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+380\_Z^2-1\right)}\_R\ln\right.}{4}$
default	$\frac{\left(7+3\sqrt{5}\right)\sqrt{5}\,\mathrm{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}+\frac{\left(-7+3\sqrt{5}\right)\sqrt{5}}{10\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(7+3*5^(1/2))*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctanh(2*x/(2*5^(1/2)-2)^(1
/2))-1/10*(-7+3*5^(1/2))*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2)+
2)^(1/2))+1/10*(-7+3*5^(1/2))*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2*5^
(1/2)+2)^(1/2))-1/10*(7+3*5^(1/2))*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctan(2*x/(
2*5^(1/2)-2)^(1/2))-1/5/x^5-3/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/5*(15*x^4 + 1)/x^5 - 1/2*integrate((3*x^2 + 5)/(x^4 + x^2 - 1), x) - 1/2
*integrate((3*x^2 - 5)/(x^4 - x^2 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(121) = 242$.
time = 0.38, size = 292, normalized size = 1.69

$$-\frac{4\sqrt{5}x^5\sqrt{17\sqrt{5}+38}\arctan\left(-\frac{1}{4}\left(6\sqrt{5}x-\sqrt{4x^2+2\sqrt{5}-2}\left(3\sqrt{5}-7\right)-14x\right)\sqrt{17\sqrt{5}+38}\right)+4\sqrt{5}x^5\sqrt{17\sqrt{5}-38}\arctan\left(-\frac{1}{4}\left(6\sqrt{5}x-\sqrt{4x^2+2\sqrt{5}+2}\left(3\sqrt{5}+7\right)+14x\right)\sqrt{17\sqrt{5}-38}\right)+\sqrt{5}x^5\sqrt{17\sqrt{5}-38}\log\left(\sqrt{17\sqrt{5}-38}\left(5\sqrt{5}+11\right)+2x\right)-\sqrt{5}x^5\sqrt{17\sqrt{5}-38}\log\left(-\sqrt{17\sqrt{5}-38}\left(5\sqrt{5}+11\right)+2x\right)-\sqrt{5}x^5\sqrt{17\sqrt{5}+38}\log\left(\sqrt{17\sqrt{5}+38}\left(5\sqrt{5}-11\right)+2x\right)+\sqrt{5}x^5\sqrt{17\sqrt{5}+38}\log\left(-\sqrt{17\sqrt{5}+38}\left(5\sqrt{5}-11\right)+2x\right)+60x^4+4}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] -1/20*(4*sqrt(5)*x^5*sqrt(17*sqrt(5) + 38)*arctan(-1/4*(6*sqrt(5)*x - sqrt(
4*x^2 + 2*sqrt(5) - 2)*(3*sqrt(5) - 7) - 14*x)*sqrt(17*sqrt(5) + 38)) + 4*s
qrt(5)*x^5*sqrt(17*sqrt(5) - 38)*arctan(-1/4*(6*sqrt(5)*x - sqrt(4*x^2 + 2*
sqrt(5) + 2)*(3*sqrt(5) + 7) + 14*x)*sqrt(17*sqrt(5) - 38)) + sqrt(5)*x^5*s
qrt(17*sqrt(5) - 38)*log(sqrt(17*sqrt(5) - 38)*(5*sqrt(5) + 11) + 2*x) - sq
rt(5)*x^5*sqrt(17*sqrt(5) - 38)*log(-sqrt(17*sqrt(5) - 38)*(5*sqrt(5) + 11)
 + 2*x) - sqrt(5)*x^5*sqrt(17*sqrt(5) + 38)*log(sqrt(17*sqrt(5) + 38)*(5*sq
rt(5) - 11) + 2*x) + sqrt(5)*x^5*sqrt(17*sqrt(5) + 38)*log(-sqrt(17*sqrt(5)
 + 38)*(5*sqrt(5) - 11) + 2*x) + 60*x^4 + 4)/x^5
```

Sympy [A]
time = 0.75, size = 73, normalized size = 0.42

$$\operatorname{RootSum}\left(6400t^4-6080t^2-1,\left(t\mapsto t\log\left(\frac{215808000t^7}{323}-\frac{194833880t^3}{323}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+6080t^2-1,\left(t\mapsto t\log\left(\frac{215808000t^7}{323}-\frac{194833880t^3}{323}+x\right)\right)\right)+\frac{-15x^4-1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 6080*_t**2 - 1, Lambda(_t, _t*log(215808000*_t**7/323
- 194833880*_t**3/323 + x))) + RootSum(6400*_t**4 + 6080*_t**2 - 1, Lambda(
_t, _t*log(215808000*_t**7/323 - 194833880*_t**3/323 + x))) + (-15*x**4 - 1
)/(5*x**5)
```

Giac [A]
time = 3.96, size = 159, normalized size = 0.92

$$\frac{1}{10}\sqrt{85\sqrt{5}-190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)-\frac{1}{10}\sqrt{85\sqrt{5}+190}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{20}\sqrt{85\sqrt{5}-190}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{85\sqrt{5}-190}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{20}\sqrt{85\sqrt{5}+190}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{20}\sqrt{85\sqrt{5}+190}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{15x^4+1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(85*sqrt(5) - 190)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/10*sqrt(8
5*sqrt(5) + 190)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/20*sqrt(85*sqrt(5) -
 190)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) - 190)*l
og(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/20*sqrt(85*sqrt(5) + 190)*log(abs(
x + sqrt(1/2*sqrt(5) - 1/2))) - 1/20*sqrt(85*sqrt(5) + 190)*log(abs(x - sqr
t(1/2*sqrt(5) - 1/2))) - 1/5*(15*x^4 + 1)/x^5
```

Mupad [B]
time = 1.49, size = 257, normalized size = 1.49

$$-\frac{3x^4+\frac{1}{5}}{x^5}-\frac{\mathrm{atan}\left(\frac{x\sqrt{-85\sqrt{5}-190}\,372096\mathrm{i}}{2550408\sqrt{5}+5702888}+\frac{\sqrt{5}\,x\sqrt{-85\sqrt{5}-190}\,832048\mathrm{i}}{5\left(2550408\sqrt{5}+5702888\right)}\right)\sqrt{-85\sqrt{5}-190}\,1\mathrm{i}}{10}+\frac{\mathrm{atan}\left(\frac{x\sqrt{190-85\sqrt{5}}\,372096\mathrm{i}}{2550408\sqrt{5}-5702888}-\frac{\sqrt{5}\,x\sqrt{190-85\sqrt{5}}\,832048\mathrm{i}}{5\left(2550408\sqrt{5}-5702888\right)}\right)\sqrt{190-85\sqrt{5}}\,1\mathrm{i}}{10}+\frac{\mathrm{atan}\left(\frac{x\sqrt{85\sqrt{5}-190}\,372096\mathrm{i}}{2550408\sqrt{5}-5702888}-\frac{\sqrt{5}\,x\sqrt{85\sqrt{5}-190}\,832048\mathrm{i}}{5\left(2550408\sqrt{5}-5702888\right)}\right)\sqrt{85\sqrt{5}-190}\,1\mathrm{i}}{10}-\frac{\mathrm{atan}\left(\frac{x\sqrt{85\sqrt{5}+190}\,372096\mathrm{i}}{2550408\sqrt{5}+5702888}+\frac{\sqrt{5}\,x\sqrt{85\sqrt{5}+190}\,832048\mathrm{i}}{5\left(2550408\sqrt{5}+5702888\right)}\right)\sqrt{85\sqrt{5}+190}\,1\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (atan((x*(190 - 85*5^(1/2))^(1/2)*372096i)/(2550408*5^(1/2) - 5702888) - (5
^(1/2)*x*(190 - 85*5^(1/2))^(1/2)*832048i)/(5*(2550408*5^(1/2) - 5702888)))
*(190 - 85*5^(1/2))^(1/2)*1i)/10 - (atan((x*(- 85*5^(1/2) - 190)^(1/2)*3720
96i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(- 85*5^(1/2) - 190)^(1/2)*83
2048i)/(5*(2550408*5^(1/2) + 5702888)))*(- 85*5^(1/2) - 190)^(1/2)*1i)/10 +
 (atan((x*(85*5^(1/2) - 190)^(1/2)*372096i)/(2550408*5^(1/2) - 5702888) - (
5^(1/2)*x*(85*5^(1/2) - 190)^(1/2)*832048i)/(5*(2550408*5^(1/2) - 5702888))
)*(85*5^(1/2) - 190)^(1/2)*1i)/10 - (atan((x*(85*5^(1/2) + 190)^(1/2)*37209
6i)/(2550408*5^(1/2) + 5702888) + (5^(1/2)*x*(85*5^(1/2) + 190)^(1/2)*83204
8i)/(5*(2550408*5^(1/2) + 5702888)))*(85*5^(1/2) + 190)^(1/2)*1i)/10 - (3*x
^4 + 1/5)/x^5
```

3.404 $\int \frac{1}{x^8(1-3x^4+x^8)}\,dx$

Optimal. Leaf size=189

$$-\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\,\tan^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3\right.}\right.}{2\sqrt{5}}$$

```
[Out] -1/7/x^7-1/x^3-1/20*arctan(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*(39603-17711*5^
(1/2))^(1/4)*2^(3/4)*5^(1/2)-1/20*arctanh(2^(1/4)*x*(1/(3+5^(1/2)))^(1/4))*
(39603-17711*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctan(1/2*x*(3+5^(1/2))^(
1/4)*2^(3/4))*(39603+17711*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)+1/20*arctanh(1/2*
x*(3+5^(1/2))^(1/4)*2^(3/4))*(39603+17711*5^(1/2))^(1/4)*2^(3/4)*5^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1382, 1518, 1436, 218, 212, 209}

$$-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\,\text{ArcTan}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}-\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\,\tanh^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603+17711\sqrt{5}\right)}\,\tanh^{-1}\left(\sqrt[4]{\frac{1}{2}\left(3+\sqrt{5}\right)}\,x\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(1 - 3*x^4 + x^8)),x]
```

```
[Out] -1/7*1/x^7 - x^(-3) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*ArcTan[(2/(3 + Sqr
t[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])/2)^(1/4)*ArcTan[((
3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5]) - (((39603 - 17711*Sqrt[5])/2)^(1/4)*
ArcTanh[(2/(3 + Sqrt[5]))^(1/4)*x])/(2*Sqrt[5]) + (((39603 + 17711*Sqrt[5])
/2)^(1/4)*ArcTanh[((3 + Sqrt[5])/2)^(1/4)*x])/(2*Sqrt[5])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
```

```
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1382

```
Int[((d_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_
Symbol] :> Simp[(d*x)^(m + 1)*((a + b*x^n + c*x^(2*n))^(p + 1)/(a*d*(m + 1)
)), x] - Dist[1/(a*d^n*(m + 1)), Int[(d*x)^(m + n)*(b*(m + n*(p + 1) + 1) +
 c*(m + 2*n*(p + 1) + 1)*x^n)*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a
, b, c, d, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && L
tQ[m, -1] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1}{x^8(1-3x^4+x^8)}\,dx = -\frac{1}{7x^7}+\frac{1}{7}\int \frac{21-7x^4}{x^4(1-3x^4+x^8)}\,dx$$
$$= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{1}{21}\int \frac{-168+63x^4}{1-3x^4+x^8}\,dx$$
$$= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{1}{10}\left(15-7\sqrt{5}\right)\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^4}\,dx-\frac{1}{10}\left(15+7\sqrt{5}\right)\int \frac{}{-\frac{3}{2}+}$$
$$= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\left(-15+7\sqrt{5}\right)\int \frac{1}{\sqrt{3+\sqrt{5}}-\sqrt{2}\,x^2}\,dx}{10\sqrt{3+\sqrt{5}}}-\frac{\left(-15+7\sqrt{5}\right)\int \frac{}{\sqrt{3+}}}{10\sqrt{3+\sqrt{}}}$$
$$= -\frac{1}{7x^7}-\frac{1}{x^3}-\frac{\sqrt[4]{\frac{1}{2}\left(39603-17711\sqrt{5}\right)}\tan^{-1}\left(\sqrt[4]{\frac{2}{3+\sqrt{5}}}\,x\right)}{2\sqrt{5}}+\frac{\sqrt[4]{\frac{1}{2}\left(39603-\right.}}{}$$

Mathematica [A]
time = 0.17, size = 189, normalized size = 1.00

$$-\frac{1}{7x^7}-\frac{1}{x^3}+\frac{\left(11+5\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}}+\frac{\left(11-5\sqrt{5}\right)\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}-\frac{\left(-11-5\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(-1+\sqrt{5}\right)}}-\frac{\left(-11+5\sqrt{5}\right)\tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}\,x\right)}{2\sqrt{10\left(1+\sqrt{5}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(1 - 3*x^4 + x^8)),x]
[Out] -1/7*1/x^7 - x^(-3) + ((11 + 5*Sqrt[5])*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*x])/(
2*Sqrt[10*(-1 + Sqrt[5])]) + ((11 - 5*Sqrt[5])*ArcTan[Sqrt[2/(1 + Sqrt[5])]
*x])/(2*Sqrt[10*(1 + Sqrt[5])]) - ((-11 - 5*Sqrt[5])*ArcTanh[Sqrt[2/(-1 + S
qrt[5])]*x])/(2*Sqrt[10*(-1 + Sqrt[5])]) - ((-11 + 5*Sqrt[5])*ArcTanh[Sqrt[
2/(1 + Sqrt[5])]*x])/(2*Sqrt[10*(1 + Sqrt[5])])
```

Maple [A]
time = 0.06, size = 148, normalized size = 0.78

method	result
risch	$\frac{-x^4-\frac{1}{7}}{x^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4+995\_Z^2-1\right)}\_R\ln\left(-90\_R^3-3571\_R+89x\right)\right)}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(25\_Z^4-995\_Z^2-1\right)}\_R\ln\left(\right.\right.}{4}$

default	$\frac{\left(11+5\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{2x}{\sqrt{2\sqrt{5}-2}}\right)}{10\sqrt{2\sqrt{5}-2}}-\frac{\left(-11+5\sqrt{5}\right)\sqrt{5}\,\arctan\left(\frac{2x}{\sqrt{2\sqrt{5}+2}}\right)}{10\sqrt{2\sqrt{5}+2}}-\frac{\left(-11+5\sqrt{5}\right)\sqrt{5}}{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(x^8-3*x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(11+5*5^(1/2))*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctanh(2*x/(2*5^(1/2)-2)^(
1/2))-1/10*(-11+5*5^(1/2))*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctan(2*x/(2*5^(1/2
)+2)^(1/2))-1/10*(-11+5*5^(1/2))*5^(1/2)/(2*5^(1/2)+2)^(1/2)*arctanh(2*x/(2
*5^(1/2)+2)^(1/2))+1/10*(11+5*5^(1/2))*5^(1/2)/(2*5^(1/2)-2)^(1/2)*arctan(2
*x/(2*5^(1/2)-2)^(1/2))-1/7/x^7-1/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="maxima")
```

```
[Out] -1/7*(7*x^4 + 1)/x^7 - 1/2*integrate((5*x^2 + 8)/(x^4 + x^2 - 1), x) + 1/2*
integrate((5*x^2 - 8)/(x^4 - x^2 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(133) = 266.
time = 0.35, size = 324, normalized size = 1.71

$$\frac{28\sqrt{10}\,x^7\sqrt{89\sqrt{5}+199}\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2}\sqrt{2x^2+\sqrt{5}-1}\left(11\sqrt{5}-25\right)-2\sqrt{10}\left(11\sqrt{5}x-25x\right)\right)\sqrt{89\sqrt{5}+199}\right)+28\sqrt{10}\,x^7\sqrt{89\sqrt{5}-199}\arctan\left(\frac{1}{40}\left(\sqrt{10}\sqrt{2}\sqrt{2x^2+\sqrt{5}+1}\left(11\sqrt{5}+25\right)-2\sqrt{10}\left(11\sqrt{5}x+25x\right)\right)\sqrt{89\sqrt{5}-199}\right)-7\sqrt{10}\,x^7\sqrt{89\sqrt{5}-199}\log\left(\sqrt{10}\sqrt{89\sqrt{5}-199}\left(9\sqrt{5}+20\right)+10x\right)+7\sqrt{10}\,x^7\sqrt{89\sqrt{5}-199}\log\left(-\sqrt{10}\sqrt{89\sqrt{5}-199}\left(9\sqrt{5}+20\right)+10x\right)+7\sqrt{10}\,x^7\sqrt{89\sqrt{5}+199}\log\left(\sqrt{10}\sqrt{89\sqrt{5}+199}\left(9\sqrt{5}-20\right)+10x\right)-7\sqrt{10}\,x^7\sqrt{89\sqrt{5}+199}\log\left(-\sqrt{10}\sqrt{89\sqrt{5}+199}\left(9\sqrt{5}-20\right)+10x\right)-280x^4-40}{280x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="fricas")
```

```
[Out] 1/280*(28*sqrt(10)*x^7*sqrt(89*sqrt(5) + 199)*arctan(1/40*(sqrt(10)*sqrt(2)
*sqrt(2*x^2 + sqrt(5) - 1)*(11*sqrt(5) - 25) - 2*sqrt(10)*(11*sqrt(5)*x - 2
5*x))*sqrt(89*sqrt(5) + 199)) + 28*sqrt(10)*x^7*sqrt(89*sqrt(5) - 199)*arct
an(1/40*(sqrt(10)*sqrt(2)*sqrt(2*x^2 + sqrt(5) + 1)*(11*sqrt(5) + 25) - 2*s
qrt(10)*(11*sqrt(5)*x + 25*x))*sqrt(89*sqrt(5) - 199)) - 7*sqrt(10)*x^7*sqr
t(89*sqrt(5) - 199)*log(sqrt(10)*sqrt(89*sqrt(5) - 199)*(9*sqrt(5) + 20) +
10*x) + 7*sqrt(10)*x^7*sqrt(89*sqrt(5) - 199)*log(-sqrt(10)*sqrt(89*sqrt(5)
 - 199)*(9*sqrt(5) + 20) + 10*x) + 7*sqrt(10)*x^7*sqrt(89*sqrt(5) + 199)*lo
g(sqrt(10)*sqrt(89*sqrt(5) + 199)*(9*sqrt(5) - 20) + 10*x) - 7*sqrt(10)*x^7
*sqrt(89*sqrt(5) + 199)*log(-sqrt(10)*sqrt(89*sqrt(5) + 199)*(9*sqrt(5) - 2
0) + 10*x) - 280*x^4 - 40)/x^7
```

Sympy [A]

time = 0.86, size = 70, normalized size = 0.37

$$\operatorname{RootSum}\left(6400t^4-15920t^2-1,\left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right)+\operatorname{RootSum}\left(6400t^4+15920t^2-1,\left(t\mapsto t\log\left(\frac{460800t^5}{17711}-\frac{2842588t}{17711}+x\right)\right)\right)+\frac{-7x^4-1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(x**8-3*x**4+1),x)
```

```
[Out] RootSum(6400*_t**4 - 15920*_t**2 - 1, Lambda(_t, _t*log(460800*_t**5/17711
- 2842588*_t/17711 + x))) + RootSum(6400*_t**4 + 15920*_t**2 - 1, Lambda(_t
, _t*log(460800*_t**5/17711 - 2842588*_t/17711 + x))) + (-7*x**4 - 1)/(7*x*
*7)
```

Giac [A]

time = 4.04, size = 159, normalized size = 0.84

$$-\frac{1}{20}\sqrt{890\sqrt{5}-1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{20}\sqrt{890\sqrt{5}+1990}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}}\right)-\frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{890\sqrt{5}-1990}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right|\right)+\frac{1}{40}\sqrt{890\sqrt{5}+1990}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{1}{40}\sqrt{890\sqrt{5}+1990}\log\left(\left|x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right|\right)-\frac{7x^4+1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(x^8-3*x^4+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(890*sqrt(5) - 1990)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/20*sqr
t(890*sqrt(5) + 1990)*arctan(x/sqrt(1/2*sqrt(5) - 1/2)) - 1/40*sqrt(890*sqr
t(5) - 1990)*log(abs(x + sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(5)
- 1990)*log(abs(x - sqrt(1/2*sqrt(5) + 1/2))) + 1/40*sqrt(890*sqrt(5) + 199
0)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/40*sqrt(890*sqrt(5) + 1990)*lo
g(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/7*(7*x^4 + 1)/x^7
```

Mupad [B]

time = 0.21, size = 291, normalized size = 1.54

$$-\frac{x^4+\frac{1}{7}}{x^7}+\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{-89\sqrt{5}-199}\,6677047i}{2\left(74049691\sqrt{5}+165580139\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{-89\sqrt{5}-199}\,14930373i}{10\left(74049691\sqrt{5}+165580139\right)}\right)\sqrt{-89\sqrt{5}-199}\,1i}{20}+\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{199-89\sqrt{5}}\,6677047i}{2\left(74049691\sqrt{5}-165580139\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{199-89\sqrt{5}}\,14930373i}{10\left(74049691\sqrt{5}-165580139\right)}\right)\sqrt{199-89\sqrt{5}}\,1i}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{89\sqrt{5}-199}\,6677047i}{2\left(74049691\sqrt{5}-165580139\right)}-\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{89\sqrt{5}-199}\,14930373i}{10\left(74049691\sqrt{5}-165580139\right)}\right)\sqrt{89\sqrt{5}-199}\,1i}{20}-\frac{\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\,x\,\sqrt{89\sqrt{5}+199}\,6677047i}{2\left(74049691\sqrt{5}+165580139\right)}+\frac{\sqrt{5}\,\sqrt{10}\,x\,\sqrt{89\sqrt{5}+199}\,14930373i}{10\left(74049691\sqrt{5}+165580139\right)}\right)\sqrt{89\sqrt{5}+199}\,1i}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(x^8 - 3*x^4 + 1)),x)
```

```
[Out] (10^(1/2)*atan((10^(1/2)*x*(- 89*5^(1/2) - 199)^(1/2)*6677047i)/(2*(7404969
1*5^(1/2) + 165580139)) + (5^(1/2)*10^(1/2)*x*(- 89*5^(1/2) - 199)^(1/2)*14
930373i)/(10*(74049691*5^(1/2) + 165580139)))*(- 89*5^(1/2) - 199)^(1/2)*1i
)/20 - (x^4 + 1/7)/x^7 + (10^(1/2)*atan((10^(1/2)*x*(199 - 89*5^(1/2))^(1/2
)*6677047i)/(2*(74049691*5^(1/2) - 165580139)) - (5^(1/2)*10^(1/2)*x*(199 -
 89*5^(1/2))^(1/2)*14930373i)/(10*(74049691*5^(1/2) - 165580139)))*(199 - 8
9*5^(1/2))^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(89*5^(1/2) - 199)^(1/
2)*6677047i)/(2*(74049691*5^(1/2) - 165580139)) - (5^(1/2)*10^(1/2)*x*(89*5
^(1/2) - 199)^(1/2)*14930373i)/(10*(74049691*5^(1/2) - 165580139)))*(89*5^(
1/2) - 199)^(1/2)*1i)/20 - (10^(1/2)*atan((10^(1/2)*x*(89*5^(1/2) + 199)^(1
/2)*6677047i)/(2*(74049691*5^(1/2) + 165580139)) + (5^(1/2)*10^(1/2)*x*(89*
5^(1/2) + 199)^(1/2)*14930373i)/(10*(74049691*5^(1/2) + 165580139)))*(89*5^
(1/2) + 199)^(1/2)*1i)/20
```

3.405 $\int \frac{x^3}{2+3x^4+x^8}\,dx$

Optimal. Leaf size=21

$$\frac{1}{4}\log\left(1+x^4\right)-\frac{1}{4}\log\left(2+x^4\right)$$

```
[Out] 1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

Rubi [A]

time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1366, 630, 31}

$$\frac{1}{4}\log\left(x^4+1\right)-\frac{1}{4}\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 + 3*x^4 + x^8),x]
```

```
[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{2+3x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{2+3x+x^2}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^4\right)-\frac{1}{4}\text{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^4\right)\\
&= \frac{1}{4}\log\left(1+x^4\right)-\frac{1}{4}\log\left(2+x^4\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\frac{1}{4}\log\left(1+x^4\right)-\frac{1}{4}\log\left(2+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 + 3*x^4 + x^8),x]
[Out] Log[1 + x^4]/4 - Log[2 + x^4]/4
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.86

method	result	size
default	$\frac{\ln(x^4+1)}{4}-\frac{\ln(x^4+2)}{4}$	18
norman	$\frac{\ln(x^4+1)}{4}-\frac{\ln(x^4+2)}{4}$	18
risch	$\frac{\ln(x^4+1)}{4}-\frac{\ln(x^4+2)}{4}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^8+3*x^4+2),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x^4+1)-1/4*ln(x^4+2)
```

Maxima [A]
time = 0.29, size = 17, normalized size = 0.81

$$-\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="maxima")
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Fricas [A]
time = 0.33, size = 17, normalized size = 0.81

$$-\frac{1}{4}\log\left(x^4+2\right)+\frac{1}{4}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="fricas")
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Sympy [A]
time = 0.04, size = 15, normalized size = 0.71

$$\frac{\log\left(x^{4}+1\right)}{4}-\frac{\log\left(x^{4}+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**8+3*x**4+2),x)
[Out] log(x**4 + 1)/4 - log(x**4 + 2)/4
```

Giac [A]
time = 3.93, size = 17, normalized size = 0.81

$$-\frac{1}{4}\log\left(x^{4}+2\right)+\frac{1}{4}\log\left(x^{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^8+3*x^4+2),x, algorithm="giac")
[Out] -1/4*log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Mupad [B]
time = 0.06, size = 16, normalized size = 0.76

$$-\frac{\operatorname{atanh}\left(\frac{256}{9\,\left(144\,x^4+160\right)}-\frac{7}{9}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4 + x^8 + 2),x)
[Out] -atanh(256/(9*(144*x^4 + 160)) - 7/9)/2
```

3.406 $\int \frac{x^{11}}{2+3x^4+x^8}\,dx$

Optimal. Leaf size=26

$$\frac{x^4}{4}+\frac{1}{4}\log\left(1+x^4\right)-\log\left(2+x^4\right)$$

[Out] 1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 717, 646, 31}

$$\frac{x^4}{4}+\frac{1}{4}\log\left(x^4+1\right)-\log\left(x^4+2\right)$$

Antiderivative was successfully verified.

[In] Int[x^11/(2 + 3*x^4 + x^8),x]

[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
```

```
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{2+3x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{2+3x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{4}\text{Subst}\left(\int \frac{-2-3x}{2+3x+x^2}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^4\right)-\text{Subst}\left(\int \frac{1}{2+x}\,dx,x,x^4\right)\\ &= \frac{x^4}{4}+\frac{1}{4}\log\left(1+x^4\right)-\log\left(2+x^4\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.00

$$\frac{x^4}{4}+\frac{1}{4}\log\left(1+x^4\right)-\log\left(2+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(2 + 3*x^4 + x^8),x]
[Out] x^4/4 + Log[1 + x^4]/4 - Log[2 + x^4]
```

Maple [A]
time = 0.02, size = 23, normalized size = 0.88

method	result	size
default	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23
norman	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23
risch	$\frac{x^4}{4}+\frac{\ln(x^4+1)}{4}-\ln\left(x^4+2\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(x^8+3*x^4+2),x,method=_RETURNVERBOSE)
[Out] 1/4*x^4+1/4*ln(x^4+1)-ln(x^4+2)
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.85

$$\frac{1}{4}\,x^4-\log\left(x^4+2\right)+\frac{1}{4}\,\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Fricas [A]
time = 0.36, size = 22, normalized size = 0.85

$$\frac{1}{4}x^4 - \log\left(x^4 + 2\right) + \frac{1}{4}\log\left(x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Sympy [A]
time = 0.04, size = 19, normalized size = 0.73

$$\frac{x^4}{4} + \frac{\log\left(x^4 + 1\right)}{4} - \log\left(x^4 + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(x**8+3*x**4+2),x)
```

```
[Out] x**4/4 + log(x**4 + 1)/4 - log(x**4 + 2)
```

Giac [A]
time = 3.94, size = 22, normalized size = 0.85

$$\frac{1}{4}x^4 - \log\left(x^4 + 2\right) + \frac{1}{4}\log\left(x^4 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(x^8+3*x^4+2),x, algorithm="giac")
```

```
[Out] 1/4*x^4 - log(x^4 + 2) + 1/4*log(x^4 + 1)
```

Mupad [B]
time = 1.32, size = 22, normalized size = 0.85

$$\frac{\ln\left(x^4 + 1\right)}{4} - \ln\left(x^4 + 2\right) + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(3*x^4 + x^8 + 2),x)
```

```
[Out] log(x^4 + 1)/4 - log(x^4 + 2) + x^4/4
```

3.407 $\int \frac{x^9}{2+x^5+x^{10}} \, dx$

Optimal. Leaf size=37

$$-\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}+\frac{1}{10}\log\left(2+x^5+x^{10}\right)$$

```
[Out] 1/10*ln(x^10+x^5+2)-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1371, 648, 632, 210, 642}

$$\frac{1}{10}\log\left(x^{10}+x^5+2\right)-\frac{\text{ArcTan}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(2 + x^5 + x^10),x]
```

```
[Out] -1/5*ArcTan[(1 + 2*x^5)/Sqrt[7]]/Sqrt[7] + Log[2 + x^5 + x^10]/10
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^9}{2+x^5+x^{10}}\,dx &= \frac{1}{5}\text{Subst}\left(\int \frac{x}{2+x+x^2}\,dx,x,x^5\right)\\
&= -\left(\frac{1}{10}\text{Subst}\left(\int \frac{1}{2+x+x^2}\,dx,x,x^5\right)\right)+\frac{1}{10}\text{Subst}\left(\int \frac{1+2x}{2+x+x^2}\,dx,x,x^5\right)\\
&= \frac{1}{10}\log\left(2+x^5+x^{10}\right)+\frac{1}{5}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,1+2x^5\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}+\frac{1}{10}\log\left(2+x^5+x^{10}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.00

$$
-\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}+\frac{1}{10}\log\left(2+x^5+x^{10}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(2 + x^5 + x^10),x]
[Out] -1/5*ArcTan[(1 + 2*x^5)/Sqrt[7]]/Sqrt[7] + Log[2 + x^5 + x^10]/10
```

Maple [A]
time = 0.02, size = 31, normalized size = 0.84

method	result	size
default	$\frac{\ln\left(x^{10}+x^5+2\right)}{10}-\frac{\arctan\left(\frac{\left(2x^5+1\right)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	31

risch	$\frac{\ln(4x^{10}+4x^5+8)}{10}-\frac{\arctan\left(\frac{(2x^5+1)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^10+x^5+2),x,method=_RETURNVERBOSE)
[Out] 1/10*ln(x^10+x^5+2)-1/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Maxima [A]
time = 0.51, size = 30, normalized size = 0.81

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="maxima")
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

Fricas [A]
time = 0.34, size = 30, normalized size = 0.81

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="fricas")
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

Sympy [A]
time = 0.05, size = 37, normalized size = 1.00

$$\frac{\log\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**10+x**5+2),x)
[Out] log(x**10 + x**5 + 2)/10 - sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

Giac [A]
time = 4.91, size = 30, normalized size = 0.81

$$-\frac{1}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)+\frac{1}{10}\log\left(x^{10}+x^5+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^10+x^5+2),x, algorithm="giac")
```

```
[Out] -1/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1)) + 1/10*log(x^10 + x^5 + 2)
```

Mupad [B]
time = 1.35, size = 32, normalized size = 0.86

$$\frac{\ln\left(x^{10}+x^5+2\right)}{10}-\frac{\sqrt{7}\,\mathrm{atan}\left(\frac{2\,\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^5 + x^10 + 2),x)
```

```
[Out] log(x^5 + x^10 + 2)/10 - (7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35
```

3.408 $\int \frac{x^4}{2+x^5+x^{10}}\,dx$

Optimal. Leaf size=23

$$\frac{2\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}$$

```
[Out] 2/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1366, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{2x^5+1}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(2 + x^5 + x^10),x]
```

```
[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^4}{2+x^5+x^{10}}\,dx = \frac{1}{5}\text{Subst}\left(\int \frac{1}{2+x+x^2}\,dx, x, x^5\right)$$
$$= -\left(\frac{2}{5}\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, 1+2x^5\right)\right)$$
$$= \frac{2\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{1+2x^5}{\sqrt{7}}\right)}{5\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(2 + x^5 + x^10),x]
[Out] (2*ArcTan[(1 + 2*x^5)/Sqrt[7]])/(5*Sqrt[7])
```

Maple [A]
time = 0.02, size = 19, normalized size = 0.83

method	result	size
default	$\frac{2\arctan\left(\frac{(2x^5+1)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	19
risch	$\frac{2\arctan\left(\frac{(2x^5+1)\sqrt{7}}{7}\right)\sqrt{7}}{35}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^10+x^5+2),x,method=_RETURNVERBOSE)
[Out] 2/35*arctan(1/7*(2*x^5+1)*7^(1/2))*7^(1/2)
```

Maxima [A]
time = 0.50, size = 18, normalized size = 0.78

$$\frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\left(2x^5+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="maxima")
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Fricas [A]
time = 0.35, size = 18, normalized size = 0.78

$$\frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x^5+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="fricas")
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Sympy [A]
time = 0.06, size = 27, normalized size = 1.17

$$\frac{2\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**10+x**5+2),x)
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x**5/7 + sqrt(7)/7)/35
```

Giac [A]
time = 5.21, size = 18, normalized size = 0.78

$$\frac{2}{35}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}\,(2\,x^5+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+x^5+2),x, algorithm="giac")
[Out] 2/35*sqrt(7)*arctan(1/7*sqrt(7)*(2*x^5 + 1))
```

Mupad [B]
time = 1.33, size = 20, normalized size = 0.87

$$\frac{2\,\sqrt{7}\,\operatorname{atan}\left(\frac{2\,\sqrt{7}\,x^5}{7}+\frac{\sqrt{7}}{7}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^5 + x^10 + 2),x)
[Out] (2*7^(1/2)*atan(7^(1/2)/7 + (2*7^(1/2)*x^5)/7))/35
```

3.409 $\int \frac{1}{x(1+x^5+x^{10})}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

```
[Out] ln(x)-1/10*ln(x^10+x^5+1)-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1371, 719, 29, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^5 + x^10)),x]
```

```
[Out] -1/5*ArcTan[(1 + 2*x^5)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[1 + x^5 + x^10]/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(1+x^5+x^{10})}\,dx &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x(1+x+x^2)}\,dx, x, x^5\right) \\
&= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^5\right) + \frac{1}{5}\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx, x, x^5\right) \\
&= \log(x) - \frac{1}{10}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^5\right) - \frac{1}{10}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^5\right) \\
&= \log(x) - \frac{1}{10}\log\left(1+x^5+x^{10}\right) + \frac{1}{5}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^5\right) \\
&= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}} + \log(x) - \frac{1}{10}\log\left(1+x^5+x^{10}\right)
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 197, normalized size = 5.05

$$
\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{5\sqrt{3}} + \log(x) - \frac{1}{10}\log\left(1+x+x^2\right) - \frac{1}{5}\text{RootSum}\left[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7+\#1^8\&, \frac{-\log(x-\#1)\#1+2\log(x-\#1)\#1^2-\log(x-\#1)\#1^3+3\log(x-\#1)\#1^4-\log(x-\#1)\#1^5-3\log(x-\#1)\#1^6+4\log(x-\#1)\#1^7}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8\#1^7}\&\right]
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^5 + x^10)),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Root
Sum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*Lo
g[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^5 -
 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 -
7*#1^6 + 8*#1^7) & ]/5
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 131, normalized size = 3.36

method	result
risch	$\ln(x)-\frac{\ln(x^{10}+x^5+1)}{10}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^5+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{15}$
default	$-\frac{\ln(x^2+x+1)}{10}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{15}-\frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{6}\right)\ln\left(2x^4+\left(-1+i\sqrt{3}\right)x^3+\left(-1-i\sqrt{3}\right)x^2+2x-1+i\sqrt{3}\right)}{5}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^10+x^5+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*ln(x^2+x+1)+1/15*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/5*(1/2+1/6*I*3
^(1/2))*ln(2*x^4+(-1+I*3^(1/2))*x^3+(-1-I*3^(1/2))*x^2+2*x-1+I*3^(1/2))-1/5
*(1/2-1/6*I*3^(1/2))*ln(2*x^4+(-1-I*3^(1/2))*x^3+(-1+I*3^(1/2))*x^2+2*x-1-I
*3^(1/2))+ln(x)
```

Maxima [A]
time = 0.53, size = 36, normalized size = 0.92

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\frac{1}{5}\log\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
1/5*log(x^5)
```

Fricas [A]
time = 0.34, size = 32, normalized size = 0.82

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(x)
```

Sympy [A]
time = 0.06, size = 41, normalized size = 1.05

$$\log(x) - \frac{\log(x^{10} + x^5 + 1)}{10} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\,x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**10+x**5+1),x)
```

```
[Out] log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3)
/3)/15
```

Giac [A]
time = 6.49, size = 33, normalized size = 0.85

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5 + 1\right)\right) - \frac{1}{10}\log\left(x^{10} + x^5 + 1\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^10+x^5+1),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(abs(x))
```

Mupad [B]
time = 0.06, size = 34, normalized size = 0.87

$$\ln(x) - \frac{\ln(x^{10} + x^5 + 1)}{10} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\,x^5}{3} + \frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^5 + x^10 + 1)),x)
```

```
[Out] log(x) - log(x^5 + x^10 + 1)/10 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)
/3))/15
```

3.410 $\int \frac{1}{x^6\left(1+x^5+x^{10}\right)}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{5x^5}-\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}-\log(x)+\frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

```
[Out] -1/5/x^5-ln(x)+1/10*ln(x^10+x^5+1)-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1371, 723, 814, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\frac{1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] -1/5*1/x^5 - ArcTan[(1 + 2*x^5)/Sqrt[3]]/(5*Sqrt[3]) - Log[x] + Log[1 + x^5
 + x^10]/10
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\left(1+x^5+x^{10}\right)} \, dx &= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x^2\left(1+x+x^2\right)} \, dx,x,x^5\right) \\
&= -\frac{1}{5x^5}+\frac{1}{5}\text{Subst}\left(\int \frac{-1-x}{x\left(1+x+x^2\right)} \, dx,x,x^5\right) \\
&= -\frac{1}{5x^5}+\frac{1}{5}\text{Subst}\left(\int \left(-\frac{1}{x}+\frac{x}{1+x+x^2}\right) \, dx,x,x^5\right) \\
&= -\frac{1}{5x^5}-\log(x)+\frac{1}{5}\text{Subst}\left(\int \frac{x}{1+x+x^2} \, dx,x,x^5\right) \\
&= -\frac{1}{5x^5}-\log(x)-\frac{1}{10}\text{Subst}\left(\int \frac{1}{1+x+x^2} \, dx,x,x^5\right)+\frac{1}{10}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2} \, dx\right. \\
&= -\frac{1}{5x^5}-\log(x)+\frac{1}{10}\log\left(1+x^5+x^{10}\right)+\frac{1}{5}\text{Subst}\left(\int \frac{1}{-3-x^2} \, dx,x,1+2x^5\right) \\
&= -\frac{1}{5x^5}-\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}-\log(x)+\frac{1}{10}\log\left(1+x^5+x^{10}\right)
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 208, normalized size = 4.33

$$
\frac{1}{30}\left(-\frac{6}{x^5}+2\sqrt{3}\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)-30\log(x)+3\log\left(1+x+x^2\right)+6\text{RootSum}\left[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1+\log(x-\#1)\#1^2-3\log(x-\#1)\#1^3+2\log(x-\#1)\#1^4+\log(x-\#1)\#1^5-4\log(x-\#1)\#1^6+4\log(x-\#1)\#1^7}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8\#1^7}\&\right]\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(1 + x^5 + x^10)),x]
```

```
[Out] (-6/x^5 + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 30*Log[x] + 3*Log[1 + x + x
^2] + 6*RootSum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-Log[x - #1]
 + Log[x - #1]*#1 + Log[x - #1]*#1^2 - 3*Log[x - #1]*#1^3 + 2*Log[x - #1]*#
1^4 + Log[x - #1]*#1^5 - 4*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#
1^2 - 4*#1^3 + 5*#1^4 - 7*#1^6 + 8*#1^7) & ])/30
```

Maple [C] Result contains complex when optimal does not.
time = 0.04, size = 138, normalized size = 2.88

method	result
risch	$-\frac{1}{5x^5}-\ln\left(x\right)+\frac{\ln\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^5+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{15}$

default	$\frac{\ln(x^2+x+1)}{10}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{15}+\frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{6}\right)\ln\left(2x^4+\left(-1-i\sqrt{3}\right)x^3+\left(-1+i\sqrt{3}\right)x^2+2x-1-i\sqrt{3}\right)}{5}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^10+x^5+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*ln(x^2+x+1)+1/15*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)+1/5*(1/2+1/6*I*3^
(1/2))*ln(2*x^4+(-1-I*3^(1/2))*x^3+(-1+I*3^(1/2))*x^2+2*x-1-I*3^(1/2))+1/5*
(1/2-1/6*I*3^(1/2))*ln(2*x^4+(-1+I*3^(1/2))*x^3+(-1-I*3^(1/2))*x^2+2*x-1+I*
3^(1/2))-1/5/x^5-ln(x)
```

Maxima [A]
time = 0.51, size = 41, normalized size = 0.85

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-\frac{1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\frac{1}{5}\log\left(x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/5/x^5 + 1/10*log(x^10 + x
^5 + 1) - 1/5*log(x^5)
```

Fricas [A]
time = 0.36, size = 49, normalized size = 1.02

$$-\frac{2\sqrt{3}x^5\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)-3x^5\log\left(x^{10}+x^5+1\right)+30x^5\log\left(x\right)+6}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="fricas")
```

```
[Out] -1/30*(2*sqrt(3)*x^5*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 3*x^5*log(x^10 + x^5
 + 1) + 30*x^5*log(x) + 6)/x^5
```

Sympy [A]
time = 0.08, size = 48, normalized size = 1.00

$$-\log\left(x\right)+\frac{\log\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**10+x**5+1),x)
```

```
[Out] -log(x) + log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3
)/3)/15 - 1/(5*x**5)
```

Giac [A]
time = 3.32, size = 45, normalized size = 0.94

$$-\frac{1}{15}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^5+1\right)\right)+\frac{x^5-1}{5x^5}+\frac{1}{10}\log\left(x^{10}+x^5+1\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^10+x^5+1),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) + 1/5*(x^5 - 1)/x^5 + 1/10*lo
g(x^10 + x^5 + 1) - log(abs(x))
```

Mupad [B]
time = 1.37, size = 41, normalized size = 0.85

$$\frac{\ln\left(x^{10}+x^5+1\right)}{10}-\ln\left(x\right)-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(x^5 + x^10 + 1)),x)
```

```
[Out] log(x^5 + x^10 + 1)/10 - log(x) - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)
/3))/15 - 1/(5*x^5)
```

3.411 $\int \frac{1}{x+x^6+x^{11}}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)$$

```
[Out] ln(x)-1/10*ln(x^10+x^5+1)-1/15*arctan(1/3*(2*x^5+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {1608, 1371, 719, 29, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{2x^5+1}{\sqrt{3}}\right)}{5\sqrt{3}}-\frac{1}{10}\log\left(x^{10}+x^5+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^6 + x^11)^(-1),x]
```

```
[Out] -1/5*ArcTan[(1 + 2*x^5)/Sqrt[3]]/Sqrt[3] + Log[x] - Log[1 + x^5 + x^10]/10
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x+x^6+x^{11}}\,dx &= \int \frac{1}{x\left(1+x^5+x^{10}\right)}\,dx\\
&= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x\left(1+x+x^2\right)}\,dx,x,x^5\right)\\
&= \frac{1}{5}\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^5\right)+\frac{1}{5}\text{Subst}\left(\int \frac{-1-x}{1+x+x^2}\,dx,x,x^5\right)\\
&= \log(x)-\frac{1}{10}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx,x,x^5\right)-\frac{1}{10}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx,x,x^5\right)\\
&= \log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)+\frac{1}{5}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x^5\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x^5+x^{10}\right)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 197, normalized size = 5.05

$$\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{5\sqrt{3}}+\log(x)-\frac{1}{10}\log\left(1+x+x^2\right)-\frac{1}{5}\text{RootSum}\left[1-\#1+\#1^3-\#1^4+\#1^5-\#1^7+\#1^8\&,\frac{-\log(x-\#1)\#1+2\log(x-\#1)\#1^2-\log(x-\#1)\#1^3+3\log(x-\#1)\#1^4-\log(x-\#1)\#1^5-3\log(x-\#1)\#1^6+4\log(x-\#1)\#1^7}{-1+3\#1^2-4\#1^3+5\#1^4-7\#1^6+8\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^6 + x^11)^(-1),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/(5*Sqrt[3]) + Log[x] - Log[1 + x + x^2]/10 - Root
Sum[1 - #1 + #1^3 - #1^4 + #1^5 - #1^7 + #1^8 & , (-(Log[x - #1]*#1) + 2*Lo
g[x - #1]*#1^2 - Log[x - #1]*#1^3 + 3*Log[x - #1]*#1^4 - Log[x - #1]*#1^5 -
 3*Log[x - #1]*#1^6 + 4*Log[x - #1]*#1^7)/(-1 + 3*#1^2 - 4*#1^3 + 5*#1^4 -
7*#1^6 + 8*#1^7) & ]/5
```

Maple [C] Result contains complex when optimal does not.
time = 0.03, size = 131, normalized size = 3.36

method	result
risch	$\ln(x)-\frac{\ln(x^{10}+x^5+1)}{10}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^5+\frac{1}{2}\right)\sqrt{3}}{3}\right)}{15}$
default	$-\frac{\ln(x^2+x+1)}{10}+\frac{\arctan\left(\frac{(2x+1)\sqrt{3}}{3}\right)\sqrt{3}}{15}-\frac{\left(\frac{1}{2}+\frac{i\sqrt{3}}{6}\right)\ln\left(2x^4+\left(-1+i\sqrt{3}\right)x^3+\left(-1-i\sqrt{3}\right)x^2+2x-1+i\sqrt{3}\right)}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^11+x^6+x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*ln(x^2+x+1)+1/15*arctan(1/3*(2*x+1)*3^(1/2))*3^(1/2)-1/5*(1/2+1/6*I*3
^(1/2))*ln(2*x^4+(-1+I*3^(1/2))*x^3+(-1-I*3^(1/2))*x^2+2*x-1+I*3^(1/2))-1/5
*(1/2-1/6*I*3^(1/2))*ln(2*x^4+(-1-I*3^(1/2))*x^3+(-1+I*3^(1/2))*x^2+2*x-1-I
*3^(1/2))+ln(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="maxima")
```

```
[Out] 1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/5*integrate((4*x^7 - 3*x^6 -
 x^5 + 3*x^4 - x^3 + 2*x^2 - x)/(x^8 - x^7 + x^5 - x^4 + x^3 - x + 1), x) -
 1/10*log(x^2 + x + 1) + log(x)
```

Fricas [A]
time = 0.35, size = 32, normalized size = 0.82

$$-\frac{1}{15}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^{5}+1\right)\right)-\frac{1}{10}\,\log\left(x^{10}+x^{5}+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(x)
```

Sympy [A]
time = 0.08, size = 41, normalized size = 1.05

$$\log\left(x\right)-\frac{\log\left(x^{10}+x^{5}+1\right)}{10}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^{5}}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**11+x**6+x),x)
```

```
[Out] log(x) - log(x**10 + x**5 + 1)/10 - sqrt(3)*atan(2*sqrt(3)*x**5/3 + sqrt(3)
/3)/15
```

Giac [A]
time = 3.65, size = 33, normalized size = 0.85

$$-\frac{1}{15}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^{5}+1\right)\right)-\frac{1}{10}\,\log\left(x^{10}+x^{5}+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^11+x^6+x),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^5 + 1)) - 1/10*log(x^10 + x^5 + 1) +
log(abs(x))
```

Mupad [B]
time = 0.03, size = 34, normalized size = 0.87

$$\ln\left(x\right)-\frac{\ln\left(x^{10}+x^5+1\right)}{10}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x^5}{3}+\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^6 + x^11),x)
```

```
[Out] log(x) - log(x^5 + x^10 + 1)/10 - (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^5)
/3))/15
```

3.412 $\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx$

Optimal. Leaf size=147

$$-\frac{b(b^2-2ac)\,x}{c^4}+\frac{(b^2-ac)\,x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{b(b^4-5ab^2c+5a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}+\frac{(b^4-3ab^2c+a^2c^2)}{2c}$$

```
[Out] -b*(-2*a*c+b^2)*x/c^4+1/2*(-a*c+b^2)*x^2/c^3-1/3*b*x^3/c^2+1/4*x^4/c+1/2*(a
^2*c^2-3*a*b^2*c+b^4)*ln(c*x^2+b*x+a)/c^5+b*(5*a^2*c^2-5*a*b^2*c+b^4)*arcta
nh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1368, 715, 648, 632, 212, 642}

$$\frac{(a^2c^2-3ab^2c+b^4)\log(a+bx+cx^2)}{2c^5}+\frac{b(5a^2c^2-5ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}-\frac{bx(b^2-2ac)}{c^4}+\frac{x^2(b^2-ac)}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(c + a/x^2 + b/x),x]
```

```
[Out] -((b*(b^2 - 2*a*c)*x)/c^4) + ((b^2 - a*c)*x^2)/(2*c^3) - (b*x^3)/(3*c^2) +
x^4/(4*c) + (b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + b*
x + c*x^2])/(2*c^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{x^3}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx = \int \frac{x^5}{a+bx+cx^2}\,dx$$
$$= \int \left(-\frac{b(b^2-2ac)}{c^4}+\frac{(b^2-ac)\,x}{c^3}-\frac{bx^2}{c^2}+\frac{x^3}{c}+\frac{ab(b^2-2ac)+(b^4-3ab^2c+a^2c^2)\,x}{c^4\,(a+bx+cx^2)}\right)\,dx$$
$$= -\frac{b(b^2-2ac)\,x}{c^4}+\frac{(b^2-ac)\,x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{\int \frac{ab(b^2-2ac)+(b^4-3ab^2c+a^2c^2)x}{a+bx+cx^2}\,dx}{c^4}$$
$$= -\frac{b(b^2-2ac)\,x}{c^4}+\frac{(b^2-ac)\,x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{(b^4-3ab^2c+a^2c^2)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^5}-\frac{(b(b^4-\cdots}{\cdots}$$
$$= -\frac{b(b^2-2ac)\,x}{c^4}+\frac{(b^2-ac)\,x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{(b^4-3ab^2c+a^2c^2)\log\left(a+bx+cx^2\right)}{2c^5}+\frac{(b\cdots}{\cdots}$$
$$= -\frac{b(b^2-2ac)\,x}{c^4}+\frac{(b^2-ac)\,x^2}{2c^3}-\frac{bx^3}{3c^2}+\frac{x^4}{4c}+\frac{b(b^4-5ab^2c+5a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^5\sqrt{b^2-4ac}}\cdots$$

Mathematica [A]
time = 0.08, size = 140, normalized size = 0.95

$$\frac{cx(-12b^3+6b^2cx-4bc(-6a+cx^2)+3c^2x(-2a+cx^2))-\frac{12b(b^4-5ab^2c+5a^2c^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+6(b^4-3ab^2c+a^2c^2)\log(a+x(b+cx))}{12c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(-12*b^3 + 6*b^2*c*x - 4*b*c*(-6*a + c*x^2) + 3*c^2*x*(-2*a + c*x^2))
- (12*b*(b^4 - 5*a*b^2*c + 5*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/Sqrt[-b^2 + 4*a*c] + 6*(b^4 - 3*a*b^2*c + a^2*c^2)*Log[a + x*(b + c*x)])
/(12*c^5)
```

Maple [A]
time = 0.06, size = 164, normalized size = 1.12

method	result
default	$\frac{\frac{1}{4}c^3x^4-\frac{1}{3}b\,c^2x^3-\frac{1}{2}a\,c^2x^2+\frac{1}{2}b^2c\,x^2+2abcx-b^3x}{c^4}+\frac{\frac{\left(a^2c^2-3a\,b^2c+b^4\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-2a^2bc+a\,b^3-\frac{\left(a^2c^2-3a\,b^2c+b^4\right)b}{2c}\right)\arctan\left(\right.}{\sqrt{4ac-b^2}}}{c^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*c^3*x^4-1/3*b*c^2*x^3-1/2*a*c^2*x^2+1/2*b^2*c*x^2+2*a*b*c*x-b^3*
x)+1/c^4*(1/2*(a^2*c^2-3*a*b^2*c+b^4)/c*ln(c*x^2+b*x+a)+2*(-2*a^2*b*c+a*b^3
-1/2*(a^2*c^2-3*a*b^2*c+b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c
-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.36, size = 466, normalized size = 3.17

$$\left[\frac{3\,(b^2c^4-4\,ac^5)x^4-4\,(b^3c^3-4\,abc^4)x^3+6\,(b^4c^2-5\,ab^2c^3+4\,a^2c^4)x^2+6\,(b^5-5\,ab^3c+5\,a^2bc^2)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-12\,(b^5c-6\,ab^3c^2+8\,a^2bc^3)x+6\,(b^6-7\,ab^4c+13\,a^2b^2c^2-4\,a^3c^3)\log\left(cx^2+bx+a\right)}{12\,(b^2c^5-4\,ac^6)},\ \frac{3\,(b^2c^4-4\,ac^5)x^4-4\,(b^3c^3-4\,abc^4)x^3+6\,(b^4c^2-5\,ab^2c^3+4\,a^2c^4)x^2+12\,(b^5-5\,ab^3c+5\,a^2bc^2)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)-12\,(b^5c-6\,ab^3c^2+8\,a^2bc^3)x+6\,(b^6-7\,ab^4c+13\,a^2b^2c^2-4\,a^3c^3)\log\left(cx^2+bx+a\right)}{12\,(b^2c^5-4\,ac^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3 + 6*(b^4*c^2
 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 6*(b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*
x + b))/(c*x^2 + b*x + a)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(
b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b^2*c^
5 - 4*a*c^6), 1/12*(3*(b^2*c^4 - 4*a*c^5)*x^4 - 4*(b^3*c^3 - 4*a*b*c^4)*x^3
 + 6*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x^2 + 12*(b^5 - 5*a*b^3*c + 5*a^2*
b*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 12*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*x + 6*(b^6 - 7*a*b^4*c + 13*a
^2*b^2*c^2 - 4*a^3*c^3)*log(c*x^2 + b*x + a))/(b^2*c^5 - 4*a*c^6)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(144) = 288$.
time = 0.70, size = 605, normalized size = 4.12

$$-\frac{bx^3}{3c^2}+x^2\left(-\frac{a}{2c^2}+\frac{b^2}{2c^3}\right)+x\left(\frac{2ab}{c^3}-\frac{b^3}{c^4}\right)+\left(-\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)}+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)\log\left(x+\frac{2a^3c^2-4a^2b^2c+ab^4-4ac^5\left(-\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)}+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)+b^2c^4\left(-\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)}+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)}{5a^2bc^2-5ab^3c+b^5}\right)+\left(\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)}+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)\log\left(x+\frac{2a^3c^2-4a^2b^2c+ab^4-4ac^5\left(\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)}+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)+b^2c^4\left(\frac{b\sqrt{-4ac+b^2}\cdot(5a^2c^2-5ab^2c+b^4)}{2c^5\cdot(4ac-b^2)}+\frac{a^2c^2-3ab^2c+b^4}{2c^5}\right)}{5a^2bc^2-5ab^3c+b^5}\right)+\frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c+a/x**2+b/x),x)
```

```
[Out] -b*x**3/(3*c**2) + x**2*(-a/(2*c**2) + b**2/(2*c**3)) + x*(2*a*b/c**3 - b**
3/c**4) + (-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5
*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5))*log(x + (2*a**
3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c**5*(-b*sqrt(-4*a*c + b**2)*(5*a**2*
c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c
 + b**4)/(2*c**5)) + b**2*c**4*(-b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b
**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*
c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**5)) + (b*sqrt(-4*a*c + b**2)*(5*a*
*2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (a**2*c**2 - 3*a*b**
2*c + b**4)/(2*c**5))*log(x + (2*a**3*c**2 - 4*a**2*b**2*c + a*b**4 - 4*a*c
**5*(b*sqrt(-4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c
 - b**2)) + (a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)) + b**2*c**4*(b*sqrt(-
4*a*c + b**2)*(5*a**2*c**2 - 5*a*b**2*c + b**4)/(2*c**5*(4*a*c - b**2)) + (
a**2*c**2 - 3*a*b**2*c + b**4)/(2*c**5)))/(5*a**2*b*c**2 - 5*a*b**3*c + b**
5)) + x**4/(4*c)
```

Giac [A]
time = 3.97, size = 145, normalized size = 0.99

$$\frac{3\,c^3x^4-4\,bc^2x^3+6\,b^2cx^2-6\,ac^2x^2-12\,b^3x+24\,abcx}{12\,c^4}+\frac{(b^4-3\,ab^2c+a^2c^2)\log\left(cx^2+bx+a\right)}{2\,c^5}-\frac{(b^5-5\,ab^3c+5\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/12*(3*c^3*x^4 - 4*b*c^2*x^3 + 6*b^2*c*x^2 - 6*a*c^2*x^2 - 12*b^3*x + 24*a
*b*c*x)/c^4 + 1/2*(b^4 - 3*a*b^2*c + a^2*c^2)*log(c*x^2 + b*x + a)/c^5 - (b
^5 - 5*a*b^3*c + 5*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(
-b^2 + 4*a*c)*c^5)
```

Mupad [B]
time = 0.16, size = 183, normalized size = 1.24

$$x\left(\frac{b\left(\frac{a}{c^2}-\frac{b^2}{c^3}\right)}{c}+\frac{a\,b}{c^3}\right)+\frac{x^4}{4\,c}-x^2\left(\frac{a}{2\,c^2}-\frac{b^2}{2\,c^3}\right)-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-4\,a^3\,c^3+13\,a^2\,b^2\,c^2-7\,a\,b^4\,c+b^6\right)}{2\,\left(4\,a\,c^6-b^2\,c^5\right)}-\frac{b\,x^3}{3\,c^2}-\frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(5\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{c^5\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c + a/x^2 + b/x),x)
```

```
[Out] x*((b*(a/c^2 - b^2/c^3))/c + (a*b)/c^3) + x^4/(4*c) - x^2*(a/(2*c^2) - b^2/
(2*c^3)) - (log(a + b*x + c*x^2)*(b^6 - 4*a^3*c^3 + 13*a^2*b^2*c^2 - 7*a*b^
4*c))/(2*(4*a*c^6 - b^2*c^5)) - (b*x^3)/(3*c^2) - (b*atan((b + 2*c*x)/(4*a*
c - b^2)^(1/2))*(b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c^5*(4*a*c - b^2)^(1/2))
```

3.413 $\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=118

$$\frac{(b^2-ac)\,x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}$$

```
[Out] (-a*c+b^2)*x/c^3-1/2*b*x^2/c^2+1/3*x^3/c-1/2*b*(-2*a*c+b^2)*ln(c*x^2+b*x+a)
/c^4-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-
4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1368, 715, 648, 632, 212, 642}

$$-\frac{(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}+\frac{x(b^2-ac)}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] ((b^2 - a*c)*x)/c^3 - (b*x^2)/(2*c^2) + x^3/(3*c) - ((b^4 - 4*a*b^2*c + 2*a
^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) - (
b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx &= \int \frac{x^4}{a+bx+cx^2}\,dx\\
&= \int \left(\frac{b^2-ac}{c^3}-\frac{bx}{c^2}+\frac{x^2}{c}-\frac{a(b^2-ac)+b(b^2-2ac)\,x}{c^3\,(a+bx+cx^2)}\right)dx\\
&= \frac{(b^2-ac)\,x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{\int \frac{a(b^2-ac)+b(b^2-2ac)x}{a+bx+cx^2}\,dx}{c^3}\\
&= \frac{(b^2-ac)\,x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{(b(b^2-2ac))\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^4}+\frac{(b^4-4ab^2c+2a^2c^2)\int \frac{1}{a+bx+cx^2}}{2c^4}\\
&= \frac{(b^2-ac)\,x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2c^4}-\frac{(b^4-4ab^2c+2a^2c^2)\,\mathrm{Subst}\big(}{c^4}\\
&= \frac{(b^2-ac)\,x}{c^3}-\frac{bx^2}{2c^2}+\frac{x^3}{3c}-\frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log}{2}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 112, normalized size = 0.95

$$\frac{cx(6b^2-6ac-3bcx+2c^2x^2)+\frac{6(b^4-4ab^2c+2a^2c^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-3(b^3-2abc)\log(a+x(b+cx))}{6c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(6*b^2 - 6*a*c - 3*b*c*x + 2*c^2*x^2) + (6*(b^4 - 4*a*b^2*c + 2*a^2*c^
2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 3*(b^3 - 2*
a*b*c)*Log[a + x*(b + c*x)])/(6*c^4)
```

Maple [A]
time = 0.05, size = 128, normalized size = 1.08

method	result
default	$-\frac{-\frac{1}{3}c^2x^3+\frac{1}{2}bc\,x^2+acx-b^2x}{c^3}+\frac{\frac{\left(2abc-b^3\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(a^2c-a\,b^2-\frac{\left(2abc-b^3\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c^3}$
risch	$\frac{x^3}{3c}-\frac{b\,x^2}{2c^2}-\frac{ax}{c^2}+\frac{b^2x}{c^3}+\frac{4\ln\left(8a^3c^3-18a^2b^2c^2+8a\,b^4c-b^6-2\sqrt{-\left(4ac-b^2\right)\left(2a^2c^2-4a\,b^2c+b^4\right)^2}\,cx-\sqrt{-(4\right.}{c^2\left(4ac-b^2\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/3*c^2*x^3+1/2*b*c*x^2+a*c*x-b^2*x)+1/c^3*(1/2*(2*a*b*c-b^3)/c*ln
(c*x^2+b*x+a)+2*(a^2*c-a*b^2-1/2*(2*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 383, normalized size = 3.25

$$\left[\frac{2\,(b^2c^3-4\,ac^4)x^3-3\,(b^3c^2-4\,abc^3)x^2+3\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+6\,(b^4c-5\,ab^2c^2+4\,a^2c^3)x-3\,(b^5-6\,ab^3c+8\,a^2bc^2)\log\left(cx^2+bx+a\right)}{6\,(b^2c^4-4\,ac^5)},\ \frac{2\,(b^2c^3-4\,ac^4)x^3-3\,(b^3c^2-4\,abc^3)x^2-6\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+6\,(b^4c-5\,ab^2c^2+4\,a^2c^3)x-3\,(b^5-6\,ab^3c+8\,a^2bc^2)\log\left(cx^2+bx+a\right)}{6\,(b^2c^4-4\,ac^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b*c^3)*x^2 + 3*(b^4 - 4*
a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 6*(b^4*c - 5*a*b^2
*c^2 + 4*a^2*c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a
))/(b^2*c^4 - 4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*x^3 - 3*(b^3*c^2 - 4*a*b
*c^3)*x^2 - 6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt
(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 6*(b^4*c - 5*a*b^2*c^2 + 4*a^2*
c^3)*x - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*log(c*x^2 + b*x + a))/(b^2*c^4 -
 4*a*c^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(110) = 220$.
time = 0.65, size = 498, normalized size = 4.22

$$-\frac{bx^2}{2c^2} + x\left(-\frac{a}{c^2} + \frac{b^2}{c^3}\right) + \left(\frac{b(2ac-b^2)}{2c^4} - \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)\log\left(x + \frac{-3a^2bc + ab^3 + 4ac^4\left(\frac{b(2ac-b^2)}{2c^4} - \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right) - b^2c^3\left(\frac{b(2ac-b^2)}{2c^4} - \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)}{2a^2c^2-4ab^2c+b^4}\right) + \left(\frac{b(2ac-b^2)}{2c^4} + \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)\log\left(x + \frac{-3a^2bc + ab^3 + 4ac^4\left(\frac{b(2ac-b^2)}{2c^4} + \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right) - b^2c^3\left(\frac{b(2ac-b^2)}{2c^4} + \frac{\sqrt{-4ac+b^2}\cdot(2a^2c^2-4ab^2c+b^4)}{2c^4\cdot(4ac-b^2)}\right)}{2a^2c^2-4ab^2c+b^4}\right) + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c+a/x**2+b/x),x)
```

```
[Out] -b*x**2/(2*c**2) + x*(-a/c**2 + b**2/c**3) + (b*(2*a*c - b**2)/(2*c**4) - s
qrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)
))*log(x + (-3*a**2*b*c + a*b**3 + 4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) - sq
rt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))
) - b**2*c**3*(b*(2*a*c - b**2)/(2*c**4) - sqrt(-4*a*c + b**2)*(2*a**2*c**2
 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c +
 b**4)) + (b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4
*a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2)))*log(x + (-3*a**2*b*c + a*b**3 +
4*a*c**4*(b*(2*a*c - b**2)/(2*c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*
a*b**2*c + b**4)/(2*c**4*(4*a*c - b**2))) - b**2*c**3*(b*(2*a*c - b**2)/(2*
c**4) + sqrt(-4*a*c + b**2)*(2*a**2*c**2 - 4*a*b**2*c + b**4)/(2*c**4*(4*a*
c - b**2))))/(2*a**2*c**2 - 4*a*b**2*c + b**4)) + x**3/(3*c)
```

Giac [A]
time = 3.35, size = 113, normalized size = 0.96

$$\frac{2\,c^2x^3 - 3\,bcx^2 + 6\,b^2x - 6\,acx}{6\,c^3} - \frac{(b^3 - 2\,abc)\log\left(cx^2 + bx + a\right)}{2\,c^4} + \frac{(b^4 - 4\,ab^2c + 2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\;c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/6*(2*c^2*x^3 - 3*b*c*x^2 + 6*b^2*x - 6*a*c*x)/c^3 - 1/2*(b^3 - 2*a*b*c)*l
og(c*x^2 + b*x + a)/c^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/
sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [B]
time = 1.40, size = 151, normalized size = 1.28

$$\frac{x^3}{3\,c}-x\,\left(\frac{a}{c^2}-\frac{b^2}{c^3}\right)-\frac{b\,x^2}{2\,c^2}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(8\,a^2\,b\,c^2-6\,a\,b^3\,c+b^5\right)}{2\,\left(4\,a\,c^5-b^2\,c^4\right)}+\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c^4\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c + a/x^2 + b/x),x)
```

```
[Out] x^3/(3*c) - x*(a/c^2 - b^2/c^3) - (b*x^2)/(2*c^2) + (log(a + b*x + c*x^2)*(
b^5 + 8*a^2*b*c^2 - 6*a*b^3*c))/(2*(4*a*c^5 - b^2*c^4)) + (atan(b/(4*a*c -
b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c
^4*(4*a*c - b^2)^(1/2))
```

3.414 $\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=89

$$-\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2c^3}$$

```
[Out] -b*x/c^2+1/2*x^2/c+1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/c^3+b*(-3*a*c+b^2)*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1368, 715, 648, 632, 212, 642}

$$\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x/(c + a/x^2 + b/x),x]
```

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^
3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx &= \int \frac{x^3}{a+bx+cx^2}\,dx\\
&= \int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+(b^2-ac)\,x}{c^2\,(a+bx+cx^2)}\right)dx\\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\int \frac{ab+(b^2-ac)x}{a+bx+cx^2}\,dx}{c^2}\\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}-\frac{(b(b^2-3ac))\int \frac{1}{a+bx+cx^2}\,dx}{2c^3}+\frac{(b^2-ac)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3}\\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2c^3}+\frac{(b(b^2-3ac))\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^3}\\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2c^3}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 84, normalized size = 0.94

$$\frac{cx(-2b+cx)-\frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log(a+x(b+cx))}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(c + a/x^2 + b/x),x]
```

```
[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

Maple [A]
time = 0.04, size = 98, normalized size = 1.10

method	result
default	$-\frac{-\frac{1}{2} c\,x^{2}+b x}{c^{2}}+\frac{\frac{\left(-a c+b^{2}\right) \ln \left(c\,x^{2}+b x+a\right)}{2 c}+\frac{2 \left(a b-\frac{\left(-a c+b^{2}\right) b}{2 c}\right) \arctan \left(\frac{2 c x+b}{\sqrt{4 a c-b^{2}}}\right)}{\sqrt{4 a c-b^{2}}}}{c^{2}}$
risch	$\frac{x^{2}}{2 c}-\frac{b x}{c^{2}}-\frac{2 \ln \left(12 a^{2} b\,c^{2}-7 a\,b^{3} c+b^{5}-2 \sqrt{-b^{2} \left(4 a c-b^{2}\right) \left(3 a c-b^{2}\right)^{2}}\, c x-\sqrt{-b^{2} \left(4 a c-b^{2}\right) \left(3 a c-b^{2}\right)^{2}}\, b\right)}{c \left(4 a c-b^{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(-1/2*c*x^2+b*x)+1/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^2+b*x+a)+2*(a*b-1/2*
(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.36, size = 297, normalized size = 3.34

$$\left[\frac{(b^{2} c^{2} - 4\,a c^{3}) x^{2} - (b^{3} - 3\,a b c) \sqrt{b^{2} - 4\,a c} \log\left(\frac{2\,c^{2} x^{2} + 2\,b c x + b^{2} - 2\,a c - \sqrt{b^{2} - 4\,a c} (2\,c x + b)}{c x^{2} + b x + a}\right) - 2\,(b^{3} c - 4\,a b c^{2}) x + (b^{4} - 5\,a b^{2} c + 4\,a^{2} c^{2}) \log\left(c x^{2} + b x + a\right)}{2\,(b^{2} c^{3} - 4\,a c^{4})}, \frac{(b^{2} c^{2} - 4\,a c^{3}) x^{2} + 2\,(b^{3} - 3\,a b c) \sqrt{-b^{2} + 4\,a c} \arctan\left(-\frac{\sqrt{-b^{2} + 4\,a c} (2\,c x + b)}{b^{2} - 4\,a c}\right) - 2\,(b^{3} c - 4\,a b c^{2}) x + (b^{4} - 5\,a b^{2} c + 4\,a^{2} c^{2}) \log\left(c x^{2} + b x + a\right)}{2\,(b^{2} c^{3} - 4\,a c^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2
+ b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3 - 3*
a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 +
b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(83) = 166$.
time = 0.45, size = 381, normalized size = 4.28

$$-\frac{bx}{c^2} + \left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c - ab^2 + 4ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) - b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)\log\left(x + \frac{2a^2c - ab^2 + 4ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right) - b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)} - \frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right) + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x**2+b/x),x)
```

```
[Out] -b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
- (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-4*
a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))
 - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*
c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*
c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c
- b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2)*(3*a*c
 - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)
) + x**2/(2*c)
```

Giac [A]
time = 3.56, size = 86, normalized size = 0.97

$$\frac{cx^2 - 2\,bx}{2\,c^2} + \frac{(b^2 - ac)\log\left(cx^2 + bx + a\right)}{2\,c^3} - \frac{(b^3 - 3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3 -
 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B]
time = 0.13, size = 112, normalized size = 1.26

$$\frac{x^2}{2\,c} - \frac{\ln\left(c\,x^2 + b\,x + a\right)\,\left(4\,a^2\,c^2 - 5\,a\,b^2\,c + b^4\right)}{2\,\left(4\,a\,c^4 - b^2\,c^3\right)} - \frac{b\,x}{c^2} + \frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(3\,a\,c - b^2\right)}{c^3\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c + a/x^2 + b/x),x)
```

```
[Out] x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c^
4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a*c
 - b^2))/(c^3*(4*a*c - b^2)^(1/2))
```

3.415 $\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}} \, dx$

Optimal. Leaf size=70

$$\frac{x}{c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log(a+bx+cx^2)}{2c^2}$$

```
[Out] x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1354, 717, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log(a+bx+cx^2)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^2 + b/x)^(-1),x]
```

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
- 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{c+\frac{a}{x^2}+\frac{b}{x}}\,dx &= \int \frac{x^2}{a+bx+cx^2}\,dx\\
&= \frac{x}{c}+\frac{\int \frac{-a-bx}{a+bx+cx^2}\,dx}{c}\\
&= \frac{x}{c}-\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{(b^2-2ac)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2}\\
&= \frac{x}{c}-\frac{b\log(a+bx+cx^2)}{2c^2}-\frac{(b^2-2ac)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2}\\
&= \frac{x}{c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log(a+bx+cx^2)}{2c^2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 73, normalized size = 1.04

$$\frac{x}{c}+\frac{(b^2-2ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}}-\frac{b\log(a+bx+cx^2)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^2 + b/x)^(-1),x]
```

```
[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2
 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Maple [A]
time = 0.03, size = 75, normalized size = 1.07

method	result
default	$\frac{x}{c}+\frac{-\frac{b\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$
risch	$\frac{x}{c}-\frac{2\ln\left(-8a^{2}c^{2}+6a\,b^{2}c-b^{4}-2\sqrt{-\left(4ac-b^{2}\right)\left(2ac-b^{2}\right)^{2}}\,cx-\sqrt{-\left(4ac-b^{2}\right)\left(2ac-b^{2}\right)^{2}}\,b\right)ab}{c\left(4ac-b^{2}\right)}+\frac{\ln\left(-8a^{2}\ldots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/c*(-1/2*b/c*ln(c*x^2+b*x+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.35, size = 235, normalized size = 3.36

$$\left[-\frac{(b^{2}-2\,ac)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)-2\,(b^{2}c-4\,ac^{2})x+(b^{3}-4\,abc)\log\left(cx^{2}+bx+a\right)}{2\,(b^{2}c^{2}-4\,ac^{3})},\,-\frac{2\,(b^{2}-2\,ac)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)-2\,(b^{2}c-4\,ac^{2})x+(b^{3}-4\,abc)\log\left(cx^{2}+bx+a\right)}{2\,(b^{2}c^{2}-4\,ac^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c^2
```

```
)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2*(b
^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(
b^2*c^2 - 4*a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(65) = 130$.
time = 0.32, size = 306, normalized size = 4.37

$$\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x),x)
```

```
[Out] (-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*
log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c*
*2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)
)) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*
c - b**2))))/(2*a*c - b**2)) + x/c
```

Giac [A]
time = 3.74, size = 67, normalized size = 0.96

$$\frac{x}{c}-\frac{b\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x),x, algorithm="giac")
```

```
[Out] x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B]
time = 1.42, size = 172, normalized size = 2.46

$$\frac{x}{c}+\frac{b^3\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}-\frac{2\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}+\frac{b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c-b^2}}-\frac{2\,a\,b\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^3-b^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^2 + b/x),x)
```

```
[Out] x/c + (b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*atan(b/(4*a
*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) + (
b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c^2*(4*a*c
- b^2)^(1/2)) - (2*a*b*c*log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)
```

3.416 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx$

Optimal. Leaf size=56

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

```
[Out] 1/2*ln(c*x^2+b*x+a)/c+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2
)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1368, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
b*x + c*x^2]/(2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x}\,dx &= \int \frac{x}{a+bx+cx^2}\,dx\\
&= \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c} - \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2c}\\
&= \frac{\log\left(a+bx+cx^2\right)}{2c} + \frac{b\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx+cx^2\right)}{2c}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 1.02

$$\frac{-\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log(a+x(b+cx))}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a +
 x*(b + c*x)])/(2*c)
```

Maple [A]
time = 0.02, size = 56, normalized size = 1.00

method	result

default	$\frac{\ln(c\,x^2+bx+a)}{2c}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}}$
risch	$\frac{2\ln\left(-2\sqrt{-b^2\left(4ac-b^2\right)}\,cx-4abc+b^3-\sqrt{-b^2\left(4ac-b^2\right)}\,b\right)a}{4ac-b^2}-\frac{\ln\left(-2\sqrt{-b^2\left(4ac-b^2\right)}\,cx-4abc+b^3-\sqrt{-b\right.}}{2c(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.34, size = 185, normalized size = 3.30

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c-4\,ac^2\right)},\ \frac{2\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+\left(b^2-4\,ac\right)\log\left(cx^2+bx+a\right)}{2\left(b^2c-4\,ac^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c
 - 4*a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(49) = 98$.
time = 0.16, size = 216, normalized size = 3.86

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{2c\left(4ac-b^2\right)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-b
*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqrt(
-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*
c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*
c - b**2)) + 1/(2*c)))/b)
```

Giac [A]
time = 3.83, size = 55, normalized size = 0.98

$$-\frac{b \arctan\left(\frac{2 \, cx+b}{\sqrt{-b^{2}+4 \, ac}}\right)}{\sqrt{-b^{2}+4 \, ac} \, c}+\frac{\log\left(cx^{2}+bx+a\right)}{2 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x,x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log(
c*x^2 + b*x + a)/c
```

Mupad [B]
time = 0.17, size = 112, normalized size = 2.00

$$\frac{2\,a\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^2-b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}-\frac{b^2\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^2-b^2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + a/x^2 + b/x)),x)
```

```
[Out] (2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(1
/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a +
b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))
```

3.417 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx$

Optimal. Leaf size=36

$$\frac{2\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] 2*arctanh((b+2*a/x)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1366, 632, 212}

$$\frac{2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^2),x]
```

```
[Out] (2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^2}\,dx = -\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx, x, \frac{1}{x}\right)$$
$$= 2\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+\frac{2a}{x}\right)$$
$$= \frac{2\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 1.06

$$\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^2),x]
[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.97

method	result	size
default	$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	35
risch	$-\frac{\ln\left(b+2cx+\sqrt{-4ac+b^2}\right)}{\sqrt{-4ac+b^2}}+\frac{\ln\left(-b-2cx+\sqrt{-4ac+b^2}\right)}{\sqrt{-4ac+b^2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^2,x,method=_RETURNVERBOSE)
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.34, size = 120, normalized size = 3.33

$$\left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="fricas")
```

```
[Out] [log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(32) = 64$.
time = 0.09, size = 124, normalized size = 3.44

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

Giac [A]
time = 4.10, size = 34, normalized size = 0.94

$$\frac{2\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^2,x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B]
time = 0.05, size = 46, normalized size = 1.28

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + a/x^2 + b/x)),x)
[Out] (2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)
^(1/2)
```

3.418 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx$

Optimal. Leaf size=62

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}$$

```
[Out] ln(x)/a-1/2*ln(c*x^2+b*x+a)/a+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a/(-4
*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1368, 719, 29, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a
 - Log[a + b*x + c*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^3}\,dx &= \int \frac{1}{x\left(a+bx+cx^2\right)}\,dx\\
&= \frac{\int \frac{1}{x}\,dx}{a}+\frac{\int \frac{-b-cx}{a+bx+cx^2}\,dx}{a}\\
&= \frac{\log(x)}{a}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a}-\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2a}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.98

$$-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2\log(x)+\log(a+x(b+cx))}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^3),x]
```

```
[Out] -1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*L
og[x] + Log[a + x*(b + c*x)])/a
```

Maple [A]
time = 0.03, size = 61, normalized size = 0.98

method	result	size
default	$\frac{-\frac{\ln\left(c\,x^{2}+b x+a\right)}{2}-\frac{b \arctan\left(\frac{2 c x+b}{\sqrt{4 a c-b^{2}}}\right)}{\sqrt{4 a c-b^{2}}}}{a}+\frac{\ln\left(x\right)}{a}$	61
risch	$\frac{\ln\left(x\right)}{a}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4 a^{2} c-a\,b^{2}\right)\_Z^{2}+\left(4 a c-b^{2}\right)\_Z+c\right)}\_R\ln\left(\left(\left(6 a c-2 b^{2}\right)\_R+3 c\right) x-a b\_R+b\right)\right)$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/2*ln(c*x^2+b*x+a)-b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2)))+ln(x)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.35, size = 211, normalized size = 3.40

$$\left[\frac{\sqrt{b^{2}-4\,ac}\,b\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)-(b^{2}-4\,ac)\log\left(cx^{2}+bx+a\right)+2\,(b^{2}-4\,ac)\log\left(x\right)}{2\,(ab^{2}-4\,a^{2}c)}, \frac{2\,\sqrt{-b^{2}+4\,ac}\,b\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)-(b^{2}-4\,ac)\log\left(cx^{2}+bx+a\right)+2\,(b^{2}-4\,ac)\log\left(x\right)}{2\,(ab^{2}-4\,a^{2}c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*c)*
b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log
(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(54) = 108$.
time = 4.61, size = 564, normalized size = 9.10

$$\left(-\frac{b\sqrt{-4ac+b^{2}}}{2a\left(4ac-b^{2}\right)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^{4}c^{2}\left(-\frac{b\sqrt{-4ac+b^{2}}}{2a(4ac-b^{2})}-\frac{1}{2a}\right)^{2}-14a^{3}b^{2}c\left(-\frac{b\sqrt{-4ac+b^{2}}}{2a(4ac-b^{2})}-\frac{1}{2a}\right)^{2}-12a^{3}c^{2}\left(-\frac{b\sqrt{-4ac+b^{2}}}{2a(4ac-b^{2})}-\frac{1}{2a}\right)+2a^{2}b^{4}\left(-\frac{b\sqrt{-4ac+b^{2}}}{2a(4ac-b^{2})}-\frac{1}{2a}\right)^{2}+3a^{2}b^{2}c\left(-\frac{b\sqrt{-4ac+b^{2}}}{2a(4ac-b^{2})}-\frac{1}{2a}\right)-12a^{2}c^{2}+11ab^{2}c-2b^{4}}{9abc^{2}-2b^{3}c}\right)+\left(\frac{b\sqrt{-4ac+b^{2}}}{2a\left(4ac-b^{2}\right)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^{4}c^{2}\left(\frac{b\sqrt{-4ac+b^{2}}}{2a(4ac-b^{2})}-\frac{1}{2a}\right)^{2}-14a^{3}b^{2}c\left(\frac{b\sqrt{-4ac+b^{2}}}{2a(4ac-b^{2})}-\frac{1}{2a}\right)^{2}-12a^{3}c^{2}\left(\frac{b\sqrt{-4ac+b^{2}}}{2a(4ac-b^{2})}-\frac{1}{2a}\right)+2a^{2}b^{4}\left(\frac{b\sqrt{-4ac+b^{2}}}{2a(4ac-b^{2})}-\frac{1}{2a}\right)^{2}+3a^{2}b^{2}c\left(\frac{b\sqrt{-4ac+b^{2}}}{2a(4ac-b^{2})}-\frac{1}{2a}\right)-12a^{2}c^{2}+11ab^{2}c-2b^{4}}{9abc^{2}-2b^{3}c}\right)+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**3,x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**
2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a
*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
 b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b*
*3*c)) + log(x)/a
```

Giac [A]
time = 4.11, size = 62, normalized size = 1.00

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\sqrt{-b^{2}+4\,ac}\,a}-\frac{\log\left(cx^{2}+bx+a\right)}{2\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^3,x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log(
c*x^2 + b*x + a)/a + log(abs(x))/a
```

Mupad [B]
time = 1.72, size = 213, normalized size = 3.44

$$\frac{\ln\left(x\right)}{a}-\ln\left(b\,c-\left(x\,\left(6\,a\,c^{2}-2\,b^{2}\,c\right)-a\,b\,c\right)\,\left(\frac{1}{2\,a}-\frac{b\,\sqrt{b^{2}-4\,a\,c}}{2\,\left(a\,b^{2}-4\,a^{2}\,c\right)}\right)+3\,c^{2}\,x\right)\,\left(\frac{1}{2\,a}-\frac{b\,\sqrt{b^{2}-4\,a\,c}}{2\,\left(a\,b^{2}-4\,a^{2}\,c\right)}\right)-\ln\left(\left(x\,\left(6\,a\,c^{2}-2\,b^{2}\,c\right)-a\,b\,c\right)\,\left(\frac{1}{2\,a}+\frac{b\,\sqrt{b^{2}-4\,a\,c}}{2\,\left(a\,b^{2}-4\,a^{2}\,c\right)}\right)-b\,c-3\,c^{2}\,x\right)\,\left(\frac{1}{2\,a}+\frac{b\,\sqrt{b^{2}-4\,a\,c}}{2\,\left(a\,b^{2}-4\,a^{2}\,c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + a/x^2 + b/x)),x)
```

```
[Out] log(x)/a - log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) - (b*(b^2 - 4
*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) + 3*c^2*x)*(1/(2*a) - (b*(b^2 - 4*a*c)^
(1/2))/(2*(a*b^2 - 4*a^2*c))) - log((x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - b*c - 3*c^2*x)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))
```

3.419 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx$

Optimal. Leaf size=81

$$-\frac{1}{ax}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}$$

```
[Out] -1/a/x-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2-(-2*a*c+b^2)*arctanh((2*c*x+b)
/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1368, 723, 814, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sq
rt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^4}\,dx = \int \frac{1}{x^2\left(a+bx+cx^2\right)}\,dx$$
$$= -\frac{1}{ax}+\frac{\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx}{a}$$
$$= -\frac{1}{ax}+\frac{\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)dx}{a}$$
$$= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx}{a^2}$$
$$= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2}+\frac{(b^2-2ac)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2}$$
$$= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c$$
$$= -\frac{1}{ax}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}$$

Mathematica [A]
time = 0.05, size = 77, normalized size = 0.95

$$\frac{-\frac{2a}{x}+\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2b\log(x)+b\log(a+x(b+cx))}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^4),x]
```

```
[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-
b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [A]
time = 0.04, size = 81, normalized size = 1.00

method	result
default	$\frac{\frac{b\ln\left(c\,x^2+bx+a\right)}{2}+\frac{2\left(-ac+\frac{b^2}{2}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^2}-\frac{1}{ax}-\frac{b\ln(x)}{a^2}$

risch	$-\frac{1}{ax}+\frac{2\ln\left(\left(\left(-4a^3c^3+18a^2b^2c^2-12a\,b^4c+2b^6+4\sqrt{-\left(4ac-b^2\right)\left(2ac-b^2\right)^2}\,abc-2\sqrt{-\left(4ac-b^2\right)\left(2ac-b^2\right)^2}\right.\right.\right.}{a(4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*b*ln(c*x^2+b*x+a)+2*(-a*c+1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2)))-1/a/x-b*ln(x)/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.41, size = 269, normalized size = 3.32

$$\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\,x\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2+bx+a\right)+2\,(b^3-4\,abc)x\log\left(x\right)}{2\,(a^2b^2-4\,a^3c)x},-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\,x\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x\log\left(cx^2+bx+a\right)+2\,(b^3-4\,abc)x\log\left(x\right)}{2\,(a^2b^2-4\,a^3c)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^2*
c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((
a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4
*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*
a^3*c)*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.15, size = 79, normalized size = 0.98

$$\frac{b\log\left(cx^2+bx+a\right)}{2\,a^2}-\frac{b\log\left(|x|\right)}{a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

Mupad [B]
time = 1.81, size = 339, normalized size = 4.19

$$\frac{\ln\left(2\,a\,b^3+2\,b^4\,x-2\,a\,b^2\,\sqrt{b^2-4\,a\,c}+a^2\,c\,\sqrt{b^2-4\,a\,c}-2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c\,x+4\,a\,b\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(a\,\left(2\,b\,c-c\,\sqrt{b^2-4\,a\,c}\right)-\frac{b^3}{2}+\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2}\right)}{4\,a^3\,c-a^2\,b^2}-\frac{1}{a\,x}-\frac{\ln\left(2\,a\,b^3+2\,b^4\,x+2\,a\,b^2\,\sqrt{b^2-4\,a\,c}-a^2\,c\,\sqrt{b^2-4\,a\,c}+2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c\,x-4\,a\,b\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(\frac{b^3}{2}-a\,\left(2\,b\,c+c\,\sqrt{b^2-4\,a\,c}\right)+\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2}\right)}{4\,a^3\,c-a^2\,b^2}-\frac{b\,\ln\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + a/x^2 + b/x)),x)
```

```
[Out] (log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^(1/2) + a^2*c*(b^2 - 4*a*c)^
(1/2) - 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x
 + 4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(a*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) - b^3/
2 + (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - 1/(a*x) - (log(2*a*
b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^(1/2) - a^2*c*(b^2 - 4*a*c)^(1/2) + 2
*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x - 4*a*b*
c*x*(b^2 - 4*a*c)^(1/2))*(b^3/2 - a*(2*b*c + c*(b^2 - 4*a*c)^(1/2)) + (b^2*
(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - (b*log(x))/a^2
```

3.420 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}$$

```
[Out] -1/2/a/x^2+b/a^2/x+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/a^3+
b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1368, 723, 814, 648, 632, 212, 642}

$$-\frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3}+\frac{\log(x)(b^2-ac)}{a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^5),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c
)*Log[a + b*x + c*x^2])/(2*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^5}\,dx = \int \frac{1}{x^3\,(a+bx+cx^2)}\,dx$$
$$= -\frac{1}{2ax^2} + \frac{\int \frac{-b-cx}{x^2(a+bx+cx^2)}\,dx}{a}$$
$$= -\frac{1}{2ax^2} + \frac{\int \left(-\frac{b}{ax^2}+\frac{b^2-ac}{a^2x}+\frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)}\right)\,dx}{a}$$
$$= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} + \frac{\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2}\,dx}{a^3}$$
$$= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b(b^2-3ac))\int \frac{1}{a+bx+cx^2}\,dx}{2a^3} - \frac{(b^2-ac)\int \frac{b+2cx}{a+bx+c}}{2a^3}$$
$$= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3} + \frac{(b(b^2-3ac))\,\mathrm{Sub}}{}$$
$$= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\,\mathrm{l}}{}$$

Mathematica [A]
time = 0.09, size = 102, normalized size = 0.98

$$\frac{-\frac{a^2}{x^2}+\frac{2ab}{x}-\frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2(b^2-ac)\log(x)+(-b^2+ac)\log(a+x(b+cx))}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^5),x]
```

```
[Out] (-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a +
x*(b + c*x)])/(2*a^3)
```

Maple [A]
time = 0.06, size = 128, normalized size = 1.23

method	result	size
default	$\frac{\frac{(c^2a-b^2c)\ln(cx^2+bx+a)}{2c}+\frac{2\left(2abc-b^3-\frac{(c^2a-b^2c)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^3}-\frac{1}{2a\,x^2}+\frac{(-ac+b^2)\ln(x)}{a^3}+\frac{b}{a^2x}$	128
risch	Expression too large to display	2265

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/2*(a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(2*a*b*c-b^3-1/2*(a*c^2-b^2*c)
*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))-1/2/a/x^2+(-a*
c+b^2)*ln(x)/a^3+b/a^2/x
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.43, size = 358, normalized size = 3.44

$$\left[-\frac{(b^3-3\,abc)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+a^2b^2-4\,a^3c+(b^4-5\,ab^2c+4\,a^2c^2)x^2\log\left(cx^2+bx+a\right)-2\,(b^4-5\,ab^2c+4\,a^2c^2)x^2\log\left(x\right)-2\,(ab^3-4\,a^2bc)x}{2\,(a^3b^2-4\,a^4c)x^2},\ \frac{2\,(b^3-3\,abc)\sqrt{-b^2+4\,ac}\,x^2\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)-a^2b^2+4\,a^3c-(b^4-5\,ab^2c+4\,a^2c^2)x^2\log\left(cx^2+bx+a\right)+2\,(b^4-5\,ab^2c+4\,a^2c^2)x^2\log\left(x\right)+2\,(ab^3-4\,a^2bc)x}{2\,(a^3b^2-4\,a^4c)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2
 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 - 4*
a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4 - 5
*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a
^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a*b^2*c
 + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^
2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.40, size = 105, normalized size = 1.01

$$-\frac{(b^2-ac)\log(cx^2+bx+a)}{2a^3}+\frac{(b^2-ac)\log(|x|)}{a^3}-\frac{(b^3-3abc)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,a^3}+\frac{2abx-a^2}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^5,x, algorithm="giac")
```

```
[Out] -1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 - (
b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a
^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

Mupad [B]
time = 1.87, size = 447, normalized size = 4.30

$$\frac{\ln\left(2ab^4+2b^5x+6a^3c^2+2ab^3\sqrt{b^2-4ac}+2b^4x\sqrt{b^2-4ac}-9a^2b^2c-10ab^3cx-3a^2bc\sqrt{b^2-4ac}+9a^2bc^2x+3a^2c^2x\sqrt{b^2-4ac}-6ab^2cx\sqrt{b^2-4ac}\right)\left(\frac{b^4}{2}-a\left(\frac{5b^2c}{2}+\frac{3bc\sqrt{b^2-4ac}}{2}\right)+\frac{b^3\sqrt{b^2-4ac}}{2}+2a^2c^2\right)}{4a^4c-a^3b^2}-\frac{\ln\left(2ab^4+2b^5x+6a^3c^2-2ab^3\sqrt{b^2-4ac}-2b^4x\sqrt{b^2-4ac}-9a^2b^2c-10ab^3cx+3a^2bc\sqrt{b^2-4ac}+9a^2bc^2x-3a^2c^2x\sqrt{b^2-4ac}+6ab^2cx\sqrt{b^2-4ac}\right)\left(a\left(\frac{5b^2c}{2}-\frac{3bc\sqrt{b^2-4ac}}{2}\right)-\frac{b^4}{2}+\frac{b^3\sqrt{b^2-4ac}}{2}-2a^2c^2\right)}{4a^4c-a^3b^2}-\frac{\frac{1}{2a}-\frac{bx}{a^2}}{x^2}-\frac{\ln(x)\,(ac-b^2)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + a/x^2 + b/x)),x)
```

```
[Out] (log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^(1/2) + 2*b^4*x*
(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c)^
(1/2) + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*x*(b^2
- 4*a*c)^(1/2))*(b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^(1/2))/2) +
(b^3*(b^2 - 4*a*c)^(1/2))/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (log(2*a*b^
4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^(1/2) - 2*b^4*x*(b^2 - 4*a*
c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^(1/2) + 9*a
^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c*x*(b^2 - 4*a*c)^(1
/2))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^(1/2))/2) - b^4/2 + (b^3*(b^2 -
 4*a*c)^(1/2))/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (b*x)/a^2)/
x^2 - (log(x)*(a*c - b^2))/a^3
```

3.421 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx$

Optimal. Leaf size=137

$$-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log}{2a}$$

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2+(a*c-b^2)/a^3/x-b*(-2*a*c+b^2)*ln(x)/a^4+1/2*b*(-2
*a*c+b^2)*ln(c*x^2+b*x+a)/a^4-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh((2*c*x+b)/(
-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1368, 723, 814, 648, 632, 212, 642}

$$\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4}-\frac{b\log(x)(b^2-2ac)}{a^4}-\frac{b^2-ac}{a^3x}+\frac{b}{2a^2x^2}-\frac{(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c +
2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c])
- (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*
a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)x^6}\,dx=\int \frac{1}{x^4\left(a+bx+cx^2\right)}\,dx$$
$$=-\frac{1}{3ax^3}+\frac{\int \frac{-b-cx}{x^3(a+bx+cx^2)}\,dx}{a}$$
$$=-\frac{1}{3ax^3}+\frac{\int \left(-\frac{b}{ax^3}+\frac{b^2-ac}{a^2x^2}+\frac{-b^3+2abc}{a^3x}+\frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a^3(a+bx+cx^2)}\right)dx}{a}$$
$$=-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{\int \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a+bx+cx^2}\,dx}{a^4}$$
$$=-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{(b(b^2-2ac))\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^4}+\frac{(b$$
$$=-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^4}-$$
$$=-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{b(b^2-$$

Mathematica [A]
time = 0.07, size = 131, normalized size = 0.96

$$\frac{-\frac{2a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{6a(-b^2+ac)}{x}+\frac{6(b^4-4ab^2c+2a^2c^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-6(b^3-2abc)\log(x)+3(b^3-2abc)\log(a+x(b+cx))}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)*x^6),x]
```

```
[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
+ 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6
*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4)
```

Maple [A]
time = 0.06, size = 157, normalized size = 1.15

method	result
default	$\frac{\frac{(-2abc^2+b^3c)\ln(cx^2+bx+a)}{2c}+\frac{2\left(a^2c^2-3ab^2c+b^4-\frac{(-2abc^2+b^3c)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^4}-\frac{1}{3a\,x^3}-\frac{-ac+b^2}{a^3x}+\frac{b(2ac-b^2)}{a^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/2*(-2*a*b*c^2+b^3*c)/c*ln(c*x^2+b*x+a)+2*(a^2*c^2-3*a*b^2*c+b^4-1/
2*(-2*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2)))-1/3/a/x^3-(-a*c+b^2)/a^3/x+b*(2*a*c-b^2)/a^4*ln(x)+1/2*b/a^2/x^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 445, normalized size = 3.25

$$\left[\frac{3\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{b^2-4\,ac}\,x^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-2\,a^3b^2+8\,a^4c+3\,(b^5-6\,ab^3c+8\,a^2bc^2)x^3\log\left(cx^2+bx+a\right)-6\,(b^5-6\,ab^3c+8\,a^2bc^2)x^3\log\left(x\right)-6\,(ab^4-5\,a^2b^2c+4\,a^3c^2)x^2+3\,(a^2b^3-4\,a^3bc)x}{6\,(a^4b^2-4\,a^5c)x^3},\,-\frac{6\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{-b^2+4\,ac}\,x^3\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+2\,a^3b^2-8\,a^4c-3\,(b^5-6\,ab^3c+8\,a^2bc^2)x^3\log\left(cx^2+bx+a\right)+6\,(b^5-6\,ab^3c+8\,a^2bc^2)x^3\log\left(x\right)+6\,(ab^4-5\,a^2b^2c+4\,a^3c^2)x^2-3\,(a^2b^3-4\,a^3bc)x}{6\,(a^4b^2-4\,a^5c)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
 - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 +
b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*a^2*
b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^
3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c - 3*(b^5
- 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*b^3*c +
8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 - 3*(a^2*
b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)/x**6,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.59, size = 136, normalized size = 0.99

$$\frac{(b^3-2\,abc)\log\left(cx^2+bx+a\right)}{2\,a^4}-\frac{(b^3-2\,abc)\log\left(|x|\right)}{a^4}+\frac{(b^4-4\,ab^2c+2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a^4}+\frac{3\,a^2bx-2\,a^3-6\,(ab^2-a^2c)x^2}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))/
a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^2)/
(a^4*x^3)
```

Mupad [B]
time = 1.92, size = 524, normalized size = 3.82

$$\ln\left(2\,a\,b^4\,\sqrt{b^2-4\,a\,c}-2\,b^6\,x-2\,a\,b^5+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}+11\,a^2\,b^3\,c-13\,a^3\,b\,c^2+2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}-17\,a^2\,b^2\,c^2\,x+12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c}-8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(\frac{b^3}{2\,a^4}-\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}+\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right)+\ln\left(2\,a\,b^5+2\,b^6\,x+2\,a\,b^4\,\sqrt{b^2-4\,a\,c}+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}-11\,a^2\,b^3\,c+13\,a^3\,b\,c^2-2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}+17\,a^2\,b^2\,c^2\,x-12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c}-8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\right)\,\left(\frac{b^3}{2\,a^4}+\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}-\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right)+\frac{\frac{x^2\,\left(a\,c-b^2\right)}{a^3}-\frac{1}{3\,a}+\frac{b\,x}{2\,a^2}}{x^3}+\frac{b\,\ln\left(x\right)\,\left(2\,a\,c-b^2\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + a/x^2 + b/x)),x)
```

```
[Out] log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c)
^(1/2) + 11*a^2*b^3*c - 13*a^3*b*c^2 + 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^
(1/2) - 17*a^2*b^2*c^2*x + 12*a*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) -
 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/
(2*a^4) - (b^2*(b^2 - 4*a*c)^(1/2))/(2*a^4) - (b*c)/a^3 + (a^2*c^2*(b^2 - 4
*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(b^2 -
4*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^3*c + 13*a^3*b*c^2 -
2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c^2*x - 12*a*b^4*c*x
 - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^
2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/(2*a^4) + (b^2*(b^2 - 4*a*c)^(1/2))/(2*
a^4) - (b*c)/a^3 - (a^2*c^2*(b^2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + ((x
^2*(a*c - b^2))/a^3 - 1/(3*a) + (b*x)/(2*a^2))/x^3 + (b*log(x)*(2*a*c - b^2
))/a^4
```

3.422 $\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=196

$$-\frac{b(3b^2-11ac)\,x}{c^3\,(b^2-4ac)}+\frac{(3b^2-8ac)\,x^2}{2c^2\,(b^2-4ac)}-\frac{bx^3}{c\,(b^2-4ac)}+\frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{b(3b^4-20ab^2c+30a^2c^2)\tanh^{-}}{c^4\,(b^2-4ac)^{3/2}}$$

```
[Out] -b*(-11*a*c+3*b^2)*x/c^3/(-4*a*c+b^2)+1/2*(-8*a*c+3*b^2)*x^2/c^2/(-4*a*c+b^
2)-b*x^3/c/(-4*a*c+b^2)+x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(30*a^2*
c^2-20*a*b^2*c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2
)^(3/2)+1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/c^4
```

Rubi [A]
time = 0.15, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {1368, 752, 814, 648, 632, 212, 642}

$$\frac{b(30a^2c^2-20ab^2c+3b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\,(b^2-4ac)^{3/2}}+\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2c^4}-\frac{bx(3b^2-11ac)}{c^3\,(b^2-4ac)}+\frac{x^2(3b^2-8ac)}{2c^2\,(b^2-4ac)}-\frac{bx^3}{c\,(b^2-4ac)}+\frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(c + a/x^2 + b/x)^2,x]
```

```
[Out] -((b*(3*b^2 - 11*a*c)*x)/(c^3*(b^2 - 4*a*c))) + ((3*b^2 - 8*a*c)*x^2)/(2*c^
2*(b^2 - 4*a*c)) - (b*x^3)/(c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x))/((b^2 - 4*
a*c)*(a + b*x + c*x^2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Lo
g[a + b*x + c*x^2])/(2*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{x}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx=\int \frac{x^5}{(a+bx+cx^2)^2}\,dx$$

$$=\frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{\int \frac{x^3(8a+3bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$=\frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{\int \left(\frac{b(3b^2-11ac)}{c^3}-\frac{(3b^2-8ac)x}{c^2}+\frac{3bx^2}{c}-\frac{ab(3b^2-11ac)+(b^2-4ac)\,(3b^2}{c^3(a+bx+cx^2)}\right.}{-b^2+4ac}$$

$$=-\frac{b(3b^2-11ac)\,x}{c^3\,(b^2-4ac)}+\frac{(3b^2-8ac)\,x^2}{2c^2\,(b^2-4ac)}-\frac{bx^3}{c\,(b^2-4ac)}+\frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{\int \frac{ab}{}}{}$$

$$=-\frac{b(3b^2-11ac)\,x}{c^3\,(b^2-4ac)}+\frac{(3b^2-8ac)\,x^2}{2c^2\,(b^2-4ac)}-\frac{bx^3}{c\,(b^2-4ac)}+\frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{(3b^2}{}$$

$$=-\frac{b(3b^2-11ac)\,x}{c^3\,(b^2-4ac)}+\frac{(3b^2-8ac)\,x^2}{2c^2\,(b^2-4ac)}-\frac{bx^3}{c\,(b^2-4ac)}+\frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{(3b^2}{}$$

$$=-\frac{b(3b^2-11ac)\,x}{c^3\,(b^2-4ac)}+\frac{(3b^2-8ac)\,x^2}{2c^2\,(b^2-4ac)}-\frac{bx^3}{c\,(b^2-4ac)}+\frac{x^4(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{b(3b}{}$$

Mathematica [A]

time = 0.15, size = 163, normalized size = 0.83

$$\frac{-4bcx+c^2x^2+\frac{2(2a^3c^2+b^5x+ab^3(b-5cx)+a^2bc(-4b+5cx))}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(3b^4-20ab^2c+30a^2c^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+(3b^2-2ac)\log(a+x(b+cx))}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(c + a/x^2 + b/x)^2,x]
```

```
[Out] (-4*b*c*x + c^2*x^2 + (2*(2*a^3*c^2 + b^5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(
-4*b + 5*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*
c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2
) + (3*b^2 - 2*a*c)*Log[a + x*(b + c*x)])/(2*c^4)
```

Maple [A]

time = 0.06, size = 238, normalized size = 1.21

method	result
default	$-\frac{-\frac{1}{2}cx^2+2bx}{c^3}+\frac{\frac{-\frac{b(5a^2c^2-5ab^2c+b^4)x}{c(4ac-b^2)}-\frac{a(2a^2c^2-4ab^2c+b^4)}{c(4ac-b^2)}}{cx^2+bx+a}+\frac{\frac{(-8a^2c^2+14ab^2c-3b^4)\ln(cx^2+bx+a)}{2c}+\frac{2\left(11a^2bc-3ab^3-\frac{(-8a^2c^2+1}{}\right.}{}}{4ac-b^2}}{c^3}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c+a/x^2+b/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-1/2*c*x^2+2*b*x)+1/c^3*((-b*(5*a^2*c^2-5*a*b^2*c+b^4)/c/(4*a*c-b^2
)*x-a/c*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*
(1/2*(-8*a^2*c^2+14*a*b^2*c-3*b^4)/c*ln(c*x^2+b*x+a)+2*(11*a^2*b*c-3*a*b^3-
1/2*(-8*a^2*c^2+14*a*b^2*c-3*b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(
4*a*c-b^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(188) = 376.
time = 0.35, size = 1029, normalized size = 5.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3 - 8*a
*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^3 -
 (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 - (3*a*b^5 - 20
*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^2 + (
3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 +
2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) +
 2*(b^7 - 11*a*b^5*c + 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2
*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2
*c^3 - 32*a^3*c^4)*x^2 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^
3)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6
)*x), 1/2*(2*a*b^6 - 16*a^2*b^4*c + 36*a^3*b^2*c^2 - 16*a^4*c^3 + (b^4*c^3
```

```
- 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*
x^3 - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3 - 16*a^3*c^4)*x^2 + 2*(3*a*b
^5 - 20*a^2*b^3*c + 30*a^3*b*c^2 + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*
x^2 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^7 - 11*a*b^5*c + 41*a^2
*b^3*c^2 - 52*a^3*b*c^3)*x + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*
a^4*c^3 + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^2 + (3*b
^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x)*log(c*x^2 + b*x + a))/(
a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)*x^2 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(180) = 360$.

time = 1.48, size = 1012, normalized size = 5.16

$$-\frac{2bx}{c^3}+\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(30a^2c^2-20ab^2c+3b^4)}{2c^4\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)\log\left(x+\frac{16a^3c^2-17a^2b^2c+16a^2c^5\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(30a^2c^2-20ab^2c+3b^4)}{2c^4\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)+3ab^4-8ab^2c^4\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(30a^2c^2-20ab^2c+3b^4)}{2c^4\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)+b^4c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(30a^2c^2-20ab^2c+3b^4)}{2c^4\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)}{30a^2bc^2-20ab^3c+3b^5}\right)+\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(30a^2c^2-20ab^2c+3b^4)}{2c^4\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)\log\left(x+\frac{16a^3c^2-17a^2b^2c+16a^2c^5\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(30a^2c^2-20ab^2c+3b^4)}{2c^4\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)+3ab^4-8ab^2c^4\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(30a^2c^2-20ab^2c+3b^4)}{2c^4\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)+b^4c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(30a^2c^2-20ab^2c+3b^4)}{2c^4\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}-\frac{2ac-3b^2}{2c^4}\right)}{30a^2bc^2-20ab^3c+3b^5}\right)+\frac{-2a^3c^2+4a^2b^2c-ab^4+x(-5a^2bc^2+5ab^3c-b^5)}{4a^2c^5-ab^2c^4+x^2\cdot(4ac^6-b^2c^5)+x(4abc^5-b^3c^4)}+\frac{x^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x**2+b/x)**2,x)
```

```
[Out] -2*b*x/c**3 + (-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*
b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2
*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**2 - 17*a**2*b**2*c + 16*a**2*c
**5*(-b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c
**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b
**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(-b*sqrt(-(4*a*c - b**2)**3)*(30*
a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2
 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + b**4*c**3*(-b*sqrt(-
(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c
**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))
)/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) + (b*sqrt(-(4*a*c - b**2)**3)*(3
0*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c*
*2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4))*log(x + (16*a**3*c**
2 - 17*a**2*b**2*c + 16*a**2*c**5*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2
 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c**4)) + 3*a*b**4 - 8*a*b**2*c**4*(b*s
qrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c + 3*b**4)/(2*c**4*(64*a
**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - (2*a*c - 3*b**2)/(2*c
**4)) + b**4*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(30*a**2*c**2 - 20*a*b**2*c +
 3*b**4)/(2*c**4*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) -
 (2*a*c - 3*b**2)/(2*c**4)))/(30*a**2*b*c**2 - 20*a*b**3*c + 3*b**5)) + (-2
*a**3*c**2 + 4*a**2*b**2*c - a*b**4 + x*(-5*a**2*b*c**2 + 5*a*b**3*c - b**5
))/(4*a**2*c**5 - a*b**2*c**4 + x**2*(4*a*c**6 - b**2*c**5) + x*(4*a*b*c**5
 - b**3*c**4)) + x**2/(2*c**2)
```

Giac [A]
time = 3.28, size = 188, normalized size = 0.96

$$-\frac{\left(3 b^{5}-20 a b^{3} c+30 a^{2} b c^{2}\right) \arctan \left(\frac{2 c x+b}{\sqrt{-b^{2}+4 a c}}\right)}{\left(b^{2} c^{4}-4 a c^{5}\right) \sqrt{-b^{2}+4 a c}}+\frac{\left(3 b^{2}-2 a c\right) \log \left(c x^{2}+b x+a\right)}{2 c^{4}}+\frac{c^{2} x^{2}-4 b c x}{2 c^{4}}+\frac{a b^{4}-4 a^{2} b^{2} c+2 a^{3} c^{2}+\left(b^{5}-5 a b^{3} c+5 a^{2} b c^{2}\right) x}{\left(c x^{2}+b x+a\right)\left(b^{2}-4 a c\right) c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/c^4 + 1/2*(c^2*x^2 - 4*b*c*x)/c^4 + (a*b^4 - 4*a^2*b^2*c + 2*a^3*
c^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c
^4)
```

Mupad [B]
time = 1.82, size = 382, normalized size = 1.95

$$\frac{x^2}{2\,c^2}-\frac{\frac{a\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}+\frac{b\,x\,\left(5\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}}{c^4\,x^2+b\,c^3\,x+a\,c^3}-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(128\,a^4\,c^4-288\,a^3\,b^2\,c^3+168\,a^2\,b^4\,c^2-38\,a\,b^6\,c+3\,b^8\right)}{2\,\left(64\,a^3\,c^7-48\,a^2\,b^2\,c^6+12\,a\,b^4\,c^5-b^6\,c^4\right)}-\frac{2\,b\,x}{c^3}+\frac{b\,\mathrm{atan}\left(\frac{c^4\,\left(\frac{2\,b\,x\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^3}-\frac{b\,\left(b^3\,c^3-4\,a\,b\,c^4\right)\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^7\,{\left(4\,a\,c-b^2\right)}^4}\right)\,{\left(4\,a\,c-b^2\right)}^{5/2}}{30\,a^2\,b\,c^2-20\,a\,b^3\,c+3\,b^5}\right)\,\left(30\,a^2\,c^2-20\,a\,b^2\,c+3\,b^4\right)}{c^4\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c + a/x^2 + b/x)^2,x)
```

```
[Out] x^2/(2*c^2) - ((a*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c*(4*a*c - b^2)) + (b*x*(
b^4 + 5*a^2*c^2 - 5*a*b^2*c))/(c*(4*a*c - b^2)))/(a*c^3 + c^4*x^2 + b*c^3*x
) - (log(a + b*x + c*x^2)*(3*b^8 + 128*a^4*c^4 + 168*a^2*b^4*c^2 - 288*a^3*
b^2*c^3 - 38*a*b^6*c))/(2*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2
*c^6)) - (2*b*x)/c^3 + (b*atan((c^4*((2*b*x*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*
c))/(c^3*(4*a*c - b^2)^3) - (b*(b^3*c^3 - 4*a*b*c^4)*(3*b^4 + 30*a^2*c^2 -
20*a*b^2*c))/(c^7*(4*a*c - b^2)^4))*(4*a*c - b^2)^(5/2))/(3*b^5 + 30*a^2*b*
c^2 - 20*a*b^3*c))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(c^4*(4*a*c - b^2)^(3
/2))
```

3.423 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx$

Optimal. Leaf size=150

$$\frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)}-\frac{bx^2}{c\,(b^2-4ac)}+\frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{2(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}}-\frac{b\log$$

[Out] 2*(-3*a*c+b^2)*x/c^2/(-4*a*c+b^2)-b*x^2/c/(-4*a*c+b^2)+x^3*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-b*ln(c*x^2+b*x+a)/c^3

Rubi [A]
time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1354, 752, 814, 648, 632, 212, 642}

$$-\frac{2(6a^2c^2-6ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}}+\frac{2x(b^2-3ac)}{c^2\,(b^2-4ac)}-\frac{bx^2}{c\,(b^2-4ac)}+\frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{b\log(a+bx+cx^2)}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(c + a/x^2 + b/x)^(-2),x]

[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) - (b*Log[a + b*x + c*x^2])/c^3

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}\,dx = \int \frac{x^4}{(a+bx+cx^2)^2}\,dx$$

$$= \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{\int \left(-\frac{2(b^2-3ac)}{c^2}+\frac{2bx}{c}+\frac{2(a(b^2-3ac)+b(b^2-4ac)x)}{c^2(a+bx+cx^2)}\right)dx}{-b^2+4ac}$$

$$= \frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)} - \frac{bx^2}{c\,(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{2\int \frac{a(b^2-3ac)+b(b^2-4ac)x}{a+bx+cx^2}\,dx}{c^2\,(b^2-4ac)}$$

$$= \frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)} - \frac{bx^2}{c\,(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{c^3} + \frac{(b^4-6}{}$$

$$= \frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)} - \frac{bx^2}{c\,(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{b\log(a+bx+cx^2)}{c^3} - \frac{(2(}{}$$

$$= \frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)} - \frac{bx^2}{c\,(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\tan}{c^3\,(b^2-4a}$$

Mathematica [A]

time = 0.12, size = 132, normalized size = 0.88

$$\frac{cx + \frac{-b^4x-ab^2(b-4cx)+a^2c(3b-2cx)}{(b^2-4ac)(a+x(b+cx))} - \frac{2(b^4-6ab^2c+6a^2c^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} - b\log(a+x(b+cx))}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^2 + b/x)^(-2),x]
```

```
[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

Maple [A]

time = 0.06, size = 198, normalized size = 1.32

method	result
default	$\frac{x}{c^2} - \frac{\frac{-\frac{(2a^2c^2-4a\,b^2c+b^4)x}{c(4ac-b^2)}+\frac{ba(3ac-b^2)}{c(4ac-b^2)}}{c\,x^2+bx+a} + \frac{\frac{(4abc-b^3)\ln(c\,x^2+bx+a)}{c}+\frac{4\left(3a^2c-a\,b^2-\frac{(4abc-b^3)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{c^2}$

risch	$\frac{x}{c^2}+\frac{\frac{\left(2a^2c^2-4a\,b^2c+b^4\right)x}{c\left(4ac-b^2\right)}-\frac{ba\left(3ac-b^2\right)}{c\left(4ac-b^2\right)}}{c^2(c\,x^2+bx+a)}-\frac{16\ln\left(-24a^3c^3+30a^2b^2c^2-10a\,b^4c+b^6-2\sqrt{-\left(4ac-b^2\right)\left(6a^2c^2-6a\,b^2c+\right.}\right.}{c(4ac-b^2)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^2-1/c^2*((-(2*a^2*c^2-4*a*b^2*c+b^4)/c/(4*a*c-b^2)*x+b*a/c*(3*a*c-b^2)/
(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(4*a*b*c-b^3)/c*ln(c*x^2+b*x+
a)+2*(3*a^2*c-a*b^2-1/2*(4*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(146) = 292$.
time = 0.40, size = 837, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(
b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a^2*b^
2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x
^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), -(a*b^5
 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^3 -
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c
```

```
^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2
)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^
3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(141) = 282$.
time = 1.01, size = 842, normalized size = 5.61

$$\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)+2ab^3+8ab^2c^3\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)-b^4c^2\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)}{12a^2c^2-12ab^2c+2b^4}\right)+\left(-\frac{b}{c^3}+\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}+\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)+2ab^3+8ab^2c^3\left(-\frac{b}{c^3}+\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)-b^4c^2\left(-\frac{b}{c^3}+\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)}{12a^2c^2-12ab^2c+2b^4}\right)+\frac{-3a^2bc+ab^3+x\left(2a^2c^2-4ab^2c+b^4\right)}{4a^2c^4-ab^2c^3+x^2\cdot\left(4ac^5-b^2c^4\right)+x\left(4abc^4-b^3c^3\right)}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2,x)
```

```
[Out] (-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**
2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a
*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2
*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**
2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*
(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a
*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - b**
4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4
)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(12*a**2
*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 + x*(2*a**2*c**2 - 4
*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*a*c**5 - b**2*c**4)
 + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

Giac [A]
time = 2.89, size = 161, normalized size = 1.07

$$\frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}}+\frac{x}{c^2}-\frac{b\log\left(cx^2+bx+a\right)}{c^3}-\frac{\frac{(b^4-4\,ab^2c+2\,a^2c^2)x}{c}+\frac{ab^3-3\,a^2bc}{c}}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^
2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3 -
 ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b*x
+ a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B]
time = 1.80, size = 261, normalized size = 1.74

$$\frac{x}{c^2}+\frac{\frac{a\,\left(b^3-3\,a\,b\,c\right)}{c\,\left(4\,a\,c-b^2\right)}+\frac{x\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}}{c^3\,x^2+b\,c^2\,x+a\,c^2}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-24\,a\,b^5\,c+2\,b^7\right)}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+12\,a\,b^4\,c^4-b^6\,c^3\right)}-\frac{2\,\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(6\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^2 + b/x)^2,x)
```

```
[Out] x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a*
b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c*x
^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^6 -
 b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b^2)^
(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*c^2 -
 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))
```

3.424 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx$

Optimal. Leaf size=114

$$-\frac{bx}{c\left(b^2-4ac\right)}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] -b*x/c/(-4*a*c+b^2)+x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)
*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*ln(c*x^2+
b*x+a)/c^2
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1368, 752, 787, 648, 632, 212, 642}

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\left(b^2-4ac\right)^{3/2}}+\frac{x^2(2a+bx)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{bx}{c\left(b^2-4ac\right)}+\frac{\log\left(a+bx+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x),x]
```

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2
- 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x}\,dx = \int \frac{x^3}{(a+bx+cx^2)^2}\,dx$$

$$= \frac{x^2(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$

$$= -\frac{bx}{c\,(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} - \frac{\int \frac{-ab+(-b^2+4ac)x}{a+bx+cx^2}\,dx}{c\,(b^2-4ac)}$$

$$= -\frac{bx}{c\,(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2} - \frac{(b(b^2-6ac))\int \frac{1}{a+bx+c}}{2c^2\,(b^2-4ac)}$$

$$= -\frac{bx}{c\,(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{\log(a+bx+cx^2)}{2c^2} + \frac{(b(b^2-6ac))\,\mathrm{Sub}}{}$$

$$= -\frac{bx}{c\,(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\,(b^2-4ac)^{3/2}} + \frac{\mathrm{l}}{}$$

Mathematica [A]
time = 0.09, size = 109, normalized size = 0.96

$$\frac{\frac{2(-2a^2c+b^3x+ab(b-3cx))}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + \log(a+x(b+cx))}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x),x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x)))
 + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c
)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

Maple [A]
time = 0.05, size = 169, normalized size = 1.48

method	result
default	$\frac{\frac{b(3ac-b^2)x}{c^2(4ac-b^2)}+\frac{a(2ac-b^2)}{(4ac-b^2)c^2}}{c\,x^2+bx+a}+\frac{\frac{(4ac-b^2)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(-ab-\frac{(4ac-b^2)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c(4ac-b^2)}$

risch	$\frac{\frac{b\left(3ac-b^2\right)x}{c^2\left(4ac-b^2\right)}+\frac{a\left(2ac-b^2\right)}{\left(4ac-b^2\right)c^2}}{c\,x^2+bx+a}+\frac{8\ln\left(-24a^2b\,c^2+10a\,b^3c-b^5-2\sqrt{-b^2\left(4ac-b^2\right)\left(6ac-b^2\right)^2}\,cx-\sqrt{-b^2\left(4ac-b^2\right)\left(\right.}\right.}{\left(4ac-b^2\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] (b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+
a)+1/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^2+b*x+a)+2*(-a*b-1/2*(4*a*c-b^
2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(108) = 216$.
time = 0.34, size = 635, normalized size = 5.57

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\,(b^5-7\,ab^3c+12\,a^2bc^2)x+(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x)\log\left(cx^2+bx+a\right)}{2\,(ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4+(b^4c^3-8\,ab^2c^4+16\,a^2c^5)x^2+(b^5c^2-8\,ab^3c^3+16\,a^2bc^4)x)},\ \frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\,(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^5-7\,ab^3c+12\,a^2bc^2)x+(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x)\log\left(cx^2+bx+a\right)}{2\,(ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4+(b^4c^3-8\,ab^2c^4+16\,a^2c^5)x^2+(b^5c^2-8\,ab^3c^3+16\,a^2bc^4)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x),
1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^
2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 -
```

```
8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^
5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(104) = 208$.
time = 0.76, size = 729, normalized size = 6.39

$$\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)-ab^2-b^4c\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)}{6abc-b^3}\right)+\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)-ab^2-b^4c\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)}{6abc-b^3}\right)+\frac{2a^2c-ab^2+x(3abc-b^3)}{4a^2c^3-ab^2c^2+x^2\cdot(4ac^4-b^2c^3)+x(4abc^3-b^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqrt(-(4*
a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*a*b*c - b**3))/(
4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b
**3*c**2))
```

Giac [A]
time = 2.78, size = 125, normalized size = 1.10

$$-\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{ab^2-2\,a^2c+(b^3-3\,abc)x}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3
)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c + (
b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B]
time = 1.86, size = 279, normalized size = 2.45

$$\frac{\frac{a\,\left(2\,a\,c-b^2\right)}{c^2\,\left(4\,a\,c-b^2\right)}+\frac{b\,x\,\left(3\,a\,c-b^2\right)}{c^2\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}{2\,\left(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2\right)}+\frac{b\,\mathrm{atan}\left(\frac{c^2\,{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(\frac{2\,b\,x\,\left(6\,a\,c-b^2\right)}{c\,{\left(4\,a\,c-b^2\right)}^3}+\frac{b^2\,\left(4\,a\,c^2-b^2\,c\right)\,\left(6\,a\,c-b^2\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^4}\right)}{b^3-6\,a\,b\,c}\right)\,\left(6\,a\,c-b^2\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + a/x^2 + b/x)^2),x)
```

```
[Out] ((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c -
b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a^2
*b^2*c^2 - 12*a*b^4*c))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^
2*c^4)) + (b*atan((c^2*(4*a*c - b^2)^(5/2)*((2*b*x*(6*a*c - b^2))/(c*(4*a*c
 - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^4)))/
(b^3 - 6*a*b*c))*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^(3/2))
```

3.425 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2} dx$

Optimal. Leaf size=71

$$\frac{b+\frac{2a}{x}}{(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)}-\frac{4a\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (b+2*a/x)/(-4*a*c+b^2)/(c+a/x^2+b/x)-4*a*arctanh((b+2*a/x)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1366, 628, 632, 212}

$$\frac{\frac{2a}{x}+b}{(b^2-4ac)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}-\frac{4a\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^2),x]
```

```
[Out] (b + (2*a)/x)/((b^2 - 4*a*c)*(c + a/x^2 + b/x)) - (4*a*ArcTanh[(b + (2*a)/x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^2}\,dx &= -\text{Subst}\left(\int \frac{1}{(c+bx+ax^2)^2}\,dx,x,\frac{1}{x}\right)\\ &= \frac{b+\frac{2a}{x}}{(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)}+\frac{(2a)\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,dx,x,\frac{1}{x}\right)}{b^2-4ac}\\ &= \frac{b+\frac{2a}{x}}{(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)}-\frac{(4a)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+\frac{2a}{x}\right)}{b^2-4ac}\\ &= \frac{b+\frac{2a}{x}}{(b^2-4ac)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)}-\frac{4a\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 81, normalized size = 1.14

$$\frac{b^2x+a(b-2cx)}{c\left(-b^2+4ac\right)(a+x(b+cx))}+\frac{4a\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^2),x]
[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A]
time = 0.04, size = 97, normalized size = 1.37

method	result
default	$\frac{-\frac{(2ac-b^2)x}{c(4ac-b^2)}+\frac{ab}{c(4ac-b^2)}}{c\,x^2+bx+a}+\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$

risch	$\frac{-\frac{\left(2ac-b^2\right)x}{c\left(4ac-b^2\right)}+\frac{ab}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2a\ln\left(\left(-8c^2a+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2a\ln\left(\left(8c^2a-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(2*a*c-b^2)/c/(4*a*c-b^2)*x+a*b/c/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(67) = 134$.
time = 0.37, size = 387, normalized size = 5.45

$$\left[-\frac{ab^3-4\,a^2bc+2\,(ac^2x^2+abcx+a^2c)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+(b^4-6\,ab^2c+8\,a^2c^2)x}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x},\,-\frac{ab^3-4\,a^2bc-4\,(ac^2x^2+abcx+a^2c)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+(b^4-6\,ab^2c+8\,a^2c^2)x}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*lo
g((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 +
 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x + a^2*c
)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
+ (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 +
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(60) = 120$.
time = 0.35, size = 280, normalized size = 3.94

$$-2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right)+2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right)+\frac{ab+x(-2ac+b^2)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x\,(4abc^2-b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**2,x)
```

```
[Out] -2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*a*
c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (3
2*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c - b*
*2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + (a*b + x*
(-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*
(4*a*b*c**2 - b**3*c))
```

Giac [A]
time = 2.93, size = 88, normalized size = 1.24

$$-\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{b^2x-2\,acx+ab}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^2,x, algorithm="giac")
```

```
[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B]
time = 1.37, size = 135, normalized size = 1.90

$$-\frac{\frac{x\,\left(2\,a\,c-b^2\right)}{c\,\left(4\,a\,c-b^2\right)}-\frac{a\,b}{c\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a}-\frac{4\,a\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{4\,a\,c\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,a}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + a/x^2 + b/x)^2),x)
```

```
[Out] - ((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x
+ c*x^2) - (4*a*atan((((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*a*c*x
)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^(3/2)
```

3.426 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3} \, dx$

Optimal. Leaf size=66

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1368, 652, 632, 212}

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^3),x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^3}\,dx &= \int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx\\ &= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\ &= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2b)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}\\ &= \frac{2a+bx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 69, normalized size = 1.05

$$\frac{2a+bx}{\left(b^2-4ac\right)(a+x(b+cx))}-\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^3),x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A]
time = 0.03, size = 70, normalized size = 1.06

method	result	size
default	$\frac{-bx-2a}{(4ac-b^2)(cx^2+bx+a)}-\frac{2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	70

risch	$\frac{-\frac{bx}{4ac-b^2}-\frac{2a}{4ac-b^2}}{c\,x^2+bx+a}+\frac{b\ln\left(\left(-8c^2a+2b^2c\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{b\ln\left(\left(8c^2a-2b^2c\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$	148

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(62) = 124.
time = 0.35, size = 338, normalized size = 5.12

$$\left[\frac{2\,ab^2-8\,a^2c-\left(bcx^2+b^2x+ab\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+\left(b^3-4\,abc\right)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x},\frac{2\,ab^2-8\,a^2c-2\left(bcx^2+b^2x+ab\right)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+\left(b^3-4\,abc\right)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="fricas")
```

```
[Out] [(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b^2 -
 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(60) = 120.
time = 0.29, size = 253, normalized size = 3.83

$$b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)-b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)+\frac{-2a-bx}{4a^2c-ab^2+x^2\cdot\left(4ac^2-b^2c\right)+x\left(4abc-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**3,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2
)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**2)
**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*c**
2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5
*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c - a*b
**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Giac [A]
time = 3.43, size = 76, normalized size = 1.15

$$\frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}+\frac{bx+2\,a}{(cx^2+bx+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^3,x, algorithm="giac")
```

```
[Out] 2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B]
time = 1.37, size = 110, normalized size = 1.67

$$-\frac{\frac{2\,a}{4\,a\,c-b^2}+\frac{b\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{2\,b\,\mathrm{atan}\left(\frac{\left(\frac{b^2}{{\left(4\,a\,c-b^2\right)}^{3/2}}+\frac{2\,b\,c\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{b}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + a/x^2 + b/x)^2),x)
```

```
[Out] - ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*atan
(((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/
b))/(4*a*c - b^2)^(3/2)
```

3.427 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4} \, dx$

Optimal. Leaf size=66

$$-\frac{b+2cx}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1368, 628, 632, 212}

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)\,(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^4}\,dx &= \int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx\\ &= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2c)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\ &= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(4c)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}\\ &= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 70, normalized size = 1.06

$$-\frac{\frac{b+2cx}{a+x(b+cx)}+\frac{4c\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{b^2-4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^4),x]
```

```
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Maple [A]
time = 0.03, size = 68, normalized size = 1.03

method	result	size
default	$\frac{2cx+b}{(4ac-b^2)(c x^2+bx+a)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	68

risch	$\frac{\frac{2cx}{4ac-b^2}+\frac{b}{4ac-b^2}}{c\,x^2+bx+a}+\frac{2c\ln\left(\left(-8c^2a+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2c\ln\left(\left(8c^2a-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$	144

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(62) = 124$.
time = 0.37, size = 341, normalized size = 5.17

$$\left[-\frac{b^3-4\,abc+2\,(c^2x^2+bcx+ac)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x},-\frac{b^3-4\,abc-4\,(c^2x^2+bcx+ac)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="fricas")
```

```
[Out] [-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 -
 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(61) = 122$.
time = 0.31, size = 265, normalized size = 4.02

$$-2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right)+2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-16ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+2b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2bc}{4c^2}\right)+\frac{b+2cx}{4a^2c-ab^2+x^2\cdot(4ac^2-b^2c)+x\,(4abc-b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**4,x)
```

```
[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*a*
c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (
32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*
c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Giac [A]
time = 6.50, size = 76, normalized size = 1.15

$$-\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,cx+b}{(cx^2+bx+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^4,x, algorithm="giac")
```

```
[Out] -4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B]
time = 0.08, size = 119, normalized size = 1.80

$$\frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{4\,c^2\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,c}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + a/x^2 + b/x)^2),x)
```

```
[Out] (b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan((((
2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))*(
4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

3.428 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx$

Optimal. Leaf size=108

$$\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}$$

```
[Out] (b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)*arctanh((2*c*
x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/2*ln(c*x^2+b*x+
a)/a^2
```

Rubi [A]
time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1368, 754, 814, 648, 632, 212, 642}

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^5),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x
]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^5}\,dx = \int \frac{1}{x\,(a+bx+cx^2)^2}\,dx$$
$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)} - \frac{\int \frac{-b^2+4ac-bcx}{x(a+bx+cx^2)}\,dx}{a\,(b^2-4ac)}$$
$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)} - \frac{\int \left(\frac{-b^2+4ac}{ax}+\frac{b(b^2-5ac)+c(b^2-4ac)x}{a(a+bx+cx^2)}\right)dx}{a\,(b^2-4ac)}$$
$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b(b^2-5ac)+c(b^2-4ac)x}{a+bx+cx^2}\,dx}{a^2\,(b^2-4ac)}$$
$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2} - \frac{(b(b^2-6ac))\int \frac{1}{a+bx+cx^2}\,dx}{2a^2\,(b^2-4ac)}$$
$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{\log(x)}{a^2} - \frac{\log(a+bx+cx^2)}{2a^2} + \frac{(b(b^2-6ac))\,\mathrm{Subst}\left(\int\right.}{a^2\,(b}$$
$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\,(b^2-4ac)^{3/2}} + \frac{\log(x)}{a^2} - \log(a$$

Mathematica [A]
time = 0.12, size = 107, normalized size = 0.99

$$\frac{\frac{2a(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + 2\log(x) - \log(a+x(b+cx))}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^5),x]
[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*L
og[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [A]
time = 0.06, size = 177, normalized size = 1.64

method	result	size
default	$-\frac{\frac{abcx}{4ac-b^2}-\frac{a(2ac-b^2)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{\left(4c^2a-b^2c\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(5abc-b^3-\frac{\left(4c^2a-b^2c\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^2}+\frac{\ln(x)}{a^2}$	177

risch	Expression too large to display	2292

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*((a*b*c/(4*a*c-b^2)*x-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*
a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(5*a*b*c-b^3-1/2*(4*a*c^2
-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))+ln(x)/
a^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(102) = 204$.
time = 0.44, size = 781, normalized size = 7.23

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\,(ab^3c-4\,a^2bc^2)x-(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x)\log\left(cx^2+bx+a\right)+2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x)\log\left(x\right)}{2\,(a^3b^4-8\,a^4b^2c+16\,a^5c^2+(a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3)x^2+(a^2b^5-8\,a^3b^3c+16\,a^4bc^2)x)},\ \frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\,(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+2\,(ab^3c-4\,a^2bc^2)x-(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x)\log\left(cx^2+bx+a\right)+2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x)\log\left(x\right)}{2\,(a^3b^4-8\,a^4b^2c+16\,a^5c^2+(a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3)x^2+(a^2b^5-8\,a^3b^3c+16\,a^4bc^2)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2
 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^
3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
```

```
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2
+ (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.71, size = 126, normalized size = 1.17

$$-\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}}-\frac{\log\left(cx^2+bx+a\right)}{2\,a^2}+\frac{\log\left(|x|\right)}{a^2}+\frac{abcx+ab^2-2\,a^2c}{(cx^2+bx+a)(b^2-4\,ac)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^5,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c
)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 + (a
*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

Mupad [B]
time = 2.10, size = 620, normalized size = 5.74

$$\frac{\ln(x)}{a^2}+\frac{\frac{2ac-b^2}{a(4ac-b^2)}-\frac{bcx}{a(4ac-b^2)}}{cx^2+bx+a}+\frac{\ln\left(2ab^6+2b^7x-96a^4c^3+2ab^3\sqrt{-(4ac-b^2)^3}-23a^2b^4c+2b^4x\sqrt{-(4ac-b^2)^3}+84a^3b^2c^2+94a^2b^3c^2x+12a^2c^2x\sqrt{-(4ac-b^2)^3}-24ab^5cx-9a^2bc\sqrt{-(4ac-b^2)^3}-120a^3bc^3x-12ab^2cx\sqrt{-(4ac-b^2)^3}\right)\left(b^6-64a^3c^3+b^3\sqrt{-(4ac-b^2)^3}+48a^2b^2c^2-12ab^4c-6abc\sqrt{-(4ac-b^2)^3}\right)}{2a^2(4ac-b^2)^3}+\frac{\ln\left(96a^4c^3-2b^7x-2ab^6+2ab^3\sqrt{-(4ac-b^2)^3}+23a^2b^4c+2b^4x\sqrt{-(4ac-b^2)^3}-84a^3b^2c^2-94a^2b^3c^2x+12a^2c^2x\sqrt{-(4ac-b^2)^3}+24ab^5cx-9a^2bc\sqrt{-(4ac-b^2)^3}+120a^3bc^3x-12ab^2cx\sqrt{-(4ac-b^2)^3}\right)\left(b^6-64a^3c^3-b^3\sqrt{-(4ac-b^2)^3}+48a^2b^2c^2-12ab^4c+6abc\sqrt{-(4ac-b^2)^3}\right)}{2a^2(4ac-b^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + a/x^2 + b/x)^2),x)
```

```
[Out] log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))/
(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a^3*
b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) - 84*a^3*
```

```
b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3)
```

3.429 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx$

Optimal. Leaf size=148

$$-\frac{2(b^2-3ac)}{a^2(b^2-4ac)x}+\frac{b^2-2ac+bcx}{a(b^2-4ac)x(a+bx+cx^2)}-\frac{2(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}}-\frac{2b\log(x)}{a^3}+\frac{b}{}$$

```
[Out] -2*(-3*a*c+b^2)/a^2/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^
2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/
a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+b*ln(c*x^2+b*x+a)/a^3
```

Rubi [A]
time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1368, 754, 814, 648, 632, 212, 642}

$$\frac{b\log(a+bx+cx^2)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{2(b^2-3ac)}{a^2x(b^2-4ac)}-\frac{2(6a^2c^2-6ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^6),x]
```

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 -
4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 +
(b*Log[a + b*x + c*x^2])/a^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^6}\,dx = \int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{\int \frac{-2(b^2-3ac)-2bcx}{x^2(a+bx+cx^2)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{2(-b^2+3ac)}{ax^2} - \frac{2b(-b^2+4ac)}{a^2x} + \frac{2(-b^4+5ab^2c-3a^2c^2-bc(b^2-}{a^2(a+bx+cx^2)}\right.}{a\left(b^2-4ac\right)}$$

$$= -\frac{2(b^2-3ac)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} - \frac{2\int \frac{-b^4+5ab^2c-3a^2c^2-}{a+bx+cx}}{a^3\left(b^2-4\right.}$$

$$= -\frac{2(b^2-3ac)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} + \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{a^3} + \frac{(}{}$$

$$= -\frac{2(b^2-3ac)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(a+bx+cx^2\right)}{a^3}$$

$$= -\frac{2(b^2-3ac)}{a^2\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)} - \frac{2\left(b^4-6ab^2c+6a^2c^2\right)\tanh^{-1}\left(\right.}{a^3\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A]

time = 0.18, size = 131, normalized size = 0.89

$$-\frac{\frac{a}{x}+\frac{a\left(b^3-3abc+b^2cx-2ac^2x\right)}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)}+\frac{2\left(b^4-6ab^2c+6a^2c^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{3/2}}+2b\log(x)-b\log(a+x(b+cx))}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^6),x]
```

```
[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(b
 + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^3)
```

Maple [A]

time = 0.06, size = 205, normalized size = 1.39

method	result
default	$-\frac{\frac{\frac{ac\left(2ac-b^2\right)x}{4ac-b^2}+\frac{ab\left(3ac-b^2\right)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{\left(-4ab\,c^2+b^3c\right)\ln\left(c\,x^2+bx+a\right)}{c}+\frac{4\left(3a^2c^2-5a\,b^2c+b^4-\frac{\left(-4ab\,c^2+b^3c\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^3}-\frac{1}{a^2 x}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x+a*b*(3*a*c-b^2)/(4*a*c-b^2))/(c*x^2+
b*x+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3*c)/c*ln(c*x^2+b*x+a)+2*(3*a^2*c^2
-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))))-1/a^2/x-2*b*ln(x)/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(144) = 288$.
time = 0.48, size = 975, normalized size = 6.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="fricas")
```

```
[Out] [-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3
*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a
^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*log(
(2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2
+ b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5
*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c
- 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^
3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)
```

```
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)
) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^
3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5*b^2*c + 1
6*a^6*c^2)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**6,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.75, size = 171, normalized size = 1.16

$$\frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}}-\frac{2\,b^2cx^2-6\,ac^2x^2+2\,b^3x-7\,abcx+ab^2-4\,a^2c}{(a^2b^2-4\,a^3c)(cx^3+bx^2+ax)}+\frac{b\log\left(cx^2+bx+a\right)}{a^3}-\frac{2\,b\log\left(|x|\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^6,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3*x
 - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*x))
 + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

Mupad [**B**]
time = 2.13, size = 775, normalized size = 5.24

$$\ln\left(2\,a\,b^7+2\,b^8\,x+2\,a\,b^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-23\,a^2\,b^5\,c-108\,a^4\,b\,c^3+24\,a^4\,c^4\,x+2\,b^5\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+87\,a^3\,b^3\,c^2+3\,a^3\,c^2\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-9\,a^2\,b^2\,c\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+97\,a^2\,b^4\,c^2\,x-138\,a^3\,b^2\,c^3\,x-24\,a\,b^6\,c\,x-12\,a\,b^3\,c\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+15\,a^2\,b\,c^2\,x\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}\right)$$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + a/x^2 + b/x)^2),x)
```

```
[Out] log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^5*c - 1
08*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) + 87*a^3*b^3
*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^
(1/2) + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*c*x*
(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(
```

```
-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c))/(a^2*(4*a*c - b^2)) + (2*c*x^2*
(3*a*c - b^2))/(a^2*(4*a*c - b^2)))/(a*x + b*x^2 + c*x^3) - log(2*a*b^4*(-(
4*a*c - b^2)^3)^(1/2) - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^4*b*c^3 -
24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) - 87*a^3*b^3*c^2 + 3*a^3*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 97*a^2*
b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-(4*a*c - b^2)
^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(-(4*a*c - b^2)^
3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - b/a^3) -
(2*b*log(x))/a^3
```

3.430 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx$

Optimal. Leaf size=202

$$-\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2}+\frac{b(3b^2-11ac)}{a^3\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)}+\frac{b(3b^4-20ab^2c+30a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*(8*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x^2+b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^2)/x
+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^2+b*x+a)+b*(30*a^2*c^2-20*a*b^2*
c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(-2*a
*c+3*b^2)*ln(x)/a^4-1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/a^4
```

Rubi [A]
time = 0.17, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1368, 754, 814, 648, 632, 212, 642}

$$-\frac{(3b^2-2ac)\log\left(a+bx+cx^2\right)}{2a^4}+\frac{\log(x)\left(3b^2-2ac\right)}{a^4}+\frac{b(3b^2-11ac)}{a^3x\left(b^2-4ac\right)}-\frac{3b^2-8ac}{2a^2x^2\left(b^2-4ac\right)}+\frac{b(30a^2c^2-20ab^2c+3b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx}{ax^2\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^2*x^7),x]
```

```
[Out] -1/2*(3*b^2 - 8*a*c)/(a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b
^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^
2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4
*a*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2
- 2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2 x^7}\,dx = \int \frac{1}{x^3\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} - \frac{\int \frac{-3b^2+8ac-3bcx}{x^3(a+bx+cx^2)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-3b^2+8ac}{ax^3}+\frac{3b^3-11abc}{a^2x^2}+\frac{(b^2-4ac)(-3b^2+2ac)}{a^3x}+\frac{b(3b^4-}{}\right.}{a\left(b^2-4ac\right)}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b(3b^2-11ac)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{(3b^2-2ac}{a^4}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b(3b^2-11ac)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{(3b^2-2ac}{a^4}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b(3b^2-11ac)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{(3b^2-2ac}{a^4}$$

$$= -\frac{3b^2-8ac}{2a^2\left(b^2-4ac\right)x^2} + \frac{b(3b^2-11ac)}{a^3\left(b^2-4ac\right)x} + \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^2\left(a+bx+cx^2\right)} + \frac{b(3b^4-20}{}$$

Mathematica [A]
time = 0.23, size = 175, normalized size = 0.87

$$\frac{-\frac{a^2}{x^2}+\frac{4ab}{x}+\frac{2a(b^4-4ab^2c+2a^2c^2+b^3cx-3abc^2x)}{(b^2-4ac)(a+x(b+cx))}+\frac{2b(3b^4-20ab^2c+30a^2c^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+2(3b^2-2ac)\log(x)+(-3b^2+2ac)\log(a+x(b+cx))}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^2*x^7),x]
```

```
[Out] (-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a
*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 3
0*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4)
```

Maple [A]
time = 0.06, size = 255, normalized size = 1.26

method	result
default	$\frac{\frac{\frac{acb(3ac-b^2)x}{4ac-b^2}-\frac{a(2a^2c^2-4ab^2c+b^4)}{4ac-b^2}}{cx^2+bx+a}+\frac{\frac{(8a^2c^3-14ab^2c^2+3b^4c)\ln(cx^2+bx+a)}{2c}+\frac{2\left(19a^2bc^2-17ab^3c+3b^5-\frac{(8a^2c^3-14ab^2c^2+3b^4c)b}{2c}\right)\arctan}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^4}$

risch	$\frac{\frac{bc(11ac-3b^2)x^3}{a^3(4ac-b^2)}-\frac{(8a^2c^2-25a\,b^2c+6b^4)x^2}{2a^3(4ac-b^2)}+\frac{3bx}{2a^2}-\frac{1}{2a}}{x^2(c\,x^2+bx+a)}-\frac{2\ln(x)c}{a^3}+\frac{3b^2\ln(x)}{a^4}+\left(\sum_{\_R=\mathrm{RootOf}((64a^7c^3-48a^6b^2c^2+12a^5b^4c-a^4b$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*((a*c*b*(3*a*c-b^2)/(4*a*c-b^2)*x-a*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-
b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)/c*l
n(c*x^2+b*x+a)+2*(19*a^2*b*c^2-17*a*b^3*c+3*b^5-1/2*(8*a^2*c^3-14*a*b^2*c^2
+3*b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))-1/2/
a^2/x^2+(-2*a*c+3*b^2)*ln(x)/a^4+2*b/a^3/x
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(194) = 388$.
time = 0.53, size = 1226, normalized size = 6.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 +
 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3
)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c
+ 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c
*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x +
((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a
*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*
a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^
4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3
```

```
*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^
5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*
c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a
*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)
*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64
*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*
a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2
)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a
^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3
*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^
2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**2/x**7,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 7.05, size = 229, normalized size = 1.13

$$-\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^4b^2-4\,a^5c)\sqrt{-b^2+4\,ac}}-\frac{(3\,b^2-2\,ac)\log(cx^2+bx+a)}{2\,a^4}+\frac{(3\,b^2-2\,ac)\log(|x|)}{a^4}-\frac{a^3b^2-4\,a^4c-2\,(3\,ab^3c-11\,a^2bc^2)x^3-(6\,ab^4-25\,a^2b^2c+8\,a^3c^2)x^2-3\,(a^2b^3-4\,a^3bc)x}{2\,(cx^2+bx+a)(b^2-4\,ac)a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^2/x^7,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c -
2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2
 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)
```

Mupad [**B**]
time = 2.30, size = 914, normalized size = 4.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + a/x^2 + b/x)^2),x)
```

```
[Out] (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^(1/2) - 7
3*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b^4*c^2 - 492*a^4*
b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(4*a*
c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4*a*c
- b^2)^3)^(1/2) - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(3*b^8 + 128*a
^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3
 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b^3*c*(-(4*a*c
 - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (log(x)*(2*a*c - 3*b^2))/a^4 -
 (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*c))/(2*a^3
*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2)))/(a*x^2 +
b*x^3 + c*x^4) + (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5*(-(4*a*c -
b^2)^3)^(1/2) - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b
^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 27*a^3*b
*c^2*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x - 24*
a^3*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x - 40*a*
b^4*c*x*(-(4*a*c - b^2)^3)^(1/2) + 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2
))*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2
- 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 20
*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3)
```

3.431 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx$

Optimal. Leaf size=238

$$\frac{3(b^4-7ab^2c+10a^2c^2)\,x}{c^3\,(b^2-4ac)^2}-\frac{3b(b^2-6ac)\,x^2}{2c^2\,(b^2-4ac)^2}+\frac{x^5(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2}+\frac{x^3(a(b^2-10ac)+b(b^2-7ac)\,x)}{c\,(b^2-4ac)^2\,(a+bx+cx^2)}-$$

```
[Out] 3*(10*a^2*c^2-7*a*b^2*c+b^4)*x/c^3/(-4*a*c+b^2)^2-3/2*b*(-6*a*c+b^2)*x^2/c^
2/(-4*a*c+b^2)^2+1/2*x^5*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+x^3*(a*(-10
*a*c+b^2)+b*(-7*a*c+b^2)*x)/c/(-4*a*c+b^2)^2/(c*x^2+b*x+a)-3*(-20*a^3*c^3+3
0*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4
*a*c+b^2)^(5/2)-3/2*b*ln(c*x^2+b*x+a)/c^4
```

Rubi [A]
time = 0.20, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {1354, 752, 832, 814, 648, 632, 212, 642}

$$\frac{3x(10a^2c^2-7ab^2c+b^4)}{c^3\,(b^2-4ac)^2}-\frac{3(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^4\,(b^2-4ac)^{5/2}}-\frac{3bx^2(b^2-6ac)}{2c^2\,(b^2-4ac)^2}+\frac{x^5(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2}+\frac{x^3(bx(b^2-7ac)+a(b^2-10ac))}{c\,(b^2-4ac)^2\,(a+bx+cx^2)}-\frac{3b\log\left(a+bx+cx^2\right)}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^2 + b/x)^(-3),x]
```

```
[Out] (3*(b^4 - 7*a*b^2*c + 10*a^2*c^2)*x)/(c^3*(b^2 - 4*a*c)^2) - (3*b*(b^2 - 6*
a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a +
 b*x + c*x^2)^2) + (x^3*(a*(b^2 - 10*a*c) + b*(b^2 - 7*a*c)*x))/(c*(b^2 - 4
*a*c)^2*(a + b*x + c*x^2)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3
*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^4*(b^2 - 4*a*c)^(5/2)) - (
3*b*Log[a + b*x + c*x^2])/(2*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3}\,dx &= \int \frac{x^6}{\left(a+bx+cx^2\right)^3}\,dx \\
&= \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int \frac{x^4(10a+2bx)}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)} \\
&= \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2-10ac\right)+b\left(b^2-7ac\right)x\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{\int \frac{x^2(6a(b^2-10ac)+6b(b}{a+bx+cx^2}}{2c\left(b^2-4a\right.} \\
&= \frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3\left(a\left(b^2-10ac\right)+b\left(b^2-7ac\right)x\right)}{c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{\int \left(-\frac{6(b^4-7ab^2c+10a}{c^2}\right.}{} \\
&= \frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2}-\frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3(a(b^2}{c\left(b^2\right.} \\
&= \frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2}-\frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3(a(b^2}{c\left(b^2\right.} \\
&= \frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2}-\frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3(a(b^2}{c\left(b^2\right.} \\
&= \frac{3\left(b^4-7ab^2c+10a^2c^2\right)x}{c^3\left(b^2-4ac\right)^2}-\frac{3b\left(b^2-6ac\right)x^2}{2c^2\left(b^2-4ac\right)^2}+\frac{x^5(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^3(a(b^2}{c\left(b^2\right.}
\end{aligned}$$

Mathematica [A]
time = 0.24, size = 260, normalized size = 1.09

$$\frac{2c^2x+\frac{b^7-14ab^5c+61a^2b^3c^2-78a^3bc^3-6b^6cx+48ab^4c^2x-102a^2b^2c^3x+36a^3c^4x}{\left(b^2-4ac\right)^2(a+x(b+cx))}+\frac{-b^6x+a^2b^2c(5b-9cx)-ab^4(b-6cx)+a^3c^2(-5b+2cx)}{\left(b^2-4ac\right)(a+x(b+cx))^2}+\frac{6c\left(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}-3bc\log(a+x(b+cx))}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^2 + b/x)^(-3),x]
```

```
[Out] (2*c^2*x + (b^7 - 14*a*b^5*c + 61*a^2*b^3*c^2 - 78*a^3*b*c^3 - 6*b^6*c*x +
48*a*b^4*c^2*x - 102*a^2*b^2*c^3*x + 36*a^3*c^4*x)/((b^2 - 4*a*c)^2*(a + x*
(b + c*x))) + (-(b^6*x) + a^2*b^2*c*(5*b - 9*c*x) - a*b^4*(b - 6*c*x) + a^3
```

```
*c^2*(-5*b + 2*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (6*c*(b^6 - 10*a
*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]
])/(-b^2 + 4*a*c)^(5/2) - 3*b*c*Log[a + x*(b + c*x)])/(2*c^5)
```

Maple [A]
time = 0.08, size = 401, normalized size = 1.68

method	result
default	$\frac{x}{c^3} - \frac{\frac{-\frac{3(6a^3c^3-17a^2b^2c^2+8a\,b^4c-b^6)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{b(42a^3c^3+41a^2b^2c^2-34a\,b^4c+5b^6)x^2}{2(16a^2c^2-8a\,b^2c+b^4)c}-\frac{a(14a^3c^3-71a^2b^2c^2+38a\,b^4c-5b^6)x}{c(16a^2c^2-8a\,b^2c+b^4)}+\frac{b\,a^2(58a^2c^2-36a}{2c(16a^2c^2-8a}}{(c\,x^2+bx+a)^2}}{c^3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^3-1/c^3*((-3*(6*a^3*c^3-17*a^2*b^2*c^2+8*a*b^4*c-b^6)/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^3+1/2*b*(42*a^3*c^3+41*a^2*b^2*c^2-34*a*b^4*c+5*b^6)/(16*a^2*c^
2-8*a*b^2*c+b^4)/c*x^2-a/c*(14*a^3*c^3-71*a^2*b^2*c^2+38*a*b^4*c-5*b^6)/(16
*a^2*c^2-8*a*b^2*c+b^4)*x+1/2*b*a^2/c*(58*a^2*c^2-36*a*b^2*c+5*b^4)/(16*a^2
*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*
a^2*b*c^2-8*a*b^3*c+b^5)/c*ln(c*x^2+b*x+a)+2*(10*a^3*c^2-7*a^2*b^2*c+b^4*a-
1/2*(16*a^2*b*c^2-8*a*b^3*c+b^5)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 953 vs. 2(228) = 456.
time = 0.38, size = 1926, normalized size = 8.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="fricas")

[Out] [-1/2*(5*a^2*b^7 - 56*a^3*b^5*c + 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 123*a^2*b^4*c^3 - 254*a^3*b^2*c^4 + 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c + 225*a^2*b^5*c^2 - 314*a^3*b^3*c^3 + 88*a^4*b*c^4)*x^2 + 3*(a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^3 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^4*c^2 - 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^3 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^2 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^5 + 24*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*a^2*b^5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x), -1/2*(5*a^2*b^7 - 56*a^3*b^5*c + 202*a^4*b^3*c^2 - 232*a^5*b*c^3 - 2*(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^5 - 4*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(2*b^8*c - 26*a*b^6*c^2 + 123*a^2*b^4*c^3 - 254*a^3*b^2*c^4 + 200*a^4*c^5)*x^3 + (5*b^9 - 58*a*b^7*c + 225*a^2*b^5*c^2 - 314*a^3*b^3*c^3 + 88*a^4*b*c^4)*x^2 + 6*(a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^4 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^3 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^2 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(5*a*b^8 - 59*a^2*b^6*c + 235*a^3*b^4*c^2 - 346*a^4*b^2*c^3 + 120*a^5*c^4)*x + 3*(a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3 + (b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^4 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^3 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^2 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^4 + 2*(b^7*c^5 - 12*a*b^5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^3 + (b^8*c^4 - 10*a*b^6*c^5 + 24*a^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^2 + 2*(a*b^7*c^4 - 12*a^2*b^5*c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1714 vs. $2(236) = 472$.

time = 3.13, size = 1714, normalized size = 7.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3,x)
```

```
[Out] (-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c*
*2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 64
0*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66
*a**3*b*c**2 - 64*a**3*c**6*(-3*b/(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20
*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**
5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b*
*8*c - b**10))) + 27*a**2*b**3*c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) - 3*sqr
t(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**
6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160
*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/
(2*c**4) - 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 1
0*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))))/(60*a**3*c**3 - 9
0*a**2*b**2*c**2 + 30*a*b**4*c - 3*b**6)) + (-3*b/(2*c**4) + 3*sqrt(-(4*a*c
 - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**
4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**
6*c**2 + 20*a*b**8*c - b**10)))*log(x + (-66*a**3*b*c**2 - 64*a**3*c**6*(-3
*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2
+ 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a
**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))) + 27*a**2*b**3*
c + 48*a**2*b**2*c**5*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**2)**5)*(20*a**3*
c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(1024*a**5*c**5 - 12
80*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c -
 b**10))) - 3*a*b**5 - 12*a*b**4*c**4*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10))) + b**6*c**3*(-3*b/(2*c**4) + 3*sqrt(-(4*a*c - b**
2)**5)*(20*a**3*c**3 - 30*a**2*b**2*c**2 + 10*a*b**4*c - b**6)/(2*c**4*(102
4*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
 + 20*a*b**8*c - b**10))))/(60*a**3*c**3 - 90*a**2*b**2*c**2 + 30*a*b**4*c
- 3*b**6)) + (-58*a**4*b*c**2 + 36*a**3*b**3*c - 5*a**2*b**5 + x**3*(36*a**
3*c**4 - 102*a**2*b**2*c**3 + 48*a*b**4*c**2 - 6*b**6*c) + x**2*(-42*a**3*b
*c**3 - 41*a**2*b**3*c**2 + 34*a*b**5*c - 5*b**7) + x*(28*a**4*c**3 - 142*a
**3*b**2*c**2 + 76*a**2*b**4*c - 10*a*b**6))/(32*a**4*c**6 - 16*a**3*b**2*c
**5 + 2*a**2*b**4*c**4 + x**4*(32*a**2*c**8 - 16*a*b**2*c**7 + 2*b**4*c**6)
```

```
 + x**3*(64*a**2*b*c**7 - 32*a*b**3*c**6 + 4*b**5*c**5) + x**2*(64*a**3*c**
7 - 12*a*b**4*c**5 + 2*b**6*c**4) + x*(64*a**3*b*c**6 - 32*a**2*b**3*c**5 +
 4*a*b**5*c**4)) + x/c**3
```

Giac [A]
time = 5.91, size = 282, normalized size = 1.18

$$\frac{3\,{\left(b^{6} - 10\,a b^{4} c + 30\,a^{2} b^{2} c^{2} - 20\,a^{3} c^{3}\right)} \arctan\left(\frac{2\,c x + b}{\sqrt{-b^{2} + 4\,a c}}\right)}{{\left(b^{4} c^{4} - 8\,a b^{2} c^{5} + 16\,a^{2} c^{6}\right)} \sqrt{-b^{2} + 4\,a c}} + \frac{x}{c^{3}} - \frac{3\,b \log\left(c x^{2} + b x + a\right)}{2\,c^{4}} - \frac{5\,a^{2} b^{5} - 36\,a^{3} b^{3} c + 58\,a^{4} b c^{2} + 6\,{\left(b^{6} c - 8\,a b^{4} c^{2} + 17\,a^{2} b^{2} c^{3} - 6\,a^{3} c^{4}\right)} x^{3} + {\left(5\,b^{7} - 34\,a b^{5} c + 41\,a^{2} b^{3} c^{2} + 42\,a^{3} b c^{3}\right)} x^{2} + 2\,{\left(5\,a b^{6} - 38\,a^{2} b^{4} c + 71\,a^{3} b^{2} c^{2} - 14\,a^{4} c^{3}\right)} x}{2\,{\left(c x^{2} + b x + a\right)}^{2} {\left(b^{2} - 4\,a c\right)}^{2} c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3,x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(
-b^2 + 4*a*c))/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) +
x/c^3 - 3/2*b*log(c*x^2 + b*x + a)/c^4 - 1/2*(5*a^2*b^5 - 36*a^3*b^3*c + 58
*a^4*b*c^2 + 6*(b^6*c - 8*a*b^4*c^2 + 17*a^2*b^2*c^3 - 6*a^3*c^4)*x^3 + (5*
b^7 - 34*a*b^5*c + 41*a^2*b^3*c^2 + 42*a^3*b*c^3)*x^2 + 2*(5*a*b^6 - 38*a^2
*b^4*c + 71*a^3*b^2*c^2 - 14*a^4*c^3)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)
^2*c^4)
```

Mupad [B]
time = 2.00, size = 705, normalized size = 2.96

$$\frac{x}{c^3}-\frac{\frac{3\,x^3\,\left(-6\,a^3\,c^3+17\,a^2\,b^2\,c^2-8\,a\,b^4\,c+b^6\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{x^2\,\left(42\,a^3\,b\,c^3+41\,a^2\,b^3\,c^2-34\,a\,b^5\,c+5\,b^7\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(58\,a^2\,b\,c^2-36\,a\,b^3\,c+5\,b^5\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x\,\left(-14\,a^3\,c^3+71\,a^2\,b^2\,c^2-38\,a\,b^4\,c+5\,b^6\right)}{c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{a^2\,c^3+c^5\,x^4+x^2\,\left(b^2\,c^3+2\,a\,c^4\right)+2\,b\,c^4\,x^3+2\,a\,b\,c^3\,x}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-3072\,a^5\,b\,c^5+3840\,a^4\,b^3\,c^4-1920\,a^3\,b^5\,c^3+480\,a^2\,b^7\,c^2-60\,a\,b^9\,c+3\,b^{11}\right)}{2\,\left(1024\,a^5\,c^9-1280\,a^4\,b^2\,c^8+640\,a^3\,b^4\,c^7-160\,a^2\,b^6\,c^6+20\,a\,b^8\,c^5-b^{10}\,c^4\right)}+\frac{3\,\mathrm{atan}\left(\frac{\left(\frac{3\,x\,\left(-20\,a^3\,c^3+30\,a^2\,b^2\,c^2-10\,a\,b^4\,c+b^6\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^5}+\frac{3\,\left(16\,a^2\,b\,c^5-8\,a\,b^3\,c^4+b^5\,c^3\right)\,\left(-20\,a^3\,c^3+30\,a^2\,b^2\,c^2-10\,a\,b^4\,c+b^6\right)}{2\,c^7\,{\left(4\,a\,c-b^2\right)}^5\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\,\left(32\,a^2\,c^6\,{\left(4\,a\,c-b^2\right)}^{5/2}+2\,b^4\,c^4\,{\left(4\,a\,c-b^2\right)}^{5/2}-16\,a\,b^2\,c^5\,{\left(4\,a\,c-b^2\right)}^{5/2}\right)}{-60\,a^3\,c^3+90\,a^2\,b^2\,c^2-30\,a\,b^4\,c+3\,b^6}\right)\,\left(-20\,a^3\,c^3+30\,a^2\,b^2\,c^2-10\,a\,b^4\,c+b^6\right)}{c^4\,{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^2 + b/x)^3,x)
```

```
[Out] x/c^3 - ((3*x^3*(b^6 - 6*a^3*c^3 + 17*a^2*b^2*c^2 - 8*a*b^4*c))/(b^4 + 16*a
^2*c^2 - 8*a*b^2*c) + (x^2*(5*b^7 + 42*a^3*b*c^3 + 41*a^2*b^3*c^2 - 34*a*b^
5*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(5*b^5 + 58*a^2*b*c^2 - 3
6*a*b^3*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(5*b^6 - 14*a^3*c^3
 + 71*a^2*b^2*c^2 - 38*a*b^4*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(a^2*c
^3 + c^5*x^4 + x^2*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^3 + 2*a*b*c^3*x) + (log(
a + b*x + c*x^2)*(3*b^11 - 3072*a^5*b*c^5 + 480*a^2*b^7*c^2 - 1920*a^3*b^5*
c^3 + 3840*a^4*b^3*c^4 - 60*a*b^9*c))/(2*(1024*a^5*c^9 - b^10*c^4 + 20*a*b^
8*c^5 - 160*a^2*b^6*c^6 + 640*a^3*b^4*c^7 - 1280*a^4*b^2*c^8)) + (3*atan(((
(3*x*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^3*(4*a*c - b^2)^5
) + (3*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*(b^6 - 20*a^3*c^3 + 30*a^2*b^
2*c^2 - 10*a*b^4*c))/(2*c^7*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
)*(32*a^2*c^6*(4*a*c - b^2)^(5/2) + 2*b^4*c^4*(4*a*c - b^2)^(5/2) - 16*a*b^
2*c^5*(4*a*c - b^2)^(5/2)))/(3*b^6 - 60*a^3*c^3 + 90*a^2*b^2*c^2 - 30*a*b^4
*c))*(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c))/(c^4*(4*a*c - b^2)^(
5/2))
```

3.432 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx$

Optimal. Leaf size=190

$$-\frac{b(b^2-7ac)\,x}{c^2\left(b^2-4ac\right)^2}+\frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x)}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{b(b^4-10ab^2c+30a^2c^2)}{c^3\,(b^2-}$$

```
[Out] -b*(-7*a*c+b^2)*x/c^2/(-4*a*c+b^2)^2+1/2*x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+
b*x+a)^2+1/2*x^2*(a*(-16*a*c+b^2)+b*(-10*a*c+b^2)*x)/c/(-4*a*c+b^2)^2/(c*x^
2+b*x+a)+b*(30*a^2*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/c^3/(-4*a*c+b^2)^(5/2)+1/2*ln(c*x^2+b*x+a)/c^3
```

Rubi [A]
time = 0.18, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1368, 752, 832, 787, 648, 632, 212, 642}

$$\frac{b(30a^2c^2-10ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx(b^2-7ac)}{c^2\left(b^2-4ac\right)^2}+\frac{x^2(bx(b^2-10ac)+a(b^2-16ac))}{2c\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{x^4(2a+bx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{\log\left(a+bx+cx^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] -((b*(b^2 - 7*a*c)*x)/(c^2*(b^2 - 4*a*c)^2)) + (x^4*(2*a + b*x))/(2*(b^2 -
4*a*c)*(a + b*x + c*x^2)^2) + (x^2*(a*(b^2 - 16*a*c) + b*(b^2 - 10*a*c)*x))
/(2*c*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^
2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(5/2)) + Log[
a + b*x + c*x^2]/(2*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
```

```
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x}\,dx = \int \frac{x^5}{(a+bx+cx^2)^3}\,dx$$

$$= \frac{x^4(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2} - \frac{\int \frac{x^3(8a+bx)}{(a+bx+cx^2)^2}\,dx}{2\,(b^2-4ac)}$$

$$= \frac{x^4(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2} + \frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x)}{2c\,(b^2-4ac)^2\,(a+bx+cx^2)} - \frac{\int \frac{x(2a(b^2-16ac)-}{a+bx-}}{2c\,(b^2-}$$

$$= -\frac{b(b^2-7ac)\,x}{c^2\,(b^2-4ac)^2} + \frac{x^4(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2} + \frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x}{2c\,(b^2-4ac)^2\,(a+bx+cx^2)}$$

$$= -\frac{b(b^2-7ac)\,x}{c^2\,(b^2-4ac)^2} + \frac{x^4(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2} + \frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x}{2c\,(b^2-4ac)^2\,(a+bx+cx^2)}$$

$$= -\frac{b(b^2-7ac)\,x}{c^2\,(b^2-4ac)^2} + \frac{x^4(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2} + \frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x}{2c\,(b^2-4ac)^2\,(a+bx+cx^2)}$$

$$= -\frac{b(b^2-7ac)\,x}{c^2\,(b^2-4ac)^2} + \frac{x^4(2a+bx)}{2\,(b^2-4ac)\,(a+bx+cx^2)^2} + \frac{x^2(a(b^2-16ac)+b(b^2-10ac)\,x}{2c\,(b^2-4ac)^2\,(a+bx+cx^2)}$$

Mathematica [A]
time = 0.20, size = 221, normalized size = 1.16

$$\frac{\frac{-b^6+11ab^4c-39a^2b^2c^2+32a^3c^3+4b^5cx-30ab^3c^2x+50a^2bc^3x}{(b^2-4ac)^2(a+x(b+cx))}+\frac{2a^3c^2+b^5x+ab^3(b-5cx)+a^2bc(-4b+5cx)}{(b^2-4ac)(a+x(b+cx))^2}-\frac{2bc(b^4-10ab^2c+30a^2c^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}+c\log(a+x(b+cx))}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x),x]
```

```
[Out] ((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x - 30*a*b^3*c^
2*x + 50*a^2*b*c^3*x)/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (2*a^3*c^2 + b^
5*x + a*b^3*(b - 5*c*x) + a^2*b*c*(-4*b + 5*c*x))/((b^2 - 4*a*c)*(a + x*(b
+ c*x))^2) - (2*b*c*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt
[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + x*(b + c*x)])/(2*c^4)
```

Maple [A]
time = 0.07, size = 357, normalized size = 1.88

method	result
default	$\frac{\frac{b\left(25a^{2}c^{2}-15a\,b^{2}c+2b^{4}\right)x^{3}}{c^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(32a^{3}c^{3}+11a^{2}b^{2}c^{2}-19a\,b^{4}c+3b^{6}\right)x^{2}}{2c^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{ab\left(31a^{2}c^{2}-22a\,b^{2}c+3b^{4}\right)x}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{3}}+\frac{3a^{2}\left(8a^{2}c^{2}-7a\,b^{2}c+b^{4}\right)}{2c^{3}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{2}+bx+a\right)^{2}}+\frac{\left(16a^{2}c^{2}-8a\,b^{2}c-\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] (1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*(
32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^2+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^3*x+3/2*
a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)
^2+1/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*a^2*c^2-8*a*b^2*c+b^4)/c*ln(c*
x^2+b*x+a)+2*(-7*a^2*b*c+a*b^3-1/2*(16*a^2*c^2-8*a*b^2*c+b^4)*b/c)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 792 vs. 2(180) = 360.
time = 0.39, size = 1603, normalized size = 8.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="fricas")
```

```
[Out] [1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(2*b^7*c
- 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 31*a*b^6*c
+ 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^
3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c
```

```
 - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 +
 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2
- 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x
+ b))/(c*x^2 + b*x + a)) + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 12
4*a^4*b*c^3)*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*
b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b
^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a
^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b
^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8)*x^4 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^
3*b*c^7)*x^3 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 -
128*a^4*c^7)*x^2 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*
b*c^6)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2
*(2*b^7*c - 23*a*b^5*c^2 + 85*a^2*b^3*c^3 - 100*a^3*b*c^4)*x^3 + (3*b^8 - 3
1*a*b^6*c + 87*a^2*b^4*c^2 - 12*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a^2*b^5
 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^
4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a
^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(3*a*b^7 - 34*a^2*b^5*c + 119*a^3*b^3*c^2 - 124*a^4*b*c^3)*x + (a^2*b^
6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
- 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^
3)*x)*log(c*x^2 + b*x + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5
- 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^4 +
 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^3 + (b^8*c^3
- 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^2 + 2*(a*
b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1510 vs. $2(180) = 360$.

time = 2.04, size = 1510, normalized size = 7.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1
024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c*
*2 + 20*a*b**8*c - b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(-b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
```

```
**8*c - b**10)) + 1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(-b*sqrt(-
(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c
**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*
b**8*c - b**10)) + 1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(-b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c*
*5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b
**8*c - b**10)) + 1/(2*c**3)) + a*b**4 + b**6*c**2*(-b*sqrt(-(4*a*c - b**2)
**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**
4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10
)) + 1/(2*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (b*sqrt(-(4*a*c
- b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1
280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c
- b**10)) + 1/(2*c**3))*log(x + (-64*a**3*c**5*(b*sqrt(-(4*a*c - b**2)**5)*
(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**
2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) +
1/(2*c**3)) + 32*a**3*c**2 + 48*a**2*b**2*c**4*(b*sqrt(-(4*a*c - b**2)**5)*
(30*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**
2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) +
1/(2*c**3)) - 9*a**2*b**2*c - 12*a*b**4*c**3*(b*sqrt(-(4*a*c - b**2)**5)*(3
0*a**2*c**2 - 10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*
c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/
(2*c**3)) + a*b**4 + b**6*c**2*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 -
10*a*b**2*c + b**4)/(2*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**
3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(2*c**3)))/(30
*a**2*b*c**2 - 10*a*b**3*c + b**5)) + (24*a**4*c**2 - 21*a**3*b**2*c + 3*a*
*2*b**4 + x**3*(50*a**2*b*c**3 - 30*a*b**3*c**2 + 4*b**5*c) + x**2*(32*a**3
*c**3 + 11*a**2*b**2*c**2 - 19*a*b**4*c + 3*b**6) + x*(62*a**3*b*c**2 - 44*
a**2*b**3*c + 6*a*b**5))/(32*a**4*c**5 - 16*a**3*b**2*c**4 + 2*a**2*b**4*c*
*3 + x**4*(32*a**2*c**7 - 16*a*b**2*c**6 + 2*b**4*c**5) + x**3*(64*a**2*b*c
**6 - 32*a*b**3*c**5 + 4*b**5*c**4) + x**2*(64*a**3*c**6 - 12*a*b**4*c**4 +
 2*b**6*c**3) + x*(64*a**3*b*c**5 - 32*a**2*b**3*c**4 + 4*a*b**5*c**3))
```

Giac [A]
time = 5.39, size = 245, normalized size = 1.29

$$-\frac{\left(b^{5}-10\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(b^{4}c^{3}-8\,ab^{2}c^{4}+16\,a^{2}c^{5}\right)\sqrt{-b^{2}+4\,ac}}+\frac{\log\left(cx^{2}+bx+a\right)}{2\,c^{3}}+\frac{3\,a^{2}b^{4}-21\,a^{3}b^{2}c+24\,a^{4}c^{2}+2\left(2\,b^{5}c-15\,ab^{3}c^{2}+25\,a^{2}bc^{3}\right)x^{3}+\left(3\,b^{6}-19\,ab^{4}c+11\,a^{2}b^{2}c^{2}+32\,a^{3}c^{3}\right)x^{2}+2\left(3\,ab^{5}-22\,a^{2}b^{3}c+31\,a^{3}bc^{2}\right)x}{2\left(cx^{2}+bx+a\right)^{2}\left(b^{2}-4\,ac\right)^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x,x, algorithm="giac")
```

```
[Out] -(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 +
b*x + a)/c^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 2*(2*b^5*c - 15
*a*b^3*c^2 + 25*a^2*b*c^3)*x^3 + (3*b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 + 32*
a^3*c^3)*x^2 + 2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2)*x)/((c*x^2 + b*x +
 a)^2*(b^2 - 4*a*c)^2*c^3)
```

Mupad [B]

time = 2.20, size = 620, normalized size = 3.26

$$\frac{\frac{3\,a^2\,\left(8\,a^2\,c^2-7\,a\,b^2\,c+b^4\right)}{2\,c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\,\left(32\,a^3\,c^3+11\,a^2\,b^2\,c^2-19\,a\,b^4\,c+3\,b^6\right)}{2\,c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b\,x^3\,\left(25\,a^2\,c^2-15\,a\,b^2\,c+2\,b^4\right)}{c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,b\,x\,\left(31\,a^2\,c^2-22\,a\,b^2\,c+3\,b^4\right)}{c^3\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-1024\,a^5\,c^5+1280\,a^4\,b^2\,c^4-640\,a^3\,b^4\,c^3+160\,a^2\,b^6\,c^2-20\,a\,b^8\,c+b^{10}\right)}{2\,\left(1024\,a^5\,c^8-1280\,a^4\,b^2\,c^7+640\,a^3\,b^4\,c^6-160\,a^2\,b^6\,c^5+20\,a\,b^8\,c^4-b^{10}\,c^3\right)}-\frac{b\,\mathrm{atan}\left(\frac{\left(\frac{b\,x\,\left(30\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^5}+\frac{b^2\,\left(16\,a^2\,c^4-8\,a\,b^2\,c^3+b^4\,c^2\right)\,\left(30\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{2\,c^5\,{\left(4\,a\,c-b^2\right)}^5\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\,\left(32\,a^2\,c^5\,{\left(4\,a\,c-b^2\right)}^{5/2}+2\,b^4\,c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}-16\,a\,b^2\,c^4\,{\left(4\,a\,c-b^2\right)}^{5/2}\right)}{30\,a^2\,b\,c^2-10\,a\,b^3\,c+b^5}\right)\,\left(30\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + a/x^2 + b/x)^3),x)
```

```
[Out] ((3*a^2*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (x^2*(3*b^6 + 32*a^3*c^3 + 11*a^2*b^2*c^2 - 19*a*b^4*c))/(2*c^3*(b^4 +
 16*a^2*c^2 - 8*a*b^2*c)) + (b*x^3*(2*b^4 + 25*a^2*c^2 - 15*a*b^2*c))/(c^2*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*b*x*(3*b^4 + 31*a^2*c^2 - 22*a*b^2*c))
/(c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 +
 2*a*b*x + 2*b*c*x^3) - (log(a + b*x + c*x^2)*(b^10 - 1024*a^5*c^5 + 160*a^
2*b^6*c^2 - 640*a^3*b^4*c^3 + 1280*a^4*b^2*c^4 - 20*a*b^8*c))/(2*(1024*a^5*
c^8 - b^10*c^3 + 20*a*b^8*c^4 - 160*a^2*b^6*c^5 + 640*a^3*b^4*c^6 - 1280*a^
4*b^2*c^7)) - (b*atan((((b*x*(b^4 + 30*a^2*c^2 - 10*a*b^2*c))/(c^2*(4*a*c -
 b^2)^5) + (b^2*(16*a^2*c^4 + b^4*c^2 - 8*a*b^2*c^3)*(b^4 + 30*a^2*c^2 - 10
*a*b^2*c))/(2*c^5*(4*a*c - b^2)^5*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(32*a^2*
c^5*(4*a*c - b^2)^(5/2) + 2*b^4*c^3*(4*a*c - b^2)^(5/2) - 16*a*b^2*c^4*(4*a
*c - b^2)^(5/2)))/(b^5 + 30*a^2*b*c^2 - 10*a*b^3*c))*(b^4 + 30*a^2*c^2 - 10
*a*b^2*c))/(c^3*(4*a*c - b^2)^(5/2))
```

3.433 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2} \, dx$

Optimal. Leaf size=111

$$\frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2}-\frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)}+\frac{12a^2\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/2*(b+2*a/x)/(-4*a*c+b^2)/(c+a/x^2+b/x)^2-3*a*(b+2*a/x)/(-4*a*c+b^2)^2/(c+
a/x^2+b/x)+12*a^2*arctanh((b+2*a/x)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1366, 628, 632, 212}

$$\frac{12a^2\tanh^{-1}\left(\frac{\frac{2a}{x}+b}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}-\frac{3a\left(\frac{2a}{x}+b\right)}{\left(b^2-4ac\right)^2\left(\frac{a}{x^2}+\frac{b}{x}+c\right)}+\frac{\frac{2a}{x}+b}{2\left(b^2-4ac\right)\left(\frac{a}{x^2}+\frac{b}{x}+c\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] (b + (2*a)/x)/(2*(b^2 - 4*a*c)*(c + a/x^2 + b/x)^2) - (3*a*(b + (2*a)/x))/(
(b^2 - 4*a*c)^2*(c + a/x^2 + b/x)) + (12*a^2*ArcTanh[(b + (2*a)/x)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^2}\,dx = -\text{Subst}\left(\int \frac{1}{(c+bx+ax^2)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} + \frac{(3a)\text{Subst}\left(\int \frac{1}{(c+bx+ax^2)^2}\,dx, x, \frac{1}{x}\right)}{b^2-4ac}$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} - \frac{\left(6a^2\right)\text{Subst}\left(\int \frac{1}{c+bx+ax^2}\,d\right.}{\left(b^2-4ac\right)^2}$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{\left(12a^2\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\right.}{\left(b^2-4ac\right)^2}$$

$$= \frac{b+\frac{2a}{x}}{2\left(b^2-4ac\right)\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^2} - \frac{3a\left(b+\frac{2a}{x}\right)}{\left(b^2-4ac\right)^2\left(c+\frac{a}{x^2}+\frac{b}{x}\right)} + \frac{12a^2\tanh^{-1}\left(\frac{b+\frac{2a}{x}}{\sqrt{b^2-4ac}}\right.}{\left(b^2-4ac\right)^{5/2}}$$

Mathematica [A]
time = 0.12, size = 174, normalized size = 1.57

$$\frac{1}{2}\left(\frac{b^5-8ab^3c+22a^2bc^2-2b^4cx+16ab^2c^2x-20a^2c^3x}{c^3\left(b^2-4ac\right)^2(a+x(b+cx))} + \frac{b^4x+ab^2(b-4cx)+a^2c(-3b+2cx)}{c^3\left(-b^2+4ac\right)(a+x(b+cx))^2} + \frac{24a^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^2),x]
```

```
[Out] ((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x + 16*a*b^2*c^2*x - 20*a^2*c^3*
x)/(c^3*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (b^4*x + a*b^2*(b - 4*c*x) + a
^2*c*(-3*b + 2*c*x))/(c^3*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + (24*a^2*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(105) = 210.
time = 0.05, size = 260, normalized size = 2.34

method	result
default	$\frac{-\frac{\left(10a^2c^2-8a\,b^2c+b^4\right)x^3}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{b\left(2a^2c^2+8a\,b^2c-b^4\right)x^2}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(6a^2c^2-10a\,b^2c+b^4\right)x}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{a^2b\left(10ac-b^2\right)}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}+\frac{12a^2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(16a^2c^2-8a\,b^2c+b^4\right)\sqrt{4ac-}}$
risch	$\frac{-\frac{\left(10a^2c^2-8a\,b^2c+b^4\right)x^3}{c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{b\left(2a^2c^2+8a\,b^2c-b^4\right)x^2}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(6a^2c^2-10a\,b^2c+b^4\right)x}{\left(16a^2c^2-8a\,b^2c+b^4\right)c^2}+\frac{a^2b\left(10ac-b^2\right)}{2c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^2+bx+a\right)^2}-\frac{6a^2\ln\left(\left(32a^2c^3-16a\,b^2c^2+2b^4\right.\right.}{(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*b*(2*a^
2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-a*(6*a^2*c^2-10*a*b
^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x+1/2*a^2*b*(10*a*c-b^2)/c^2/(16*a
^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+12*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4
*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(105) = 210.
time = 0.37, size = 953, normalized size = 8.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4*c^2 + 42
*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^2 + 8*a^3
*b*c^3)*x^2 - 12*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x + a^4*c^2 +
 (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^
```

```
6 - 14*a^2*b^4*c + 46*a^3*b^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^
4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*
c^6 - 64*a^3*c^7)*x^4 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3
*b*c^6)*x^3 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 1
28*a^4*c^6)*x^2 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b
*c^5)*x), -1/2*(a^2*b^5 - 14*a^3*b^3*c + 40*a^4*b*c^2 + 2*(b^6*c - 12*a*b^4
*c^2 + 42*a^2*b^2*c^3 - 40*a^3*c^4)*x^3 + (b^7 - 12*a*b^5*c + 30*a^2*b^3*c^
2 + 8*a^3*b*c^3)*x^2 + 24*(a^2*c^4*x^4 + 2*a^2*b*c^3*x^3 + 2*a^3*b*c^2*x +
a^4*c^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b
^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^6 - 14*a^2*b^4*c + 46*a^3*b
^2*c^2 - 24*a^4*c^3)*x)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64
*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3*c^7)*x^4 + 2*(
b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^3 + (b^8*c^2 - 10
*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*x^2 + 2*(a*b^7*
c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(94) = 188$.
time = 0.80, size = 547, normalized size = 4.93

$$-6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}+6a^2b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}+6a^2b}{12a^2c}\right)+6a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+72a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}-6a^2b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}+6a^2b}{12a^2c}\right)+\frac{10a^3bc-a^2b^3+x^3(-20a^2c^3+16ab^2c^2-2b^4c)+x^2\cdot(2a^2bc^2+8ab^3c-b^5)+x(-12a^3c^2+20a^2b^2c-2ab^4)}{32a^4c^4-16a^3b^2c^3+2a^2b^4c^2+x^4\cdot(32a^2c^6-16ab^2c^5+2b^4c^4)+x^3\cdot(64a^2bc^5-32ab^3c^4+4b^5c^3)+x^2\cdot(64a^3c^5-12ab^4c^3+2b^6c^2)+x\left(64a^3bc^4-32a^2b^3c^3+4ab^5c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**2,x)
```

```
[Out] -6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**5*c**3*sqrt(-1/(4*a*c -
 b**2)**5) + 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) - 72*a**3*b**4*c
*sqrt(-1/(4*a*c - b**2)**5) + 6*a**2*b**6*sqrt(-1/(4*a*c - b**2)**5) + 6*a*
*2*b)/(12*a**2*c)) + 6*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**5*c*
*3*sqrt(-1/(4*a*c - b**2)**5) - 288*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**
5) + 72*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 6*a**2*b**6*sqrt(-1/(4*a*c
 - b**2)**5) + 6*a**2*b)/(12*a**2*c)) + (10*a**3*b*c - a**2*b**3 + x**3*(-2
0*a**2*c**3 + 16*a*b**2*c**2 - 2*b**4*c) + x**2*(2*a**2*b*c**2 + 8*a*b**3*c
 - b**5) + x*(-12*a**3*c**2 + 20*a**2*b**2*c - 2*a*b**4))/(32*a**4*c**4 - 1
6*a**3*b**2*c**3 + 2*a**2*b**4*c**2 + x**4*(32*a**2*c**6 - 16*a*b**2*c**5 +
 2*b**4*c**4) + x**3*(64*a**2*b*c**5 - 32*a*b**3*c**4 + 4*b**5*c**3) + x**2
*(64*a**3*c**5 - 12*a*b**4*c**3 + 2*b**6*c**2) + x*(64*a**3*b*c**4 - 32*a**
2*b**3*c**3 + 4*a*b**5*c**2))
```

Giac [A]
time = 3.68, size = 202, normalized size = 1.82

$$\frac{12\,a^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{2\,b^4cx^3-16\,ab^2c^2x^3+20\,a^2c^3x^3+b^5x^2-8\,ab^3cx^2-2\,a^2bc^2x^2+2\,ab^4x-20\,a^2b^2cx+12\,a^3c^2x+a^2b^3-10\,a^3bc}{2\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)(cx^2+bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^2,x, algorithm="giac")
```

```
[Out] 12*a^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^4*c*x^3 - 16*a*b^2*c^2*x^3 + 20*a^2*c^3*x
^3 + b^5*x^2 - 8*a*b^3*c*x^2 - 2*a^2*b*c^2*x^2 + 2*a*b^4*x - 20*a^2*b^2*c*x
 + 12*a^3*c^2*x + a^2*b^3 - 10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*(c*x^2 + b*x + a)^2)
```

Mupad [B]
time = 0.19, size = 343, normalized size = 3.09

$$\frac{12\,a^2\,\mathrm{atan}\left(\frac{\left(\frac{6\,a^2\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{12\,a^2\,c\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,a^2}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{\frac{x^3\,\left(10\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(b^3-10\,a\,b\,c\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{x^2\,\left(2\,a^2\,b\,c^2+8\,a\,b^3\,c-b^5\right)}{2\,c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x\,\left(6\,a^2\,c^2-10\,a\,b^2\,c+b^4\right)}{c^2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + a/x^2 + b/x)^3),x)
```

```
[Out] (12*a^2*atan((((6*a^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2
)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (12*a^2*c*x)/(4*a*c - b^2)^(5/2))*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c))/(6*a^2)))/(4*a*c - b^2)^(5/2) - ((x^3*(b^4 + 10*
a^2*c^2 - 8*a*b^2*c))/(c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*(b^3 - 10*a
*b*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^2*(2*a^2*b*c^2 - b^5 + 8
*a*b^3*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x*(b^4 + 6*a^2*c^2 -
 10*a*b^2*c))/(c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^
2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3)
```

3.434 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3} \, dx$

Optimal. Leaf size=107

$$-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] -1/2*x^3*(2*c*x+b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3/2*b*x*(b*x+2*a)/(-4*a*c+b
^2)^2/(c*x^2+b*x+a)+6*a*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1368, 742, 736, 632, 212}

$$\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{6ab\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] -1/2*(x^3*(b + 2*c*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*b*x*(2*a +
b*x))/(2*(b^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*a*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
```

```
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] + Dist[m*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c))), Int[(
d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^3}\,dx &= \int \frac{x^3}{(a+bx+cx^2)^3}\,dx \\
&= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)(a+bx+cx^2)^2}+\frac{(3b)\int \frac{x^2}{(a+bx+cx^2)^2}\,dx}{2\left(b^2-4ac\right)} \\
&= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)(a+bx+cx^2)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2(a+bx+cx^2)}-\frac{(3ab)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2} \\
&= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)(a+bx+cx^2)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2(a+bx+cx^2)}+\frac{(6ab)\text{Subst}\left(\int \frac{1}{b^2-4a
\end{aligned}$$

$$= -\frac{x^3(b+2cx)}{2\left(b^2-4ac\right)(a+bx+cx^2)^2}+\frac{3bx(2a+bx)}{2\left(b^2-4ac\right)^2(a+bx+cx^2)}+\frac{6ab\tanh^{-1}\left(\frac{b+2}{\sqrt{b^2-}}\right.}{\left(b^2-4ac\right)^{5/}}$$

Mathematica [A]
time = 0.13, size = 126, normalized size = 1.18

$$-\frac{8a^3c+b^4x^2+abx(2b^2+bcx+6c^2x^2)+a^2(b^2+10bcx+16c^2x^2)}{2c\left(b^2-4ac\right)^2(a+x(b+cx))^2}-\frac{6ab\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^3),x]
```

```
[Out] -1/2*(8*a^3*c + b^4*x^2 + a*b*x*(2*b^2 + b*c*x + 6*c^2*x^2) + a^2*(b^2 + 10
*b*c*x + 16*c^2*x^2))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^2) - (6*a*b*ArcT
an[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(99) = 198$.
time = 0.04, size = 223, normalized size = 2.08

method	result
default	$\frac{-\frac{3abc\,x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{\left(16a^{2}c^{2}+a\,b^{2}c+b^{4}\right)x^{2}}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5ac+b^{2}\right)abx}{c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a^{2}\left(8ac+b^{2}\right)}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{2}+bx+a\right)^{2}}-\frac{6ab\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}$
risch	$\frac{-\frac{3abc\,x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{\left(16a^{2}c^{2}+a\,b^{2}c+b^{4}\right)x^{2}}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5ac+b^{2}\right)abx}{c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a^{2}\left(8ac+b^{2}\right)}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{2}+bx+a\right)^{2}}-\frac{3ba\ln\left(\left(32a^{2}c^{3}-16a\,b^{2}c^{2}+2b^{4}c\right)x-(\right.}{(-4ac-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*(16*a^2*c^2+a*b^2*c+b^4)/c/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^2-(5*a*c+b^2)*a*b/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x-
1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2-6*a*b/(16
*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(99) = 198$.
time = 0.39, size = 872, normalized size = 8.15

$$\left[-\frac{a^{2}b^{4}+4\,a^{3}b^{2}c-32\,a^{4}c^{2}+6\,(ab^{3}c^{2}-4\,a^{2}bc^{3})x^{3}+(b^{6}-3\,ab^{4}c+12\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3})x^{2}-6\,(abc^{3}x^{4}+2\,ab^{2}c^{2}x^{3}+2\,a^{2}b^{2}cx+a^{3}bc+(ab^{3}c+2\,a^{2}bc^{2})x^{2})\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)+2\,(ab^{5}+a^{2}b^{3}c-20\,a^{3}bc^{2})x}{2\,(a^{2}b^{6}c-12\,a^{3}b^{4}c^{2}+48\,a^{4}b^{2}c^{3}-64\,a^{5}c^{4}+(b^{6}c^{3}-12\,ab^{4}c^{4}+48\,a^{2}b^{2}c^{5}-64\,a^{3}c^{6})x^{4}+2\,(b^{7}c^{2}-12\,ab^{5}c^{3}+48\,a^{2}b^{3}c^{4}-64\,a^{3}bc^{5})x^{3}+(b^{8}c-10\,ab^{6}c^{2}+24\,a^{2}b^{4}c^{3}+32\,a^{3}b^{2}c^{4}-128\,a^{4}c^{5})x^{2}+2\,(ab^{7}c-12\,a^{2}b^{5}c^{2}+48\,a^{3}b^{3}c^{3}-64\,a^{4}bc^{4})x)},\ -\frac{a^{2}b^{4}+4\,a^{3}b^{2}c-32\,a^{4}c^{2}+6\,(ab^{3}c^{2}-4\,a^{2}bc^{3})x^{3}+(b^{6}-3\,ab^{4}c+12\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3})x^{2}-12\,(abc^{3}x^{4}+2\,ab^{2}c^{2}x^{3}+2\,a^{2}b^{2}cx+a^{3}bc+(ab^{3}c+2\,a^{2}bc^{2})x^{2})\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)+2\,(ab^{5}+a^{2}b^{3}c-20\,a^{3}bc^{2})x}{2\,(a^{2}b^{6}c-12\,a^{3}b^{4}c^{2}+48\,a^{4}b^{2}c^{3}-64\,a^{5}c^{4}+(b^{6}c^{3}-12\,ab^{4}c^{4}+48\,a^{2}b^{2}c^{5}-64\,a^{3}c^{6})x^{4}+2\,(b^{7}c^{2}-12\,ab^{5}c^{3}+48\,a^{2}b^{3}c^{4}-64\,a^{3}bc^{5})x^{3}+(b^{8}c-10\,ab^{6}c^{2}+24\,a^{2}b^{4}c^{3}+32\,a^{3}b^{2}c^{4}-128\,a^{4}c^{5})x^{2}+2\,(ab^{7}c-12\,a^{2}b^{5}c^{2}+48\,a^{3}b^{3}c^{3}-64\,a^{4}bc^{4})x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b^3*c^2 - 4*a^2*b*c^3)*x^3
 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)*x^2 - 6*(a*b*c^3*x^4 + 2
*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3*c + 2*a^2*b*c^2)*x^2)*sqr
t(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(
2*c*x + b))/(c*x^2 + b*x + a)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x)/(a
^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b
^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*
a^2*b^3*c^4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 +
32*a^3*b^2*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^
3*c^3 - 64*a^4*b*c^4)*x), -1/2*(a^2*b^4 + 4*a^3*b^2*c - 32*a^4*c^2 + 6*(a*b
^3*c^2 - 4*a^2*b*c^3)*x^3 + (b^6 - 3*a*b^4*c + 12*a^2*b^2*c^2 - 64*a^3*c^3)
*x^2 - 12*(a*b*c^3*x^4 + 2*a*b^2*c^2*x^3 + 2*a^2*b^2*c*x + a^3*b*c + (a*b^3
*c + 2*a^2*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x
 + b)/(b^2 - 4*a*c)) + 2*(a*b^5 + a^2*b^3*c - 20*a^3*b*c^2)*x)/(a^2*b^6*c -
 12*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6)*x^4 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^
4 - 64*a^3*b*c^5)*x^3 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2
*c^4 - 128*a^4*c^5)*x^2 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64
*a^4*b*c^4)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(102) = 204$.
time = 0.66, size = 513, normalized size = 4.79

$$3ab\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{-192a^4bc^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-36a^2b^5c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^7\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^2}{6abc}\right)-3ab\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\log\left(x+\frac{192a^4bc^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-144a^3b^3c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+36a^2b^5c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}-3ab^7\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}+3ab^2}{6abc}\right)+\frac{-8a^3c-a^2b^2-6abc^2x^3+x^2\left(-16a^2c^2-ab^2c-b^4\right)+x\left(-10a^2bc-2ab^3\right)}{32a^4c^3-16a^3b^2c^2+2a^2b^4c+x^4\cdot\left(32a^2c^5-16ab^2c^4+2b^4c^3\right)+x^3\cdot\left(64a^2bc^4-32ab^3c^3+4b^5c^2\right)+x^2\cdot\left(64a^3c^4-12ab^4c^2+2b^6c\right)+x\left(64a^3bc^3-32a^2b^3c^2+4ab^5c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**3,x)
```

```
[Out] 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**4*b*c**3*sqrt(-1/(4*a*c -
 b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) - 36*a**2*b**5*c
*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**
2)/(6*a*b*c)) - 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**4*b*c**3*s
qrt(-1/(4*a*c - b**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) +
 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*c - b**2
)**5) + 3*a*b**2)/(6*a*b*c)) + (-8*a**3*c - a**2*b**2 - 6*a*b*c**2*x**3 + x
**2*(-16*a**2*c**2 - a*b**2*c - b**4) + x*(-10*a**2*b*c - 2*a*b**3))/(32*a*
*4*c**3 - 16*a**3*b**2*c**2 + 2*a**2*b**4*c + x**4*(32*a**2*c**5 - 16*a*b**
2*c**4 + 2*b**4*c**3) + x**3*(64*a**2*b*c**4 - 32*a*b**3*c**3 + 4*b**5*c**2
) + x**2*(64*a**3*c**4 - 12*a*b**4*c**2 + 2*b**6*c) + x*(64*a**3*b*c**3 - 3
2*a**2*b**3*c**2 + 4*a*b**5*c))
```

Giac [A]
time = 3.35, size = 163, normalized size = 1.52

$$-\frac{6\,ab\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{6\,abc^2x^3+b^4x^2+ab^2cx^2+16\,a^2c^2x^2+2\,ab^3x+10\,a^2bcx+a^2b^2+8\,a^3c}{2\,(b^4c-8\,ab^2c^2+16\,a^2c^3)(cx^2+bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^3,x, algorithm="giac")
```

```
[Out] -6*a*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*a*b*c^2*x^3 + b^4*x^2 + a*b^2*c*x^2 + 16*a^
2*c^2*x^2 + 2*a*b^3*x + 10*a^2*b*c*x + a^2*b^2 + 8*a^3*c)/((b^4*c - 8*a*b^2
*c^2 + 16*a^2*c^3)*(c*x^2 + b*x + a)^2)
```

Mupad [B]
time = 1.43, size = 271, normalized size = 2.53

$$-\frac{\frac{x^2\,\left(16\,a^2\,c^2+a\,b^2\,c+b^4\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\,\left(b^2+8\,a\,c\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,a\,b\,c\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{a\,b\,x\,\left(b^2+5\,a\,c\right)}{c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}-\frac{6\,a\,b\,\mathrm{atan}\left(\frac{\left(\frac{3\,a\,b^2}{{\left(4\,a\,c-b^2\right)}^{5/2}}+\frac{6\,a\,b\,c\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{3\,a\,b}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + a/x^2 + b/x)^3),x)
```

```
[Out] - ((x^2*(b^4 + 16*a^2*c^2 + a*b^2*c))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (a^2*(8*a*c + b^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*a*b*c*x^3)/
(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (a*b*x*(5*a*c + b^2))/(c*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) -
 (6*a*b*atan((((3*a*b^2)/(4*a*c - b^2)^(5/2) + (6*a*b*c*x)/(4*a*c - b^2)^(5
/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(3*a*b)))/(4*a*c - b^2)^(5/2)
```

3.435 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4} dx$

Optimal. Leaf size=115

$$\frac{x(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}+\frac{3ab+(b^2+2ac)x}{(b^2-4ac)^2(a+bx+cx^2)}-\frac{2(b^2+2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] 1/2*x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+(3*a*b+(2*a*c+b^2)*x)/(-4*a*c+
b^2)^2/(c*x^2+b*x+a)-2*(2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-
4*a*c+b^2)^(5/2)
```

Rubi [A]
time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1368, 752, 652, 632, 212}

$$\frac{x(2a+bx)}{2(b^2-4ac)(a+bx+cx^2)^2}+\frac{x(2ac+b^2)+3ab}{(b^2-4ac)^2(a+bx+cx^2)}-\frac{2(2ac+b^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] (x*(2*a + b*x))/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*a*b + (b^2 + 2*a
*c)*x)/((b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (2*(b^2 + 2*a*c)*ArcTanh[(b +
2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
```

```
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^4}\,dx = \int \frac{x^2}{(a+bx+cx^2)^3}\,dx$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)(a+bx+cx^2)^2} - \frac{\int \frac{2a-2bx}{(a+bx+cx^2)^2}\,dx}{2\left(b^2-4ac\right)}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)(a+bx+cx^2)^2} + \frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2(a+bx+cx^2)} + \frac{\left(b^2+2ac\right)\int \frac{1}{a+bx+cx^2}}{\left(b^2-4ac\right)^2}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)(a+bx+cx^2)^2} + \frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2(a+bx+cx^2)} - \frac{(2(b^2+2ac))\,\mathrm{Subst}(}{(b^2}$$

$$= \frac{x(2a+bx)}{2\left(b^2-4ac\right)(a+bx+cx^2)^2} + \frac{3ab+\left(b^2+2ac\right)x}{\left(b^2-4ac\right)^2(a+bx+cx^2)} - \frac{2(b^2+2ac)\tanh^{-1}\left(\right.}{(b^2-4ac)}$$

Mathematica [A]
time = 0.09, size = 131, normalized size = 1.14

$$\frac{1}{2}\left(\frac{b^2x+a(b-2cx)}{c\left(-b^2+4ac\right)(a+x(b+cx))^2}+\frac{\left(b^2+2ac\right)(b+2cx)}{c\left(b^2-4ac\right)^2(a+x(b+cx))}+\frac{4(b^2+2ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^4),x]
```

```
[Out] ((b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))^2) + ((b^2 + 2
*a*c)*(b + 2*c*x))/(c*(b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (4*(b^2 + 2*a*c)
*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/2
```

Maple [A]
time = 0.05, size = 210, normalized size = 1.83

method	result
default	$\frac{\frac{c\left(2ac+b^{2}\right)x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3b\left(2ac+b^{2}\right)x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(2ac-5b^{2}\right)x}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3a^{2}b}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}}{\left(c\,x^{2}+bx+a\right)^{2}}+\frac{2\left(2ac+b^{2}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)\sqrt{4ac-b^{2}}}$
risch	$\frac{\frac{c\left(2ac+b^{2}\right)x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3b\left(2ac+b^{2}\right)x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(2ac-5b^{2}\right)x}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{3a^{2}b}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}}{\left(c\,x^{2}+bx+a\right)^{2}}-\frac{2\ln\left(\left(32a^{2}c^{3}-16a\,b^{2}c^{2}+2b^{4}c\right)x+\left(-4ac+b^{2}\right)^{\frac{5}{2}}+\right.}{\left(-4ac+b^{2}\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] (c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+3/2*b*(2*a*c+b^2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^2-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+3*a^2*b/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+2*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2
*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(109) = 218$.
time = 0.37, size = 887, normalized size = 7.71

$$\left[\frac{6\,a^{2}b^{3}-24\,a^{3}bc+2\,(b^{4}c-2\,ab^{2}c^{2}-8\,a^{2}c^{3})x^{3}+3\,(b^{5}-2\,ab^{3}c-8\,a^{2}bc^{2})x^{2}+2\,((b^{2}c^{2}+2\,ac^{3})x^{4}+a^{2}b^{2}+2\,a^{3}c+2\,(b^{3}c+2\,abc^{2})x^{3}+(b^{4}+4\,ab^{2}c+4\,a^{2}c^{2})x^{2}+2\,(ab^{3}+2\,a^{2}bc)x)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac-\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)+2\,(5\,ab^{4}-22\,a^{2}b^{2}c+8\,a^{3}c^{2})x}{2\,(a^{2}b^{6}-12\,a^{3}b^{4}c+48\,a^{4}b^{2}c^{2}-64\,a^{5}c^{3}+(b^{6}c^{2}-12\,ab^{4}c^{3}+48\,a^{2}b^{2}c^{4}-64\,a^{3}c^{5})x^{4}+2\,(b^{7}c-12\,ab^{5}c^{2}+48\,a^{2}b^{3}c^{3}-64\,a^{3}bc^{4})x^{3}+(b^{8}-10\,ab^{6}c+24\,a^{2}b^{4}c^{2}+32\,a^{3}b^{2}c^{3}-128\,a^{4}c^{4})x^{2}+2\,(ab^{7}-12\,a^{2}b^{5}c+48\,a^{3}b^{3}c^{2}-64\,a^{4}bc^{3})x)},\ \frac{6\,a^{2}b^{3}-24\,a^{3}bc+2\,(b^{4}c-2\,ab^{2}c^{2}-8\,a^{2}c^{3})x^{3}+3\,(b^{5}-2\,ab^{3}c-8\,a^{2}bc^{2})x^{2}-4\,((b^{2}c^{2}+2\,ac^{3})x^{4}+a^{2}b^{2}+2\,a^{3}c+2\,(b^{3}c+2\,abc^{2})x^{3}+(b^{4}+4\,ab^{2}c+4\,a^{2}c^{2})x^{2}+2\,(ab^{3}+2\,a^{2}bc)x)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)+2\,(5\,ab^{4}-22\,a^{2}b^{2}c+8\,a^{3}c^{2})x}{2\,(a^{2}b^{6}-12\,a^{3}b^{4}c+48\,a^{4}b^{2}c^{2}-64\,a^{5}c^{3}+(b^{6}c^{2}-12\,ab^{4}c^{3}+48\,a^{2}b^{2}c^{4}-64\,a^{3}c^{5})x^{4}+2\,(b^{7}c-12\,ab^{5}c^{2}+48\,a^{2}b^{3}c^{3}-64\,a^{3}bc^{4})x^{3}+(b^{8}-10\,ab^{6}c+24\,a^{2}b^{4}c^{2}+32\,a^{3}b^{2}c^{3}-128\,a^{4}c^{4})x^{2}+2\,(ab^{7}-12\,a^{2}b^{5}c+48\,a^{3}b^{3}c^{2}-64\,a^{4}bc^{3})x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*
(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x^2 + 2*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2
+ 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 +
 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2
- 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(5*a*b^4 -
22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), 1/2*(6*a^2*b^3 - 24*a^3*b*c + 2*
(b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*x^3 + 3*(b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*x
^2 - 4*((b^2*c^2 + 2*a*c^3)*x^4 + a^2*b^2 + 2*a^3*c + 2*(b^3*c + 2*a*b*c^2)
*x^3 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^2 + 2*(a*b^3 + 2*a^2*b*c)*x)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(5*a*
b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
+ 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*
a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(107) = 214$.
time = 0.74, size = 570, normalized size = 4.96

$$- \sqrt{- \frac{1}{\left(4ac - b^{2}\right)^{5}}} \cdot \left(2ac + b^{2}\right) \log{\left(x + \frac{- 64a^{3}c^{3}\sqrt{- \frac{1}{\left(4ac - b^{2}\right)^{5}}} \cdot \left(2ac + b^{2}\right) + 48a^{2}b^{2}c^{2}\sqrt{- \frac{1}{\left(4ac - b^{2}\right)^{5}}} \cdot \left(2ac + b^{2}\right) - 12ab^{4}c\sqrt{- \frac{1}{\left(4ac - b^{2}\right)^{5}}} \cdot \left(2ac + b^{2}\right) + 2abc + b^{6}\sqrt{- \frac{1}{\left(4ac - b^{2}\right)^{5}}} \cdot \left(2ac + b^{2}\right) + b^{3}}{4ac^{2} + 2b^{2}c} \right)} + \sqrt{- \frac{1}{\left(4ac - b^{2}\right)^{5}}} \cdot \left(2ac + b^{2}\right) \log{\left(x + \frac{64a^{3}c^{3}\sqrt{- \frac{1}{\left(4ac - b^{2}\right)^{5}}} \cdot \left(2ac + b^{2}\right) - 48a^{2}b^{2}c^{2}\sqrt{- \frac{1}{\left(4ac - b^{2}\right)^{5}}} \cdot \left(2ac + b^{2}\right) + 12ab^{4}c\sqrt{- \frac{1}{\left(4ac - b^{2}\right)^{5}}} \cdot \left(2ac + b^{2}\right) + 2abc - b^{6}\sqrt{- \frac{1}{\left(4ac - b^{2}\right)^{5}}} \cdot \left(2ac + b^{2}\right) + b^{3}}{4ac^{2} + 2b^{2}c} \right)} + \frac{6a^{2}b + x^{3} \cdot \left(4ac^{2} + 2b^{2}c\right) + x^{2} \cdot \left(6abc + 3b^{3}\right) + x \left(- 4a^{2}c + 10ab^{2}\right)}{32a^{4}c^{2} - 16a^{3}b^{2}c + 2a^{2}b^{4} + x^{4} \cdot \left(32a^{2}c^{4} - 16ab^{2}c^{3} + 2b^{4}c^{2}\right) + x^{3} \cdot \left(64a^{2}bc^{3} - 32ab^{3}c^{2} + 4b^{5}c\right) + x^{2} \cdot \left(64a^{3}c^{3} - 12ab^{4}c + 2b^{6}\right) + x \left(64a^{3}bc^{2} - 32a^{2}b^{3}c + 4ab^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**4,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (-64*a**3*c**3*sqrt(-1/(
4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)
 + 2*a*b*c + b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + b**3)/(4*a*c*
*2 + 2*b**2*c)) + sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x + (64*a**
3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**2*sqrt(-
1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5
)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)
 + b**3)/(4*a*c**2 + 2*b**2*c)) + (6*a**2*b + x**3*(4*a*c**2 + 2*b**2*c) +
x**2*(6*a*b*c + 3*b**3) + x*(-4*a**2*c + 10*a*b**2))/(32*a**4*c**2 - 16*a**
3*b**2*c + 2*a**2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2)
 + x**3*(64*a**2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 -
 12*a*b**4*c + 2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Giac [**A**]

time = 3.23, size = 154, normalized size = 1.34

$$\frac{2\,(b^2+2\,ac)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}+\frac{2\,b^2cx^3+4\,ac^2x^3+3\,b^3x^2+6\,abcx^2+10\,ab^2x-4\,a^2cx+6\,a^2b}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^4,x, algorithm="giac")
```

```
[Out] 2*(b^2 + 2*a*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c +
16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(2*b^2*c*x^3 + 4*a*c^2*x^3 + 3*b^3*x^
2 + 6*a*b*c*x^2 + 10*a*b^2*x - 4*a^2*c*x + 6*a^2*b)/((b^4 - 8*a*b^2*c + 16*
a^2*c^2)*(c*x^2 + b*x + a)^2)
```

Mupad [B]

time = 1.50, size = 313, normalized size = 2.72

$$\frac{\frac{3\,a^2\,b}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{a\,x\,\left(2\,a\,c-5\,b^2\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,b\,x^2\,\left(b^2+2\,a\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{c\,x^3\,\left(b^2+2\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}+\frac{2\,\mathrm{atan}\left(\frac{\left(\frac{\left(b^2+2\,a\,c\right)\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{2\,c\,x\,\left(b^2+2\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{b^2+2\,a\,c}\right)\,\left(b^2+2\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + a/x^2 + b/x)^3),x)
```

```
[Out] ((3*a^2*b)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (a*x*(2*a*c - 5*b^2))/(b^4 + 16
*a^2*c^2 - 8*a*b^2*c) + (3*b*x^2*(2*a*c + b^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) + (c*x^3*(2*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c
 + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (2*atan(((((2*a*c + b^2)*(
b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) + (2*c*x*(2*a*c + b^2))/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 -
 8*a*b^2*c))/(2*a*c + b^2))*(2*a*c + b^2))/(4*a*c - b^2)^(5/2)
```

3.436 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx$

Optimal. Leaf size=103

$$\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2-3/2*b*(2*c*x+b)/(-4*a*c+b^2)^2/(
c*x^2+b*x+a)+6*b*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1368, 652, 628, 632, 212}

$$\frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^5),x]
```

```
[Out] (2*a + b*x)/(2*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) - (3*b*(b + 2*c*x))/(2*(b
^2 - 4*a*c)^2*(a + b*x + c*x^2)) + (6*b*c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*
a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^5}\,dx &= \int \frac{x}{\left(a+bx+cx^2\right)^3}\,dx\\
&= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{(3b)\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx}{2\left(b^2-4ac\right)}\\
&= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{(3bc)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}\\
&= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{(6bc)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-\dots}\right.}{\left(b^2-4\dots\right.}\\
&= \frac{2a+bx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{3b(b+2cx)}{2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{6bc\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4\dots}}\right.}{\left(b^2-4ac\right)^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 102, normalized size = 0.99

$$
\frac{\frac{\left(b^2-4ac\right)(2a+bx)}{(a+x(b+cx))^2}-\frac{3b(b+2cx)}{a+x(b+cx)}-\frac{12bc\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2\left(b^2-4ac\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^5),x]
```

```
[Out] (((b^2 - 4*a*c)*(2*a + b*x))/(a + x*(b + c*x))^2 - (3*b*(b + 2*c*x))/(a + x
*(b + c*x)) - (12*b*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c])/(2*(b^2 - 4*a*c)^2)
```

Maple [A]
time = 0.05, size = 118, normalized size = 1.15

method	result
default	$\frac{-bx-2a}{2(4ac-b^{2})(c\,x^{2}+bx+a)^{2}}-\frac{3b\left(\frac{2cx+b}{\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}\right)}{2(4ac-b^{2})}$
risch	$\frac{-\frac{3b\,c^{2}x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{9b^{2}c\,x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5ac+b^{2}\right)bx}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{a\left(8ac+b^{2}\right)}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{(c\,x^{2}+bx+a)^{2}}-\frac{3cb\ln\left(\left(32a^{2}c^{3}-16a\,b^{2}c^{2}+2b^{4}c\right)x-(-4ac\right.}{(-4ac+b^{2})^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)^2-3/2*b/(4*a*c-b^2)*((2*c*x+b)/(4*
a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(
1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(95) = 190.
time = 0.35, size = 788, normalized size = 7.65

$$\left[-\frac{ab^{4}+4\,a^{2}b^{2}c-32\,a^{3}c^{2}+6\,(b^{3}c^{2}-4\,abc^{3})x^{3}+9\,(b^{4}c-4\,ab^{2}c^{2})x^{2}-6\,(bc^{3}x^{4}+2\,b^{2}c^{2}x^{3}+2\,ab^{2}cx+a^{2}bc+(b^{3}c+2\,abc^{2})x^{2})\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)+2\,(b^{5}+ab^{3}c-20\,a^{2}bc^{2})x}{2\,(a^{2}b^{6}-12\,a^{3}b^{4}c+48\,a^{4}b^{2}c^{2}-64\,a^{5}c^{3}+(b^{6}c^{2}-12\,ab^{4}c^{3}+48\,a^{2}b^{2}c^{4}-64\,a^{3}c^{5})x^{4}+2\,(b^{7}c-12\,ab^{5}c^{2}+48\,a^{2}b^{3}c^{3}-64\,a^{3}bc^{4})x^{3}+(b^{8}-10\,ab^{6}c+24\,a^{2}b^{4}c^{2}+32\,a^{3}b^{2}c^{3}-128\,a^{4}c^{4})x^{2}+2\,(ab^{7}-12\,a^{2}b^{5}c+48\,a^{3}b^{3}c^{2}-64\,a^{4}bc^{3})x)},\ -\frac{ab^{4}+4\,a^{2}b^{2}c-32\,a^{3}c^{2}+6\,(b^{3}c^{2}-4\,abc^{3})x^{3}+9\,(b^{4}c-4\,ab^{2}c^{2})x^{2}-12\,(bc^{3}x^{4}+2\,b^{2}c^{2}x^{3}+2\,ab^{2}cx+a^{2}bc+(b^{3}c+2\,abc^{2})x^{2})\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)+2\,(b^{5}+ab^{3}c-20\,a^{2}bc^{2})x}{2\,(a^{2}b^{6}-12\,a^{3}b^{4}c+48\,a^{4}b^{2}c^{2}-64\,a^{5}c^{3}+(b^{6}c^{2}-12\,ab^{4}c^{3}+48\,a^{2}b^{2}c^{4}-64\,a^{3}c^{5})x^{4}+2\,(b^{7}c-12\,ab^{5}c^{2}+48\,a^{2}b^{3}c^{3}-64\,a^{3}bc^{4})x^{3}+(b^{8}-10\,ab^{6}c+24\,a^{2}b^{4}c^{2}+32\,a^{3}b^{2}c^{3}-128\,a^{4}c^{4})x^{2}+2\,(ab^{7}-12\,a^{2}b^{5}c+48\,a^{3}b^{3}c^{2}-64\,a^{4}bc^{3})x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(
b^4*c - 4*a*b^2*c^2)*x^2 - 6*(b*c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2
*b*c + (b^3*c + 2*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x
+ b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*(b^5
+ a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b
^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(a*b^4 + 4*a^2*b^2*c - 32*a
^3*c^2 + 6*(b^3*c^2 - 4*a*b*c^3)*x^3 + 9*(b^4*c - 4*a*b^2*c^2)*x^2 - 12*(b*
c^3*x^4 + 2*b^2*c^2*x^3 + 2*a*b^2*c*x + a^2*b*c + (b^3*c + 2*a*b*c^2)*x^2)*
sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) +
2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(95) = 190$.
time = 0.62, size = 481, normalized size = 4.67

$$3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^7c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^2c}{6bc^2}\right)-3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-3b^7c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^2c}{6bc^2}\right)+\frac{-8a^2c-ab^2-9b^2cx^2-6bc^2x^3+x(-10abc-2b^3)}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(64a^3c^3-12ab^4c+2b^6)+x(64a^3bc^2-32a^2b^3c+4ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**5,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-192*a**3*b*c**4*sqrt(-1/(4*a*c -
 b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 36*a*b**5*c**2
*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*
c)/(6*b*c**2)) - 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x + (192*a**3*b*c**4*
sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5)
+ 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c - b**
2)**5) + 3*b**2*c)/(6*b*c**2)) + (-8*a**2*c - a*b**2 - 9*b**2*c*x**2 - 6*b*
c**2*x**3 + x*(-10*a*b*c - 2*b**3))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a**2
*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**2*
b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c + 2*
b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Giac [A]
time = 4.05, size = 135, normalized size = 1.31

$$-\frac{6\,bc\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2x^3+9\,b^2cx^2+2\,b^3x+10\,abcx+ab^2+8\,a^2c}{2\,(b^4-8\,ab^2c+16\,a^2c^2)(cx^2+bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^5,x, algorithm="giac")
```

```
[Out] -6*b*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) - 1/2*(6*b*c^2*x^3 + 9*b^2*c*x^2 + 2*b^3*x + 10*a*b*
c*x + a*b^2 + 8*a^2*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2)
```

Mupad [B]
time = 1.43, size = 253, normalized size = 2.46

$$-\frac{\frac{8\,c\,a^2+a\,b^2}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b^2\,c\,x^2}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{3\,b\,c^2\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{b\,x\,\left(b^2+5\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}-\frac{6\,b\,c\,\mathrm{atan}\left(\frac{\left(\frac{3\,b^2\,c}{{\left(4\,a\,c-b^2\right)}^{5/2}}+\frac{6\,b\,c^2\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{3\,b\,c}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + a/x^2 + b/x)^3),x)
```

```
[Out] - ((a*b^2 + 8*a^2*c)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^2*c*x^2)/(2*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*b*c^2*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2
*c) + (b*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2
) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (6*b*c*atan((((3*b^2*c)/(4*a*c -
 b^2)^(5/2) + (6*b*c^2*x)/(4*a*c - b^2)^(5/2))*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c))/(3*b*c)))/(4*a*c - b^2)^(5/2)
```

3.437 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6} dx$

Optimal. Leaf size=103

$$\frac{-b-2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/2*(-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+3*c*(2*c*x+b)/(-4*a*c+b^2)^2/(c
*x^2+b*x+a)-12*c^2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A]
time = 0.03, antiderivative size = 101, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1368, 628, 632, 212}

$$-\frac{12c^2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}}+\frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}-\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] -1/2*(b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (3*c*(b + 2*c*x))/((
b^2 - 4*a*c)^2*(a + b*x + c*x^2)) - (12*c^2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^6}\,dx = \int \frac{1}{(a+bx+cx^2)^3}\,dx$$
$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} - \frac{(3c)\int \frac{1}{(a+bx+cx^2)^2}\,dx}{b^2-4ac}$$
$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} + \frac{(6c^2)\int \frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2}$$
$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{(12c^2)\,\mathrm{Subst}\left(\int \frac{}{b^2-\cdots}\right.}{(b^2-\cdots}$$
$$= -\frac{b+2cx}{2\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2} + \frac{3c(b+2cx)}{\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)} - \frac{12c^2\tanh^{-1}\left(\frac{b+\cdots}{\sqrt{b^2\cdots}}\right.}{(b^2-4ac)^{5/\cdots}}$$

Mathematica [A]
time = 0.06, size = 97, normalized size = 0.94

$$\frac{-\frac{(b+2cx)\left(b^2-6bcx-2c\left(5a+3cx^2\right)\right)}{(a+x(b+cx))^2}+\frac{24c^2\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^6),x]
```

```
[Out] (-(((b + 2*c*x)*(b^2 - 6*b*c*x - 2*c*(5*a + 3*c*x^2)))/(a + x*(b + c*x))^2)
 + (24*c^2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(
b^2 - 4*a*c)^2)
```

Maple [A]
time = 0.05, size = 116, normalized size = 1.13

method	result
default	$\frac{2cx+b}{2(4ac-b^2)(cx^2+bx+a)^2}+\frac{3c\left(\frac{2cx+b}{\left(4ac-b^2\right)\left(cx^2+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}\right)}{4ac-b^2}$
risch	$\frac{\frac{6c^3x^3}{16a^2c^2-8ab^2c+b^4}+\frac{9bc^2x^2}{16a^2c^2-8ab^2c+b^4}+\frac{2\left(5ac+b^2\right)cx}{16a^2c^2-8ab^2c+b^4}+\frac{b\left(10ac-b^2\right)}{32a^2c^2-16ab^2c+2b^4}}{(cx^2+bx+a)^2}-\frac{6c^2\ln\left(\left(32a^2c^3-16ab^2c^2+2b^4c\right)x+\left(-4ac+b^2\right)^{\frac{5}{2}}+\right.}{(-4ac+b^2)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+3*c/(4*a*c-b^2)*((2*c*x+b)/(4*a*c
-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2
)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(95) = 190$.
time = 0.37, size = 785, normalized size = 7.62

$$\left[-\frac{b^5-14ab^3c+40a^2bc^2-12(b^2c^3-4ac^4)x^3-18(b^3c^2-4abc^3)x^2-12\left(c^4x^4+2bc^3x^3+2abc^2x+a^2c^2+(b^2c^2+2ac^3)x^2\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^2+2bcx+b^2-2ac-\sqrt{b^2-4ac}(2cx+b)}{cx^2+bx+a}\right)-4(b^4c+ab^2c^2-20a^2c^3)x}{2\left(a^2b^6-12a^3b^4c+48a^4b^2c^2-64a^5c^3+(b^6c^2-12ab^4c^3+48a^2b^2c^4-64a^3c^5)x^4+2(b^7c-12ab^5c^2+48a^2b^3c^3-64a^3bc^4)x^3+(b^8-10ab^6c+24a^2b^4c^2+32a^3b^2c^3-128a^4c^4)x^2+2(ab^7-12a^2b^5c+48a^3b^3c^2-64a^4bc^3)x\right)},\ -\frac{b^5-14ab^3c+40a^2bc^2-12(b^2c^3-4ac^4)x^3-18(b^3c^2-4abc^3)x^2+24\left(c^4x^4+2bc^3x^3+2abc^2x+a^2c^2+(b^2c^2+2ac^3)x^2\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\sqrt{-b^2+4ac}(2cx+b)}{b^2-4ac}\right)-4(b^4c+ab^2c^2-20a^2c^3)x}{2\left(a^2b^6-12a^3b^4c+48a^4b^2c^2-64a^5c^3+(b^6c^2-12ab^4c^3+48a^2b^2c^4-64a^3c^5)x^4+2(b^7c-12ab^5c^2+48a^2b^3c^3-64a^3bc^4)x^3+(b^8-10ab^6c+24a^2b^4c^2+32a^3b^2c^3-128a^4c^4)x^2+2(ab^7-12a^2b^5c+48a^3b^3c^2-64a^4bc^3)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b
^3*c^2 - 4*a*b*c^3)*x^2 - 12*(c^4*x^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2
 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^
2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 4*(b^4*c +
a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
```

```
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x), -1/2*(b^5 - 14*a*b^3*c + 40*a^2*b*
c^2 - 12*(b^2*c^3 - 4*a*c^4)*x^3 - 18*(b^3*c^2 - 4*a*b*c^3)*x^2 + 24*(c^4*x
^4 + 2*b*c^3*x^3 + 2*a*b*c^2*x + a^2*c^2 + (b^2*c^2 + 2*a*c^3)*x^2)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 4*(b^4*
c + a*b^2*c^2 - 20*a^2*c^3)*x)/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*
(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^
6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^
2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(95) = 190$.
time = 0.72, size = 474, normalized size = 4.60

$$-6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{-384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}+288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+6b^6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+6bc^2}{12c^3}\right)+6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x+\frac{384a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}-288a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+72ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-6b^6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+6bc^2}{12c^3}\right)+\frac{10abc-b^3+18bc^2x^2+12c^3x^3+x(20ac^2+4b^2c)}{32a^4c^2-16a^3b^2c+2a^2b^4+x^4\cdot(32a^2c^4-16ab^2c^3+2b^4c^2)+x^3\cdot(64a^2bc^3-32ab^3c^2+4b^5c)+x^2\cdot(64a^3c^3-12ab^4c+2b^6)+x\left(64a^3bc^2-32a^2b^3c+4ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**6,x)
```

```
[Out] -6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (-384*a**3*c**5*sqrt(-1/(4*a*c -
 b**2)**5) + 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 72*a*b**4*c**3
*sqrt(-1/(4*a*c - b**2)**5) + 6*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 6*b*
c**2)/(12*c**3)) + 6*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x + (384*a**3*c**5
*sqrt(-1/(4*a*c - b**2)**5) - 288*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5)
 + 72*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 6*b**6*c**2*sqrt(-1/(4*a*c -
 b**2)**5) + 6*b*c**2)/(12*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x**2 + 12*
c**3*x**3 + x*(20*a*c**2 + 4*b**2*c))/(32*a**4*c**2 - 16*a**3*b**2*c + 2*a*
*2*b**4 + x**4*(32*a**2*c**4 - 16*a*b**2*c**3 + 2*b**4*c**2) + x**3*(64*a**
2*b*c**3 - 32*a*b**3*c**2 + 4*b**5*c) + x**2*(64*a**3*c**3 - 12*a*b**4*c +
2*b**6) + x*(64*a**3*b*c**2 - 32*a**2*b**3*c + 4*a*b**5))
```

Giac [**A**]
time = 3.32, size = 136, normalized size = 1.32

$$\frac{12\,c^2\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx+20\,ac^2x-b^3+10\,abc}{2\,\left(b^4-8\,ab^2c+16\,a^2c^2\right)\left(cx^2+bx+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^6,x, algorithm="giac")
```

```
[Out] 12*c^2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c + 16*a^2*c^
2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x + 20*a*
```

```
c^2*x - b^3 + 10*a*b*c)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*(c*x^2 + b*x + a)^2
)
```

Mupad [B]
time = 1.42, size = 285, normalized size = 2.77

$$\frac{\frac{6\,c^3\,x^3}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}-\frac{b^3-10\,a\,b\,c}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b\,c^2\,x^2}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{2\,c\,x\,\left(b^2+5\,a\,c\right)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}}{x^2\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^4+2\,a\,b\,x+2\,b\,c\,x^3}+\frac{12\,c^2\,\mathrm{atan}\left(\frac{\left(\frac{12\,c^3\,x}{{\left(4\,a\,c-b^2\right)}^{5/2}}+\frac{6\,c^2\,\left(16\,a^2\,b\,c^2-8\,a\,b^3\,c+b^5\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,c^2}\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + a/x^2 + b/x)^3),x)
```

```
[Out] ((6*c^3*x^3)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) - (b^3 - 10*a*b*c)/(2*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*x^2)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (2*
c*x*(5*a*c + b^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(2*a*c + b^2) + a^2
 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) + (12*c^2*atan((((12*c^3*x)/(4*a*c - b^2)
^(5/2) + (6*c^2*(b^5 + 16*a^2*b*c^2 - 8*a*b^3*c))/((4*a*c - b^2)^(5/2)*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2)))/(4*a
*c - b^2)^(5/2)
```

3.438 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx$

Optimal. Leaf size=185

$$\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{b\left(b^4-10ab^2c+30a^2c^2\right)\tanh^{-1}\left(\right.}{a^3\left(b^2-4ac\right)^{5/2}}$$

```
[Out] 1/2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)^2+1/2*(2*b^4-15*a*b^2*c+
16*a^2*c^2+2*b*c*(-7*a*c+b^2)*x)/a^2/(-4*a*c+b^2)^2/(c*x^2+b*x+a)+b*(30*a^2
*c^2-10*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)
^(5/2)+ln(x)/a^3-1/2*ln(c*x^2+b*x+a)/a^3
```

Rubi [A]

time = 0.16, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1368, 754, 836, 814, 648, 632, 212, 642}

$$-\frac{\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)}{a^3}+\frac{16a^2c^2+2bcx\left(b^2-7ac\right)-15ab^2c+2b^4}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^7),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(2*a*(b^2 - 4*a*c)*(a + b*x + c*x^2)^2) + (2*b^4 - 15
*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*(a +
b*x + c*x^2)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a + b*x + c*x^
2]/(2*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
```

```
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^7}\,dx=\int \frac{1}{x\left(a+bx+cx^2\right)^3}\,dx$$

$$=\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}-\frac{\int \frac{-2\left(b^2-4ac\right)-3bcx}{x\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$=\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{\int \frac{2(b^2}{}}{}$$

$$=\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{\int \left(\frac{2(}{}\right.}{}$$

$$=\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^3}$$

$$=\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^3}$$

$$=\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{\log(x)}{a^3}$$

$$=\frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)\left(a+bx+cx^2\right)^2}+\frac{2b^4-15ab^2c+16a^2c^2+2bc(b^2-7ac)\,x}{2a^2\left(b^2-4ac\right)^2\left(a+bx+cx^2\right)}+\frac{b(b^4-}{}$$

Mathematica [**A**]
time = 0.23, size = 178, normalized size = 0.96

$$\frac{\frac{a^2\left(b^2-2ac+bcx\right)}{\left(b^2-4ac\right)\left(a+x(b+cx)\right)^2}+\frac{a\left(2b^4-15ab^2c+16a^2c^2+2b^3cx-14abc^2x\right)}{\left(b^2-4ac\right)^2\left(a+x(b+cx)\right)}-\frac{2b\left(b^4-10ab^2c+30a^2c^2\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}+2\log(x)-\log(a+x(b+cx))}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^7),x]
```

```
[Out] ((a^2*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))^2) + (a*(2*b^
4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x - 14*a*b*c^2*x))/((b^2 - 4*a*c)^2*(
a + x*(b + c*x))) - (2*b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + 2*Log[x] - Log[a + x*(b + c*x)
])/(2*a^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(175) = 350$.
time = 0.06, size = 352, normalized size = 1.90

method	result
default	$-\frac{\frac{abc^2(7ac-b^2)x^3}{16a^2c^2-8ab^2c+b^4}-\frac{ca(16a^2c^2-29ab^2c+4b^4)x^2}{2(16a^2c^2-8ab^2c+b^4)}+\frac{ab(a^2c^2+6ab^2c-b^4)x}{16a^2c^2-8ab^2c+b^4}-\frac{3a^2(8a^2c^2-7ab^2c+b^4)}{2(16a^2c^2-8ab^2c+b^4)}}{(cx^2+bx+a)^2}+\frac{\frac{(16a^2c^3-8ab^2c^2+b^4c)\ln(cx^2+bx+a)}{2c}}{a^3}$
risch	$\frac{-\frac{bc^2(7ac-b^2)x^3}{a^2(16a^2c^2-8ab^2c+b^4)}+\frac{c(16a^2c^2-29ab^2c+4b^4)x^2}{2(16a^2c^2-8ab^2c+b^4)a^2}-\frac{b(a^2c^2+6ab^2c-b^4)x}{a^2(16a^2c^2-8ab^2c+b^4)}+\frac{12a^2c^2-\frac{21}{2}ab^2c+\frac{3}{2}b^4}{a(16a^2c^2-8ab^2c+b^4)}}{(cx^2+bx+a)^2}+\frac{\ln(x)}{a^3}+\Big(\sum_{\_R=\text{RootOf}((102}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((a*b*c^2*(7*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*c*a*(16*a^2
*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+a*b*(a^2*c^2+6*a*b^2*
c-b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-3/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)/(16*a
^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(1
6*a^2*c^3-8*a*b^2*c^2+b^4*c)/c*ln(c*x^2+b*x+a)+2*(23*a^2*b*c^2-9*a*b^3*c+b^
5-1/2*(16*a^2*c^3-8*a*b^2*c^2+b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b
)/(4*a*c-b^2)^(1/2))))+ln(x)/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(175) = 350$.
time = 0.61, size = 1985, normalized size = 10.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="fricas")
```

```
[Out] [1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^
2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*
a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + (a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + (b
^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^
2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^2 + 2*
(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2
 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)
) + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^2*b^6 -
12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x
)*log(c*x^2 + b*x + a) + 2*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7
*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^
5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(x))/(a^5*b^6 - 12*a^6*b^4*c + 4
8*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4
 - 64*a^6*c^5)*x^4 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^
6*b*c^4)*x^3 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 -
128*a^7*c^4)*x^2 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^
3)*x), 1/2*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*
b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^3 + (4*a*b^6*c - 45*a^2*b^4*c^2
+ 132*a^3*b^2*c^3 - 64*a^4*c^4)*x^2 + 2*(a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*
c^2 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^4 + 2*(b^6*c - 10*a*b^4*c^2
 + 30*a^2*b^2*c^3)*x^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*
x^2 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x)*sqrt(-b^2 + 4*a*c)*arcta
n(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(a*b^7 - 10*a^2*b^5*c
+ 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*x - (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4
 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10
*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 -
12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x)*log(c*x^2 + b*x + a) + 2*(
a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + (b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b
^3*c^3 - 64*a^3*b*c^4)*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^
2*c^3 - 128*a^4*c^4)*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^
4*b*c^3)*x)*log(x))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 +
 (a^3*b^6*c^2 - 12*a^4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^4 + 2*(a^3*
b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^3 + (a^3*b^8 - 10
*a^4*b^6*c + 24*a^5*b^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^2 + 2*(a^4*b^
7 - 12*a^5*b^5*c + 48*a^6*b^3*c^2 - 64*a^7*b*c^3)*x)]
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**7,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.94, size = 239, normalized size = 1.29

$$-\frac{(b^5-10\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^4-8\,a^4b^2c+16\,a^5c^2)\sqrt{-b^2+4\,ac}}-\frac{\log\left(cx^2+bx+a\right)}{2\,a^3}+\frac{\log\left(|x|\right)}{a^3}+\frac{3\,a^2b^4-21\,a^3b^2c+24\,a^4c^2+2\,(ab^3c^2-7\,a^2bc^3)x^3+(4\,ab^4c-29\,a^2b^2c^2+16\,a^3c^3)x^2+2\,(ab^5-6\,a^2b^3c-a^3bc^2)x}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^7,x, algorithm="giac")
```

```
[Out] -(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 +
b*x + a)/a^3 + log(abs(x))/a^3 + 1/2*(3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2
 + 2*(a*b^3*c^2 - 7*a^2*b*c^3)*x^3 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c
^3)*x^2 + 2*(a*b^5 - 6*a^2*b^3*c - a^3*b*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2
- 4*a*c)^2*a^3)
```

Mupad [B]
time = 2.46, size = 1089, normalized size = 5.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + a/x^2 + b/x)^3),x)
```

```
[Out] log(x)/a^3 + ((3*(b^4 + 8*a^2*c^2 - 7*a*b^2*c))/(2*a*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)) + (x^2*(4*b^4*c + 16*a^2*c^3 - 29*a*b^2*c^2))/(2*a^2*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) - (b*x*(a^2*c^2 - b^4 + 6*a*b^2*c))/(a^2*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c)) - (b*c^2*x^3*(7*a*c - b^2))/(a^2*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)))/(x^2*(2*a*c + b^2) + a^2 + c^2*x^4 + 2*a*b*x + 2*b*c*x^3) - (log(1
536*a^6*c^5 - 2*b^11*x - 2*a*b^10 + 2*a*b^5*(-(4*a*c - b^2)^5)^(1/2) + 39*a
^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^(1/2) - 303*a^3*b^6*c^2 + 1160*a^4*b^
4*c^3 - 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(-(4*a*c - b^2)^5)^(1/2) + 39*a^3*b
*c^2*(-(4*a*c - b^2)^5)^(1/2) - 321*a^2*b^7*c^2*x + 1286*a^3*b^5*c^3*x - 25
60*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^9*c*x + 2
016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2*b^2*c^2*x*
(-(4*a*c - b^2)^5)^(1/2))*(1024*a^5*c^5 - b^10 + b^5*(-(4*a*c - b^2)^5)^(1/
2) - 160*a^2*b^6*c^2 + 640*a^3*b^4*c^3 - 1280*a^4*b^2*c^4 + 20*a*b^8*c + 30
```

```
*a^2*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a*b^3*c*(-(4*a*c - b^2)^5)^(1/2)))
/(2*a^3*(4*a*c - b^2)^5) + (log(2*a*b^10 + 2*b^11*x - 1536*a^6*c^5 + 2*a*b^
5*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^8*c + 2*b^6*x*(-(4*a*c - b^2)^5)^(1/2
) + 303*a^3*b^6*c^2 - 1160*a^4*b^4*c^3 + 2160*a^5*b^2*c^4 - 17*a^2*b^3*c*(-
(4*a*c - b^2)^5)^(1/2) + 39*a^3*b*c^2*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^
7*c^2*x - 1286*a^3*b^5*c^3*x + 2560*a^4*b^3*c^4*x - 48*a^3*c^3*x*(-(4*a*c -
 b^2)^5)^(1/2) - 40*a*b^9*c*x - 2016*a^5*b*c^5*x - 20*a*b^4*c*x*(-(4*a*c -
b^2)^5)^(1/2) + 63*a^2*b^2*c^2*x*(-(4*a*c - b^2)^5)^(1/2))*(b^10 - 1024*a^5
*c^5 + b^5*(-(4*a*c - b^2)^5)^(1/2) + 160*a^2*b^6*c^2 - 640*a^3*b^4*c^3 + 1
280*a^4*b^2*c^4 - 20*a*b^8*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a
*b^3*c*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^3*(4*a*c - b^2)^5)
```

3.439 $\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8} \, dx$

Optimal. Leaf size=239

$$-\frac{3(b^2-5ac)\,(b^2-2ac)}{a^3\,(b^2-4ac)^2\,x}+\frac{b^2-2ac+bcx}{2a\,(b^2-4ac)\,x\,(a+bx+cx^2)^2}+\frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,x}{2a^2\,(b^2-4ac)^2\,x\,(a+bx+cx^2)}-\frac{3(b^6-$$

```
[Out] -3*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/x+1/2*(b*c*x-2*a*c+b^2)/a/(
-4*a*c+b^2)/x/(c*x^2+b*x+a)^2+1/2*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*b*c*(-6*a*
c+b^2)*x)/a^2/(-4*a*c+b^2)^2/x/(c*x^2+b*x+a)-3*(-20*a^3*c^3+30*a^2*b^2*c^2-
10*a*b^4*c+b^6)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(5/2
)-3*b*ln(x)/a^4+3/2*b*ln(c*x^2+b*x+a)/a^4
```

Rubi [A]
time = 0.19, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1368, 754, 836, 814, 648, 632, 212, 642}

$$\frac{3b\log\left(a+bx+cx^2\right)}{2a^4}-\frac{3b\log(x)}{a^4}-\frac{3(b^2-5ac)\,(b^2-2ac)}{a^3x\,(b^2-4ac)^2}+\frac{20a^2c^2+3bcx(b^2-6ac)-20ab^2c+3b^4}{2a^2x\,(b^2-4ac)^2\,(a+bx+cx^2)}-\frac{3(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\,(b^2-4ac)^{5/2}}+\frac{-2ac+b^2+bcx}{2ax\,(b^2-4ac)\,(a+bx+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(a^3*(b^2 - 4*a*c)^2*x) + (b^2 - 2*a*c + b
*c*x)/(2*a*(b^2 - 4*a*c)*x*(a + b*x + c*x^2)^2) + (3*b^4 - 20*a*b^2*c + 20*
a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x)/(2*a^2*(b^2 - 4*a*c)^2*x*(a + b*x + c*x^2)
) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3*b*Lo
g[a + b*x + c*x^2])/(2*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(c+\frac{a}{x^2}+\frac{b}{x}\right)^3 x^8}\,dx = \int \frac{1}{x^2\left(a+bx+cx^2\right)^3}\,dx$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} - \frac{\int \frac{-3b^2+10ac-4bcx}{x^2\left(a+bx+cx^2\right)^2}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)} + \frac{\int \frac{6(b^2}{}}{}$$

$$= \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2+3bc(b^2-6ac)\,x}{2a^2\left(b^2-4ac\right)^2x\left(a+bx+cx^2\right)} + \frac{\int \left(\frac{6(}{}\right.}{}$$

$$= -\frac{3(b^2-5ac)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2}{2a^2\left(b^2-4ac\right)^2x}$$

$$= -\frac{3(b^2-5ac)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2}{2a^2\left(b^2-4ac\right)^2x}$$

$$= -\frac{3(b^2-5ac)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2}{2a^2\left(b^2-4ac\right)^2x}$$

$$= -\frac{3(b^2-5ac)\left(b^2-2ac\right)}{a^3\left(b^2-4ac\right)^2x} + \frac{b^2-2ac+bcx}{2a\left(b^2-4ac\right)x\left(a+bx+cx^2\right)^2} + \frac{3b^4-20ab^2c+20a^2c^2}{2a^2\left(b^2-4ac\right)^2x}$$

Mathematica [A]
time = 0.28, size = 221, normalized size = 0.92

$$\frac{-\frac{2a}{x}+\frac{a^2\left(b^3-3abc+b^2cx-2ac^2x\right)}{\left(-b^2+4ac\right)\left(a+x(b+cx)\right)^2}-\frac{a\left(4b^5-29ab^3c+46a^2bc^2+4b^4cx-26ab^2c^2x+28a^2c^3x\right)}{\left(b^2-4ac\right)^2\left(a+x(b+cx)\right)}+\frac{6\left(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}-6b\log(x)+3b\log\left(a+x(b+cx)\right)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c + a/x^2 + b/x)^3*x^8),x]
```

```
[Out] ((-2*a)/x + (a^2*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((-b^2 + 4*a*c)*(a
+ x*(b + c*x))^2) - (a*(4*b^5 - 29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x - 26*
a*b^2*c^2*x + 28*a^2*c^3*x))/((b^2 - 4*a*c)^2*(a + x*(b + c*x))) + (6*(b^6
```

```
- 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/(-b^2 + 4*a*c)^(5/2) - 6*b*Log[x] + 3*b*Log[a + x*(b + c*x)])/(2*a
^4)
```

Maple [A]
time = 0.06, size = 404, normalized size = 1.69

method	result
default	$-\frac{\frac{\frac{c^2 a \left(14a^2c^2-13a\,b^2c+2b^4\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{abc\left(74a^2c^2-55a\,b^2c+8b^4\right)x^2}{32a^2c^2-16a\,b^2c+2b^4}+\frac{a\left(18a^3c^3+7a^2b^2c^2-12a\,b^4c+2b^6\right)x}{16a^2c^2-8a\,b^2c+b^4}+\frac{a^2b\left(58a^2c^2-36a\,b^2c+5b^4\right)}{32a^2c^2-16a\,b^2c+2b^4}}{\left(c\,x^2+bx+a\right)^2}+\frac{3\left(-16a^2\right.}{}}{a^4}$
risch	$\frac{-\frac{3c^2\left(10a^2c^2-7a\,b^2c+b^4\right)x^4}{a^3\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3bc\left(46a^2c^2-29a\,b^2c+4b^4\right)x^3}{2\left(16a^2c^2-8a\,b^2c+b^4\right)a^3}-\frac{\left(50a^3c^3+7a^2b^2c^2-18a\,b^4c+3b^6\right)x^2}{a^3\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{b\left(122a^2c^2-68a\,b^2c+9b^4\right)x}{2a^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{1}{a}}{x(c\,x^2+bx+a)^2}-\frac{3b\ln(}{a^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^2+b/x)^3/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^4*((c^2*a*(14*a^2*c^2-13*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3
+1/2*a*b*c*(74*a^2*c^2-55*a*b^2*c+8*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+a*(
18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/2
*a^2*b*(58*a^2*c^2-36*a*b^2*c+5*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^2+b*x
+a)^2+3/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c)/c
*ln(c*x^2+b*x+a)+2*(10*a^3*c^3-23*a^2*b^2*c^2+9*a*b^4*c-b^6-1/2*(-16*a^2*b*
c^3+8*a*b^3*c^2-b^5*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2))))-1/a^3/x-3*b*ln(x)/a^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. 2(229) = 458.
time = 0.76, size = 2280, normalized size = 9.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="fricas")

[Out] [-1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*x + a) + 6*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x), -1/2*(2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^4 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*x^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*x^2 + 6*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^5 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^4 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*x - 3*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 64*a^5*b*c^3)*x)*log(c*x^2 + b*x + a) + 6*((b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^5 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c^3 - 64*a^3*b^2*c^4)*x^4 + (b^9 - 10*a*b^7*c + 24*a^2*b^5*c^2

```
 + 32*a^3*b^3*c^3 - 128*a^4*b*c^4)*x^3 + 2*(a*b^8 - 12*a^2*b^6*c + 48*a^3*b
^4*c^2 - 64*a^4*b^2*c^3)*x^2 + (a^2*b^7 - 12*a^3*b^5*c + 48*a^4*b^3*c^2 - 6
4*a^5*b*c^3)*x)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 6
4*a^7*c^5)*x^5 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48*a^6*b^3*c^3 - 64*a^7*b*
c^4)*x^4 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2 + 32*a^7*b^2*c^3 - 128*
a^8*c^4)*x^3 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b^3*c^2 - 64*a^8*b*c^3)*x
^2 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**2+b/x)**3/x**8,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.15, size = 309, normalized size = 1.29

$$\frac{3\,(b^6-10\,ab^4c+30\,a^2b^2c^2-20\,a^3c^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^4b^4-8\,a^5b^2c+16\,a^6c^2)\sqrt{-b^2+4\,ac}}+\frac{3\,b\log\left(cx^2+bx+a\right)}{2\,a^4}-\frac{3\,b\log\left(|x|\right)}{a^4}-\frac{2\,a^3b^4-16\,a^4b^2c+32\,a^5c^2+6\,(ab^4c^2-7\,a^2b^2c^3+10\,a^3c^4)x^4+3\,(4\,ab^5c-29\,a^2b^3c^2+46\,a^3bc^3)x^3+2\,(3\,ab^6-18\,a^2b^4c+7\,a^3b^2c^2+50\,a^4c^3)x^2+(9\,a^2b^5-68\,a^3b^3c+122\,a^4bc^2)x}{2\,(cx^2+bx+a)^2(b^2-4\,ac)^2a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^2+b/x)^3/x^8,x, algorithm="giac")
```

```
[Out] 3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*x + b)/sqrt(
-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) +
3/2*b*log(c*x^2 + b*x + a)/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(2*a^3*b^4 - 16*
a^4*b^2*c + 32*a^5*c^2 + 6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*x^4 + 3
*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*x^3 + 2*(3*a*b^6 - 18*a^2*b^4*
c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*x^2 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b
*c^2)*x)/((c*x^2 + b*x + a)^2*(b^2 - 4*a*c)^2*a^4*x)
```

Mupad [**B**]
time = 2.55, size = 1255, normalized size = 5.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(c + a/x^2 + b/x)^3),x)
```

```
[Out] - (1/a + (x^2*(3*b^6 + 50*a^3*c^3 + 7*a^2*b^2*c^2 - 18*a*b^4*c))/(a^3*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c)) + (x*(9*b^5 + 122*a^2*b*c^2 - 68*a*b^3*c))/(2*a^
2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*x^3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2
*b*c^3))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^2*x^4*(b^4 + 10*a^2*
```

c^2 - 7*a*b^2*c))/(a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^3*(2*a*c + b^2) + a^2*x + c^2*x^5 + 2*a*b*x^2 + 2*b*c*x^4) - (3*b*log(x))/a^4 - (3*log(2*a*b^11 + 2*b^12*x + 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c - 1696*a^6*b*c^5 + 320*a^6*c^6*x + 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a^3*b^7*c^2 - 1170*a^4*b^5*c^3 + 2240*a^5*b^3*c^4 - 10*a^4*c^3*(-(4*a*c - b^2)^5)^(1/2) - 17*a^2*b^4*c*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^8*c^2*x - 1296*a^3*b^6*c^3*x + 2660*a^4*b^4*c^4*x - 2336*a^5*b^2*c^5*x - 40*a*b^10*c*x + 39*a^3*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 20*a*b^5*c*x*(-(4*a*c - b^2)^5)^(1/2) - 58*a^3*b*c^3*x*(-(4*a*c - b^2)^5)^(1/2) + 63*a^2*b^3*c^2*x*(-(4*a*c - b^2)^5)^(1/2))*(b^11 + b^6*(-(4*a*c - b^2)^5)^(1/2) - 1024*a^5*b*c^5 + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 - 20*a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 20*a*b^9*c + 30*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 10*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^4*(4*a*c - b^2)^5) - (3*log(2*a*b^11 + 2*b^12*x - 2*a*b^6*(-(4*a*c - b^2)^5)^(1/2) - 39*a^2*b^9*c - 1696*a^6*b*c^5 + 320*a^6*c^6*x - 2*b^7*x*(-(4*a*c - b^2)^5)^(1/2) + 303*a^3*b^7*c^2 - 1170*a^4*b^5*c^3 + 2240*a^5*b^3*c^4 + 10*a^4*c^3*(-(4*a*c - b^2)^5)^(1/2) + 17*a^2*b^4*c*(-(4*a*c - b^2)^5)^(1/2) + 321*a^2*b^8*c^2*x - 1296*a^3*b^6*c^3*x + 2660*a^4*b^4*c^4*x - 2336*a^5*b^2*c^5*x - 40*a*b^10*c*x - 39*a^3*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 20*a*b^5*c*x*(-(4*a*c - b^2)^5)^(1/2) + 58*a^3*b*c^3*x*(-(4*a*c - b^2)^5)^(1/2) - 63*a^2*b^3*c^2*x*(-(4*a*c - b^2)^5)^(1/2))*(b^11 - b^6*(-(4*a*c - b^2)^5)^(1/2) - 1024*a^5*b*c^5 + 160*a^2*b^7*c^2 - 640*a^3*b^5*c^3 + 1280*a^4*b^3*c^4 + 20*a^3*c^3*(-(4*a*c - b^2)^5)^(1/2) - 20*a*b^9*c - 30*a^2*b^2*c^2*(-(4*a*c - b^2)^5)^(1/2) + 10*a*b^4*c*(-(4*a*c - b^2)^5)^(1/2)))/(2*a^4*(4*a*c - b^2)^5)

3.440 $\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$

Optimal. Leaf size=40

$$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

```
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1368, 715, 646, 31}

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}-\frac{16}{567}\log(3x+2)+\frac{\log(5x+1)}{4375}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
```

```
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx &= \int \frac{x^4}{2+13x+15x^2}\,dx\\
&= \int \left(\frac{139}{3375}-\frac{13x}{225}+\frac{x^2}{15}-\frac{278+1417x}{3375\,(2+13x+15x^2)}\right)dx\\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{\int \frac{278+1417x}{2+13x+15x^2}\,dx}{3375}\\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}+\frac{3}{875}\int \frac{1}{3+15x}\,dx-\frac{80}{189}\int \frac{1}{10+15x}\,dx\\
&= \frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 40, normalized size = 1.00

$$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16}{567}\log(2+3x)+\frac{\log(1+5x)}{4375}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(15 + 2/x^2 + 13/x),x]
[Out] (139*x)/3375 - (13*x^2)/450 + x^3/45 - (16*Log[2 + 3*x])/567 + Log[1 + 5*x]
/4375
```

Maple [A]
time = 0.02, size = 31, normalized size = 0.78

method	result	size
default	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(2+3x)}{567}+\frac{\ln(1+5x)}{4375}$	31
norman	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(2+3x)}{567}+\frac{\ln(1+5x)}{4375}$	31
risch	$\frac{139x}{3375}-\frac{13x^2}{450}+\frac{x^3}{45}-\frac{16\ln(2+3x)}{567}+\frac{\ln(1+5x)}{4375}$	31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(15+2/x^2+13/x),x,method=_RETURNVERBOSE)
[Out] 139/3375*x-13/450*x^2+1/45*x^3-16/567*ln(2+3*x)+1/4375*ln(1+5*x)
```

Maxima [A]
time = 0.32, size = 30, normalized size = 0.75

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="maxima")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Fricas [A]
time = 0.39, size = 30, normalized size = 0.75

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(5\,x+1\right)-\frac{16}{567}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="fricas")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(5*x + 1) - 16/567*log(3*x +
 2)
```

Sympy [A]
time = 0.05, size = 34, normalized size = 0.85

$$\frac{x^3}{45}-\frac{13x^2}{450}+\frac{139x}{3375}+\frac{\log\left(x+\frac{1}{5}\right)}{4375}-\frac{16\log\left(x+\frac{2}{3}\right)}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(15+2/x**2+13/x),x)
```

```
[Out] x**3/45 - 13*x**2/450 + 139*x/3375 + log(x + 1/5)/4375 - 16*log(x + 2/3)/56
7
```

Giac [A]
time = 2.84, size = 32, normalized size = 0.80

$$\frac{1}{45}\,x^3-\frac{13}{450}\,x^2+\frac{139}{3375}\,x+\frac{1}{4375}\,\log\left(|5\,x+1|\right)-\frac{16}{567}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(15+2/x^2+13/x),x, algorithm="giac")
```

```
[Out] 1/45*x^3 - 13/450*x^2 + 139/3375*x + 1/4375*log(abs(5*x + 1)) - 16/567*log(
abs(3*x + 2))
```

Mupad [B]
time = 0.05, size = 26, normalized size = 0.65

$$\frac{139\,x}{3375}-\frac{16\,\ln\left(x+\frac{2}{3}\right)}{567}+\frac{\ln\left(x+\frac{1}{5}\right)}{4375}-\frac{13\,x^2}{450}+\frac{x^3}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(13/x + 2/x^2 + 15),x)
```

```
[Out] (139*x)/3375 - (16*log(x + 2/3))/567 + log(x + 1/5)/4375 - (13*x^2)/450 + x
^3/45
```

3.441 $\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}} \, dx$

Optimal. Leaf size=33

$$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

```
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1368, 715, 646, 31}

$$\frac{x^2}{30}-\frac{13x}{225}+\frac{8}{189}\log(3x+2)-\frac{1}{875}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(15 + 2/x^2 + 13/x),x]
```

```
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
```

```
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx &= \int \frac{x^3}{2+13x+15x^2}\,dx\\
&= \int \left(-\frac{13}{225}+\frac{x}{15}+\frac{26+139x}{225\,(2+13x+15x^2)}\right)dx\\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{1}{225}\int \frac{26+139x}{2+13x+15x^2}\,dx\\
&= -\frac{13x}{225}+\frac{x^2}{30}-\frac{3}{175}\int \frac{1}{3+15x}\,dx+\frac{40}{63}\int \frac{1}{10+15x}\,dx\\
&= -\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 1.00

$$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8}{189}\log(2+3x)-\frac{1}{875}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(15 + 2/x^2 + 13/x),x]
[Out] (-13*x)/225 + x^2/30 + (8*Log[2 + 3*x])/189 - Log[1 + 5*x]/875
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.79

method	result	size
default	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(2+3x)}{189}-\frac{\ln(1+5x)}{875}$	26
norman	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(2+3x)}{189}-\frac{\ln(1+5x)}{875}$	26
risch	$-\frac{13x}{225}+\frac{x^2}{30}+\frac{8\ln(2+3x)}{189}-\frac{\ln(1+5x)}{875}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(15+2/x^2+13/x),x,method=_RETURNVERBOSE)
[Out] -13/225*x+1/30*x^2+8/189*ln(2+3*x)-1/875*ln(1+5*x)
```

Maxima [A]
time = 0.31, size = 25, normalized size = 0.76

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="maxima")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Fricas [A]
time = 0.36, size = 25, normalized size = 0.76

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(5\,x+1\right)+\frac{8}{189}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="fricas")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(5*x + 1) + 8/189*log(3*x + 2)
```

Sympy [A]
time = 0.04, size = 27, normalized size = 0.82

$$\frac{x^2}{30}-\frac{13x}{225}-\frac{\log\left(x+\frac{1}{5}\right)}{875}+\frac{8\log\left(x+\frac{2}{3}\right)}{189}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x**2+13/x),x)
[Out] x**2/30 - 13*x/225 - log(x + 1/5)/875 + 8*log(x + 2/3)/189
```

Giac [A]
time = 4.19, size = 27, normalized size = 0.82

$$\frac{1}{30}\,x^2-\frac{13}{225}\,x-\frac{1}{875}\,\log\left(|5\,x+1|\right)+\frac{8}{189}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(15+2/x^2+13/x),x, algorithm="giac")
[Out] 1/30*x^2 - 13/225*x - 1/875*log(abs(5*x + 1)) + 8/189*log(abs(3*x + 2))
```

Mupad [B]
time = 1.31, size = 21, normalized size = 0.64

$$\frac{8\,\ln\left(x+\frac{2}{3}\right)}{189}-\frac{13\,x}{225}-\frac{\ln\left(x+\frac{1}{5}\right)}{875}+\frac{x^2}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(13/x + 2/x^2 + 15),x)
[Out] (8*log(x + 2/3))/189 - (13*x)/225 - log(x + 1/5)/875 + x^2/30
```

3.442 $\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx$

Optimal. Leaf size=26

$$\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

```
[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {1354, 717, 646, 31}

$$\frac{x}{15}-\frac{4}{63}\log(3x+2)+\frac{1}{175}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(15 + 2/x^2 + 13/x)^(-1),x]
```

```
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
```

```
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{15+\frac{2}{x^2}+\frac{13}{x}}\,dx &= \int \frac{x^2}{2+13x+15x^2}\,dx\\ &= \frac{x}{15}+\frac{1}{15}\int \frac{-2-13x}{2+13x+15x^2}\,dx\\ &= \frac{x}{15}+\frac{3}{35}\int \frac{1}{3+15x}\,dx-\frac{20}{21}\int \frac{1}{10+15x}\,dx\\ &= \frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 26, normalized size = 1.00

$$\frac{x}{15}-\frac{4}{63}\log(2+3x)+\frac{1}{175}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(15 + 2/x^2 + 13/x)^(-1),x]
[Out] x/15 - (4*Log[2 + 3*x])/63 + Log[1 + 5*x]/175
```

Maple [A]
time = 0.02, size = 21, normalized size = 0.81

method	result	size
default	$\frac{x}{15}-\frac{4\ln(2+3x)}{63}+\frac{\ln(1+5x)}{175}$	21
norman	$\frac{x}{15}-\frac{4\ln(2+3x)}{63}+\frac{\ln(1+5x)}{175}$	21
risch	$\frac{x}{15}-\frac{4\ln(2+3x)}{63}+\frac{\ln(1+5x)}{175}$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x),x,method=_RETURNVERBOSE)
[Out] 1/15*x-4/63*ln(2+3*x)+1/175*ln(1+5*x)
```

Maxima [A]
time = 0.30, size = 20, normalized size = 0.77

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="maxima")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Fricas [A]
time = 0.36, size = 20, normalized size = 0.77

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(5\,x+1\right)-\frac{4}{63}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="fricas")
[Out] 1/15*x + 1/175*log(5*x + 1) - 4/63*log(3*x + 2)
```

Sympy [A]
time = 0.04, size = 20, normalized size = 0.77

$$\frac{x}{15}+\frac{\log\left(x+\frac{1}{5}\right)}{175}-\frac{4\log\left(x+\frac{2}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x),x)
[Out] x/15 + log(x + 1/5)/175 - 4*log(x + 2/3)/63
```

Giac [A]
time = 4.82, size = 22, normalized size = 0.85

$$\frac{1}{15}\,x+\frac{1}{175}\,\log\left(|5\,x+1|\right)-\frac{4}{63}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x),x, algorithm="giac")
[Out] 1/15*x + 1/175*log(abs(5*x + 1)) - 4/63*log(abs(3*x + 2))
```

Mupad [B]
time = 0.08, size = 16, normalized size = 0.62

$$\frac{x}{15}-\frac{4\,\ln\left(x+\frac{2}{3}\right)}{63}+\frac{\ln\left(x+\frac{1}{5}\right)}{175}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(13/x + 2/x^2 + 15),x)
[Out] x/15 - (4*log(x + 2/3))/63 + log(x + 1/5)/175
```

3.443 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx$

Optimal. Leaf size=21

$$\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1368, 646, 31}

$$\frac{2}{21}\log(3x+2)-\frac{1}{35}\log(5x+1)$$

Antiderivative was successfully verified.

[In] Int[1/((15 + 2/x^2 + 13/x)*x),x]

[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x}\,dx=\int \frac{x}{2+13x+15x^2}\,dx$$
$$=-\left(\frac{3}{7}\int \frac{1}{3+15x}\,dx\right)+\frac{10}{7}\int \frac{1}{10+15x}\,dx$$
$$=\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\frac{2}{21}\log(2+3x)-\frac{1}{35}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x),x]
```

```
[Out] (2*Log[2 + 3*x])/21 - Log[1 + 5*x]/35
```

Maple [A]
time = 0.01, size = 18, normalized size = 0.86

method	result	size
default	$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$	18
norman	$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$	18
risch	$\frac{2\ln(2+3x)}{21}-\frac{\ln(1+5x)}{35}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2/21*ln(2+3*x)-1/35*ln(1+5*x)
```

Maxima [A]
time = 0.31, size = 17, normalized size = 0.81

$$-\frac{1}{35}\log\left(5x+1\right)+\frac{2}{21}\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="maxima")
```

```
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Fricas [A]
time = 0.34, size = 17, normalized size = 0.81

$$-\frac{1}{35}\,\log\left(5\,x+1\right)+\frac{2}{21}\,\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="fricas")
[Out] -1/35*log(5*x + 1) + 2/21*log(3*x + 2)
```

Sympy [A]
time = 0.03, size = 17, normalized size = 0.81

$$-\frac{\log\left(x+\frac{1}{5}\right)}{35}+\frac{2\log\left(x+\frac{2}{3}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x,x)
[Out] -log(x + 1/5)/35 + 2*log(x + 2/3)/21
```

Giac [A]
time = 4.16, size = 19, normalized size = 0.90

$$-\frac{1}{35}\,\log\left(|5\,x+1|\right)+\frac{2}{21}\,\log\left(|3\,x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x,x, algorithm="giac")
[Out] -1/35*log(abs(5*x + 1)) + 2/21*log(abs(3*x + 2))
```

Mupad [B]
time = 0.07, size = 13, normalized size = 0.62

$$\frac{2\,\ln\left(x+\frac{2}{3}\right)}{21}-\frac{\ln\left(x+\frac{1}{5}\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(13/x + 2/x^2 + 15)),x)
[Out] (2*log(x + 2/3))/21 - log(x + 1/5)/35
```

3.444 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{7}\log\left(5+\frac{1}{x}\right)-\frac{1}{7}\log\left(3+\frac{2}{x}\right)$$

```
[Out] 1/7*ln(5+1/x)-1/7*ln(3+2/x)
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1366, 630, 31}

$$\frac{1}{7}\log\left(\frac{1}{x}+5\right)-\frac{1}{7}\log\left(\frac{2}{x}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

```
[Out] Log[5 + x^(-1)]/7 - Log[3 + 2/x]/7
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 630

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^2}\,dx = -\text{Subst}\left(\int \frac{1}{15+13x+2x^2}\,dx, x, \frac{1}{x}\right)$$
$$= -\left(\frac{2}{7}\text{Subst}\left(\int \frac{1}{3+2x}\,dx, x, \frac{1}{x}\right)\right) + \frac{2}{7}\text{Subst}\left(\int \frac{1}{10+2x}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{1}{7}\log\left(5+\frac{1}{x}\right) - \frac{1}{7}\log\left(3+\frac{2}{x}\right)$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 0.91

$$-\frac{1}{7}\log(2+3x)+\frac{1}{7}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^2),x]
```

```
[Out] -1/7*Log[2 + 3*x] + Log[1 + 5*x]/7
```

Maple [A]
time = 0.03, size = 18, normalized size = 0.78

method	result	size
default	$\frac{\ln(1+5x)}{7}-\frac{\ln(2+3x)}{7}$	18
norman	$\frac{\ln(1+5x)}{7}-\frac{\ln(2+3x)}{7}$	18
risch	$\frac{\ln(1+5x)}{7}-\frac{\ln(2+3x)}{7}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*ln(1+5*x)-1/7*ln(2+3*x)
```

Maxima [A]
time = 0.29, size = 17, normalized size = 0.74

$$\frac{1}{7}\log\left(5\,x+1\right)-\frac{1}{7}\log\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Fricas [A]
time = 0.35, size = 17, normalized size = 0.74

$$\frac{1}{7} \log (5x+1) - \frac{1}{7} \log (3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="fricas")
[Out] 1/7*log(5*x + 1) - 1/7*log(3*x + 2)
```

Sympy [A]
time = 0.03, size = 15, normalized size = 0.65

$$\frac{\log \left(x+\frac{1}{5}\right)}{7} - \frac{\log \left(x+\frac{2}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**2,x)
[Out] log(x + 1/5)/7 - log(x + 2/3)/7
```

Giac [A]
time = 3.77, size = 19, normalized size = 0.83

$$\frac{1}{7} \log (|5x+1|) - \frac{1}{7} \log (|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^2,x, algorithm="giac")
[Out] 1/7*log(abs(5*x + 1)) - 1/7*log(abs(3*x + 2))
```

Mupad [B]
time = 1.37, size = 8, normalized size = 0.35

$$-\frac{2 \operatorname{atanh}\left(\frac{30x}{7}+\frac{13}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(13/x + 2/x^2 + 15)),x)
[Out] -(2*atanh((30*x)/7 + 13/7))/7
```

3.445 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx$

Optimal. Leaf size=27

$$\frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)$$

```
[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1368, 719, 29, 646, 31}

$$\frac{\log(x)}{2}+\frac{3}{14}\log(3x+2)-\frac{5}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^3),x]
```

```
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^3}\,dx &= \int \frac{1}{x\left(2+13x+15x^2\right)}\,dx\\ &= \frac{1}{2}\int \frac{1}{x}\,dx+\frac{1}{2}\int \frac{-13-15x}{2+13x+15x^2}\,dx\\ &= \frac{\log(x)}{2}+\frac{45}{14}\int \frac{1}{10+15x}\,dx-\frac{75}{7}\int \frac{1}{3+15x}\,dx\\ &= \frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$\frac{\log(x)}{2}+\frac{3}{14}\log(2+3x)-\frac{5}{7}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^3),x]
[Out] Log[x]/2 + (3*Log[2 + 3*x])/14 - (5*Log[1 + 5*x])/7
```

Maple [A]
time = 0.02, size = 22, normalized size = 0.81

method	result	size
default	$\frac{\ln(x)}{2}+\frac{3\ln(2+3x)}{14}-\frac{5\ln(1+5x)}{7}$	22
norman	$\frac{\ln(x)}{2}+\frac{3\ln(2+3x)}{14}-\frac{5\ln(1+5x)}{7}$	22
risch	$\frac{\ln(x)}{2}+\frac{3\ln(2+3x)}{14}-\frac{5\ln(1+5x)}{7}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^3,x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x)+3/14*ln(2+3*x)-5/7*ln(1+5*x)
```

Maxima [A]
time = 0.30, size = 21, normalized size = 0.78

$$-\frac{5}{7}\log(5x+1)+\frac{3}{14}\log(3x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="maxima")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Fricas [A]
time = 0.39, size = 21, normalized size = 0.78

$$-\frac{5}{7}\log(5x+1)+\frac{3}{14}\log(3x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="fricas")
[Out] -5/7*log(5*x + 1) + 3/14*log(3*x + 2) + 1/2*log(x)
```

Sympy [A]
time = 0.05, size = 24, normalized size = 0.89

$$\frac{\log(x)}{2}-\frac{5\log\left(x+\frac{1}{5}\right)}{7}+\frac{3\log\left(x+\frac{2}{3}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**3,x)
[Out] log(x)/2 - 5*log(x + 1/5)/7 + 3*log(x + 2/3)/14
```

Giac [A]
time = 3.83, size = 24, normalized size = 0.89

$$-\frac{5}{7}\log(|5x+1|)+\frac{3}{14}\log(|3x+2|)+\frac{1}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^3,x, algorithm="giac")
[Out] -5/7*log(abs(5*x + 1)) + 3/14*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

Mupad [B]
time = 1.39, size = 17, normalized size = 0.63

$$\frac{3\ln\left(x+\frac{2}{3}\right)}{14}-\frac{5\ln\left(x+\frac{1}{5}\right)}{7}+\frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(13/x + 2/x^2 + 15)),x)
[Out] (3*log(x + 2/3))/14 - (5*log(x + 1/5))/7 + log(x)/2
```

3.446 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

```
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1368, 723, 814}

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(3x+2)+\frac{25}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^4),x]
```

```
[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^4}\,dx = \int \frac{1}{x^2\left(2+13x+15x^2\right)}\,dx$$
$$= -\frac{1}{2x}+\frac{1}{2}\int \frac{-13-15x}{x\left(2+13x+15x^2\right)}\,dx$$
$$= -\frac{1}{2x}+\frac{1}{2}\int \left(-\frac{13}{2x}-\frac{27}{14(2+3x)}+\frac{250}{7(1+5x)}\right)dx$$
$$= -\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

Mathematica [A]
time = 0.00, size = 34, normalized size = 1.00

$$-\frac{1}{2x}-\frac{13\log(x)}{4}-\frac{9}{28}\log(2+3x)+\frac{25}{7}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^4),x]
[Out] -1/2*1/x - (13*Log[x])/4 - (9*Log[2 + 3*x])/28 + (25*Log[1 + 5*x])/7
```

Maple [A]
time = 0.02, size = 27, normalized size = 0.79

method	result	size
default	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(2+3x)}{28}+\frac{25\ln(1+5x)}{7}$	27
norman	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(2+3x)}{28}+\frac{25\ln(1+5x)}{7}$	27
risch	$-\frac{1}{2x}-\frac{13\ln(x)}{4}-\frac{9\ln(2+3x)}{28}+\frac{25\ln(1+5x)}{7}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^4,x,method=_RETURNVERBOSE)
[Out] -1/2/x-13/4*ln(x)-9/28*ln(2+3*x)+25/7*ln(1+5*x)
```

Maxima [A]
time = 0.30, size = 26, normalized size = 0.76

$$-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(5\,x+1\right)-\frac{9}{28}\,\log\left(3\,x+2\right)-\frac{13}{4}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="maxima")
```

```
[Out] -1/2/x + 25/7*log(5*x + 1) - 9/28*log(3*x + 2) - 13/4*log(x)
```

Fricas [A]
time = 0.35, size = 30, normalized size = 0.88

$$\frac{100\,x\log\left(5\,x+1\right)-9\,x\log\left(3\,x+2\right)-91\,x\log\left(x\right)-14}{28\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="fricas")
[Out] 1/28*(100*x*log(5*x + 1) - 9*x*log(3*x + 2) - 91*x*log(x) - 14)/x
```

Sympy [A]
time = 0.06, size = 31, normalized size = 0.91

$$-\frac{13\log\left(x\right)}{4}+\frac{25\log\left(x+\frac{1}{5}\right)}{7}-\frac{9\log\left(x+\frac{2}{3}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**4,x)
[Out] -13*log(x)/4 + 25*log(x + 1/5)/7 - 9*log(x + 2/3)/28 - 1/(2*x)
```

Giac [A]
time = 4.21, size = 29, normalized size = 0.85

$$-\frac{1}{2\,x}+\frac{25}{7}\,\log\left(|5\,x+1|\right)-\frac{9}{28}\,\log\left(|3\,x+2|\right)-\frac{13}{4}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^4,x, algorithm="giac")
[Out] -1/2/x + 25/7*log(abs(5*x + 1)) - 9/28*log(abs(3*x + 2)) - 13/4*log(abs(x))
```

Mupad [B]
time = 0.04, size = 22, normalized size = 0.65

$$\frac{25\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{9\,\ln\left(x+\frac{2}{3}\right)}{28}-\frac{13\,\ln\left(x\right)}{4}-\frac{1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(13/x + 2/x^2 + 15)),x)
[Out] (25*log(x + 1/5))/7 - (9*log(x + 2/3))/28 - (13*log(x))/4 - 1/(2*x)
```

3.447 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1368, 723, 814}

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(3x+2)-\frac{125}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

```
[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^5}\,dx = \int \frac{1}{x^3\left(2+13x+15x^2\right)}\,dx$$
$$= -\frac{1}{4x^2}+\frac{1}{2}\int \frac{-13-15x}{x^2\left(2+13x+15x^2\right)}\,dx$$
$$= -\frac{1}{4x^2}+\frac{1}{2}\int \left(-\frac{13}{2x^2}+\frac{139}{4x}+\frac{81}{28(2+3x)}-\frac{1250}{7(1+5x)}\right)dx$$
$$= -\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

Mathematica [A]
time = 0.00, size = 41, normalized size = 1.00

$$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\log(x)}{8}+\frac{27}{56}\log(2+3x)-\frac{125}{7}\log(1+5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^5),x]
```

```
[Out] -1/4*1/x^2 + 13/(4*x) + (139*Log[x])/8 + (27*Log[2 + 3*x])/56 - (125*Log[1
+ 5*x])/7
```

Maple [A]
time = 0.02, size = 32, normalized size = 0.78

method	result	size
risch	$\frac{\frac{13x}{4}-\frac{1}{4}}{x^2}+\frac{139\ln(x)}{8}+\frac{27\ln(2+3x)}{56}-\frac{125\ln(1+5x)}{7}$	31
default	$-\frac{1}{4x^2}+\frac{13}{4x}+\frac{139\ln(x)}{8}+\frac{27\ln(2+3x)}{56}-\frac{125\ln(1+5x)}{7}$	32
norman	$\frac{-\frac{1}{4}x^2+\frac{13}{4}x^3}{x^4}+\frac{139\ln(x)}{8}+\frac{27\ln(2+3x)}{56}-\frac{125\ln(1+5x)}{7}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/x^2+13/4/x+139/8*ln(x)+27/56*ln(2+3*x)-125/7*ln(1+5*x)
```

Maxima [A]
time = 0.30, size = 31, normalized size = 0.76

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\log\left(5\,x+1\right)+\frac{27}{56}\,\log\left(3\,x+2\right)+\frac{139}{8}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(5*x + 1) + 27/56*log(3*x + 2) + 139/8*log(x)
```

Fricas [A]
time = 0.33, size = 39, normalized size = 0.95

$$-\frac{1000\,x^2\log\left(5\,x+1\right)-27\,x^2\log\left(3\,x+2\right)-973\,x^2\log\left(x\right)-182\,x+14}{56\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/56*(1000*x^2*log(5*x + 1) - 27*x^2*log(3*x + 2) - 973*x^2*log(x) - 182*x
 + 14)/x^2
```

Sympy [A]
time = 0.06, size = 36, normalized size = 0.88

$$\frac{139\log\left(x\right)}{8}-\frac{125\log\left(x+\frac{1}{5}\right)}{7}+\frac{27\log\left(x+\frac{2}{3}\right)}{56}+\frac{13x-1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**5,x)
```

```
[Out] 139*log(x)/8 - 125*log(x + 1/5)/7 + 27*log(x + 2/3)/56 + (13*x - 1)/(4*x**2
)
```

Giac [A]
time = 3.25, size = 34, normalized size = 0.83

$$\frac{13\,x-1}{4\,x^2}-\frac{125}{7}\,\log\left(\left|5\,x+1\right|\right)+\frac{27}{56}\,\log\left(\left|3\,x+2\right|\right)+\frac{139}{8}\,\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*(13*x - 1)/x^2 - 125/7*log(abs(5*x + 1)) + 27/56*log(abs(3*x + 2)) + 13
9/8*log(abs(x))
```

Mupad [B]
time = 1.31, size = 26, normalized size = 0.63

$$\frac{27\,\ln\left(x+\frac{2}{3}\right)}{56}-\frac{125\,\ln\left(x+\frac{1}{5}\right)}{7}+\frac{139\,\ln\left(x\right)}{8}+\frac{\frac{13\,x}{4}-\frac{1}{4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(13/x + 2/x^2 + 15)),x)
```

```
[Out] (27*log(x + 2/3))/56 - (125*log(x + 1/5))/7 + (139*log(x))/8 + ((13*x)/4 -
1/4)/x^2
```

3.448 $\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)$$

```
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Rubi [A]
time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1368, 723, 814}

$$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(3x+2)+\frac{625}{7}\log(5x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((15 + 2/x^2 + 13/x)*x^6),x]
```

```
[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1368

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Int[x^(m + 2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n
}, x] && EqQ[n2, 2*n] && ILtQ[p, 0] && NegQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(15+\frac{2}{x^2}+\frac{13}{x}\right)x^6}\,dx &= \int \frac{1}{x^4\left(2+13x+15x^2\right)}\,dx \\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \frac{-13-15x}{x^3\left(2+13x+15x^2\right)}\,dx \\
&= -\frac{1}{6x^3}+\frac{1}{2}\int \left(-\frac{13}{2x^3}+\frac{139}{4x^2}-\frac{1417}{8x}-\frac{243}{56(2+3x)}+\frac{6250}{7(1+5x)}\right)dx \\
&= -\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 48, normalized size = 1.00

$$
-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\log(x)}{16}-\frac{81}{112}\log(2+3x)+\frac{625}{7}\log(1+5x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((15 + 2/x^2 + 13/x)*x^6),x]
[Out] -1/6*1/x^3 + 13/(8*x^2) - 139/(8*x) - (1417*Log[x])/16 - (81*Log[2 + 3*x])/
112 + (625*Log[1 + 5*x])/7
```

Maple [A]
time = 0.02, size = 37, normalized size = 0.77

method	result	size
risch	$\frac{-\frac{139}{8}x^2+\frac{13}{8}x-\frac{1}{6}}{x^3}-\frac{1417\ln(x)}{16}-\frac{81\ln(2+3x)}{112}+\frac{625\ln(1+5x)}{7}$	36
default	$-\frac{1}{6x^3}+\frac{13}{8x^2}-\frac{139}{8x}-\frac{1417\ln(x)}{16}-\frac{81\ln(2+3x)}{112}+\frac{625\ln(1+5x)}{7}$	37
norman	$\frac{-\frac{1}{6}x^2+\frac{13}{8}x^3-\frac{139}{8}x^4}{x^5}-\frac{1417\ln(x)}{16}-\frac{81\ln(2+3x)}{112}+\frac{625\ln(1+5x)}{7}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(15+2/x^2+13/x)/x^6,x,method=_RETURNVERBOSE)
[Out] -1/6/x^3+13/8/x^2-139/8/x-1417/16*ln(x)-81/112*ln(2+3*x)+625/7*ln(1+5*x)
```

Maxima [A]
time = 0.30, size = 36, normalized size = 0.75

$$
-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(5\,x+1\right)-\frac{81}{112}\,\log\left(3\,x+2\right)-\frac{1417}{16}\,\log\left(x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(5*x + 1) - 81/112*log(3*x + 2) -
 1417/16*log(x)
```

Fricas [A]
time = 0.35, size = 44, normalized size = 0.92

$$\frac{30000\,x^3\log\left(5\,x+1\right)-243\,x^3\log\left(3\,x+2\right)-29757\,x^3\log\left(x\right)-5838\,x^2+546\,x-56}{336\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/336*(30000*x^3*log(5*x + 1) - 243*x^3*log(3*x + 2) - 29757*x^3*log(x) - 5
838*x^2 + 546*x - 56)/x^3
```

Sympy [A]
time = 0.07, size = 41, normalized size = 0.85

$$-\frac{1417\log\left(x\right)}{16}+\frac{625\log\left(x+\frac{1}{5}\right)}{7}-\frac{81\log\left(x+\frac{2}{3}\right)}{112}+\frac{-417x^2+39x-4}{24x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x**2+13/x)/x**6,x)
```

```
[Out] -1417*log(x)/16 + 625*log(x + 1/5)/7 - 81*log(x + 2/3)/112 + (-417*x**2 + 3
9*x - 4)/(24*x**3)
```

Giac [A]
time = 3.15, size = 39, normalized size = 0.81

$$-\frac{417\,x^2-39\,x+4}{24\,x^3}+\frac{625}{7}\,\log\left(\left|5\,x+1\right|\right)-\frac{81}{112}\,\log\left(\left|3\,x+2\right|\right)-\frac{1417}{16}\,\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(15+2/x^2+13/x)/x^6,x, algorithm="giac")
```

```
[Out] -1/24*(417*x^2 - 39*x + 4)/x^3 + 625/7*log(abs(5*x + 1)) - 81/112*log(abs(3
*x + 2)) - 1417/16*log(abs(x))
```

Mupad [B]
time = 0.05, size = 32, normalized size = 0.67

$$\frac{625\,\ln\left(x+\frac{1}{5}\right)}{7}-\frac{81\,\ln\left(x+\frac{2}{3}\right)}{112}-\frac{1417\,\ln\left(x\right)}{16}-\frac{\frac{139\,x^2}{8}-\frac{13\,x}{8}+\frac{1}{6}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(13/x + 2/x^2 + 15)),x)
```

```
[Out] (625*log(x + 1/5))/7 - (81*log(x + 2/3))/112 - (1417*log(x))/16 - ((139*x^2
)/8 - (13*x)/8 + 1/6)/x^3
```

3.449 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx$

Optimal. Leaf size=204

$$-\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right)-\frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right)}{64c}+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}x+\frac{5}{2}a^3$$

```
[Out] -5/24*(a+c/x^2+b/x)^(3/2)*(7*b+6*c/x)+(a+c/x^2+b/x)^(5/2)*x+5/2*a^(3/2)*b*a
rctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))+5/128*(-48*a^2*c^2-24*a*b
^2*c+b^4)*arctanh(1/2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))/c^(3/2)-5/64*(
b*(44*a*c+b^2)+2*c*(12*a*c+b^2)/x)*(a+c/x^2+b/x)^(1/2)/c
```

Rubi [A]
time = 0.16, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1356, 746, 828, 857, 635, 212, 738}

$$\frac{5}{2}a^{3/2}b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)+\frac{5(-48a^2c^2-24ab^2c+b^4)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{128c^{3/2}}-\frac{5\left(\frac{2c(12ac+b^2)}{x}+b(44ac+b^2)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{64c}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}-\frac{5}{24}\left(7b+\frac{6c}{x}\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(5/2),x]
```

```
[Out] (-5*(a + c/x^2 + b/x)^(3/2)*(7*b + (6*c)/x))/24 - (5*Sqrt[a + c/x^2 + b/x]*
(b*(b^2 + 44*a*c) + (2*c*(b^2 + 12*a*c))/x))/(64*c) + (a + c/x^2 + b/x)^(5/
2)*x + (5*a^(3/2)*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])
/2 + (5*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sq
rt[a + c/x^2 + b/x])])/(128*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} dx = -\text{Subst}\left(\int \frac{(a+bx+cx^2)^{5/2}}{x^2} dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} x - \frac{5}{2}\text{Subst}\left(\int \frac{(b+2cx)\,(a+bx+cx^2)^{3/2}}{x} dx, x, \frac{1}{x}\right)$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) + \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2} x + \frac{5\text{Subst}\left(\int \frac{(-8abc-c(b^2}{}\right.}{}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) - \frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right.}{64c}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) - \frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right.}{64c}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) - \frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right.}{64c}$$

$$= -\frac{5}{24}\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}\left(7b+\frac{6c}{x}\right) - \frac{5\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(b(b^2+44ac)+\frac{2c(b^2+12ac)}{x}\right.}{64c}$$

Mathematica [A]
time = 1.24, size = 208, normalized size = 1.02

$$-\frac{\sqrt{a+\frac{c+bx}{x^2}}\left(15(b^4-24ab^2c-48a^2c^2)\,x^4\tanh^{-1}\left(\frac{\sqrt{a}\,x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)+\sqrt{c}\left(\sqrt{c+x(b+ax)}\,(48c^3+15b^3x^3+8c^2x(17b+27ax)+2cx^2(59b^2+278abx-96a^2x^2))+480a^{3/2}bcx^4\log\left(b+2ax-2\sqrt{a}\,\sqrt{c+x(b+ax)}\right)\right)\right)}{192c^{3/2}x^3\sqrt{c+x(b+ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(5/2),x]
```

```
[Out] -1/192*(Sqrt[a + (c + b*x)/x^2]*(15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*x^4*Arc
Tanh[(Sqrt[a]*x - Sqrt[c + x*(b + a*x)])/Sqrt[c]] + Sqrt[c]*(Sqrt[c + x*(b
```

```
+ a*x)]*(48*c^3 + 15*b^3*x^3 + 8*c^2*x*(17*b + 27*a*x) + 2*c*x^2*(59*b^2 +
278*a*b*x - 96*a^2*x^2)) + 480*a^(3/2)*b*c*x^4*Log[b + 2*a*x - 2*Sqrt[a]*Sq
rt[c + x*(b + a*x)]])))/(c^(3/2)*x^3*Sqrt[c + x*(b + a*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(174) = 348$.
time = 0.06, size = 701, normalized size = 3.44

method	result
risch	$-\frac{(556abc\,x^3+15b^3x^3+216a\,c^2x^2+118b^2c\,x^2+136b\,c^2x+48c^3)\sqrt{\frac{a\,x^2+bx+c}{x^2}}}{192x^3c}+\frac{\left(a^2\sqrt{a\,x^2+bx+c}+\frac{5a^{\frac{3}{2}}b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\ldots}\right)}{2}\ldots\right.}{\ldots}$
default	$\frac{\left(\frac{a\,x^2+bx+c}{x^2}\right)^{\frac{5}{2}}x\left(-6a^{\frac{3}{2}}\left(a\,x^2+bx+c\right)^{\frac{5}{2}}b^4x^4-96\left(a\,x^2+bx+c\right)^{\frac{7}{2}}c^3a^{\frac{3}{2}}-360\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+bx+c}}{x}\right)a^{\frac{5}{2}}c^{\frac{7}{2}}b^2x^4+152a\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*((a*x^2+b*x+c)/x^2)^(5/2)*x*(-6*a^(3/2)*(a*x^2+b*x+c)^(5/2)*b^4*x^4-9
6*(a*x^2+b*x+c)^(7/2)*c^3*a^(3/2)-360*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(
1/2))/x)*a^(5/2)*c^(7/2)*b^2*x^4+152*a^(7/2)*(a*x^2+b*x+c)^(5/2)*b*c*x^5-15
2*a^(5/2)*(a*x^2+b*x+c)^(7/2)*b*c*x^3+148*a^(5/2)*(a*x^2+b*x+c)^(5/2)*b^2*c
*x^4+280*a^(7/2)*(a*x^2+b*x+c)^(3/2)*b*c^2*x^5-10*a^(5/2)*(a*x^2+b*x+c)^(3/
2)*b^3*c*x^5+6*a^(3/2)*(a*x^2+b*x+c)^(7/2)*b^3*x^3-144*a^(5/2)*(a*x^2+b*x+c
)^(7/2)*c^2*x^2+4*a^(3/2)*(a*x^2+b*x+c)^(7/2)*b^2*c*x^2+240*a^(7/2)*(a*x^2+
b*x+c)^(3/2)*c^3*x^4-10*a^(3/2)*(a*x^2+b*x+c)^(3/2)*b^4*c*x^4+16*a^(3/2)*(a
*x^2+b*x+c)^(7/2)*b*c^2*x+720*a^(7/2)*(a*x^2+b*x+c)^(1/2)*c^4*x^4-30*a^(3/2
)*(a*x^2+b*x+c)^(1/2)*b^4*c^2*x^4-6*a^(5/2)*(a*x^2+b*x+c)^(5/2)*b^3*x^5+144
*a^(7/2)*(a*x^2+b*x+c)^(5/2)*c^2*x^4-720*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c
)^(1/2))/x)*a^(7/2)*c^(9/2)*x^4+15*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2
))/x)*a^(3/2)*c^(5/2)*b^4*x^4+260*a^(5/2)*(a*x^2+b*x+c)^(3/2)*b^2*c^2*x^4+6
00*a^(7/2)*(a*x^2+b*x+c)^(1/2)*b*c^3*x^5-30*a^(5/2)*(a*x^2+b*x+c)^(1/2)*b^3
*c^2*x^5+960*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b*
c^4*x^4+660*a^(5/2)*(a*x^2+b*x+c)^(1/2)*b^2*c^3*x^4)/(a*x^2+b*x+c)^(5/2)/c^
4/a^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

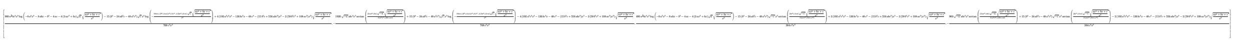
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(5/2), x)
```

Fricas [A]
time = 0.48, size = 959, normalized size = 4.70



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(960*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2
*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) - 15*(b^4 - 24*a*b^2*c -
 48*a^2*c^2)*sqrt(c)*x^3*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x
^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) + 4*(192*a^2*c^2*x^4
- 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108
*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/768*(1920*sqrt(-a)*
a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)
/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(c)*x^3*
log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt(
(a*x^2 + b*x + c)/x^2))/x^2) - 4*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 -
(15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2
+ b*x + c)/x^2))/(c^2*x^3), 1/384*(480*a^(3/2)*b*c^2*x^3*log(-8*a^2*x^2 - 8
*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2
)) - 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*
c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(192
*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (15*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*
b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c^2*x^3), -1/384*(9
60*sqrt(-a)*a*b*c^2*x^3*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b
*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 15*(b^4 - 24*a*b^2*c - 48*a^2*c^2)*
sqrt(-c)*x^3*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2
)/(a*c*x^2 + b*c*x + c^2)) - 2*(192*a^2*c^2*x^4 - 136*b*c^3*x - 48*c^4 - (1
5*b^3*c + 556*a*b*c^2)*x^3 - 2*(59*b^2*c^2 + 108*a*c^3)*x^2)*sqrt((a*x^2 +
b*x + c)/x^2))/(c^2*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x} + \frac{c}{x^2}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(5/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(5/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x + c/x^2)^(5/2),x)
```

```
[Out] int((a + b/x + c/x^2)^(5/2), x)
```

3.450 $\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx$

Optimal. Leaf size=145

$$-\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right)+\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}x+\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)-\frac{3(b^2+4ac)\,t}{}$$

```
[Out] (a+c/x^2+b/x)^(3/2)*x+3/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/
2))*a^(1/2)-3/8*(4*a*c+b^2)*arctanh(1/2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/
2))/c^(1/2)-3/4*(3*b+2*c/x)*(a+c/x^2+b/x)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {1356, 746, 828, 857, 635, 212, 738}

$$-\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{8\sqrt{c}}+x\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}-\frac{3}{4}\left(3b+\frac{2c}{x}\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{3}{2}\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (-3*Sqrt[a + c/x^2 + b/x]*(3*b + (2*c)/x))/4 + (a + c/x^2 + b/x)^(3/2)*x +
(3*Sqrt[a]*b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/2 - (3
*(b^2 + 4*a*c)*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x])])/(8
*Sqrt[c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} dx = -\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^2} dx, x, \frac{1}{x}\right)$$

$$= \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x - \frac{3}{2}\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x} dx, x, \frac{1}{x}\right)$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right) + \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + \frac{3\text{Subst}\left(\int \frac{-4abc-c(b^2+4ac}{x\sqrt{a+bx+c}}\right.}{8c}$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right) + \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3ab)\text{Subst}\left(\int \frac{}{x\sqrt{a+}}\right.$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right) + \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + (3ab)\text{Subst}\left(\int \frac{1}{4a-x^2}\right.$$

$$= -\frac{3}{4}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\left(3b+\frac{2c}{x}\right) + \left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2} x + \frac{3}{2}\sqrt{a}\, b \tanh^{-1}\left(\frac{2}{2\sqrt{a}\sqrt{}}\right.$$

Mathematica [A]
time = 0.54, size = 158, normalized size = 1.09

$$\frac{\sqrt{a+\frac{c+bx}{x^2}}\left(3(b^2+4ac)x^2\tanh^{-1}\left(\frac{\sqrt{a}\,x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)-\sqrt{c}\left((2c+x(5b-4ax))\sqrt{c+x(b+ax)}+6\sqrt{a}\,bx^2\log\left(b+2ax-2\sqrt{a}\,\sqrt{c+x(b+ax)}\right)\right)\right)}{4\sqrt{c}\,x\sqrt{c+x(b+ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(3/2),x]
```

```
[Out] (Sqrt[a + (c + b*x)/x^2]*(3*(b^2 + 4*a*c)*x^2*ArcTanh[(Sqrt[a]*x - Sqrt[c +
 x*(b + a*x)])/Sqrt[c]] - Sqrt[c]*((2*c + x*(5*b - 4*a*x))*Sqrt[c + x*(b +
a*x)] + 6*Sqrt[a]*b*x^2*Log[b + 2*a*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]])))
/(4*Sqrt[c]*x*Sqrt[c + x*(b + a*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(119) = 238.
time = 0.04, size = 334, normalized size = 2.30

method	result
risch	$-\frac{(5bx+2c)\sqrt{\frac{a\,x^2+bx+c}{x^2}}}{4x}+\frac{\left(a\sqrt{a\,x^2+bx+c}+\frac{3\sqrt{a}\,b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^2+bx+c}\right)}{2}-\frac{3\sqrt{c}\,\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{}}{}\right)}{2}\right.}{\sqrt{a\,x^2+bx}}$
default	$-\frac{\left(\frac{a\,x^2+bx+c}{x^2}\right)^{\frac{3}{2}}x\left(12a^{\frac{5}{2}}c^{\frac{5}{2}}\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+bx+c}}{x}\right)x^2-2a^{\frac{5}{2}}\left(a\,x^2+bx+c\right)^{\frac{3}{2}}b\,x^3-4a^{\frac{5}{2}}\left(a\,x^2+bx+c\right)^{\frac{3}{2}}c\,x^2-6a^{\frac{5}{2}}\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*((a*x^2+b*x+c)/x^2)^(3/2)*x*(12*a^(5/2)*c^(5/2)*ln((2*c+b*x+2*c^(1/2)*
(a*x^2+b*x+c)^(1/2))/x)*x^2-2*a^(5/2)*(a*x^2+b*x+c)^(3/2)*b*x^3-4*a^(5/2)*(
a*x^2+b*x+c)^(3/2)*c*x^2-6*a^(5/2)*(a*x^2+b*x+c)^(1/2)*b*c*x^3+3*a^(3/2)*c^
(3/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*x+c)^(1/2))/x)*b^2*x^2-12*a^(5/2)*(a*x
^2+b*x+c)^(1/2)*c^2*x^2+2*a^(3/2)*(a*x^2+b*x+c)^(5/2)*b*x-2*a^(3/2)*(a*x^2+
b*x+c)^(3/2)*b^2*x^2+4*(a*x^2+b*x+c)^(5/2)*c*a^(3/2)-6*a^(3/2)*(a*x^2+b*x+c
)^(1/2)*b^2*c*x^2-12*a^2*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))*b*c^2*x^2)/(a*x^2+b*x+c)^(3/2)/c^2/a^(3/2)
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(3/2), x)
```

Fricas [**A**]

time = 0.42, size = 709, normalized size = 4.89

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(12*sqrt(a)*b*c*x*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2
 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 3*(b^2 + 4*a*c)*sqrt(c)*x*lo
g(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a
```

```
*x^2 + b*x + c)/x^2))/x^2) + 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2)*sqrt((a*x^2 +
b*x + c)/x^2))/(c*x), -1/16*(24*sqrt(-a)*b*c*x*arctan(1/2*(2*a*x^2 + b*x)*s
qrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - 3*(b^2 + 4*a
*c)*sqrt(c)*x*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)
*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2) - 4*(4*a*c*x^2 - 5*b*c*x - 2*c^2
)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), 1/8*(6*sqrt(a)*b*c*x*log(-8*a^2*x^2 -
 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x
^2)) + 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt(
(a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + 2*(4*a*c*x^2 - 5*b*c*x -
2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x), -1/8*(12*sqrt(-a)*b*c*x*arctan(1
/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x +
a*c)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-c)*sqrt
((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) - 2*(4*a*c*x^2 - 5*b*c*x -
 2*c^2)*sqrt((a*x^2 + b*x + c)/x^2))/(c*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(3/2),x)
[Out] Integral((a + b/x + c/x**2)**(3/2), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x + c/x^2)^(3/2),x)
[Out] int((a + b/x + c/x^2)^(3/2), x)
```

3.451 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx$

Optimal. Leaf size=105

$$\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x+\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2\sqrt{a}}-\sqrt{c}\,\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)$$

```
[Out] 1/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(1/2)-arctanh(1/
2*(b+2*c/x)/c^(1/2)/(a+c/x^2+b/x)^(1/2))*c^(1/2)+x*(a+c/x^2+b/x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.375$, Rules used = {1356, 746, 857, 635, 212, 738}

$$x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\sqrt{a}}-\sqrt{c}\,\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] Sqrt[a + c/x^2 + b/x]*x + (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2
+ b/x])])/(2*Sqrt[a]) - Sqrt[c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c
/x^2 + b/x])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x - \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x - \frac{1}{2}b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right) - c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+}}\right.$$
$$= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x + b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right) - (2c)\text{Subst}\left(\int \frac{1}{4c-}\right.$$
$$= \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x + \frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2\sqrt{a}} - \sqrt{c}\tanh^{-1}\left(\frac{b+\frac{2c}{x}}{2\sqrt{c}\sqrt{a+\frac{c}{x^2}}}\right.$$

Mathematica [A]
time = 0.18, size = 131, normalized size = 1.25

$$\frac{x\sqrt{a+\frac{c+bx}{x^2}}\left(2\sqrt{a}\sqrt{c+x(b+ax)}+4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{a}\,x-\sqrt{c+x(b+ax)}}{\sqrt{c}}\right)-b\log\left(b+2ax-2\sqrt{a}\sqrt{c+x(b+ax)}\right)\right)}{2\sqrt{a}\sqrt{c+x(b+ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*(2*Sqrt[a]*Sqrt[c + x*(b + a*x)] + 4*Sqrt[a]*Sqr
t[c]*ArcTanh[(Sqrt[a]*x - Sqrt[c + x*(b + a*x)])/Sqrt[c]] - b*Log[b + 2*a*x
 - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]]))/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])
```

Maple [A]
time = 0.02, size = 121, normalized size = 1.15

method	result
default	$\frac{\sqrt{\frac{a\,x^2+bx+c}{x^2}}\,x\left(-2\sqrt{c}\,\ln\left(\frac{2c+bx+2\sqrt{c}\,\sqrt{a\,x^2+bx+c}}{x}\right)\sqrt{a}+b\ln\left(\frac{2\sqrt{a\,x^2+bx+c}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)+2\sqrt{a\,x\right.}{2\sqrt{a\,x^2+bx+c}\,\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x^2+b*x+c)/x^2)^(1/2)*x*(-2*c^(1/2)*ln((2*c+b*x+2*c^(1/2)*(a*x^2+b*
x+c)^(1/2))/x)*a^(1/2)+b*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))+2*(a*x^2+b*x+c)^(1/2)*a^(1/2))/(a*x^2+b*x+c)^(1/2)/a^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x + c/x^2), x)
```

Fricas [A]
time = 0.40, size = 590, normalized size = 5.62

$$\left[\frac{4ax\sqrt{\frac{ax^2+bx+c}{x^2}}+\sqrt{a}\,b\log\left(-8a^2x^2-8abx-b^2-4ac-4(2ax^2+bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)+2a\sqrt{c}\log\left(-\frac{8bcx+(b^2+4ac)x^2+8c^2-4(bx^2+2cx)\sqrt{c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{x^2}\right)}{4a},\ \frac{2ax\sqrt{\frac{ax^2+bx+c}{x^2}}-\sqrt{-a}\,b\arctan\left(\frac{(2ax^2+bx)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2(a^2x^2+abx+ac)}\right)+a\sqrt{c}\log\left(-\frac{8bcx+(b^2+4ac)x^2+8c^2-4(bx^2+2cx)\sqrt{c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{x^2}\right)}{2a},\ \frac{4ax\sqrt{\frac{ax^2+bx+c}{x^2}}+4a\sqrt{-c}\arctan\left(\frac{(bx^2+2cx)\sqrt{-c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2(acx^2+bcx+c^2)}\right)+\sqrt{a}\,b\log\left(-8a^2x^2-8abx-b^2-4ac-4(2ax^2+bx)\sqrt{a}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)}{4a},\ \frac{2ax\sqrt{\frac{ax^2+bx+c}{x^2}}-\sqrt{-a}\,b\arctan\left(\frac{(2ax^2+bx)\sqrt{-a}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2(a^2x^2+abx+ac)}\right)+2a\sqrt{-c}\arctan\left(\frac{(bx^2+2cx)\sqrt{-c}\sqrt{\frac{ax^2+bx+c}{x^2}}}{2(acx^2+bcx+c^2)}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*
x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) +
2*a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*s
qrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x +
 c)/x^2) - sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x
 + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + a*sqrt(c)*log(-(8*b*c*x + (b^2 + 4*a*
c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*sqrt(c)*sqrt((a*x^2 + b*x + c)/x^2))/x^2
))/a, 1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + 4*a*sqrt(-c)*arctan(1/2*(b*x
^2 + 2*c*x)*sqrt(-c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) +
 sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*sqrt(
a)*sqrt((a*x^2 + b*x + c)/x^2)))/a, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2)
- sqrt(-a)*b*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2
)/(a^2*x^2 + a*b*x + a*c)) + 2*a*sqrt(-c)*arctan(1/2*(b*x^2 + 2*c*x)*sqrt(-
c)*sqrt((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x + c/x**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**B**]
time = 0.13, size = 100, normalized size = 0.95

$$x\sqrt{\frac{1}{x^2}}\,\sqrt{a\,x^2+b\,x+c}-\sqrt{c}\,x\,\ln\left(\frac{2\,c+2\,\sqrt{c}\,\sqrt{a\,x^2+b\,x+c}+b\,x}{x}\right)\sqrt{\frac{1}{x^2}}+\frac{b\,x\,\ln\left(\frac{\frac{b}{2}+\sqrt{a}\,\sqrt{a\,x^2+b\,x+c}+a\,x}{\sqrt{a}}\right)\sqrt{\frac{1}{x^2}}}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x + c/x^2)^(1/2),x)
```

```
[Out] x*(1/x^2)^(1/2)*(c + b*x + a*x^2)^(1/2) - c^(1/2)*x*log((2*c + 2*c^(1/2)*(c
 + b*x + a*x^2)^(1/2) + b*x)/x)*(1/x^2)^(1/2) + (b*x*log((b/2 + a^(1/2)*(c
+ b*x + a*x^2)^(1/2) + a*x)/a^(1/2))*(1/x^2)^(1/2))/(2*a^(1/2))
```

3.452 $\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a}-\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{3/2}}$$

```
[Out] -1/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(3/2)+x*(a+c/x^
2+b/x)^(1/2)/a
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1356, 744, 738, 212}

$$\frac{x\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (Sqrt[a + c/x^2 + b/x]*x)/a - (b*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/
x^2 + b/x])])/(2*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\,dx &= -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx,x,\frac{1}{x}\right)\\ &= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a}+\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,\frac{1}{x}\right)}{2a}\\ &= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a}-\frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+\frac{b}{x}}{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{a}\\ &= \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a}-\frac{b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 88, normalized size = 1.31

$$\frac{2\sqrt{a}\,(c+x(b+ax))+b\sqrt{c+x(b+ax)}\,\log\left(a\left(b+2ax-2\sqrt{a}\,\sqrt{c+x(b+ax)}\right)\right)}{2a^{3/2}x\sqrt{a+\frac{c+bx}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + c/x^2 + b/x],x]
```

```
[Out] (2*Sqrt[a]*(c + x*(b + a*x)) + b*Sqrt[c + x*(b + a*x)]*Log[a*(b + 2*a*x - 2
*Sqrt[a]*Sqrt[c + x*(b + a*x)]])])/(2*a^(3/2)*x*Sqrt[a + (c + b*x)/x^2])
```

Maple [A]
time = 0.05, size = 88, normalized size = 1.31

method	result	size
default	$\frac{\sqrt{a\,x^{2}+bx+c}\,\left(2\sqrt{a\,x^{2}+bx+c}\,a^{\frac{3}{2}}-b\ln\left(\frac{2\sqrt{a\,x^{2}+bx+c}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)a\right)}{2\sqrt{\frac{a\,x^{2}+bx+c}{x^{2}}}\,x\,a^{\frac{5}{2}}}$	88
risch	$\frac{a\,x^{2}+bx+c}{a\sqrt{\frac{a\,x^{2}+bx+c}{x^{2}}}\,x}-\frac{b\ln\left(\frac{\frac{b}{2}+ax}{\sqrt{a}}+\sqrt{a\,x^{2}+bx+c}\right)\sqrt{a\,x^{2}+bx+c}}{2a^{\frac{3}{2}}\sqrt{\frac{a\,x^{2}+bx+c}{x^{2}}}\,x}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*x^2+b*x+c)^(1/2)*(2*(a*x^2+b*x+c)^(1/2)*a^(3/2)-b*ln(1/2*(2*(a*x^2+b
*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a)/((a*x^2+b*x+c)/x^2)^(1/2)/x/a^(5/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a + b/x + c/x^2), x)
```

Fricas [A]
time = 0.35, size = 171, normalized size = 2.55

$$\left[\frac{4\,ax\sqrt{\frac{ax^{2}+bx+c}{x^{2}}}+\sqrt{a}\,b\log\left(-8\,a^{2}x^{2}-8\,abx-b^{2}-4\,ac+4\,(2\,ax^{2}+bx)\sqrt{a}\,\sqrt{\frac{ax^{2}+bx+c}{x^{2}}}\right)}{4\,a^{2}},\ \frac{2\,ax\sqrt{\frac{ax^{2}+bx+c}{x^{2}}}+\sqrt{-a}\,b\arctan\left(\frac{(2\,ax^{2}+bx)\sqrt{-a}\,\sqrt{\frac{ax^{2}+bx+c}{x^{2}}}}{2\,(a^{2}x^{2}+abx+ac)}\right)}{2\,a^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(a)*b*log(-8*a^2*x^2 - 8*a*b*
x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)))/a
```

```
^2, 1/2*(2*a*x*sqrt((a*x^2 + b*x + c)/x^2) + sqrt(-a)*b*arctan(1/2*(2*a*x^2
 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)))/a^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b/x + c/x**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [B]
time = 1.45, size = 53, normalized size = 0.79

$$\frac{x\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a}-\frac{b\,\mathrm{atanh}\left(\frac{a+\frac{b}{2\,x}}{\sqrt{a}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x + c/x^2)^(1/2),x)
```

```
[Out] (x*(a + b/x + c/x^2)^(1/2))/a - (b*atanh((a + b/(2*x))/(a^(1/2)*(a + b/x +
c/x^2)^(1/2))))/(2*a^(3/2))
```

3.453 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=133

$$\frac{(3b^2-8ac)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2(b^2-4ac)}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a(b^2-4ac)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}\right)}{2a^{5/2}}$$

```
[Out] -3/2*b*arctanh(1/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(5/2)-2*(b^2-2*
a*c+b*c/x)*x/a/(-4*a*c+b^2)/(a+c/x^2+b/x)^(1/2)+(-8*a*c+3*b^2)*x*(a+c/x^2+b
/x)^(1/2)/a^2/(-4*a*c+b^2)
```

Rubi [A]
time = 0.07, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1356, 754, 820, 738, 212}

$$-\frac{3b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{5/2}}+\frac{x(3b^2-8ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{a^2(b^2-4ac)}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{a(b^2-4ac)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(-3/2),x]
```

```
[Out] ((3*b^2 - 8*a*c)*Sqrt[a + c/x^2 + b/x]*x)/(a^2*(b^2 - 4*a*c)) - (2*(b^2 - 2
*a*c + (b*c)/x)*x)/(a*(b^2 - 4*a*c)*Sqrt[a + c/x^2 + b/x]) - (3*b*ArcTanh[(
2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} + \frac{2\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3b^2+8ac\right)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, \frac{1}{x}\right)}{a\left(b^2-4ac\right)}$$

$$= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2\left(b^2-4ac\right)} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} + \frac{(3b)\text{Subst}\left(\int \frac{}{x\sqrt{a+}}\right.}{2}$$

$$= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2\left(b^2-4ac\right)} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{(3b)\text{Subst}\left(\int \frac{1}{4a-x^2}\right.}{}$$

$$= \frac{\left(3b^2-8ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{a^2\left(b^2-4ac\right)} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{a\left(b^2-4ac\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \frac{3b\tanh^{-1}\left(\frac{}{2\sqrt{a}\sqrt{}}\right.}{2a^{5/2}}$$

Mathematica [A]
time = 0.44, size = 140, normalized size = 1.05

$$-\frac{2\sqrt{a}\left(-3b^3x+10abcx+4ac(2c+ax^2)-b^2(3c+ax^2)\right)-3b(b^2-4ac)\sqrt{c+x(b+ax)}\log\left(a^2\left(b+2ax-2\sqrt{a}\sqrt{c+x(b+ax)}\right)\right)}{2a^{5/2}\left(b^2-4ac\right)x\sqrt{a+\frac{c+bx}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(-3/2),x]
```

```
[Out] -1/2*(2*Sqrt[a]*(-3*b^3*x + 10*a*b*c*x + 4*a*c*(2*c + a*x^2) - b^2*(3*c + a
*x^2)) - 3*b*(b^2 - 4*a*c)*Sqrt[c + x*(b + a*x)]*Log[a^2*(b + 2*a*x - 2*Sqr
t[a]*Sqrt[c + x*(b + a*x)])])/(a^(5/2)*(b^2 - 4*a*c)*x*Sqrt[a + (c + b*x)/x
^2])
```

Maple [A]
time = 0.06, size = 197, normalized size = 1.48

method	result
default	$\frac{(a\,x^2+bx+c)\left(8a^{\frac{7}{2}}c\,x^2-2a^{\frac{5}{2}}b^2x^2+20a^{\frac{5}{2}}bcx-6a^{\frac{3}{2}}b^3x+16a^{\frac{5}{2}}c^2-6a^{\frac{3}{2}}b^2c-12\ln\left(\frac{2\sqrt{a\,x^2+bx+c}\,\sqrt{a}+2ax+b}{2\sqrt{a}}\right)\sqrt{a\,x^2+b}\right.}{2a^{\frac{7}{2}}\left(\frac{a\,x^2+bx+c}{x^2}\right)^{\frac{3}{2}}x^3(4ac-b^2)}$
risch	$\frac{a\,x^2+bx+c}{a^2\sqrt{\frac{a\,x^2+bx+c}{x^2}}\,x}+\frac{\left(\frac{3bx}{2a^2\sqrt{a\,x^2+bx+c}}-\frac{b^2}{4a^3\sqrt{a\,x^2+bx+c}}-\frac{b^3x}{2a^2(4ac-b^2)\sqrt{a\,x^2+bx+c}}-\frac{}{4a^3(4ac-b^2)\sqrt{}}\right.}{\sqrt{\frac{a\,x^2}{}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+c/x^2+b/x)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2*(a*x^2+b*x+c)/a^(7/2)*(8*a^(7/2)*c*x^2-2*a^(5/2)*b^2*x^2+20*a^(5/2)*b*c*x-6*a^(3/2)*b^3*x+16*a^(5/2)*c^2-6*a^(3/2)*b^2*c-12*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a^2*b*c+3*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(1/2)*a*b^3)/((a*x^2+b*x+c)/x^2)^(3/2)/x^3/(4*a*c-b^2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="maxima")

[Out] integrate((a + b/x + c/x^2)^(-3/2), x)

Fricas [A]
time = 0.40, size = 465, normalized size = 3.50

$$\left[\frac{3\,(b^3c-4\,abc^2+(ab^3-4\,a^2bc)x^2+(b^4-4\,ab^2c)x)\sqrt{a}\,\log\left(-8\,a^2x^2-8\,abx-b^2-4\,ac+4\,(2\,ax^2+bx)\sqrt{a}\,\sqrt{\frac{ax^2+bx+c}{x^2}}\right)+4\,((a^2b^2-4\,a^3c)x^3+(3\,ab^3-10\,a^2bc)x^2+(3\,ab^2c-8\,a^2c^2)x)\sqrt{\frac{ax^2+bx+c}{x^2}}}{4\,(a^3b^2c-4\,a^4c^2+(a^4b^2-4\,a^5c)x^2+(a^3b^3-4\,a^4bc)x)},\ \frac{3\,(b^3c-4\,abc^2+(ab^3-4\,a^2bc)x^2+(b^4-4\,ab^2c)x)\sqrt{-a}\,\arctan\left(\frac{(2\,ax^2+bx)\sqrt{-a}\,\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\,(a^2x^2+abx+ac)}\right)+2\,((a^2b^2-4\,a^3c)x^3+(3\,ab^3-10\,a^2bc)x^2+(3\,ab^2c-8\,a^2c^2)x)\sqrt{\frac{ax^2+bx+c}{x^2}}}{2\,(a^3b^2c-4\,a^4c^2+(a^4b^2-4\,a^5c)x^2+(a^3b^3-4\,a^4bc)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="fricas")

[Out] [1/4*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)*sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*sqrt((a*x^2 + b*x + c)/x^2)) + 4*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*a^2*b*c)*x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3*b^2*c - 4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x), 1/

```
2*(3*(b^3*c - 4*a*b*c^2 + (a*b^3 - 4*a^2*b*c)*x^2 + (b^4 - 4*a*b^2*c)*x)*sq
rt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*x^2 + b*x + c)/x^2)/(a^2
*x^2 + a*b*x + a*c)) + 2*((a^2*b^2 - 4*a^3*c)*x^3 + (3*a*b^3 - 10*a^2*b*c)*
x^2 + (3*a*b^2*c - 8*a^2*c^2)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^3*b^2*c -
4*a^4*c^2 + (a^4*b^2 - 4*a^5*c)*x^2 + (a^3*b^3 - 4*a^4*b*c)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(3/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(-3/2), x)
```

Giac [A]
time = 5.56, size = 237, normalized size = 1.78

$$-\frac{\left(3\,b^3\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)-12\,abc\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)+6\,\sqrt{a}\,b^2\sqrt{c}-16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{2\left(a^{\frac{5}{2}}b^2-4\,a^{\frac{7}{2}}c\right)}+\frac{\left(\frac{(ab^2-4\,a^2c)x}{a^2b^2\mathrm{sgn}(x)-4\,a^3c\mathrm{sgn}(x)}+\frac{3\,b^3-10\,abc}{a^2b^2\mathrm{sgn}(x)-4\,a^3c\mathrm{sgn}(x)}\right)x+\frac{3\,b^2c-8\,ac^2}{a^2b^2\mathrm{sgn}(x)-4\,a^3c\mathrm{sgn}(x)}}{\sqrt{ax^2+bx+c}}+\frac{3\,b\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx+c}\right)\sqrt{a}-b\right|\right)}{2\,a^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*b^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(-b + 2*sqrt
(a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/(a^(5/2)
*b^2 - 4*a^(7/2)*c) + (((a*b^2 - 4*a^2*c)*x/(a^2*b^2*sgn(x) - 4*a^3*c*sgn(x
)) + (3*b^3 - 10*a*b*c)/(a^2*b^2*sgn(x) - 4*a^3*c*sgn(x)))*x + (3*b^2*c - 8
*a*c^2)/(a^2*b^2*sgn(x) - 4*a^3*c*sgn(x)))/sqrt(a*x^2 + b*x + c) + 3/2*b*lo
g(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x + c))*sqrt(a) - b))/(a^(5/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x + c/x^2)^(3/2),x)
```

```
[Out] int(1/(a + b/x + c/x^2)^(3/2), x)
```

3.454 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=220

$$\frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{3a^3\left(b^2-4ac\right)^2}-\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}}-\frac{2\left(5b^4-32ab^2c+32a^2c^2+\frac{bc(5}{}\right.}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{c}{x^2}}}$$

```
[Out] -2/3*(b^2-2*a*c+b*c/x)*x/a/(-4*a*c+b^2)/(a+c/x^2+b/x)^(3/2)-5/2*b*arctanh(1
/2*(2*a+b/x)/a^(1/2)/(a+c/x^2+b/x)^(1/2))/a^(7/2)-2/3*(5*b^4-32*a*b^2*c+32*
a^2*c^2+b*c*(-28*a*c+5*b^2)/x)*x/a^2/(-4*a*c+b^2)^2/(a+c/x^2+b/x)^(1/2)+1/3
*(128*a^2*c^2-100*a*b^2*c+15*b^4)*x*(a+c/x^2+b/x)^(1/2)/a^3/(-4*a*c+b^2)^2
```

Rubi [A]
time = 0.13, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1356, 754, 836, 820, 738, 212}

$$-\frac{5b\tanh^{-1}\left(\frac{2a+\frac{b}{x}}{2\sqrt{a}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}\right)}{2a^{7/2}}-\frac{2x\left(32a^2c^2+\frac{bc(5b^2-28ac)}{x}-32ab^2c+5b^4\right)}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}+\frac{x(128a^2c^2-100ab^2c+15b^4)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{3a^3\left(b^2-4ac\right)^2}-\frac{2x\left(-2ac+b^2+\frac{bc}{x}\right)}{3a\left(b^2-4ac\right)\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a + c/x^2 + b/x]*x)/(3*a^3*(b^2
- 4*a*c)^2) - (2*(b^2 - 2*a*c + (b*c)/x)*x)/(3*a*(b^2 - 4*a*c)*(a + c/x^2 +
 b/x)^(3/2)) - (2*(5*b^4 - 32*a*b^2*c + 32*a^2*c^2 + (b*c*(5*b^2 - 28*a*c))
/x)*x)/(3*a^2*(b^2 - 4*a*c)^2*Sqrt[a + c/x^2 + b/x]) - (5*b*ArcTanh[(2*a +
b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x])])/(2*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
 EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^{5/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} + \frac{2\text{Subst}\left(\int \frac{\frac{1}{2}\left(-5b^2+16ac\right)-3bcx}{x^2\left(a+bx+cx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a\left(b^2-4ac\right)}$$

$$= -\frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - \frac{2\left(5b^4-32ab^2c+32a^2c^2+\frac{bc\left(5b^2-28ac\right)}{x}\right)x}{3a^2\left(b^2-4ac\right)^2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}} - \text{4Sub}$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - 2\Big(5$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - 2\Big(5$$

$$= \frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x}{3a^3\left(b^2-4ac\right)^2} - \frac{2\left(b^2-2ac+\frac{bc}{x}\right)x}{3a\left(b^2-4ac\right)\left(a+\frac{c}{x^2}+\frac{b}{x}\right)^{3/2}} - 2\Big(5$$

Mathematica [A]
time = 1.10, size = 246, normalized size = 1.12

$$\frac{\frac{2\sqrt{a}\,(c+x(b+ax))\left(15b^6x^2+8a^2bc^2x\left(39c+32ax^2\right)-2ab^3cx\left(105c+74ax^2\right)+10b^5\left(3cx+2ax^3\right)+3b^4\left(5c^2-30acx^2+a^2x^4\right)+16a^2c^2\left(8c^2+12acx^2+3a^2x^4\right)-4ab^2c\left(25c^2-12acx^2+6a^2x^4\right)\right)}{\left(b^2-4ac\right)^2}+15b(c+x(b+ax))^{5/2}\log\left(a^3\left(b+2ax-2\sqrt{a}\sqrt{c+x(b+ax)}\right)\right)}{6a^{7/2}x^5\left(a+\frac{c+bx}{x^2}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + c/x^2 + b/x)^(-5/2),x]
```

```
[Out] ((2*Sqrt[a]*(c + x*(b + a*x))*(15*b^6*x^2 + 8*a^2*b*c^2*x*(39*c + 32*a*x^2)
 - 2*a*b^3*c*x*(105*c + 74*a*x^2) + 10*b^5*(3*c*x + 2*a*x^3) + 3*b^4*(5*c^2
 - 30*a*c*x^2 + a^2*x^4) + 16*a^2*c^2*(8*c^2 + 12*a*c*x^2 + 3*a^2*x^4) - 4*
a*b^2*c*(25*c^2 - 12*a*c*x^2 + 6*a^2*x^4)))/(b^2 - 4*a*c)^2 + 15*b*(c + x*(
```

```
b + a*x))^(5/2)*Log[a^3*(b + 2*a*x - 2*Sqrt[a]*Sqrt[c + x*(b + a*x)]])])/(6*
a^(7/2)*x^5*(a + (c + b*x)/x^2)^(5/2))
```

Maple [A]
time = 0.10, size = 376, normalized size = 1.71

method	result
default	$\frac{(a\,x^2+bx+c)\left(96a^{\frac{13}{2}}c^2x^4-48a^{\frac{11}{2}}b^2c\,x^4+512a^{\frac{11}{2}}b\,c^2x^3+6a^{\frac{9}{2}}b^4x^4+384a^{\frac{11}{2}}c^3x^2-296a^{\frac{9}{2}}b^3c\,x^3+96a^{\frac{9}{2}}b^2c^2x^2+40a^{\frac{7}{2}}b^5x^3+624a^{\frac{9}{2}}b\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(a*x^2+b*x+c)*(96*a^(13/2)*c^2*x^4-48*a^(11/2)*b^2*c*x^4+512*a^(11/2)*b
*c^2*x^3+6*a^(9/2)*b^4*x^4+384*a^(11/2)*c^3*x^2-296*a^(9/2)*b^3*c*x^3+96*a^
(9/2)*b^2*c^2*x^2+40*a^(7/2)*b^5*x^3+624*a^(9/2)*b*c^3*x-180*a^(7/2)*b^4*c*
x^2+256*a^(9/2)*c^4-420*a^(7/2)*b^3*c^2*x+30*a^(5/2)*b^6*x^2-200*a^(7/2)*b^
2*c^3+60*a^(5/2)*b^5*c*x+30*a^(5/2)*b^4*c^2-240*ln(1/2*(2*(a*x^2+b*x+c)^(1/
2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(3/2)*a^4*b*c^2+120*ln(1/2*(2*(a
*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(3/2)*a^3*b^3*c-1
5*ln(1/2*(2*(a*x^2+b*x+c)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*(a*x^2+b*x+c)^(3/
2)*a^2*b^5)/a^(11/2)/((a*x^2+b*x+c)/x^2)^(5/2)/x^5/(4*a*c-b^2)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x + c/x^2)^(-5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(198) = 396.
time = 0.54, size = 1081, normalized size = 4.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^2*b^5 - 8*a^3*b^3*c +
16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x^3 + (b^7 - 6
*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x)*
sqrt(a)*log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c + 4*(2*a*x^2 + b*x)*sqrt(a)*
sqrt((a*x^2 + b*x + c)/x^2)) + 4*(3*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*x^
5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*a*b^6 - 30*a^2*b
^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35*a^2*b^3*c^2 + 5
2*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^3*c^4)*x)*sqrt((
a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4 + (a^6*b^4
 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)
*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^5*c - 8*a^5*b^3*
c^2 + 16*a^6*b*c^3)*x), 1/6*(15*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^4 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b
^2*c^2)*x^3 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^2 + 2*(b^6*c - 8*a*b^4*c^2
 + 16*a^2*b^2*c^3)*x)*sqrt(-a)*arctan(1/2*(2*a*x^2 + b*x)*sqrt(-a)*sqrt((a*
x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) + 2*(3*(a^3*b^4 - 8*a^4*b^2*c
+ 16*a^5*c^2)*x^5 + 4*(5*a^2*b^5 - 37*a^3*b^3*c + 64*a^4*b*c^2)*x^4 + 3*(5*
a*b^6 - 30*a^2*b^4*c + 16*a^3*b^2*c^2 + 64*a^4*c^3)*x^3 + 6*(5*a*b^5*c - 35
*a^2*b^3*c^2 + 52*a^3*b*c^3)*x^2 + (15*a*b^4*c^2 - 100*a^2*b^2*c^3 + 128*a^
3*c^4)*x)*sqrt((a*x^2 + b*x + c)/x^2))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^
6*c^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^4 + 2*(a^5*b^5 - 8*a^6*b^3*c
 + 16*a^7*b*c^2)*x^3 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^2 + 2*(a^4*b^
5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)**(5/2),x)
```

```
[Out] Integral((a + b/x + c/x**2)**(-5/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(198) = 396.
time = 5.42, size = 507, normalized size = 2.30

$$-\frac{\left(15\,b^5\sqrt{c}\,\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)-120\,ab^3c^{\frac{3}{2}}\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)+240\,a^2bc^{\frac{5}{2}}\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)+30\,\sqrt{a}\,b^4c-200\,a^{\frac{3}{2}}b^2c^2+256\,a^{\frac{5}{2}}c^3\right)\mathrm{sgn}(x)}{6\left(a^{\frac{7}{2}}b^4\sqrt{c}-8\,a^{\frac{9}{2}}b^2c^{\frac{3}{2}}+16\,a^{\frac{11}{2}}c^{\frac{5}{2}}\right)}+\frac{\left(\left(\left(\frac{3\,(a^2b^4-8\,a^3b^2c+16\,a^4c^2)x}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x)}+\frac{4\,(5\,ab^5-37\,a^2b^3c+64\,a^3bc^2)}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x)}\right)x+\frac{3\,(5\,b^6-30\,ab^4c+16\,a^2b^2c^2+64\,a^3c^3)}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x)}\right)x+\frac{6\,(5\,b^5c-35\,ab^3c^2+52\,a^2bc^3)}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x)}\right)x+\frac{15\,b^4c^2-100\,ab^2c^3+128\,a^2c^4}{a^3b^4\mathrm{sgn}(x)-8\,a^4b^2c\mathrm{sgn}(x)+16\,a^5c^2\mathrm{sgn}(x)}}{3\,(ax^2+bx+c)^{\frac{3}{2}}}+\frac{5\,b\log\left(\left|-2\left(\sqrt{a}\,x-\sqrt{ax^2+bx+c}\right)\sqrt{a}-b\right|\right)}{2\,a^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(15*b^5*sqrt(c)*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 120*a*b^3*c^(3/2)*l
og(abs(-b + 2*sqrt(a)*sqrt(c))) + 240*a^2*b*c^(5/2)*log(abs(-b + 2*sqrt(a)*
```

```
sqrt(c))) + 30*sqrt(a)*b^4*c - 200*a^(3/2)*b^2*c^2 + 256*a^(5/2)*c^3)*sgn(x
)/(a^(7/2)*b^4*sqrt(c) - 8*a^(9/2)*b^2*c^(3/2) + 16*a^(11/2)*c^(5/2)) + 1/3
*((((3*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*x/(a^3*b^4*sgn(x) - 8*a^4*b^2*c
*sgn(x) + 16*a^5*c^2*sgn(x)) + 4*(5*a*b^5 - 37*a^2*b^3*c + 64*a^3*b*c^2)/(a
^3*b^4*sgn(x) - 8*a^4*b^2*c*sgn(x) + 16*a^5*c^2*sgn(x)))*x + 3*(5*b^6 - 30*
a*b^4*c + 16*a^2*b^2*c^2 + 64*a^3*c^3)/(a^3*b^4*sgn(x) - 8*a^4*b^2*c*sgn(x)
 + 16*a^5*c^2*sgn(x)))*x + 6*(5*b^5*c - 35*a*b^3*c^2 + 52*a^2*b*c^3)/(a^3*b
^4*sgn(x) - 8*a^4*b^2*c*sgn(x) + 16*a^5*c^2*sgn(x)))*x + (15*b^4*c^2 - 100*
a*b^2*c^3 + 128*a^2*c^4)/(a^3*b^4*sgn(x) - 8*a^4*b^2*c*sgn(x) + 16*a^5*c^2*
sgn(x)))/(a*x^2 + b*x + c)^(3/2) + 5/2*b*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2
 + b*x + c))*sqrt(a) - b))/(a^(7/2)*sgn(x))
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a+\frac{b}{x}+\frac{c}{x^2}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/x + c/x^2)^(5/2),x)
```

```
[Out] int(1/(a + b/x + c/x^2)^(5/2), x)
```

3.455 $\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx$

Optimal. Leaf size=73

$$\frac{a\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,x}{a+\frac{b}{x}}-\frac{b\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,\log\left(\frac{1}{x}\right)}{a+\frac{b}{x}}$$

```
[Out] a*x*(a^2+b^2/x^2+2*a*b/x)^(1/2)/(a+b/x)-b*ln(1/x)*(a^2+b^2/x^2+2*a*b/x)^(1/
2)/(a+b/x)
```

Rubi [A]

time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1356, 660, 45}

$$\frac{ax\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}-\frac{b\log\left(\frac{1}{x}\right)\sqrt{a^2+\frac{2ab}{x}+\frac{b^2}{x^2}}}{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
```

```
[Out] (a*Sqrt[a^2 + b^2/x^2 + (2*a*b)/x]*x)/(a + b/x) - (b*Sqrt[a^2 + b^2/x^2 + (
2*a*b)/x]*Log[x^(-1)])/(a + b/x)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1356

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[
Int[(a + b/x^n + c/x^(2*n))^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, p}, x] &&
```

```
EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a^2+2abx+b^2x^2}}{x^2}\,dx,x,\frac{1}{x}\right)\\ &= -\frac{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,\text{Subst}\left(\int \frac{ab+b^2x}{x^2}\,dx,x,\frac{1}{x}\right)}{ab+\frac{b^2}{x}}\\ &= -\frac{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,\text{Subst}\left(\int \left(\frac{ab}{x^2}+\frac{b^2}{x}\right)\,dx,x,\frac{1}{x}\right)}{ab+\frac{b^2}{x}}\\ &= \frac{a\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,x}{a+\frac{b}{x}}+\frac{b\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,\log(x)}{a+\frac{b}{x}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.44

$$\frac{x\sqrt{\frac{(b+ax)^2}{x^2}}\,(ax+b\log(x))}{b+ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + b^2/x^2 + (2*a*b)/x],x]
[Out] (x*Sqrt[(b + a*x)^2/x^2]*(a*x + b*Log[x]))/(b + a*x)
```

Maple [A]
time = 0.04, size = 40, normalized size = 0.55

method	result	size
default	$\frac{\sqrt{\frac{a^2x^2+2abx+b^2}{x^2}}\,x(ax+b\ln(x))}{ax+b}$	40
risch	$\frac{\sqrt{\frac{(ax+b)^2}{x^2}}\,x^2a}{ax+b}+\frac{\sqrt{\frac{(ax+b)^2}{x^2}}\,xb\ln(x)}{ax+b}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^2+2*a*b/x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a^2*x^2+2*a*b*x+b^2)/x^2)^(1/2)/(a*x+b)*x*(a*x+b*ln(x))
```

Maxima [A]
time = 0.30, size = 8, normalized size = 0.11

$$ax + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="maxima")
[Out] a*x + b*log(x)
```

Fricas [A]
time = 0.42, size = 8, normalized size = 0.11

$$ax + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="fricas")
[Out] a*x + b*log(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + \frac{2ab}{x} + \frac{b^2}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**2+2*a*b/x)**(1/2),x)
[Out] Integral(sqrt(a**2 + 2*a*b/x + b**2/x**2), x)
```

Giac [A]
time = 4.39, size = 29, normalized size = 0.40

$$ax\operatorname{sgn}(ax^2 + bx) + b\log(|x|)\operatorname{sgn}(ax^2 + bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^2+2*a*b/x)^(1/2),x, algorithm="giac")
[Out] a*x*sgn(a*x^2 + b*x) + b*log(abs(x))*sgn(a*x^2 + b*x)
```

Mupad [B]
time = 0.11, size = 134, normalized size = 1.84

$$x\sqrt{\frac{1}{x^2}}\sqrt{a^2x^2 + 2abx + b^2} - x\ln\left(\frac{2\sqrt{b^2}\sqrt{a^2x^2 + 2abx + b^2} + 2b^2 + 2abx}{x}\right)\sqrt{b^2}\sqrt{\frac{1}{x^2}} + \frac{abx\ln\left(\frac{ab + \sqrt{a^2}\sqrt{a^2x^2 + 2abx + b^2} + a^2x}{\sqrt{a^2}}\right)\sqrt{\frac{1}{x^2}}}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^2 + (2*a*b)/x)^(1/2),x)
```

```
[Out] x*(1/x^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) - x*log((2*(b^2)^(1/2)*(b^2
 + a^2*x^2 + 2*a*b*x)^(1/2) + 2*b^2 + 2*a*b*x)/x)*(b^2)^(1/2)*(1/x^2)^(1/2)
 + (a*b*x*log((a*b + (a^2)^(1/2)*(b^2 + a^2*x^2 + 2*a*b*x)^(1/2) + a^2*x)/(
a^2)^(1/2))*(1/x^2)^(1/2))/(a^2)^(1/2)
```

3.456 $\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx$

Optimal. Leaf size=179

$$\frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arc
tan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c
+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1354, 1136, 1180, 211}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
```

```
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{c+\frac{a}{x^4}+\frac{b}{x^2}}\,dx &= \int \frac{x^4}{a+bx^2+cx^4}\,dx \\
&= \frac{x}{c}-\frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4a}}}{2c} \\
&= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2}}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 202, normalized size = 1.13

$$\frac{x}{c}-\frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^4 + b/x^2)^(-1),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A]
time = 0.04, size = 169, normalized size = 0.94

method	result
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(-b\_R^2-a\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{x}{c}-\frac{\left(b^2-2ac-b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-b\sqrt{-4ac+b^2}+2ac-b^2\right)\sqrt{}}{2\sqrt{-4ac+b^2}\,c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a/x^4+b/x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] x/c-1/2*(b^2-2*a*c-b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))+1/2*(-b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(143) = 286.
time = 0.38, size = 1059, normalized size = 5.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b
*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2
*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

Sympy [A]
time = 1.37, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{2}c^{5}-128ab^{2}c^{4}+16b^{4}c^{3}\right)+t^{2}\cdot\left(48a^{2}bc^{2}-28ab^{3}c+4b^{5}\right)+a^{3},\left(t\mapsto t\log\left(x+\frac{32t^{3}abc^{4}-8t^{3}b^{3}c^{3}-4ta^{2}c^{2}+8tab^{2}c-2tb^{4}}{a^{2}c-ab^{2}}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**4+b/x**2),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2109 vs. 2(143) = 286.
time = 4.27, size = 2109, normalized size = 11.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^4+b/x^2),x, algorithm="giac")
```

```
[Out] x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*
b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 -
2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a
*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*
c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)
)/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*
b^2*c^5 - 4*a^2*c^6)*c^2) + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2
- 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^
2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b
```

```
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3
- 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^
2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*
c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)
```

Mupad [B]
time = 2.08, size = 3026, normalized size = 16.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^4 + b/x^2),x)
```

```
[Out] x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x
*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4
*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4
*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c
))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2
*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c -
```

(2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i

3.457 $\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx$

Optimal. Leaf size=631

$$\frac{x}{c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

```
[Out] x/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(b+(2*a*c-b^2)/(
-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(
2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+
b^2)^(1/2))^(2/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-
4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^
2)^(1/2))^(1/3))*3^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3
)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+
b^2)^(1/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(
-4*a*c+b^2)^(1/2))^(2/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(
-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a*c+b^2)/(-4*
a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/6*arctan(1/3
*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(b+(-2*a*c+b
^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3
)
```

Rubi [A]
time = 0.79, antiderivative size = 631, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {1354, 1381, 1436, 206, 31, 648, 631, 210, 642}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^6 + b/x^3)^(-1),x]
```

```
[Out] x/c + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)
*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b -
Sqrt[b^2 - 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)
*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((b - (b^2 - 2*a*c)/Sqrt[
b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2
```

```
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 -
 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b - Sqrt[b^2 -
 4*a*c])^(2/3)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2
- 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)
*c^(2/3)*x^2])/(6*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{c+\frac{a}{x^6}+\frac{b}{x^3}}\,dx = \int \frac{x^6}{a+bx^3+cx^6}\,dx$$

$$= \frac{x}{c} - \frac{\int \frac{a+bx^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{}{\left(b-\sqrt{b^2\right.}}}{3\sqrt[3]{2}\,c}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log}{3\sqrt[3]{2}\,c^{4/3}}$$

$$= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log}{3\sqrt[3]{2}\,c^{4/3}}$$

$$= \frac{x}{c} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b+\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 70, normalized size = 0.11

$$\frac{x}{c} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^6 + b/x^3)^(-1),x]
```

```
[Out] x/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*Log[x - #1] + b*Log[x - #1]*#1^3)/
(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.20, size = 59, normalized size = 0.09

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-\_R^{3}b-a\right)\ln\left(x-\_R\right)}{2\_R^{5}c+b\_R^{2}}}{3c}$	59
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-\_R^{3}b-a\right)\ln\left(x-\_R\right)}{2\_R^{5}c+b\_R^{2}}}{3c}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^6+b/x^3),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/3/c*sum((-_R^3*b-a)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*
b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^3 + a)/(c*x^6 + b*x^3 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5841 vs. 2(495) = 990.
time = 1.67, size = 5841, normalized size = 9.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b
^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12
*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*ar
```

ctan(1/6*(sqrt(2)*(1/2)^(2/3)*sqrt(2*(a^2*b^8 - 8*a^3*b^6*c + 20*a^4*b^4*c^2 - 16*a^5*b^2*c^3 + 4*a^6*c^4)*x^2 + (1/2)^(2/3)*(b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6 - (b^11*c^4 - 16*a*b^9*c^5 + 98*a^2*b^7*c^6 - 280*a^3*b^5*c^7 + 352*a^4*b^3*c^8 - 128*a^5*b*c^9)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) + (1/2)^(1/3)*((a*b^9*c^4 - 12*a^2*b^7*c^5 + 50*a^3*b^5*c^6 - 80*a^4*b^3*c^7 + 32*a^5*b*c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - (a*b^10 - 12*a^2*b^8*c + 52*a^3*b^6*c^2 - 96*a^4*b^4*c^3 + 68*a^5*b^2*c^4 - 16*a^6*c^5)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3))*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b^2*c^7 + 64*a^4*c^8)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4))*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - 2*(1/2)^(2/3)*(sqrt(3)*(a*b^12*c^4 - 17*a^2*b^10*c^5 + 114*a^3*b^8*c^6 - 378*a^4*b^6*c^7 + 632*a^5*b^4*c^8 - 480*a^6*b^2*c^9 + 128*a^7*c^10)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) - sqrt(3)*(a*b^13 - 15*a^2*b^11*c + 88*a^3*b^9*c^2 - 252*a^4*b^7*c^3 + 356*a^5*b^5*c^4 - 220*a^6*b^3*c^5 + 48*a^7*b*c^6)*x)*(-(b^3 - 2*a*b*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - 2*sqrt(3)*(a^4*b^8 - 8*a^5*b^6*c + 20*a^6*b^4*c^2 - 16*a^7*b^2*c^3 + 4*a^8*c^4))/(a^4*b^8 - 8*a^5*b^6*c + 20*a^6*b^4*c^2 - 16*a^7*b^2*c^3 + 4*a^8*c^4)) - 4*sqrt(3)*(1/2)^(1/3)*c*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(1/3)*arctan(1/6*(sqrt(2)*(1/2)^(2/3)*sqrt(2*(a^2*b^8 - 8*a^3*b^6*c + 20*a^4*b^4*c^2 - 16*a^5*b^2*c^3 + 4*a^6*c^4)*x^2 + (1/2)^(2/3)*(b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6 + (b^11*c^4 - 16*a*b^9*c^5 + 98*a^2*b^7*c^6 - 280*a^3*b^5*c^7 + 352*a^4*b^3*c^8 - 128*a^5*b*c^9)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - (1/2)^(1/3)*((a*b^9*c^4 - 12*a^2*b^7*c^5 + 50*a^3*b^5*c^6 - 80*a^4*b^3*c^7 + 32*a^5*b*c^8)*x*sqrt((b^8 - 8*a*b^6*c + 20

```
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2
*b^2*c^10 - 64*a^3*c^11)) + (a*b^10 - 12*a^2*b^8*c + 52*a^3*b^6*c^2 - 96*a^
4*b^4*c^3 + 68*a^5*b^2*c^4 - 16*a^6*c^5)*x)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4
*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4
)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)))/(b^2*c^4 - 4*a
*c^5))^(1/3))*(sqrt(3)*(b^8*c^4 - 13*a*b^6*c^5 + 60*a^2*b^4*c^6 - 112*a^3*b
^2*c^7 + 64*a^4*c^8)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^
3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^10 - 64*a^3*c^11)) +
sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b*c^4)
)*(-(b^3 - 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)))/(b^2*c^4 - 4*a*c^5))^(2/3) - 2*(1/2)^(2/3)*(sqrt(3)*(a*b
^12*c^4 - 17*a^2*b^10*c^5 + 114*a^3*b^8*c^6 - 378*a^4*b^6*c^7 + 632*a^5*b^4
*c^8 - 480*a^6*b^2*c^9 + 128*a^7*c^10)*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4
*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^9 + 48*a^2*b^2*c^1
0 - 64*a^3*c^11)) + sqrt(3)*(a*b^13 - 15*a^2*b^11*c + 88*a^3*b^9*c^2 - 252*
a^4*b^7*c^3 + 356*a^5*b^5*c^4 - 220*a^6*b^3*c^5 + 48*a^7*b*c^6)*x)*(-(b^3 -
 2*a*b*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*
a^3*b^2*c^3 + 4*a^4*c^4)/(b^6*c^8 - 12*a*b^4*c^...
```

Sympy [A]
time = 52.83, size = 196, normalized size = 0.31

$$\operatorname{RootSum}\left(t^{6}\cdot\left(46656a^{3}c^{7}-34992a^{2}b^{2}c^{6}+8748ab^{4}c^{5}-729b^{6}c^{4}\right)+t^{3}\cdot\left(864a^{3}bc^{3}-864a^{2}b^{3}c^{2}+270ab^{5}c-27b^{7}\right)+a^{4},\left(t\mapsto t\log\left(x+\frac{1296t^{4}a^{2}bc^{6}-648t^{4}ab^{3}c^{5}+81t^{4}b^{5}c^{4}-12ta^{3}c^{3}+39ta^{2}b^{2}c^{2}-21tab^{4}c+3tb^{6}}{2a^{3}c^{2}-4a^{2}b^{2}c+ab^{4}}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**6+b/x**3),x)
```

```
[Out] RootSum(_t**6*(46656*a**3*c**7 - 34992*a**2*b**2*c**6 + 8748*a*b**4*c**5 -
729*b**6*c**4) + _t**3*(864*a**3*b*c**3 - 864*a**2*b**3*c**2 + 270*a*b**5*c
 - 27*b**7) + a**4, Lambda(_t, _t*log(x + (1296*_t**4*a**2*b*c**6 - 648*_t*
*4*a*b**3*c**5 + 81*_t**4*b**5*c**4 - 12*_t*a**3*c**3 + 39*_t*a**2*b**2*c**
2 - 21*_t*a*b**4*c + 3*_t*b**6)/(2*a**3*c**2 - 4*a**2*b**2*c + a*b**4)))) +
 x/c
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^6+b/x^3),x, algorithm="giac")
```

```
[Out] integrate(1/(c + b/x^3 + a/x^6), x)
```

Mupad [B]
time = 4.54, size = 2280, normalized size = 3.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^6 + b/x^3),x)
```

```
[Out] log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(-(b^4*(-(4*a*
c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(
4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^
(1/2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*(-(b^4*(-(4*a*c
 - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + x/c + log((3*
a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*((b^7 + b^4*(-(4*a*c
- b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c -
b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) -
b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(4*c*(4*a*c - b^2)))*((b^7 + b^4*(-(4*a*c -
b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7
- b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*
a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i - 1)*((b^7 + b^4*(-(4*a*
c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^
2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c
- b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2)
- b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2
)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2 + 2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 - log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c - (3*2^(2/3)*a*(3^(1/2)*1i
 + 1)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*c^3 + 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*
(-(4*a*c - b^2)^3)^(1/2) - b^4 - 16*a^2*c^2 + 8*a*b^2*c))/(8*c*(4*a*c - b^2
)))*((3^(1/2)*1i)/2 + 1/2)*((b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 32*a^3*b*
c^3 + 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c - 4*
a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5
- 48*a^2*b^2*c^6)))^(1/3) + log((3*a^2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c -
 (3*2^(2/3)*a*(3^(1/2)*1i - 1)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a
^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c
 - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^3))^(1/3)*(b^4 +
2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/2) + b^4 + 16*a^2*c^2 - 8*a
```

```
*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 - 1/2)*(-(b^4*(-(4*a*c - b^2)
^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3*c^
7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3) - log((3*a^2*x*(b^4 +
2*a^2*c^2 - 4*a*b^2*c))/c + (3*2^(2/3)*a*(3^(1/2)*1i + 1)*(-(b^4*(-(4*a*c -
 b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2 + 2*a^2*c^2*(-(4*a*c -
 b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a
*c - b^2)^3))^(1/3)*(b^4 + 2*a^2*c^2 - 4*a*b^2*c)*(b*(-(4*a*c - b^2)^3)^(1/
2) + b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(8*c*(4*a*c - b^2)))*((3^(1/2)*1i)/2 +
1/2)*(-(b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 32*a^3*b*c^3 - 32*a^2*b^3*c^2
+ 2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 10*a*b^5*c - 4*a*b^2*c*(-(4*a*c - b^
2)^3)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(
1/3)
```

3.458 $\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx$

Optimal. Leaf size=376

$$\frac{x}{c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}-$$

```
[Out] x/c+1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+
b^2)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*
arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-
4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctan(
2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2
)^(1/2))*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*arctanh(2^(1/4)*
c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))
*2^(3/4)/c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A]

time = 0.44, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1354, 1381, 1436, 218, 214, 211}

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + a/x^8 + b/x^4)^(-1),x]
```

```
[Out] x/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])
^(3/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*
c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4
*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1}{c+\frac{a}{x^8}+\frac{b}{x^4}}\,dx=\int \frac{x^8}{a+bx^4+cx^8}\,dx$$
$$=\frac{x}{c}-\frac{\int \frac{a+bx^4}{a+bx^4+cx^8}\,dx}{c}$$
$$=\frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}}$$
$$=\frac{x}{c}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int -}{2c\sqrt{-}}$$
$$=\frac{x}{c}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b\right.}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 70, normalized size = 0.19

$$\frac{x}{c}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{a\log(x-\#1)+b\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + a/x^8 + b/x^4)^(-1),x]
```

```
[Out] x/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[x - #1] + b*Log[x - #1]*#1^4)/
(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 59, normalized size = 0.16

method	result	size
default	$\frac{x}{c}+\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^4b-a\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	59

risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-\_R^4b-a\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4c}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+a/x^8+b/x^4),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/4/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*
b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^4 + a)/(c*x^8 + b*x^4 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5310 vs. $2(296) = 592$.
time = 1.03, size = 5310, normalized size = 14.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 -
8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(-1/2*(sqrt(1/2)*(b^11 - 13
*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^
5 + (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352*a^4*b
^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*
sqrt((a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*x^2
 + 1/2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 1
25*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6 + (b^11*c^5 - 15*a*b^9*c^6 + 85
*a^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqrt((b^8
 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b
*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c
```

```
^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*
a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))
 - sqrt(1/2)*((a*b^14*c^5 - 19*a^2*b^12*c^6 + 147*a^3*b^10*c^7 - 590*a^4*b^
8*c^8 + 1290*a^5*b^6*c^9 - 1464*a^6*b^4*c^10 + 736*a^7*b^2*c^11 - 128*a^8*c
^12)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)) + (a*b^15 - 16*a^
2*b^13*c + 103*a^3*b^11*c^2 - 340*a^4*b^9*c^3 + 605*a^5*b^7*c^4 - 554*a^6*b
^5*c^5 + 224*a^7*b^3*c^6 - 32*a^8*b*c^7)*x)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*
b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(sqrt(1/2)
*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7
)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^
10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7)))/(a^5*b^8 - 6*a^6*b^6*c + 11*a^7*b^4*c^2 - 6*a^8*b^2*c^3 +
 a^9*c^4)) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4
*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6
*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/2*(sqrt(1/2)*(b^1
1 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^
5*b*c^5 - (b^10*c^5 - 16*a*b^8*c^6 + 98*a^2*b^6*c^7 - 280*a^3*b^4*c^8 + 352
*a^4*b^2*c^9 - 128*a^5*c^10)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^
13)))*sqrt((a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^
4)*x^2 + 1/2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c
^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6 - (b^11*c^5 - 15*a*b^9*c^
6 + 85*a^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 1
1*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(sqrt
(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^
2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b
^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*
a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))
/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)) + sqrt(1/2)*((a*b^14*c^5 - 19*a^2*b^
12*c^6 + 147*a^3*b^10*c^7 - 590*a^4*b^8*c^8 + 1290*a^5*b^6*c^9 - 1464*a^6*b
^4*c^10 + 736*a^7*b^2*c^11 - 128*a^8*c^12)*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)) - (a*b^15 - 16*a^2*b^13*c + 103*a^3*b^11*c^2 - 340*a^4
```

```
*b^9*c^3 + 605*a^5*b^7*c^4 - 554*a^6*b^5*c^5 + 224*a^7*b^3*c^6 - 32*a^8*b*c
^7)*x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a
*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2
 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x**8+b/x**4),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+a/x^8+b/x^4),x, algorithm="giac")
[Out] integrate(1/(c + b/x^4 + a/x^8), x)
```

Mupad [**B**]
time = 3.78, size = 2500, normalized size = 6.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + a/x^8 + b/x^4),x)
[Out] atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*
(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120
*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-
(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2
*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 204
8*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 6
1*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b
^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16
*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*
a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b
*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2)
```

- 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i - (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i)/((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c + (4*x*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)))*(-(b^9 + b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 + a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c - 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*2i + atan(((((16*(a^3*b^6 - 4*a^6*c^3 - 7*a^4*b^4*c + 13*a^5*b^2*c^2))/c - (4*x*(-(b^9 - b^

4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(3/4)*(4096*a^5*b*c^6 + 256*a^3*b^5*c^4 - 2048*a^4*b^3*c^5))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) - (4*x*(a^4*b^4 + 2*a^6*c^2 - 4*a^5*b^2*c))/c)*(-(b^9 - b^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4 + 61*a^2*b^5*c^2 - 120*a^3*b^3*c^3 - a^2*c^2*(-(4*a*c - b^2)^5)^(1/2) - 13*a*b^7*c + 3*a*b^2*c*(-(4*a*c - b^2)^5)^(1/2))/(512*(2...

3.459 $\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx$

Optimal. Leaf size=106

$$2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

```
[Out] -2*arctanh(1/2*(2*a+b*x^(1/2))/a^(1/2)/(a+c*x+b*x^(1/2))^(1/2))*a^(1/2)+b*a
rctanh(1/2*(b+2*c*x^(1/2))/c^(1/2)/(a+c*x+b*x^(1/2))^(1/2))/c^(1/2)+2*(a+c*
x+b*x^(1/2))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1371, 748, 857, 635, 212, 738}

$$2\sqrt{a+b\sqrt{x}+cx}-2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2c\sqrt{x}}{2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] - 2*Sqrt[a]*ArcTanh[(2*a + b*Sqrt[x])/(2*Sqrt[a
]*Sqrt[a + b*Sqrt[x] + c*x])] + (b*ArcTanh[(b + 2*c*Sqrt[x])/(2*Sqrt[c]*Sqr
t[a + b*Sqrt[x] + c*x])])/Sqrt[c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sqrt{x}+cx}}{x}\,dx = 2\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, \sqrt{x}\right)$$

$$= 2\sqrt{a+b\sqrt{x}+cx} - \text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, \sqrt{x}\right)$$

$$= 2\sqrt{a+b\sqrt{x}+cx} + (2a)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, \sqrt{x}\right) + b\text{Subst}\Bigg(\int$$

$$= 2\sqrt{a+b\sqrt{x}+cx} - (4a)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+b\sqrt{x}}{\sqrt{a+b\sqrt{x}+cx}}\right) + (2b)\text{Su}$$

$$= 2\sqrt{a+b\sqrt{x}+cx} - 2\sqrt{a}\tanh^{-1}\left(\frac{2a+b\sqrt{x}}{2\sqrt{a}\sqrt{a+b\sqrt{x}+cx}}\right) + \frac{b\tanh^{-1}\Bigg(\frac{}{2\sqrt{\cdot}}}{}$$

Mathematica [A]
time = 0.15, size = 105, normalized size = 0.99

$$2\sqrt{a+b\sqrt{x}+cx} + 4\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{x}-\sqrt{a+b\sqrt{x}+cx}}{\sqrt{a}}\right) - \frac{b\log\left(b+2c\sqrt{x}-2\sqrt{c}\sqrt{a+b\sqrt{x}+cx}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[x] + c*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*Sqrt[x] + c*x] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[x] - Sqrt[a +
 b*Sqrt[x] + c*x])/Sqrt[a]] - (b*Log[b + 2*c*Sqrt[x] - 2*Sqrt[c]*Sqrt[a + b
*Sqrt[x] + c*x]])/Sqrt[c]
```

Maple [A]
time = 0.02, size = 84, normalized size = 0.79

method	result
derivativedivides	$2\sqrt{a+cx+b\sqrt{x}} + \frac{b\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+cx+b\sqrt{x}}\right)}{\sqrt{c}} - 2\sqrt{a}\,\ln\left(\frac{2a+b\sqrt{x}+2\sqrt{a}\,\sqrt{a}}{\sqrt{x}}\right.$
default	$2\sqrt{a+cx+b\sqrt{x}} + \frac{b\ln\left(\frac{\frac{b}{2}+c\sqrt{x}}{\sqrt{c}}+\sqrt{a+cx+b\sqrt{x}}\right)}{\sqrt{c}} - 2\sqrt{a}\,\ln\left(\frac{2a+b\sqrt{x}+2\sqrt{a}\,\sqrt{a}}{\sqrt{x}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*x+b*x^(1/2))^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a+c*x+b*x^(1/2))^(1/2)+b*ln((1/2*b+c*x^(1/2))/c^(1/2)+(a+c*x+b*x^(1/2))^
(1/2))/c^(1/2)-2*a^(1/2)*ln((2*a+b*x^(1/2)+2*a^(1/2)*(a+c*x+b*x^(1/2))^(1/2
))/x^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x + b*sqrt(x) + a)/x, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{x} + cx}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*sqrt(x) + c*x)/x, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*x+b*x^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+c\,x+b\,\sqrt{x}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x + b*x^(1/2))^(1/2)/x,x)
```

```
[Out] int((a + c*x + b*x^(1/2))^(1/2)/x, x)
```

3.460 $\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx$

Optimal. Leaf size=40

$$-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}+\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}$$

```
[Out] -1/160*b*(b+2*c*x^(1/2))^5/c^4+1/192*(b+2*c*x^(1/2))^6/c^4
```

Rubi [A]
time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {28, 196, 45}

$$\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}-\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

```
[Out] -1/160*(b*(b + 2*c*Sqrt[x])^5)/c^4 + (b + 2*c*Sqrt[x])^6/(192*c^4)
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 196

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rubi steps

$$\int \left(\frac{b^2}{4c}+b\sqrt{x}+cx\right)^2 dx = \frac{\int \left(\frac{b}{2}+c\sqrt{x}\right)^4 dx}{c^2}$$
$$= \frac{2\text{Subst}\left(\int x\left(\frac{b}{2}+cx\right)^4 dx, x, \sqrt{x}\right)}{c^2}$$
$$= \frac{2\text{Subst}\left(\int \left(-\frac{b\left(\frac{b}{2}+cx\right)^4}{2c}+\frac{\left(\frac{b}{2}+cx\right)^5}{c}\right) dx, x, \sqrt{x}\right)}{c^2}$$
$$= -\frac{b\left(b+2c\sqrt{x}\right)^5}{160c^4}+\frac{\left(b+2c\sqrt{x}\right)^6}{192c^4}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.38

$$\frac{15b^4x+80b^3cx^{3/2}+180b^2c^2x^2+192bc^3x^{5/2}+80c^4x^3}{240c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2/(4*c) + b*Sqrt[x] + c*x)^2,x]
```

```
[Out] (15*b^4*x + 80*b^3*c*x^(3/2) + 180*b^2*c^2*x^2 + 192*b*c^3*x^(5/2) + 80*c^4
*x^3)/(240*c^2)
```

Maple [A]
time = 0.06, size = 52, normalized size = 1.30

method	result	size
derivativedivides	$\frac{\frac{8c^4x^3}{3}+\frac{32b\,c^3x^{\frac{5}{2}}}{5}+6b^2c^2x^2+\frac{8b^3c\,x^{\frac{3}{2}}}{3}+\frac{b^4x}{2}}{8c^2}$	50
default	$\frac{b^2x^2}{2}+\frac{b\left(\frac{8c^2x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{3}{2}}}{3}\right)}{2c}+\frac{\left(\frac{b^2}{4c}+cx\right)^3}{3c}$	52
trager	$\frac{\frac{\left(16c^4x^2+36b^2x\,c^2+16c^4x+3b^4+36b^2c^2+16c^4\right)(-1+x)}{3}+\frac{16bc\,x^{\frac{3}{2}}\left(12c^2x+5b^2\right)}{15}}{16c^2}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/4/c*b^2+c*x+b*x^(1/2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b^2*x^2+1/2*b/c*(8/5*c^2*x^(5/2)+2/3*b^2*x^(3/2))+1/3*(1/4/c*b^2+c*x)^3
/c
```

Maxima [A]
time = 0.28, size = 54, normalized size = 1.35

$$\frac{1}{3}\,c^2x^3+\frac{4}{5}\,bcx^{\frac{5}{2}}+\frac{1}{2}\,b^2x^2+\frac{b^4x}{16\,c^2}+\frac{\left(3\,cx^2+4\,bx^{\frac{3}{2}}\right)b^2}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*b^2/c+c*x+b*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 4/5*b*c*x^(5/2) + 1/2*b^2*x^2 + 1/16*b^4*x/c^2 + 1/12*(3*c*x^
2 + 4*b*x^(3/2))*b^2/c
```

Fricas [A]
time = 0.41, size = 53, normalized size = 1.32

$$\frac{80\,c^4x^3+180\,b^2c^2x^2+15\,b^4x+16\,(12\,bc^3x^2+5\,b^3cx)\sqrt{x}}{240\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*b^2/c+c*x+b*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(80*c^4*x^3 + 180*b^2*c^2*x^2 + 15*b^4*x + 16*(12*b*c^3*x^2 + 5*b^3*c
*x)*sqrt(x))/c^2
```

Sympy [A]
time = 0.09, size = 51, normalized size = 1.28

$$\frac{b^4x}{16c^2}+\frac{b^3x^{\frac{3}{2}}}{3c}+\frac{3b^2x^2}{4}+\frac{4bcx^{\frac{5}{2}}}{5}+\frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*b**2/c+c*x+b*x**(1/2))**2,x)
```

```
[Out] b**4*x/(16*c**2) + b**3*x**(3/2)/(3*c) + 3*b**2*x**2/4 + 4*b*c*x**(5/2)/5 +
 c**2*x**3/3
```

Giac [A]
time = 3.81, size = 49, normalized size = 1.22

$$\frac{80\,c^4x^3+192\,bc^3x^{\frac{5}{2}}+180\,b^2c^2x^2+80\,b^3cx^{\frac{3}{2}}+15\,b^4x}{240\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/4*b^2/c+c*x+b*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/240*(80*c^4*x^3 + 192*b*c^3*x^(5/2) + 180*b^2*c^2*x^2 + 80*b^3*c*x^(3/2)
+ 15*b^4*x)/c^2
```

Mupad [B]
time = 0.04, size = 44, normalized size = 1.10

$$\frac{3\,b^2\,x^2}{4}+\frac{c^2\,x^3}{3}+\frac{b^4\,x}{16\,c^2}+\frac{b^3\,x^{3/2}}{3\,c}+\frac{4\,b\,c\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x + b*x^(1/2) + b^2/(4*c))^2,x)
[Out] (3*b^2*x^2)/4 + (c^2*x^3)/3 + (b^4*x)/(16*c^2) + (b^3*x^(3/2))/(3*c) + (4*b
*c*x^(5/2))/5
```

3.461 $\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

```
[Out] -2*a*ln(a+b*x^(1/2))*(a+b*x^(1/2))/b^2/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2)+2*(a
^2+b^2*x+2*a*b*x^(1/2))^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1355, 654, 622, 31}

$$\frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2}-\frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
```

```
[Out] (2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])/b^2 - (2*a*(a + b*Sqrt[x])*Log[a + b*
Sqrt[x]])/(b^2*Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 622

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx = 2\text{Subst}\left(\int \frac{x}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{(2a)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt{x}\right)}{b}$$

$$= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{\left(2a\left(a+b\sqrt{x}\right)\right)\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, \sqrt{x}\right)}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

$$= \frac{2\sqrt{a^2+2ab\sqrt{x}+b^2x}}{b^2} - \frac{2a\left(a+b\sqrt{x}\right)\log\left(a+b\sqrt{x}\right)}{b^2\sqrt{a^2+2ab\sqrt{x}+b^2x}}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.67

$$\frac{2\left(a+b\sqrt{x}\right)\left(b\sqrt{x}-a\log\left(a+b\sqrt{x}\right)\right)}{b^2\sqrt{\left(a+b\sqrt{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*Sqrt[x] + b^2*x],x]
[Out] (2*(a + b*Sqrt[x])*(b*Sqrt[x] - a*Log[a + b*Sqrt[x]]))/(b^2*Sqrt[(a + b*Sqr
t[x])^2])
```

Maple [A]
time = 0.06, size = 50, normalized size = 0.67

method	result	size
derivativedivides	$-\frac{2\left(a+b\sqrt{x}\right)\left(a\ln\left(a+b\sqrt{x}\right)-b\sqrt{x}\right)}{\sqrt{\left(a+b\sqrt{x}\right)^2}\,b^2}$	41

default	$\frac{2\sqrt{a^2+b^2x+2ab\sqrt{x}}\,\left(b\sqrt{x}-a\ln\left(a+b\sqrt{x}\right)\right)}{\left(a+b\sqrt{x}\right)b^2}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a^2+b^2*x+2*a*b*x^(1/2))^(1/2)*(b*x^(1/2)-a*ln(a+b*x^(1/2)))/(a+b*x^(1/2
))/b^2
```

Maxima [A]
time = 0.34, size = 23, normalized size = 0.31

$$-\frac{2\,a\log\left(b\sqrt{x}+a\right)}{b^2}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*log(b*sqrt(x) + a)/b^2 + 2*sqrt(x)/b
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a^2+2ab\sqrt{x}+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2*x+2*a*b*x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b*sqrt(x) + b**2*x), x)
```

Giac [A]
time = 4.73, size = 45, normalized size = 0.60

$$-\frac{2\,|a|\log\left(\left|\sqrt{b^2x}\,\mathrm{sgn}(a)\,\mathrm{sgn}(b)+|a|\right|\right)}{b^2}+\frac{2\,\sqrt{b^2x}}{b^2\mathrm{sgn}\left(a\right)\mathrm{sgn}\left(b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*x+2*a*b*x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] -2*abs(a)*log(abs(sqrt(b^2*x)*sgn(a)*sgn(b) + abs(a)))/b^2 + 2*sqrt(b^2*x)/
(b^2*sgn(a)*sgn(b))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b^2\,x+a^2+2\,a\,b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2),x)
```

```
[Out] int(1/(b^2*x + a^2 + 2*a*b*x^(1/2))^(1/2), x)
```

3.462 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2} dx$

Optimal. Leaf size=137

$$\frac{3a^2\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{3b^3}+\frac{3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+\cdots}}{10b\cdots}$$

```
[Out] 3/8*a^2*(a+b*x^(1/3))^7*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-2/3*a*(a+
b*x^(1/3))^8*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+3/10*(a+b*x^(1/3))^9
*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 660, 45}

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^9}{10b^3}-\frac{2a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^8}{3b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3) -
 (2*a*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(3*b^3) +
(3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(10*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
```

ctionQ[n]

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}\,dx = 3\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{7/2}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2(ab+b^2x)^7\,dx, x, \sqrt[3]{x}\right)}{b^7\left(a+b\sqrt[3]{x}\right)}$$
$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int \left(\frac{a^2(ab+b^2x)^7}{b^2}-\frac{2a(ab+b^2x)^8}{b^3}+\frac{(ab+b^2\cdots}{b^4}\right.\right.}{b^7\left(a+b\sqrt[3]{x}\right)}$$
$$= \frac{3a^2\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{8b^3}-\frac{2a\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{\cdots}}}{3b^3}$$

Mathematica [A]
time = 0.03, size = 117, normalized size = 0.85

$$\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{7/2}\left(120a^7x+630a^6bx^{4/3}+1512a^5b^2x^{5/3}+2100a^4b^3x^2+1800a^3b^4x^{7/3}+945a^2b^5x^{8/3}+280ab^6x^3+36b^7x^{10/3}\right)}{120\left(a+b\sqrt[3]{x}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(7/2),x]
```

```
[Out] (((a + b*x^(1/3))^2)^(7/2)*(120*a^7*x + 630*a^6*b*x^(4/3) + 1512*a^5*b^2*x^
(5/3) + 2100*a^4*b^3*x^2 + 1800*a^3*b^4*x^(7/3) + 945*a^2*b^5*x^(8/3) + 280
*a*b^6*x^3 + 36*b^7*x^(10/3)))/(120*(a + b*x^(1/3))^7)
```

Maple [A]
time = 0.09, size = 109, normalized size = 0.80

method	result
derivativedivides	$\frac{\left(\left(a+b\,x^{\frac{1}{3}}\right)^2\right)^{\frac{7}{2}}x\left(36b^7x^{\frac{7}{3}}+280a\,b^6x^2+945a^2b^5x^{\frac{5}{3}}+1800a^3b^4x^{\frac{4}{3}}+2100a^4b^3x+1512a^5b^2x^{\frac{2}{3}}+630a^6b\,x^{\frac{1}{3}}+120a^7\right)}{120\left(a+b\,x^{\frac{1}{3}}\right)^7}$
default	$\frac{\sqrt{a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}}\left(36b^7x^{\frac{10}{3}}+945a^2b^5x^{\frac{8}{3}}+1800a^3b^4x^{\frac{7}{3}}+1512a^5b^2x^{\frac{5}{3}}+630a^6b\,x^{\frac{4}{3}}+280a\,b^6x^3+2100a^4b^3x^2\cdots\right.}{120a+120b\,x^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(36*b^7*x^(10/3)+945*a^2*b^5*x^
(8/3)+1800*a^3*b^4*x^(7/3)+1512*a^5*b^2*x^(5/3)+630*a^6*b*x^(4/3)+280*a*b^6
*x^3+2100*a^4*b^3*x^2+120*a^7*x)/(a+b*x^(1/3))
```

Maxima [A]
time = 0.30, size = 114, normalized size = 0.83

$$\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a^2x^{\frac{1}{3}}}{8\,b^2}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a^3}{8\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{9}{2}}x^{\frac{1}{3}}}{10\,b^2}-\frac{11\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{9}{2}}a}{30\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
```

```
[Out] 3/8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a^2*x^(1/3)/b^2 + 3/8*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^(7/2)*a^3/b^3 + 3/10*(b^2*x^(2/3) + 2*a*b*x^(1
/3) + a^2)^(9/2)*x^(1/3)/b^2 - 11/30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(9
/2)*a/b^3
```

Fricas [A]
time = 0.40, size = 84, normalized size = 0.61

$$\frac{7}{3}\,ab^6x^3+\frac{35}{2}\,a^4b^3x^2+a^7x+\frac{63}{40}\,\left(5\,a^2b^5x^2+8\,a^5b^2x\right)x^{\frac{2}{3}}+\frac{3}{20}\,\left(2\,b^7x^3+100\,a^3b^4x^2+35\,a^6bx\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")
```

```
[Out] 7/3*a*b^6*x^3 + 35/2*a^4*b^3*x^2 + a^7*x + 63/40*(5*a^2*b^5*x^2 + 8*a^5*b^2
*x)*x^(2/3) + 3/20*(2*b^7*x^3 + 100*a^3*b^4*x^2 + 35*a^6*b*x)*x^(1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(7/2), x)
```

Giac [A]
time = 4.66, size = 140, normalized size = 1.02

$$\frac{3}{10}\,b^7x^{\frac{10}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{7}{3}\,ab^6x^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{63}{8}\,a^2b^5x^{\frac{8}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+15\,a^3b^4x^{\frac{7}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{35}{2}\,a^4b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{63}{5}\,a^5b^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{21}{4}\,a^6bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^7x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
```

```
[Out] 3/10*b^7*x^(10/3)*sgn(b*x^(1/3) + a) + 7/3*a*b^6*x^3*sgn(b*x^(1/3) + a) + 6
3/8*a^2*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15*a^3*b^4*x^(7/3)*sgn(b*x^(1/3) +
 a) + 35/2*a^4*b^3*x^2*sgn(b*x^(1/3) + a) + 63/5*a^5*b^2*x^(5/3)*sgn(b*x^(1
/3) + a) + 21/4*a^6*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^7*x*sgn(b*x^(1/3) + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2),x)
```

```
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2), x)
```

3.463 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}\,dx$

Optimal. Leaf size=137

$$\frac{a^2\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{7b^3}+\frac{3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+}}{8b^3}$$

```
[Out] 1/2*a^2*(a+b*x^(1/3))^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-6/7*a*(a+
b*x^(1/3))^6*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+3/8*(a+b*x^(1/3))^7*
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

Rubi [A]
time = 0.05, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 660, 45}

$$\frac{3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^7}{8b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
```

```
[Out] (a^2*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3) - (
6*a*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(7*b^3) + (3
*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(8*b^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
```

`ctionQ[n]`

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2} dx = 3\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{5/2} dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2(ab+b^2x)^5 \, dx, x, \sqrt[3]{x}\right)}{b^5\left(a+b\sqrt[3]{x}\right)}$$

$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int \left(\frac{a^2(ab+b^2x)^5}{b^2}-\frac{2a(ab+b^2x)^6}{b^3}+\frac{(ab+b^2}{b^4}\right.\right.}{b^5\left(a+b\sqrt[3]{x}\right)}$$

$$= \frac{a^2\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{2b^3} - \frac{6a\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{}}}{7b^3}$$

Mathematica [A]
time = 0.03, size = 93, normalized size = 0.68

$$\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{5/2}\left(56a^5x+210a^4bx^{4/3}+336a^3b^2x^{5/3}+280a^2b^3x^2+120ab^4x^{7/3}+21b^5x^{8/3}\right)}{56\left(a+b\sqrt[3]{x}\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(5/2),x]
[Out] (((a + b*x^(1/3))^2)^(5/2)*(56*a^5*x + 210*a^4*b*x^(4/3) + 336*a^3*b^2*x^(5
/3) + 280*a^2*b^3*x^2 + 120*a*b^4*x^(7/3) + 21*b^5*x^(8/3)))/(56*(a + b*x^(
1/3))^5)
```

Maple [A]
time = 0.06, size = 87, normalized size = 0.64

method	result	size
derivativedivides	$\frac{\left(\left(a+b\,x^{\frac{1}{3}}\right)^2\right)^{\frac{5}{2}} x\left(21b^5x^{\frac{5}{3}}+120b^4a\,x^{\frac{4}{3}}+280a^2b^3x+336b^2a^3x^{\frac{2}{3}}+210b\,a^4x^{\frac{1}{3}}+56a^5\right)}{56\left(a+b\,x^{\frac{1}{3}}\right)^5}$	76
default	$\frac{\sqrt{a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}}\left(21b^5x^{\frac{8}{3}}+120b^4a\,x^{\frac{7}{3}}+336b^2a^3x^{\frac{5}{3}}+210b\,a^4x^{\frac{4}{3}}+280a^2b^3x^2+56a^5x\right)}{56a+56b\,x^{\frac{1}{3}}}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/56*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(21*b^5*x^(8/3)+120*b^4*a*x^(7/3
)+336*b^2*a^3*x^(5/3)+210*b*a^4*x^(4/3)+280*a^2*b^3*x^2+56*a^5*x)/(a+b*x^(1
/3))
```

Maxima [A]
time = 0.29, size = 114, normalized size = 0.83

$$\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a^2x^{\frac{1}{3}}}{2\,b^2}+\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a^3}{2\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}x^{\frac{1}{3}}}{8\,b^2}-\frac{27\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{7}{2}}a}{56\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^2*x^(1/3)/b^2 + 1/2*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2)*a^3/b^3 + 3/8*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^(7/2)*x^(1/3)/b^2 - 27/56*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(7/
2)*a/b^3
```

Fricas [A]
time = 0.33, size = 61, normalized size = 0.45

$$5\,a^2b^3x^2+a^5x+\frac{3}{8}\left(b^5x^2+16\,a^3b^2x\right)x^{\frac{2}{3}}+\frac{15}{28}\left(4\,ab^4x^2+7\,a^4bx\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
```

```
[Out] 5*a^2*b^3*x^2 + a^5*x + 3/8*(b^5*x^2 + 16*a^3*b^2*x)*x^(2/3) + 15/28*(4*a*b
^4*x^2 + 7*a^4*b*x)*x^(1/3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(5/2), x)
```

Giac [A]
time = 5.12, size = 102, normalized size = 0.74

$$\frac{3}{8}\,b^5x^{\frac{8}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{7}\,ab^4x^{\frac{7}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+5\,a^2b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+6\,a^3b^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{15}{4}\,a^4bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^5x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
```

```
[Out] 3/8*b^5*x^(8/3)*sgn(b*x^(1/3) + a) + 15/7*a*b^4*x^(7/3)*sgn(b*x^(1/3) + a)
+ 5*a^2*b^3*x^2*sgn(b*x^(1/3) + a) + 6*a^3*b^2*x^(5/3)*sgn(b*x^(1/3) + a) +
 15/4*a^4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^5*x*sgn(b*x^(1/3) + a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)
```

```
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2), x)
```

3.464 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2} dx$

Optimal. Leaf size=137

$$\frac{3a^2\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4b^3}-\frac{6a\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{5b^3}+\frac{\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+\cdots}}{2b^3}$$

```
[Out] 3/4*a^2*(a+b*x^(1/3))^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3-6/5*a*(a+
b*x^(1/3))^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3+1/2*(a+b*x^(1/3))^5*
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/b^3
```

Rubi [A]
time = 0.04, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1355, 659}

$$\frac{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^5}{2b^3}-\frac{6a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^4}{5b^3}+\frac{3a^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\left(a+b\sqrt[3]{x}\right)^3}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(4*b^3) -
 (6*a*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(5*b^3) +
((a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])/(2*b^3)
```

Rule 659

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c
, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&
!IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}\,dx = 3\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^{3/2}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int\left(\frac{a^2(ab+b^2x)^3}{b^2}-\frac{2a(ab+b^2x)^4}{b^3}+\frac{(ab+b^2}{b^4}\right.\right.}{b^3\left(a+b\sqrt[3]{x}\right)}$$

$$= \frac{3a^2\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4b^3} - \frac{6a\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{}}}{5b^3}$$

Mathematica [A]
time = 0.02, size = 67, normalized size = 0.49

$$\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{3/2}\left(20a^3x+45a^2bx^{4/3}+36ab^2x^{5/3}+10b^3x^2\right)}{20\left(a+b\sqrt[3]{x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(3/2),x]
```

```
[Out] (((a + b*x^(1/3))^2)^(3/2)*(20*a^3*x + 45*a^2*b*x^(4/3) + 36*a*b^2*x^(5/3)
+ 10*b^3*x^2))/(20*(a + b*x^(1/3))^3)
```

Maple [A]
time = 0.05, size = 65, normalized size = 0.47

method	result	size
derivativedivides	$\frac{\left(\left(a+b\,x^{\frac{1}{3}}\right)^2\right)^{\frac{3}{2}} x\left(10b^3x+36a\,b^2x^{\frac{2}{3}}+45a^2b\,x^{\frac{1}{3}}+20a^3\right)}{20\left(a+b\,x^{\frac{1}{3}}\right)^3}$	54
default	$\frac{\sqrt{a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}}\left(36a\,b^2x^{\frac{5}{3}}+45a^2b\,x^{\frac{4}{3}}+10b^3x^2+20a^3x\right)}{20a+20b\,x^{\frac{1}{3}}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(36*a*b^2*x^(5/3)+45*a^2*b*x^(4/
3)+10*b^3*x^2+20*a^3*x)/(a+b*x^(1/3))
```

Maxima [A]
time = 0.30, size = 114, normalized size = 0.83

$$\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}a^2x^{\frac{1}{3}}}{4\,b^2}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}a^3}{4\,b^3}+\frac{\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}x^{\frac{1}{3}}}{2\,b^2}-\frac{7\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{5}{2}}a}{10\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

```
[Out] 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^2*x^(1/3)/b^2 + 3/4*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a^3/b^3 + 1/2*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^(5/2)*x^(1/3)/b^2 - 7/10*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(5/2
)*a/b^3
```

Fricas [A]
time = 0.34, size = 32, normalized size = 0.23

$$\frac{1}{2}\,b^3x^2+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{4}\,a^2bx^{\frac{4}{3}}+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*b^3*x^2 + 9/5*a*b^2*x^(5/3) + 9/4*a^2*b*x^(4/3) + a^3*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(3/2), x)
```

Giac [A]
time = 4.75, size = 64, normalized size = 0.47

$$\frac{1}{2}\,b^3x^2\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{9}{5}\,ab^2x^{\frac{5}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+\frac{9}{4}\,a^2bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+a^3x\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sgn(b*x^(1/3) + a) + 9/5*a*b^2*x^(5/3)*sgn(b*x^(1/3) + a) + 9/4
*a^2*b*x^(4/3)*sgn(b*x^(1/3) + a) + a^3*x*sgn(b*x^(1/3) + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)
```

```
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

3.465 $\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx$

Optimal. Leaf size=88

$$\frac{a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x}{a+b\sqrt[3]{x}}+\frac{3b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x^{4/3}}{4\left(a+b\sqrt[3]{x}\right)}$$

```
[Out] a*x*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/(a+b*x^(1/3))+3/4*b*x^(4/3)*(a^2+
2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)/(a+b*x^(1/3))
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {1355, 660, 45}

$$\frac{3bx^{4/3}\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{4\left(a+b\sqrt[3]{x}\right)}+\frac{ax\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}{a+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (a*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x)/(a + b*x^(1/3)) + (3*b*Sqrt[a
^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]*x^(4/3))/(4*(a + b*x^(1/3)))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
```

```
ctionQ[n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,dx &= 3\text{Subst}\left(\int x^2\sqrt{a^2+2abx+b^2x^2}\,dx, x, \sqrt[3]{x}\right)\\ &= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int x^2(ab+b^2x)\,dx, x, \sqrt[3]{x}\right)}{b\left(a+b\sqrt[3]{x}\right)}\\ &= \frac{\left(3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\right)\text{Subst}\left(\int (abx^2+b^2x^3)\,dx, x, \sqrt[3]{x}\right)}{b\left(a+b\sqrt[3]{x}\right)}\\ &= \frac{a\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x}{a+b\sqrt[3]{x}}+\frac{3b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}\,x^{4/3}}{4\left(a+b\sqrt[3]{x}\right)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.49

$$\frac{\sqrt{\left(a+b\sqrt[3]{x}\right)^2}\,\left(4ax+3bx^{4/3}\right)}{4\left(a+b\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
[Out] (Sqrt[(a + b*x^(1/3))^2]*(4*a*x + 3*b*x^(4/3)))/(4*(a + b*x^(1/3)))
```

Maple [A]
time = 0.09, size = 43, normalized size = 0.49

method	result	size
derivativedivides	$\frac{\mathrm{csgn}\left(a+b\,x^{\frac{1}{3}}\right)\left(a+b\,x^{\frac{1}{3}}\right)^{2}\left(3b^{2}x^{\frac{2}{3}}-2ab\,x^{\frac{1}{3}}+a^{2}\right)}{4b^{3}}$	42
default	$\frac{\sqrt{a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}}\,\left(3b\,x^{\frac{4}{3}}+4ax\right)}{4a+4b\,x^{\frac{1}{3}}}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(3*b*x^(4/3)+4*a*x)/(a+b*x^(1/3))
```

Maxima [A]
time = 0.42, size = 114, normalized size = 1.30

$$\frac{3\sqrt{b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2}\,a^2x^{\frac{1}{3}}}{2\,b^2}+\frac{3\sqrt{b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2}\,a^3}{2\,b^3}+\frac{3\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}x^{\frac{1}{3}}}{4\,b^2}-\frac{5\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^{\frac{3}{2}}a}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^2*x^(1/3)/b^2 + 3/2*sqrt(b^2*
x^(2/3) + 2*a*b*x^(1/3) + a^2)*a^3/b^3 + 3/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
 a^2)^(3/2)*x^(1/3)/b^2 - 5/4*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^(3/2)*a/b
^3
```

Fricas [A]
time = 0.32, size = 10, normalized size = 0.11

$$\frac{3}{4}\,bx^{\frac{4}{3}}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
```

```
[Out] 3/4*b*x^(4/3) + a*x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), x)
```

Giac [A]
time = 4.42, size = 26, normalized size = 0.30

$$\frac{3}{4}\,bx^{\frac{4}{3}}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)+ax\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*b*x^(4/3)*sgn(b*x^(1/3) + a) + a*x*sgn(b*x^(1/3) + a)
```

Mupad [B]
time = 1.56, size = 71, normalized size = 0.81

$$\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^3-4\,a^2\,b\,x^{1/3}-5\,a\,b^2\,x^{2/3}+3\,b\,x^{1/3}\,\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2),x)
```

```
[Out] ((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^3 - 4*a^2*b*x^(1/3) - 5*a*b^2
*x^(2/3) + 3*b*x^(1/3)*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))))/(4*b^3)
```

3.466 $\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx$

Optimal. Leaf size=147

$$-\frac{3a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)x^{2/3}}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] -3*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3/2*(a
+b*x^(1/3))*x^(2/3)/b/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3*a^2*(a+b*x^(1
/3))*ln(a+b*x^(1/3))/b^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 660, 45}

$$-\frac{3a\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3x^{2/3}\left(a+b\sqrt[3]{x}\right)}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (-3*a*(a + b*x^(1/3))*x^(1/3))/(b^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]
) + (3*(a + b*x^(1/3))*x^(2/3))/(2*b*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)
]) + (3*a^2*(a + b*x^(1/3))*Log[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/
3) + b^2*x^(2/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\,dx = 3\text{Subst}\left(\int \frac{x^2}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3b\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+b^2x}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= \frac{\left(3b\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(-\frac{a}{b^3}+\frac{x}{b^2}+\frac{a^2}{b^3(a+bx)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$
$$= -\frac{3a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{b^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)x^{2/3}}{2b\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3a^2\left(a+b\right.}{b^3\sqrt{a^2-}}$$

Mathematica [A]
time = 0.03, size = 65, normalized size = 0.44

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(b\left(-2a+b\sqrt[3]{x}\right)\sqrt[3]{x}+2a^2\log\left(a+b\sqrt[3]{x}\right)\right)}{2b^3\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)],x]
```

```
[Out] (3*(a + b*x^(1/3))*(b*(-2*a + b*x^(1/3))*x^(1/3) + 2*a^2*Log[a + b*x^(1/3)]
))/(2*b^3*Sqrt[(a + b*x^(1/3))^2])
```

Maple [A]
time = 0.06, size = 103, normalized size = 0.70

method	result	si
derivativedivides	$\frac{3\left(a+b\,x^{\frac{1}{3}}\right)\left(b^2x^{\frac{2}{3}}+2a^2\ln\left(a+b\,x^{\frac{1}{3}}\right)-2ab\,x^{\frac{1}{3}}\right)}{2\sqrt{\left(a+b\,x^{\frac{1}{3}}\right)^2}\,b^3}$	52

default	$\frac{\sqrt{a^{2}+2ab\,x^{\frac{1}{3}}+b^{2}x^{\frac{2}{3}}}\,\left(3b^{2}x^{\frac{2}{3}}-6ab\,x^{\frac{1}{3}}+2a^{2}\ln\left(b^{3}x+a^{3}\right)+4a^{2}\ln\left(a+b\,x^{\frac{1}{3}}\right)-2a^{2}\ln\left(b^{2}x^{\frac{2}{3}}-ab\,x^{\frac{1}{3}}+a^{2}\right)\right)}{2\left(a+b\,x^{\frac{1}{3}}\right)b^{3}}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(3*b^2*x^(2/3)-6*a*b*x^(1/3)+2*a^
2*ln(b^3*x+a^3)+4*a^2*ln(a+b*x^(1/3))-2*a^2*ln(b^2*x^(2/3)-a*b*x^(1/3)+a^2)
)/(a+b*x^(1/3))/b^3
```

Maxima [A]
time = 0.32, size = 36, normalized size = 0.24

$$\frac{3\,a^{2}\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{b^{3}}+\frac{3\,x^{\frac{2}{3}}}{2\,b}-\frac{3\,ax^{\frac{1}{3}}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="maxima")
[Out] 3*a^2*log(x^(1/3) + a/b)/b^3 + 3/2*x^(2/3)/b - 3*a*x^(1/3)/b^2
```

Fricas [A]
time = 0.36, size = 33, normalized size = 0.22

$$\frac{3\left(2\,a^{2}\log\left(bx^{\frac{1}{3}}+a\right)+b^{2}x^{\frac{2}{3}}-2\,abx^{\frac{1}{3}}\right)}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="fricas")
[Out] 3/2*(2*a^2*log(b*x^(1/3) + a) + b^2*x^(2/3) - 2*a*b*x^(1/3))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{a^{2}+2ab\sqrt[3]{x}+b^{2}x^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(1/2),x)
[Out] Integral(1/sqrt(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3)), x)
```

Giac [A]
time = 3.89, size = 61, normalized size = 0.41

$$\frac{3\left(bx^{\frac{2}{3}}\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)-2\,ax^{\frac{1}{3}}\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)\right)}{2\,b^2}+\frac{3\,a^2\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^3\operatorname{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*(b*x^(2/3)*sgn(b*x^(1/3) + a) - 2*a*x^(1/3)*sgn(b*x^(1/3) + a))/b^2 + 3
*a^2*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2), x)
```

3.467 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] 6*a/b^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/2*a^2/b^3/(a+b*x^(1/3))/(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+3*(a+b*x^(1/3))*ln(a+b*x^(1/3))/b^3/(a^2
+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 660, 45}

$$-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{3\left(a+b\sqrt[3]{x}\right)\log\left(a+b\sqrt[3]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
```

```
[Out] (6*a)/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - (3*a^2)/(2*b^3*(a + b
*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) + (3*(a + b*x^(1/3))*Log
[a + b*x^(1/3)])/(b^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
```

```
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{3/2}}\,dx &= 3\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{\left(3b^3\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= \frac{\left(3b^3\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^5(a+bx)^3}-\frac{2a}{b^5(a+bx)^2}+\frac{1}{b^5(a+bx)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= \frac{6a}{b^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3a^2}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 0.55

$$\frac{3a\left(3a+4b\sqrt[3]{x}\right)+6\left(a+b\sqrt[3]{x}\right)^2\log\left(a+b\sqrt[3]{x}\right)}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{\left(a+b\sqrt[3]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-3/2),x]
[Out] (3*a*(3*a + 4*b*x^(1/3)) + 6*(a + b*x^(1/3))^2*Log[a + b*x^(1/3)])/(2*b^3*(
a + b*x^(1/3))*Sqrt[(a + b*x^(1/3))^2])
```

Maple [A]
time = 0.04, size = 92, normalized size = 0.71

method	result	size
derivativedivides	$\frac{3\left(2\ln\left(a+b\,x^{\frac{1}{3}}\right)b^2x^{\frac{2}{3}}+4\ln\left(a+b\,x^{\frac{1}{3}}\right)ab\,x^{\frac{1}{3}}+2a^2\ln\left(a+b\,x^{\frac{1}{3}}\right)+4ab\,x^{\frac{1}{3}}+3a^2\right)\left(a+b\,x^{\frac{1}{3}}\right)}{2b^3\left(\left(a+b\,x^{\frac{1}{3}}\right)^2\right)^{\frac{3}{2}}}$	81
default	$\frac{3\sqrt{a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}}\left(2\ln\left(a+b\,x^{\frac{1}{3}}\right)b^2x^{\frac{2}{3}}+4\ln\left(a+b\,x^{\frac{1}{3}}\right)ab\,x^{\frac{1}{3}}+2a^2\ln\left(a+b\,x^{\frac{1}{3}}\right)+4ab\,x^{\frac{1}{3}}+3a^2\right)}{2\left(a+b\,x^{\frac{1}{3}}\right)^3b^3}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(2*ln(a+b*x^(1/3))*b^2*x^(2/3)+4*
ln(a+b*x^(1/3))*a*b*x^(1/3)+2*a^2*ln(a+b*x^(1/3))+4*a*b*x^(1/3)+3*a^2)/(a+b
*x^(1/3))^3/b^3
```

Maxima [A]
time = 0.41, size = 55, normalized size = 0.42

$$\frac{3\,\log\left(x^{\frac{1}{3}}+\frac{a}{b}\right)}{b^3}+\frac{6\,ax^{\frac{1}{3}}}{b^4\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}+\frac{9\,a^2}{2\,b^5\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="maxima")
```

```
[Out] 3*log(x^(1/3) + a/b)/b^3 + 6*a*x^(1/3)/(b^4*(x^(1/3) + a/b)^2) + 9/2*a^2/(b
^5*(x^(1/3) + a/b)^2)
```

Fricas [A]
time = 0.36, size = 113, normalized size = 0.87

$$\frac{3\left(6\,a^3b^3x+3\,a^6+2\,(b^6x^2+2\,a^3b^3x+a^6)\log\left(bx^{\frac{1}{3}}+a\right)+(4\,ab^5x+a^4b^2)x^{\frac{2}{3}}-(5\,a^2b^4x+2\,a^5b)x^{\frac{1}{3}}\right)}{2\,(b^9x^2+2\,a^3b^6x+a^6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="fricas")
```

```
[Out] 3/2*(6*a^3*b^3*x + 3*a^6 + 2*(b^6*x^2 + 2*a^3*b^3*x + a^6)*log(b*x^(1/3) +
a) + (4*a*b^5*x + a^4*b^2)*x^(2/3) - (5*a^2*b^4*x + 2*a^5*b)*x^(1/3))/(b^9*
x^2 + 2*a^3*b^6*x + a^6*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-3/2), x)
```

Giac [A]
time = 4.08, size = 64, normalized size = 0.49

$$\frac{3\,\log\left(\left|bx^{\frac{1}{3}}+a\right|\right)}{b^{3}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}+\frac{3\left(4\,ax^{\frac{1}{3}}+\frac{3\,a^{2}}{b}\right)}{2\left(bx^{\frac{1}{3}}+a\right)^{2}b^{2}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(3/2),x, algorithm="giac")
```

```
[Out] 3*log(abs(b*x^(1/3) + a))/(b^3*sgn(b*x^(1/3) + a)) + 3/2*(4*a*x^(1/3) + 3*a
^2/b)/((b*x^(1/3) + a)^2*b^2*sgn(b*x^(1/3) + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(3/2), x)
```

3.468 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

```
[Out] -3/4*a^2/b^3/(a+b*x^(1/3))^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+2*a/b^3/
(a+b*x^(1/3))^2/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/2/b^3/(a+b*x^(1/3))
/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 660, 45}

$$-\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{2b^3\left(a+b\sqrt[3]{x}\right)\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
```

```
[Out] (-3*a^2)/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
+ (2*a)/(b^3*(a + b*x^(1/3))^2*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3
/(2*b^3*(a + b*x^(1/3))*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
```

```
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{5/2}}\,dx &= 3\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, \sqrt[3]{x}\right)\\
&= \frac{\left(3b^5\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^5}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= \frac{\left(3b^5\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^7(a+bx)^5}-\frac{2a}{b^7(a+bx)^4}+\frac{1}{b^7(a+bx)^3}\right)dx, x, \sqrt[3]{x}\right.}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= -\frac{3a^2}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{b^3\left(a+b\sqrt[3]{x}\right)^2\sqrt{a^2+2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.41

$$\frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-4ab\sqrt[3]{x}-6b^2x^{2/3}\right)}{4b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-5/2),x]
[Out] ((a + b*x^(1/3))*(-a^2 - 4*a*b*x^(1/3) - 6*b^2*x^(2/3)))/(4*b^3*((a + b*x^(
1/3))^2)^(5/2))
```

Maple [A]
time = 0.06, size = 54, normalized size = 0.40

method	result	size
derivativedivides	$-\frac{\left(6b^2x^{\frac{2}{3}}+4ab\,x^{\frac{1}{3}}+a^2\right)\left(a+b\,x^{\frac{1}{3}}\right)}{4b^3\left(\left(a+b\,x^{\frac{1}{3}}\right)^2\right)^{\frac{5}{2}}}$	43
default	$-\frac{\sqrt{a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}}\left(6b^2x^{\frac{2}{3}}+4ab\,x^{\frac{1}{3}}+a^2\right)}{4\left(a+b\,x^{\frac{1}{3}}\right)^5b^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(6*b^2*x^(2/3)+4*a*b*x^(1/3)+a^2
)/(a+b*x^(1/3))^5/b^3
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.39

$$-\frac{3}{2\,b^5\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^2}+\frac{2\,a}{b^6\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^3}-\frac{3\,a^2}{4\,b^7\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="maxima")
```

```
[Out] -3/2/(b^5*(x^(1/3) + a/b)^2) + 2*a/(b^6*(x^(1/3) + a/b)^3) - 3/4*a^2/(b^7*(
x^(1/3) + a/b)^4)
```

Fricas [A]
time = 0.38, size = 136, normalized size = 1.01

$$\frac{20\,ab^9x^3-60\,a^4b^6x^2-a^{10}-9\,(5\,a^2b^8x^2-4\,a^5b^5x)x^{\frac{2}{3}}-3\,(2\,b^{10}x^3-20\,a^3b^7x^2+5\,a^6b^4x)x^{\frac{1}{3}}}{4\,(b^{15}x^4+4\,a^3b^{12}x^3+6\,a^6b^9x^2+4\,a^9b^6x+a^{12}b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4*(20*a*b^9*x^3 - 60*a^4*b^6*x^2 - a^10 - 9*(5*a^2*b^8*x^2 - 4*a^5*b^5*x)
*x^(2/3) - 3*(2*b^10*x^3 - 20*a^3*b^7*x^2 + 5*a^6*b^4*x)*x^(1/3))/(b^15*x^4
 + 4*a^3*b^12*x^3 + 6*a^6*b^9*x^2 + 4*a^9*b^6*x + a^12*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-5/2), x)
```

Giac [A]
time = 2.60, size = 43, normalized size = 0.32

$$-\frac{6\,b^2x^{\frac{2}{3}}+4\,abx^{\frac{1}{3}}+a^2}{4\left(bx^{\frac{1}{3}}+a\right)^4b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*(6*b^2*x^(2/3) + 4*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^4*b^3*sgn(b*x^(
1/3) + a))
```

Mupad [B]
time = 2.80, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+6\,b^2\,x^{2/3}+4\,a\,b\,x^{1/3}\right)}{4\,b^3\,{\left(a+b\,x^{1/3}\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(5/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 6*b^2*x^(2/3) + 4*a*b*x^
(1/3)))/(4*b^3*(a + b*x^(1/3))^5)
```

3.469 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{}}$$

```
[Out] -1/2*a^2/b^3/(a+b*x^(1/3))^5/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+6/5*a/b^
3/(a+b*x^(1/3))^4/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/4/b^3/(a+b*x^(1/3
))^3/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$-\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{4b^3\left(a+b\sqrt[3]{x}\right)^3\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

```
[Out] -1/2*a^2/(b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) +
(6*a)/(5*b^3*(a + b*x^(1/3))^4*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) - 3
/(4*b^3*(a + b*x^(1/3))^3*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
```

```
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{7/2}}\,dx &= 3\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{7/2}}\,dx,x,\sqrt[3]{x}\right)\\
&= \frac{\left(3b^7\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^7}\,dx,x,\sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= \frac{\left(3b^7\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^9(a+bx)^7}-\frac{2a}{b^9(a+bx)^6}+\frac{1}{b^9(a+bx)^5}\right)\,dx,x,\sqrt[3]{x}\right.}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}\\
&= -\frac{a^2}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{5b^3\left(a+b\sqrt[3]{x}\right)^4\sqrt{a^2+}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.41

$$\frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-6ab\sqrt[3]{x}-15b^2x^{2/3}\right)}{20b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-7/2),x]
```

```
[Out] ((a + b*x^(1/3))*(-a^2 - 6*a*b*x^(1/3) - 15*b^2*x^(2/3)))/(20*b^3*((a + b*x
^(1/3))^2)^(7/2))
```

Maple [A]
time = 0.05, size = 54, normalized size = 0.39

method	result	size
derivativedivides	$-\frac{\left(15b^2x^{\frac{2}{3}}+6abx^{\frac{1}{3}}+a^2\right)\left(a+bx^{\frac{1}{3}}\right)}{20b^3\left(\left(a+bx^{\frac{1}{3}}\right)^2\right)^{\frac{7}{2}}}$	43
default	$-\frac{\sqrt{a^2+2abx^{\frac{1}{3}}+b^2x^{\frac{2}{3}}}\left(15b^2x^{\frac{2}{3}}+6abx^{\frac{1}{3}}+a^2\right)}{20\left(a+bx^{\frac{1}{3}}\right)^7b^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/20*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(15*b^2*x^(2/3)+6*a*b*x^(1/3)+a
^2)/(a+b*x^(1/3))^7/b^3
```

Maxima [A]
time = 0.42, size = 53, normalized size = 0.39

$$-\frac{3}{4\,b^{7}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{4}}+\frac{6\,a}{5\,b^{8}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{5}}-\frac{a^{2}}{2\,b^{9}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="maxima")
```

```
[Out] -3/4/(b^7*(x^(1/3) + a/b)^4) + 6/5*a/(b^8*(x^(1/3) + a/b)^5) - 1/2*a^2/(b^9
*(x^(1/3) + a/b)^6)
```

Fricas [A]
time = 0.40, size = 209, normalized size = 1.53

$$-\frac{280\,a^{2}b^{12}x^{4}-1400\,a^{5}b^{9}x^{3}+735\,a^{8}b^{6}x^{2}-14\,a^{11}b^{3}x+a^{14}+3\,(5\,b^{14}x^{4}-210\,a^{3}b^{11}x^{3}+483\,a^{6}b^{8}x^{2}-112\,a^{9}b^{5}x)x^{\frac{2}{3}}-3\,(28\,ab^{13}x^{4}-357\,a^{4}b^{10}x^{3}+390\,a^{7}b^{7}x^{2}-35\,a^{10}b^{4}x)x^{\frac{1}{3}}}{20\,(b^{21}x^{6}+6\,a^{3}b^{18}x^{5}+15\,a^{6}b^{15}x^{4}+20\,a^{9}b^{12}x^{3}+15\,a^{12}b^{9}x^{2}+6\,a^{15}b^{6}x+a^{18}b^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="fricas")
```

```
[Out] -1/20*(280*a^2*b^12*x^4 - 1400*a^5*b^9*x^3 + 735*a^8*b^6*x^2 - 14*a^11*b^3*
x + a^14 + 3*(5*b^14*x^4 - 210*a^3*b^11*x^3 + 483*a^6*b^8*x^2 - 112*a^9*b^5
*x)*x^(2/3) - 3*(28*a*b^13*x^4 - 357*a^4*b^10*x^3 + 390*a^7*b^7*x^2 - 35*a^
10*b^4*x)*x^(1/3))/(b^21*x^6 + 6*a^3*b^18*x^5 + 15*a^6*b^15*x^4 + 20*a^9*b^
12*x^3 + 15*a^12*b^9*x^2 + 6*a^15*b^6*x + a^18*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2ab\sqrt[3]{x}+b^{2}x^{\frac{2}{3}}\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(7/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-7/2), x)
```

Giac [A]
time = 3.52, size = 43, normalized size = 0.31

$$-\frac{15\,b^{2}x^{\frac{2}{3}}+6\,abx^{\frac{1}{3}}+a^{2}}{20\left(bx^{\frac{1}{3}}+a\right)^{6}b^{3}\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(7/2),x, algorithm="giac")
```

```
[Out] -1/20*(15*b^2*x^(2/3) + 6*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^6*b^3*sgn(b*x
^(1/3) + a))
```

Mupad [B]
time = 3.23, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+15\,b^2\,x^{2/3}+6\,a\,b\,x^{1/3}\right)}{20\,b^3\,{\left(a+b\,x^{1/3}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(7/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 15*b^2*x^(2/3) + 6*a*b*x
^(1/3)))/(20*b^3*(a + b*x^(1/3))^7)
```

3.470 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{}}$$

```
[Out] -3/8*a^2/b^3/(a+b*x^(1/3))^7/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+6/7*a/b^
3/(a+b*x^(1/3))^6/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-1/2/b^3/(a+b*x^(1/3
))^5/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 660, 45}

$$-\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{1}{2b^3\left(a+b\sqrt[3]{x}\right)^5\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
```

```
[Out] (-3*a^2)/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
+ (6*a)/(7*b^3*(a + b*x^(1/3))^6*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)]) -
 1/(2*b^3*(a + b*x^(1/3))^5*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
```

```
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{9/2}}\,dx = 3\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{9/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^9\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^9}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{\left(3b^9\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^{11}(a+bx)^9}-\frac{2a}{b^{11}(a+bx)^8}+\frac{1}{b^{11}(a+bx)^7}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= -\frac{3a^2}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{6a}{7b^3\left(a+b\sqrt[3]{x}\right)^6\sqrt{a^2+}}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.41

$$\frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-8ab\sqrt[3]{x}-28b^2x^{2/3}\right)}{56b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-9/2),x]
[Out] ((a + b*x^(1/3))*(-a^2 - 8*a*b*x^(1/3) - 28*b^2*x^(2/3)))/(56*b^3*((a + b*x
^(1/3))^2)^(9/2))
```

Maple [A]
time = 0.04, size = 54, normalized size = 0.39

method	result	size
derivativedivides	$-\frac{\left(28b^2x^{\frac{2}{3}}+8ab\,x^{\frac{1}{3}}+a^2\right)\left(a+b\,x^{\frac{1}{3}}\right)}{56b^3\left(\left(a+b\,x^{\frac{1}{3}}\right)^2\right)^{\frac{9}{2}}}$	43
default	$-\frac{\sqrt{a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}}\left(28b^2x^{\frac{2}{3}}+8ab\,x^{\frac{1}{3}}+a^2\right)}{56\left(a+b\,x^{\frac{1}{3}}\right)^9b^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/56*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(28*b^2*x^(2/3)+8*a*b*x^(1/3)+a
^2)/(a+b*x^(1/3))^9/b^3
```

Maxima [A]
time = 0.31, size = 53, normalized size = 0.39

$$-\frac{1}{2\,b^9\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^6}+\frac{6\,a}{7\,b^{10}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^7}-\frac{3\,a^2}{8\,b^{11}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2/(b^9*(x^(1/3) + a/b)^6) + 6/7*a/(b^10*(x^(1/3) + a/b)^7) - 3/8*a^2/(b^
11*(x^(1/3) + a/b)^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(107) = 214.
time = 0.39, size = 275, normalized size = 2.01

$$-\frac{28\,b^{18}x^6-2856\,a^3b^{15}x^5+18186\,a^6b^{12}x^4-20608\,a^9b^9x^3+4200\,a^{12}b^6x^2-48\,a^{15}b^3x+a^{18}-27\,(8\,ab^{17}x^5-244\,a^4b^{14}x^4+840\,a^7b^{11}x^3-553\,a^{10}b^8x^2+56\,a^{13}b^5x)x^{\frac{2}{3}}+27\,(35\,a^2b^{16}x^5-448\,a^5b^{13}x^4+876\,a^8b^{10}x^3-328\,a^{11}b^7x^2+14\,a^{14}b^4x)x^{\frac{1}{3}}}{56\,(b^{27}x^8+8\,a^3b^{24}x^7+28\,a^6b^{21}x^6+56\,a^9b^{18}x^5+70\,a^{12}b^{15}x^4+56\,a^{15}b^{12}x^3+28\,a^{18}b^9x^2+8\,a^{21}b^6x+a^{24}b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="fricas")
```

```
[Out] -1/56*(28*b^18*x^6 - 2856*a^3*b^15*x^5 + 18186*a^6*b^12*x^4 - 20608*a^9*b^9
*x^3 + 4200*a^12*b^6*x^2 - 48*a^15*b^3*x + a^18 - 27*(8*a*b^17*x^5 - 244*a^
4*b^14*x^4 + 840*a^7*b^11*x^3 - 553*a^10*b^8*x^2 + 56*a^13*b^5*x)*x^(2/3) +
 27*(35*a^2*b^16*x^5 - 448*a^5*b^13*x^4 + 876*a^8*b^10*x^3 - 328*a^11*b^7*x
^2 + 14*a^14*b^4*x)*x^(1/3))/(b^27*x^8 + 8*a^3*b^24*x^7 + 28*a^6*b^21*x^6 +
 56*a^9*b^18*x^5 + 70*a^12*b^15*x^4 + 56*a^15*b^12*x^3 + 28*a^18*b^9*x^2 +
8*a^21*b^6*x + a^24*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{\frac{2}{3}}\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(9/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-9/2), x)
```

Giac [A]
time = 4.85, size = 43, normalized size = 0.31

$$-\frac{28\,b^2x^{\frac{2}{3}}+8\,abx^{\frac{1}{3}}+a^2}{56\left(bx^{\frac{1}{3}}+a\right)^8b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(9/2),x, algorithm="giac")
```

```
[Out] -1/56*(28*b^2*x^(2/3) + 8*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^8*b^3*sgn(b*x
^(1/3) + a))
```

Mupad [B]
time = 3.65, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+28\,b^2\,x^{2/3}+8\,a\,b\,x^{1/3}\right)}{56\,b^3\,{\left(a+b\,x^{1/3}\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(9/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 28*b^2*x^(2/3) + 8*a*b*x
^(1/3)))/(56*b^3*(a + b*x^(1/3))^9)
```

3.471 $\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{}}$$

```
[Out] -3/10*a^2/b^3/(a+b*x^(1/3))^9/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)+2/3*a/b
^3/(a+b*x^(1/3))^8/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)-3/8/b^3/(a+b*x^(1/
3))^7/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1355, 660, 45}

$$-\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}-\frac{3}{8b^3\left(a+b\sqrt[3]{x}\right)^7\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

```
[Out] (-3*a^2)/(10*b^3*(a + b*x^(1/3))^9*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
 + (2*a)/(3*b^3*(a + b*x^(1/3))^8*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
- 3/(8*b^3*(a + b*x^(1/3))^7*Sqrt[a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
```

```
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^{11/2}}\,dx = 3\text{Subst}\left(\int \frac{x^2}{\left(a^2+2abx+b^2x^2\right)^{11/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^{11}\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \frac{x^2}{(ab+b^2x)^{11}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= \frac{\left(3b^{11}\left(a+b\sqrt[3]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^2}{b^{13}(a+bx)^{11}}-\frac{2a}{b^{13}(a+bx)^{10}}+\frac{1}{b^{13}(a+bx)^9}\right)\,dx, x\right.}{\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}$$

$$= -\frac{3a^2}{10b^3\left(a+b\sqrt[3]{x}\right)^9\sqrt{a^2+2ab\sqrt[3]{x}+b^2x^{2/3}}}+\frac{2a}{3b^3\left(a+b\sqrt[3]{x}\right)^8\sqrt{a^2}}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 0.41

$$\frac{\left(a+b\sqrt[3]{x}\right)\left(-a^2-10ab\sqrt[3]{x}-45b^2x^{2/3}\right)}{120b^3\left(\left(a+b\sqrt[3]{x}\right)^2\right)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^(-11/2),x]
```

```
[Out] ((a + b*x^(1/3))*(-a^2 - 10*a*b*x^(1/3) - 45*b^2*x^(2/3)))/(120*b^3*((a + b
*x^(1/3))^2)^(11/2))
```

Maple [A]
time = 0.05, size = 54, normalized size = 0.39

method	result	size
derivativedivides	$-\frac{\left(45b^2x^{\frac{2}{3}}+10abx^{\frac{1}{3}}+a^2\right)\left(a+bx^{\frac{1}{3}}\right)}{120b^3\left(\left(a+bx^{\frac{1}{3}}\right)^2\right)^{\frac{11}{2}}}$	43
default	$-\frac{\sqrt{a^2+2abx^{\frac{1}{3}}+b^2x^{\frac{2}{3}}}\left(45b^2x^{\frac{2}{3}}+10abx^{\frac{1}{3}}+a^2\right)}{120\left(a+bx^{\frac{1}{3}}\right)^{11}b^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(1/2)*(45*b^2*x^(2/3)+10*a*b*x^(1/3)
+a^2)/(a+b*x^(1/3))^11/b^3
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.39

$$-\frac{3}{8\,b^{11}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{8}}+\frac{2\,a}{3\,b^{12}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{9}}-\frac{3\,a^{2}}{10\,b^{13}\left(x^{\frac{1}{3}}+\frac{a}{b}\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="maxima")
```

```
[Out] -3/8/(b^11*(x^(1/3) + a/b)^8) + 2/3*a/(b^12*(x^(1/3) + a/b)^9) - 3/10*a^2/(
b^13*(x^(1/3) + a/b)^10)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(107) = 214.
time = 0.39, size = 343, normalized size = 2.50

$$\frac{440\,ab^{21}x^{7}-25630\,a^{4}b^{18}x^{6}+186252\,a^{7}b^{15}x^{5}-326150\,a^{10}b^{12}x^{4}+154000\,a^{13}b^{9}x^{3}-16005\,a^{16}b^{6}x^{2}+110\,a^{19}b^{3}x-a^{22}-27\,(88\,a^{2}b^{20}x^{6}-2200\,a^{5}b^{17}x^{5}+9625\,a^{8}b^{14}x^{4}-10910\,a^{11}b^{11}x^{3}+3245\,a^{14}b^{8}x^{2}-176\,a^{17}b^{5}x)x^{\frac{2}{3}}-9\,(5\,b^{22}x^{7}-990\,a^{3}b^{19}x^{6}+12705\,a^{6}b^{16}x^{5}-34760\,a^{9}b^{13}x^{4}+25542\,a^{12}b^{10}x^{3}-4620\,a^{15}b^{7}x^{2}+110\,a^{18}b^{4}x)x^{\frac{1}{3}}}{120\,(b^{33}x^{10}+10\,a^{3}b^{30}x^{9}+45\,a^{6}b^{27}x^{8}+120\,a^{9}b^{24}x^{7}+210\,a^{12}b^{21}x^{6}+252\,a^{15}b^{18}x^{5}+210\,a^{18}b^{15}x^{4}+120\,a^{21}b^{12}x^{3}+45\,a^{24}b^{9}x^{2}+10\,a^{27}b^{6}x+a^{30}b^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="fricas")
```

```
[Out] 1/120*(440*a*b^21*x^7 - 25630*a^4*b^18*x^6 + 186252*a^7*b^15*x^5 - 326150*a
^10*b^12*x^4 + 154000*a^13*b^9*x^3 - 16005*a^16*b^6*x^2 + 110*a^19*b^3*x -
a^22 - 27*(88*a^2*b^20*x^6 - 2200*a^5*b^17*x^5 + 9625*a^8*b^14*x^4 - 10910*
a^11*b^11*x^3 + 3245*a^14*b^8*x^2 - 176*a^17*b^5*x)*x^(2/3) - 9*(5*b^22*x^7
 - 990*a^3*b^19*x^6 + 12705*a^6*b^16*x^5 - 34760*a^9*b^13*x^4 + 25542*a^12*
b^10*x^3 - 4620*a^15*b^7*x^2 + 110*a^18*b^4*x)*x^(1/3))/(b^33*x^10 + 10*a^3
*b^30*x^9 + 45*a^6*b^27*x^8 + 120*a^9*b^24*x^7 + 210*a^12*b^21*x^6 + 252*a^
15*b^18*x^5 + 210*a^18*b^15*x^4 + 120*a^21*b^12*x^3 + 45*a^24*b^9*x^2 + 10*
a^27*b^6*x + a^30*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^{2}+2ab\sqrt[3]{x}+b^{2}x^{\frac{2}{3}}\right)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**(11/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**(-11/2), x)
```

Giac [A]
time = 5.21, size = 43, normalized size = 0.31

$$-\frac{45\,b^2x^{\frac{2}{3}}+10\,abx^{\frac{1}{3}}+a^2}{120\left(bx^{\frac{1}{3}}+a\right)^{10}b^3\mathrm{sgn}\left(bx^{\frac{1}{3}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^(11/2),x, algorithm="giac")
```

```
[Out] -1/120*(45*b^2*x^(2/3) + 10*a*b*x^(1/3) + a^2)/((b*x^(1/3) + a)^10*b^3*sgn(
b*x^(1/3) + a))
```

Mupad [B]
time = 4.34, size = 53, normalized size = 0.39

$$-\frac{\sqrt{a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}}\,\left(a^2+45\,b^2\,x^{2/3}+10\,a\,b\,x^{1/3}\right)}{120\,b^3\,{\left(a+b\,x^{1/3}\right)}^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(11/2),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^(1/2)*(a^2 + 45*b^2*x^(2/3) + 10*a*b*
x^(1/3)))/(120*b^3*(a + b*x^(1/3))^11)
```

3.472 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx$

Optimal. Leaf size=77

$$\frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x(dx)^m\,{}_2F_1\left(3(1+m),-2p;4+3m;-\frac{b\sqrt[3]{x}}{a}\right)}{1+m}$$

```
[Out] (a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x*(d*x)^m*hypergeom([-2*p, 3+3*m],[4+3*m]
,-b*x^(1/3)/a)/(1+m)/((1+b*x^(1/3)/a)^(2*p))
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1370, 350, 348, 66}

$$\frac{x(dx)^m\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(3(m+1),-2p;3m+4;-\frac{b\sqrt[3]{x}}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

```
[Out] ((a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x*(d*x)^m*Hypergeometric2F1[3*(1 + m
), -2*p, 4 + 3*m, -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 348

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 350

```
Int[((c_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[c^Int
Part[m]*((c*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a + b*x^n)^p, x], x] /;
 FreeQ[{a, b, c, m, p}, x] && FractionQ[n]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p (dx)^m\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int \left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}(dx$$

$$= \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^{-m}(dx)^m\right)\int \left(1+\frac{b\right.$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^{-m}(dx)^m\right)\text{Subst}\left(\int\right.$$

$$= \frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x(dx)^m\,{}_2F_1\left(3(1+m),-\right.}{1+m}$$

Mathematica [A]
time = 0.07, size = 68, normalized size = 0.88

$$\frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}x(dx)^m\,{}_2F_1\left(3(1+m),-2p;1+3(1+m);-\frac{b\sqrt[3]{x}}{a}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*(d*x)^m,x]
```

```
[Out] (((a + b*x^(1/3))^2)^p*x*(d*x)^m*Hypergeometric2F1[3*(1 + m), -2*p, 1 + 3*(
1 + m), -((b*x^(1/3))/a)])/((1 + m)*(1 + (b*x^(1/3))/a)^(2*p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p (dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)
```

[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="maxima")

[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: alglo
gextint: unimplemented

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*(d*x)**m,x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*(d*x)^m,x, algorithm="giac")

[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*(d*x)^m, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)

[Out] int((d*x)^m*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p, x)

3.473 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx$

Optimal. Leaf size=468

$$\frac{3a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+2p)}-\frac{12a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+p)}+\frac{84a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)}{}$$

```
[Out] 3*a^9*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(1+2*p)-12*a^9*
(1+b*x^(1/3)/a)^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(1+p)+84*a^9*(1+b*x
^(1/3)/a)^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(3+2*p)-84*a^9*(1+b*x^(1/
3)/a)^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(2+p)+210*a^9*(1+b*x^(1/3)/a)
^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(5+2*p)-84*a^9*(1+b*x^(1/3)/a)^6*(
a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(3+p)+84*a^9*(1+b*x^(1/3)/a)^7*(a^2+2*
a*b*x^(1/3)+b^2*x^(2/3))^p/b^9/(7+2*p)-12*a^9*(1+b*x^(1/3)/a)^8*(a^2+2*a*b*
x^(1/3)+b^2*x^(2/3))^p/b^9/(4+p)+3*a^9*(1+b*x^(1/3)/a)^9*(a^2+2*a*b*x^(1/3)
+b^2*x^(2/3))^p/b^9/(9+2*p)
```

Rubi [A]
time = 0.16, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1370, 272, 45}

$$\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^9\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+9)}-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^8\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+4)}+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^7\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+7)}-\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+3)}+\frac{210a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+5)}-\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+2)}+\frac{84a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+3)}-\frac{12a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(p+1)}+\frac{3a^9\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

```
[Out] (3*a^9*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(1 +
 2*p)) - (12*a^9*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^
p)/(b^9*(1 + p)) + (84*a^9*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) + b^2
*x^(2/3))^p)/(b^9*(3 + 2*p)) - (84*a^9*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*b*x
^(1/3) + b^2*x^(2/3))^p)/(b^9*(2 + p)) + (210*a^9*(1 + (b*x^(1/3))/a)^5*(a^
2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(5 + 2*p)) - (84*a^9*(1 + (b*x^(1/
3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(3 + p)) + (84*a^9*(1
+ (b*x^(1/3))/a)^7*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*(7 + 2*p)) -
 (12*a^9*(1 + (b*x^(1/3))/a)^8*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^9*
(4 + p)) + (3*a^9*(1 + (b*x^(1/3))/a)^9*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))
^p)/(b^9*(9 + 2*p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$
\begin{aligned}
\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x^2\,dx &= \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}x^2\,dx \\
&= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^8\left(1+\frac{bx}{a}\right)^{2p} \right. \\
&= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(\frac{a^8\left(1+\frac{bx}{a}\right)^{2p}}{b^8}\right.\right. \\
&= \frac{3a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^9(1+2p)}-\frac{12a^9\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2a\right.}{b^9(1+p)}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 207, normalized size = 0.44

$$\frac{3\left(\frac{a^8}{1+2p}-\frac{4a^7\left(a+b\sqrt[3]{x}\right)}{1+p}+\frac{28a^6\left(a+b\sqrt[3]{x}\right)^2}{3+2p}-\frac{28a^5\left(a+b\sqrt[3]{x}\right)^3}{2+p}+\frac{70a^4\left(a+b\sqrt[3]{x}\right)^4}{5+2p}-\frac{28a^3\left(a+b\sqrt[3]{x}\right)^5}{3+p}+\frac{28a^2\left(a+b\sqrt[3]{x}\right)^6}{7+2p}-\frac{4a\left(a+b\sqrt[3]{x}\right)^7}{4+p}+\frac{\left(a+b\sqrt[3]{x}\right)^8}{9+2p}\right)\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x^2,x]
```

```
[Out] (3*(a^8/(1 + 2*p) - (4*a^7*(a + b*x^(1/3)))/(1 + p) + (28*a^6*(a + b*x^(1/3
))^2)/(3 + 2*p) - (28*a^5*(a + b*x^(1/3))^3)/(2 + p) + (70*a^4*(a + b*x^(1/
3))^4)/(5 + 2*p) - (28*a^3*(a + b*x^(1/3))^5)/(3 + p) + (28*a^2*(a + b*x^(1
```

/3))^6)/(7 + 2*p) - (4*a*(a + b*x^(1/3))^7)/(4 + p) + (a + b*x^(1/3))^8/(9 + 2*p))*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p)/b^9

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2}+2 a b \,x^{\frac{1}{3}}+b^{2} x^{\frac{2}{3}}\right)^{p} x^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)

[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x)

Maxima [A]
time = 0.30, size = 362, normalized size = 0.77

$$\frac{3\left((16\,p^8+288\,p^7+2184\,p^6+9072\,p^5+22449\,p^4+33642\,p^3+29531\,p^2+13698\,p+2520)b^9x^3+(16\,p^8+224\,p^7+1288\,p^6+3920\,p^5+6769\,p^4+6566\,p^3+3267\,p^2+630\,p)ab^8x^{\frac{8}{3}}-8\,(8\,p^7+84\,p^6+350\,p^5+735\,p^4+812\,p^3+441\,p^2+90\,p)a^2b^7x^{\frac{7}{3}}+28\,(8\,p^6+60\,p^5+170\,p^4+225\,p^3+137\,p^2+30\,p)a^3b^6x^2-168\,(4\,p^5+20\,p^4+35\,p^3+25\,p^2+6\,p)a^4b^5x^{\frac{5}{3}}+420\,(4\,p^4+12\,p^3+11\,p^2+3\,p)a^5b^4x^{\frac{4}{3}}-1680\,(2\,p^3+3\,p^2+p)a^6b^3x+2520\,(2\,p^2+p)a^7b^2x^{\frac{2}{3}}-5040\,a^8bpx^{\frac{1}{3}}+2520\,a^9\right)\left(bx^{\frac{1}{3}}+a\right)^{2p}}{(32\,p^9+720\,p^8+6960\,p^7+37800\,p^6+126546\,p^5+269325\,p^4+361840\,p^3+293175\,p^2+128322\,p+22680)b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="maxima")

[Out] 3*((16*p^8 + 288*p^7 + 2184*p^6 + 9072*p^5 + 22449*p^4 + 33642*p^3 + 29531*p^2 + 13698*p + 2520)*b^9*x^3 + (16*p^8 + 224*p^7 + 1288*p^6 + 3920*p^5 + 6769*p^4 + 6566*p^3 + 3267*p^2 + 630*p)*a*b^8*x^(8/3) - 8*(8*p^7 + 84*p^6 + 350*p^5 + 735*p^4 + 812*p^3 + 441*p^2 + 90*p)*a^2*b^7*x^(7/3) + 28*(8*p^6 + 60*p^5 + 170*p^4 + 225*p^3 + 137*p^2 + 30*p)*a^3*b^6*x^2 - 168*(4*p^5 + 20*p^4 + 35*p^3 + 25*p^2 + 6*p)*a^4*b^5*x^(5/3) + 420*(4*p^4 + 12*p^3 + 11*p^2 + 3*p)*a^5*b^4*x^(4/3) - 1680*(2*p^3 + 3*p^2 + p)*a^6*b^3*x + 2520*(2*p^2 + p)*a^7*b^2*x^(2/3) - 5040*a^8*b*p*x^(1/3) + 2520*a^9)*(b*x^(1/3) + a)^(2*p)/((32*p^9 + 720*p^8 + 6960*p^7 + 37800*p^6 + 126546*p^5 + 269325*p^4 + 361840*p^3 + 293175*p^2 + 128322*p + 22680)*b^9)

Fricas [A]
time = 0.48, size = 579, normalized size = 1.24

$$\frac{3\left(2520\,a^9+(16\,b^9p^8+288\,b^9p^7+2184\,b^9p^6+9072\,b^9p^5+22449\,b^9p^4+33642\,b^9p^3+29531\,b^9p^2+13698\,b^9p+2520\,b^9)x^3+28\,(8\,a^3b^6p^6+60\,a^3b^6p^5+170\,a^3b^6p^4+225\,a^3b^6p^3+137\,a^3b^6p^2+30\,a^3b^6p)x^2-1680\,(2\,a^6b^3p^3+3\,a^6b^3p^2+a^6b^3p)x+(5040\,a^7b^2p^2+2520\,a^7b^2p+(16\,ab^8p^8+224\,ab^8p^7+1288\,ab^8p^6+3920\,ab^8p^5+6769\,ab^8p^4+6566\,ab^8p^3+3267\,ab^8p^2+630\,ab^8p)x^2-168\,(4\,a^4b^5p^5+20\,a^4b^5p^4+35\,a^4b^5p^3+25\,a^4b^5p^2+6\,a^4b^5p)x)x^{\frac{2}{3}}-4\,(1260\,a^8bp+2\,(8\,a^2b^7p^7+84\,a^2b^7p^6+350\,a^2b^7p^5+735\,a^2b^7p^4+812\,a^2b^7p^3+441\,a^2b^7p^2+90\,a^2b^7p)x^2-105\,(4\,a^5b^4p^4+12\,a^5b^4p^3+11\,a^5b^4p^2+3\,a^5b^4p)x)x^{\frac{1}{3}}\right)\left(b^2x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^2\right)^p}{32\,b^9p^9+720\,b^9p^8+6960\,b^9p^7+37800\,b^9p^6+126546\,b^9p^5+269325\,b^9p^4+361840\,b^9p^3+293175\,b^9p^2+128322\,b^9p+22680\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="fricas")

[Out] 3*(2520*a^9 + (16*b^9*p^8 + 288*b^9*p^7 + 2184*b^9*p^6 + 9072*b^9*p^5 + 22449*b^9*p^4 + 33642*b^9*p^3 + 29531*b^9*p^2 + 13698*b^9*p + 2520*b^9)*x^3 + 28*(8*a^3*b^6*p^6 + 60*a^3*b^6*p^5 + 170*a^3*b^6*p^4 + 225*a^3*b^6*p^3 + 137*a^3*b^6*p^2 + 30*a^3*b^6*p)*x^2 - 1680*(2*a^6*b^3*p^3 + 3*a^6*b^3*p^2 + a^6*b^3*p)*x + (5040*a^7*b^2*p^2 + 2520*a^7*b^2*p + (16*a*b^8*p^8 + 224*a*b^

```
8*p^7 + 1288*a*b^8*p^6 + 3920*a*b^8*p^5 + 6769*a*b^8*p^4 + 6566*a*b^8*p^3 +
 3267*a*b^8*p^2 + 630*a*b^8*p)*x^2 - 168*(4*a^4*b^5*p^5 + 20*a^4*b^5*p^4 +
35*a^4*b^5*p^3 + 25*a^4*b^5*p^2 + 6*a^4*b^5*p)*x)*x^(2/3) - 4*(1260*a^8*b*p
 + 2*(8*a^2*b^7*p^7 + 84*a^2*b^7*p^6 + 350*a^2*b^7*p^5 + 735*a^2*b^7*p^4 +
812*a^2*b^7*p^3 + 441*a^2*b^7*p^2 + 90*a^2*b^7*p)*x^2 - 105*(4*a^5*b^4*p^4
+ 12*a^5*b^4*p^3 + 11*a^5*b^4*p^2 + 3*a^5*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p^7 + 37800*b^
9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293175*b^9*p^2 +
 128322*b^9*p + 22680*b^9)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. $2(414) = 828$.
time = 4.00, size = 1564, normalized size = 3.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x^2,x, algorithm="giac")
```

```
[Out] 3*(16*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^8*x^3 + 16*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^8*x^(8/3) + 288*(b^2*x^(2/3) + 2*a*b*x^(1/3
) + a^2)^p*b^9*p^7*x^3 + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^
7*x^(8/3) - 64*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^7*x^(7/3) +
2184*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^6*x^3 + 1288*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^6*x^(8/3) - 672*(b^2*x^(2/3) + 2*a*b*x^(1/
3) + a^2)^p*a^2*b^7*p^6*x^(7/3) + 224*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^3*b^6*p^6*x^2 + 9072*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*p^5*x^3 +
 3920*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^5*x^(8/3) - 2800*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^5*x^(7/3) + 1680*(b^2*x^(2/3) + 2
*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^5*x^2 + 22449*(b^2*x^(2/3) + 2*a*b*x^(1/3)
+ a^2)^p*b^9*p^4*x^3 - 672*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p^
5*x^(5/3) + 6769*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^4*x^(8/3) -
5880*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^4*x^(7/3) + 4760*(b^2*
x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^4*x^2 + 33642*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*b^9*p^3*x^3 - 3360*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^
p*a^4*b^5*p^4*x^(5/3) + 6566*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p^
```

```
3*x^(8/3) + 1680*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^4*x^(4/3)
- 6496*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^3*x^(7/3) + 6300*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^6*p^3*x^2 + 29531*(b^2*x^(2/3) + 2
*a*b*x^(1/3) + a^2)^p*b^9*p^2*x^3 - 5880*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2
)^p*a^4*b^5*p^3*x^(5/3) + 3267*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*
p^2*x^(8/3) + 5040*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^3*x^(4/3
) - 3528*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p^2*x^(7/3) - 3360*(
b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^6*b^3*p^3*x + 3836*(b^2*x^(2/3) + 2*
a*b*x^(1/3) + a^2)^p*a^3*b^6*p^2*x^2 + 13698*(b^2*x^(2/3) + 2*a*b*x^(1/3) +
 a^2)^p*b^9*p*x^3 - 4200*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p^2*
x^(5/3) + 630*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^8*p*x^(8/3) + 4620*
(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^5*b^4*p^2*x^(4/3) - 720*(b^2*x^(2/3
) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^7*p*x^(7/3) - 5040*(b^2*x^(2/3) + 2*a*b*x^
(1/3) + a^2)^p*a^6*b^3*p^2*x + 840*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^
3*b^6*p*x^2 + 2520*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^9*x^3 + 5040*(b^
2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^7*b^2*p^2*x^(2/3) - 1008*(b^2*x^(2/3)
+ 2*a*b*x^(1/3) + a^2)^p*a^4*b^5*p*x^(5/3) + 1260*(b^2*x^(2/3) + 2*a*b*x^(1
/3) + a^2)^p*a^5*b^4*p*x^(4/3) - 1680*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^6*b^3*p*x + 2520*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^7*b^2*p*x^(2/3)
 - 5040*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^8*b*p*x^(1/3) + 2520*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^9)/(32*b^9*p^9 + 720*b^9*p^8 + 6960*b^9*p
^7 + 37800*b^9*p^6 + 126546*b^9*p^5 + 269325*b^9*p^4 + 361840*b^9*p^3 + 293
175*b^9*p^2 + 128322*b^9*p + 22680*b^9)
```

Mupad [B]
time = 3.52, size = 777, normalized size = 1.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^3*(13698*p + 29531*p^2 + 33642*
p^3 + 22449*p^4 + 9072*p^5 + 2184*p^6 + 288*p^7 + 16*p^8 + 2520))/(128322*p
 + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7
 + 720*p^8 + 32*p^9 + 22680) + (7560*a^9)/(b^9*(128322*p + 293175*p^2 + 361
840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9
 + 22680)) - (15120*a^8*p*x^(1/3))/(b^8*(128322*p + 293175*p^2 + 361840*p^3
 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 2268
0)) + (3*a*p*x^(8/3)*(3267*p + 6566*p^2 + 6769*p^3 + 3920*p^4 + 1288*p^5 +
224*p^6 + 16*p^7 + 630))/(b*(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^
4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)) + (84*a^
3*p*x^2*(137*p + 225*p^2 + 170*p^3 + 60*p^4 + 8*p^5 + 30))/(b^3*(128322*p +
 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 +
 720*p^8 + 32*p^9 + 22680)) - (5040*a^6*p*x*(3*p + 2*p^2 + 1))/(b^6*(128322
```

*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p
^7 + 720*p^8 + 32*p^9 + 22680)) - (24*a^2*p*x^(7/3)*(441*p + 812*p^2 + 735*
p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90))/(b^2*(128322*p + 293175*p^2 + 361840*
p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 2
2680)) + (7560*a^7*p*x^(2/3)*(2*p + 1))/(b^7*(128322*p + 293175*p^2 + 36184
0*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 +
 22680)) + (1260*a^5*p*x^(4/3)*(11*p + 12*p^2 + 4*p^3 + 3))/(b^5*(128322*p
+ 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546*p^5 + 37800*p^6 + 6960*p^7
+ 720*p^8 + 32*p^9 + 22680)) - (504*a^4*p*x^(5/3)*(25*p + 35*p^2 + 20*p^3 +
 4*p^4 + 6))/(b^4*(128322*p + 293175*p^2 + 361840*p^3 + 269325*p^4 + 126546
*p^5 + 37800*p^6 + 6960*p^7 + 720*p^8 + 32*p^9 + 22680)))

3.474 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx$

Optimal. Leaf size=315

$$-\frac{3a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(1+2p)}+\frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(1+p)}-\frac{30a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right.}{}$$

```
[Out] -3*a^6*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(1+2*p)+15/2*a
^6*(1+b*x^(1/3)/a)^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(1+p)-30*a^6*(1+
b*x^(1/3)/a)^3*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(3+2*p)+15*a^6*(1+b*x^
(1/3)/a)^4*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(2+p)-15*a^6*(1+b*x^(1/3)/
a)^5*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(5+2*p)+3/2*a^6*(1+b*x^(1/3)/a)^
6*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^6/(3+p)
```

Rubi [A]

time = 0.09, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1370, 272, 45}

$$\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^6\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+3)}-\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^5\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+5)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^4\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(p+2)}-\frac{30a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+3)}+\frac{15a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{2b^6(p+1)}-\frac{3a^6\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

```
[Out] (-3*a^6*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(1
+ 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))
^p)/(2*b^6*(1 + p)) - (30*a^6*(1 + (b*x^(1/3))/a)^3*(a^2 + 2*a*b*x^(1/3) +
b^2*x^(2/3))^p)/(b^6*(3 + 2*p)) + (15*a^6*(1 + (b*x^(1/3))/a)^4*(a^2 + 2*a*
b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(2 + p)) - (15*a^6*(1 + (b*x^(1/3))/a)^5*(
a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^6*(5 + 2*p)) + (3*a^6*(1 + (b*x^(1
/3))/a)^6*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(2*b^6*(3 + p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p x\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p} x\,dx$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^5\left(1+\frac{bx}{a}\right)^{2p}\right.$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(-\frac{a^5\left(1+\frac{bx}{a}\right)^{2}}{b^5}\right.\right.$$

$$= -\frac{3a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^6(1+2p)}+\frac{15a^6\left(1+\frac{b\sqrt[3]{x}}{a}\right)^2\left(a^2+2\right.}{2b^6(1+p}$$

Mathematica [A]
time = 0.16, size = 143, normalized size = 0.45

$$\frac{3\left(-\frac{2a^5}{1+2p}+\frac{5a^4\left(a+b\sqrt[3]{x}\right)}{1+p}-\frac{20a^3\left(a+b\sqrt[3]{x}\right)^2}{3+2p}+\frac{10a^2\left(a+b\sqrt[3]{x}\right)^3}{2+p}-\frac{10a\left(a+b\sqrt[3]{x}\right)^4}{5+2p}+\frac{\left(a+b\sqrt[3]{x}\right)^5}{3+p}\right)\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*x,x]
```

```
[Out] (3*((-2*a^5)/(1 + 2*p) + (5*a^4*(a + b*x^(1/3)))/(1 + p) - (20*a^3*(a + b*x
^(1/3))^2)/(3 + 2*p) + (10*a^2*(a + b*x^(1/3))^3)/(2 + p) - (10*a*(a + b*x^
(1/3))^4)/(5 + 2*p) + (a + b*x^(1/3))^5/(3 + p))*(a + b*x^(1/3))*((a + b*x^
(1/3))^2)^p)/(2*b^6)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)

[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x)

Maxima [A]
time = 0.31, size = 198, normalized size = 0.63

$$\frac{3\left((8p^5+60p^4+170p^3+225p^2+137p+30)b^6x^2+2(4p^5+20p^4+35p^3+25p^2+6p)ab^5x^{\frac{5}{3}}-5(4p^4+12p^3+11p^2+3p)a^2b^4x^{\frac{4}{3}}+20(2p^3+3p^2+p)a^3b^3x-30(2p^2+p)a^4b^2x^{\frac{2}{3}}+60a^5bpx^{\frac{1}{3}}-30a^6\right)\left(bx^{\frac{1}{3}}+a\right)^{2p}}{2(8p^6+84p^5+350p^4+735p^3+812p^2+441p+90)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="maxima")

[Out] 3/2*((8*p^5 + 60*p^4 + 170*p^3 + 225*p^2 + 137*p + 30)*b^6*x^2 + 2*(4*p^5 + 20*p^4 + 35*p^3 + 25*p^2 + 6*p)*a*b^5*x^(5/3) - 5*(4*p^4 + 12*p^3 + 11*p^2 + 3*p)*a^2*b^4*x^(4/3) + 20*(2*p^3 + 3*p^2 + p)*a^3*b^3*x - 30*(2*p^2 + p)*a^4*b^2*x^(2/3) + 60*a^5*b*p*x^(1/3) - 30*a^6)*(b*x^(1/3) + a)^(2*p)/((8*p^6 + 84*p^5 + 350*p^4 + 735*p^3 + 812*p^2 + 441*p + 90)*b^6)

Fricas [A]
time = 0.45, size = 297, normalized size = 0.94

$$-\frac{3\left(30a^6-(8b^6p^5+60b^6p^4+170b^6p^3+225b^6p^2+137b^6p+30b^6)x^2-20(2a^3b^3p^3+3a^3b^3p^2+a^3b^3p)x+2(30a^4b^2p^2+15a^4b^2p-(4ab^5p^5+20ab^5p^4+35ab^5p^3+25ab^5p^2+6ab^5p)x)x^{\frac{2}{3}}-5(12a^5bp-(4a^2b^4p^4+12a^2b^4p^3+11a^2b^4p^2+3a^2b^4p)x)x^{\frac{1}{3}}\right)\left(b^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+a^2\right)^p}{2(8b^6p^6+84b^6p^5+350b^6p^4+735b^6p^3+812b^6p^2+441b^6p+90b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="fricas")

[Out] -3/2*(30*a^6 - (8*b^6*p^5 + 60*b^6*p^4 + 170*b^6*p^3 + 225*b^6*p^2 + 137*b^6*p + 30*b^6)*x^2 - 20*(2*a^3*b^3*p^3 + 3*a^3*b^3*p^2 + a^3*b^3*p)*x + 2*(30*a^4*b^2*p^2 + 15*a^4*b^2*p - (4*a*b^5*p^5 + 20*a*b^5*p^4 + 35*a*b^5*p^3 + 25*a*b^5*p^2 + 6*a*b^5*p)*x)*x^(2/3) - 5*(12*a^5*b*p - (4*a^2*b^4*p^4 + 12*a^2*b^4*p^3 + 11*a^2*b^4*p^2 + 3*a^2*b^4*p)*x)*x^(1/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 812*b^6*p^2 + 441*b^6*p + 90*b^6)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p*x,x)

[Out] Exception raised: HeuristicGCDFailed >> no luck

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(275) = 550.
time = 4.50, size = 745, normalized size = 2.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*x,x, algorithm="giac")
```

```
[Out] 3/2*(8*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^5*x^2 + 8*(b^2*x^(2/3) +
 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^5*x^(5/3) + 60*(b^2*x^(2/3) + 2*a*b*x^(1/3)
 + a^2)^p*b^6*p^4*x^2 + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^4*
x^(5/3) - 20*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b^4*p^4*x^(4/3) + 17
0*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^3*x^2 + 70*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*a*b^5*p^3*x^(5/3) - 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a
^2)^p*a^2*b^4*p^3*x^(4/3) + 40*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3*b^
3*p^3*x + 225*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p^2*x^2 + 50*(b^2*x
^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p^2*x^(5/3) - 55*(b^2*x^(2/3) + 2*a*b
*x^(1/3) + a^2)^p*a^2*b^4*p^2*x^(4/3) + 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a
^2)^p*a^3*b^3*p^2*x + 137*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*p*x^2 -
 60*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p^2*x^(2/3) + 12*(b^2*x^(
2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^5*p*x^(5/3) - 15*(b^2*x^(2/3) + 2*a*b*x^(
1/3) + a^2)^p*a^2*b^4*p*x^(4/3) + 20*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*
a^3*b^3*p*x + 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^6*x^2 - 30*(b^2*x^
(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^4*b^2*p*x^(2/3) + 60*(b^2*x^(2/3) + 2*a*b*
x^(1/3) + a^2)^p*a^5*b*p*x^(1/3) - 30*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p
*a^6)/(8*b^6*p^6 + 84*b^6*p^5 + 350*b^6*p^4 + 735*b^6*p^3 + 812*b^6*p^2 + 4
41*b^6*p + 90*b^6)
```

Mupad [B]
time = 2.17, size = 390, normalized size = 1.24

$$\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,\left(\frac{3\,x^2\,\left(8\,p^5+60\,p^4+170\,p^3+225\,p^2+137\,p+30\right)}{2\,\left(8\,p^6+84\,p^5+350\,p^4+735\,p^3+812\,p^2+441\,p+90\right)}-\frac{45\,a^6}{b^6\,\left(8\,p^6+84\,p^5+350\,p^4+735\,p^3+812\,p^2+441\,p+90\right)}+\frac{90\,a^5\,p\,x^{1/3}}{b^5\,\left(8\,p^6+84\,p^5+350\,p^4+735\,p^3+812\,p^2+441\,p+90\right)}-\frac{15\,a^2\,p\,x^{4/3}\,\left(4\,p^3+12\,p^2+11\,p+3\right)}{2\,b^2\,\left(8\,p^6+84\,p^5+350\,p^4+735\,p^3+812\,p^2+441\,p+90\right)}+\frac{30\,a^3\,p\,x\,\left(2\,p^2+3\,p+1\right)}{b^3\,\left(8\,p^6+84\,p^5+350\,p^4+735\,p^3+812\,p^2+441\,p+90\right)}-\frac{45\,a^4\,p\,x^{2/3}\,\left(2\,p+1\right)}{b^4\,\left(8\,p^6+84\,p^5+350\,p^4+735\,p^3+812\,p^2+441\,p+90\right)}+\frac{3\,a\,p\,x^{5/3}\,\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}{b\,\left(8\,p^6+84\,p^5+350\,p^4+735\,p^3+812\,p^2+441\,p+90\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x^2*(137*p + 225*p^2 + 170*p^3 +
60*p^4 + 8*p^5 + 30))/(2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*
p^6 + 90)) - (45*a^6)/(b^6*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 +
8*p^6 + 90)) + (90*a^5*p*x^(1/3))/(b^5*(441*p + 812*p^2 + 735*p^3 + 350*p^4
 + 84*p^5 + 8*p^6 + 90)) - (15*a^2*p*x^(4/3)*(11*p + 12*p^2 + 4*p^3 + 3))/(
2*b^2*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (30*a^
3*p*x*(3*p + 2*p^2 + 1))/(b^3*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5
 + 8*p^6 + 90)) - (45*a^4*p*x^(2/3)*(2*p + 1))/(b^4*(441*p + 812*p^2 + 735*
p^3 + 350*p^4 + 84*p^5 + 8*p^6 + 90)) + (3*a*p*x^(5/3)*(25*p + 35*p^2 + 20*
p^3 + 4*p^4 + 6))/(b*(441*p + 812*p^2 + 735*p^3 + 350*p^4 + 84*p^5 + 8*p^6
+ 90)))
```

3.475 $\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p \, dx$

Optimal. Leaf size=142

$$\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+2p)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+p)}+\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2a\right.}{b^3(3+2}$$

```
[Out] 3*a^2*(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(1+2*p)-3*a*(a+b*
x^(1/3))^2*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(1+p)+3*(a+b*x^(1/3))^3*(a
^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/b^3/(3+2*p)
```

Rubi [A]
time = 0.05, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1355, 660, 45}

$$\frac{3\left(a+b\sqrt[3]{x}\right)^3\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+3)}-\frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(p+1)}+\frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
```

```
[Out] (3*a^2*(a + b*x^(1/3))*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1 + 2*p
)) - (3*a*(a + b*x^(1/3))^2*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(1
+ p)) + (3*(a + b*x^(1/3))^3*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(b^3*(3
 + 2*p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
```

`ctionQ[n]`

Rubi steps

$$\int \left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,dx = 3\text{Subst}\left(\int x^2\left(a^2+2abx+b^2x^2\right)^p\,dx, x, \sqrt[3]{x}\right)$$
$$= \left(3\left(b\left(a+b\sqrt[3]{x}\right)\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^{2p}\,d\right.$$
$$= \left(3\left(b\left(a+b\sqrt[3]{x}\right)\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^{2p}}{b^2}\right.\right.$$
$$= \frac{3a^2\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{b^3(1+2p)} - \frac{3a\left(a+b\sqrt[3]{x}\right)^2\left(a^2+2ab\sqrt[3]{x}\right.}{b^3(1+p)}$$

Mathematica [A]
time = 0.07, size = 83, normalized size = 0.58

$$\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(a^2-ab(1+2p)\sqrt[3]{x}+b^2\left(1+3p+2p^2\right)x^{2/3}\right)}{b^3(1+p)(1+2p)(3+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p,x]
[Out] (3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(a^2 - a*b*(1 + 2*p)*x^(1/3) + b^2
*(1 + 3*p + 2*p^2)*x^(2/3)))/(b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x)
```

Maxima [A]
time = 0.28, size = 77, normalized size = 0.54

$$\frac{3\left(\left(2\,p^2+3\,p+1\right)b^3x+\left(2\,p^2+p\right)ab^2x^{\frac{2}{3}}-2\,a^2bpx^{\frac{1}{3}}+a^3\right)\left(bx^{\frac{1}{3}}+a\right)^{2\,p}}{\left(4\,p^3+12\,p^2+11\,p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="maxima")
```

```
[Out] 3*((2*p^2 + 3*p + 1)*b^3*x + (2*p^2 + p)*a*b^2*x^(2/3) - 2*a^2*b*p*x^(1/3)
+ a^3)*(b*x^(1/3) + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Fricas [A]
time = 0.39, size = 110, normalized size = 0.77

$$-\frac{3\left(2\,a^{2}bpx^{\frac{1}{3}}-a^{3}-(2\,b^{3}p^{2}+3\,b^{3}p+b^{3})x-(2\,ab^{2}p^{2}+ab^{2}p)x^{\frac{2}{3}}\right)\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}}{4\,b^{3}p^{3}+12\,b^{3}p^{2}+11\,b^{3}p+3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="fricas")
```

```
[Out] -3*(2*a^2*b*p*x^(1/3) - a^3 - (2*b^3*p^2 + 3*b^3*p + b^3)*x - (2*a*b^2*p^2
+ a*b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(4*b^3*p^3 + 12*b
^3*p^2 + 11*b^3*p + 3*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^{2}+2ab\sqrt[3]{x}+b^{2}x^{\frac{2}{3}}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p,x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p, x)
```

Giac [A]
time = 3.01, size = 229, normalized size = 1.61

$$\frac{3\left(2\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}b^{3}p^{2}x+2\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}ab^{2}p^{2}x^{\frac{2}{3}}+3\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}b^{3}px+\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}ab^{2}px^{\frac{2}{3}}-2\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}a^{2}bpx^{\frac{1}{3}}+\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}b^{3}x+\left(b^{2}x^{\frac{2}{3}}+2\,abx^{\frac{1}{3}}+a^{2}\right)^{p}a^{3}\right)}{4\,b^{3}p^{3}+12\,b^{3}p^{2}+11\,b^{3}p+3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p,x, algorithm="giac")
```

```
[Out] 3*(2*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*p^2*x + 2*(b^2*x^(2/3) + 2*a
*b*x^(1/3) + a^2)^p*a*b^2*p^2*x^(2/3) + 3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^
2)^p*b^3*p*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a*b^2*p*x^(2/3) - 2*(b
^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^2*b*p*x^(1/3) + (b^2*x^(2/3) + 2*a*b*
x^(1/3) + a^2)^p*b^3*x + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*a^3)/(4*b^3*
p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Mupad [B]
time = 1.54, size = 138, normalized size = 0.97

$$\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p\,\left(\frac{3\,x\,\left(2\,p^2+3\,p+1\right)}{4\,p^3+12\,p^2+11\,p+3}+\frac{3\,a^3}{b^3\,\left(4\,p^3+12\,p^2+11\,p+3\right)}-\frac{6\,a^2\,p\,x^{1/3}}{b^2\,\left(4\,p^3+12\,p^2+11\,p+3\right)}+\frac{3\,a\,p\,x^{2/3}\,\left(2\,p+1\right)}{b\,\left(4\,p^3+12\,p^2+11\,p+3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p,x)
```

```
[Out] (a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((3*x*(3*p + 2*p^2 + 1))/(11*p + 12*p
^2 + 4*p^3 + 3) + (3*a^3)/(b^3*(11*p + 12*p^2 + 4*p^3 + 3)) - (6*a^2*p*x^(1
/3))/(b^2*(11*p + 12*p^2 + 4*p^3 + 3)) + (3*a*p*x^(2/3)*(2*p + 1))/(b*(11*p
 + 12*p^2 + 4*p^3 + 3)))
```

3.476 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx$

Optimal. Leaf size=69

$$-\frac{3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(1,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right)}{1+2p}$$

```
[Out] -3*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*hypergeom([1, 1+2*p],[
2+2*p],1+b*x^(1/3)/a)/(1+2*p)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1370, 272, 67}

$$-\frac{3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}\,b}{a}+1\right)}{2p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
```

```
[Out] (-3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeometri
c2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(1 + 2*p)
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
```

```
!IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x}\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int \frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}}{x}\,dx$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x}\,dx, x,\right.$$

$$= -\frac{3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(1,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right.}{1+2p}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 0.84

$$-\frac{3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p {}_2F_1\left(1,1+2p;2+2p;1+\frac{b\sqrt[3]{x}}{a}\right)}{a(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x,x]
[Out] (-3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[1, 1 + 2*p, 2 +
 2*p, 1 + (b*x^(1/3))/a])/(a*(1 + 2*p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x,x)
```

```
[Out] Integral(((a + b*x**(1/3))**2)**p/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x,x)
```

```
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x, x)
```

3.477 $\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx$

Optimal. Leaf size=75

$$\frac{3b^3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(4,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right)}{a^3(1+2p)}$$

```
[Out] 3*b^3*(1+b*x^(1/3)/a)*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p*hypergeom([4, 1+2*p
],[2+2*p],1+b*x^(1/3)/a)/a^3/(1+2*p)
```

Rubi [A]
time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1370, 272, 67}

$$\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p {}_2F_1\left(4,2p+1;2(p+1);\frac{\sqrt[3]{x}\,b}{a}+1\right)}{a^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
```

```
[Out] (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p*Hypergeome
tric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 + 2*p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
```

```
!IntegerQ[2*p]
```

Rubi steps

$$\int \frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}\,dx = \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\int \frac{\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{2p}}{x^2}\,dx$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx}{a}\right)^{2p}}{x^4}\,dx, x$$

$$= \frac{3b^3\left(1+\frac{b\sqrt[3]{x}}{a}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\ {}_2F_1\left(4,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right.}{a^3(1+2p)}$$

Mathematica [A]
time = 0.07, size = 61, normalized size = 0.81

$$\frac{3b^3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\ {}_2F_1\left(4,1+2p;2+2p;1+\frac{b\sqrt[3]{x}}{a}\right)}{a^4(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2,x]
[Out] (3*b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*Hypergeometric2F1[4, 1 + 2*p,
2 + 2*p, 1 + (b*x^(1/3))/a])/(a^4*(1 + 2*p))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="maxima")
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="fricas")
[Out] integral((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(a + b\sqrt[3]{x}\right)^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2,x)
[Out] Integral(((a + b*x**(1/3))**2)**p/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2,x, algorithm="giac")
[Out] integrate((b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + b^2\,x^{2/3} + 2\,a\,b\,x^{1/3}\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2,x)
[Out] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2, x)
```

3.478 $\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2} - \frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2\right.}{3a^3x}\right.$

Optimal. Leaf size=146

$$-\frac{\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{ax}+\frac{b(1-p)\left(a+b\sqrt[3]{x}\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{a^2x^{2/3}}-\frac{b^2(1-2p)(1-p)\,(}{}$$

```
[Out] -(a+b*x^(1/3))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a/x+b*(1-p)*(a+b*x^(1/3))*
(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^2/x^(2/3)-b^2*(1-2*p)*(1-p)*(a+b*x^(1/3
))*(a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x^(1/3)
```

Rubi [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 3, integrand size = 77, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {1370, 272, 67}

$$\frac{2b^3(1-2p)(1-p)p\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(1,2p+1;2(p+1);\frac{\sqrt[3]{x}\,b}{a}+1\right)}{a^3(2p+1)}+\frac{3b^3\left(\frac{b\sqrt[3]{x}}{a}+1\right)\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p\,{}_2F_1\left(4,2p+1;2(p+1);\frac{\sqrt[3]{x}\,b}{a}+1\right)}{a^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 - p)*p*
(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

```
[Out] (2*b^3*(1 - 2*p)*(1 - p)*p*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x
^(2/3))^p*Hypergeometric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3
*(1 + 2*p)) + (3*b^3*(1 + (b*x^(1/3))/a)*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3)
)^p*Hypergeometric2F1[4, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a])/(a^3*(1 +
2*p))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int \left(\frac{\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{x^2}-\frac{2b^3(1-2p)(1-p)p\left(a^2+2ab\sqrt[3]{x}+b^2x^{2/3}\right)^p}{3a^3x}\right)dx = -\frac{(2b^3(1-2p)(1-p)p}{}$$

$$= \left(\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}(a^2$$

$$= \left(3\left(1+\frac{b\sqrt[3]{x}}{a}\right)^{-2p}(a$$

$$= \frac{2b^3(1-2p)(1-p)p\Big(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.25, size = 101, normalized size = 0.69

$$\frac{b^3\left(a+b\sqrt[3]{x}\right)\left(\left(a+b\sqrt[3]{x}\right)^2\right)^p\left(2p(1-3p+2p^2)\,{}_2F_1\left(1,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right)+3\,{}_2F_1\left(4,1+2p;2(1+p);1+\frac{b\sqrt[3]{x}}{a}\right)\right)}{a^3(a+2ap)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p/x^2 - (2*b^3*(1 - 2*p)*(1 -
 p)*p*(a^2 + 2*a*b*x^(1/3) + b^2*x^(2/3))^p)/(3*a^3*x),x]
```

```
[Out] (b^3*(a + b*x^(1/3))*((a + b*x^(1/3))^2)^p*(2*p*(1 - 3*p + 2*p^2)*Hypergeom
etric2F1[1, 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a] + 3*Hypergeometric2F1[4,
 1 + 2*p, 2*(1 + p), 1 + (b*x^(1/3))/a]))/(a^3*(a + 2*a*p))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p}{x^2}-\frac{2b^3(1-2p)\,(1-p)\,p\left(a^2+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{2}{3}}\right)^p}{3a^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a*
b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x)
```

```
[Out] int((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^2+2*a*
b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="maxima")
```

```
[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*
p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Fricas [A]
time = 0.56, size = 82, normalized size = 0.56

$$-\frac{\left(a^{2} b p x^{\frac{1}{3}} + a^{3} + \left(2 \, b^{3} p^{2} - 3 \, b^{3} p + b^{3}\right) x + 2 \, \left(a b^{2} p^{2} - a b^{2} p\right) x^{\frac{2}{3}}\right) \left(b^{2} x^{\frac{2}{3}} + 2 \, a b x^{\frac{1}{3}} + a^{2}\right)^{p}}{a^{3} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="fricas")
```

```
[Out] -(a^2*b*p*x^(1/3) + a^3 + (2*b^3*p^2 - 3*b^3*p + b^3)*x + 2*(a*b^2*p^2 - a*
b^2*p)*x^(2/3))*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/(a^3*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \left(-\frac{3 a^{3} \left(a^{2} + 2 a b \sqrt[3]{x} + b^{2} x^{\frac{2}{3}}\right)^{p}}{x^{2}}\right)\, dx + \int \frac{2 b^{3} p \left(a^{2} + 2 a b \sqrt[3]{x} + b^{2} x^{\frac{2}{3}}\right)^{p}}{x}\, dx + \int \left(-\frac{6 b^{3} p^{2} \left(a^{2} + 2 a b \sqrt[3]{x} + b^{2} x^{\frac{2}{3}}\right)^{p}}{x}\right)\, dx + \int \frac{4 b^{3} p^{3} \left(a^{2} + 2 a b \sqrt[3]{x} + b^{2} x^{\frac{2}{3}}\right)^{p}}{x}\, dx}{3 a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/x**2-2/3*b**3*(1-2*p)*(1-p
)*p*(a**2+2*a*b*x**(1/3)+b**2*x**(2/3))**p/a**3/x,x)
```

```
[Out] -(Integral(-3*a**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x**2, x) + In
tegral(2*b**3*p*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integral
```

```
(-6*b**3*p**2*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x) + Integral(4
*b**3*p**3*(a**2 + 2*a*b*x**(1/3) + b**2*x**(2/3))**p/x, x))/(3*a**3)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/x^2-2/3*b^3*(1-2*p)*(1-p)*p*(a^
2+2*a*b*x^(1/3)+b^2*x^(2/3))^p/a^3/x,x, algorithm="giac")
```

```
[Out] integrate(-2/3*(b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p*b^3*(2*p - 1)*(p - 1)*
p/(a^3*x) + (b^2*x^(2/3) + 2*a*b*x^(1/3) + a^2)^p/x^2, x)
```

Mupad [B]
time = 1.65, size = 69, normalized size = 0.47

$$-\frac{\left(a^2+b^2\,x^{2/3}+2\,a\,b\,x^{1/3}\right)}^p\,\left(\frac{b^3\,x\,\left(2\,p^2-3\,p+1\right)}{a^3}+\frac{b\,p\,x^{1/3}}{a}+\frac{2\,b^2\,p\,x^{2/3}\,\left(p-1\right)}{a^2}+1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p/x^2 - (2*b^3*p*(2*p - 1)*(p - 1)*
(a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p)/(3*a^3*x),x)
```

```
[Out] -((a^2 + b^2*x^(2/3) + 2*a*b*x^(1/3))^p*((b^3*x*(2*p^2 - 3*p + 1))/a^3 + (b
*p*x^(1/3))/a + (2*b^2*p*x^(2/3)*(p - 1))/a^2 + 1))/x
```

3.479 $\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\left(a+b\sqrt[4]{x}\right)\sqrt[4]{x}}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a}{b^4}$$

```
[Out] -12*a^2/b^4/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)+2*a^3/b^4/(a+b*x^(1/4))/(
a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)+4*(a+b*x^(1/4))*x^(1/4)/b^3/(a^2+2*a*b
*x^(1/4)+b^2*x^(1/2))^(1/2)-12*a*(a+b*x^(1/4))*ln(a+b*x^(1/4))/b^4/(a^2+2*a
*b*x^(1/4)+b^2*x^(1/2))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 660, 45}

$$-\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}-\frac{12a\left(a+b\sqrt[4]{x}\right)\log\left(a+b\sqrt[4]{x}\right)}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{4\sqrt[4]{x}\left(a+b\sqrt[4]{x}\right)}{b^3\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

```
[Out] (-12*a^2)/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (2*a^3)/(b^4*(a +
 b*x^(1/4))*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) + (4*(a + b*x^(1/4))*x
^(1/4))/(b^3*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]]) - (12*a*(a + b*x^(1/4
))*Log[a + b*x^(1/4)])/(b^4*Sqrt[a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x]])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}\right)^{3/2}}\,dx = 4\text{Subst}\left(\int \frac{x^3}{(a^2+2abx+b^2x^2)^{3/2}}\,dx, x, \sqrt[4]{x}\right)$$
$$= \frac{\left(4b^3\left(a+b\sqrt[4]{x}\right)\right)\text{Subst}\left(\int \frac{x^3}{(ab+b^2x)^3}\,dx, x, \sqrt[4]{x}\right)}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$
$$= \frac{\left(4b^3\left(a+b\sqrt[4]{x}\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^6}-\frac{a^3}{b^6(a+bx)^3}+\frac{3a^2}{b^6(a+bx)^2}-\frac{3a}{b^6(a+bx)}\right)dx, x, \sqrt[4]{x}\right)}{\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$
$$= -\frac{12a^2}{b^4\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}+\frac{2a^3}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{a^2+2ab\sqrt[4]{x}+b^2\sqrt{x}}}$$

Mathematica [A]
time = 0.07, size = 93, normalized size = 0.53

$$\frac{2\left(-5a^3-4a^2b\sqrt[4]{x}+4ab^2\sqrt{x}+2b^3x^{3/4}-6a\left(a+b\sqrt[4]{x}\right)^2\log\left(a+b\sqrt[4]{x}\right)\right)}{b^4\left(a+b\sqrt[4]{x}\right)\sqrt{\left(a+b\sqrt[4]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/4) + b^2*Sqrt[x])^(-3/2),x]
```

```
[Out] (2*(-5*a^3 - 4*a^2*b*x^(1/4) + 4*a*b^2*Sqrt[x] + 2*b^3*x^(3/4) - 6*a*(a + b
*x^(1/4))^2*Log[a + b*x^(1/4)]))/(b^4*(a + b*x^(1/4))*Sqrt[(a + b*x^(1/4))^
2])
```

Maple [A]
time = 0.08, size = 114, normalized size = 0.65

method	result
derivativedivides	$-\frac{2\left(6\ln\left(a+b\,x^{\frac{1}{4}}\right)a\,b^2\sqrt{x}-2b^3x^{\frac{3}{4}}+12\ln\left(a+b\,x^{\frac{1}{4}}\right)a^2b\,x^{\frac{1}{4}}-4a\,b^2\sqrt{x}+6\ln\left(a+b\,x^{\frac{1}{4}}\right)a^3+4a^2b\,x^{\frac{1}{4}}+5a^3\right)\left(a+b\,x^{\frac{1}{4}}\right)}{b^4\left(\left(a+b\,x^{\frac{1}{4}}\right)^2\right)^{\frac{3}{2}}}$

default	$\frac{2\sqrt{a^{2}+2ab\,x^{\frac{1}{4}}+b^{2}\sqrt{x}}\,\left(2b^{3}x^{\frac{3}{4}}-6\ln\left(a+b\,x^{\frac{1}{4}}\right)a\,b^{2}\sqrt{x}+4a\,b^{2}\sqrt{x}-12\ln\left(a+b\,x^{\frac{1}{4}}\right)a^{2}b\,x^{\frac{1}{4}}-4a^{2}b\,x^{\frac{1}{4}}-6\,\mathrm{l}\right.}{\left(a+b\,x^{\frac{1}{4}}\right)^{3}b^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(1/2)*(2*b^3*x^(3/4)-6*ln(a+b*x^(1/4))*a*
b^2*x^(1/2)+4*a*b^2*x^(1/2)-12*ln(a+b*x^(1/4))*a^2*b*x^(1/4)-4*a^2*b*x^(1/4
)-6*ln(a+b*x^(1/4))*a^3-5*a^3)/(a+b*x^(1/4))^3/b^4
```

Maxima [A]
time = 0.29, size = 114, normalized size = 0.65

$$\frac{4\sqrt{x}}{\sqrt{b^{2}\sqrt{x}+2\,abx^{\frac{1}{4}}+a^{2}}\,b^{2}}-\frac{12\,a\log\left(x^{\frac{1}{4}}+\frac{a}{b}\right)}{b^{4}}+\frac{8\,a^{2}}{\sqrt{b^{2}\sqrt{x}+2\,abx^{\frac{1}{4}}+a^{2}}\,b^{4}}-\frac{24\,a^{2}x^{\frac{1}{4}}}{b^{5}\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^{2}}-\frac{22\,a^{3}}{b^{6}\left(x^{\frac{1}{4}}+\frac{a}{b}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] 4*sqrt(x)/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^2) - 12*a*log(x^(1/4)
+ a/b)/b^4 + 8*a^2/(sqrt(b^2*sqrt(x) + 2*a*b*x^(1/4) + a^2)*b^4) - 24*a^2*x
^(1/4)/(b^5*(x^(1/4) + a/b)^2) - 22*a^3/(b^6*(x^(1/4) + a/b)^2)
```

Fricas [A]
time = 2.12, size = 147, normalized size = 0.84

$$\frac{2\left(9\,a^{5}b^{4}x-5\,a^{9}-6\,(ab^{8}x^{2}-2\,a^{5}b^{4}x+a^{9})\log\left(bx^{\frac{1}{4}}+a\right)-2\,(3\,a^{2}b^{7}x-a^{6}b^{3})x^{\frac{3}{4}}+(7\,a^{3}b^{6}x-3\,a^{7}b^{2})\sqrt{x}+2\,(b^{9}x^{2}-6\,a^{4}b^{5}x+3\,a^{8}b)x^{\frac{1}{4}}\right)}{b^{12}x^{2}-2\,a^{4}b^{8}x+a^{8}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(9*a^5*b^4*x - 5*a^9 - 6*(a*b^8*x^2 - 2*a^5*b^4*x + a^9)*log(b*x^(1/4) +
a) - 2*(3*a^2*b^7*x - a^6*b^3)*x^(3/4) + (7*a^3*b^6*x - 3*a^7*b^2)*sqrt(x)
+ 2*(b^9*x^2 - 6*a^4*b^5*x + 3*a^8*b)*x^(1/4))/(b^12*x^2 - 2*a^4*b^8*x + a^
8*b^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^{2}+2ab\sqrt[4]{x}+b^{2}\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/4)+b**2*x**(1/2))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**(1/4) + b**2*sqrt(x))**(-3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/4)+b^2*x^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a^2+b^2\,\sqrt{x}+2\,a\,b\,x^{1/4}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(1/2) + 2*a*b*x^(1/4))^(3/2), x)
```

3.480 $\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=268

$$-\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}}}$$

```
[Out] -60*a^2/b^6/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)+3/2*a^5/b^6/(a+b*x^(1/6))
^3/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)-10*a^4/b^6/(a+b*x^(1/6))^2/(a^2+2*
a*b*x^(1/6)+b^2*x^(1/3))^(1/2)+30*a^3/b^6/(a+b*x^(1/6))/(a^2+2*a*b*x^(1/6)+
b^2*x^(1/3))^(1/2)+6*(a+b*x^(1/6))*x^(1/6)/b^5/(a^2+2*a*b*x^(1/6)+b^2*x^(1/
3))^(1/2)-30*a*(a+b*x^(1/6))*ln(a+b*x^(1/6))/b^6/(a^2+2*a*b*x^(1/6)+b^2*x^(
1/3))^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 660, 45}

$$-\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{30a\left(a+b\sqrt[6]{x}\right)\log\left(a+b\sqrt[6]{x}\right)}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{6\sqrt[6]{x}\left(a+b\sqrt[6]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}-\frac{10a^4}{b^6\left(a+b\sqrt[6]{x}\right)^2\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{30a^3}{b^6\left(a+b\sqrt[6]{x}\right)\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

```
[Out] (-60*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (3*a^5)/(2*b^6*(a
 + b*x^(1/6))^3*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (10*a^4)/(b^6*(a
 + b*x^(1/6))^2*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (30*a^3)/(b^6*(a
 + b*x^(1/6))*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) + (6*(a + b*x^(1/6))
*x^(1/6))/(b^5*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)]) - (30*a*(a + b*x^(1
/6))*Log[a + b*x^(1/6)])/(b^6*Sqrt[a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
```

```
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}\right)^{5/2}}\,dx = 6\text{Subst}\left(\int \frac{x^5}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, \sqrt[6]{x}\right)$$
$$= \frac{\left(6b^5\left(a+b\sqrt[6]{x}\right)\right)\text{Subst}\left(\int \frac{x^5}{(ab+b^2x)^5}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$
$$= \frac{\left(6b^5\left(a+b\sqrt[6]{x}\right)\right)\text{Subst}\left(\int \left(\frac{1}{b^{10}}-\frac{a^5}{b^{10}(a+bx)^5}+\frac{5a^4}{b^{10}(a+bx)^4}-\frac{10a^3}{b^{10}(a+bx)^3}+\frac{1}{b^{10}(\cdots\right.\right.}{\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}$$
$$= -\frac{60a^2}{b^6\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}+\frac{3a^5}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{a^2+2ab\sqrt[6]{x}+b^2\sqrt[3]{x}}}\cdots$$

Mathematica [A]
time = 0.11, size = 121, normalized size = 0.45

$$\frac{-77a^5-248a^4b\sqrt[6]{x}-252a^3b^2\sqrt[3]{x}-48a^2b^3\sqrt{x}+48ab^4x^{2/3}+12b^5x^{5/6}-60a\left(a+b\sqrt[6]{x}\right)^4\log\left(a+b\sqrt[6]{x}\right)}{2b^6\left(a+b\sqrt[6]{x}\right)^3\sqrt{\left(a+b\sqrt[6]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/6) + b^2*x^(1/3))^(-5/2),x]
```

```
[Out] (-77*a^5 - 248*a^4*b*x^(1/6) - 252*a^3*b^2*x^(1/3) - 48*a^2*b^3*Sqrt[x] + 4
8*a*b^4*x^(2/3) + 12*b^5*x^(5/6) - 60*a*(a + b*x^(1/6))^4*Log[a + b*x^(1/6)
])/(2*b^6*(a + b*x^(1/6))^3*Sqrt[(a + b*x^(1/6))^2])
```

Maple [A]
time = 0.10, size = 174, normalized size = 0.65

method	result

derivativedivides	$-\frac{\left(60\ln\left(a+b\,x^{\frac{1}{6}}\right)a\,b^{4}x^{\frac{2}{3}}-12b^{5}x^{\frac{5}{6}}+240\ln\left(a+b\,x^{\frac{1}{6}}\right)a^{2}b^{3}\sqrt{x}-48a\,b^{4}x^{\frac{2}{3}}+360\ln\left(a+b\,x^{\frac{1}{6}}\right)a^{3}b^{2}x^{\frac{1}{3}}+48a^{2}b^{3}\sqrt{x}+2}{2b^{6}\left(\left(a+b\,x^{\frac{1}{6}}\right)^{2}\right)^{\frac{5}{2}}}$
default	$-\frac{\sqrt{a^{2}+2ab\,x^{\frac{1}{6}}+b^{2}x^{\frac{1}{3}}}\,\left(60\ln\left(a+b\,x^{\frac{1}{6}}\right)a\,b^{4}x^{\frac{2}{3}}-12b^{5}x^{\frac{5}{6}}+240\ln\left(a+b\,x^{\frac{1}{6}}\right)a^{2}b^{3}\sqrt{x}-48a\,b^{4}x^{\frac{2}{3}}+360\ln\left(a+\right.\right.}{2\left(a+b\,x^{\frac{1}{6}}\right)^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(1/2)*(60*ln(a+b*x^(1/6))*a*b^4*x^(2/3
)-12*b^5*x^(5/6)+240*ln(a+b*x^(1/6))*a^2*b^3*x^(1/2)-48*a*b^4*x^(2/3)+360*l
n(a+b*x^(1/6))*a^3*b^2*x^(1/3)+48*a^2*b^3*x^(1/2)+240*ln(a+b*x^(1/6))*a^4*b
*x^(1/6)+252*a^3*b^2*x^(1/3)+60*ln(a+b*x^(1/6))*a^5+248*a^4*b*x^(1/6)+77*a^
5)/(a+b*x^(1/6))^5/b^6
```

Maxima [A]
time = 0.31, size = 119, normalized size = 0.44

$$\frac{12\,b^{5}x^{\frac{5}{6}}+48\,ab^{4}x^{\frac{2}{3}}-48\,a^{2}b^{3}\sqrt{x}-252\,a^{3}b^{2}x^{\frac{1}{3}}-248\,a^{4}bx^{\frac{1}{6}}-77\,a^{5}}{2\left(b^{10}x^{\frac{2}{3}}+4\,ab^{9}\sqrt{x}+6\,a^{2}b^{8}x^{\frac{1}{3}}+4\,a^{3}b^{7}x^{\frac{1}{6}}+a^{4}b^{6}\right)}-\frac{30\,a\log\left(bx^{\frac{1}{6}}+a\right)}{b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^5*x^(5/6) + 48*a*b^4*x^(2/3) - 48*a^2*b^3*sqrt(x) - 252*a^3*b^2*x
^(1/3) - 248*a^4*b*x^(1/6) - 77*a^5)/(b^10*x^(2/3) + 4*a*b^9*sqrt(x) + 6*a^
2*b^8*x^(1/3) + 4*a^3*b^7*x^(1/6) + a^4*b^6) - 30*a*log(b*x^(1/6) + a)/b^6
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/6)+b**2*x**(1/3))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.40, size = 105, normalized size = 0.39

$$-\frac{30\,a\log\left(\left|bx^{\frac{1}{6}}+a\right|\right)}{b^{6}\mathrm{sgn}\left(bx^{\frac{1}{6}}+a\right)}+\frac{6\,x^{\frac{1}{6}}}{b^{5}\mathrm{sgn}\left(bx^{\frac{1}{6}}+a\right)}-\frac{120\,a^{2}b^{3}\sqrt{x}+300\,a^{3}b^{2}x^{\frac{1}{3}}+260\,a^{4}bx^{\frac{1}{6}}+77\,a^{5}}{2\left(bx^{\frac{1}{6}}+a\right)^{4}b^{6}\mathrm{sgn}\left(bx^{\frac{1}{6}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/6)+b^2*x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] -30*a*log(abs(b*x^(1/6) + a))/(b^6*sgn(b*x^(1/6) + a)) + 6*x^(1/6)/(b^5*sgn
(b*x^(1/6) + a)) - 1/2*(120*a^2*b^3*sqrt(x) + 300*a^3*b^2*x^(1/3) + 260*a^4
*b*x^(1/6) + 77*a^5)/((b*x^(1/6) + a)^4*b^6*sgn(b*x^(1/6) + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(a^2+b^2\,x^{1/3}+2\,a\,b\,x^{1/6}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(1/3) + 2*a*b*x^(1/6))^(5/2), x)
```

3.481 $\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx$

Optimal. Leaf size=179

$$-\frac{2b^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}}{\left(a+\frac{b}{\sqrt{x}}\right)\sqrt{x}}+\frac{6a^2b\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\sqrt{x}}{a+\frac{b}{\sqrt{x}}}+\frac{a^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\,x}{a+\frac{b}{\sqrt{x}}}+\frac{6ab^2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\log}{a+\frac{b}{\sqrt{x}}}$$

```
[Out] a^3*x*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)/(a+b/x^(1/2))+3*a*b^2*ln(x)*(a^2+b^2/
x+2*a*b/x^(1/2))^(1/2)/(a+b/x^(1/2))-2*b^3*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)/
(a+b/x^(1/2))/x^(1/2)+6*a^2*b*x^(1/2)*(a^2+b^2/x+2*a*b/x^(1/2))^(1/2)/(a+b/
x^(1/2))
```

Rubi [A]
time = 0.06, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1355, 1369, 269, 45}

$$\frac{6a^2b\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}+\frac{6ab^2\log\left(\sqrt{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}-\frac{2b^3\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{\sqrt{x}\left(a+\frac{b}{\sqrt{x}}\right)}+\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt{x}}+\frac{b^2}{x}}}{a+\frac{b}{\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

```
[Out] (-2*b^3*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]])/((a + b/Sqrt[x])*Sqrt[x]) + (6
*a^2*b*Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*Sqrt[x])/(a + b/Sqrt[x]) + (a^3*
Sqrt[a^2 + b^2/x + (2*a*b)/Sqrt[x]]*x)/(a + b/Sqrt[x]) + (6*a*b^2*Sqrt[a^2
+ b^2/x + (2*a*b)/Sqrt[x]]*Log[Sqrt[x]])/(a + b/Sqrt[x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}\right)^{3/2} dx = 2\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2} x\,dx, x, \sqrt{x}\right)$$

$$= \frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^3 x\,dx, x, \sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$

$$= \frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^3}{x^2}\,dx, x, \sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$

$$= \frac{\left(2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\right)\text{Subst}\left(\int \left(3a^2b^4+\frac{b^6}{x^2}+\frac{3ab^5}{x}+a^3b^3x\right)dx, x, \sqrt{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt{x}}\right)}$$

$$= -\frac{2b^4\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}}{\left(ab+\frac{b^2}{\sqrt{x}}\right)\sqrt{x}} + \frac{6a^2b^2\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}\,\sqrt{x}}{ab+\frac{b^2}{\sqrt{x}}} + \frac{a^3\sqrt{a^2+\frac{b^2}{x}+\frac{2ab}{\sqrt{x}}}}{a+\frac{b}{\sqrt{x}}}$$

Mathematica [A]

time = 0.03, size = 66, normalized size = 0.37

$$\frac{\sqrt{\frac{\left(b+a\sqrt{x}\right)^2}{x}}\left(-2b^3+6a^2bx+a^3x^{3/2}+3ab^2\sqrt{x}\,\log(x)\right)}{b+a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x + (2*a*b)/Sqrt[x])^(3/2),x]
```

```
[Out] (Sqrt[(b + a*Sqrt[x])^2/x]*(-2*b^3 + 6*a^2*b*x + a^3*x^(3/2) + 3*a*b^2*Sqrt
[x]*Log[x]))/(b + a*Sqrt[x])
```

Maple [A]
time = 0.06, size = 68, normalized size = 0.38

method	result	size
derivativedivides	$\frac{\left(\frac{a^2x+b^2+2ab\sqrt{x}}{x}\right)^{\frac{3}{2}} x\left(a^3x^{\frac{3}{2}}+3a\,b^2\ln(x)\sqrt{x}+6a^2bx-2b^3\right)}{\left(a\sqrt{x}+b\right)^3}$	65
default	$\frac{\sqrt{\frac{a^2x^{\frac{3}{2}}+b^2\sqrt{x}+2abx}{x^{\frac{3}{2}}}}\left(a^3x^{\frac{3}{2}}+3a\,b^2\ln(x)\sqrt{x}+6a^2bx-2b^3\right)}{a\sqrt{x}+b}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a^2*x^(3/2)+b^2*x^(1/2)+2*a*b*x)/x^(3/2))^(1/2)*(a^3*x^(3/2)+3*a*b^2*ln(x
)*x^(1/2)+6*a^2*b*x-2*b^3)/(a*x^(1/2)+b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="maxima")
```

```
[Out] a^3*x + 3*a*b^2*integrate(1/x, x) + 6*a^2*b*sqrt(x) - 2*b^3/sqrt(x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + \frac{2ab}{\sqrt{x}} + \frac{b^2}{x}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x+2*a*b/x**(1/2))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/sqrt(x) + b**2/x)**(3/2), x)
```

Giac [A]
time = 4.15, size = 80, normalized size = 0.45

$$a^3x\mathrm{sgn}\left(ax + b\sqrt{x}\right)\mathrm{sgn}(x) + 3\,ab^2\log\left(|x|\right)\mathrm{sgn}\left(ax + b\sqrt{x}\right)\mathrm{sgn}(x) + 6\,a^2b\sqrt{x}\,\mathrm{sgn}\left(ax + b\sqrt{x}\right)\mathrm{sgn}(x) - \frac{2\,b^3\mathrm{sgn}\left(ax + b\sqrt{x}\right)\mathrm{sgn}(x)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x+2*a*b/x^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] a^3*x*sgn(a*x + b*sqrt(x))*sgn(x) + 3*a*b^2*log(abs(x))*sgn(a*x + b*sqrt(x)
)*sgn(x) + 6*a^2*b*sqrt(x)*sgn(a*x + b*sqrt(x))*sgn(x) - 2*b^3*sgn(a*x + b*
sqrt(x))*sgn(x)/sqrt(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a^2 + \frac{b^2}{x} + \frac{2\,a\,b}{\sqrt{x}}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2),x)
```

```
[Out] int((a^2 + b^2/x + (2*a*b)/x^(1/2))^(3/2), x)
```

3.482 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx$

Optimal. Leaf size=391

$$-\frac{3b^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{4\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{4/3}}-\frac{7ab^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}-\frac{63a^2b^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}-\frac{105a^3b^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}$$

```
[Out] -3/4*b^7*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(4/3)-7*a*b^
6*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x-63/2*a^2*b^5*(a^2+b
^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(2/3)-105*a^3*b^4*(a^2+b^2/
x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(1/3)+63*a^5*b^2*x^(1/3)*(a^2+
b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+21/2*a^6*b*x^(2/3)*(a^2+b^2/
x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a^7*x*(a^2+b^2/x^(2/3)+2*a*b/x^(
1/3))^(1/2)/(a+b/x^(1/3))+35*a^4*b^3*ln(x)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^
(1/2)/(a+b/x^(1/3))
```

Rubi [A]
time = 0.13, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$-\frac{3b^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{4x^{4/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{7ab^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{63a^2b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{21a^6bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{63a^5b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{105a^4b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}-\frac{105a^3b^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (-3*b^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(4*(a + b/x^(1/3))*x^(4/
3)) - (7*a*b^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))*
x) - (63*a^2*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1/
3))*x^(2/3)) - (105*a^3*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a
+ b/x^(1/3))*x^(1/3)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)]*x^(1/3))/(a + b/x^(1/3)) + (21*a^6*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^
(1/3)]*x^(2/3))/(2*(a + b/x^(1/3))) + (a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)
/x^(1/3)]*x)/(a + b/x^(1/3)) + (105*a^4*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^(1/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{7/2} dx = 3\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{7/2} x^2\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^7 x^2\,dx, x, \sqrt[3]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^7}{x^5}\,dx, x, \sqrt[3]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(21a^5b^9+\frac{b^{14}}{x^5}+\frac{7ab^{13}}{x^4}+\frac{21a^2b^{12}}{x^3}+\frac{35a^3b^{11}}{x^2}\right.\right.}{b^6\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$
$$= -\frac{3b^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{4/3}} - \frac{7ab^7\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x} - \frac{63a^2b^6\sqrt{a^2+\frac{b^2}{x^{2}}}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right.}$$

Mathematica [A]
time = 0.05, size = 125, normalized size = 0.32

$$\frac{\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\left(-3b^7-28ab^6\sqrt[3]{x}-126a^2b^5x^{2/3}-420a^3b^4x+252a^5b^2x^{5/3}+42a^6bx^2+4a^7x^{7/3}+140a^4b^3x^{4/3}\log(x)\right)}{4\left(b+a\sqrt[3]{x}\right)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*(-3*b^7 - 28*a*b^6*x^(1/3) - 126*a^2*b^5*x
^(2/3) - 420*a^3*b^4*x + 252*a^5*b^2*x^(5/3) + 42*a^6*b*x^2 + 4*a^7*x^(7/3)
 + 140*a^4*b^3*x^(4/3)*Log[x]))/(4*(b + a*x^(1/3))*x)
```

Maple [A]
time = 0.07, size = 115, normalized size = 0.29

method	result

derivativedivides	$\frac{\left(\frac{a^{2} x^{\frac{2}{3}}+2 a b\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}} x \left(4 a^{7} x^{\frac{7}{3}}+42 a^{6} b\,x^{2}+140 a^{4} b^{3} \ln \left(x \right) x^{\frac{4}{3}}+252 a^{5} b^{2} x^{\frac{5}{3}}-420 a^{3} b^{4} x -126 a^{2} b^{5} x^{\frac{2}{3}}-28 a\,b^{6} x^{\frac{1}{3}}-3 b^{7}\right)}{4 \left(b +a\,x^{\frac{1}{3}}\right)^{7}}$
default	$\frac{\left(\frac{a^{2} x^{\frac{2}{3}}+2 a b\,x^{\frac{1}{3}}+b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}} \left(42 a^{6} b\,x^{3}+252 a^{5} b^{2} x^{\frac{8}{3}}+140 a^{4} b^{3} \ln \left(x \right) x^{\frac{7}{3}}+4 a^{7} x^{\frac{10}{3}}-28 a\,b^{6} x^{\frac{4}{3}}-420 a^{3} b^{4} x^{2}-126 a^{2} b^{5} x^{\frac{5}{3}}-3 b^{7} x \right)}{4 \left(b +a\,x^{\frac{1}{3}}\right)^{7}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(7/2)*(42*a^6*b*x^3+252*a^5*b
^2*x^(8/3)+140*a^4*b^3*ln(x)*x^(7/3)+4*a^7*x^(10/3)-28*a*b^6*x^(4/3)-420*a^
3*b^4*x^2-126*a^2*b^5*x^(5/3)-3*b^7*x)/(b+a*x^(1/3))^7
```

Maxima [A]
time = 0.33, size = 79, normalized size = 0.20

$$35\,a^4b^3\log\left(x\right)+\frac{4\,a^7x^{\frac{7}{3}}+42\,a^6bx^2+252\,a^5b^2x^{\frac{5}{3}}-420\,a^3b^4x-126\,a^2b^5x^{\frac{2}{3}}-28\,ab^6x^{\frac{1}{3}}-3\,b^7}{4\,x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="maxima")
```

```
[Out] 35*a^4*b^3*log(x) + 1/4*(4*a^7*x^(7/3) + 42*a^6*b*x^2 + 252*a^5*b^2*x^(5/3)
 - 420*a^3*b^4*x - 126*a^2*b^5*x^(2/3) - 28*a*b^6*x^(1/3) - 3*b^7)/x^(4/3)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(7/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(7/2), x)
```

Giac [A]
time = 4.53, size = 173, normalized size = 0.44

$$a^7x\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+35\,a^4b^3\log\left(|x|\right)\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+\frac{21}{2}\,a^6bx^{\frac{2}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+63\,a^5b^2x^{\frac{1}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)-\frac{420\,a^3b^4x\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+126\,a^2b^5x^{\frac{2}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+28\,ab^6x^{\frac{1}{3}}\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)+3\,b^7\operatorname{sgn}\left(ax+bx^{\frac{2}{3}}\right)\operatorname{sgn}(x)}{4\,x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(7/2),x, algorithm="giac")
```

```
[Out] a^7*x*sgn(a*x + b*x^(2/3))*sgn(x) + 35*a^4*b^3*log(abs(x))*sgn(a*x + b*x^(2
/3))*sgn(x) + 21/2*a^6*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 63*a^5*b^2*x
^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 1/4*(420*a^3*b^4*x*sgn(a*x + b*x^(2/3)
)*sgn(x) + 126*a^2*b^5*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 28*a*b^6*x^(1/
3)*sgn(a*x + b*x^(2/3))*sgn(x) + 3*b^7*sgn(a*x + b*x^(2/3))*sgn(x))/x^(4/3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(7/2), x)
```

3.483 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx$

Optimal. Leaf size=291

$$-\frac{3b^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}-\frac{15ab^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}+\frac{30a^3b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\sqrt[3]{x}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{15a^4b\sqrt{a^2+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right.}$$

```
[Out] -3/2*b^5*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(2/3)-15*a*b
^4*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))/x^(1/3)+30*a^3*b^2*x
^(1/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+15/2*a^4*b*x^(2/
3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a^5*x*(a^2+b^2/x^(2/
3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+10*a^2*b^3*ln(x)*(a^2+b^2/x^(2/3)+2*a
*b/x^(1/3))^(1/2)/(a+b/x^(1/3))
```

Rubi [A]
time = 0.09, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$-\frac{3b^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2x^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}-\frac{15ab^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^2b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{15a^4bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{30a^3b^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

```
[Out] (-3*b^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/(2*(a + b/x^(1/3))*x^(2/
3)) - (15*a*b^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])/((a + b/x^(1/3))
*x^(1/3)) + (30*a^3*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/
(a + b/x^(1/3)) + (15*a^4*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/
3))/(2*(a + b/x^(1/3))) + (a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)
/(a + b/x^(1/3)) + (30*a^2*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Lo
g[x^(1/3)])/(a + b/x^(1/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2} dx = 3\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^2\, dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^2\, dx, x, \sqrt[3]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^5}{x^3}\, dx, x, \sqrt[3]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(10a^3b^7+\frac{b^{10}}{x^3}+\frac{5ab^9}{x^2}+\frac{10a^2b^8}{x}+5a^4b^6x \cdots\right.\right.}{b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= -\frac{3b^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}} - \frac{15ab^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}{\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\sqrt[3]{x}} + \frac{30a^3b^3\sqrt{a^2+\cdots}}{ab+\cdots}$$

Mathematica [A]
time = 0.05, size = 99, normalized size = 0.34

$$\frac{\left(b+a\sqrt[3]{x}\right)\left(-3b^5-30ab^4\sqrt[3]{x}+60a^3b^2x+15a^4bx^{4/3}+2a^5x^{5/3}+20a^2b^3x^{2/3}\log(x)\right)}{2\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-3*b^5 - 30*a*b^4*x^(1/3) + 60*a^3*b^2*x + 15*a^4*b*x^(4/
3) + 2*a^5*x^(5/3) + 20*a^2*b^3*x^(2/3)*Log[x]))/(2*Sqrt[(b + a*x^(1/3))^2/
x^(2/3)]*x)
```

Maple [A]
time = 0.04, size = 91, normalized size = 0.31

method	result	size
derivativedivides	$\frac{\left(\frac{a^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}x\left(2a^5x^{\frac{5}{3}}+15b\,a^4x^{\frac{4}{3}}+20a^2b^3\ln(x)x^{\frac{2}{3}}+60b^2a^3x-30b^4a\,x^{\frac{1}{3}}-3b^5\right)}{2\left(b+a\,x^{\frac{1}{3}}\right)^5}$	91
default	$\frac{\left(\frac{a^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}x\left(2a^5x^{\frac{5}{3}}+15b\,a^4x^{\frac{4}{3}}+20a^2b^3\ln(x)x^{\frac{2}{3}}+60b^2a^3x-30b^4a\,x^{\frac{1}{3}}-3b^5\right)}{2\left(b+a\,x^{\frac{1}{3}}\right)^5}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)*x*(2*a^5*x^(5/3)+15*b*a
^4*x^(4/3)+20*a^2*b^3*ln(x)*x^(2/3)+60*b^2*a^3*x-30*b^4*a*x^(1/3)-3*b^5)/(b
+a*x^(1/3))^5
```

Maxima [A]
time = 0.34, size = 57, normalized size = 0.20

$$10\,a^2b^3\log\left(x\right)+\frac{2\,a^5x^{\frac{5}{3}}+15\,a^4bx^{\frac{4}{3}}+60\,a^3b^2x-30\,ab^4x^{\frac{1}{3}}-3\,b^5}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 10*a^2*b^3*log(x) + 1/2*(2*a^5*x^(5/3) + 15*a^4*b*x^(4/3) + 60*a^3*b^2*x -
30*a*b^4*x^(1/3) - 3*b^5)/x^(2/3)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} + \frac{2ab}{\sqrt[3]{x}} + \frac{b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(5/2), x)
```

Giac [**A**]
time = 3.15, size = 128, normalized size = 0.44

$$a^{5}x\mathrm{sgn}\left(ax + bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right) + 10\,a^{2}b^{3}\log\left(|x|\right)\mathrm{sgn}\left(ax + bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right) + \frac{15}{2}\,a^{4}bx^{\frac{2}{3}}\mathrm{sgn}\left(ax + bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right) + 30\,a^{3}b^{2}x^{\frac{1}{3}}\mathrm{sgn}\left(ax + bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right) - \frac{3\left(10\,ab^{4}x^{\frac{1}{3}}\mathrm{sgn}\left(ax + bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right) + b^{5}\mathrm{sgn}\left(ax + bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)\right)}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(2/3))*sgn(x) + 10*a^2*b^3*log(abs(x))*sgn(a*x + b*x^(2
/3))*sgn(x) + 15/2*a^4*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 30*a^3*b^2*x
^(1/3)*sgn(a*x + b*x^(2/3))*sgn(x) - 3/2*(10*a*b^4*x^(1/3)*sgn(a*x + b*x^(2
/3))*sgn(x) + b^5*sgn(a*x + b*x^(2/3))*sgn(x))/x^(2/3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)
```

3.484 $\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}\,dx$

Optimal. Leaf size=189

$$\frac{9ab^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,\sqrt[3]{x}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{9a^2b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x}{a+\frac{b}{\sqrt[3]{x}}}+\frac{3b^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\cdots}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] 9*a*b^2*x^(1/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+9/2*a^2
*b*x^(2/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a^3*x*(a^2+b
^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+b^3*ln(x)*(a^2+b^2/x^(2/3)+2*
a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))
```

Rubi [A]

time = 0.07, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$\frac{9a^2bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{9ab^2\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{3b^3\log\left(\sqrt[3]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}+\frac{a^3x\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

```
[Out] (9*a*b^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(1/3))/(a + b/x^(1/3))
 + (9*a^2*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^
(1/3))) + (a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))
 + (3*b^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*Log[x^(1/3)])/(a + b/x^
(1/3))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2} dx = 3\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2} x^2\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^3 x^2\,dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^3}{x}\,dx, x, \sqrt[3]{x}\right)}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(3ab^5+\frac{b^6}{x}+3a^2b^4x+a^3b^3x^2\right) dx, x, \sqrt[3]{\ }\right.}{b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}$$

$$= \frac{9ab^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,\sqrt[3]{x}}{ab+\frac{b^2}{\sqrt[3]{x}}} + \frac{9a^2b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)} + \frac{a^3\sqrt{a^2 -}}{a}$$

Mathematica [A]

time = 0.03, size = 75, normalized size = 0.40

$$\frac{\left(b+a\sqrt[3]{x}\right)\left(18ab^2\sqrt[3]{x}+9a^2bx^{2/3}+2a^3x+2b^3\log(x)\right)}{2\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x]
```

```
[Out] ((b + a*x^(1/3))*(18*a*b^2*x^(1/3) + 9*a^2*b*x^(2/3) + 2*a^3*x + 2*b^3*Log[
x]))/(2*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3))
```

Maple [A]
time = 0.04, size = 69, normalized size = 0.37

method	result	size
derivativedivides	$\frac{\left(\frac{a^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}x\left(2a^3x+9a^2bx^{\frac{2}{3}}+2b^3\ln(x)+18ab^2x^{\frac{1}{3}}\right)}{2\left(b+ax^{\frac{1}{3}}\right)^3}$	69
default	$\frac{\left(\frac{a^2x^{\frac{2}{3}}+2abx^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}x\left(2a^3x+9a^2bx^{\frac{2}{3}}+2b^3\ln(x)+18ab^2x^{\frac{1}{3}}\right)}{2\left(b+ax^{\frac{1}{3}}\right)^3}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)*x*(2*a^3*x+9*a^2*b*x^(2
/3)+2*b^3*ln(x)+18*a*b^2*x^(1/3))/(b+a*x^(1/3))^3
```

Maxima [A]
time = 0.29, size = 30, normalized size = 0.16

$$a^3x+b^3\log(x)+\frac{9}{2}a^2bx^{\frac{2}{3}}+9ab^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
```

```
[Out] a^3*x + b^3*log(x) + 9/2*a^2*b*x^(2/3) + 9*a*b^2*x^(1/3)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} + \frac{2ab}{\sqrt[3]{x}} + \frac{b^{2}}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(3/2), x)
```

Giac [A]
time = 3.50, size = 79, normalized size = 0.42

$$a^{3} x \operatorname{sgn}\left(a x + b x^{\frac{2}{3}}\right) \operatorname{sgn}(x) + b^{3} \log\left(|x|\right) \operatorname{sgn}\left(a x + b x^{\frac{2}{3}}\right) \operatorname{sgn}(x) + \frac{9}{2}\, a^{2} b x^{\frac{2}{3}} \operatorname{sgn}\left(a x + b x^{\frac{2}{3}}\right) \operatorname{sgn}(x) + 9\, a b^{2} x^{\frac{1}{3}} \operatorname{sgn}\left(a x + b x^{\frac{2}{3}}\right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

```
[Out] a^3*x*sgn(a*x + b*x^(2/3))*sgn(x) + b^3*log(abs(x))*sgn(a*x + b*x^(2/3))*sg
n(x) + 9/2*a^2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x) + 9*a*b^2*x^(1/3)*sgn(
a*x + b*x^(2/3))*sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)
```

3.485 $\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=88

$$\frac{3b\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x^{2/3}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x}{a+\frac{b}{\sqrt[3]{x}}}$$

```
[Out] 3/2*b*x^(2/3)*(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))+a*x*(a^2+
b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)/(a+b/x^(1/3))
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {1355, 1369, 14}

$$\frac{3bx^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{2\left(a+\frac{b}{\sqrt[3]{x}}\right)}+\frac{ax\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}{a+\frac{b}{\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (3*b*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x^(2/3))/(2*(a + b/x^(1/3)))
 + (a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*x)/(a + b/x^(1/3))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,dx &= 3\text{Subst}\left(\int \sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}\,x^2\,dx,x,\sqrt[3]{x}\right)\\ &= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)x^2\,dx,x,\sqrt[3]{x}\right)}{ab+\frac{b^2}{\sqrt[3]{x}}}\\ &= \frac{\left(3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\right)\text{Subst}\left(\int \left(b^2x+abx^2\right)\,dx,x,\sqrt[3]{x}\right)}{ab+\frac{b^2}{\sqrt[3]{x}}}\\ &= \frac{3b^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x^{2/3}}{2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)}+\frac{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,x}{a+\frac{b}{\sqrt[3]{x}}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 49, normalized size = 0.56

$$\frac{\left(3b+2a\sqrt[3]{x}\right)\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\,x}{2\left(b+a\sqrt[3]{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
[Out] ((3*b + 2*a*x^(1/3))*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x)/(2*(b + a*x^(1/3)))
```

Maple [A]
time = 0.05, size = 50, normalized size = 0.57

method	result	size
derivativedivides	$\frac{\sqrt{\frac{a^2 x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}}\, x\left(2a\,x^{\frac{1}{3}}+3b\right)}{2b+2a\,x^{\frac{1}{3}}}$	47
default	$\frac{\sqrt{\frac{a^2 x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}}\, x^{\frac{1}{3}}\left(3b\,x^{\frac{2}{3}}+2ax\right)}{2b+2a\,x^{\frac{1}{3}}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)*x^(1/3)*(3*b*x^(2/3)+2*
a*x)/(b+a*x^(1/3))
```

Maxima [A]
time = 0.29, size = 10, normalized size = 0.11

$$ax + \frac{3}{2}\,bx^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
```

```
[Out] a*x + 3/2*b*x^(2/3)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

```
[Out] Integral(sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

Giac [A]
time = 3.18, size = 34, normalized size = 0.39

$$ax\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)+\frac{3}{2}\,bx^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
[Out] a*x*sgn(a*x + b*x^(2/3))*sgn(x) + 3/2*b*x^(2/3)*sgn(a*x + b*x^(2/3))*sgn(x)
```

Mupad [B]
time = 1.43, size = 39, normalized size = 0.44

$$\frac{x\,\left(a+\frac{3\,b}{2\,x^{1/3}}\right)\,\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}}}{a+\frac{b}{x^{1/3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2),x)
[Out] (x*(a + (3*b)/(2*x^(1/3)))*(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2))/(a
+ b/x^(1/3))
```

3.486 $\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx$

Optimal. Leaf size=190

$$\frac{3b^2\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{3b\left(a+\frac{b}{\sqrt[3]{x}}\right)x^{2/3}}{2a^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}+\frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(b+a\sqrt[3]{x}\right)}{a^4\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

```
[Out] 3*b^2*(a+b/x^(1/3))*x^(1/3)/a^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-3/2*b
*(a+b/x^(1/3))*x^(2/3)/a^2/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)+(a+b/x^(1/
3))*x/a/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-3*b^3*(a+b/x^(1/3))*ln(b+a*x^
(1/3))/a^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$-\frac{3bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^2\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{3b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{3b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] (3*b^2*(a + b/x^(1/3))*x^(1/3))/(a^3*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)]) - (3*b*(a + b/x^(1/3))*x^(2/3))/(2*a^2*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/
3)]) - (3*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^4*Sqrt[a^2 + b^2/x^(2/
3) + (2*a*b)/x^(1/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}\,dx = 3\text{Subst}\left(\int \frac{x^2}{\sqrt{a^2+\frac{b^2}{x^2}+\frac{2ab}{x}}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{ab+\frac{b^2}{x}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^3}{b^2+abx}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{b}{a^3}-\frac{x}{a^2}+\frac{x^2}{ab}-\frac{b^2}{a^3(b+ax)}\right)dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^2+\frac{b^3}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^3\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} - \frac{3\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)x^{2/3}}{2a^2\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} + \frac{\left(a+\frac{b}{\sqrt[3]{x}}\right)x}{a\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}} -$$

Mathematica [A]

time = 0.03, size = 86, normalized size = 0.45

$$\frac{\left(b+a\sqrt[3]{x}\right)\left(6ab^2\sqrt[3]{x}-3a^2bx^{2/3}+2a^3x-6b^3\log\left(b+a\sqrt[3]{x}\right)\right)}{2a^4\sqrt{\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)],x]
```

```
[Out] ((b + a*x^(1/3))*(6*a*b^2*x^(1/3) - 3*a^2*b*x^(2/3) + 2*a^3*x - 6*b^3*Log[b
 + a*x^(1/3)]))/(2*a^4*Sqrt[(b + a*x^(1/3))^2/x^(2/3)]*x^(1/3))
```

Maple [A]
time = 0.04, size = 78, normalized size = 0.41

method	result	size
derivativedivides	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(-2a^3x+3a^2b\,x^{\frac{2}{3}}+6b^3\ln\left(b+a\,x^{\frac{1}{3}}\right)-6a\,b^2x^{\frac{1}{3}}\right)}{2\sqrt{\frac{a^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}}\,x^{\frac{1}{3}}a^4}$	78
default	$-\frac{\left(b+a\,x^{\frac{1}{3}}\right)\left(-2a^3x+3a^2b\,x^{\frac{2}{3}}+6b^3\ln\left(b+a\,x^{\frac{1}{3}}\right)-6a\,b^2x^{\frac{1}{3}}\right)}{2\sqrt{\frac{a^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}}\,x^{\frac{1}{3}}a^4}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b+a*x^(1/3))*(-2*a^3*x+3*a^2*b*x^(2/3)+6*b^3*ln(b+a*x^(1/3))-6*a*b^2*
x^(1/3))/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(1/2)/x^(1/3)/a^4
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.23

$$-\frac{3\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^4}+\frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="maxima")
```

```
[Out] -3*b^3*log(a*x^(1/3) + b)/a^4 + 1/2*(2*a^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3
))/a^3
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3)), x)
```

Giac [A]
time = 3.48, size = 77, normalized size = 0.41

$$-\frac{3\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^4\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}+\frac{2\,a^2x-3\,abx^{\frac{2}{3}}+6\,b^2x^{\frac{1}{3}}}{2\,a^3\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2),x, algorithm="giac")
```

```
[Out] -3*b^3*log(abs(a*x^(1/3) + b))/(a^4*sgn(a*x + b*x^(2/3))*sgn(x)) + 1/2*(2*a
^2*x - 3*a*b*x^(2/3) + 6*b^2*x^(1/3))/(a^3*sgn(a*x + b*x^(2/3))*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2),x)
```

```
[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(1/2), x)
```

3.487 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx$

Optimal. Leaf size=300

$$\frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,\left(b+a\sqrt[3]{x}\right)^2}-\frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\,\left(b+a\sqrt[3]{x}\right)}+\frac{18b^2\left(a+\frac{b}{\sqrt[3]{x}}\right)\sqrt[3]{x}}{a^5\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}-\frac{9b\left(\cdots\right.}{2a^4\sqrt{a\cdots}}$$

```
[Out] 3/2*b^5*(a+b/x^(1/3))/a^6/(b+a*x^(1/3))^2/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(
1/2)-15*b^4*(a+b/x^(1/3))/a^6/(b+a*x^(1/3))/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))
^(1/2)+18*b^2*(a+b/x^(1/3))*x^(1/3)/a^5/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/
2)-9/2*b*(a+b/x^(1/3))*x^(2/3)/a^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)+(a
+b/x^(1/3))*x/a^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-30*b^3*(a+b/x^(1/3)
)*ln(b+a*x^(1/3))/a^6/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$\frac{3b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\,\left(a\sqrt[3]{x}+b\right)^2}-\frac{15b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\,\left(a\sqrt[3]{x}+b\right)}-\frac{30b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{18b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{9bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^4\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^3\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

```
[Out] (3*b^5*(a + b/x^(1/3)))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b
 + a*x^(1/3))^2) - (15*b^4*(a + b/x^(1/3)))/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (
2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (18*b^2*(a + b/x^(1/3))*x^(1/3))/(a^5*Sq
rt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (9*b*(a + b/x^(1/3))*x^(2/3))/(2
*a^4*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^3*
Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]) - (30*b^3*(a + b/x^(1/3))*Log[b
+ a*x^(1/3)])/(a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{3/2}}\,dx = 3\text{Subst}\left(\int \frac{x^2}{\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+\frac{b^2}{x}\right)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^5}{(b^2+abx)^3}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^2\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{6}{a^5b}-\frac{3x}{a^4b^2}+\frac{x^2}{a^3b^3}-\frac{b^2}{a^5(b+ax)^3}+\frac{5b}{a^5(b+ax)^2}-\frac{1}{a^5(b\cdots}\right.\right.}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^5+\frac{b^6}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^2} - \frac{15\left(ab^4+\frac{b^5}{\sqrt[3]{x}}\right)}{a^6\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)}$$

Mathematica [A]
time = 0.08, size = 126, normalized size = 0.42

$$\frac{\left(b+a\sqrt[3]{x}\right)\left(-27b^5+6ab^4\sqrt[3]{x}+63a^2b^3x^{2/3}+20a^3b^2x-5a^4bx^{4/3}+2a^5x^{5/3}-60b^3\left(b+a\sqrt[3]{x}\right)^2\log\left(b+a\sqrt[3]{x}\right)\right)}{2a^6\left(\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}\right)^{3/2}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-3/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-27*b^5 + 6*a*b^4*x^(1/3) + 63*a^2*b^3*x^(2/3) + 20*a^3*b
^2*x - 5*a^4*b*x^(4/3) + 2*a^5*x^(5/3) - 60*b^3*(b + a*x^(1/3))^2*Log[b + a
*x^(1/3)]))/(2*a^6*((b + a*x^(1/3))^2/x^(2/3))^(3/2)*x)
```

Maple [A]
time = 0.04, size = 141, normalized size = 0.47

method	result
derivativedivides	$-\frac{\left(-2a^5x^{\frac{5}{3}}+5b\,a^4x^{\frac{4}{3}}+60\ln\left(b+a\,x^{\frac{1}{3}}\right)a^2b^3x^{\frac{2}{3}}-20b^2a^3x+120\ln\left(b+a\,x^{\frac{1}{3}}\right)a\,b^4x^{\frac{1}{3}}-63b^3x^{\frac{2}{3}}a^2+60\ln\left(b+a\,x^{\frac{1}{3}}\right)b^5-6b^4a}{2a^6x\left(\frac{a^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}$
default	$\frac{\left(2a^5x^{\frac{5}{3}}-5b\,a^4x^{\frac{4}{3}}-60\ln\left(b+a\,x^{\frac{1}{3}}\right)a^2b^3x^{\frac{2}{3}}+63b^3x^{\frac{2}{3}}a^2-120\ln\left(b+a\,x^{\frac{1}{3}}\right)a\,b^4x^{\frac{1}{3}}+6b^4a\,x^{\frac{1}{3}}-60\ln\left(b+a\,x^{\frac{1}{3}}\right)b^5+20b^2a^3x}{2\left(\frac{a^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}x\,a^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(3/2)/x*(2*a^5*x^(5/3)-5*b*a^
4*x^(4/3)-60*ln(b+a*x^(1/3))*a^2*b^3*x^(2/3)+63*b^3*x^(2/3)*a^2-120*ln(b+a*
x^(1/3))*a*b^4*x^(1/3)+6*b^4*a*x^(1/3)-60*ln(b+a*x^(1/3))*b^5+20*b^2*a^3*x-
27*b^5)*(b+a*x^(1/3))/a^6
```

Maxima [A]
time = 0.30, size = 97, normalized size = 0.32

$$\frac{2\,a^5x^{\frac{5}{3}}-5\,a^4bx^{\frac{4}{3}}+20\,a^3b^2x+63\,a^2b^3x^{\frac{2}{3}}+6\,ab^4x^{\frac{1}{3}}-27\,b^5}{2\left(a^8x^{\frac{2}{3}}+2\,a^7bx^{\frac{1}{3}}+a^6b^2\right)}-\frac{30\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^5*x^(5/3) - 5*a^4*b*x^(4/3) + 20*a^3*b^2*x + 63*a^2*b^3*x^(2/3) +
6*a*b^4*x^(1/3) - 27*b^5)/(a^8*x^(2/3) + 2*a^7*b*x^(1/3) + a^6*b^2) - 30*b^
3*log(a*x^(1/3) + b)/a^6
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-3/2), x)
```

Giac [A]
time = 4.27, size = 127, normalized size = 0.42

$$-\frac{30\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^6\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}-\frac{3\left(10\,ab^4x^{\frac{1}{3}}+9\,b^5\right)}{2\left(ax^{\frac{1}{3}}+b\right)^2a^6\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}+\frac{2\,a^6x-9\,a^5bx^{\frac{2}{3}}+36\,a^4b^2x^{\frac{1}{3}}}{2\,a^9\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(3/2),x, algorithm="giac")
```

```
[Out] -30*b^3*log(abs(a*x^(1/3) + b))/(a^6*sgn(a*x + b*x^(2/3))*sgn(x)) - 3/2*(10
*a*b^4*x^(1/3) + 9*b^5)/((a*x^(1/3) + b)^2*a^6*sgn(a*x + b*x^(2/3))*sgn(x))
 + 1/2*(2*a^6*x - 9*a^5*b*x^(2/3) + 36*a^4*b^2*x^(1/3))/(a^9*sgn(a*x + b*x^
(2/3))*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(3/2), x)
```

3.488 $\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx$

Optimal. Leaf size=410

$$\frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^4}-\frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^3}+\frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+\right.}$$

```
[Out] 3/4*b^7*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))^4/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(
1/2)-7*b^6*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))^3/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3)
)^(1/2)+63/2*b^5*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))^2/(a^2+b^2/x^(2/3)+2*a*b/x
^(1/3))^(1/2)-105*b^4*(a+b/x^(1/3))/a^8/(b+a*x^(1/3))/(a^2+b^2/x^(2/3)+2*a*
b/x^(1/3))^(1/2)+45*b^2*(a+b/x^(1/3))*x^(1/3)/a^7/(a^2+b^2/x^(2/3)+2*a*b/x^
(1/3))^(1/2)-15/2*b*(a+b/x^(1/3))*x^(2/3)/a^6/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3
))^(1/2)+(a+b/x^(1/3))*x/a^5/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)-105*b^3*
(a+b/x^(1/3))*ln(b+a*x^(1/3))/a^8/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$\frac{3b^7\left(a+\frac{b}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^4}-\frac{7b^6\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^3}+\frac{63b^5\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)^2}-\frac{105b^4\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}\left(a\sqrt[3]{x}+b\right)}-\frac{105b^3\left(a+\frac{b}{\sqrt[3]{x}}\right)\log\left(a\sqrt[3]{x}+b\right)}{a^8\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{45b^2\sqrt[3]{x}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^7\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}-\frac{15bx^{2/3}\left(a+\frac{b}{\sqrt[3]{x}}\right)}{2a^6\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}+\frac{x\left(a+\frac{b}{\sqrt[3]{x}}\right)}{a^5\sqrt{a^2+\frac{2ab}{\sqrt[3]{x}}+\frac{b^2}{x^{2/3}}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] (3*b^7*(a + b/x^(1/3)))/(4*a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b
 + a*x^(1/3))^4) - (7*b^6*(a + b/x^(1/3)))/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2
*a*b)/x^(1/3)]*(b + a*x^(1/3))^3) + (63*b^5*(a + b/x^(1/3)))/(2*a^8*Sqrt[a^
2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))^2) - (105*b^4*(a + b/x^(
1/3)))/(a^8*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3)]*(b + a*x^(1/3))) + (4
5*b^2*(a + b/x^(1/3))*x^(1/3))/(a^7*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3
)]) - (15*b*(a + b/x^(1/3))*x^(2/3))/(2*a^6*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b
)/x^(1/3)]) + ((a + b/x^(1/3))*x)/(a^5*Sqrt[a^2 + b^2/x^(2/3) + (2*a*b)/x^(
1/3)]) - (105*b^3*(a + b/x^(1/3))*Log[b + a*x^(1/3)])/(a^8*Sqrt[a^2 + b^2/x
^(2/3) + (2*a*b)/x^(1/3)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}\right)^{5/2}}\,dx = 3\text{Subst}\left(\int \frac{x^2}{\left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2}}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^2}{\left(ab+\frac{b^2}{x}\right)^5}\,dx,x,\sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \frac{x^7}{(b^2+abx)^5}\,dx,x,\sqrt[3]{x}\right)}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{\left(3b^4\left(ab+\frac{b^2}{\sqrt[3]{x}}\right)\right)\text{Subst}\left(\int \left(\frac{15}{a^7b^3}-\frac{5x}{a^6b^4}+\frac{x^2}{a^5b^5}-\frac{b^2}{a^7(b+ax)^5}+\frac{7b}{a^7(b+ax)^4}-\right.\right.}{\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}}$$

$$= \frac{3\left(ab^7+\frac{b^8}{\sqrt[3]{x}}\right)}{4a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)^4} - \frac{7\left(ab^6+\frac{b^7}{\sqrt[3]{x}}\right)}{a^8\sqrt{a^2+\frac{b^2}{x^{2/3}}+\frac{2ab}{\sqrt[3]{x}}}\left(b+a\sqrt[3]{x}\right)}$$

Mathematica [A]
time = 0.08, size = 152, normalized size = 0.37

$$\frac{\left(b+a\sqrt[3]{x}\right)\left(-319b^7-856ab^6\sqrt[3]{x}-444a^2b^5x^{2/3}+544a^3b^4x+556a^4b^3x^{4/3}+84a^5b^2x^{5/3}-14a^6bx^2+4a^7x^{7/3}-420b^3\left(b+a\sqrt[3]{x}\right)^4\log\left(b+a\sqrt[3]{x}\right)\right)}{4a^8\left(\frac{\left(b+a\sqrt[3]{x}\right)^2}{x^{2/3}}\right)^{5/2}x^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(-5/2),x]
```

```
[Out] ((b + a*x^(1/3))*(-319*b^7 - 856*a*b^6*x^(1/3) - 444*a^2*b^5*x^(2/3) + 544*
a^3*b^4*x + 556*a^4*b^3*x^(4/3) + 84*a^5*b^2*x^(5/3) - 14*a^6*b*x^2 + 4*a^7
*x^(7/3) - 420*b^3*(b + a*x^(1/3))^4*Log[b + a*x^(1/3)]))/(4*a^8*((b + a*x^
(1/3))^2/x^(2/3))^(5/2)*x^(5/3))
```

Maple [A]
time = 0.04, size = 199, normalized size = 0.49

method	result
derivativedivides	$-\frac{\left(-4a^7x^{\frac{7}{3}}+14a^6b\,x^2+420\ln\left(b+a\,x^{\frac{1}{3}}\right)a^4b^3x^{\frac{4}{3}}-84a^5b^2x^{\frac{5}{3}}+1680\ln\left(b+a\,x^{\frac{1}{3}}\right)a^3b^4x-556x^{\frac{4}{3}}b^3a^4+2520\ln\left(b+a\,x^{\frac{1}{3}}\right)a^2\right.}{4a^8x^{\frac{5}{3}}\left(\frac{a^2x^{\frac{2}{3}}+2ab\,x}{x^{\frac{2}{3}}}\right.}$
default	$\frac{\left(4a^7x^{\frac{7}{3}}+84a^5b^2x^{\frac{5}{3}}-420\ln\left(b+a\,x^{\frac{1}{3}}\right)a^4b^3x^{\frac{4}{3}}+556x^{\frac{4}{3}}b^3a^4-2520\ln\left(b+a\,x^{\frac{1}{3}}\right)a^2b^5x^{\frac{2}{3}}-444a^2b^5x^{\frac{2}{3}}-1680\ln\left(b+a\,x^{\frac{1}{3}}\right)a\,b\right.}{4\left(\frac{a^2x^{\frac{2}{3}}+2ab\,x^{\frac{1}{3}}+b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/((a^2*x^(2/3)+2*a*b*x^(1/3)+b^2)/x^(2/3))^(5/2)/x^(5/3)*(4*a^7*x^(7/3)+
84*a^5*b^2*x^(5/3)-420*ln(b+a*x^(1/3))*a^4*b^3*x^(4/3)+556*x^(4/3)*b^3*a^4-
2520*ln(b+a*x^(1/3))*a^2*b^5*x^(2/3)-444*a^2*b^5*x^(2/3)-1680*ln(b+a*x^(1/3
))*a*b^6*x^(1/3)-1680*ln(b+a*x^(1/3))*a^3*b^4*x-14*a^6*b*x^2-856*a*b^6*x^(1
/3)-420*ln(b+a*x^(1/3))*b^7+544*a^3*b^4*x-319*b^7)*(b+a*x^(1/3))/a^8
```

Maxima [A]
time = 0.29, size = 139, normalized size = 0.34

$$\frac{4\,a^7x^{\frac{7}{3}}-14\,a^6bx^2+84\,a^5b^2x^{\frac{5}{3}}+556\,a^4b^3x^{\frac{4}{3}}+544\,a^3b^4x-444\,a^2b^5x^{\frac{2}{3}}-856\,ab^6x^{\frac{1}{3}}-319\,b^7}{4\left(a^{12}x^{\frac{4}{3}}+4\,a^{11}bx+6\,a^{10}b^2x^{\frac{2}{3}}+4\,a^9b^3x^{\frac{1}{3}}+a^8b^4\right)}-\frac{105\,b^3\log\left(ax^{\frac{1}{3}}+b\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^7*x^(7/3) - 14*a^6*b*x^2 + 84*a^5*b^2*x^(5/3) + 556*a^4*b^3*x^(4/3
) + 544*a^3*b^4*x - 444*a^2*b^5*x^(2/3) - 856*a*b^6*x^(1/3) - 319*b^7)/(a^1
2*x^(4/3) + 4*a^11*b*x + 6*a^10*b^2*x^(2/3) + 4*a^9*b^3*x^(1/3) + a^8*b^4)
- 105*b^3*log(a*x^(1/3) + b)/a^8
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2 + \frac{2ab}{\sqrt[3]{x}} + \frac{b^2}{x^{\frac{2}{3}}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2/x**(2/3)+2*a*b/x**(1/3))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/3) + b**2/x**(2/3))**(-5/2), x)
```

Giac [A]
time = 4.70, size = 147, normalized size = 0.36

$$-\frac{105\,b^3\log\left(\left|ax^{\frac{1}{3}}+b\right|\right)}{a^8\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)} - \frac{420\,a^3b^4x + 1134\,a^2b^5x^{\frac{2}{3}} + 1036\,ab^6x^{\frac{1}{3}} + 319\,b^7}{4\left(ax^{\frac{1}{3}}+b\right)^4 a^8\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)} + \frac{2\,a^{10}x - 15\,a^9bx^{\frac{2}{3}} + 90\,a^8b^2x^{\frac{1}{3}}}{2\,a^{15}\mathrm{sgn}\left(ax+bx^{\frac{2}{3}}\right)\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2/x^(2/3)+2*a*b/x^(1/3))^(5/2),x, algorithm="giac")
```

```
[Out] -105*b^3*log(abs(a*x^(1/3) + b))/(a^8*sgn(a*x + b*x^(2/3))*sgn(x)) - 1/4*(4
20*a^3*b^4*x + 1134*a^2*b^5*x^(2/3) + 1036*a*b^6*x^(1/3) + 319*b^7)/((a*x^(
1/3) + b)^4*a^8*sgn(a*x + b*x^(2/3))*sgn(x)) + 1/2*(2*a^10*x - 15*a^9*b*x^(
2/3) + 90*a^8*b^2*x^(1/3))/(a^15*sgn(a*x + b*x^(2/3))*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+\frac{b^2}{x^{2/3}}+\frac{2\,a\,b}{x^{1/3}}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2),x)
```

```
[Out] int(1/(a^2 + b^2/x^(2/3) + (2*a*b)/x^(1/3))^(5/2), x)
```

3.489 $\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx$

Optimal. Leaf size=289

$$-\frac{4b^5\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}}{\left(a+\frac{b}{\sqrt[4]{x}}\right)\sqrt[4]{x}}+\frac{40a^2b^3\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt[4]{x}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^3b^2\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\sqrt{x}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^4b\sqrt{a^2+}}{3\left(\right.}$$

```
[Out] -4*b^5*(a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))/x^(1/4)+40*a^2*b
^3*x^(1/4)*(a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+20/3*a^4*b*x
^(3/4)*(a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+a^5*x*(a^2+2*a*b
/x^(1/4)+b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+5*a*b^4*ln(x)*(a^2+2*a*b/x^(1/4)+
b^2/x^(1/2))^(1/2)/(a+b/x^(1/4))+20*a^3*b^2*x^(1/2)*(a^2+2*a*b/x^(1/4)+b^2/
x^(1/2))^(1/2)/(a+b/x^(1/4))
```

Rubi [A]
time = 0.09, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$-\frac{4b^5\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{\sqrt[4]{x}\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{20ab^4\log\left(\sqrt[4]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{40a^2b^3\sqrt[4]{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}+\frac{20a^4bx^{3/4}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{3\left(a+\frac{b}{\sqrt[4]{x}}\right)}+\frac{20a^3b^2\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[4]{x}}+\frac{b^2}{\sqrt{x}}}}{a+\frac{b}{\sqrt[4]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

```
[Out] (-4*b^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)])/((a + b/x^(1/4))*x^(1/4)
) + (40*a^2*b^3*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(1/4))/(a + b/x
^(1/4)) + (20*a^3*b^2*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Sqrt[x])/(a
 + b/x^(1/4)) + (20*a^4*b*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x^(3/4)
)/(3*(a + b/x^(1/4))) + (a^5*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*x)/(
a + b/x^(1/4)) + (20*a*b^4*Sqrt[a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4)]*Log[x^
(1/4)])/(a + b/x^(1/4))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}\right)^{5/2} dx = 4\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^3\,dx, x, \sqrt[4]{x}\right)$$

$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^3\,dx, x, \sqrt[4]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$

$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^5}{x^2}\,dx, x, \sqrt[4]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$

$$= \frac{\left(4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\right)\text{Subst}\left(\int \left(10a^2b^8+\frac{b^{10}}{x^2}+\frac{5ab^9}{x}+10a^3b^7x+5a^4b\right.\right.}{b^4\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)}$$

$$= -\frac{4b^6\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}}{\left(ab+\frac{b^2}{\sqrt[4]{x}}\right)\sqrt[4]{x}} + \frac{40a^2b^4\sqrt{a^2+\frac{b^2}{\sqrt{x}}+\frac{2ab}{\sqrt[4]{x}}}\,\sqrt[4]{x}}{ab+\frac{b^2}{\sqrt[4]{x}}} + \frac{20a^3b^3\sqrt{\ldots}}{}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 0.34

$$\frac{\sqrt{\frac{\left(b+a\sqrt[4]{x}\right)^2}{\sqrt{x}}}\left(-12b^5+120a^2b^3\sqrt{x}+60a^3b^2x^{3/4}+20a^4bx+3a^5x^{5/4}+15ab^4\sqrt[4]{x}\log(x)\right)}{3\left(b+a\sqrt[4]{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/Sqrt[x] + (2*a*b)/x^(1/4))^(5/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/4))^2/Sqrt[x]]*(-12*b^5 + 120*a^2*b^3*Sqrt[x] + 60*a^3*b^
2*x^(3/4) + 20*a^4*b*x + 3*a^5*x^(5/4) + 15*a*b^4*x^(1/4)*Log[x]))/(3*(b +
a*x^(1/4)))
```

Maple [A]
time = 0.07, size = 94, normalized size = 0.33

method	result	size
derivativedivides	$\frac{\left(\frac{a^2\sqrt{x}+2ab\,x^{\frac{1}{4}}+b^2}{\sqrt{x}}\right)^{\frac{5}{2}} x\left(3a^5x^{\frac{5}{4}}+20b\,a^4x+60b^2a^3x^{\frac{3}{4}}+15b^4a\ln(x)x^{\frac{1}{4}}+120a^2b^3\sqrt{x}-12b^5\right)}{3\left(a\,x^{\frac{1}{4}}+b\right)^5}$	91
default	$\frac{\sqrt{\frac{a^2x^{\frac{3}{4}}+2ab\sqrt{x}+b^2x^{\frac{1}{4}}}{x^{\frac{3}{4}}}}\left(3a^5x^{\frac{5}{4}}+20b\,a^4x+60b^2a^3x^{\frac{3}{4}}+15b^4a\ln(x)x^{\frac{1}{4}}+120a^2b^3\sqrt{x}-12b^5\right)}{3a\,x^{\frac{1}{4}}+3b}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((a^2*x^(3/4)+2*a*b*x^(1/2)+b^2*x^(1/4))/x^(3/4))^(1/2)*(3*a^5*x^(5/4)+
20*b*a^4*x+60*b^2*a^3*x^(3/4)+15*b^4*a*ln(x)*x^(1/4)+120*a^2*b^3*x^(1/2)-12
*b^5)/(a*x^(1/4)+b)
```

Maxima [A]
time = 0.29, size = 57, normalized size = 0.20

$$5\,ab^4\log\left(x\right)+\frac{3\,a^5x^{\frac{5}{4}}+20\,a^4bx+60\,a^3b^2x^{\frac{3}{4}}+120\,a^2b^3\sqrt{x}-12\,b^5}{3\,x^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="maxima")
```

```
[Out] 5*a*b^4*log(x) + 1/3*(3*a^5*x^(5/4) + 20*a^4*b*x + 60*a^3*b^2*x^(3/4) + 120
*a^2*b^3*sqrt(x) - 12*b^5)/x^(1/4)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b/x**(1/4)+b**2/x**(1/2))**(5/2),x)
[Out] Timed out
```

Giac [**A**]
time = 5.21, size = 126, normalized size = 0.44

$$a^5x\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)+5\,ab^4\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)+\frac{20}{3}\,a^4bx^{\frac{3}{4}}\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)+20\,a^3b^2\sqrt{x}\,\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)+40\,a^2b^3x^{\frac{1}{4}}\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)-\frac{4\,b^5\mathrm{sgn}\left(ax+bx^{\frac{3}{4}}\right)\mathrm{sgn}(x)}{x^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b/x^(1/4)+b^2/x^(1/2))^(5/2),x, algorithm="giac")
[Out] a^5*x*sgn(a*x + b*x^(3/4))*sgn(x) + 5*a*b^4*log(abs(x))*sgn(a*x + b*x^(3/4)
)*sgn(x) + 20/3*a^4*b*x^(3/4)*sgn(a*x + b*x^(3/4))*sgn(x) + 20*a^3*b^2*sqrt
(x)*sgn(a*x + b*x^(3/4))*sgn(x) + 40*a^2*b^3*x^(1/4)*sgn(a*x + b*x^(3/4))*s
gn(x) - 4*b^5*sgn(a*x + b*x^(3/4))*sgn(x)/x^(1/4)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2+\frac{b^2}{\sqrt{x}}+\frac{2\,a\,b}{x^{1/4}}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2),x)
[Out] int((a^2 + b^2/x^(1/2) + (2*a*b)/x^(1/4))^(5/2), x)
```

3.490 $\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx$

Optimal. Leaf size=291

$$\frac{25ab^4\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\sqrt[5]{x}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^2b^3\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{2/5}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{50a^3b^2\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}x^{3/5}}{3\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{25a^4b\sqrt{a}}{4}$$

```
[Out] 25*a*b^4*x^(1/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+25*a^2
*b^3*x^(2/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+50/3*a^3*b
^2*x^(3/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+25/4*a^4*b*x
^(4/5)*(a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+a^5*x*(a^2+b^2/x
^(2/5)+2*a*b/x^(1/5))^(1/2)/(a+b/x^(1/5))+b^5*ln(x)*(a^2+b^2/x^(2/5)+2*a*b/
x^(1/5))^(1/2)/(a+b/x^(1/5))
```

Rubi [A]
time = 0.09, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$\frac{5b^5\log\left(\sqrt[5]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25ab^4\sqrt[5]{x}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^2b^3x^{2/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{a^5x\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{a+\frac{b}{\sqrt[5]{x}}}+\frac{25a^4bx^{4/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{4\left(a+\frac{b}{\sqrt[5]{x}}\right)}+\frac{50a^3b^2x^{3/5}\sqrt{a^2+\frac{2ab}{\sqrt[5]{x}}+\frac{b^2}{x^{2/5}}}}{3\left(a+\frac{b}{\sqrt[5]{x}}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

```
[Out] (25*a*b^4*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(1/5))/(a + b/x^(1/5)
) + (25*a^2*b^3*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(2/5))/(a + b/x
^(1/5)) + (50*a^3*b^2*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(3/5))/(3
*(a + b/x^(1/5))) + (25*a^4*b*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*x^(
4/5))/(4*(a + b/x^(1/5))) + (a^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*
x)/(a + b/x^(1/5)) + (5*b^5*Sqrt[a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5)]*Log[x
^(1/5)])/(a + b/x^(1/5))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}\right)^{5/2} dx = 5\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{5/2} x^4\,dx, x, \sqrt[5]{x}\right)$$

$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^5 x^4\,dx, x, \sqrt[5]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^5}{x}\,dx, x, \sqrt[5]{x}\right)}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

$$= \frac{\left(5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\right)\text{Subst}\left(\int \left(5ab^9+\frac{b^{10}}{x}+10a^2b^8x+10a^3b^7x^2+5a\right.\right.}{b^4\left(ab+\frac{b^2}{\sqrt[5]{x}}\right)}$$

$$= \frac{25ab^5\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\sqrt[5]{x}}{ab+\frac{b^2}{\sqrt[5]{x}}} + \frac{25a^2b^4\sqrt{a^2+\frac{b^2}{x^{2/5}}+\frac{2ab}{\sqrt[5]{x}}}\,x^{2/5}}{ab+\frac{b^2}{\sqrt[5]{x}}} + \frac{50a^3b^{\cdots}}{\cdots}$$

Mathematica [A]
time = 0.05, size = 101, normalized size = 0.35

$$\frac{\left(b+a\sqrt[5]{x}\right)\left(300ab^4\sqrt[5]{x}+300a^2b^3x^{2/5}+200a^3b^2x^{3/5}+75a^4bx^{4/5}+12a^5x+12b^5\log(x)\right)}{12\sqrt{\frac{\left(b+a\sqrt[5]{x}\right)^2}{x^{2/5}}}\sqrt[5]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x]
```

```
[Out] ((b + a*x^(1/5))*(300*a*b^4*x^(1/5) + 300*a^2*b^3*x^(2/5) + 200*a^3*b^2*x^(
3/5) + 75*a^4*b*x^(4/5) + 12*a^5*x + 12*b^5*Log[x]))/(12*Sqrt[(b + a*x^(1/5
))^2/x^(2/5)]*x^(1/5))
```

Maple [A]
time = 0.06, size = 91, normalized size = 0.31

method	result	size
derivativedivides	$\frac{\left(\frac{a^{2} x^{\frac{2}{5}}+2 a b\, x^{\frac{1}{5}}+b^{2}}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}} x \left(12 a^{5} x+75 b\, a^{4} x^{\frac{4}{5}}+200 b^{2} a^{3} x^{\frac{3}{5}}+300 a^{2} b^{3} x^{\frac{2}{5}}+12 b^{5} \ln \left(x \right)+300 b^{4} a\, x^{\frac{1}{5}}\right)}{12 \left(a\, x^{\frac{1}{5}}+b \right)^{5}}$	91
default	$\frac{\left(\frac{a^{2} x^{\frac{2}{5}}+2 a b\, x^{\frac{1}{5}}+b^{2}}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}} x \left(12 a^{5} x+75 b\, a^{4} x^{\frac{4}{5}}+200 b^{2} a^{3} x^{\frac{3}{5}}+300 a^{2} b^{3} x^{\frac{2}{5}}+12 b^{5} \ln \left(x \right)+300 b^{4} a\, x^{\frac{1}{5}}\right)}{12 \left(a\, x^{\frac{1}{5}}+b \right)^{5}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*((a^2*x^(2/5)+2*a*b*x^(1/5)+b^2)/x^(2/5))^(5/2)*x*(12*a^5*x+75*b*a^4*x
^(4/5)+200*b^2*a^3*x^(3/5)+300*a^2*b^3*x^(2/5)+12*b^5*ln(x)+300*b^4*a*x^(1/
5))/(a*x^(1/5)+b)^5
```

Maxima [A]
time = 0.29, size = 52, normalized size = 0.18

$$a^5x+b^5\log\left(x\right)+\frac{25}{4}\,a^4bx^{\frac{4}{5}}+\frac{50}{3}\,a^3b^2x^{\frac{3}{5}}+25\,a^2b^3x^{\frac{2}{5}}+25\,ab^4x^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="maxima")
```

```
[Out] a^5*x + b^5*log(x) + 25/4*a^4*b*x^(4/5) + 50/3*a^3*b^2*x^(3/5) + 25*a^2*b^3
*x^(2/5) + 25*a*b^4*x^(1/5)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2} + \frac{2ab}{\sqrt[5]{x}} + \frac{b^{2}}{x^{\frac{2}{5}}}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(2/5)+2*a*b/x**(1/5))**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b/x**(1/5) + b**2/x**(2/5))**(5/2), x)
```

Giac [**A**]
time = 6.53, size = 125, normalized size = 0.43

$$a^{5} x \operatorname{sgn}\left(a x + b x^{\frac{4}{5}}\right) \operatorname{sgn}(x) + b^{5} \log\left(|x|\right) \operatorname{sgn}\left(a x + b x^{\frac{4}{5}}\right) \operatorname{sgn}(x) + \frac{25}{4} \, a^{4} b x^{\frac{4}{5}} \operatorname{sgn}\left(a x + b x^{\frac{4}{5}}\right) \operatorname{sgn}(x) + \frac{50}{3} \, a^{3} b^{2} x^{\frac{3}{5}} \operatorname{sgn}\left(a x + b x^{\frac{4}{5}}\right) \operatorname{sgn}(x) + 25 \, a^{2} b^{3} x^{\frac{2}{5}} \operatorname{sgn}\left(a x + b x^{\frac{4}{5}}\right) \operatorname{sgn}(x) + 25 \, a b^{4} x^{\frac{1}{5}} \operatorname{sgn}\left(a x + b x^{\frac{4}{5}}\right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(2/5)+2*a*b/x^(1/5))^(5/2),x, algorithm="giac")
```

```
[Out] a^5*x*sgn(a*x + b*x^(4/5))*sgn(x) + b^5*log(abs(x))*sgn(a*x + b*x^(4/5))*sg
n(x) + 25/4*a^4*b*x^(4/5)*sgn(a*x + b*x^(4/5))*sgn(x) + 50/3*a^3*b^2*x^(3/5
)*sgn(a*x + b*x^(4/5))*sgn(x) + 25*a^2*b^3*x^(2/5)*sgn(a*x + b*x^(4/5))*sgn
(x) + 25*a*b^4*x^(1/5)*sgn(a*x + b*x^(4/5))*sgn(x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+\frac{b^2}{x^{2/5}}+\frac{2\,a\,b}{x^{1/5}}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2),x)
```

```
[Out] int((a^2 + b^2/x^(2/5) + (2*a*b)/x^(1/5))^(5/2), x)
```

3.491 $\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx$

Optimal. Leaf size=222

$$\frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

```
[Out] 20*a/b^5/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)-5/4*a^4/b^5/(a+b*x^(1/5))^3/
(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)+20/3*a^3/b^5/(a+b*x^(1/5))^2/(a^2+2*a
*b*x^(1/5)+b^2*x^(2/5))^(1/2)-15*a^2/b^5/(a+b*x^(1/5))/(a^2+2*a*b*x^(1/5)+b
^2*x^(2/5))^(1/2)+5*(a+b*x^(1/5))*ln(a+b*x^(1/5))/b^5/(a^2+2*a*b*x^(1/5)+b^
2*x^(2/5))^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1355, 660, 45}

$$-\frac{15a^2}{b^5\left(a+b\sqrt[5]{x}\right)\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{5\left(a+b\sqrt[5]{x}\right)\log\left(a+b\sqrt[5]{x}\right)}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}-\frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}+\frac{20a^3}{3b^5\left(a+b\sqrt[5]{x}\right)^2\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] (20*a)/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (5*a^4)/(4*b^5*(a +
b*x^(1/5))^3*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (20*a^3)/(3*b^5*(a
+ b*x^(1/5))^2*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) - (15*a^2)/(b^5*(a
+ b*x^(1/5))*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)]) + (5*(a + b*x^(1/5))*
Log[a + b*x^(1/5)])/(b^5*Sqrt[a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rubi steps

$$\int \frac{1}{\left(a^2+2ab\sqrt[5]{x}+b^2x^{2/5}\right)^{5/2}}\,dx = 5\text{Subst}\left(\int \frac{x^4}{\left(a^2+2abx+b^2x^2\right)^{5/2}}\,dx, x, \sqrt[5]{x}\right)$$

$$= \frac{\left(5b^5\left(a+b\sqrt[5]{x}\right)\right)\text{Subst}\left(\int \frac{x^4}{(ab+b^2x)^5}\,dx, x, \sqrt[5]{x}\right)}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$= \frac{\left(5b^5\left(a+b\sqrt[5]{x}\right)\right)\text{Subst}\left(\int \left(\frac{a^4}{b^9(a+bx)^5}-\frac{4a^3}{b^9(a+bx)^4}+\frac{6a^2}{b^9(a+bx)^3}-\frac{4a}{b^9(a+bx)^2}+\right.\right.}{\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

$$= \frac{20a}{b^5\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}} - \frac{5a^4}{4b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{a^2+2ab\sqrt[5]{x}+b^2x^{2/5}}}$$

Mathematica [A]
time = 0.07, size = 98, normalized size = 0.44

$$\frac{5a\left(25a^3+88a^2b\sqrt[5]{x}+108ab^2x^{2/5}+48b^3x^{3/5}\right)+60\left(a+b\sqrt[5]{x}\right)^4\log\left(a+b\sqrt[5]{x}\right)}{12b^5\left(a+b\sqrt[5]{x}\right)^3\sqrt{\left(a+b\sqrt[5]{x}\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^(1/5) + b^2*x^(2/5))^(-5/2),x]
```

```
[Out] (5*a*(25*a^3 + 88*a^2*b*x^(1/5) + 108*a*b^2*x^(2/5) + 48*b^3*x^(3/5)) + 60*
(a + b*x^(1/5))^4*Log[a + b*x^(1/5)])/(12*b^5*(a + b*x^(1/5))^3*Sqrt[(a + b
*x^(1/5))^2])
```

Maple [A]
time = 0.05, size = 152, normalized size = 0.68

method	result

derivativedivides	$\frac{5\left(12\ln\left(a+b\,x^{\frac{1}{5}}\right)b^{4}x^{\frac{4}{5}}+48\ln\left(a+b\,x^{\frac{1}{5}}\right)a\,b^{3}x^{\frac{3}{5}}+72\ln\left(a+b\,x^{\frac{1}{5}}\right)a^{2}b^{2}x^{\frac{2}{5}}+48a\,b^{3}x^{\frac{3}{5}}+48\ln\left(a+b\,x^{\frac{1}{5}}\right)a^{3}b\,x^{\frac{1}{5}}+108a^{2}b^{2}x^{\frac{2}{5}}+\right.}{12b^{5}\left(\left(a+b\,x^{\frac{1}{5}}\right)^{2}\right)^{\frac{5}{2}}}$
default	$\frac{5\sqrt{a^{2}+2ab\,x^{\frac{1}{5}}+b^{2}x^{\frac{2}{5}}}\,\left(12\ln\left(a+b\,x^{\frac{1}{5}}\right)b^{4}x^{\frac{4}{5}}+48\ln\left(a+b\,x^{\frac{1}{5}}\right)a\,b^{3}x^{\frac{3}{5}}+72\ln\left(a+b\,x^{\frac{1}{5}}\right)a^{2}b^{2}x^{\frac{2}{5}}+48a\,b^{3}x^{\frac{3}{5}}+48\ln\right.}{12\left(a+b\,x^{\frac{1}{5}}\right)^{5}b^{5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 5/12*(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(1/2)*(12*ln(a+b*x^(1/5))*b^4*x^(4/5)+
48*ln(a+b*x^(1/5))*a*b^3*x^(3/5)+72*ln(a+b*x^(1/5))*a^2*b^2*x^(2/5)+48*a*b^
3*x^(3/5)+48*ln(a+b*x^(1/5))*a^3*b*x^(1/5)+108*a^2*b^2*x^(2/5)+12*ln(a+b*x^
(1/5))*a^4+88*a^3*b*x^(1/5)+25*a^4)/(a+b*x^(1/5))^5/b^5
```

Maxima [A]
time = 0.28, size = 99, normalized size = 0.45

$$\frac{5\left(48\,ab^{3}x^{\frac{3}{5}}+108\,a^{2}b^{2}x^{\frac{2}{5}}+88\,a^{3}bx^{\frac{1}{5}}+25\,a^{4}\right)}{12\left(b^{9}x^{\frac{4}{5}}+4\,ab^{8}x^{\frac{3}{5}}+6\,a^{2}b^{7}x^{\frac{2}{5}}+4\,a^{3}b^{6}x^{\frac{1}{5}}+a^{4}b^{5}\right)}+\frac{5\,\log\left(bx^{\frac{1}{5}}+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="maxima")
```

```
[Out] 5/12*(48*a*b^3*x^(3/5) + 108*a^2*b^2*x^(2/5) + 88*a^3*b*x^(1/5) + 25*a^4)/(
b^9*x^(4/5) + 4*a*b^8*x^(3/5) + 6*a^2*b^7*x^(2/5) + 4*a^3*b^6*x^(1/5) + a^4
*b^5) + 5*log(b*x^(1/5) + a)/b^5
```

Fricas [A]
time = 0.45, size = 302, normalized size = 1.36

$$\frac{5\left(300\,a^{5}b^{15}x^{3}+100\,a^{15}b^{5}x+25\,a^{20}+12\left(b^{20}x^{4}+4\,a^{5}b^{15}x^{3}+6\,a^{10}b^{10}x^{2}+4\,a^{15}b^{5}x+a^{20}\right)\log\left(bx^{\frac{1}{5}}+a\right)+\left(48\,ab^{19}x^{3}-226\,a^{6}b^{14}x^{2}+104\,a^{11}b^{9}x+3\,a^{16}b^{4}\right)x^{\frac{4}{5}}-\left(84\,a^{2}b^{18}x^{3}-228\,a^{7}b^{13}x^{2}+67\,a^{12}b^{8}x+4\,a^{17}b^{3}\right)x^{\frac{3}{5}}+\left(136\,a^{3}b^{17}x^{3}-197\,a^{8}b^{12}x^{2}+48\,a^{13}b^{7}x+6\,a^{18}b^{2}\right)x^{\frac{2}{5}}-\left(207\,a^{4}b^{16}x^{3}-124\,a^{9}b^{11}x^{2}+56\,a^{14}b^{6}x+12\,a^{19}b\right)x^{\frac{1}{5}}\right)}{12\left(b^{25}x^{4}+4\,a^{5}b^{20}x^{3}+6\,a^{10}b^{15}x^{2}+4\,a^{15}b^{10}x+a^{20}b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="fricas")
```

```
[Out] 5/12*(300*a^5*b^15*x^3 + 100*a^15*b^5*x + 25*a^20 + 12*(b^20*x^4 + 4*a^5*b^
15*x^3 + 6*a^10*b^10*x^2 + 4*a^15*b^5*x + a^20)*log(b*x^(1/5) + a) + (48*a*
b^19*x^3 - 226*a^6*b^14*x^2 + 104*a^11*b^9*x + 3*a^16*b^4)*x^(4/5) - (84*a^
2*b^18*x^3 - 228*a^7*b^13*x^2 + 67*a^12*b^8*x + 4*a^17*b^3)*x^(3/5) + (136*
a^3*b^17*x^3 - 197*a^8*b^12*x^2 + 48*a^13*b^7*x + 6*a^18*b^2)*x^(2/5) - (20
7*a^4*b^16*x^3 - 124*a^9*b^11*x^2 + 56*a^14*b^6*x + 12*a^19*b)*x^(1/5))/(b^
25*x^4 + 4*a^5*b^20*x^3 + 6*a^10*b^15*x^2 + 4*a^15*b^10*x + a^20*b^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^{2}+2ab\sqrt[5]{x}+b^{2}x^{\frac{2}{5}}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**(1/5)+b**2*x**(2/5))**(5/2),x)
[Out] Integral((a**2 + 2*a*b*x**(1/5) + b**2*x**(2/5))**(-5/2), x)
```

Giac [A]
time = 6.02, size = 84, normalized size = 0.38

$$\frac{5\,\log\left(\left|bx^{\frac{1}{5}}+a\right|\right)}{b^{5}\mathrm{sgn}\left(bx^{\frac{1}{5}}+a\right)}+\frac{5\left(48\,ab^{2}x^{\frac{3}{5}}+108\,a^{2}bx^{\frac{2}{5}}+88\,a^{3}x^{\frac{1}{5}}+\frac{25\,a^{4}}{b}\right)}{12\left(bx^{\frac{1}{5}}+a\right)^{4}b^{4}\mathrm{sgn}\left(bx^{\frac{1}{5}}+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^(1/5)+b^2*x^(2/5))^(5/2),x, algorithm="giac")
[Out] 5*log(abs(b*x^(1/5) + a))/(b^5*sgn(b*x^(1/5) + a)) + 5/12*(48*a*b^2*x^(3/5)
 + 108*a^2*b*x^(2/5) + 88*a^3*x^(1/5) + 25*a^4/b)/((b*x^(1/5) + a)^4*b^4*sg
n(b*x^(1/5) + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a^2+b^2\,x^{2/5}+2\,a\,b\,x^{1/5}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2),x)
[Out] int(1/(a^2 + b^2*x^(2/5) + 2*a*b*x^(1/5))^(5/2), x)
```

3.492 $\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2}\,dx$

Optimal. Leaf size=391

$$-\frac{6b^7\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}}{\left(a+\frac{b}{\sqrt[6]{x}}\right)\sqrt[6]{x}}+\frac{126a^2b^5\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\,\sqrt[6]{x}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{105a^3b^4\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\,\sqrt[3]{x}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{70a^4b^3\sqrt{a}}{}$$

```
[Out] -6*b^7*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))/x^(1/6)+126*a^2*
b^5*x^(1/6)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+105*a^3*b^4
*x^(1/3)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+63/2*a^5*b^2*x
^(2/3)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+42/5*a^6*b*x^(5/
6)*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+a^7*x*(a^2+b^2/x^(1/
3)+2*a*b/x^(1/6))^(1/2)/(a+b/x^(1/6))+7*a*b^6*ln(x)*(a^2+b^2/x^(1/3)+2*a*b/
x^(1/6))^(1/2)/(a+b/x^(1/6))+70*a^4*b^3*(a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(1/
2)*x^(1/2)/(a+b/x^(1/6))
```

Rubi [A]
time = 0.12, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1355, 1369, 269, 45}

$$-\frac{6b^7\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{\sqrt[6]{x}\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{42ab^6\log\left(\sqrt[6]{x}\right)\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{126a^2b^5\sqrt[6]{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{a^7x\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{42a^6bx^{5/6}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{5\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{63a^5b^2x^{2/3}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{2\left(a+\frac{b}{\sqrt[6]{x}}\right)}+\frac{70a^4b^3\sqrt{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}+\frac{105a^3b^4\sqrt[3]{x}\sqrt{a^2+\frac{2ab}{\sqrt[6]{x}}+\frac{b^2}{\sqrt[3]{x}}}}{a+\frac{b}{\sqrt[6]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

```
[Out] (-6*b^7*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)])/((a + b/x^(1/6))*x^(1/6)
) + (126*a^2*b^5*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/6))/(a + b/
x^(1/6)) + (105*a^3*b^4*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*x^(1/3))/
(a + b/x^(1/6)) + (70*a^4*b^3*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6)]*Sqr
t[x])/(a + b/x^(1/6)) + (63*a^5*b^2*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6
)]*x^(2/3))/(2*(a + b/x^(1/6))) + (42*a^6*b*Sqrt[a^2 + b^2/x^(1/3) + (2*a*b
)/x^(1/6)]*x^(5/6))/(5*(a + b/x^(1/6))) + (a^7*Sqrt[a^2 + b^2/x^(1/3) + (2*
a*b)/x^(1/6)]*x)/(a + b/x^(1/6)) + (42*a*b^6*Sqrt[a^2 + b^2/x^(1/3) + (2*a*
b)/x^(1/6)]*Log[x^(1/6)])/(a + b/x^(1/6))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
```

```
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 1355

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k
 = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n
))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && Fra
ctionQ[n]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \left(a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}\right)^{7/2}dx = 6\text{Subst}\left(\int \left(a^2+\frac{b^2}{x^2}+\frac{2ab}{x}\right)^{7/2}x^5\,dx,x,\sqrt[6]{x}\right)$$

$$= \frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int \left(ab+\frac{b^2}{x}\right)^7x^5\,dx,x,\sqrt[6]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$= \frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int \frac{(b^2+abx)^7}{x^2}\,dx,x,\sqrt[6]{x}\right)}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$= \frac{\left(6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\right)\text{Subst}\left(\int \left(21a^2b^{12}+\frac{b^{14}}{x^2}+\frac{7ab^{13}}{x}+35a^3b^{11}x+35a\right.\right.}{b^6\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)}$$

$$= -\frac{6b^8\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}}{\left(ab+\frac{b^2}{\sqrt[6]{x}}\right)\sqrt[6]{x}}+\frac{126a^2b^6\sqrt{a^2+\frac{b^2}{\sqrt[3]{x}}+\frac{2ab}{\sqrt[6]{x}}}\sqrt[6]{x}}{ab+\frac{b^2}{\sqrt[6]{x}}}+\frac{105a^3b^5\sqrt{}}{}$$

Mathematica [A]
time = 0.05, size = 124, normalized size = 0.32

$$\frac{\sqrt{\frac{\left(b+a\sqrt[6]{x}\right)^2}{\sqrt[3]{x}}}\left(-60b^7+1260a^2b^5\sqrt[3]{x}+1050a^3b^4\sqrt{x}+700a^4b^3x^{2/3}+315a^5b^2x^{5/6}+84a^6bx+10a^7x^{7/6}+70ab^6\sqrt[6]{x}\log(x)\right)}{10\left(b+a\sqrt[6]{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x]
```

```
[Out] (Sqrt[(b + a*x^(1/6))^2/x^(1/3)]*(-60*b^7 + 1260*a^2*b^5*x^(1/3) + 1050*a^3
*b^4*Sqrt[x] + 700*a^4*b^3*x^(2/3) + 315*a^5*b^2*x^(5/6) + 84*a^6*b*x + 10*
a^7*x^(7/6) + 70*a*b^6*x^(1/6)*Log[x]))/(10*(b + a*x^(1/6)))
```

Maple [A]
time = 0.07, size = 116, normalized size = 0.30

method	result

derivativedivides	$\frac{\left(\frac{x^{\frac{1}{3}}a^2+2ab\,x^{\frac{1}{6}}+b^2}{x^{\frac{1}{3}}}\right)^{\frac{7}{2}} x\left(10a^7x^{\frac{7}{6}}+84a^6bx+315a^5b^2x^{\frac{5}{6}}+700a^4b^3x^{\frac{2}{3}}+1050a^3b^4\sqrt{x}+70a\,b^6\ln(x)x^{\frac{1}{6}}+1260a^2b^5x^{\frac{1}{3}}-6\right.}{10\left(a\,x^{\frac{1}{6}}+b\right)^7}$
default	$\frac{\sqrt{\frac{a^2\sqrt{x}+2ab\,x^{\frac{1}{3}}+b^2x^{\frac{1}{6}}}{\sqrt{x}}}\left(10a^7x^{\frac{7}{6}}+84a^6bx+315a^5b^2x^{\frac{5}{6}}+700a^4b^3x^{\frac{2}{3}}+1050a^3b^4\sqrt{x}+70a\,b^6\ln(x)x^{\frac{1}{6}}+1260a^2b^5\right.}{10a\,x^{\frac{1}{6}}+10b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*((a^2*x^(1/2)+2*a*b*x^(1/3)+b^2*x^(1/6))/x^(1/2))^(1/2)*(10*a^7*x^(7/6
)+84*a^6*b*x+315*a^5*b^2*x^(5/6)+700*a^4*b^3*x^(2/3)+1050*a^3*b^4*x^(1/2)+7
0*a*b^6*ln(x)*x^(1/6)+1260*a^2*b^5*x^(1/3)-60*b^7)/(a*x^(1/6)+b)
```

Maxima [A]
time = 0.30, size = 79, normalized size = 0.20

$$7\,ab^6\log\left(x\right)+\frac{10\,a^7x^{\frac{7}{6}}+84\,a^6bx+315\,a^5b^2x^{\frac{5}{6}}+700\,a^4b^3x^{\frac{2}{3}}+1050\,a^3b^4\sqrt{x}+1260\,a^2b^5x^{\frac{1}{3}}-60\,b^7}{10\,x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="maxima")
```

```
[Out] 7*a*b^6*log(x) + 1/10*(10*a^7*x^(7/6) + 84*a^6*b*x + 315*a^5*b^2*x^(5/6) +
700*a^4*b^3*x^(2/3) + 1050*a^3*b^4*sqrt(x) + 1260*a^2*b^5*x^(1/3) - 60*b^7)
/x^(1/6)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/x**(1/3)+2*a*b/x**(1/6))**(7/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.49, size = 172, normalized size = 0.44

$$a^7x\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)+7\,ab^6\log\left(|x|\right)\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)+\frac{42}{5}\,a^6bx^{\frac{5}{6}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)+\frac{63}{2}\,a^5b^2x^{\frac{2}{3}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)+70\,a^4b^3\sqrt{x}\,\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)+105\,a^3b^4x^{\frac{1}{3}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)+126\,a^2b^5x^{\frac{1}{6}}\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)-\frac{6\,b^7\mathrm{sgn}\left(ax+bx^{\frac{5}{6}}\right)\mathrm{sgn}(x)}{x^{\frac{1}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/x^(1/3)+2*a*b/x^(1/6))^(7/2),x, algorithm="giac")
```

```
[Out] a^7*x*sgn(a*x + b*x^(5/6))*sgn(x) + 7*a*b^6*log(abs(x))*sgn(a*x + b*x^(5/6)
)*sgn(x) + 42/5*a^6*b*x^(5/6)*sgn(a*x + b*x^(5/6))*sgn(x) + 63/2*a^5*b^2*x^
(2/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 70*a^4*b^3*sqrt(x)*sgn(a*x + b*x^(5/6))
*sgn(x) + 105*a^3*b^4*x^(1/3)*sgn(a*x + b*x^(5/6))*sgn(x) + 126*a^2*b^5*x^(
1/6)*sgn(a*x + b*x^(5/6))*sgn(x) - 6*b^7*sgn(a*x + b*x^(5/6))*sgn(x)/x^(1/6
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a^2+\frac{b^2}{x^{1/3}}+\frac{2\,a\,b}{x^{1/6}}\right)^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2),x)
```

```
[Out] int((a^2 + b^2/x^(1/3) + (2*a*b)/x^(1/6))^(7/2), x)
```

3.493 $\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=46

$$-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b^2\log\left(b+cx^n\right)}{c^3n}$$

```
[Out] -b*x^n/c^2/n+1/2*x^(2*n)/c/n+b^2*ln(b+c*x^n)/c^3/n
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 272, 45}

$$\frac{b^2\log\left(b+cx^n\right)}{c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b^2*Log[b + c*x^n])/(c^3*n)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1+4n}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1+3n}}{b+cx^n}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{b+cx}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{b^2}{c^2(b+cx)}\right)\,dx, x, x^n\right)}{n}$$
$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b^2\log(b+cx^n)}{c^3n}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.83

$$\frac{cx^n(-2b+cx^n)+2b^2\log(b+cx^n)}{2c^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (c*x^n*(-2*b + c*x^n) + 2*b^2*Log[b + c*x^n])/(2*c^3*n)
```

Maple [A]
time = 0.22, size = 47, normalized size = 1.02

method	result	size
risch	$\frac{x^{2n}}{2cn}-\frac{b\,x^n}{c^2n}+\frac{b^2\ln\left(x^n+\frac{b}{c}\right)}{c^3n}$	47
norman	$\left(\frac{\mathrm{e}^{3n\ln(x)}}{2cn}-\frac{b\,\mathrm{e}^{2n\ln(x)}}{c^2n}\right)\mathrm{e}^{-n\ln(x)}+\frac{b^2\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{c^3n}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c/n*(x^n)^2-b*x^n/c^2/n+b^2/c^3/n*ln(x^n+b/c)
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.98

$$\frac{b^2\log\left(\frac{cx^n+b}{c}\right)}{c^3n}+\frac{cx^{2\,n}-2\,bx^n}{2\,c^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] b^2*log((c*x^n + b)/c)/(c^3*n) + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n)
```

Fricas [A]
time = 0.45, size = 38, normalized size = 0.83

$$\frac{c^2 x^{2\,n} - 2\,bcx^n + 2\,b^2 \log\left(cx^n + b\right)}{2\,c^3 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*x^(2*n) - 2*b*c*x^n + 2*b^2*log(c*x^n + b))/(c^3*n)
```

Sympy [A]
time = 7.25, size = 42, normalized size = 0.91

$$\frac{b^2 \left(\begin{cases} \frac{x^n}{b} & \text{for}\ c = 0 \\ \frac{\log\left(b + cx^n\right)}{c} & \text{otherwise} \end{cases}\right)}{c^2 n} - \frac{bx^n}{c^2 n} + \frac{x^{2n}}{2cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] b**2*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(c**2*n) - b*
x**n/(c**2*n) + x**(2*n)/(2*c*n)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{4\,n-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(4*n - 1)/(b*x^n + c*x^(2*n)), x)
```

3.494 $\int \frac{x^{-1+3n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=28

$$\frac{x^n}{cn} - \frac{b\log\left(b+cx^n\right)}{c^2n}$$

[Out] x^n/c/n-b*ln(b+c*x^n)/c^2/n

Rubi [A]
time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1598, 272, 45}

$$\frac{x^n}{cn} - \frac{b\log\left(b+cx^n\right)}{c^2n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]

[Out] x^n/(c*n) - (b*Log[b + c*x^n])/(c^2*n)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1+3n}}{bx^n + cx^{2n}}\,dx = \int \frac{x^{-1+2n}}{b + cx^n}\,dx$$
$$= \frac{\operatorname{Subst}\left(\int \frac{x}{b+cx}\,dx, x, x^n\right)}{n}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{c} - \frac{b}{c(b+cx)}\right)\,dx, x, x^n\right)}{n}$$
$$= \frac{x^n}{cn} - \frac{b\log\left(b + cx^n\right)}{c^2 n}$$

Mathematica [A]
time = 0.03, size = 27, normalized size = 0.96

$$\frac{cx^n - b\log\left(cn(b + cx^n)\right)}{c^2 n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(b*x^n + c*x^(2*n)),x]
[Out] (c*x^n - b*Log[c*n*(b + c*x^n)])/(c^2*n)
```

Maple [A]
time = 0.22, size = 31, normalized size = 1.11

method	result	size
risch	$\frac{x^n}{cn} - \frac{b\ln\left(x^n + \frac{b}{c}\right)}{c^2 n}$	31
norman	$\frac{e^{n\ln(x)}}{cn} - \frac{b\ln\left(c\,e^{n\ln(x)} + b\right)}{c^2 n}$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] x^n/c/n-b/c^2/n*ln(x^n+b/c)
```

Maxima [A]
time = 0.28, size = 32, normalized size = 1.14

$$\frac{x^n}{cn} - \frac{b\log\left(\frac{cx^n + b}{c}\right)}{c^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] x^n/(c*n) - b*log((c*x^n + b)/c)/(c^2*n)
```

Fricas [A]
time = 0.45, size = 24, normalized size = 0.86

$$\frac{cx^n - b\log\left(cx^n + b\right)}{c^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] (c*x^n - b*log(c*x^n + b))/(c^2*n)
```

Sympy [A]
time = 6.82, size = 26, normalized size = 0.93

$$-\frac{b\left(\begin{cases}\frac{x^n}{b} & \text{for}\: c = 0\\\frac{\log\left(b + cx^n\right)}{c} & \text{otherwise}\end{cases}\right)}{cn} + \frac{x^n}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -b*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(c*n) + x**n/(c
*n)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{3\,n-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(3*n - 1)/(b*x^n + c*x^(2*n)), x)
```

3.495 $\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(b+cx^n\right)}{cn}$$

```
[Out] ln(b+c*x^n)/c/n
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1598, 266}

$$\frac{\log\left(b+cx^n\right)}{cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[b + c*x^n]/(c*n)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1+n}}{b+cx^n}\,dx\\ &= \frac{\log\left(b+cx^n\right)}{cn}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 1.20

$$\frac{\log\left(bn+cnx^n\right)}{cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[b*n + c*n*x^n]/(c*n)
```

Maple [**A**]
time = 0.20, size = 18, normalized size = 1.20

method	result	size
norman	$\frac{\ln\left(c\,{\mathrm e}^{n\ln\left(x\right)}+b\right)}{c n}$	18
risch	$\frac{\ln\left(x^{n}+\frac{b}{c}\right)}{c n}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c/n*ln(c*exp(n*ln(x))+b)
```

Maxima [**A**]
time = 0.28, size = 19, normalized size = 1.27

$$\frac{\log\left(\frac{cx^n+b}{c}\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log((c*x^n + b)/c)/(c*n)
```

Fricas [**A**]
time = 0.42, size = 15, normalized size = 1.00

$$\frac{\log\left(cx^n+b\right)}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] log(c*x^n + b)/(c*n)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(10) = 20$.

time = 3.02, size = 73, normalized size = 4.87

$$\begin{cases} \tilde{\infty} \log(x) & \text{for } b = 0 \wedge c = 0 \wedge n = 0 \\ \frac{x^n}{bn} & \text{for } c = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ \frac{\frac{2n \log\left(x^{2n}\right)}{4n^2-2n} - \frac{\log\left(x^{2n}\right)}{4n^2-2n}}{c} & \text{for } b = 0 \\ -\frac{\log(x)}{c} + \frac{\log\left(x^n + \frac{cx^{2n}}{b}\right)}{cn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x**n/(b*n), Eq(c,
0)), (log(x)/(b + c), Eq(n, 0)), ((2*n*log(x**(2*n))/(4*n**2 - 2*n) - log(x
**(2*n))/(4*n**2 - 2*n))/c, Eq(b, 0)), (-log(x)/c + log(x**n + c*x**(2*n)/b
)/(c*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x^{2\,n-1}}{b\,x^n + c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(2*n - 1)/(b*x^n + c*x^(2*n)), x)
```

3.496 $\int \frac{x^{-1+n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{b}-\frac{\log(b+cx^n)}{bn}$$

```
[Out] ln(x)/b-ln(b+c*x^n)/b/n
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1598, 272, 36, 29, 31}

$$\frac{\log(x)}{b}-\frac{\log(b+cx^n)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[x]/b - Log[b + c*x^n]/(b*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1+n}}{bx^n + cx^{2n}}\,dx &= \int \frac{1}{x\,(b+cx^n)}\,dx \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x(b+cx)}\,dx, x, x^n\right)}{n} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{bn} - \frac{c\operatorname{Subst}\left(\int \frac{1}{b+cx}\,dx, x, x^n\right)}{bn} \\
&= \frac{\log(x)}{b} - \frac{\log\left(b+cx^n\right)}{bn}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 25, normalized size = 1.09

$$\frac{\log\left(x^n\right) - \log\left(bn(b+cx^n)\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(b*x^n + c*x^(2*n)),x]
[Out] (Log[x^n] - Log[b*n*(b + c*x^n)])/(b*n)
```

Maple [A]
time = 0.20, size = 26, normalized size = 1.13

method	result	size
norman	$\frac{\ln(x)}{b} - \frac{\ln\left(c\,e^{n\ln(x)}+b\right)}{bn}$	26
risch	$\frac{\ln(x)}{b} - \frac{\ln\left(x^n+\frac{b}{c}\right)}{bn}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] ln(x)/b-1/b/n*ln(c*exp(n*ln(x))+b)
```

Maxima [A]
time = 0.28, size = 27, normalized size = 1.17

$$\frac{\log\left(x\right)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] log(x)/b - log((c*x^n + b)/c)/(b*n)
```

Fricas [A]
time = 0.37, size = 22, normalized size = 0.96

$$\frac{n\log(x) - \log(cx^n + b)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] (n*log(x) - log(c*x^n + b))/(b*n)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(15) = 30$.
time = 2.76, size = 42, normalized size = 1.83

$$\begin{cases} \frac{\log(x)}{c} & \text{for } b = 0 \wedge n = 0 \\ -\frac{x^{-n}}{cn} & \text{for } b = 0 \\ \frac{\log(x)}{b+c} & \text{for } n = 0 \\ \frac{2\log(x)}{b} - \frac{\log\left(x^n + \frac{cx^{2n}}{b}\right)}{bn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(b*x**n+c*x**(2*n)),x)
[Out] Piecewise((log(x)/c, Eq(b, 0) & Eq(n, 0)), (-1/(c*n*x**n), Eq(b, 0)), (log(
x)/(b + c), Eq(n, 0)), (2*log(x)/b - log(x**n + c*x**(2*n)/b)/(b*n), True))
```

Giac [A]
time = 5.83, size = 25, normalized size = 1.09

$$\frac{\log(|x|)}{b} - \frac{\log(|cx^n + b|)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] log(abs(x))/b - log(abs(c*x^n + b))/(b*n)
```

Mupad [B]
time = 1.37, size = 20, normalized size = 0.87

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x^n}{b} + 1\right)}{b\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(b*x^n + c*x^(2*n)),x)
[Out] -(2*atanh((2*c*x^n)/b + 1))/(b*n)
```

3.497 $\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=57

$$-\frac{x^{-2n}}{2bn}+\frac{cx^{-n}}{b^2n}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log\left(b+cx^n\right)}{b^3n}$$

```
[Out] -1/2/b/n/(x^(2*n))+c/b^2/n/(x^n)+c^2*ln(x)/b^3-c^2*ln(b+c*x^n)/b^3/n
```

Rubi [A]
time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {1598, 272, 46}

$$-\frac{c^2\log\left(b+cx^n\right)}{b^3n}+\frac{c^2\log(x)}{b^3}+\frac{cx^{-n}}{b^2n}-\frac{x^{-2n}}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/2*1/(b*n*x^(2*n)) + c/(b^2*n*x^n) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*x^
n])/(b^3*n)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1-n}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-2n}}{b+cx^n}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^3(b+cx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^3}-\frac{c}{b^2x^2}+\frac{c^2}{b^3x}-\frac{c^3}{b^3(b+cx)}\right)\,dx, x, x^n\right)}{n}$$
$$= -\frac{x^{-2n}}{2bn}+\frac{cx^{-n}}{b^2n}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log(b+cx^n)}{b^3n}$$

Mathematica [A]
time = 0.06, size = 48, normalized size = 0.84

$$-\frac{bx^{-2n}(b-2cx^n)-2c^2\log(x^n)+2c^2\log(b+cx^n)}{2b^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/2*((b*(b - 2*c*x^n))/x^(2*n) - 2*c^2*Log[x^n] + 2*c^2*Log[b + c*x^n])/(b
^3*n)
```

Maple [A]
time = 0.20, size = 58, normalized size = 1.02

method	result	size
risch	$\frac{c\,x^{-n}}{b^2n}-\frac{x^{-2n}}{2bn}+\frac{c^2\ln(x)}{b^3}-\frac{c^2\ln\left(x^n+\frac{b}{c}\right)}{b^3n}$	58
norman	$\left(\frac{c\,\mathrm{e}^{n\ln(x)}}{b^2n}-\frac{1}{2bn}+\frac{c^2\ln(x)\mathrm{e}^{2n\ln(x)}}{b^3}\right)\mathrm{e}^{-2n\ln(x)}-\frac{c^2\ln(c\,\mathrm{e}^{n\ln(x)}+b)}{b^3n}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] c/b^2/n/(x^n)-1/2/b/n/(x^n)^2+c^2*ln(x)/b^3-c^2/b^3/n*ln(x^n+b/c)
```

Maxima [A]
time = 0.30, size = 58, normalized size = 1.02

$$\frac{c^2\log(x)}{b^3}-\frac{c^2\log\left(\frac{cx^n+b}{c}\right)}{b^3n}+\frac{2\,cx^n-b}{2\,b^2nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*log(x)/b^3 - c^2*log((c*x^n + b)/c)/(b^3*n) + 1/2*(2*c*x^n - b)/(b^2*n*
x^(2*n))
```

Fricas [A]
time = 0.41, size = 59, normalized size = 1.04

$$\frac{2\,c^2nx^{2\,n}\log\left(x\right)-2\,c^2x^{2\,n}\log\left(cx^n+b\right)+2\,bcx^n-b^2}{2\,b^3nx^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*c^2*n*x^(2*n)*log(x) - 2*c^2*x^(2*n)*log(c*x^n + b) + 2*b*c*x^n - b^
2)/(b^3*n*x^(2*n))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{n+1}\,\left(b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n + 1)*(b*x^n + c*x^(2*n))), x)
```

3.498 $\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=76

$$-\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(b+cx^n\right)}{b^4n}$$

```
[Out] -1/3/b/n/(x^(3*n))+1/2*c/b^2/n/(x^(2*n))-c^2/b^3/n/(x^n)-c^3*ln(x)/b^4+c^3*
ln(b+c*x^n)/b^4/n
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1598, 272, 46}

$$\frac{c^3\log\left(b+cx^n\right)}{b^4n}-\frac{c^3\log(x)}{b^4}-\frac{c^2x^{-n}}{b^3n}+\frac{cx^{-2n}}{2b^2n}-\frac{x^{-3n}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/3*1/(b*n*x^(3*n)) + c/(2*b^2*n*x^(2*n)) - c^2/(b^3*n*x^n) - (c^3*Log[x])
/b^4 + (c^3*Log[b + c*x^n])/(b^4*n)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1-2n}}{bx^n+cx^{2n}}\,dx=\int \frac{x^{-1-3n}}{b+cx^n}\,dx$$
$$=\frac{\text{Subst}\left(\int \frac{1}{x^4(b+cx)}\,dx,x,x^n\right)}{n}$$
$$=\frac{\text{Subst}\left(\int \left(\frac{1}{bx^4}-\frac{c}{b^2x^3}+\frac{c^2}{b^3x^2}-\frac{c^3}{b^4x}+\frac{c^4}{b^4(b+cx)}\right)\,dx,x,x^n\right)}{n}$$
$$=-\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}-\frac{c^3\log(x)}{b^4}+\frac{c^3\log(b+cx^n)}{b^4n}$$

Mathematica [A]
time = 0.07, size = 63, normalized size = 0.83

$$-\frac{bx^{-3n}(2b^2-3bcx^n+6c^2x^{2n})+6c^3\log(x^n)-6c^3\log(b+cx^n)}{6b^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/6*((b*(2*b^2 - 3*b*c*x^n + 6*c^2*x^(2*n)))/x^(3*n) + 6*c^3*Log[x^n] - 6*
c^3*Log[b + c*x^n])/(b^4*n)
```

Maple [A]
time = 0.20, size = 75, normalized size = 0.99

method	result	size
risch	$-\frac{c^2x^{-n}}{b^3n}+\frac{c\,x^{-2n}}{2b^2n}-\frac{x^{-3n}}{3bn}-\frac{c^3\ln(x)}{b^4}+\frac{c^3\ln\left(x^n+\frac{b}{c}\right)}{b^4n}$	75
norman	$\left(-\frac{1}{3bn}+\frac{c\,\mathrm{e}^{n\ln(x)}}{2b^2n}-\frac{c^2\mathrm{e}^{2n\ln(x)}}{b^3n}-\frac{c^3\ln(x)\mathrm{e}^{3n\ln(x)}}{b^4}\right)\mathrm{e}^{-3n\ln(x)}+\frac{c^3\ln(c\,\mathrm{e}^{n\ln(x)}+b)}{b^4n}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -c^2/b^3/n/(x^n)+1/2*c/b^2/n/(x^n)^2-1/3/b/n/(x^n)^3-c^3*ln(x)/b^4+c^3/b^4/
n*ln(x^n+b/c)
```

Maxima [A]
time = 0.29, size = 71, normalized size = 0.93

$$-\frac{c^3\log(x)}{b^4}+\frac{c^3\log\left(\frac{cx^n+b}{c}\right)}{b^4n}-\frac{6\,c^2x^{2\,n}-3\,bcx^n+2\,b^2}{6\,b^3nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c^3*log(x)/b^4 + c^3*log((c*x^n + b)/c)/(b^4*n) - 1/6*(6*c^2*x^(2*n) - 3*b
*c*x^n + 2*b^2)/(b^3*n*x^(3*n))
```

Fricas [A]
time = 0.37, size = 72, normalized size = 0.95

$$-\frac{6\,c^3nx^{3\,n}\log\left(x\right)-6\,c^3x^{3\,n}\log\left(cx^n+b\right)+6\,bc^2x^{2\,n}-3\,b^2cx^n+2\,b^3}{6\,b^4nx^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/6*(6*c^3*n*x^(3*n)*log(x) - 6*c^3*x^(3*n)*log(c*x^n + b) + 6*b*c^2*x^(2*
n) - 3*b^2*c*x^n + 2*b^3)/(b^4*n*x^(3*n))
```

Sympy [A]
time = 21.50, size = 73, normalized size = 0.96

$$-\frac{x^{-3n}}{3bn}+\frac{cx^{-2n}}{2b^2n}-\frac{c^2x^{-n}}{b^3n}+\frac{c^4\left(\begin{cases}\frac{x^n}{b} & \text{for } c=0\\ \frac{\log\left(b+cx^n\right)}{c} & \text{otherwise}\end{cases}\right)}{b^4n}-\frac{c^3\log\left(x^n\right)}{b^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -1/(3*b*n*x**(3*n)) + c/(2*b**2*n*x**(2*n)) - c**2/(b**3*n*x**n) + c**4*Pie
cewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, True))/(b**4*n) - c**3*log(x
**n)/(b**4*n)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2\,n+1}\,\left(b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(2*n + 1)*(b*x^n + c*x^(2*n))), x)
```

3.499 $\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=93

$$-\frac{x^{-4n}}{4bn}+\frac{cx^{-3n}}{3b^2n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}+\frac{c^4\log(x)}{b^5}-\frac{c^4\log(b+cx^n)}{b^5n}$$

```
[Out] -1/4/b/n/(x^(4*n))+1/3*c/b^2/n/(x^(3*n))-1/2*c^2/b^3/n/(x^(2*n))+c^3/b^4/n/
(x^n)+c^4*ln(x)/b^5-c^4*ln(b+c*x^n)/b^5/n
```

Rubi [A]

time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1598, 272, 46}

$$-\frac{c^4\log(b+cx^n)}{b^5n}+\frac{c^4\log(x)}{b^5}+\frac{c^3x^{-n}}{b^4n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{cx^{-3n}}{3b^2n}-\frac{x^{-4n}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/4*1/(b*n*x^(4*n)) + c/(3*b^2*n*x^(3*n)) - c^2/(2*b^3*n*x^(2*n)) + c^3/(b
^4*n*x^n) + (c^4*Log[x])/b^5 - (c^4*Log[b + c*x^n])/(b^5*n)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1-3n}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-4n}}{b+cx^n}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x^5(b+cx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{bx^5}-\frac{c}{b^2x^4}+\frac{c^2}{b^3x^3}-\frac{c^3}{b^4x^2}+\frac{c^4}{b^5x}-\frac{c^5}{b^5(b+cx)}\right)dx, x, x^n\right)}{n}$$
$$= -\frac{x^{-4n}}{4bn}+\frac{cx^{-3n}}{3b^2n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}+\frac{c^4\log(x)}{b^5}-\frac{c^4\log(b+cx^n)}{b^5n}$$

Mathematica [A]
time = 0.09, size = 76, normalized size = 0.82

$$-\frac{bx^{-4n}(3b^3-4b^2cx^n+6bc^2x^{2n}-12c^3x^{3n})-12c^4\log(x^n)+12c^4\log(b+cx^n)}{12b^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/12*((b*(3*b^3 - 4*b^2*c*x^n + 6*b*c^2*x^(2*n) - 12*c^3*x^(3*n)))/x^(4*n)
 - 12*c^4*Log[x^n] + 12*c^4*Log[b + c*x^n])/(b^5*n)
```

Maple [A]
time = 0.20, size = 90, normalized size = 0.97

method	result	size
risch	$\frac{c^3x^{-n}}{b^4n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c\,x^{-3n}}{3b^2n}-\frac{x^{-4n}}{4bn}+\frac{c^4\ln(x)}{b^5}-\frac{c^4\ln\left(x^n+\frac{b}{c}\right)}{b^5n}$	90
norman	$\left(\frac{c^3\mathrm{e}^{3n\ln(x)}}{b^4n}-\frac{1}{4bn}+\frac{c\,\mathrm{e}^{n\ln(x)}}{3b^2n}-\frac{c^2\mathrm{e}^{2n\ln(x)}}{2b^3n}+\frac{c^4\ln(x)\mathrm{e}^{4n\ln(x)}}{b^5}\right)\mathrm{e}^{-4n\ln(x)}-\frac{c^4\ln\left(c\,\mathrm{e}^{n\ln(x)}+b\right)}{b^5n}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] c^3/b^4/n/(x^n)-1/2*c^2/b^3/n/(x^n)^2+1/3*c/b^2/n/(x^n)^3-1/4/b/n/(x^n)^4+c
^4*ln(x)/b^5-c^4/b^5/n*ln(x^n+b/c)
```

Maxima [A]
time = 0.28, size = 84, normalized size = 0.90

$$\frac{c^4\log(x)}{b^5}-\frac{c^4\log\left(\frac{cx^n+b}{c}\right)}{b^5n}+\frac{12\,c^3x^{3\,n}-6\,bc^2x^{2\,n}+4\,b^2cx^n-3\,b^3}{12\,b^4nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^4*log(x)/b^5 - c^4*log((c*x^n + b)/c)/(b^5*n) + 1/12*(12*c^3*x^(3*n) - 6*
b*c^2*x^(2*n) + 4*b^2*c*x^n - 3*b^3)/(b^4*n*x^(4*n))
```

Fricas [A]
time = 0.38, size = 85, normalized size = 0.91

$$\frac{12\,c^4nx^{4\,n}\log\left(x\right)-12\,c^4x^{4\,n}\log\left(cx^n+b\right)+12\,bc^3x^{3\,n}-6\,b^2c^2x^{2\,n}+4\,b^3cx^n-3\,b^4}{12\,b^5nx^{4\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*c^4*n*x^(4*n)*log(x) - 12*c^4*x^(4*n)*log(c*x^n + b) + 12*b*c^3*x^
(3*n) - 6*b^2*c^2*x^(2*n) + 4*b^3*c*x^n - 3*b^4)/(b^5*n*x^(4*n))
```

Sympy [A]
time = 40.51, size = 88, normalized size = 0.95

$$-\frac{x^{-4n}}{4bn}+\frac{cx^{-3n}}{3b^2n}-\frac{c^2x^{-2n}}{2b^3n}+\frac{c^3x^{-n}}{b^4n}-\frac{c^5\left(\begin{cases}\frac{x^n}{b} & \text{for } c=0\\ \frac{\log\left(b+cx^n\right)}{c} & \text{otherwise}\end{cases}\right)}{b^5n}+\frac{c^4\log\left(x^n\right)}{b^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] -1/(4*b*n*x**(4*n)) + c/(3*b**2*n*x**(3*n)) - c**2/(2*b**3*n*x**(2*n)) + c*
*3/(b**4*n*x**n) - c**5*Piecewise((x**n/b, Eq(c, 0)), (log(b + c*x**n)/c, T
rue))/(b**5*n) + c**4*log(x**n)/(b**5*n)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3\,n+1}\,\left(b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(3*n + 1)*(b*x^n + c*x^(2*n))), x)
```

3.500 $\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=236

$$-\frac{4x^{-3n/4}}{3bn}+\frac{\sqrt{2}\,c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n}-\frac{\sqrt{2}\,c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n}+\frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\,\sqrt[4]{b}\ \cdots\right.}{\sqrt{2}\,b^{7/\cdots}}$$

```
[Out] -4/3/b/n/(x^(3/4*n))+1/2*c^(3/4)*ln(-b^(1/4)*c^(1/4)*x^(1/4*n)*2^(1/2)+b^(1
/2)+x^(1/2*n)*c^(1/2))/b^(7/4)/n*2^(1/2)-1/2*c^(3/4)*ln(b^(1/4)*c^(1/4)*x^(
1/4*n)*2^(1/2)+b^(1/2)+x^(1/2*n)*c^(1/2))/b^(7/4)/n*2^(1/2)-c^(3/4)*arctan(
-1+c^(1/4)*x^(1/4*n)*2^(1/2)/b^(1/4))*2^(1/2)/b^(7/4)/n-c^(3/4)*arctan(1+c^
(1/4)*x^(1/4*n)*2^(1/2)/b^(1/4))*2^(1/2)/b^(7/4)/n
```

Rubi [A]
time = 0.15, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1598, 369, 352, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt{2}\,c^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n}-\frac{\sqrt{2}\,c^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{b}}+1\right)}{b^{7/4}n}+\frac{c^{3/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,x^{n/4}+\sqrt{b}+\sqrt{c}\,x^{n/2}\right)}{\sqrt{2}\,b^{7/4}n}-\frac{c^{3/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,x^{n/4}+\sqrt{b}+\sqrt{c}\,x^{n/2}\right)}{\sqrt{2}\,b^{7/4}n}-\frac{4x^{-3n/4}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(3*b*n*x^((3*n)/4)) + (Sqrt[2]*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x^(n/
4))/b^(1/4)])/(b^(7/4)*n) - (Sqrt[2]*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x^
(n/4))/b^(1/4)])/(b^(7/4)*n) + (c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n) - (c^(3/4)*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*x^(n/4) + Sqrt[c]*x^(n/2)])/(Sqrt[2]*b^(7/4)*n)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 369

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-\frac{3n}{4}}}{b+cx^n}\,dx$$
$$= -\frac{4x^{-3n/4}}{3bn} - \frac{c\int \frac{x^{\frac{1}{4}(-4+n)}}{b+cx^n}\,dx}{b}$$
$$= -\frac{4x^{-3n/4}}{3bn} - \frac{(4c)\text{Subst}\left(\int \frac{1}{b+cx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{bn}$$
$$= -\frac{4x^{-3n/4}}{3bn} - \frac{(2c)\text{Subst}\left(\int \frac{\sqrt{b}-\sqrt{c}\,x^2}{b+cx^4}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n} - \frac{(2c)\text{Subst}\left(\int \frac{\sqrt{b}+\sqrt{c}\,x^2}{b+cx^4}\,dx,\right.}{b^{3/2}n}$$
$$= -\frac{4x^{-3n/4}}{3bn} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt{b}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{c}}+x^2}\,dx, x, x^{1+\frac{1}{4}(-4+n)}\right)}{b^{3/2}n} - \frac{\sqrt{c}\,\text{Subst}\left(\int \frac{}{\frac{\sqrt{b}}{\sqrt{c}}+}\right.}{}$$
$$= -\frac{4x^{-3n/4}}{3bn} + \frac{c^{3/4}\log\left(\sqrt{b}-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\,x^{n/4}+\sqrt{c}\,x^{n/2}\right)}{\sqrt{2}\,b^{7/4}n} - \frac{c^{3/4}\log\left(\sqrt{b}+\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\right.}{\sqrt{2}\,b^{7/4}n}$$
$$= -\frac{4x^{-3n/4}}{3bn} + \frac{\sqrt{2}\,c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} - \frac{\sqrt{2}\,c^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{b}}\right)}{b^{7/4}n} + \frac{c}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 34, normalized size = 0.14

$$-\frac{4x^{-3n/4}\,{}_2F_1\left(-\frac{3}{4},1;\frac{1}{4};-\frac{cx^n}{b}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-4*Hypergeometric2F1[-3/4, 1, 1/4, -((c*x^n)/b)])/(3*b*n*x^((3*n)/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.25, size = 54, normalized size = 0.23

method	result	size

risch	$-\frac{4x^{-\frac{3n}{4}}}{3bn}+\left(\sum_{\_R=\mathrm{RootOf}\left(b^7n^4\_Z^4+c^3\right)}\_R\ln\left(x^{\frac{n}{4}}-\frac{b^2n\_R}{c}\right)\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -4/3/b/n/(x^(1/4*n))^3+sum(_R*ln(x^(1/4*n)-b^2*n/c*_R),_R=RootOf(_Z^4*b^7*n
^4+c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(1/4*n)/(b*c*x*x^n + b^2*x), x) - 4/3/(b*n*x^(3/4*n))
```

Fricas [A]
time = 0.40, size = 272, normalized size = 1.15

$$-\frac{12\,bnx^3x^{\frac{3}{4}\,n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^5cn^3xx^{\frac{1}{4}\,n-1}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{3}{4}}-b^5n^3x\sqrt{\frac{b^4n^2\sqrt{-\frac{c^3}{b^7n^4}}+c^2x^2x^{\frac{1}{2}\,n-2}}{x^2}}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{3}{4}}}{c^3}\right)+3\,bnx^3x^{\frac{3}{4}\,n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2n\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}+cxx^{\frac{1}{4}\,n-1}}{x}\right)-3\,bnx^3x^{\frac{3}{4}\,n-3}\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}\log\left(-\frac{b^2n\left(-\frac{c^3}{b^7n^4}\right)^{\frac{1}{4}}-cxx^{\frac{1}{4}\,n-1}}{x}\right)+4}{3\,bnx^3x^{\frac{3}{4}\,n-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/3*(12*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*arctan(-(b^5*c*n^3*x*
x^(1/4*n - 1)*(-c^3/(b^7*n^4))^(3/4) - b^5*n^3*x*sqrt((b^4*n^2*sqrt(-c^3/(b
^7*n^4)) + c^2*x^2*x^(1/2*n - 2))/x^2)*(-c^3/(b^7*n^4))^(3/4))/c^3) + 3*b*n
*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*log((b^2*n*(-c^3/(b^7*n^4))^(1/4)
 + c*x*x^(1/4*n - 1))/x) - 3*b*n*x^3*x^(3/4*n - 3)*(-c^3/(b^7*n^4))^(1/4)*l
og(-(b^2*n*(-c^3/(b^7*n^4))^(1/4) - c*x*x^(1/4*n - 1))/x) + 4)/(b*n*x^3*x^(
3/4*n - 3))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4374 deep
```

Giac [A]
time = 5.04, size = 203, normalized size = 0.86

$$-\frac{\frac{6\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,(x^n)^{\frac{1}{4}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{6\sqrt{2}\,(bc^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,(x^n)^{\frac{1}{4}}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{3\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(x^{\frac{1}{2}n}+\sqrt{2}\,(x^n)^{\frac{1}{4}}\left(\frac{b}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{b}{c}}\right)}{b^2}-\frac{3\sqrt{2}\,(bc^3)^{\frac{1}{4}}\log\left(x^{\frac{1}{2}n}-\sqrt{2}\,(x^n)^{\frac{1}{4}}\left(\frac{b}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{b}{c}}\right)}{b^2}+\frac{8}{bx^{\frac{3}{4}n}}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] -1/6*(6*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*(
x^n)^(1/4))/(b/c)^(1/4))/b^2 + 6*sqrt(2)*(b*c^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(b/c)^(1/4) - 2*(x^n)^(1/4))/(b/c)^(1/4))/b^2 + 3*sqrt(2)*(b*c^3)^
(1/4)*log(x^(1/2*n) + sqrt(2)*(x^n)^(1/4)*(b/c)^(1/4) + sqrt(b/c))/b^2 - 3*
sqrt(2)*(b*c^3)^(1/4)*log(x^(1/2*n) - sqrt(2)*(x^n)^(1/4)*(b/c)^(1/4) + sqr
t(b/c))/b^2 + 8/(b*x^(3/4*n)))/n
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{\frac{n}{4}-1}}{b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(n/4 - 1)/(b*x^n + c*x^(2*n)), x)
```

3.501 $\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=160

$$-\frac{3x^{-2n/3}}{2bn}+\frac{\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}\,x^{n/3}}{\sqrt{3}\,\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}\,x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x^{n/3}+c^{2/}\right.}{2b^{5/3}n}$$

```
[Out] -3/2/b/n/(x^(2/3*n))-c^(2/3)*ln(b^(1/3)+c^(1/3)*x^(1/3*n))/b^(5/3)/n+1/2*c^
(2/3)*ln(b^(2/3)-b^(1/3)*c^(1/3)*x^(1/3*n)+c^(2/3)*x^(2/3*n))/b^(5/3)/n+c^(
2/3)*arctan(1/3*(b^(1/3)-2*c^(1/3)*x^(1/3*n))/b^(1/3)*3^(1/2))*3^(1/2)/b^(5
/3)/n
```

Rubi [A]
time = 0.09, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {1598, 369, 352, 206, 31, 648, 631, 210, 642}

$$\frac{\sqrt{3}\,c^{2/3}\text{ArcTan}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}\,x^{n/3}}{\sqrt{3}\,\sqrt[3]{b}}\right)}{b^{5/3}n}-\frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}\,x^{n/3}\right)}{b^{5/3}n}+\frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n}-\frac{3x^{-2n/3}}{2bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(2*b*n*x^((2*n)/3)) + (Sqrt[3]*c^(2/3)*ArcTan[(b^(1/3) - 2*c^(1/3)*x^(n/
3))/(Sqrt[3]*b^(1/3))])/(b^(5/3)*n) - (c^(2/3)*Log[b^(1/3) + c^(1/3)*x^(n/3
)])/(b^(5/3)*n) + (c^(2/3)*Log[b^(2/3) - b^(1/3)*c^(1/3)*x^(n/3) + c^(2/3)*
x^((2*n)/3)])/(2*b^(5/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 369

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{3}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-\frac{2n}{3}}}{b+cx^n}\,dx$$
$$= -\frac{3x^{-2n/3}}{2bn} - \frac{c\int \frac{x^{\frac{1}{3}(-3+n)}}{b+cx^n}\,dx}{b}$$
$$= -\frac{3x^{-2n/3}}{2bn} - \frac{(3c)\text{Subst}\left(\int \frac{1}{b+cx^3}\,dx, x, x^{1+\frac{1}{3}(-3+n)}\right)}{bn}$$
$$= -\frac{3x^{-2n/3}}{2bn} - \frac{c\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{c}\,x}\,dx, x, x^{1+\frac{1}{3}(-3+n)}\right)}{b^{5/3}n} - \frac{c\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{c}\,x}{b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}\,x+c^{2/3}x^2}\right.}{b^{5/3}n}$$
$$= -\frac{3x^{-2n/3}}{2bn} - \frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}\,x^{n/3}\right)}{b^{5/3}n} + \frac{c^{2/3}\text{Subst}\left(\int \frac{-\sqrt[3]{b}\sqrt[3]{c}+2c^{2/3}x}{b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx, x, x^{1+\frac{1}{3}(\cdot}\right.}{2b^{5/3}n}$$
$$= -\frac{3x^{-2n/3}}{2bn} - \frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}\,x^{n/3}\right)}{b^{5/3}n} + \frac{c^{2/3}\log\left(b^{2/3}-\sqrt[3]{b}\sqrt[3]{c}\,x^{n/3}+c^{2/3}x^{2n/3}\right)}{2b^{5/3}n} - \cdots$$
$$= -\frac{3x^{-2n/3}}{2bn} + \frac{\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{c}\,x^{n/3}}{\sqrt{3}\sqrt[3]{b}}\right)}{b^{5/3}n} - \frac{c^{2/3}\log\left(\sqrt[3]{b}+\sqrt[3]{c}\,x^{n/3}\right)}{b^{5/3}n} + \frac{c^{2/3}\log\left(b\cdots\right.}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 34, normalized size = 0.21

$$-\frac{3x^{-2n/3}\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{cx^n}{b}\right)}{2bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/3)/(b*x^n + c*x^(2*n)),x]
[Out] (-3*Hypergeometric2F1[-2/3, 1, 1/3, -((c*x^n)/b)])/(2*b*n*x^((2*n)/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.20, size = 54, normalized size = 0.34

method	result	size
risch	$-\frac{3x^{-\frac{2n}{3}}}{2bn}+\left(\sum_{\_R=\text{RootOf}\left(b^5n^3\_Z^3+c^2\right)}\_R\ln\left(x^{\frac{n}{3}}-\frac{b^2n\_R}{c}\right)\right)$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/b/n/(x^(1/3*n))^2+sum(_R*ln(x^(1/3*n)-b^2*n/c*_R),_R=RootOf(_Z^3*b^5*n
^3+c^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(1/3*n)/(b*c*x*x^n + b^2*x), x) - 3/2/(b*n*x^(2/3*n))
```

Fricas [A]
time = 0.37, size = 212, normalized size = 1.32

$$\frac{2\sqrt{3}\,x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bxx^{\frac{1}{3}n-1}\left(-\frac{c^2}{b^2}\right)^{\frac{2}{3}}-\sqrt{3}\,c}{3c}\right)+2x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{cxx^{\frac{1}{3}n-1}-b\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}}{x}\right)-x^2x^{\frac{2}{3}n-2}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{c^2x^2x^{\frac{2}{3}n-2}+bcxx^{\frac{1}{3}n-1}\left(-\frac{c^2}{b^2}\right)^{\frac{1}{3}}+b^2\left(-\frac{c^2}{b^2}\right)^{\frac{2}{3}}}{x^2}\right)-3}{2bnx^2x^{\frac{2}{3}n-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*x^2*x^(2/3*n - 2)*(-c^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x
*x^(1/3*n - 1)*(-c^2/b^2)^(2/3) - sqrt(3)*c)/c) + 2*x^2*x^(2/3*n - 2)*(-c^2
/b^2)^(1/3)*log((c*x*x^(1/3*n - 1) - b*(-c^2/b^2)^(1/3))/x) - x^2*x^(2/3*n
- 2)*(-c^2/b^2)^(1/3)*log((c^2*x^2*x^(2/3*n - 2) + b*c*x*x^(1/3*n - 1)*(-c^
2/b^2)^(1/3) + b^2*(-c^2/b^2)^(2/3))/x^2) - 3)/(b*n*x^2*x^(2/3*n - 2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 6.08, size = 136, normalized size = 0.85

$$\frac{\frac{2c\left(-\frac{b}{c}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}n}-\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right|\right)}{b^2}-\frac{2\sqrt{3}\left(-bc^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}n}+\left(-\frac{b}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{b}{c}\right)^{\frac{1}{3}}}\right)}{b^2}-\frac{\left(-bc^2\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}n}\left(-\frac{b}{c}\right)^{\frac{1}{3}}+\left(x^n\right)^{\frac{2}{3}}+\left(-\frac{b}{c}\right)^{\frac{2}{3}}\right)}{b^2}-\frac{3}{b\left(x^n\right)^{\frac{2}{3}}}}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] 1/2*(2*c*(-b/c)^(1/3)*log(abs(x^(1/3*n) - (-b/c)^(1/3)))/b^2 - 2*sqrt(3)*(-
b*c^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3*n) + (-b/c)^(1/3))/(-b/c)^(1/3))/
b^2 - (-b*c^2)^(1/3)*log(x^(1/3*n)*(-b/c)^(1/3) + (x^n)^(2/3) + (-b/c)^(2/3
))/b^2 - 3/(b*(x^n)^(2/3)))/n
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{n}{3}-1}}{b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(n/3 - 1)/(b*x^n + c*x^(2*n)), x)
```

3.502 $\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=50

$$-\frac{2x^{-n/2}}{bn}+\frac{2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}$$

```
[Out] -2/b/n/(x^(1/2*n))+2*arctan(b^(1/2)/(x^(1/2*n))/c^(1/2))*c^(1/2)/b^(3/2)/n
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1598, 352, 199, 327, 211}

$$\frac{2\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}-\frac{2x^{-n/2}}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(b*n*x^(n/2)) + (2*Sqrt[c]*ArcTan[Sqrt[b]/(Sqrt[c]*x^(n/2))])/(b^(3/2)*n
)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
```

```
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{-1+\frac{n}{2}}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-\frac{n}{2}}}{b+cx^n}\,dx\\
&= -\frac{2\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^2}}\,dx, x, x^{-n/2}\right)}{n}\\
&= -\frac{2\text{Subst}\left(\int \frac{x^2}{c+bx^2}\,dx, x, x^{-n/2}\right)}{n}\\
&= -\frac{2x^{-n/2}}{bn}+\frac{(2c)\text{Subst}\left(\int \frac{1}{c+bx^2}\,dx, x, x^{-n/2}\right)}{bn}\\
&= -\frac{2x^{-n/2}}{bn}+\frac{2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{3/2}n}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 32, normalized size = 0.64

$$-\frac{2x^{-n/2}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{cx^n}{b}\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)/(b*x^n + c*x^(2*n)),x]
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, -((c*x^n)/b)])/(b*n*x^(n/2))
```

Maple [A]
time = 0.19, size = 79, normalized size = 1.58

method	result	size
risch	$-\frac{2x^{-\frac{n}{2}}}{bn}+\frac{\sqrt{-bc}\,\ln\left(x^{\frac{n}{2}}-\frac{\sqrt{-bc}}{c}\right)}{b^2 n}-\frac{\sqrt{-bc}\,\ln\left(x^{\frac{n}{2}}+\frac{\sqrt{-bc}}{c}\right)}{b^2 n}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b/n/(x^(1/2*n))+1/b^2*(-b*c)^(1/2)/n*ln(x^(1/2*n)-1/c*(-b*c)^(1/2))-1/b^
2*(-b*c)^(1/2)/n*ln(x^(1/2*n)+1/c*(-b*c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -c*integrate(x^(1/2*n)/(b*c*x*x^n + b^2*x), x) - 2/(b*n*x^(1/2*n))
```

Fricas [A]
time = 0.49, size = 151, normalized size = 3.02

$$\left[\frac{xx^{\frac{1}{2}\,n-1}\sqrt{-\frac{c}{b}}\,\log\left(\frac{cx^2x^{n-2}-2\,bxx^{\frac{1}{2}\,n-1}\sqrt{-\frac{c}{b}}-b}{cx^2x^{n-2}+b}\right)-2}{bnxx^{\frac{1}{2}\,n-1}},\ \frac{2\left(xx^{\frac{1}{2}\,n-1}\sqrt{\frac{c}{b}}\,\arctan\left(\frac{b\sqrt{\frac{c}{b}}}{cxx^{\frac{1}{2}\,n-1}}\right)-1\right)}{bnxx^{\frac{1}{2}\,n-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [(x*x^(1/2*n - 1)*sqrt(-c/b)*log((c*x^2*x^(n - 2) - 2*b*x*x^(1/2*n - 1)*sqr
t(-c/b) - b)/(c*x^2*x^(n - 2) + b)) - 2)/(b*n*x*x^(1/2*n - 1)), 2*(x*x^(1/2
*n - 1)*sqrt(c/b)*arctan(b*sqrt(c/b)/(c*x*x^(1/2*n - 1))) - 1)/(b*n*x*x^(1/
2*n - 1))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)/(b*x**n+c*x**(2*n)),x)
```

[Out] Timed out

Giac [A]
time = 6.36, size = 38, normalized size = 0.76

$$-\frac{2\left(\frac{c\arctan\left(\frac{c\sqrt{x^n}}{\sqrt{bc}}\right)}{\sqrt{bc}\,b}+\frac{1}{b\sqrt{x^n}}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] -2*(c*arctan(c*sqrt(x^n)/sqrt(b*c))/(sqrt(b*c)*b) + 1/(b*sqrt(x^n)))/n
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{\frac{n}{2}-1}}{b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/2 - 1)/(b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(n/2 - 1)/(b*x^n + c*x^(2*n)), x)
```

3.503 $\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=68

$$-\frac{2x^{-3n/2}}{3bn}+\frac{2cx^{-n/2}}{b^2n}-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}$$

```
[Out] -2/3/b/n/(x^(3/2*n))+2*c/b^2/n/(x^(1/2*n))-2*c^(3/2)*arctan(b^(1/2)/(x^(1/2
*n))/c^(1/2))/b^(5/2)/n
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1598, 369, 352, 199, 327, 211}

$$-\frac{2c^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}+\frac{2cx^{-n/2}}{b^2n}-\frac{2x^{-3n/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(3*b*n*x^((3*n)/2)) + (2*c)/(b^2*n*x^(n/2)) - (2*c^(3/2)*ArcTan[Sqrt[b]/
(Sqrt[c]*x^(n/2))])/(b^(5/2)*n)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 369

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{n}{2}}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-\frac{3n}{2}}}{b+cx^n}\,dx\\
&= -\frac{2x^{-3n/2}}{3bn}-\frac{c\int \frac{x^{-1-\frac{n}{2}}}{b+cx^n}\,dx}{b}\\
&= -\frac{2x^{-3n/2}}{3bn}+\frac{(2c)\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^2}}\,dx,x,x^{-n/2}\right)}{bn}\\
&= -\frac{2x^{-3n/2}}{3bn}+\frac{(2c)\text{Subst}\left(\int \frac{x^2}{c+bx^2}\,dx,x,x^{-n/2}\right)}{bn}\\
&= -\frac{2x^{-3n/2}}{3bn}+\frac{2cx^{-n/2}}{b^2n}-\frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{c+bx^2}\,dx,x,x^{-n/2}\right)}{b^2n}\\
&= -\frac{2x^{-3n/2}}{3bn}+\frac{2cx^{-n/2}}{b^2n}-\frac{2c^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^{-n/2}}{\sqrt{c}}\right)}{b^{5/2}n}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 34, normalized size = 0.50

$$-\frac{2x^{-3n/2}\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{cx^n}{b}\right)}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/2)/(b*x^n + c*x^(2*n)),x]
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, -((c*x^n)/b)])/(3*b*n*x^((3*n)/2))
```

Maple [A]
time = 0.20, size = 97, normalized size = 1.43

method	result	size
risch	$\frac{2 c x^{-\frac{n}{2}}}{b^{2} n}-\frac{2 x^{-\frac{3 n}{2}}}{3 b n}+\frac{\sqrt{-b c}\, c \ln \left(x^{\frac{n}{2}}+\frac{\sqrt{-b c}}{c}\right)}{b^{3} n}-\frac{\sqrt{-b c}\, c \ln \left(x^{\frac{n}{2}}-\frac{\sqrt{-b c}}{c}\right)}{b^{3} n}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] 2*c/b^2/n/(x^(1/2*n))-2/3/b/n/(x^(1/2*n))^3+1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/
2*n)+1/c*(-b*c)^(1/2))-1/b^3*(-b*c)^(1/2)*c/n*ln(x^(1/2*n)-1/c*(-b*c)^(1/2)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] c^2*integrate(x^(1/2*n)/(b^2*c*x*x^n + b^3*x), x) + 2/3*(3*c*x^n - b)/(b^2*
n*x^(3/2*n))
```

Fricas [A]
time = 0.38, size = 161, normalized size = 2.37

$$\left[-\frac{2\, b x^{3} x^{-\frac{3}{2}\, n-3}-6\, c x x^{-\frac{1}{2}\, n-1}-3\, c \sqrt{-\frac{c}{b}} \log \left(\frac{b x^{2} x^{-n-2}-2\, b x x^{-\frac{1}{2}\, n-1} \sqrt{-\frac{c}{b}}-c}{b x^{2} x^{-n-2}+c}\right)}{3\, b^{2} n}, -\frac{2 \left(b x^{3} x^{-\frac{3}{2}\, n-3}-3\, c x x^{-\frac{1}{2}\, n-1}-3\, c \sqrt{\frac{c}{b}} \arctan \left(\frac{\sqrt{\frac{c}{b}}}{x x^{-\frac{1}{2}\, n-1}}\right)\right)}{3\, b^{2} n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] [-1/3*(2*b*x^3*x^(-3/2*n - 3) - 6*c*x*x^(-1/2*n - 1) - 3*c*sqrt(-c/b)*log((
b*x^2*x^(-n - 2) - 2*b*x*x^(-1/2*n - 1)*sqrt(-c/b) - c)/(b*x^2*x^(-n - 2) +
 c)))/(b^2*n), -2/3*(b*x^3*x^(-3/2*n - 3) - 3*c*x*x^(-1/2*n - 1) - 3*c*sqrt
(c/b)*arctan(sqrt(c/b)/(x*x^(-1/2*n - 1))))/(b^2*n)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{\frac{n}{2}+1}\,\left(b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/2 + 1)*(b*x^n + c*x^(2*n))), x)
```

3.504 $\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=176

$$-\frac{3x^{-4n/3}}{4bn}+\frac{3cx^{-n/3}}{b^2n}+\frac{\sqrt{3}\,c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}\,x^{-n/3}}{\sqrt{3}\,\sqrt[3]{c}}\right)}{b^{7/3}n}-\frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}\,x^{-n/3}\right)}{b^{7/3}n}+\frac{c^{4/3}\log\left(c^{2/3}+b^{2/3}x^{-2}\right.}{2b^{7/}}$$

```
[Out] -3/4/b/n/(x^(4/3*n))+3*c/b^2/n/(x^(1/3*n))-c^(4/3)*ln(c^(1/3)+b^(1/3)/(x^(1
/3*n)))/b^(7/3)/n+1/2*c^(4/3)*ln(c^(2/3)+b^(2/3)/(x^(2/3*n))-b^(1/3)*c^(1/3
)/(x^(1/3*n)))/b^(7/3)/n+c^(4/3)*arctan(1/3*(1-2*b^(1/3)/c^(1/3)/(x^(1/3*n)
))*3^(1/2))*3^(1/2)/b^(7/3)/n
```

Rubi [A]
time = 0.10, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1598, 369, 352, 199, 327, 206, 31, 648, 631, 210, 642}

$$\frac{\sqrt{3}\,c^{4/3}\text{ArcTan}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{b}\,x^{-n/3}}{\sqrt{3}\,\sqrt[3]{c}}\right)}{b^{7/3}n}-\frac{c^{4/3}\log\left(\sqrt[3]{b}\,x^{-n/3}+\sqrt[3]{c}\right)}{b^{7/3}n}+\frac{c^{4/3}\log\left(b^{2/3}x^{-2n/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x^{-n/3}+c^{2/3}\right)}{2b^{7/3}n}+\frac{3cx^{-n/3}}{b^2n}-\frac{3x^{-4n/3}}{4bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(4*b*n*x^((4*n)/3)) + (3*c)/(b^2*n*x^(n/3)) + (Sqrt[3]*c^(4/3)*ArcTan[(c
^(1/3) - (2*b^(1/3))/x^(n/3))/(Sqrt[3]*c^(1/3))])/(b^(7/3)*n) - (c^(4/3)*Lo
g[c^(1/3) + b^(1/3)/x^(n/3)])/(b^(7/3)*n) + (c^(4/3)*Log[c^(2/3) + b^(2/3)/
x^((2*n)/3) - (b^(1/3)*c^(1/3))/x^(n/3)])/(2*b^(7/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 369

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{n}{3}}}{bx^n+cx^{2n}}\,dx &= \int \frac{x^{-1-\frac{4n}{3}}}{b+cx^n}\,dx \\
&= -\frac{3x^{-4n/3}}{4bn} - \frac{c\int \frac{x^{-1-\frac{n}{3}}}{b+cx^n}\,dx}{b} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{(3c)\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^3}}\,dx, x, x^{-n/3}\right)}{bn} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{(3c)\text{Subst}\left(\int \frac{x^3}{c+bx^3}\,dx, x, x^{-n/3}\right)}{bn} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{(3c^2)\,\text{Subst}\left(\int \frac{1}{c+bx^3}\,dx, x, x^{-n/3}\right)}{b^2n} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{b}\,x}\,dx, x, x^{-n/3}\right)}{b^2n} - \frac{c^{4/3}\text{Subst}\left(\int \frac{2\sqrt[3]{c}-}{c^{2/3}-\sqrt[3]{b}\,\sqrt[3]{}}\right.}{b^2} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}\,x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\text{Subst}\left(\int \frac{-\sqrt[3]{b}\,\sqrt[3]{c}+2b^{2/3}x}{c^{2/3}-\sqrt[3]{b}\,\sqrt[3]{c}\,x+b^{2/3}x^2}\,d\right.}{2b^{7/3}n} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}\,x^{-n/3}\right)}{b^{7/3}n} + \frac{c^{4/3}\log\left(c^{2/3}+b^{2/3}x^{-2n/3}-\sqrt[3]{b}\,\sqrt[3]{}\right.}{2b^{7/3}n} \\
&= -\frac{3x^{-4n/3}}{4bn} + \frac{3cx^{-n/3}}{b^2n} + \frac{\sqrt{3}\,c^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x^{-n/3}}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{b^{7/3}n} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{b}\,x^{-n/3}\right)}{b^{7/3}n} +
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 34, normalized size = 0.19

$$-\frac{3x^{-4n/3}\,{}_2F_1\left(-\frac{4}{3},1;-\frac{1}{3};-\frac{cx^n}{b}\right)}{4bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/3)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] (-3*Hypergeometric2F1[-4/3, 1, -1/3, -((c*x^n)/b)])/(4*b*n*x^((4*n)/3))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.21, size = 73, normalized size = 0.41

method	result	size
risch	$\frac{3c\,x^{-\frac{n}{3}}}{b^2 n}-\frac{3x^{-\frac{4n}{3}}}{4bn}+\left(\sum_{\_R=\mathrm{RootOf}\left(b^7 n^3\_Z^3+c^4\right)}\_R\ln\left(x^{\frac{n}{3}}+\frac{b^5 n^2\_R^2}{c^3}\right)\right)$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 3*c/b^2/n/(x^(1/3*n))-3/4/b/n/(x^(1/3*n))^4+sum(_R*ln(x^(1/3*n)+b^5*n^2/c^3
*_R^2),_R=RootOf(_Z^3*b^7*n^3+c^4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^(2/3*n)/(b^2*c*x*x^n + b^3*x), x) + 3/4*(4*c*x^n - b)/(b^2*
n*x^(4/3*n))
```

Fricas [A]
time = 0.36, size = 171, normalized size = 0.97

$$-\frac{3\,bx^4x^{-\frac{4}{3}n-4}-12\,cxx^{-\frac{1}{3}n-1}-4\sqrt{3}\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bxx^{-\frac{1}{3}n-1}\left(-\frac{c}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,c}{3\,c}\right)-4\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\log\left(\frac{xx^{-\frac{1}{3}n-1}-\left(-\frac{c}{b}\right)^{\frac{1}{3}}}{x}\right)+2\,c\left(-\frac{c}{b}\right)^{\frac{1}{3}}\log\left(\frac{x^2x^{-\frac{2}{3}n-2}+xx^{-\frac{1}{3}n-1}\left(-\frac{c}{b}\right)^{\frac{1}{3}}+\left(-\frac{c}{b}\right)^{\frac{2}{3}}}{x^2}\right)}{4\,b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/4*(3*b*x^4*x^(-4/3*n - 4) - 12*c*x*x^(-1/3*n - 1) - 4*sqrt(3)*c*(-c/b)^(
1/3)*arctan(1/3*(2*sqrt(3)*b*x*x^(-1/3*n - 1)*(-c/b)^(2/3) - sqrt(3)*c)/c)
- 4*c*(-c/b)^(1/3)*log((x*x^(-1/3*n - 1) - (-c/b)^(1/3))/x) + 2*c*(-c/b)^(1
/3)*log((x^2*x^(-2/3*n - 2) + x*x^(-1/3*n - 1)*(-c/b)^(1/3) + (-c/b)^(2/3))
/x^2))/(b^2*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/3*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{\frac{n}{3}+1}\,\left(b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/3 + 1)*(b*x^n + c*x^(2*n))), x)
```

3.505 $\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=252

$$-\frac{4x^{-5n/4}}{5bn}+\frac{4cx^{-n/4}}{b^2n}+\frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}-\frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{b}\,x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}+\frac{c^{5/4}\log\left(\sqrt{\right.}}{}$$

```
[Out] -4/5/b/n/(x^(5/4*n))+4*c/b^2/n/(x^(1/4*n))+1/2*c^(5/4)*ln(-b^(1/4)*c^(1/4)*
2^(1/2)/(x^(1/4*n))+b^(1/2)/(x^(1/2*n))+c^(1/2))/b^(9/4)/n*2^(1/2)-1/2*c^(5
/4)*ln(b^(1/4)*c^(1/4)*2^(1/2)/(x^(1/4*n))+b^(1/2)/(x^(1/2*n))+c^(1/2))/b^(
9/4)/n*2^(1/2)+c^(5/4)*arctan(1-b^(1/4)*2^(1/2)/c^(1/4)/(x^(1/4*n)))*2^(1/2
)/b^(9/4)/n-c^(5/4)*arctan(1+b^(1/4)*2^(1/2)/c^(1/4)/(x^(1/4*n)))*2^(1/2)/b
^(9/4)/n
```

Rubi [A]
time = 0.16, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {1598, 369, 352, 199, 327, 217, 1179, 642, 1176, 631, 210}

$$\frac{\sqrt{2}\,c^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n}-\frac{\sqrt{2}\,c^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{b}\,x^{-n/4}}{\sqrt[4]{c}}+1\right)}{b^{9/4}n}+\frac{c^{5/4}\log\left(-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,x^{-n/4}+\sqrt{b}\,x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}\,b^{9/4}n}-\frac{c^{5/4}\log\left(\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,x^{-n/4}+\sqrt{b}\,x^{-n/2}+\sqrt{c}\right)}{\sqrt{2}\,b^{9/4}n}+\frac{4cx^{-n/4}}{b^2n}-\frac{4x^{-5n/4}}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(5*b*n*x^((5*n)/4)) + (4*c)/(b^2*n*x^(n/4)) + (Sqrt[2]*c^(5/4)*ArcTan[1
- (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) - (Sqrt[2]*c^(5/4)*ArcT
an[1 + (Sqrt[2]*b^(1/4))/(c^(1/4)*x^(n/4))])/(b^(9/4)*n) + (c^(5/4)*Log[Sqr
t[c] + Sqrt[b]/x^(n/2) - (Sqrt[2]*b^(1/4)*c^(1/4))/x^(n/4)])/(Sqrt[2]*b^(9/
4)*n) - (c^(5/4)*Log[Sqrt[c] + Sqrt[b]/x^(n/2) + (Sqrt[2]*b^(1/4)*c^(1/4))/
x^(n/4)])/(Sqrt[2]*b^(9/4)*n)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 352

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/(m + 1),
Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{
a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

Rule 369

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[x^(m + 1)/(a*(m +
 1)), x] - Dist[b/a, Int[x^Simplify[m + n]/(a + b*x^n), x], x] /; FreeQ[{a,
 b, m, n}, x] && FractionQ[Simplify[(m + 1)/n]] && SumSimplerQ[m, n]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{4}}}{bx^n+cx^{2n}}\,dx = \int \frac{x^{-1-\frac{5n}{4}}}{b+cx^n}\,dx$$

$$= -\frac{4x^{-5n/4}}{5bn} - \frac{c\int \frac{x^{-1-\frac{n}{4}}}{b+cx^n}\,dx}{b}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{(4c)\text{Subst}\left(\int \frac{1}{b+\frac{c}{x^4}}\,dx, x, x^{-n/4}\right)}{bn}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{(4c)\text{Subst}\left(\int \frac{x^4}{c+bx^4}\,dx, x, x^{-n/4}\right)}{bn}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} - \frac{(4c^2)\,\text{Subst}\left(\int \frac{1}{c+bx^4}\,dx, x, x^{-n/4}\right)}{b^2n}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} - \frac{(2c^{3/2})\,\text{Subst}\left(\int \frac{\sqrt{c}-\sqrt{b}\,x^2}{c+bx^4}\,dx, x, x^{-n/4}\right)}{b^2n} - \frac{(2c^{3/2})\,\text{Subst}\Big(\int \cdots}{\cdots}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{c^{5/4}\text{Subst}\left(\int \frac{\frac{\sqrt{2}\,\sqrt[4]{c}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{c}}{\sqrt{b}}-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{b}}-x^2}\,dx, x, x^{-n/4}\right)}{\sqrt{2}\,b^{9/4}n} + \frac{c^{5/4}\text{Subst}\Big(\cdots}{\cdots}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{c^{5/4}\log\left(\sqrt{c}+\sqrt{b}\,x^{-n/2}-\sqrt{2}\,\sqrt[4]{b}\,\sqrt[4]{c}\,x^{-n/4}\right)}{\sqrt{2}\,b^{9/4}n} - \frac{c^{5/4}\log\Big(\sqrt{c}\cdots}{\cdots}$$

$$= -\frac{4x^{-5n/4}}{5bn} + \frac{4cx^{-n/4}}{b^2n} + \frac{\sqrt{2}\,c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{b}\,x^{-n/4}}{\sqrt[4]{c}}\right)}{b^{9/4}n} - \frac{\sqrt{2}\,c^{5/4}\tan^{-1}\Big(1+\frac{\sqrt{2}\cdots}{\cdots}}{b^{9/4}n}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.03, size = 34, normalized size = 0.13

$$-\frac{4x^{-5n/4}\,{}_2F_1\left(-\frac{5}{4},1;-\frac{1}{4};-\frac{cx^n}{b}\right)}{5bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/4)/(b*x^n + c*x^(2*n)),x]
[Out] (-4*Hypergeometric2F1[-5/4, 1, -1/4, -((c*x^n)/b)])/(5*b*n*x^((5*n)/4))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 73, normalized size = 0.29

method	result	size
risch	$\frac{4c\,x^{-\frac{n}{4}}}{b^2 n}-\frac{4x^{-\frac{5n}{4}}}{5bn}+\left(\sum_{\_R=\mathrm{RootOf}(b^9n^4\_Z^4+c^5)}\_R\ln\left(x^{\frac{n}{4}}+\frac{b^7n^3\_R^3}{c^4}\right)\right)$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] 4*c/b^2/n/(x^(1/4*n))-4/5/b/n/(x^(1/4*n))^5+sum(_R*ln(x^(1/4*n)+b^7*n^3/c^4
*_R^3),_R=RootOf(_Z^4*b^9*n^4+c^5))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] c^2*integrate(x^(3/4*n)/(b^2*c*x*x^n + b^3*x), x) + 4/5*(5*c*x^n - b)/(b^2*
n*x^(5/4*n))
```

Fricas [A]
time = 0.36, size = 259, normalized size = 1.03

$$-\frac{4\,bx^5x^{-\frac{5}{4}\,n-5}+20\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{b^7cn^3xx^{-\frac{1}{4}\,n-1}\left(-\frac{c^5}{b^9n^4}\right)^{\frac{3}{4}}-b^7n^3x\sqrt{\frac{b^4n^2\sqrt{-\frac{c^5}{b^9n^4}}+c^2x^2x^{-\frac{1}{2}\,n-2}}{x^2}}\left(-\frac{c^5}{b^9n^4}\right)^{\frac{3}{4}}}{c^5}\right)+5\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\log\left(\frac{b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}+cxx^{-\frac{1}{4}\,n-1}}{x}\right)-5\,b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}\log\left(-\frac{b^2n\left(-\frac{c^5}{b^9n^4}\right)^{\frac{1}{4}}-cxx^{-\frac{1}{4}\,n-1}}{x}\right)-20\,cxx^{-\frac{1}{4}\,n-1}}{5\,b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] -1/5*(4*b*x^5*x^(-5/4*n - 5) + 20*b^2*n*(-c^5/(b^9*n^4))^(1/4)*arctan(-(b^7
*c*n^3*x*x^(-1/4*n - 1)*(-c^5/(b^9*n^4))^(3/4) - b^7*n^3*x*sqrt((b^4*n^2*sq
rt(-c^5/(b^9*n^4)) + c^2*x^2*x^(-1/2*n - 2))/x^2)*(-c^5/(b^9*n^4))^(3/4))/c
^5) + 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log((b^2*n*(-c^5/(b^9*n^4))^(1/4) + c*
x*x^(-1/4*n - 1))/x) - 5*b^2*n*(-c^5/(b^9*n^4))^(1/4)*log(-(b^2*n*(-c^5/(b^
9*n^4))^(1/4) - c*x*x^(-1/4*n - 1))/x) - 20*c*x*x^(-1/4*n - 1))/(b^2*n)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/4*n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{4}+1}\,\left(b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/4 + 1)*(b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/4 + 1)*(b*x^n + c*x^(2*n))), x)
```

3.506 $\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=37

$$\frac{x^{-n(1+p)}\left(bx^n+cx^{2n}\right)^{1+p}}{cn(1+p)}$$

```
[Out] (b*x^n+c*x^(2*n))^(1+p)/c/n/(1+p)/(x^(n*(1+p)))
```

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2039}

$$\frac{x^{-n(p+1)}\left(bx^n+cx^{2n}\right)^{p+1}}{cn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(c*n*(1 + p)*x^(n*(1 + p)))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1-n(-1+p)}\left(bx^n+cx^{2n}\right)^p\,dx=\frac{x^{-n(1+p)}\left(bx^n+cx^{2n}\right)^{1+p}}{cn(1+p)}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 1.03

$$\frac{x^{-np}(b+cx^n)\left(x^n(b+cx^n)\right)^p}{cn(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*(-1 + p))*(b*x^n + c*x^(2*n))^p,x]
[Out] ((b + c*x^n)*(x^n*(b + c*x^n))^p)/(c*n*(1 + p)*x^(n*p))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{-1-n(-1+p)} \left(b\,x^{n}+c\,x^{2n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
[Out] int(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x)
```

Maxima [A]
time = 0.48, size = 43, normalized size = 1.16

$$\frac{(cx^n+b)e^{(-np\log(x)+p\log(cx^n+b)+p\log(x^n))}}{cn(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
[Out] (c*x^n + b)*e^(-n*p*log(x) + p*log(c*x^n + b) + p*log(x^n))/(c*n*(p + 1))
```

Fricas [A]
time = 0.40, size = 59, normalized size = 1.59

$$\frac{(cxx^{-np+n-1}x^n+bxx^{-np+n-1})(cx^{2\,n}+bx^n)^p}{(cnp+cn)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
[Out] (c*x*x^(-n*p + n - 1)*x^n + b*x*x^(-n*p + n - 1))*(c*x^(2*n) + b*x^n)^p/((c
*n*p + c*n)*x^n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*(-1+p))*(b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(p - 1) - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(b\,x^n+c\,x^{2\,n})^p}{x^{n\,(p-1)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1),x)
```

```
[Out] int((b*x^n + c*x^(2*n))^p/x^(n*(p - 1) + 1), x)
```

3.507 $\int x^{-1-n(1+2p)}(bx^n+cx^{2n})^p\,dx$

Optimal. Leaf size=38

$$-\frac{x^{-2n(1+p)}(bx^n+cx^{2n})^{1+p}}{bn(1+p)}$$

```
[Out] -(b*x^n+c*x^(2*n))^(1+p)/b/n/(1+p)/(x^(2*n*(1+p)))
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2039}

$$-\frac{x^{-2n(p+1)}(bx^n+cx^{2n})^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
[Out] -((b*x^n + c*x^(2*n))^(1 + p)/(b*n*(1 + p)*x^(2*n*(1 + p))))
```

Rule 2039

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1-n(1+2p)}\left(bx^n+cx^{2n}\right)^p\,dx=-\frac{x^{-2n(1+p)}\left(bx^n+cx^{2n}\right)^{1+p}}{bn(1+p)}$$

Mathematica [A]
time = 0.07, size = 43, normalized size = 1.13

$$-\frac{x^{-n(1+2p)}(b+cx^n)\left(x^n(b+cx^n)\right)^p}{bn(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*(1 + 2*p))*(b*x^n + c*x^(2*n))^p,x]
[Out] -(((b + c*x^n)*(x^n*(b + c*x^n))^p)/(b*n*(1 + p)*x^(n*(1 + 2*p))))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int x^{-1-n(1+2p)}\left(b\,x^{n}+c\,x^{2n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

Fricas [A]
time = 0.36, size = 59, normalized size = 1.55

$$-\frac{\left(cxx^{-2\,np-n-1}x^{n}+bxx^{-2\,np-n-1}\right)\left(cx^{2\,n}+bx^{n}\right)^{p}}{bnp+bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] -(c*x*x^(-2*n*p - n - 1)*x^n + b*x*x^(-2*n*p - n - 1))*(c*x^(2*n) + b*x^n)^
p/(b*n*p + b*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*(1+2*p))*(b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(1+2*p))*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n)^p*x^(-n*(2*p + 1) - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(b\,x^n+c\,x^{2\,n}\right)^p}{x^{n\,\left(2\,p+1\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + c*x^(2*n))^p/x^(n*(2*p + 1) + 1),x)
[Out] int((b*x^n + c*x^(2*n))^p/x^(n*(2*p + 1) + 1), x)
```

3.508 $\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^{5/2}\,dx$

Optimal. Leaf size=112

$$-\frac{a(a+bx^n)^6\sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\,(ab^2+b^3x^n)}+\frac{(a+bx^n)^7\sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\,(ab^2+b^3x^n)}$$

```
[Out] -1/6*a*(a+b*x^n)^6*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)+1/7*
(a+b*x^n)^7*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)
```

Rubi [A]
time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1369, 272, 45}

$$\frac{(a+bx^n)^7\sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\,(ab^2+b^3x^n)}-\frac{a(a+bx^n)^6\sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\,(ab^2+b^3x^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] -1/6*(a*(a + b*x^n)^6*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b^2 + b^3*
x^n)) + ((a + b*x^n)^7*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(7*n*(a*b^2 + b
^3*x^n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}\left(ab+b^2x^n\right)^5\,dx}{b^4\left(ab+b^2x^n\right)}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int x\left(ab+b^2x\right)^5\,dx,x,x^n\right)}{b^4n\left(ab+b^2x^n\right)}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^5}{b}+\frac{\left(ab+b^2x\right)^6}{b^2}\right)dx,x,x^n\right)}{b^4n\left(ab+b^2x^n\right)}\\ &= -\frac{a(a+bx^n)^6\sqrt{a^2+2abx^n+b^2x^{2n}}}{6n\left(ab^2+b^3x^n\right)}+\frac{(a+bx^n)^7\sqrt{a^2+2abx^n+b^2x^{2n}}}{7n\left(ab^2+b^3x^n\right)}\end{aligned}$$

Mathematica [**A**]
time = 0.05, size = 96, normalized size = 0.86

$$\frac{x^{2n}\left((a+bx^n)^2\right)^{5/2}\left(21a^5+70a^4bx^n+105a^3b^2x^{2n}+84a^2b^3x^{3n}+35ab^4x^{4n}+6b^5x^{5n}\right)}{42n\left(a+bx^n\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
[Out] (x^(2*n)*((a + b*x^n)^2)^(5/2)*(21*a^5 + 70*a^4*b*x^n + 105*a^3*b^2*x^(2*n)
 + 84*a^2*b^3*x^(3*n) + 35*a*b^4*x^(4*n) + 6*b^5*x^(5*n)))/(42*n*(a + b*x^n
)^5)
```

Maple [**A**]
time = 0.04, size = 208, normalized size = 1.86

method	result
risch	$\frac{\sqrt{(a+b\,x^n)^2}\;b^5x^{7n}}{7(a+b\,x^n)n}+\frac{5\sqrt{(a+b\,x^n)^2}\;b^4a\,x^{6n}}{6(a+b\,x^n)n}+\frac{2\sqrt{(a+b\,x^n)^2}\;a^2b^3x^{5n}}{(a+b\,x^n)n}+\frac{5\sqrt{(a+b\,x^n)^2}\;b^2a^3x^{4n}}{2(a+b\,x^n)n}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/7*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^5/n*(x^n)^7+5/6*((a+b*x^n)^2)^(1/2)/(a+
b*x^n)*b^4*a/n*(x^n)^6+2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b^3/n*(x^n)^5+5/
2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^2*a^3/n*(x^n)^4+5/3*((a+b*x^n)^2)^(1/2)/(
a+b*x^n)*b*a^4/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^5/n*(x^n)^2
```

Maxima [A]
time = 0.32, size = 74, normalized size = 0.66

$$\frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima
")
```

```
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*x
^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

Fricas [A]
time = 0.35, size = 74, normalized size = 0.66

$$\frac{6\,b^5x^{7\,n}+35\,ab^4x^{6\,n}+84\,a^2b^3x^{5\,n}+105\,a^3b^2x^{4\,n}+70\,a^4bx^{3\,n}+21\,a^5x^{2\,n}}{42\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas
")
```

```
[Out] 1/42*(6*b^5*x^(7*n) + 35*a*b^4*x^(6*n) + 84*a^2*b^3*x^(5*n) + 105*a^3*b^2*x
^(4*n) + 70*a^4*b*x^(3*n) + 21*a^5*x^(2*n))/n
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8569 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2)*x^(2*n - 1), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2\,n-1}\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

3.509 $\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

Optimal. Leaf size=112

$$-\frac{a(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\,(ab^2+b^3x^n)}+\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\,(ab^2+b^3x^n)}$$

```
[Out] -1/4*a*(a+b*x^n)^4*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)+1/5*
(a+b*x^n)^5*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b^2+b^3*x^n)
```

Rubi [A]
time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1369, 272, 45}

$$\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}{5n\,(ab^2+b^3x^n)}-\frac{a(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\,(ab^2+b^3x^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] -1/4*(a*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b^2 + b^3*
x^n)) + ((a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(5*n*(a*b^2 + b
^3*x^n))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}\left(ab+b^2x^n\right)^3\,dx}{b^2\left(ab+b^2x^n\right)}$$

$$=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int x\left(ab+b^2x\right)^3\,dx,x,x^n\right)}{b^2n\left(ab+b^2x^n\right)}$$

$$=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\text{Subst}\left(\int\left(-\frac{a\left(ab+b^2x\right)^3}{b}+\frac{\left(ab+b^2x\right)^4}{b^2}\right)\,dx,x,x^n\right)}{b^2n\left(ab+b^2x^n\right)}$$

$$=-\frac{a(a+bx^n)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}{4n\left(ab^2+b^3x^n\right)}+\frac{(a+bx^n)^5\sqrt{a^2+2abx^n+b}}{5n\left(ab^2+b^3x^n\right)}$$

Mathematica [A]
time = 0.04, size = 70, normalized size = 0.62

$$\frac{x^{2n}\left((a+bx^n)^2\right)^{3/2}\left(10a^3+20a^2bx^n+15ab^2x^{2n}+4b^3x^{3n}\right)}{20n\,(a+bx^n)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^(2*n)*((a + b*x^n)^2)^(3/2)*(10*a^3 + 20*a^2*b*x^n + 15*a*b^2*x^(2*n) +
4*b^3*x^(3*n)))/(20*n*(a + b*x^n)^3)
```

Maple [A]
time = 0.04, size = 135, normalized size = 1.21

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\;b^3x^{5n}}{5(a+b\,x^n)n}+\frac{3\sqrt{(a+b\,x^n)^2}\;a\,b^2x^{4n}}{4(a+b\,x^n)n}+\frac{\sqrt{(a+b\,x^n)^2}\;a^2b\,x^{3n}}{(a+b\,x^n)n}+\frac{\sqrt{(a+b\,x^n)^2}\;a^3x^{2n}}{2(a+b\,x^n)n}$	135

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3/n*(x^n)^5+3/4*((a+b*x^n)^2)^(1/2)/(a+
b*x^n)*a*b^2/n*(x^n)^4+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/n*(x^n)^3+1/2*((
a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/n*(x^n)^2
```

Maxima [A]
time = 0.47, size = 48, normalized size = 0.43

$$\frac{4\,b^3x^{5\,n}+15\,ab^2x^{4\,n}+20\,a^2bx^{3\,n}+10\,a^3x^{2\,n}}{20\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima
")
```

```
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n))
/n
```

Fricas [A]
time = 0.36, size = 48, normalized size = 0.43

$$\frac{4\,b^3x^{5\,n}+15\,ab^2x^{4\,n}+20\,a^2bx^{3\,n}+10\,a^3x^{2\,n}}{20\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas
")
```

```
[Out] 1/20*(4*b^3*x^(5*n) + 15*a*b^2*x^(4*n) + 20*a^2*b*x^(3*n) + 10*a^3*x^(2*n))
/n
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^(2*n - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2\,n-1}\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.510 $\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=99

$$\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n(ab+b^2x^n)}$$

```
[Out] 1/2*a*x^(2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a+b*x^n)+1/3*b^2*x^(3*n)
*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)
```

Rubi [A]
time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1369, 14}

$$\frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n(ab+b^2x^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a + b*x^n)) + (b^2*x^
(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^{-1+2n}(ab+b^2x^n)\,dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(abx^{-1+2n}+b^2x^{-1+3n}\right)dx}{ab+b^2x^n}\\ &= \frac{ax^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n(a+bx^n)}+\frac{b^2x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n(ab+b^2x^n)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.44

$$\frac{x^{2n}\sqrt{(a+bx^n)^2}\,(3a+2bx^n)}{6n\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^(2*n)*Sqrt[(a + b*x^n)^2]*(3*a + 2*b*x^n))/(6*n*(a + b*x^n))
```

Maple [A]
time = 0.02, size = 64, normalized size = 0.65

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\;b\,x^{3n}}{3(a+b\,x^n)n}+\frac{\sqrt{(a+b\,x^n)^2}\;a\,x^{2n}}{2(a+b\,x^n)n}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n*(x^n)^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*
x^n)*a/n*(x^n)^2
```

Maxima [A]
time = 0.46, size = 22, normalized size = 0.22

$$\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

Fricas [A]
time = 0.42, size = 22, normalized size = 0.22

$$\frac{2\,bx^{3\,n}+3\,ax^{2\,n}}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/6*(2*b*x^(3*n) + 3*a*x^(2*n))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2n-1}\sqrt{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(x**(2*n - 1)*sqrt((a + b*x**n)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^(2*n - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2\,n-1}\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.511 $\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=90

$$\frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log\left(a+bx^n\right)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] x^n*(a+b*x^n)/b/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-a*(a+b*x^n)*ln(a+b*x^n)
/b^2/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1369, 272, 45}

$$\frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{a(a+bx^n)\log\left(a+bx^n\right)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^n*(a + b*x^n))/(b*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - (a*(a + b*x^n
)*Log[a + b*x^n])/(b^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx = \frac{(ab+b^2x^n)\int \frac{x^{-1+2n}}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{(ab+b^2x^n)\,\mathrm{Subst}\left(\int \frac{x}{ab+b^2x}\,dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{(ab+b^2x^n)\,\mathrm{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a}{b^2(a+bx)}\right)\,dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{x^n(a+bx^n)}{bn\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{a(a+bx^n)\log(a+bx^n)}{b^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A]
time = 0.04, size = 47, normalized size = 0.52

$$\frac{(a+bx^n)\left(bx^n-a\log(bn(a+bx^n))\right)}{b^2n\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] ((a + b*x^n)*(b*x^n - a*Log[b*n*(a + b*x^n)]))/(b^2*n*Sqrt[(a + b*x^n)^2])
```

Maple [A]
time = 0.03, size = 71, normalized size = 0.79

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\;x^n}{(a+b\,x^n)bn} - \frac{\sqrt{(a+b\,x^n)^2}\;a\ln\left(x^n+\frac{a}{b}\right)}{(a+b\,x^n)b^2n}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)/b/n*x^n-((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/b^2/n
*ln(x^n+a/b)
```

Maxima [A]
time = 0.54, size = 32, normalized size = 0.36

$$\frac{x^n}{bn} - \frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima
")
```

```
[Out] x^n/(b*n) - a*log((b*x^n + a)/b)/(b^2*n)
```

Fricas [A]
time = 0.37, size = 24, normalized size = 0.27

$$\frac{bx^n - a\log\left(bx^n + a\right)}{b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas
")
```

```
[Out] (b*x^n - a*log(b*x^n + a))/(b^2*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{\left(a + bx^n\right)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)/sqrt((a + b*x**n)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.512 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{x^{2n}}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/2*x^(2*n)/a/n/(a+b*x^n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1369, 270}

$$\frac{x^{2n}}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] x^(2*n)/(2*a*n*(a + b*x^n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2\left(ab+b^2x^n\right)\right)\int \frac{x^{-1+2n}}{\left(ab+b^2x^n\right)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^{2n}}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 40, normalized size = 0.83

$$\frac{(-a-2bx^n)(a+bx^n)}{2b^2n\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] ((-a - 2*b*x^n)*(a + b*x^n))/(2*b^2*n*((a + b*x^n)^2)^(3/2))
```

Maple [A]
time = 0.03, size = 37, normalized size = 0.77

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^n)^2}\,(2b\,x^n+a)}{2(a+b\,x^n)^3b^2n}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^3*(2*b*x^n+a)/b^2/n
```

Maxima [A]
time = 0.41, size = 41, normalized size = 0.85

$$-\frac{2\,bx^n+a}{2\,(b^4nx^{2\,n}+2\,ab^3nx^n+a^2b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima
")
```

```
[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

Fricas [A]
time = 0.34, size = 41, normalized size = 0.85

$$-\frac{2\,bx^n+a}{2\,(b^4nx^{2\,n}+2\,ab^3nx^n+a^2b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas
")
```

```
[Out] -1/2*(2*b*x^n + a)/(b^4*n*x^(2*n) + 2*a*b^3*n*x^n + a^2*b^2*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral(x**(2*n - 1)/((a + b*x**n)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2\,n-1}}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.513 $\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{5/2}}\,dx$

Optimal. Leaf size=88

$$\frac{a}{4b^2n\left(a+bx^n\right)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{3b^2n\left(a+bx^n\right)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/4*a/b^2/n/(a+b*x^n)^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-1/3/b^2/n/(a+b*x^
n)^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1369, 272, 45}

$$\frac{a}{4b^2n\left(a+bx^n\right)^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{3b^2n\left(a+bx^n\right)^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
```

```
[Out] a/(4*b^2*n*(a + b*x^n)^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(3*b^2*n*
(a + b*x^n)^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{5/2}}\,dx = \frac{\left(b^4(ab+b^2x^n)\right)\int \frac{x^{-1+2n}}{(ab+b^2x^n)^5}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(b^4(ab+b^2x^n)\right)\operatorname{Subst}\left(\int \frac{x}{(ab+b^2x)^5}\,dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(b^4(ab+b^2x^n)\right)\operatorname{Subst}\left(\int \left(-\frac{a}{b^6(a+bx)^5}+\frac{1}{b^6(a+bx)^4}\right)dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{a}{4b^2n\left(a+bx^n\right)^3\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{3b^2n\left(a+bx^n\right)^2\sqrt{a^2+2abx^n+b}}$$

<u>**Mathematica**</u> [A]
time = 0.05, size = 40, normalized size = 0.45

$$\frac{(-a-4bx^n)\,(a+bx^n)}{12b^2n\left((a+bx^n)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(5/2),x]
[Out] ((-a - 4*b*x^n)*(a + b*x^n))/(12*b^2*n*((a + b*x^n)^2)^(5/2))
```

<u>**Maple**</u> [A]
time = 0.03, size = 37, normalized size = 0.42

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^n)^2}\,(4b\,x^n+a)}{12(a+b\,x^n)^5 b^2 n}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/12*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^5*(4*b*x^n+a)/b^2/n
```

<u>**Maxima**</u> [A]
time = 0.53, size = 69, normalized size = 0.78

$$-\frac{4\,bx^n+a}{12\left(b^6nx^{4\,n}+4\,ab^5nx^{3\,n}+6\,a^2b^4nx^{2\,n}+4\,a^3b^3nx^n+a^4b^2n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="maxima
")
```

```
[Out] -1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*n
) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

Fricas [A]
time = 0.34, size = 69, normalized size = 0.78

$$-\frac{4\,bx^n+a}{12\,(b^6nx^{4\,n}+4\,ab^5nx^{3\,n}+6\,a^2b^4nx^{2\,n}+4\,a^3b^3nx^n+a^4b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="fricas
")
```

```
[Out] -1/12*(4*b*x^n + a)/(b^6*n*x^(4*n) + 4*a*b^5*n*x^(3*n) + 6*a^2*b^4*n*x^(2*n
) + 4*a^3*b^3*n*x^n + a^4*b^2*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2),x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(5/2), x)
```

3.514 $\int \frac{x^{-1+2n}}{\left(a^2+2abx^n+b^2x^{2n}\right)^{7/2}} \, dx$

Optimal. Leaf size=88

$$\frac{a}{6b^2n\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\left(a+bx^n\right)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

[Out] 1/6*a/b^2/n/(a+b*x^n)^5/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-1/5/b^2/n/(a+b*x^n)^4/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1369, 272, 45}

$$\frac{a}{6b^2n\left(a+bx^n\right)^5\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{1}{5b^2n\left(a+bx^n\right)^4\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
```

```
[Out] a/(6*b^2*n*(a + b*x^n)^5*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - 1/(5*b^2*n*(a + b*x^n)^4*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a^2+2abx^n+b^2x^{2n})^{7/2}}\,dx = \frac{(b^6(ab+b^2x^n))\int \frac{x^{-1+2n}}{(ab+b^2x^n)^7}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{(b^6(ab+b^2x^n))\,\text{Subst}\left(\int \frac{x}{(ab+b^2x)^7}\,dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{(b^6(ab+b^2x^n))\,\text{Subst}\left(\int \left(-\frac{a}{b^8(a+bx)^7}+\frac{1}{b^8(a+bx)^6}\right)dx, x, x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{a}{6b^2n\,(a+bx^n)^5\sqrt{a^2+2abx^n+b^2x^{2n}}} - \frac{1}{5b^2n\,(a+bx^n)^4\sqrt{a^2+2abx^n+b^2}}$$

Mathematica [A]
time = 0.05, size = 40, normalized size = 0.45

$$\frac{(-a-6bx^n)\,(a+bx^n)}{30b^2n\left((a+bx^n)^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(7/2),x]
[Out] ((-a - 6*b*x^n)*(a + b*x^n))/(30*b^2*n*((a + b*x^n)^2)^(7/2))
```

Maple [A]
time = 0.03, size = 37, normalized size = 0.42

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^n)^2}\,(6b\,x^n+a)}{30(a+b\,x^n)^7b^2n}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x,method=_RETURNVERBOSE)
[Out] -1/30*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^7*(6*b*x^n+a)/b^2/n
```

Maxima [A]
time = 0.40, size = 97, normalized size = 1.10

$$-\frac{6\,bx^n+a}{30\,(b^8nx^{6\,n}+6\,ab^7nx^{5\,n}+15\,a^2b^6nx^{4\,n}+20\,a^3b^5nx^{3\,n}+15\,a^4b^4nx^{2\,n}+6\,a^5b^3nx^n+a^6b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="maxima
")
```

```
[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4*
n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*b^
2*n)
```

Fricas [A]
time = 0.37, size = 97, normalized size = 1.10

$$-\frac{6\,bx^{n}+a}{30\,\left(b^{8}nx^{6\,n}+6\,ab^{7}nx^{5\,n}+15\,a^{2}b^{6}nx^{4\,n}+20\,a^{3}b^{5}nx^{3\,n}+15\,a^{4}b^{4}nx^{2\,n}+6\,a^{5}b^{3}nx^{n}+a^{6}b^{2}n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="fricas
")
```

```
[Out] -1/30*(6*b*x^n + a)/(b^8*n*x^(6*n) + 6*a*b^7*n*x^(5*n) + 15*a^2*b^6*n*x^(4*
n) + 20*a^3*b^5*n*x^(3*n) + 15*a^4*b^4*n*x^(2*n) + 6*a^5*b^3*n*x^n + a^6*b^
2*n)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a**2+2*a*b*x**n+b**2*x**(2*n))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2),x)
```

```
[Out] int(x^(2*n - 1)/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(7/2), x)
```

3.515 $\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=108

$$\frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\,(a+bx^n)}+\frac{b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\,(ab+b^2x^n)}$$

```
[Out] a*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m)/(a+b*x^n)+b^2*x^(1+
n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+n)/(a*b+b^2*x^n)
```

Rubi [A]
time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1369, 14, 20, 30}

$$\frac{b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\,(ab+b^2x^n)}+\frac{a(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x^n
)) + (b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m +
n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
```

```
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \sqrt{a^2+2abx^n+b^2x^{2n}}\, dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int (dx)^m \left(ab+b^2x^n\right) dx}{ab+b^2x^n} \\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int \left(ab(dx)^m+b^2x^n(dx)^m\right) dx}{ab+b^2x^n} \\
&= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{\left(b^2\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int x^n(dx}{ab+b^2x^n} \\
&= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{\left(b^2x^{-m}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}\right.}{ab+b^2x^n} \\
&= \frac{a(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\left(ab+b^2x^n\right)}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 0.51

$$\frac{x(dx)^m\sqrt{\left(a+bx^n\right)^2}\left(a(1+m+n)+b(1+m)x^n\right)}{(1+m)(1+m+n)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(a + b*x^n)^2]*(a*(1 + m + n) + b*(1 + m)*x^n))/((1 + m)*(1
 + m + n)*(a + b*x^n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 132, normalized size = 1.22

method	result
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\;x\left(mb\,x^{n}+am+an+b\,x^{n}+a\right)\mathrm{e}^{\frac{m\left(i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}(idx)^{2}-i\pi\,\mathrm{csgn}(ix)\mathrm{csgn}(idx)\mathrm{csgn}(id)-i\pi\mathrm{csgn}(idx)^{3}+i\pi\mathrm{csgn}(idx)^{2}\mathrm{csgn}(id)}{2}}}{\left(a+b\,x^{n}\right)(1+m)(1+m+n)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x*(m*b*x^n+a*m+a*n+b*x^n+a)/(1+m)/(1+m+n)*exp
(1/2*m*(I*Pi*csgn(I*x)*csgn(I*d*x)^2-I*Pi*csgn(I*x)*csgn(I*d*x)*csgn(I*d)-I
*Pi*csgn(I*d*x)^3+I*Pi*csgn(I*d*x)^2*csgn(I*d)+2*ln(x)+2*ln(d)))
```

Maxima [A]
time = 0.31, size = 47, normalized size = 0.44

$$\frac{ad^m(m+n+1)xx^m + bd^m(m+1)xe^{(m\log(x)+n\log(x))}}{m^2+m(n+2)+n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] (a*d^m*(m + n + 1)*x*x^m + b*d^m*(m + 1)*x*e^(m*log(x) + n*log(x)))/(m^2 +
m*(n + 2) + n + 1)
```

Fricas [A]
time = 0.36, size = 57, normalized size = 0.53

$$\frac{(bm+b)xx^n e^{(m\log(d)+m\log(x))} + (am+an+a)xe^{(m\log(d)+m\log(x))}}{m^2+(m+1)n+2\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m + a*n + a)*x*e^(m*log(d) +
m*log(x)))/(m^2 + (m + 1)*n + 2*m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{(a+bx^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**n)**2), x)
```

Giac [A]
time = 3.83, size = 173, normalized size = 1.60

$$\frac{bmxx^n e^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a) + amxe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a) + bmxe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a) + anxe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a) + bxx^n e^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a) + axe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a) + bxe^{(m\log(d)+m\log(x))}\mathrm{sgn}(bx^n+a)}{m^2+mn+2\,m+n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] (b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*m*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a*n*
x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*x^n*e^(m*log(d) + m*log(x))*
sgn(b*x^n + a) + a*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a))/(m^2 + m*n + 2*m + n + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.516 $\int x^2 \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=93

$$\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(a+bx^n)}+\frac{b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)(ab+b^2x^n)}$$

```
[Out] 1/3*a*x^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(3+n)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/(3+n)/(a*b+b^2*x^n)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 14}

$$\frac{b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)(ab+b^2x^n)}+\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^2*x^(3 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2(ab+b^2x^n)\,dx}{ab+b^2x^n}\\&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int (abx^2+b^2x^{2+n})\,dx}{ab+b^2x^n}\\&=\frac{ax^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(a+bx^n)}+\frac{b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)(ab+b^2x^n)}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 46, normalized size = 0.49

$$\frac{x^3\sqrt{(a+bx^n)^2}\,(a(3+n)+3bx^n)}{3(3+n)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a*(3 + n) + 3*b*x^n))/(3*(3 + n)*(a + b*x^n))
```

Maple [A]
time = 0.04, size = 61, normalized size = 0.66

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\;a\,x^3}{3a+3b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\;b\,x^3x^n}{(a+b\,x^n)(3+n)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x^3+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(3+
n)*x^3*x^n
```

Maxima [A]
time = 0.33, size = 25, normalized size = 0.27

$$\frac{3\,bx^3x^n+a(n+3)x^3}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b*x^3*x^n + a*(n + 3)*x^3)/(n + 3)
```

Fricas [A]
time = 0.37, size = 28, normalized size = 0.30

$$\frac{3\,bx^3x^n+(an+3\,a)x^3}{3\,(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*b*x^3*x^n + (a*n + 3*a)*x^3)/(n + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{(a + bx^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a + b*x**n)**2), x)
```

Giac [A]
time = 5.89, size = 53, normalized size = 0.57

$$\frac{3\,bx^3x^n\mathrm{sgn}(bx^n + a) + anx^3\mathrm{sgn}(bx^n + a) + 3\,ax^3\mathrm{sgn}(bx^n + a)}{3\,(n + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*b*x^3*x^n*sgn(b*x^n + a) + a*n*x^3*sgn(b*x^n + a) + 3*a*x^3*sgn(b*x^
n + a))/(n + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.517 $\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=93

$$\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}+\frac{b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)(ab+b^2x^n)}$$

```
[Out] 1/2*a*x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(2+n)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/(2+n)/(a*b+b^2*x^n)
```

Rubi [A]
time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 14}

$$\frac{b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)(ab+b^2x^n)}+\frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (b^2*x^(2 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a^2+2abx^n+b^2x^{2n}}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x(ab+b^2x^n)\,dx}{ab+b^2x^n}\\ &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int (abx+b^2x^{1+n})\,dx}{ab+b^2x^n}\\ &= \frac{ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}+\frac{b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)(ab+b^2x^n)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.49

$$\frac{x^2\sqrt{(a+bx^n)^2}\,(a(2+n)+2bx^n)}{2(2+n)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a*(2 + n) + 2*b*x^n))/(2*(2 + n)*(a + b*x^n))
```

Maple [A]
time = 0.02, size = 61, normalized size = 0.66

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\;a\,x^2}{2a+2b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\;b\,x^2x^n}{(a+b\,x^n)(2+n)}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(2+
n)*x^2*x^n
```

Maxima [A]
time = 0.30, size = 25, normalized size = 0.27

$$\frac{2\,bx^2x^n+a(n+2)x^2}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^2*x^n + a*(n + 2)*x^2)/(n + 2)
```

Fricas [A]
time = 0.35, size = 28, normalized size = 0.30

$$\frac{2\,bx^2x^n+(an+2\,a)x^2}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*x^n + (a*n + 2*a)*x^2)/(n + 2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x*sqrt((a + b*x**n)**2), x)
```

Giac [A]
time = 3.89, size = 53, normalized size = 0.57

$$\frac{2\,bx^2x^n\mathrm{sgn}(bx^n+a)+anx^2\mathrm{sgn}(bx^n+a)+2\,ax^2\mathrm{sgn}(bx^n+a)}{2\,(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*x^2*x^n*sgn(b*x^n + a) + a*n*x^2*sgn(b*x^n + a) + 2*a*x^2*sgn(b*x^
n + a))/(n + 2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.518 $\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx$

Optimal. Leaf size=88

$$\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}+\frac{b^2x^{1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+n)\left(ab+b^2x^n\right)}$$

```
[Out] a*x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+b^2*x^(1+n)*(a^2+2*a*b*x^n+
b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)
```

Rubi [A]
time = 0.01, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1357}

$$\frac{b^2x^{n+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+1)\left(ab+b^2x^n\right)}+\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(a + b*x^n) + (b^2*x^(1 + n)*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + n)*(a*b + b^2*x^n))
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a^2+2abx^n+b^2x^{2n}}\,dx&=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(2ab+2b^2x^n\right)\,dx}{2ab+2b^2x^n}\\&=\frac{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}+\frac{b^2x^{1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+n)\left(ab+b^2x^n\right)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 0.44

$$\frac{x\sqrt{\left(a+bx^n\right)^2}\left(a+an+bx^n\right)}{(1+n)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*Sqrt[(a + b*x^n)^2]*(a + a*n + b*x^n))/((1 + n)*(a + b*x^n))
```

Maple [A]
time = 0.02, size = 56, normalized size = 0.64

method	result	size
risch	$\frac{\sqrt{(a+b\,x^{n})^{2}}\,ax}{a+b\,x^{n}}+\frac{\sqrt{(a+b\,x^{n})^{2}}\,bx\,x^{n}}{(a+b\,x^{n})(1+n)}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/(1+n)*x*x
^n
```

Maxima [A]
time = 0.30, size = 19, normalized size = 0.22

$$\frac{a(n+1)x+bxx^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] (a*(n + 1)*x + b*x*x^n)/(n + 1)
```

Fricas [A]
time = 0.37, size = 20, normalized size = 0.23

$$\frac{bxx^{n}+(an+a)x}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] (b*x*x^n + (a*n + a)*x)/(n + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a^{2}+2abx^{n}+b^{2}x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

Giac [A]
time = 5.67, size = 25, normalized size = 0.28

$$\left(ax+\frac{bx^{n+1}}{n+1}\right)\operatorname{sgn}(bx^n+a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] (a*x + b*x^(n + 1)/(n + 1))*sgn(b*x^n + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.519 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx$

Optimal. Leaf size=85

$$\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\,\log(x)}{a+bx^n}$$

```
[Out] b^2*x^n*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+a*ln(x)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.071$, Rules used = {1369, 14}

$$\frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\left(ab+b^2x^n\right)}+\frac{a\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
```

```
[Out] (b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (a*Sqrt
[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x}\,dx}{ab+b^2x^n}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x}+b^2x^{-1+n}\right)\,dx}{ab+b^2x^n}$$
$$= \frac{b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)} + \frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}\log(x)}{a+bx^n}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 0.45

$$\frac{\sqrt{(a+bx^n)^2}\,(bx^n+a\log(x^n))}{n\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x,x]
[Out] (Sqrt[(a + b*x^n)^2]*(b*x^n + a*Log[x^n]))/(n*(a + b*x^n))
```

Maple [A]
time = 0.03, size = 54, normalized size = 0.64

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\;a\ln(x)}{a+b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\;b\,x^n}{(a+b\,x^n)n}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*ln(x)+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b/n*x^n
```

Maxima [A]
time = 0.33, size = 13, normalized size = 0.15

$$a\log(x)+\frac{bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="maxima")
[Out] a*log(x) + b*x^n/n
```

Fricas [A]
time = 0.36, size = 15, normalized size = 0.18

$$\frac{an\log(x) + bx^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] (a*n*log(x) + b*x^n)/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(a + bx^n)^2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x, x)
```

3.520 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx$

Optimal. Leaf size=94

$$-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}-\frac{b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\,(ab+b^2x^n)}$$

```
[Out] -a*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x/(a+b*x^n)-b^2*x^(-1+n)*(a^2+2*a*b*x^
n+b^2*x^(2*n))^(1/2)/(1-n)/(a*b+b^2*x^n)
```

Rubi [A]
time = 0.02, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 14}

$$-\frac{b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\,(ab+b^2x^n)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (b^2*x^(-1 + n
)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x^2}\,dx}{ab+b^2x^n}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x^2}+b^2x^{-2+n}\right)\,dx}{ab+b^2x^n}$$
$$= -\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)} - \frac{b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\,(ab+b^2x^n)}$$

Mathematica [A]
time = 0.03, size = 42, normalized size = 0.45

$$\frac{\sqrt{(a+bx^n)^2}\,(a-an+bx^n)}{(-1+n)x\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a - a*n + b*x^n))/((-1 + n)*x*(a + b*x^n))
```

Maple [A]
time = 0.02, size = 61, normalized size = 0.65

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^n)^2}\,a}{(a+b\,x^n)x}+\frac{\sqrt{(a+b\,x^n)^2}\,b\,x^n}{(a+b\,x^n)(-1+n)x}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*b/x
*x^n
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.23

$$-\frac{a(n-1)-bx^n}{(n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -(a*(n - 1) - b*x^n)/((n - 1)*x)
```

Fricas [A]
time = 0.38, size = 23, normalized size = 0.24

$$-\frac{an - bx^n - a}{(n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
[Out] -(a*n - b*x^n - a)/((n - 1)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(a+bx^n)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**2,x)
[Out] Integral(sqrt((a + b*x**n)**2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2,x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^2, x)
```

3.521 $\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx$

Optimal. Leaf size=96

$$-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}-\frac{b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}$$

```
[Out] -1/2*a*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2/(a+b*x^n)-b^2*x^(-2+n)*(a^2+2*
a*b*x^n+b^2*x^(2*n))^(1/2)/(2-n)/(a*b+b^2*x^n)
```

Rubi [A]
time = 0.02, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{1369, 14\}$

$$-\frac{b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x^2*(a + b*x^n)) - (b^2*x^(-2
 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*(a*b + b^2*x^n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{ab+b^2x^n}{x^3}\,dx}{ab+b^2x^n}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{ab}{x^3}+b^2x^{-3+n}\right)\,dx}{ab+b^2x^n}$$
$$= -\frac{a\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2(a+bx^n)} - \frac{b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)(ab+b^2x^n)}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.49

$$\frac{\sqrt{(a+bx^n)^2}\,(-a(-2+n)+2bx^n)}{2(-2+n)x^2\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(-(a*(-2 + n)) + 2*b*x^n))/(2*(-2 + n)*x^2*(a + b*x^n)
)
```

Maple [A]
time = 0.02, size = 61, normalized size = 0.64

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^n)^2}\;a}{2(a+b\,x^n)x^2}+\frac{\sqrt{(a+b\,x^n)^2}\;b\,x^n}{(a+b\,x^n)(-2+n)x^2}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a/x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-2+
n)*b/x^2*x^n
```

Maxima [A]
time = 0.28, size = 22, normalized size = 0.23

$$-\frac{a(n-2)-2\,bx^n}{2\,(n-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a*(n - 2) - 2*b*x^n)/((n - 2)*x^2)
```

Fricas [A]
time = 0.42, size = 23, normalized size = 0.24

$$-\frac{an - 2\,bx^n - 2\,a}{2\,(n-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*n - 2*b*x^n - 2*a)/((n - 2)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(a+bx^n)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a + b*x**n)**2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)/x^3, x)
```

3.522 $\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=238

$$\frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\,(a+bx^n)}+\frac{3a^2b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1+m+n)\,(ab+b^2x^n)}+\frac{3ab^3x^{1+2n}(dx)^m\sqrt{a^2+2abx^n+b^2}}{(1+m+2n)\,(ab+b^2x^n)}$$

```
[Out] a^3*(d*x)^(1+m)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/d/(1+m)/(a+b*x^n)+3*a^2*b
^2*x^(1+n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+n)/(a*b+b^2*x^n)+
3*a*b^3*x^(1+2*n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+2*n)/(a*b+
b^2*x^n)+b^4*x^(1+3*n)*(d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+m+3*n)/
(a*b+b^2*x^n)
```

Rubi [A]

time = 0.07, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1369, 276, 20, 30}

$$\frac{3a^2b^2x^{n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+n+1)\,(ab+b^2x^n)}+\frac{b^4x^{3n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+3n+1)\,(ab+b^2x^n)}+\frac{3ab^3x^{2n+1}(dx)^m\sqrt{a^2+2abx^n+b^2x^{2n}}}{(m+2n+1)\,(ab+b^2x^n)}+\frac{a^3(dx)^{m+1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(m+1)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(d*(1 + m)*(a + b*x
^n)) + (3*a^2*b^2*x^(1 + n)*(d*x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((
1 + m + n)*(a*b + b^2*x^n)) + (3*a*b^3*x^(1 + 2*n)*(d*x)^m*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)])/((1 + m + 2*n)*(a*b + b^2*x^n)) + (b^4*x^(1 + 3*n)*(d*
x)^m*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 + m + 3*n)*(a*b + b^2*x^n))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int (dx)^m \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int (dx)^m \left(ab+b^2x^n\right)^3 dx}{b^2\left(ab+b^2x^n\right)}$$

$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}} \int \left(a^3b^3(dx)^m+3a^2b^4x^n(dx)^m+3ab^5x^{2n}(dx)\right.}{b^2\left(ab+b^2x^n\right)}$$

$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{\left(3a^2b^2\sqrt{a^2+2abx^n+b^2x^{2n}}\right)\int}{ab+b^2x^n}$$

$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{\left(3a^2b^2x^{-m}(dx)^m\sqrt{a^2+2abx^n+\right.}{ab+b^2x^n}$$

$$= \frac{a^3(dx)^{1+m}\sqrt{a^2+2abx^n+b^2x^{2n}}}{d(1+m)\left(a+bx^n\right)} + \frac{3a^2b^2x^{1+n}(dx)^m\sqrt{a^2+2abx^n+}}{(1+m+n)\left(ab+b^2x^n\right)}$$

Mathematica [**A**]
time = 0.09, size = 90, normalized size = 0.38

$$\frac{x(dx)^m\left((a+bx^n)^2\right)^{3/2}\left(\frac{a^3}{1+m}+\frac{3a^2bx^n}{1+m+n}+\frac{3ab^2x^{2n}}{1+m+2n}+\frac{b^3x^{3n}}{1+m+3n}\right)}{(a+bx^n)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*((a + b*x^n)^2)^(3/2)*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) +
 (3*a*b^2*x^(2*n))/(1 + m + 2*n) + (b^3*x^(3*n))/(1 + m + 3*n)))/(a + b*x^n
)^3
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 532, normalized size = 2.24

method	result
risch	$\frac{\sqrt{(a+b\,x^n)^2}\;x(9a^2b\,m^2x^n+a^3+3a^2b\,m^3x^n+b^3x^{3n}+3x^{2n}a\,b^2+b^3m^3x^{3n}+3b^3m^2x^{3n}+2b^3n^2x^{3n}+3m\,b^3x^{3n}+3b^3x^{3n}n+9m\,a^2b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*x*(b^3*(x^n)^3+9*a*b^2*n^2*(x^n)^2+9*a^2*b*m^
2*x^n+a^3+3*b^3*m^2*n*(x^n)^3+2*b^3*m*n^2*(x^n)^3+3*a*b^2*m^3*(x^n)^2+6*b^3
*m*n*(x^n)^3+3*a^2*b*m^3*x^n+9*m*a^2*b*x^n+15*a^2*b*n*x^n+a^3*m^3+3*a^3*m^2
+11*a^3*n^2+6*a^3*n+12*a*b^2*m^2*n*(x^n)^2+12*a*b^2*(x^n)^2*n+3*m*a^3+9*a*b
^2*m^2*(x^n)^2+6*a^3*m^2*n+11*a^3*m*n^2+12*a^3*m*n+3*(x^n)^2*a*b^2+b^3*m^3*
(x^n)^3+3*b^3*m^2*(x^n)^3+2*b^3*n^2*(x^n)^3+3*m*b^3*(x^n)^3+3*b^3*(x^n)^3*n
+3*a^2*b*x^n+18*a^2*b*n^2*x^n+9*m*a*b^2*(x^n)^2+6*a^3*n^3+9*a*b^2*m*n^2*(x^
n)^2+15*a^2*b*m^2*n*x^n+18*a^2*b*m*n^2*x^n+24*a*b^2*m*n*(x^n)^2+30*a^2*b*m*
n*x^n)/(1+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)*exp(1/2*m*(I*Pi*csgn(I*x)*csgn(I*d
*x)^2-I*Pi*csgn(I*x)*csgn(I*d*x)*csgn(I*d)-I*Pi*csgn(I*d*x)^3+I*Pi*csgn(I*d
*x)^2*csgn(I*d)+2*ln(x)+2*ln(d)))
```

Maxima [A]
time = 0.30, size = 276, normalized size = 1.16

$$\frac{(m^3+3m^2(2n+1)+6n^3+(11n^2+12n+3)m+11n^2+6n+1)a^3d^mxx^m+(m^3+3m^2(n+1)+(2n^2+6n+3)m+2n^2+3n+1)b^3d^mxe^{(m\log(x)+3n\log(x))}+3(m^3+m^2(4n+3)+(3n^2+8n+3)m+3n^2+4n+1)ab^2d^mxe^{(m\log(x)+2n\log(x))}+3(m^3+m^2(5n+3)+(6n^2+10n+3)m+6n^2+5n+1)a^2bd^mxe^{(m\log(x)+n\log(x))}}{m^4+2m^3(3n+2)+(11n^2+18n+6)m^2+6n^3+2(3n^3+11n^2+9n+2)m+11n^2+6n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] ((m^3 + 3*m^2*(2*n + 1) + 6*n^3 + (11*n^2 + 12*n + 3)*m + 11*n^2 + 6*n + 1)
*a^3*d^m*x*x^m + (m^3 + 3*m^2*(n + 1) + (2*n^2 + 6*n + 3)*m + 2*n^2 + 3*n +
 1)*b^3*d^m*x*e^(m*log(x) + 3*n*log(x)) + 3*(m^3 + m^2*(4*n + 3) + (3*n^2 +
 8*n + 3)*m + 3*n^2 + 4*n + 1)*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x)) + 3*(m
^3 + m^2*(5*n + 3) + (6*n^2 + 10*n + 3)*m + 6*n^2 + 5*n + 1)*a^2*b*d^m*x*e^
(m*log(x) + n*log(x)))/(m^4 + 2*m^3*(3*n + 2) + (11*n^2 + 18*n + 6)*m^2 + 6
*n^3 + 2*(3*n^3 + 11*n^2 + 9*n + 2)*m + 11*n^2 + 6*n + 1)
```

Fricas [A]
time = 0.39, size = 390, normalized size = 1.64

$$\frac{(b^3m^3+3b^3m^2+3b^3m+b^3+2(b^3m+b^3)n^2+3(b^3m^2+2b^3m+b^3)n)xx^{3n}e^{(m\log(d)+m\log(x))}+3(ab^2m^3+3ab^2m^2+3ab^2m+ab^2+3(ab^2m+ab^2)n^2+4(ab^2m^2+2ab^2m+ab^2)n)xx^{2n}e^{(m\log(d)+m\log(x))}+3(a^2bm^3+3a^2bm^2+3a^2bm+a^2b+6(a^2bm+a^2b)n^2+5(a^2bm^2+2a^2bm+a^2b)n)xx^ne^{(m\log(d)+m\log(x))}+(a^3m^3+6a^3n^3+3a^3m^2+3a^3m+a^3+11(a^3m+a^3)n^2+6(a^3m^2+2a^3m+a^3)n)xe^{(m\log(d)+m\log(x))}}{m^4+6(m+1)n^3+4m^3+11(m^2+2m+1)n^2+6m^2+6(m^3+3m^2+3m+1)n+4m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 3*b^3*m^2 + 3*b^3*m + b^3 + 2*(b^3*m + b^3)*n^2 + 3*(b^3*m^2 +
2*b^3*m + b^3)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*(a*b^2*m^3 + 3*a*b^
2*m^2 + 3*a*b^2*m + a*b^2 + 3*(a*b^2*m + a*b^2)*n^2 + 4*(a*b^2*m^2 + 2*a*b^
2*m + a*b^2)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^3 + 3*a^2*b*
m^2 + 3*a^2*b*m + a^2*b + 6*(a^2*b*m + a^2*b)*n^2 + 5*(a^2*b*m^2 + 2*a^2*b*
m + a^2*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^3*m^3 + 6*a^3*n^3 + 3*a^3*
m^2 + 3*a^3*m + a^3 + 11*(a^3*m + a^3)*n^2 + 6*(a^3*m^2 + 2*a^3*m + a^3)*n)
*x*e^(m*log(d) + m*log(x)))/(m^4 + 6*(m + 1)*n^3 + 4*m^3 + 11*(m^2 + 2*m +
1)*n^2 + 6*m^2 + 6*(m^3 + 3*m^2 + 3*m + 1)*n + 4*m + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left((a+bx^n)^2\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**n)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2719 vs. 2(230) = 460.
time = 5.53, size = 2719, normalized size = 11.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] (b^3*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x
^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(3*n)*e^(m*
log(d) + m*log(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^(2*n)*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b^3*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + 12*a*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3
*b^3*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m*n^2
*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x*x^(2*n)*e
^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*x^n*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + 3*a*b^2*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + b^3*m^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*m^2
*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*m^2*n*x*x^n*e^(m
*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*x^n*e^(m*log(d) + m*log(
x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n
 + a) + 9*a*b^2*m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*
m*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + a^3*m^3*x*e^(m*log(d)
+ m*log(x))*sgn(b*x^n + a) + 3*a^2*b*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^
```

n + a) + 3*a*b^2*m^3*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*m^3*x*e
^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*m^2*n*x*e^(m*log(d) + m*log(x
))*sgn(b*x^n + a) + 15*a^2*b*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
 + 12*a*b^2*m^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*n*x*
e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 11*a^3*m*n^2*x*e^(m*log(d) + m*log
(x))*sgn(b*x^n + a) + 18*a^2*b*m*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n +
a) + 9*a*b^2*m*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*m*n^2*x
*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*n^3*x*e^(m*log(d) + m*log(x
))*sgn(b*x^n + a) + 3*b^3*m^2*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n +
 a) + 6*b^3*m*n*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^
2*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m^2*x*x^(2*n)*
e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^2*x*x^(2*n)*e^(m*log(d) +
m*log(x))*sgn(b*x^n + a) + 24*a*b^2*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*s
gn(b*x^n + a) + 6*b^3*m*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
+ 9*a*b^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^2*
x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m^2*x*x^n*e^(m*l
og(d) + m*log(x))*sgn(b*x^n + a) + 9*a*b^2*m^2*x*x^n*e^(m*log(d) + m*log(x)
)*sgn(b*x^n + a) + 3*b^3*m^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) +
 30*a^2*b*m*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 24*a*b^2*m*n*x
*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*b^3*m*n*x*x^n*e^(m*log(d) +
 m*log(x))*sgn(b*x^n + a) + 18*a^2*b*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(
b*x^n + a) + 9*a*b^2*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b
^3*n^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^3*m^2*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m^2*x*e^(m*log(d) + m*log(x))*sgn(b
*x^n + a) + 9*a*b^2*m^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m^
2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a^3*m*n*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + 30*a^2*b*m*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n +
a) + 24*a*b^2*m*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*b^3*m*n*x*e^
(m*log(d) + m*log(x))*sgn(b*x^n + a) + 11*a^3*n^2*x*e^(m*log(d) + m*log(x))
*sgn(b*x^n + a) + 18*a^2*b*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9
*a*b^2*n^2*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 2*b^3*n^2*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^(3*n)*e^(m*log(d) + m*log(x))*s
gn(b*x^n + a) + 3*b^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) +
9*a*b^2*m*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^(2
*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b^2*n*x*x^(2*n)*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x))*s
gn(b*x^n + a) + 9*a^2*b*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 9*
a*b^2*m*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*x^n*e^(m*l
og(d) + m*log(x))*sgn(b*x^n + a) + 15*a^2*b*n*x*x^n*e^(m*log(d) + m*log(x))
*sgn(b*x^n + a) + 12*a*b^2*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) +
 3*b^3*n*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^3*m*x*e^(m*log(
d) + m*log(x))*sgn(b*x^n + a) + 9*a^2*b*m*x*e^(m*log(d) + m*log(x))*sgn(b*x
^n + a) + 9*a*b^2*m*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*m*x*e^
(m*log(d) + m*log(x))*sgn(b*x^n + a) + 6*a^3*n*x*e^(m*log(d) + m*log(x))*sg
n(b*x^n + a) + 15*a^2*b*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 12*a*b

```
^2*n*x*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*b^3*n*x*e^(m*log(d) + m*l
og(x))*sgn(b*x^n + a) + b^3*x*x^(3*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a
) + 3*a*b^2*x*x^(2*n)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + b^3*x*x^(2*n
)*e^(m*log(d) + m*log(x))*sgn(b*x^n + a) + 3*a^2*b*x*x^n*e^(m*log(d) + m*lo
g(x))*sgn(b*x^n + a) + 3*a*b^2*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n + a)
 + b^3*x*x^n*e^(m*log(d) + m*log(x))*sgn(b*x^n ...
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (d\,x)^m\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int((d*x)^m*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.523 $\int x^2(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

Optimal. Leaf size=212

$$\frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}+\frac{b^4x^{3(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(1+n)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{3+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3+n)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{3+2n}\sqrt{}}{(3+2}$$

```
[Out] 1/3*a^3*x^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+1/3*b^4*x^(3+3*n)*(
a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)+3*a^2*b^2*x^(3+n)*(a^2
+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(3+n)/(a*b+b^2*x^n)+3*a*b^3*x^(3+2*n)*(a^2+2*
a*b*x^n+b^2*x^(2*n))^(1/2)/(3+2*n)/(a*b+b^2*x^n)
```

Rubi [A]
time = 0.04, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.071$, Rules used = {1369, 276}

$$\frac{3a^2b^2x^{n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+3)\left(ab+b^2x^n\right)}+\frac{b^4x^{3(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(n+1)\left(ab+b^2x^n\right)}+\frac{3ab^3x^{2n+3}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2n+3)\left(ab+b^2x^n\right)}+\frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(a + b*x^n)) + (b^4*x^(3*(
1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*(1 + n)*(a*b + b^2*x^n)) +
(3*a^2*b^2*x^(3 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 + n)*(a*b + b
^2*x^n)) + (3*a*b^3*x^(3 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((3 +
2*n)*(a*b + b^2*x^n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x^2\left(ab+b^2x^n\right)^3\,dx}{b^2\left(ab+b^2x^n\right)}$$

$$=\frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int\left(a^3b^3x^2+3ab^5x^{2(1+n)}+3a^2b^4x^{2+n}+b^6x^{2+3n}\right)}{b^2\left(ab+b^2x^n\right)}$$

$$=\frac{a^3x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{3\left(a+bx^n\right)}+\frac{b^4x^{3(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3(1+n)\left(ab+b^2x^n\right)}+\frac{3a^2b^2x^{3}\cdots}{(\cdots}$$

Mathematica [A]
time = 0.07, size = 123, normalized size = 0.58

$$\frac{x^3\sqrt{(a+bx^n)^2}\left(a^3(9+18n+11n^2+2n^3)+9a^2b(3+5n+2n^2)\,x^n+9ab^2(3+4n+n^2)\,x^{2n}+b^3(9+9n+2n^2)\,x^{3n}\right)}{3(1+n)(3+n)(3+2n)\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^n)^2]*(a^3*(9 + 18*n + 11*n^2 + 2*n^3) + 9*a^2*b*(3 + 5*
n + 2*n^2)*x^n + 9*a*b^2*(3 + 4*n + n^2)*x^(2*n) + b^3*(9 + 9*n + 2*n^2)*x^
(3*n)))/(3*(1 + n)*(3 + n)*(3 + 2*n)*(a + b*x^n))
```

Maple [A]
time = 0.04, size = 146, normalized size = 0.69

method	result
risch	$\frac{\sqrt{(a+b\,x^n)^2}\;a^3x^3}{3a+3b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\;b^3x^3x^{3n}}{3(a+b\,x^n)(1+n)}+\frac{3\sqrt{(a+b\,x^n)^2}\;a\,b^2x^3x^{2n}}{(a+b\,x^n)(3+2n)}+\frac{3\sqrt{(a+b\,x^n)^2}\;a^2b\,x^3x^n}{(a+b\,x^n)(3+n)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x^3+1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)
*b^3*x^3/(1+n)*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^2/(3+2*n)*x^3*(x
^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(3+n)*x^3*x^n
```

Maxima [A]
time = 0.31, size = 108, normalized size = 0.51

$$\frac{(2\,n^2+9\,n+9)b^3x^3x^{3\,n}+9\,(n^2+4\,n+3)ab^2x^3x^{2\,n}+9\,(2\,n^2+5\,n+3)a^2bx^3x^n+(2\,n^3+11\,n^2+18\,n+9)a^3x^3}{3\,(2\,n^3+11\,n^2+18\,n+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*((2*n^2 + 9*n + 9)*b^3*x^3*x^(3*n) + 9*(n^2 + 4*n + 3)*a*b^2*x^3*x^(2*n
) + 9*(2*n^2 + 5*n + 3)*a^2*b*x^3*x^n + (2*n^3 + 11*n^2 + 18*n + 9)*a^3*x^3
)/(2*n^3 + 11*n^2 + 18*n + 9)
```

Fricas [A]
time = 0.35, size = 144, normalized size = 0.68

$$\frac{(2\,b^3n^2+9\,b^3n+9\,b^3)x^3x^{3\,n}+9\,(ab^2n^2+4\,ab^2n+3\,ab^2)x^3x^{2\,n}+9\,(2\,a^2bn^2+5\,a^2bn+3\,a^2b)x^3x^n+(2\,a^3n^3+11\,a^3n^2+18\,a^3n+9\,a^3)x^3}{3\,(2\,n^3+11\,n^2+18\,n+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*b^3*n^2 + 9*b^3*n + 9*b^3)*x^3*x^(3*n) + 9*(a*b^2*n^2 + 4*a*b^2*n +
 3*a*b^2)*x^3*x^(2*n) + 9*(2*a^2*b*n^2 + 5*a^2*b*n + 3*a^2*b)*x^3*x^n + (2*
a^3*n^3 + 11*a^3*n^2 + 18*a^3*n + 9*a^3)*x^3)/(2*n^3 + 11*n^2 + 18*n + 9)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(\left(a+bx^{n}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x**n)**2)**(3/2), x)
```

Giac [A]
time = 4.62, size = 292, normalized size = 1.38

$$\frac{2\,b^3n^2x^3x^{3\,n}\mathrm{sgn}(bx^n+a)+9\,ab^2n^2x^3x^{2\,n}\mathrm{sgn}(bx^n+a)+18\,a^2bn^2x^3x^n\mathrm{sgn}(bx^n+a)+2\,a^3n^3x^3\mathrm{sgn}(bx^n+a)+9\,b^3nx^3x^{3\,n}\mathrm{sgn}(bx^n+a)+36\,ab^2nx^3x^{2\,n}\mathrm{sgn}(bx^n+a)+45\,a^2bnx^3x^n\mathrm{sgn}(bx^n+a)+11\,a^3n^2x^3\mathrm{sgn}(bx^n+a)+9\,b^3x^3x^{3\,n}\mathrm{sgn}(bx^n+a)+27\,ab^2x^3x^{2\,n}\mathrm{sgn}(bx^n+a)+27\,a^2bx^3x^n\mathrm{sgn}(bx^n+a)+18\,a^3nx^3\mathrm{sgn}(bx^n+a)+9\,a^3x^3\mathrm{sgn}(bx^n+a)}{3\,(2\,n^3+11\,n^2+18\,n+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*b^3*n^2*x^3*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^3*x^(2*n)*sgn(b*x
^n + a) + 18*a^2*b*n^2*x^3*x^n*sgn(b*x^n + a) + 2*a^3*n^3*x^3*sgn(b*x^n + a
) + 9*b^3*n*x^3*x^(3*n)*sgn(b*x^n + a) + 36*a*b^2*n*x^3*x^(2*n)*sgn(b*x^n +
 a) + 45*a^2*b*n*x^3*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^3*sgn(b*x^n + a) + 9
*b^3*x^3*x^(3*n)*sgn(b*x^n + a) + 27*a*b^2*x^3*x^(2*n)*sgn(b*x^n + a) + 27*
a^2*b*x^3*x^n*sgn(b*x^n + a) + 18*a^3*n*x^3*sgn(b*x^n + a) + 9*a^3*x^3*sgn(
b*x^n + a))/(2*n^3 + 11*n^2 + 18*n + 9)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.524 $\int x(a^2+2abx^n+b^2x^{2n})^{3/2}\,dx$

Optimal. Leaf size=211

$$\frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}+\frac{3ab^3x^{2(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1+n)(ab+b^2x^n)}+\frac{3a^2b^2x^{2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2+n)(ab+b^2x^n)}+\frac{b^4x^{2+3n}\sqrt{\ }}{(2+}$$

```
[Out] 1/2*a^3*x^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(a+b*x^n)+3/2*a*b^3*x^(2+2*n)
*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1+n)/(a*b+b^2*x^n)+3*a^2*b^2*x^(2+n)*(a
^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2+n)/(a*b+b^2*x^n)+b^4*x^(2+3*n)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(1/2)/(2+3*n)/(a*b+b^2*x^n)
```

Rubi [A]
time = 0.04, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {1369, 276}

$$\frac{3a^2b^2x^{n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(n+2)(ab+b^2x^n)}+\frac{b^4x^{3n+2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(3n+2)(ab+b^2x^n)}+\frac{3ab^3x^{2(n+1)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(n+1)(ab+b^2x^n)}+\frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(a + b*x^n)) + (3*a*b^3*x^
(2*(1 + n))*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 + n)*(a*b + b^2*x^n)
) + (3*a^2*b^2*x^(2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 + n)*(a*b
 + b^2*x^n)) + (b^4*x^(2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 +
3*n)*(a*b + b^2*x^n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int x(ab+b^2x^n)^3\,dx}{b^2(ab+b^2x^n)}$$

$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int (a^3b^3x+3a^2b^4x^{1+n}+3ab^5x^{1+2n}+b^6x^{1+3n})\,dx}{b^2(ab+b^2x^n)}$$

$$= \frac{a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(a+bx^n)} + \frac{3ab^3x^{2(1+n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1+n)(ab+b^2x^n)} + \frac{3a^2b^2\cdots}{}$$

Mathematica [A]
time = 0.06, size = 124, normalized size = 0.59

$$\frac{x^2\sqrt{(a+bx^n)^2}\,(a^3(4+12n+11n^2+3n^3)+6a^2b(2+5n+3n^2)\,x^n+3ab^2(4+8n+3n^2)\,x^{2n}+2b^3(2+3n+n^2)\,x^{3n})}{2(1+n)(2+n)(2+3n)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^n)^2]*(a^3*(4 + 12*n + 11*n^2 + 3*n^3) + 6*a^2*b*(2 + 5*
n + 3*n^2)*x^n + 3*a*b^2*(4 + 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 + 3*n + n^2)*
x^(3*n)))/(2*(1 + n)*(2 + n)*(2 + 3*n)*(a + b*x^n))
```

Maple [A]
time = 0.02, size = 145, normalized size = 0.69

method	result
risch	$\frac{\sqrt{(a+b\,x^n)^2}\;a^3x^2}{2a+2b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\;b^3x^2x^{3n}}{(a+b\,x^n)(2+3n)}+\frac{3\sqrt{(a+b\,x^n)^2}\;a\,b^2x^2x^{2n}}{2(a+b\,x^n)(1+n)}+\frac{3\sqrt{(a+b\,x^n)^2}\;a^2b\,x^2x^n}{(a+b\,x^n)(2+n)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3
/(2+3*n)*x^2*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^2*x^2/(1+n)*(x^n
)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(2+n)*x^2*x^n
```

Maxima [A]
time = 0.28, size = 109, normalized size = 0.52

$$\frac{2\,(n^2+3\,n+2)b^3x^2x^{3\,n}+3\,(3\,n^2+8\,n+4)ab^2x^2x^{2\,n}+6\,(3\,n^2+5\,n+2)a^2bx^2x^n+(3\,n^3+11\,n^2+12\,n+4)a^3x^2}{2\,(3\,n^3+11\,n^2+12\,n+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(n^2 + 3*n + 2)*b^3*x^2*x^(3*n) + 3*(3*n^2 + 8*n + 4)*a*b^2*x^2*x^(2
*n) + 6*(3*n^2 + 5*n + 2)*a^2*b*x^2*x^n + (3*n^3 + 11*n^2 + 12*n + 4)*a^3*x
^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

Fricas [A]
time = 0.36, size = 145, normalized size = 0.69

$$\frac{2\,(b^3n^2+3\,b^3n+2\,b^3)x^2x^{3\,n}+3\,(3\,ab^2n^2+8\,ab^2n+4\,ab^2)x^2x^{2\,n}+6\,(3\,a^2bn^2+5\,a^2bn+2\,a^2b)x^2x^n+(3\,a^3n^3+11\,a^3n^2+12\,a^3n+4\,a^3)x^2}{2\,(3\,n^3+11\,n^2+12\,n+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^3*n^2 + 3*b^3*n + 2*b^3)*x^2*x^(3*n) + 3*(3*a*b^2*n^2 + 8*a*b^2*n
 + 4*a*b^2)*x^2*x^(2*n) + 6*(3*a^2*b*n^2 + 5*a^2*b*n + 2*a^2*b)*x^2*x^n + (
3*a^3*n^3 + 11*a^3*n^2 + 12*a^3*n + 4*a^3)*x^2)/(3*n^3 + 11*n^2 + 12*n + 4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(\left(a+bx^{n}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x*((a + b*x**n)**2)**(3/2), x)
```

Giac [A]
time = 2.97, size = 292, normalized size = 1.38

$$\frac{2\,b^3n^2x^2x^{3\,n}\mathrm{sgn}(bx^n+a)+9\,ab^2n^2x^2x^{2\,n}\mathrm{sgn}(bx^n+a)+18\,a^2bn^2x^2x^n\mathrm{sgn}(bx^n+a)+3\,a^3n^3x^2\mathrm{sgn}(bx^n+a)+6\,b^3nx^2x^{3\,n}\mathrm{sgn}(bx^n+a)+24\,ab^2nx^2x^{2\,n}\mathrm{sgn}(bx^n+a)+30\,a^2bnx^2x^n\mathrm{sgn}(bx^n+a)+11\,a^3n^2x^2\mathrm{sgn}(bx^n+a)+4\,b^3x^2x^{3\,n}\mathrm{sgn}(bx^n+a)+12\,ab^2x^2x^{2\,n}\mathrm{sgn}(bx^n+a)+12\,a^2bx^2x^n\mathrm{sgn}(bx^n+a)+12\,a^3nx^2\mathrm{sgn}(bx^n+a)+4\,a^3x^2\mathrm{sgn}(bx^n+a)}{2\,(3\,n^3+11\,n^2+12\,n+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*n^2*x^2*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*n^2*x^2*x^(2*n)*sgn(b*x
^n + a) + 18*a^2*b*n^2*x^2*x^n*sgn(b*x^n + a) + 3*a^3*n^3*x^2*sgn(b*x^n + a
) + 6*b^3*n*x^2*x^(3*n)*sgn(b*x^n + a) + 24*a*b^2*n*x^2*x^(2*n)*sgn(b*x^n +
 a) + 30*a^2*b*n*x^2*x^n*sgn(b*x^n + a) + 11*a^3*n^2*x^2*sgn(b*x^n + a) + 4
*b^3*x^2*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*x^2*x^(2*n)*sgn(b*x^n + a) + 12*
a^2*b*x^2*x^n*sgn(b*x^n + a) + 12*a^3*n*x^2*sgn(b*x^n + a) + 4*a^3*x^2*sgn(
b*x^n + a))/(3*n^3 + 11*n^2 + 12*n + 4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.525 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}\,dx$

Optimal. Leaf size=206

$$\frac{a^3x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{\left(a+bx^n\right)^3}+\frac{3a^2b^4x^{1+n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{\left(1+n\right)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{\left(1+2n\right)\left(ab+b^2x^n\right)^3}+\frac{b^6x^{1+3n}}{(1}$$

```
[Out] a^3*x*(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/(a+b*x^n)^3+3*a^2*b^4*x^(1+n)*(a^2+
2*a*b*x^n+b^2*x^(2*n))^(3/2)/(1+n)/(a*b+b^2*x^n)^3+3*a*b^5*x^(1+2*n)*(a^2+2
*a*b*x^n+b^2*x^(2*n))^(3/2)/(1+2*n)/(a*b+b^2*x^n)^3+b^6*x^(1+3*n)*(a^2+2*a*
b*x^n+b^2*x^(2*n))^(3/2)/(1+3*n)/(a*b+b^2*x^n)^3
```

Rubi [A]
time = 0.04, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {1357, 250}

$$\frac{b^6x^{3n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(3n+1)\left(ab+b^2x^n\right)^3}+\frac{3ab^5x^{2n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(2n+1)\left(ab+b^2x^n\right)^3}+\frac{3a^2b^4x^{n+1}\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{(n+1)\left(ab+b^2x^n\right)^3}+\frac{a^3x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{\left(a+bx^n\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/(a + b*x^n)^3 + (3*a^2*b^4*x^
(1 + n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + n)*(a*b + b^2*x^n)^3)
+ (3*a*b^5*x^(1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/((1 + 2*n)*(a
*b + b^2*x^n)^3) + (b^6*x^(1 + 3*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))/
((1 + 3*n)*(a*b + b^2*x^n)^3)
```

Rule 250

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && IGtQ[p, 0]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} dx = \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} \int \left(2ab+2b^2x^n\right)^3 dx}{\left(2ab+2b^2x^n\right)^3}$$
$$= \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2} \int \left(8a^3b^3+24a^2b^4x^n+24ab^5x^{2n}+8b^6x^{3n}\right) dx}{\left(2ab+2b^2x^n\right)^3}$$
$$= \frac{a^3x(a^2+2abx^n+b^2x^{2n})^{3/2}}{(a+bx^n)^3} + \frac{3a^2b^4x^{1+n}(a^2+2abx^n+b^2x^{2n})^{3/2}}{(1+n)\left(ab+b^2x^n\right)^3} + \frac{3ab^5x}{}$$

Mathematica [A]
time = 0.07, size = 122, normalized size = 0.59

$$\frac{x\sqrt{(a+bx^n)^2}\left(a^3(1+6n+11n^2+6n^3)+3a^2b(1+5n+6n^2)x^n+3ab^2(1+4n+3n^2)x^{2n}+b^3(1+3n+2n^2)x^{3n}\right)}{(1+n)(1+2n)(1+3n)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*Sqrt[(a + b*x^n)^2]*(a^3*(1 + 6*n + 11*n^2 + 6*n^3) + 3*a^2*b*(1 + 5*n +
 6*n^2)*x^n + 3*a*b^2*(1 + 4*n + 3*n^2)*x^(2*n) + b^3*(1 + 3*n + 2*n^2)*x^(
3*n)))/((1 + n)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n))
```

Maple [A]
time = 0.02, size = 138, normalized size = 0.67

method	result	siz
risch	$\frac{\sqrt{(a+b\,x^n)^2}\;a^3x}{a+b\,x^n}+\frac{\sqrt{(a+b\,x^n)^2}\;b^3x\,x^{3n}}{(a+b\,x^n)(1+3n)}+\frac{3\sqrt{(a+b\,x^n)^2}\;a\,b^2x\,x^{2n}}{(a+b\,x^n)(1+2n)}+\frac{3\sqrt{(a+b\,x^n)^2}\;a^2bx\,x^n}{(a+b\,x^n)(1+n)}$	138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b^3/(1+3*
n)*x*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^2/(1+2*n)*x*(x^n)^2+3*((a+
b*x^n)^2)^(1/2)/(a+b*x^n)*a^2*b/(1+n)*x*x^n
```

Maxima [A]
time = 0.32, size = 101, normalized size = 0.49

$$\frac{(2\,n^2+3\,n+1)b^3xx^{3\,n}+3\,(3\,n^2+4\,n+1)ab^2xx^{2\,n}+3\,(6\,n^2+5\,n+1)a^2bxx^n+(6\,n^3+11\,n^2+6\,n+1)a^3x}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] ((2*n^2 + 3*n + 1)*b^3*x*x^(3*n) + 3*(3*n^2 + 4*n + 1)*a*b^2*x*x^(2*n) + 3*
(6*n^2 + 5*n + 1)*a^2*b*x*x^n + (6*n^3 + 11*n^2 + 6*n + 1)*a^3*x)/(6*n^3 +
11*n^2 + 6*n + 1)
```

Fricas [A]
time = 0.41, size = 130, normalized size = 0.63

$$\frac{(2\,b^3n^2+3\,b^3n+b^3)xx^{3\,n}+3\,(3\,ab^2n^2+4\,ab^2n+ab^2)xx^{2\,n}+3\,(6\,a^2bn^2+5\,a^2bn+a^2b)xx^n+(6\,a^3n^3+11\,a^3n^2+6\,a^3n+a^3)x}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] ((2*b^3*n^2 + 3*b^3*n + b^3)*x*x^(3*n) + 3*(3*a*b^2*n^2 + 4*a*b^2*n + a*b^2
)*x*x^(2*n) + 3*(6*a^2*b*n^2 + 5*a^2*b*n + a^2*b)*x*x^n + (6*a^3*n^3 + 11*a
^3*n^2 + 6*a^3*n + a^3)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^{2}+2abx^{n}+b^{2}x^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(3/2), x)
```

Giac [A]
time = 7.47, size = 263, normalized size = 1.28

$$\frac{6\,a^3n^3x\mathrm{sgn}(bx^n+a)+2\,b^3n^2xx^{3\,n}\mathrm{sgn}(bx^n+a)+9\,ab^2n^2xx^{2\,n}\mathrm{sgn}(bx^n+a)+18\,a^2bn^2xx^n\mathrm{sgn}(bx^n+a)+11\,a^3n^2x\mathrm{sgn}(bx^n+a)+3\,b^3nxx^{3\,n}\mathrm{sgn}(bx^n+a)+12\,ab^2nxx^{2\,n}\mathrm{sgn}(bx^n+a)+15\,a^2bnxx^n\mathrm{sgn}(bx^n+a)+6\,a^3nx\mathrm{sgn}(bx^n+a)+b^3xx^{3\,n}\mathrm{sgn}(bx^n+a)+3\,ab^2xx^{2\,n}\mathrm{sgn}(bx^n+a)+3\,a^2bxx^n\mathrm{sgn}(bx^n+a)+a^3x\mathrm{sgn}(bx^n+a)}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] (6*a^3*n^3*x*sgn(b*x^n + a) + 2*b^3*n^2*x*x^(3*n)*sgn(b*x^n + a) + 9*a*b^2*
n^2*x*x^(2*n)*sgn(b*x^n + a) + 18*a^2*b*n^2*x*x^n*sgn(b*x^n + a) + 11*a^3*n
^2*x*sgn(b*x^n + a) + 3*b^3*n*x*x^(3*n)*sgn(b*x^n + a) + 12*a*b^2*n*x*x^(2*
n)*sgn(b*x^n + a) + 15*a^2*b*n*x*x^n*sgn(b*x^n + a) + 6*a^3*n*x*sgn(b*x^n +
 a) + b^3*x*x^(3*n)*sgn(b*x^n + a) + 3*a*b^2*x*x^(2*n)*sgn(b*x^n + a) + 3*a
^2*b*x*x^n*sgn(b*x^n + a) + a^3*x*sgn(b*x^n + a))/(6*n^3 + 11*n^2 + 6*n + 1
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.526 $\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x}\,dx$

Optimal. Leaf size=196

$$\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(ab+b^2x^n)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\,(ab+b^2x^n)}+\frac{a^3\sqrt{a^2+2abx}}{a\,-}$$

```
[Out] 3*a^2*b^2*x^n*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+3/2*a*b^3*x
^(2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+1/3*b^4*x^(3*n)*(a
^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/n/(a*b+b^2*x^n)+a^3*ln(x)*(a^2+2*a*b*x^n+b^
2*x^(2*n))^(1/2)/(a+b*x^n)
```

Rubi [A]
time = 0.04, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1369, 272, 45}

$$\frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n\,(ab+b^2x^n)}+\frac{b^4x^{3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{3n\,(ab+b^2x^n)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n\,(ab+b^2x^n)}+\frac{a^3\log(x)\sqrt{a^2+2abx^n+b^2x^{2n}}}{a+bx^n}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

```
[Out] (3*a^2*b^2*x^n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(n*(a*b + b^2*x^n)) + (
3*a*b^3*x^(2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*n*(a*b + b^2*x^n))
+ (b^4*x^(3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(3*n*(a*b + b^2*x^n)) +
 (a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]*Log[x])/(a + b*x^n)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
```

```
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x}\,dx &= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{(ab+b^2x^n)^3}{x}\,dx}{b^2(ab+b^2x^n)} \\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\mathrm{Subst}\left(\int \frac{(ab+b^2x)^3}{x}\,dx,x,x^n\right)}{b^2n(ab+b^2x^n)} \\
&= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\,\mathrm{Subst}\left(\int \left(3a^2b^4+\frac{a^3b^3}{x}+3ab^5x+b^6x^2\right)dx,x,x^n\right)}{b^2n(ab+b^2x^n)} \\
&= \frac{3a^2b^2x^n\sqrt{a^2+2abx^n+b^2x^{2n}}}{n(ab+b^2x^n)}+\frac{3ab^3x^{2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2n(ab+b^2x^n)}+\frac{b^4x^{3n}\sqrt{}}{3}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 67, normalized size = 0.34

$$\frac{\sqrt{(a+bx^n)^2}\left(bx^n(18a^2+9abx^n+2b^2x^{2n})+6a^3\log(x^n)\right)}{6n(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(b*x^n*(18*a^2 + 9*a*b*x^n + 2*b^2*x^(2*n)) + 6*a^3*Lo
g[x^n]))/(6*n*(a + b*x^n))
```

Maple [A]
time = 0.02, size = 127, normalized size = 0.65

method	result	size
risch	$\frac{\sqrt{(a+bx^n)^2}\,a^3\ln(x)}{a+bx^n}+\frac{\sqrt{(a+bx^n)^2}\,b^3x^{3n}}{3(a+bx^n)n}+\frac{3\sqrt{(a+bx^n)^2}\,ab^2x^{2n}}{2(a+bx^n)n}+\frac{3\sqrt{(a+bx^n)^2}\,a^2bx^n}{(a+bx^n)n}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3*ln(x)+1/3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*b
^3/n*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a*b^2/n*(x^n)^2+3*((a+b*x^n)
^2)^(1/2)/(a+b*x^n)*a^2*b/n*x^n
```

Maxima [A]
time = 0.29, size = 43, normalized size = 0.22

$$a^3 \log(x) + \frac{2\,b^3x^{3\,n} + 9\,ab^2x^{2\,n} + 18\,a^2bx^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="maxima")
[Out] a^3*log(x) + 1/6*(2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

Fricas [A]
time = 0.37, size = 44, normalized size = 0.22

$$\frac{6\,a^3n\log(x) + 2\,b^3x^{3\,n} + 9\,ab^2x^{2\,n} + 18\,a^2bx^n}{6\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="fricas")
[Out] 1/6*(6*a^3*n*log(x) + 2*b^3*x^(3*n) + 9*a*b^2*x^(2*n) + 18*a^2*b*x^n)/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a + bx^n)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x,x)
[Out] Integral(((a + b*x**n)**2)**(3/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x,x, algorithm="giac")
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x,x)
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x, x)
```

3.527 $\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2}\,dx$

Optimal. Leaf size=212

$$-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}-\frac{3a^2b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\,(ab+b^2x^n)}-\frac{3ab^3x^{-1+2n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)\,(ab+b^2x^n)}-\frac{b^4x^{-1+}}{(1}$$

```
[Out] -a^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x/(a+b*x^n)-3*a^2*b^2*x^(-1+n)*(a^2+
2*a*b*x^n+b^2*x^(2*n))^(1/2)/(1-n)/(a*b+b^2*x^n)-3*a*b^3*x^(-1+2*n)*(a^2+2*
a*b*x^n+b^2*x^(2*n))^(1/2)/(1-2*n)/(a*b+b^2*x^n)-b^4*x^(-1+3*n)*(a^2+2*a*b*
x^n+b^2*x^(2*n))^(1/2)/(1-3*n)/(a*b+b^2*x^n)
```

Rubi [A]

time = 0.05, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 276}

$$-\frac{3a^2b^2x^{n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)\,(ab+b^2x^n)}-\frac{b^4x^{3n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-3n)\,(ab+b^2x^n)}-\frac{3ab^3x^{2n-1}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-2n)\,(ab+b^2x^n)}-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x*(a + b*x^n))) - (3*a^2*b^2*x
^(-1 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - n)*(a*b + b^2*x^n)) -
(3*a*b^3*x^(-1 + 2*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 - 2*n)*(a*b
+ b^2*x^n)) - (b^4*x^(-1 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((1 -
3*n)*(a*b + b^2*x^n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{(ab+b^2x^n)^3}{x^2}\,dx}{b^2(ab+b^2x^n)}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{a^3b^3}{x^2}+3a^2b^4x^{-2+n}+3ab^5x^{2(-1+n)}+b^6x^{-2+3n}\right) dx}{b^2(ab+b^2x^n)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{x(a+bx^n)} - \frac{3a^2b^2x^{-1+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(1-n)(ab+b^2x^n)} - \frac{3ab^3x^{-1+}}{(1}$$

Mathematica [A]
time = 0.08, size = 124, normalized size = 0.58

$$\frac{\sqrt{(a+bx^n)^2}\,(a^3(1-6n+11n^2-6n^3)+3a^2b(1-5n+6n^2)\,x^n+3ab^2(1-4n+3n^2)\,x^{2n}+b^3(1-3n+2n^2)\,x^{3n})}{(-1+n)(-1+2n)(-1+3n)x\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(1 - 6*n + 11*n^2 - 6*n^3) + 3*a^2*b*(1 - 5*n + 6
*n^2)*x^n + 3*a*b^2*(1 - 4*n + 3*n^2)*x^(2*n) + b^3*(1 - 3*n + 2*n^2)*x^(3*
n)))/((-1 + n)*(-1 + 2*n)*(-1 + 3*n)*x*(a + b*x^n))
```

Maple [A]
time = 0.02, size = 147, normalized size = 0.69

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^n)^2}\,a^3}{(a+b\,x^n)x}+\frac{\sqrt{(a+b\,x^n)^2}\,b^3x^{3n}}{(a+b\,x^n)(-1+3n)x}+\frac{3\sqrt{(a+b\,x^n)^2}\,a\,b^2x^{2n}}{(a+b\,x^n)(-1+2n)x}+\frac{3\sqrt{(a+b\,x^n)^2}\,a^2b\,x^n}{(a+b\,x^n)(-1+n)x}$	147

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/x+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+3*n)
*b^3/x*(x^n)^3+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+2*n)*a*b^2/x*(x^n)^2+3*(
(a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*a^2*b/x*x^n
```

Maxima [A]
time = 0.32, size = 101, normalized size = 0.48

$$\frac{(2\,n^2-3\,n+1)b^3x^{3\,n}+3\,(3\,n^2-4\,n+1)ab^2x^{2\,n}+3\,(6\,n^2-5\,n+1)a^2bx^n-(6\,n^3-11\,n^2+6\,n-1)a^3}{(6\,n^3-11\,n^2+6\,n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*b^3*x^(3*n) + 3*(3*n^2 - 4*n + 1)*a*b^2*x^(2*n) + 3*(6*n
^2 - 5*n + 1)*a^2*b*x^n - (6*n^3 - 11*n^2 + 6*n - 1)*a^3)/((6*n^3 - 11*n^2
+ 6*n - 1)*x)
```

Fricas [A]
time = 0.37, size = 131, normalized size = 0.62

$$-\frac{6\,a^{3}n^{3}-11\,a^{3}n^{2}+6\,a^{3}n-a^{3}-(2\,b^{3}n^{2}-3\,b^{3}n+b^{3})x^{3\,n}-3\,(3\,ab^{2}n^{2}-4\,ab^{2}n+ab^{2})x^{2\,n}-3\,(6\,a^{2}bn^{2}-5\,a^{2}bn+a^{2}b)x^{n}}{(6\,n^{3}-11\,n^{2}+6\,n-1)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(6*a^3*n^3 - 11*a^3*n^2 + 6*a^3*n - a^3 - (2*b^3*n^2 - 3*b^3*n + b^3)*x^(3
*n) - 3*(3*a*b^2*n^2 - 4*a*b^2*n + a*b^2)*x^(2*n) - 3*(6*a^2*b*n^2 - 5*a^2*
b*n + a^2*b)*x^n)/((6*n^3 - 11*n^2 + 6*n - 1)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left((a+bx^{n})^{2}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**n)**2)**(3/2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a^{2}+b^{2}\,x^{2\,n}+2\,a\,b\,x^{n}\right)}^{3/2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^2, x)
```

3.528 $\int \frac{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=218

$$-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\left(ab+b^2x^n\right)}-\frac{3a^2b^2x^{-2+n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{b^4x^{-2+3n}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)\left(ab+b^2x^n\right)}$$

```
[Out] -1/2*a^3*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/x^2/(a+b*x^n)-3/2*a*b^3*(a^2+2*a
*b*x^n+b^2*x^(2*n))^(1/2)/(1-n)/(x^(2-2*n))/(a*b+b^2*x^n)-3*a^2*b^2*x^(-2+n
)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2-n)/(a*b+b^2*x^n)-b^4*x^(-2+3*n)*(a^2
+2*a*b*x^n+b^2*x^(2*n))^(1/2)/(2-3*n)/(a*b+b^2*x^n)
```

Rubi [A]

time = 0.05, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 276}

$$-\frac{3a^2b^2x^{n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-n)\left(ab+b^2x^n\right)}-\frac{b^4x^{3n-2}\sqrt{a^2+2abx^n+b^2x^{2n}}}{(2-3n)\left(ab+b^2x^n\right)}-\frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\left(ab+b^2x^n\right)}-\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] -1/2*(a^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(x^2*(a + b*x^n)) - (3*a*b^3
*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(2*(1 - n)*x^(2*(1 - n))*(a*b + b^2*x
^n)) - (3*a^2*b^2*x^(-2 + n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/((2 - n)*
(a*b + b^2*x^n)) - (b^4*x^(-2 + 3*n)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])/(
(2 - 3*n)*(a*b + b^2*x^n))
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(a^2+2abx^n+b^2x^{2n})^{3/2}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \frac{(ab+b^2x^n)^3}{x^3}\,dx}{b^2\,(ab+b^2x^n)}$$
$$= \frac{\sqrt{a^2+2abx^n+b^2x^{2n}}\int \left(\frac{a^3b^3}{x^3}+3a^2b^4x^{-3+n}+b^6x^{3(-1+n)}+3ab^5x^{-3+2n}\right) d}{b^2\,(ab+b^2x^n)}$$
$$= -\frac{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}{2x^2\,(a+bx^n)} - \frac{3ab^3x^{-2(1-n)}\sqrt{a^2+2abx^n+b^2x^{2n}}}{2(1-n)\,(ab+b^2x^n)} - \frac{3a^2b^2x}{}$$

Mathematica [A]
time = 0.08, size = 124, normalized size = 0.57

$$\frac{\sqrt{(a+bx^n)^2}\,(a^3(4-12n+11n^2-3n^3)+6a^2b(2-5n+3n^2)\,x^n+3ab^2(4-8n+3n^2)\,x^{2n}+2b^3(2-3n+n^2)\,x^{3n})}{2(-2+n)(-1+n)(-2+3n)x^2\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^n)^2]*(a^3*(4 - 12*n + 11*n^2 - 3*n^3) + 6*a^2*b*(2 - 5*n +
3*n^2)*x^n + 3*a*b^2*(4 - 8*n + 3*n^2)*x^(2*n) + 2*b^3*(2 - 3*n + n^2)*x^(3
*n)))/(2*(-2 + n)*(-1 + n)*(-2 + 3*n)*x^2*(a + b*x^n))
```

Maple [A]
time = 0.02, size = 145, normalized size = 0.67

method	result	size
risch	$-\frac{\sqrt{(a+b\,x^n)^2}\;a^3}{2(a+b\,x^n)x^2}+\frac{\sqrt{(a+b\,x^n)^2}\;b^3x^{3n}}{(a+b\,x^n)(-2+3n)x^2}+\frac{3\sqrt{(a+b\,x^n)^2}\;a\,b^2x^{2n}}{2(a+b\,x^n)(-1+n)x^2}+\frac{3\sqrt{(a+b\,x^n)^2}\;a^2b\,x^n}{(a+b\,x^n)(-2+n)x^2}$	145

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)*a^3/x^2+((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-
2+3*n)*b^3/x^2*(x^n)^3+3/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-1+n)*a*b^2/x^2*(
x^n)^2+3*((a+b*x^n)^2)^(1/2)/(a+b*x^n)/(-2+n)*a^2*b/x^2*x^n
```

Maxima [A]
time = 0.30, size = 101, normalized size = 0.46

$$\frac{2\,(n^2-3\,n+2)b^3x^{3\,n}+3\,(3\,n^2-8\,n+4)ab^2x^{2\,n}+6\,(3\,n^2-5\,n+2)a^2bx^n-(3\,n^3-11\,n^2+12\,n-4)a^3}{2\,(3\,n^3-11\,n^2+12\,n-4)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(n^2 - 3*n + 2)*b^3*x^(3*n) + 3*(3*n^2 - 8*n + 4)*a*b^2*x^(2*n) + 6*
(3*n^2 - 5*n + 2)*a^2*b*x^n - (3*n^3 - 11*n^2 + 12*n - 4)*a^3)/((3*n^3 - 11
*n^2 + 12*n - 4)*x^2)
```

Fricas [A]
time = 0.38, size = 134, normalized size = 0.61

$$-\frac{3\,a^3n^3-11\,a^3n^2+12\,a^3n-4\,a^3-2\,(b^3n^2-3\,b^3n+2\,b^3)x^{3\,n}-3\,(3\,ab^2n^2-8\,ab^2n+4\,ab^2)x^{2\,n}-6\,(3\,a^2bn^2-5\,a^2bn+2\,a^2b)x^n}{2\,(3\,n^3-11\,n^2+12\,n-4)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(3*a^3*n^3 - 11*a^3*n^2 + 12*a^3*n - 4*a^3 - 2*(b^3*n^2 - 3*b^3*n + 2*
b^3)*x^(3*n) - 3*(3*a*b^2*n^2 - 8*a*b^2*n + 4*a*b^2)*x^(2*n) - 6*(3*a^2*b*n
^2 - 5*a^2*b*n + 2*a^2*b)*x^n)/((3*n^3 - 11*n^2 + 12*n - 4)*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((a+bx^n)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**n)**2)**(3/2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3,x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)/x^3, x)
```

3.529 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=76

$$\frac{(dx)^{1+m}(a+bx^n)\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (d*x)^(1+m)*(a+b*x^n)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/d/(1+m
)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1369, 371}

$$\frac{(dx)^{m+1}(a+bx^n)\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -
((b*x^n)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{(ab+b^2x^n)\int \frac{(dx)^m}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{(dx)^{1+m}(a+bx^n)\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{ad(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 62, normalized size = 0.82

$$\frac{x(dx)^m\,(a+bx^n)\;{}_2F_1\left(1,\frac{1+m}{n};1+\frac{1+m}{n};-\frac{bx^n}{a}\right)}{a(1+m)\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[1, (1 + m)/n, 1 + (1 + m)/n, -((b*
x^n)/a)])/(a*(1 + m)*Sqrt[(a + b*x^n)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt((a + b*x**n)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.530 $\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3(a+bx^n)\ {}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/3*x^3*(a+b*x^n)*hypergeom([1, 3/n],[(3+n)/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+b
^2*x^(2*n))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 371}

$$\frac{x^3(a+bx^n)\ {}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{(ab+b^2x^n)\int \frac{x^2}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^3(a+bx^n)\ {}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.83

$$\frac{x^3(a+bx^n)\ {}_2F_1\left(1,\frac{3}{n};1+\frac{3}{n};-\frac{bx^n}{a}\right)}{3a\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[1, 3/n, 1 + 3/n, -((b*x^n)/a)])/(3*a*Sqr
t[(a + b*x^n)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] integral(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt((a + b*x**n)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.531 $\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2(a+bx^n)\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/2*x^2*(a+b*x^n)*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+b
^2*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 371}

$$\frac{x^2(a+bx^n)\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{\left(ab+b^2x^n\right)\int \frac{x}{ab+b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^2(a+bx^n)\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 0.83

$$\frac{x^2(a+bx^n)\ {}_2F_1\left(1,\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2a\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[1, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a*Sqr
t[(a + b*x^n)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] integral(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x/sqrt((a + b*x**n)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.532 $\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=55

$$\frac{x(a+bx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] x*(a+b*x^n)*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(a^2+2*a*b*x^n+b^2*x^(2*
n))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1357, 251}

$$\frac{x(a+bx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*S
qrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{(2ab+2b^2x^n)\int \frac{1}{2ab+2b^2x^n}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x(a+bx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 0.80

$$\frac{x(a+bx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)],x]
```

```
[Out] (x*(a + b*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*S
qrt[(a + b*x^n)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

```
[Out] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**2 + 2*a*b*x**n + b**2*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2), x)
```

3.533 $\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=85

$$\frac{(a+bx^n)\log(x)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (a+b*x^n)*ln(x)/a/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-(a+b*x^n)*ln(a+b*x^n)/a
/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.179$, Rules used = {1369, 272, 36, 29, 31}

$$\frac{\log(x)(a+bx^n)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] ((a + b*x^n)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) - ((a + b*x^n)
*Log[a + b*x^n])/(a*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{(ab+b^2x^n)\int \frac{1}{x(ab+b^2x^n)}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{(ab+b^2x^n)\,\mathrm{Subst}\left(\int \frac{1}{x(ab+b^2x)}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{(ab+b^2x^n)\,\mathrm{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{abn\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(b(ab+b^2x^n))\,\mathrm{Subst}\left(\int \frac{1}{ab+b^2x}\,dx,x,x^n\right)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{(a+bx^n)\log(x)}{a\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{(a+bx^n)\log(a+bx^n)}{an\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.53

$$\frac{(a+bx^n)\left(\log(x^n)-\log(an(a+bx^n))\right)}{an\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
[Out] ((a + b*x^n)*(Log[x^n] - Log[a*n*(a + b*x^n)]))/(a*n*Sqrt[(a + b*x^n)^2])
```

Maple [A]
time = 0.02, size = 66, normalized size = 0.78

method	result	size
risch	$\frac{\sqrt{(a+b\,x^n)^2}\,\ln(x)}{(a+b\,x^n)a}-\frac{\sqrt{(a+b\,x^n)^2}\,\ln\left(x^n+\frac{a}{b}\right)}{(a+b\,x^n)an}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x,method=_RETURNVERBOSE)
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*ln(x)/a-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a/n*ln(
x^n+a/b)
```

Maxima [A]
time = 0.28, size = 27, normalized size = 0.32

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{bx^n+a}{b}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] log(x)/a - log((b*x^n + a)/b)/(a*n)
```

Fricas [A]
time = 0.40, size = 22, normalized size = 0.26

$$\frac{n\log(x) - \log(bx^n + a)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] (n*log(x) - log(b*x^n + a))/(a*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
[Out] Integral(1/(x*sqrt((a + b*x**n)**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
[Out] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

3.534 $\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=65

$$-\frac{(a+bx^n)\ {}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -(a+b*x^n)*hypergeom([1, -1/n],[(-1+n)/n],-b*x^n/a)/a/x/(a^2+2*a*b*x^n+b^2*
x^(2*n))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 371}

$$-\frac{(a+bx^n)\ {}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(
a*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{(ab+b^2x^n)\int \frac{1}{x^2(ab+b^2x^n)}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\
&= -\frac{(a+bx^n)\ {}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{a^2+2abx^n+b^2x^{2n}}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 51, normalized size = 0.78

$$-\frac{(a+bx^n)\ {}_2F_1\left(1,-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{ax\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
[Out] -(((a + b*x^n)*Hypergeometric2F1[1, -n^(-1), 1 - n^(-1), -((b*x^n)/a)])/(a*
x*Sqrt[(a + b*x^n)^2]))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^2*x^2*x^(2*n) + 2*a*b*x^2*x
^n + a^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((a + b*x**n)**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

3.535 $\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx$

Optimal. Leaf size=67

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -1/2*(a+b*x^n)*hypergeom([1, -2/n],[(-2+n)/n],-b*x^n/a)/a/x^2/(a^2+2*a*b*x^
n+b^2*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 371}

$$-\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
```

```
[Out] -1/2*((a + b*x^n)*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(
a*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\,dx &= \frac{(ab+b^2x^n)\int \frac{1}{x^3(ab+b^2x^n)}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= -\frac{(a+bx^n)\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.79

$$-\frac{(a+bx^n)\ {}_2F_1\left(1,-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2ax^2\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]),x]
[Out] -1/2*((a + b*x^n)*Hypergeometric2F1[1, -2/n, 1 - 2/n, -((b*x^n)/a)])/(a*x^2
*Sqrt[(a + b*x^n)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a^2+2ab\,x^n+b^2x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
[Out] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^2*x^3*x^(2*n) + 2*a*b*x^3*x
^n + a^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{(a+bx^n)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((a + b*x**n)**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(1/2)), x)
```

3.536 $\int \frac{(dx)^m}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{(dx)^{1+m}\,(a+bx^n)\;{}_2F_1\left(3,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] (d*x)^(1+m)*(a+b*x^n)*hypergeom([3, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^3/d/(1
+m)/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1369, 371}

$$\frac{(dx)^{m+1}\,(a+bx^n)\;{}_2F_1\left(3,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -
((b*x^n)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\int \frac{(dx)^m}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx = \frac{(b^2(ab+b^2x^n))\int \frac{(dx)^m}{(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}} = \frac{(dx)^{1+m}\,(a+bx^n)\;{}_2F_1\left(3,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a^3d(1+m)\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Mathematica [A]
time = 0.03, size = 61, normalized size = 0.80

$$\frac{x(dx)^m\,(a+bx^n)\;{}_2F_1\left(3,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a^3(1+m)\sqrt{(a+bx^n)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^n)*Hypergeometric2F1[3, (1 + m)/n, (1 + m + n)/n, -((b*
x^n)/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^n)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a^2+2ab\,x^n+b^2x^{2n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*d^m*integrate(1/2*x^m/(a^2*b*n^2*x^n
+ a^3*n^2), x) - 1/2*(a*d^m*(m - 3*n + 1)*x*x^m + b*d^m*(m - 2*n + 1)*x*e^(
m*log(x) + n*log(x)))/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x)^m/(b^4*x^(4*n) + 4*a^2*b
^2*x^(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral((d*x)**m/((a + b*x**n)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((d*x)^m/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
[Out] int((d*x)^m/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.537 $\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3(a+bx^n)\ {}_2F_1\left(3,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/3*x^3*(a+b*x^n)*hypergeom([3, 3/n],[(3+n)/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 371}

$$\frac{x^3(a+bx^n)\ {}_2F_1\left(3,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(a + b*x^n)*Hypergeometric2F1[3, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^3
*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx &= \frac{\left(b^2(ab+b^2x^n)\right)\int \frac{x^2}{(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^3(a+bx^n)\ {}_2F_1\left(3,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.86

$$\frac{x^3(a+bx^n)^3\ {}_2F_1\left(3,\frac{3}{n};1+\frac{3}{n};-\frac{bx^n}{a}\right)}{3a^3\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(a + b*x^n)^3*Hypergeometric2F1[3, 3/n, 1 + 3/n, -((b*x^n)/a)])/(3*a^3
*((a + b*x^n)^2)^(3/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a^2+2ab\,x^n+b^2x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (2*n^2 - 9*n + 9)*integrate(1/2*x^2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b*
(2*n - 3)*x^3*x^n + 3*a*(n - 1)*x^3)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n
 + a^4*n^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x^2/(b^4*x^(4*n) + 4*a^2*b^2*x
^(2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left((a + bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**n)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(a^2 + b^2\,x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x^2/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.538 $\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2(a+bx^n)\ {}_2F_1\left(3,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] 1/2*x^2*(a+b*x^n)*hypergeom([3, 2/n],[(2+n)/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^n
+b^2*x^(2*n))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1369, 371}

$$\frac{x^2(a+bx^n)\ {}_2F_1\left(3,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(a + b*x^n)*Hypergeometric2F1[3, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^3
*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2(ab+b^2x^n)\right)\int \frac{x}{(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= \frac{x^2(a+bx^n)\ {}_2F_1\left(3,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.86

$$\frac{x^2(a+bx^n)^3\ {}_2F_1\left(3,\frac{2}{n};1+\frac{2}{n};-\frac{bx^n}{a}\right)}{2a^3\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(a + b*x^n)^3*Hypergeometric2F1[3, 2/n, 1 + 2/n, -((b*x^n)/a)])/(2*a^3
*((a + b*x^n)^2)^(3/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a^2+2ab\,x^n+b^2x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (n^2 - 3*n + 2)*integrate(x/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(2*b*(n - 1
)*x^2*x^n + a*(3*n - 2)*x^2)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n
^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)*x/(b^4*x^(4*n) + 4*a^2*b^2*x^(
2*n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**n)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(x/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.539 $\int \frac{1}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{x(a+bx^n)^3\ {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}$$

```
[Out] x*(a+b*x^n)^3*hypergeom([3, 1/n],[1+1/n],-b*x^n/a)/a^3/(a^2+2*a*b*x^n+b^2*x
^(2*n))^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1357, 251}

$$\frac{x(a+bx^n)^3\ {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a
^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2))
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(2ab+2b^2x^n\right)^3 \int \frac{1}{\left(2ab+2b^2x^n\right)^3}\,dx}{\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\\ &= \frac{x(a+bx^n)^3\ {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 0.81

$$\frac{x(a+bx^n)^3\ {}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(a + b*x^n)^3*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a
^3*((a + b*x^n)^2)^(3/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2+2ab\,x^n+b^2x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (2*n^2 - 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^n + a^3*n^2), x) + 1/2*(b*(2*n
 - 1)*x*x^n + a*(3*n - 1)*x)/(a^2*b^2*n^2*x^(2*n) + 2*a^3*b*n^2*x^n + a^4*n
^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^(4*n) + 4*a^2*b^2*x^(2*
n) + 4*a^3*b*x^n + a^4 + 2*(2*a*b^3*x^n + a^2*b^2)*x^(2*n)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2 + 2abx^n + b^2x^{2n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2), x)
```

3.540 $\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=159

$$\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\left(a+bx^n\right)\log(x)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{\left(a+bx^n\right)\log(}{a^3n\sqrt{a^2+2abx}}$$

```
[Out] 1/a^2/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)+1/2/a/n/(a+b*x^n)/(a^2+2*a*b*x^n+
b^2*x^(2*n))^(1/2)+(a+b*x^n)*ln(x)/a^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)-(a
+b*x^n)*ln(a+b*x^n)/a^3/n/(a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1369, 272, 46}

$$\frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{\log(x)\left(a+bx^n\right)}{a^3\sqrt{a^2+2abx^n+b^2x^{2n}}}-\frac{\left(a+bx^n\right)\log\left(a+bx^n\right)}{a^3n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] 1/(a^2*n*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]) + 1/(2*a*n*(a + b*x^n)*Sqrt[a
^2 + 2*a*b*x^n + b^2*x^(2*n)]) + ((a + b*x^n)*Log[x])/(a^3*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)]) - ((a + b*x^n)*Log[a + b*x^n])/(a^3*n*Sqrt[a^2 + 2*a*b
*x^n + b^2*x^(2*n)])
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
```

```
[p - 1/2]
```

Rubi steps

$$\int \frac{1}{x\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx = \frac{\left(b^2(ab+b^2x^n)\right)\int \frac{1}{x(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(b^2(ab+b^2x^n)\right)\text{Subst}\left(\int \frac{1}{x(ab+b^2x)^3}\,dx,x,x^n\right)}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{\left(b^2(ab+b^2x^n)\right)\text{Subst}\left(\int \left(\frac{1}{a^3b^3x}-\frac{1}{ab^2(a+bx)^3}-\frac{1}{a^2b^2(a+bx)^2}-\frac{1}{a^3b^2(a+bx)}\right)\,dx\right.}{n\sqrt{a^2+2abx^n+b^2x^{2n}}}$$
$$= \frac{1}{a^2n\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{1}{2an\left(a+bx^n\right)\sqrt{a^2+2abx^n+b^2x^{2n}}}+\frac{}{a^3\sqrt{}}$$

Mathematica [A]
time = 0.08, size = 79, normalized size = 0.50

$$\frac{a(3a+2bx^n)+2(a+bx^n)^2\log\left(x^n\right)-2(a+bx^n)^2\log\left(a+bx^n\right)}{2a^3n\left(a+bx^n\right)\sqrt{\left(a+bx^n\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] (a*(3*a + 2*b*x^n) + 2*(a + b*x^n)^2*Log[x^n] - 2*(a + b*x^n)^2*Log[a + b*x
^n])/(2*a^3*n*(a + b*x^n)*Sqrt[(a + b*x^n)^2])
```

Maple [A]
time = 0.02, size = 104, normalized size = 0.65

method	result	size
risch	$\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,\ln\left(x\right)}{\left(a+b\,x^{n}\right)a^{3}}+\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,\left(2b\,x^{n}+3a\right)}{2\left(a+b\,x^{n}\right)^{3}a^{2}n}-\frac{\sqrt{\left(a+b\,x^{n}\right)^{2}}\,\ln\left(x^{n}+\frac{a}{b}\right)}{\left(a+b\,x^{n}\right)a^{3}n}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((a+b*x^n)^2)^(1/2)/(a+b*x^n)*ln(x)/a^3+1/2*((a+b*x^n)^2)^(1/2)/(a+b*x^n)^3
*(2*b*x^n+3*a)/a^2/n-((a+b*x^n)^2)^(1/2)/(a+b*x^n)/a^3/n*ln(x^n+a/b)
```

Maxima [A]
time = 0.29, size = 70, normalized size = 0.44

$$\frac{2\,bx^n+3\,a}{2\,(a^2b^2nx^{2\,n}+2\,a^3bnx^n+a^4n)}+\frac{\log\left(x\right)}{a^3}-\frac{\log\left(\frac{bx^n+a}{b}\right)}{a^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n + 3*a)/(a^2*b^2*n*x^(2*n) + 2*a^3*b*n*x^n + a^4*n) + log(x)/a^
3 - log((b*x^n + a)/b)/(a^3*n)
```

Fricas [A]
time = 0.34, size = 106, normalized size = 0.67

$$\frac{2\,b^2nx^{2\,n}\log\left(x\right)+2\,a^2n\log\left(x\right)+3\,a^2+2\,(2\,abn\log\left(x\right)+ab)x^n-2\,(b^2x^{2\,n}+2\,abx^n+a^2)\log\left(bx^n+a\right)}{2\,(a^3b^2nx^{2\,n}+2\,a^4bnx^n+a^5n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*n*x^(2*n)*log(x) + 2*a^2*n*log(x) + 3*a^2 + 2*(2*a*b*n*log(x) +
a*b)*x^n - 2*(b^2*x^(2*n) + 2*a*b*x^n + a^2)*log(b*x^n + a))/(a^3*b^2*n*x^(
2*n) + 2*a^4*b*n*x^n + a^5*n)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\left(\left(a+bx^n\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**n)**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a^2+b^2x^{2n}+2abx^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
[Out] int(1/(x*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

3.541 $\int \frac{1}{x^2(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{(a+bx^n)\ {}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -(a+b*x^n)*hypergeom([3, -1/n],[(-1+n)/n],-b*x^n/a)/a^3/x/(a^2+2*a*b*x^n+b^
2*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 371}

$$-\frac{(a+bx^n)\ {}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -(((a + b*x^n)*Hypergeometric2F1[3, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(
a^3*x*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2(ab+b^2x^n)\right)\int \frac{1}{x^2(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= -\frac{(a+bx^n)\ {}_2F_1\left(3,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^3x\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.82

$$-\frac{(a+bx^n)^3\ {}_2F_1\left(3,-\frac{1}{n};1-\frac{1}{n};-\frac{bx^n}{a}\right)}{a^3x\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -(((a + b*x^n)^3*Hypergeometric2F1[3, -n^(-1), 1 - n^(-1), -((b*x^n)/a)])/(
a^3*x*((a + b*x^n)^2)^(3/2)))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a^2+2ab\,x^n+b^2x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (2*n^2 + 3*n + 1)*integrate(1/2/(a^2*b*n^2*x^2*x^n + a^3*n^2*x^2), x) + 1/2
*(b*(2*n + 1)*x^n + a*(3*n + 1))/(a^2*b^2*n^2*x*x^(2*n) + 2*a^3*b*n^2*x*x^n
 + a^4*n^2*x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^2*x^(4*n) + 4*a^2*b^2*x
^2*x^(2*n) + 4*a^3*b*x^2*x^n + a^4*x^2 + 2*(2*a*b^3*x^2*x^n + a^2*b^2*x^2)*
x^(2*n)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left((a+bx^n)^2 \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
[Out] Integral(1/(x**2*((a + b*x**n)**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 \left(a^2 + b^2\, x^{2\,n} + 2\,a\,b\,x^n\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
[Out] int(1/(x^2*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

3.542 $\int \frac{1}{x^3(a^2+2abx^n+b^2x^{2n})^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

```
[Out] -1/2*(a+b*x^n)*hypergeom([3, -2/n],[(-2+n)/n],-b*x^n/a)/a^3/x^2/(a^2+2*a*b*
x^n+b^2*x^(2*n))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1369, 371}

$$-\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -1/2*((a + b*x^n)*Hypergeometric2F1[3, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(
a^3*x^2*Sqrt[a^2 + 2*a*b*x^n + b^2*x^(2*n)])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1369

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a^2+2abx^n+b^2x^{2n}\right)^{3/2}}\,dx &= \frac{\left(b^2(ab+b^2x^n)\right)\int \frac{1}{x^3(ab+b^2x^n)^3}\,dx}{\sqrt{a^2+2abx^n+b^2x^{2n}}}\\ &= -\frac{(a+bx^n)\,{}_2F_1\left(3,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\sqrt{a^2+2abx^n+b^2x^{2n}}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.82

$$-\frac{(a+bx^n)^3\ {}_2F_1\left(3,-\frac{2}{n};1-\frac{2}{n};-\frac{bx^n}{a}\right)}{2a^3x^2\left((a+bx^n)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^(3/2)),x]
```

```
[Out] -1/2*((a + b*x^n)^3*Hypergeometric2F1[3, -2/n, 1 - 2/n, -((b*x^n)/a)])/(a^3
*x^2*((a + b*x^n)^2)^(3/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a^2+2ab\,x^n+b^2x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] (n^2 + 3*n + 2)*integrate(1/(a^2*b*n^2*x^3*x^n + a^3*n^2*x^3), x) + 1/2*(2*
b*(n + 1)*x^n + a*(3*n + 2))/(a^2*b^2*n^2*x^2*x^(2*n) + 2*a^3*b*n^2*x^2*x^n
 + a^4*n^2*x^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b^2*x^(2*n) + 2*a*b*x^n + a^2)/(b^4*x^3*x^(4*n) + 4*a^2*b^2*x
^3*x^(2*n) + 4*a^3*b*x^3*x^n + a^4*x^3 + 2*(2*a*b^3*x^3*x^n + a^2*b^2*x^3)*
x^(2*n)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left((a+bx^n)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**2+2*a*b*x**n+b**2*x**(2*n))**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**n)**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^2+2*a*b*x^n+b^2*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^(3/2)), x)
```

3.543 $\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p \, dx$

Optimal. Leaf size=52

$$\frac{x\left(a+bx^{\frac{1}{-1-2p}}\right)\left(a^2+2abx^{\frac{1}{-1-2p}}+b^2x^{-\frac{2}{1+2p}}\right)^p}{a}$$

```
[Out] x*(a+b*x^(1/(-1-2*p)))*(a^2+2*a*b*x^(1/(-1-2*p))+b^2/(x^(2/(1+2*p))))^p/a
```

Rubi [A]
time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1357, 197}

$$\frac{x\left(a+bx^{\frac{1}{-2p-1}}\right)\left(a^2+2abx^{\frac{1}{-2p-1}}+b^2x^{-\frac{2}{2p+1}}\right)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

```
[Out] (x*(a + b*x^(-1 - 2*p)^(-1))*(a^2 + 2*a*b*x^(-1 - 2*p)^(-1) + b^2/x^(2/(1 +
 2*p)))^p)/a
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p \, dx = \left(\left(a^2+b^2x^{-\frac{2}{1+2p}}+2abx^{-\frac{1}{1+2p}}\right)^p\left(2ab+2b^2x^{-\frac{1}{1+2p}}\right)^{-2p}\right)\int \left(2ab+2b\right.$$
$$= \frac{x\left(a+bx^{\frac{1}{-1-2p}}\right)\left(a^2+2abx^{\frac{1}{-1-2p}}+b^2x^{-\frac{2}{1+2p}}\right)^p}{a}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 1.12

$$\frac{x^{\frac{2p}{1+2p}}\left(b+ax^{\frac{1}{1+2p}}\right)\left(x^{-\frac{2}{1+2p}}\left(b+ax^{\frac{1}{1+2p}}\right)^2\right)^p}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(2/(1 + 2*p)) + (2*a*b)/x^(1 + 2*p)^(-1))^p,x]
```

```
[Out] (x^((2*p)/(1 + 2*p))*(b + a*x^(1 + 2*p)^(-1))*((b + a*x^(1 + 2*p)^(-1))^2/x
^(2/(1 + 2*p)))^p)/a
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(a^2+b^2x^{-\frac{2}{1+2p}}+2ab\,x^{-\frac{1}{1+2p}}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x)
```

```
[Out] int((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="m
axima")
```

```
[Out] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p, x)
```

Fricas [A]
time = 0.37, size = 79, normalized size = 1.52

$$\frac{\left(axx^{\left(\frac{1}{2\,p+1}\right)}+bx\right)\left(\frac{a^2x^{\frac{2}{2\,p+1}}+2\,abx^{\left(\frac{1}{2\,p+1}\right)}+b^2}{x^{\frac{2}{2\,p+1}}}\right)^p}{ax^{\left(\frac{1}{2\,p+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="f
ricas")
```

```
[Out] (a*x*x^(1/(2*p + 1)) + b*x)*((a^2*x^(2/(2*p + 1)) + 2*a*b*x^(1/(2*p + 1)) +
 b^2)/x^(2/(2*p + 1)))^p/(a*x^(1/(2*p + 1)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/(x**(2/(1+2*p)))+2*a*b/(x**(1/(1+2*p))))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(2/(1+2*p)))+2*a*b/(x^(1/(1+2*p))))^p,x, algorithm="g
iac")
```

```
[Out] integrate((a^2 + b^2/x^(2/(2*p + 1)) + 2*a*b/x^(1/(2*p + 1)))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a^2+\frac{b^2}{x^{\frac{2}{2\,p+1}}}+\frac{2\,a\,b}{x^{\frac{1}{2\,p+1}}}\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2/x^(2/(2*p + 1)) + (2*a*b)/x^(1/(2*p + 1)))^p,x)
```

```
[Out] int((a^2 + b^2/x^(2/(2*p + 1)) + (2*a*b)/x^(1/(2*p + 1)))^p, x)
```

3.544 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-n}{2n}}\,dx$

Optimal. Leaf size=43

$$\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}}{a}$$

```
[Out] x*(a+b*x^n)/a/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n))
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {1357, 197}

$$\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{n+1}{2n}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n))/(a*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((1 + n)/(2*n)))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}\,dx &= \left(\left(2ab+2b^2x^n\right)^{\frac{1+n}{n}}\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}\right)\int \left(2ab+2b^2x^n\right)^{-\frac{1+n}{n}}\,dx\\ &= \frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+n}{2n}}}{a}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 32, normalized size = 0.74

$$\frac{x(a+bx^n)\left((a+bx^n)^2\right)^{-\frac{1+n}{2n}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - n)/(2*n)),x]
[Out] (x*(a + b*x^n))/(a*((a + b*x^n)^2)^((1 + n)/(2*n)))
```

Maple [A]
time = 0.07, size = 51, normalized size = 1.19

method	result	size
norman	$\left(x+\frac{b x\,{\mathrm e}^{n \ln \left(x \right)}}{a}\right) {\mathrm e}^{\frac{\left(1+n \right) \ln \left(\frac{1}{\sqrt{a^{2}+2 a b\,{\mathrm e}^{n \ln \left(x \right)}+b^{2} {\mathrm e}^{2 n \ln \left(x \right)}}}\right)}{n}}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x,method=_RETURNVERBOSE)
[Out] (x+b/a*x*exp(n*ln(x)))/exp(1/2*(1+n)/n*ln(a^2+2*a*b*exp(n*ln(x))+b^2*exp(n*
ln(x))^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="maxim
a")
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

Fricas [A]
time = 0.44, size = 45, normalized size = 1.05

$$\frac{bxx^n + ax}{\left(b^2x^{2\,n} + 2\,abx^n + a^2\right)^{\frac{n+1}{2\,n}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="frica
s")
[Out] (b*x*x^n + a*x)/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)*a)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 + 2abx^n + b^2x^{2n}\right)^{-\frac{\frac{n}{2}+\frac{1}{2}}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+n)/n)),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**n + b**2*x**(2*n))**(-(n/2 + 1/2)/n), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+n)/n)),x, algorithm="giac"
)
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(n + 1)/n)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{\frac{\frac{n}{2}+\frac{1}{2}}{n}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n/2 + 1/2)/n),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n/2 + 1/2)/n), x)
```

3.545 $\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p\,dx$

Optimal. Leaf size=130

$$\frac{2(1+p)x\left(a+bx^{-\frac{1}{2(1+p)}}\right)\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a(1+2p)}-\frac{x\left(a+bx^{-\frac{1}{2(1+p)}}\right)^2\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a^2(1+2p)}$$

```
[Out] 2*(1+p)*x*(a+b/(x^(1/2/(1+p))))*(a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))
))^p/a/(1+2*p)-x*(a+b/(x^(1/2/(1+p))))^2*(a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1
/2/(1+p))))^p/a^2/(1+2*p)
```

Rubi [A]
time = 0.04, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {1357, 198, 197}

$$\frac{2(p+1)x\left(a+bx^{-\frac{1}{2(p+1)}}\right)\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a(2p+1)}-\frac{x\left(a+bx^{-\frac{1}{2(p+1)}}\right)^2\left(a^2+2abx^{-\frac{1}{2(p+1)}}+b^2x^{-\frac{1}{p+1}}\right)^p}{a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

```
[Out] (2*(1 + p)*x*(a + b/x^(1/(2*(1 + p))))*(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/
x^(1/(2*(1 + p))))^p)/(a*(1 + 2*p)) - (x*(a + b/x^(1/(2*(1 + p))))^2*(a^2 +
 b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p)/(a^2*(1 + 2*p))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p dx = \left(\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p \left(2ab+2b^2x^{-\frac{1}{2(1+p)}}\right)^{-2p}\right)\int \Big(2ab$$

$$= \frac{2(1+p)x\left(a+bx^{-\frac{1}{2(1+p)}}\right)\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a(1+2p)} - \frac{\Big(\Big(a^2}{}$$

$$= \frac{2(1+p)x\left(a+bx^{-\frac{1}{2(1+p)}}\right)\left(a^2+b^2x^{-\frac{1}{1+p}}+2abx^{-\frac{1}{2(1+p)}}\right)^p}{a(1+2p)} - \frac{x\Big(a+}{}$$

Mathematica [A]
time = 0.12, size = 80, normalized size = 0.62

$$\frac{x^{\frac{p}{1+p}}\left(b+ax^{\frac{1}{2+2p}}\right)\left(x^{-\frac{1}{1+p}}\left(b+ax^{\frac{1}{2+2p}}\right)^2\right)^p\left(-b+a(1+2p)x^{\frac{1}{2+2p}}\right)}{a^2(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2/x^(1 + p)^(-1) + (2*a*b)/x^(1/(2*(1 + p))))^p,x]
```

```
[Out] (x^(p/(1 + p))*(b + a*x^(2 + 2*p)^(-1))*((b + a*x^(2 + 2*p)^(-1))^2/x^(1 +
p)^(-1))^p*(-b + a*(1 + 2*p)*x^(2 + 2*p)^(-1)))/(a^2*(1 + 2*p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(a^2+b^2x^{-\frac{1}{1+p}}+2ab\,x^{-\frac{1}{2(1+p)}}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x)
```

```
[Out] int((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="max
ima")
```

[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)

Fricas [A]
time = 0.37, size = 103, normalized size = 0.79

$$\frac{\left(2\,abpxx^{\frac{1}{2\,(p+1)}}-b^2x+(2\,a^2p+a^2)xx^{\left(\frac{1}{p+1}\right)}\right)\left(\frac{2\,abx^{\frac{1}{2\,(p+1)}}+a^2x^{\left(\frac{1}{p+1}\right)}+b^2}{x^{\left(\frac{1}{p+1}\right)}}\right)^p}{(2\,a^2p+a^2)x^{\left(\frac{1}{p+1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="fri
cas")
```

```
[Out] (2*a*b*p*x*x^(1/2/(p + 1)) - b^2*x + (2*a^2*p + a^2)*x*x^(1/(p + 1)))*((2*a
*b*x^(1/2/(p + 1)) + a^2*x^(1/(p + 1)) + b^2)/x^(1/(p + 1)))^p/((2*a^2*p +
a^2)*x^(1/(p + 1)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2+b**2/(x**(1/(1+p)))+2*a*b/(x**(1/2/(1+p))))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2+b^2/(x^(1/(1+p)))+2*a*b/(x^(1/2/(1+p))))^p,x, algorithm="gia
c")
```

```
[Out] integrate((a^2 + 2*a*b/x^(1/2/(p + 1)) + b^2/x^(1/(p + 1)))^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{b^2}{x^{\frac{1}{p+1}}}+a^2+\frac{2\,a\,b}{x^{\frac{1}{2\,(p+1)}}}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2/x^(1/(p + 1)) + a^2 + (2*a*b)/x^(1/(2*(p + 1))))^p,x)
```

```
[Out] int((b^2/x^(1/(p + 1)) + a^2 + (2*a*b)/x^(1/(2*(p + 1))))^p, x)
```

3.546 $\int \left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{-1-2n}{2n}}\,dx$

Optimal. Leaf size=102

$$\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)}+\frac{nx(a+bx^n)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a^2(1+n)}$$

```
[Out] x*(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^(-1-1/2/n)/a/(1+n)+n*x*(a+b*x^n)^2*
(a^2+2*a*b*x^n+b^2*x^(2*n))^(-1-1/2/n)/a^2/(1+n)
```

Rubi [A]
time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1357, 198, 197}

$$\frac{nx(a+bx^n)^2\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a^2(n+1)}+\frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-\frac{1}{n}-2\right)}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)),x]
```

```
[Out] (x*(a + b*x^n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/(a*(1 + n
)) + (n*x*(a + b*x^n)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-2 - n^(-1))/2))/
(a^2*(1 + n))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1357

```
Int[((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_), x_Symbol] :> Dist[(
a + b*x^n + c*x^(2*n))^p/(b + 2*c*x^n)^(2*p), Int[(b + 2*c*x^n)^(2*p), x],
x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+2n}{2n}}\,dx = \left(\left(2ab+2b^2x^n\right)^{\frac{1+2n}{n}}\left(a^2+2abx^n+b^2x^{2n}\right)^{-\frac{1+2n}{2n}}\right)\int \left(2ab+2b^2x^n\right)^{-\frac{1+2n}{n}}\,d$$

$$= \frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)} + \frac{\left(n\left(2ab+2b^2x^n\right)^{\frac{1+2n}{n}}\left(a^2+2a\right.\right.}{}$$

$$= \frac{x(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^{\frac{1}{2}\left(-2-\frac{1}{n}\right)}}{a(1+n)} + \frac{nx(a+bx^n)^2\left(a^2+2abx^n+b\right.}{a^2(1+n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.05, size = 59, normalized size = 0.58

$$\frac{x\left((a+bx^n)^2\right)^{-\frac{1}{2}/n}\left(1+\frac{bx^n}{a}\right)^{\frac{1}{n}}\,{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + 2*a*b*x^n + b^2*x^(2*n))^((-1 - 2*n)/(2*n)),x]
```

```
[Out] (x*(1 + (b*x^n)/a)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1),
 -((b*x^n)/a)])/(a^2*((a + b*x^n)^2)^(1/(2*n)))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(a^2+2ab\,x^n+b^2x^{2n}\right)^{-\frac{1+2n}{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x)
```

```
[Out] int(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="max
ima")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)
```

Fricas [A]
time = 0.37, size = 82, normalized size = 0.80

$$\frac{b^2nxx^{2n} + (2\,abn + ab)xx^n + (a^2n + a^2)x}{(a^2n + a^2)(b^2x^{2n} + 2\,abx^n + a^2)^{\frac{2n+1}{2n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="fri
cas")
```

```
[Out] (b^2*n*x*x^(2*n) + (2*a*b*n + a*b)*x*x^n + (a^2*n + a^2)*x)/((a^2*n + a^2)*
(b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a**2+2*a*b*x**n+b**2*x**(2*n))**(1/2*(1+2*n)/n)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a^2+2*a*b*x^n+b^2*x^(2*n))^(1/2*(1+2*n)/n)),x, algorithm="gia
c")
```

```
[Out] integrate(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)^(1/2*(2*n + 1)/n)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^{\frac{n+\frac{1}{2}}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n + 1/2)/n),x)
```

```
[Out] int(1/(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^((n + 1/2)/n), x)
```

3.547 $\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx$

Optimal. Leaf size=117

$$-\frac{(dx)^{-2n(1+p)}\,(a+bx^n)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{adn(1+2p)}+\frac{(dx)^{-2n(1+p)}\left(a^2+2abx^n+b^2x^{2n}\right)^{1+p}}{2a^2dn(1+p)(1+2p)}$$

```
[Out] -(a+b*x^n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/a/d/n/(1+2*p)/((d*x)^(2*n*(1+p)))+
1/2*(a^2+2*a*b*x^n+b^2*x^(2*n))^(1+p)/a^2/d/n/(1+p)/(1+2*p)/((d*x)^(2*n*(1+
p)))
```

Rubi [A]
time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1370, 279, 270}

$$\frac{\left(\frac{bx^n}{a}+1\right)^2(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{2dn\left(2p^2+3p+1\right)}-\frac{\left(\frac{bx^n}{a}+1\right)(dx)^{-2n(p+1)}\left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -(((1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(d*n*(1 + 2*p)*(d*x)^
(2*n*(1 + p)))) + ((1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*
d*n*(1 + 3*p + 2*p^2)*(d*x)^(2*n*(1 + p)))
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-(
c*x)^(m + 1))*((a + b*x^n)^(p + 1)/(a*c*n*(p + 1))), x] + Dist[(m + n*(p +
1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a,
 b, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2abx^n+b^2x^{2n}\right)^p \, dx = \left(\left(1+\frac{bx^n}{a}\right)^{-2p} \left(a^2+2abx^n+b^2x^{2n}\right)^p\right) \int (dx)^{-1-2n(1+p)} \left(1\right.$$

$$= -\frac{(dx)^{-2n(1+p)} \left(1+\frac{bx^n}{a}\right) \left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(1+2p)} + \frac{\left(\left(-2n(1+\right.\right.}{}$$

$$= -\frac{(dx)^{-2n(1+p)} \left(1+\frac{bx^n}{a}\right) \left(a^2+2abx^n+b^2x^{2n}\right)^p}{dn(1+2p)} + \frac{(dx)^{-2n(1+p)}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.07, size = 75, normalized size = 0.64

$$-\frac{x(dx)^{-1-2n(1+p)} \left((a+bx^n)^2\right)^p \left(1+\frac{bx^n}{a}\right)^{-2p} {}_2F_1\left(-2p,-2(1+p);1-2(1+p);-\frac{bx^n}{a}\right)}{2n(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(-1 - 2*n*(1 + p))*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
[Out] -1/2*(x*(d*x)^(-1 - 2*n*(1 + p))*((a + b*x^n)^2)^p*Hypergeometric2F1[-2*p,
-2*(1 + p), 1 - 2*(1 + p), -((b*x^n)/a)])/(n*(1 + p)*(1 + (b*x^n)/a)^(2*p))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (dx)^{-1-2n(1+p)} \left(a^2+2ab\,x^n+b^2x^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
[Out] int((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
maxima")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

Fricas [A]
time = 0.41, size = 165, normalized size = 1.41

$$-\frac{\left(2\,abpxx^{n}e^{\left(-(2\,np+2\,n+1)\log(d)-(2\,np+2\,n+1)\log(x)\right)}-b^{2}xx^{2\,n}e^{\left(-(2\,np+2\,n+1)\log(d)-(2\,np+2\,n+1)\log(x)\right)}+(2\,a^{2}p+a^{2})xe^{\left(-(2\,np+2\,n+1)\log(d)-(2\,np+2\,n+1)\log(x)\right)}\right)(b^{2}x^{2\,n}+2\,abx^{n}+a^{2})^{p}}{2\,(2\,a^{2}np^{2}+3\,a^{2}np+a^{2}n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
fricas")
```

```
[Out] -1/2*(2*a*b*p*x*x^n*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x)
) - b^2*x*x^(2*n)*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x))
+ (2*a^2*p + a^2)*x*e^(-(2*n*p + 2*n + 1)*log(d) - (2*n*p + 2*n + 1)*log(x)
))*(b^2*x^(2*n) + 2*a*b*x^n + a^2)^p/(2*a^2*n*p^2 + 3*a^2*n*p + a^2*n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(-1-2*n*(1+p))*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(-1-2*n*(1+p))*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="
giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*(d*x)^(-2*n*(p + 1) - 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^p}{{\left(d\,x\right)}^{2\,n\,\left(p+1\right)+1}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1),x)
```

```
[Out] int((a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p/(d*x)^(2*n*(p + 1) + 1), x)
```

3.548 $\int x^{-1+2n}(a^2+2abx^n+b^2x^{2n})^p\,dx$

Optimal. Leaf size=103

$$-\frac{a^2\left(1+\frac{bx^n}{a}\right)(a^2+2abx^n+b^2x^{2n})^p}{b^2n(1+2p)}+\frac{a^2\left(1+\frac{bx^n}{a}\right)^2(a^2+2abx^n+b^2x^{2n})^p}{2b^2n(1+p)}$$

```
[Out] -a^2*(1+b*x^n/a)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/b^2/n/(1+2*p)+1/2*a^2*(1+b*x
^n/a)^2*(a^2+2*a*b*x^n+b^2*x^(2*n))^p/b^2/n/(1+p)
```

Rubi [A]
time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1370, 272, 45}

$$\frac{a^2\left(\frac{bx^n}{a}+1\right)^2(a^2+2abx^n+b^2x^{2n})^p}{2b^2n(p+1)}-\frac{a^2\left(\frac{bx^n}{a}+1\right)(a^2+2abx^n+b^2x^{2n})^p}{b^2n(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
```

```
[Out] -((a^2*(1 + (b*x^n)/a)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(b^2*n*(1 + 2*p))
) + (a^2*(1 + (b*x^n)/a)^2*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p)/(2*b^2*n*(1 +
 p))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1370

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/(1 + 2*
c*(x^n/b))^(2*FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/b))^(2*p), x], x] /;
 FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] &&
 !IntegerQ[2*p]
```

Rubi steps

$$\int x^{-1+2n}\left(a^2+2abx^n+b^2x^{2n}\right)^p\,dx=\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\int x^{-1+2n}\left(1+\frac{bx^n}{a}\right)^{2p}dx$$

$$=\frac{\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\text{Subst}\left(\int x\left(1+\frac{bx}{a}\right)^{2p}dx,x,x^n\right)}{n}$$

$$=\frac{\left(\left(1+\frac{bx^n}{a}\right)^{-2p}\left(a^2+2abx^n+b^2x^{2n}\right)^p\right)\text{Subst}\left(\int\left(-\frac{a\left(1+\frac{bx}{a}\right)^{2p}}{b}+\frac{a\left(1+\frac{bx}{a}\right.}{b}\right.\right.}{n}$$

$$=-\frac{a^2\left(1+\frac{bx^n}{a}\right)\left(a^2+2abx^n+b^2x^{2n}\right)^p}{b^2n(1+2p)}+\frac{a^2\left(1+\frac{bx^n}{a}\right)^2\left(a^2+2abx^n+b^2x\right.}{2b^2n(1+p)}$$

Mathematica [A]
time = 0.05, size = 54, normalized size = 0.52

$$\frac{(a+bx^n)\left((a+bx^n)^2\right)^p(-a+b(1+2p)x^n)}{2b^2n(1+p)(1+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)*(a^2 + 2*a*b*x^n + b^2*x^(2*n))^p,x]
[Out] ((a + b*x^n)*((a + b*x^n)^2)^p*(-a + b*(1 + 2*p)*x^n))/(2*b^2*n*(1 + p)*(1
+ 2*p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.08, size = 148, normalized size = 1.44

method	result
risch	$-\frac{\left(-2b^2p\,x^{2n}-2ap\,x^n b-b^2x^{2n}+a^2\right)e^{\frac{p\left(-i\pi\text{csgn}\left(i(a+b\,x^n)^2\right)^3+2i\pi\text{csgn}\left(i(a+b\,x^n)^2\right)^2\text{csgn}(i(a+b\,x^n))-i\pi\,\text{csgn}\left(i(a+b\,x^n)^2\right)\text{csgn}(i(a+b\,x^n))^2\right.}{2}}}{2(1+2p)(1+p)n\,b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x,method=_RETURNVERBOSE)
[Out] -1/2*(-2*b^2*p*(x^n)^2-2*a*p*x^n*b-b^2*(x^n)^2+a^2)/(1+2*p)/(1+p)/n/b^2*exp
(1/2*p*(-I*Pi*csgn(I*(a+b*x^n)^2)^3+2*I*Pi*csgn(I*(a+b*x^n)^2)^2*csgn(I*(a+
b*x^n))-I*Pi*csgn(I*(a+b*x^n)^2)*csgn(I*(a+b*x^n))^2+4*ln(a+b*x^n)))
```

Maxima [A]
time = 0.30, size = 59, normalized size = 0.57

$$\frac{(b^2(2\,p+1)x^{2\,n}+2\,abpx^n-a^2)(bx^n+a)^{2\,p}}{2\,(2\,p^2+3\,p+1)b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(2*p + 1)*x^(2*n) + 2*a*b*p*x^n - a^2)*(b*x^n + a)^(2*p)/((2*p^2 +
 3*p + 1)*b^2*n)
```

Fricas [**A**]
time = 0.35, size = 78, normalized size = 0.76

$$\frac{(2\,abpx^n - a^2 + (2\,b^2p + b^2)x^{2\,n})(b^2x^{2\,n} + 2\,abx^n + a^2)^p}{2\,(2\,b^2np^2 + 3\,b^2np + b^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*p*x^n - a^2 + (2*b^2*p + b^2)*x^(2*n))*(b^2*x^(2*n) + 2*a*b*x^n
+ a^2)^p/(2*b^2*n*p^2 + 3*b^2*n*p + b^2*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a**2+2*a*b*x**n+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a^2+2*a*b*x^n+b^2*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + 2*a*b*x^n + a^2)^p*x^(2*n - 1), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2\,n-1}\,{\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p,x)
```

```
[Out] int(x^(2*n - 1)*(a^2 + b^2*x^(2*n) + 2*a*b*x^n)^p, x)
```

3.549 $\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=111

$$-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,n}+\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}$$

```
[Out] -b*x^n/c^2/n+1/2*x^(2*n)/c/n+1/2*(-a*c+b^2)*ln(a+b*x^n+c*x^(2*n))/c^3/n+b*(
-3*a*c+b^2)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/c^3/n/(-4*a*c+b^2)^(1/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1371, 715, 648, 632, 212, 642}

$$\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3n\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}-\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((b*x^n)/(c^2*n)) + x^(2*n)/(2*c*n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^
n)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[a + b*x
^n + c*x^(2*n)])/(2*c^3*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+4n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx,x,x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+(b^2-ac)x}{c^2(a+bx+cx^2)}\right)\,dx,x,x^n\right)}{n}$$

$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{\text{Subst}\left(\int \frac{ab+(b^2-ac)x}{a+bx+cx^2}\,dx,x,x^n\right)}{c^2n}$$

$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}-\frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{2c^3n}+\frac{(b^2-ac)\,\text{Subst}\left(\int \frac{b+2}{a+bx+}\right.}{2c^3n}$$

$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}+\frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx\right.}{c^3n}$$

$$= -\frac{bx^n}{c^2n}+\frac{x^{2n}}{2cn}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,n}+\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2c^3n}$$

Mathematica [A]

time = 0.22, size = 97, normalized size = 0.87

$$\frac{cx^n(-2b+cx^n)-\frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log\left(a+x^n(b+cx^n)\right)}{2c^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 4*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (c*x^n*(-2*b + c*x^n) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x^n*(b + c*x^n)])/(2*c^3
*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(103) = 206$.

time = 0.11, size = 973, normalized size = 8.77

method	result
risch	$-\frac{\ln(x)a}{c^2}+\frac{\ln(x)b^2}{c^3}+\frac{x^{2n}}{2cn}-\frac{b\,x^n}{c^2n}+\frac{4n^2\ln(x)a^2c^2}{4a\,c^4n^2-b^2c^3n^2}-\frac{5n^2\ln(x)a\,b^2c}{4a\,c^4n^2-b^2c^3n^2}+\frac{n^2\ln(x)b^4}{4a\,c^4n^2-b^2c^3n^2}-\frac{2\ln\left(x^n+\frac{3a\,b^2c-b^4+\sqrt{-3}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*ln(x)*a+1/c^3*ln(x)*b^2+1/2/c/n*(x^n)^2-b*x^n/c^2/n+4/(4*a*c^4*n^2-b
^2*c^3*n^2)*n^2*ln(x)*a^2*c^2-5/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*a*b^2*c
+1/(4*a*c^4*n^2-b^2*c^3*n^2)*n^2*ln(x)*b^4-2/c/(4*a*c-b^2)/n*ln(x^n+1/2*(3*
a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a
*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c
^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*a*b^2-1/2/c^3/(4*
a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b
^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4+1/2/c^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a
*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*
c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)-2/c/(4*a*c-b^
2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+
b^8)^(1/2))/c/b/(3*a*c-b^2))*a^2+5/2/c^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2
*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^
2))*a*b^2-1/2/c^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3
+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^4-1/2/c^3/(4*a*c-
b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*
c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b
^8)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] (b^2 - a*c)*log(x)/c^3 + 1/2*(c*x^(2*n) - 2*b*x^n)/(c^2*n) + integrate(-(a*
b^2 - a^2*c + (b^3 - 2*a*b*c)*x^n)/(c^4*x*x^(2*n) + b*c^3*x*x^n + a*c^3*x),
 x)
```

Fricas [A]
time = 0.39, size = 353, normalized size = 3.18

$$\left[-\frac{(b^3-3\,abc)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc-\sqrt{b^2-4\,ac}\,c\right)x^n-\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right)-(b^2c^2-4\,ac^3)x^{2\,n}+2\,(b^3c-4\,abc^2)x^n-(b^4-5\,ab^2c+4\,a^2c^2)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,(b^2c^3-4\,ac^4)n}, \frac{2\,(b^3-3\,abc)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}\,cx^n+\sqrt{-b^2+4\,ac}\,b}{b^2-4\,ac}\right)+(b^2c^2-4\,ac^3)x^{2\,n}-2\,(b^3c-4\,abc^2)x^n+(b^4-5\,ab^2c+4\,a^2c^2)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,(b^2c^3-4\,ac^4)n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c +
 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^
n + a)) - (b^2*c^2 - 4*a*c^3)*x^(2*n) + 2*(b^3*c - 4*a*b*c^2)*x^n - (b^4 -
5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^3 - 4*a*c^4)*n),
 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*
x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^2*c^2 - 4*a*c^3)*x^(2*n) -
2*(b^3*c - 4*a*b*c^2)*x^n + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^(2*n) + b
*x^n + a))/((b^2*c^3 - 4*a*c^4)*n)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+4*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{4\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4*n - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^(4*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.550 $\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=87

$$\frac{x^n}{cn}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,n}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}$$

```
[Out] x^n/c/n-1/2*b*ln(a+b*x^n+c*x^(2*n))/c^2/n-(-2*a*c+b^2)*arctanh((b+2*c*x^n)/
(-4*a*c+b^2)^(1/2))/c^2/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1371, 717, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2n\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}+\frac{x^n}{cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x^n/(c*n) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c^2*S
qrt[b^2 - 4*a*c]*n) - (b*Log[a + b*x^n + c*x^(2*n)])/(2*c^2*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx,x,x^n\right)}{n}\\
&= \frac{x^n}{cn}+\frac{\operatorname{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx,x,x^n\right)}{cn}\\
&= \frac{x^n}{cn}-\frac{b\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{2c^2n}+\frac{(b^2-2ac)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{2c^2n}\\
&= \frac{x^n}{cn}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}-\frac{(b^2-2ac)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^n\right)}{c^2n}\\
&= \frac{x^n}{cn}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,n}-\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2c^2n}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 82, normalized size = 0.94

$$\frac{2cx^n+\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+x^n(b+cx^n)\right)}{2c^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*x^n + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + x^n*(b + c*x^n)])/(2*c^2*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(81) = 162$.
time = 0.09, size = 664, normalized size = 7.63

method	result
risch	$-\frac{b\ln(x)}{c^2}+\frac{x^n}{cn}+\frac{4n^2\ln(x)abc}{4a\,c^3n^2-b^2c^2n^2}-\frac{n^2\ln(x)b^3}{4a\,c^3n^2-b^2c^2n^2}-\frac{2\ln\left(x^n-\frac{-2abc+b^3+\sqrt{-16a^3c^3+20a^2b^2c^2-8a\,b^4c+b}}{2c\left(2ac-b^2\right)}\right.}{(4ac-b^2)cn}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -b/c^2*ln(x)+x^n/c/n+4/(4*a*c^3*n^2-b^2*c^2*n^2)*n^2*ln(x)*a*b*c-1/(4*a*c^3
*n^2-b^2*c^2*n^2)*n^2*ln(x)*b^3-2/(4*a*c-b^2)/c/n*ln(x^n-1/2*(-2*a*b*c+b^3+
(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*a*b+1/2/(4
*a*c-b^2)/c^2/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^
4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3+1/2/(4*a*c-b^2)/c^2/n*ln(x^n-1/2*(-2*a*b
*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-1
6*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)-2/(4*a*c-b^2)/c/n*ln(x^n+1/2*
(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2
))*a*b+1/2/(4*a*c-b^2)/c^2/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^
2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1/2/(4*a*c-b^2)/c^2/n*ln(x^n
+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*
c-b^2))*(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -b*log(x)/c^2 + x^n/(c*n) - integrate(-(a*b + (b^2 - a*c)*x^n)/(c^3*x*x^(2*
n) + b*c^2*x*x^n + a*c^2*x), x)
```

Fricas [A]
time = 0.39, size = 285, normalized size = 3.28

$$\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)x^n+\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right)-2\,(b^2c-4\,ac^2)x^n+(b^3-4\,abc)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,(b^2c^2-4\,ac^3)n},-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{2\sqrt{-b^2+4\,ac}\,cx^n+\sqrt{-b^2+4\,ac}\,b}{b^2-4\,ac}\right)-2\,(b^2c-4\,ac^2)x^n+(b^3-4\,abc)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,(b^2c^2-4\,ac^3)n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2
*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n
+ a)) - 2*(b^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a
))/((b^2*c^2 - 4*a*c^3)*n), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan
(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 2*(b
^2*c - 4*a*c^2)*x^n + (b^3 - 4*a*b*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c^2
 - 4*a*c^3)*n)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(3*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.551 $\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}} \, dx$

Optimal. Leaf size=68

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\, n}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

```
[Out] 1/2*ln(a+b*x^n+c*x^(2*n))/c/n+b*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/c/n
/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1371, 648, 632, 212, 642}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{cn\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*n) + Log[
a + b*x^n + c*x^(2*n)]/(2*c*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{2cn}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{2cn}\\ &= \frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}+\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^n\right)}{cn}\\ &= \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,n}+\frac{\log\left(a+bx^n+cx^{2n}\right)}{2cn}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 66, normalized size = 0.97

$$\frac{-\frac{2b\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+x^n(b+cx^n)\right)}{2cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + x^n*(b + c*x^n)])/(2*c*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(62) = 124$.
time = 0.07, size = 402, normalized size = 5.91

method	result
risch	$\frac{\ln(x)}{c}-\frac{4n^2\ln(x)ac}{4a\,c^2n^2-b^2c\,n^2}+\frac{n^2\ln(x)b^2}{4a\,c^2n^2-b^2c\,n^2}+\frac{2\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)a}{(4ac-b^2)n}-\frac{\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right.}{2c(4ac-b^2)n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*ln(x)-4/(4*a*c^2*n^2-b^2*c*n^2)*n^2*ln(x)*a*c+1/(4*a*c^2*n^2-b^2*c*n^2)
*n^2*ln(x)*b^2+2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c
)*a-1/2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2+1
/2/c/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*
c+b^4)^(1/2)+2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*a
-1/2/c/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/c
/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4
)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log(x)/c - integrate((b*x^n + a)/(c^2*x*x^(2*n) + b*c*x*x^n + a*c*x), x)
```

Fricas [A]
time = 0.36, size = 231, normalized size = 3.40

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc+\sqrt{b^2-4\,ac}\,c\right)x^n+\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right)+\left(b^2-4\,ac\right)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,\left(b^2c-4\,ac^2\right)n},\ \frac{2\,\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}\,cx^n+\sqrt{-b^2+4\,ac}\,b}{b^2-4\,ac}\right)+\left(b^2-4\,ac\right)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,\left(b^2c-4\,ac^2\right)n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(
b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + (b^2
- 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a*c^2)*n), 1/2*(2*sqrt(-b^
2 + 4*a*c)*b*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b
^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((b^2*c - 4*a*c^2)
*n)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(2*n - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.552 $\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=39

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,n}$$

```
[Out] -2*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1366, 632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{n\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{x^{-1+n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{n}$$
$$= -\frac{2\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^n\right)}{n}$$
$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,n}$$

Mathematica [A]
time = 0.06, size = 43, normalized size = 1.10

$$\frac{2\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(35) = 70.
time = 0.06, size = 113, normalized size = 2.90

method	result	size
risch	$-\frac{\ln\left(x^n+\frac{b^2-4ac+b\sqrt{-4ac+b^2}}{2c\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}\,n}+\frac{\ln\left(x^n+\frac{b\sqrt{-4ac+b^2}+4ac-b^2}{2c\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}\,n}$	113

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))/c/(-4*a
*c+b^2)^(1/2))+1/(-4*a*c+b^2)^(1/2)/n*ln(x^n+1/2*(b*(-4*a*c+b^2)^(1/2)+4*a*
c-b^2)/c/(-4*a*c+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate(x^(n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.
time = 0.35, size = 159, normalized size = 4.08

$$\left[\frac{\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc-\sqrt{b^2-4\,ac}\,c\right)x^n-\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right)}{\sqrt{b^2-4\,ac}\,n}, -\frac{2\,\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}\,cx^n+\sqrt{-b^2+4\,ac}\,b}{b^2-4\,ac}\right)}{(b^2-4\,ac)n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] [log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqr
t(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a))/(sqrt(b^2 - 4*a*c)*n), -2*sqrt(-
b^2 + 4*a*c)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b
^2 - 4*a*c))/((b^2 - 4*a*c)*n)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

Giac [A]
time = 4.31, size = 39, normalized size = 1.00

$$\frac{2\,\arctan\left(\frac{2\,cx^n+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] 2*arctan((2*c*x^n + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*n)
```

Mupad [B]
time = 1.47, size = 39, normalized size = 1.00

$$\frac{2\,\mathrm{atan}\left(\frac{b+2\,c\,x^n}{\sqrt{4\,a\,c-b^2}}\right)}{n\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] (2*atan((b + 2*c*x^n)/(4*a*c - b^2)^(1/2)))/(n*(4*a*c - b^2)^(1/2))
```

3.553 $\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=98

$$-\frac{x^{-n}}{an}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\,n}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}$$

```
[Out] -1/a/n/(x^n)-b*ln(x)/a^2+1/2*b*ln(a+b*x^n+c*x^(2*n))/a^2/n-(-2*a*c+b^2)*arc
tanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/a^2/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1371, 723, 814, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2n\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{b\log(x)}{a^2}-\frac{x^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(1/(a*n*x^n)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(
a^2*Sqrt[b^2 - 4*a*c]*n) - (b*Log[x])/a^2 + (b*Log[a + b*x^n + c*x^(2*n)])/
(2*a^2*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1-n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)}\,dx,x,x^n\right)}{n}$$
$$= -\frac{x^{-n}}{an}+\frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx,x,x^n\right)}{an}$$
$$= -\frac{x^{-n}}{an}+\frac{\text{Subst}\left(\int \left(-\frac{b}{ax}+\frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx,x,x^n\right)}{an}$$
$$= -\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx,x,x^n\right)}{a^2n}$$
$$= -\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{2a^2n}+\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,\right.}{2a^2n}$$
$$= -\frac{x^{-n}}{an}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+\right.}{a^2n}$$
$$= -\frac{x^{-n}}{an}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\,n}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^n+cx^{2n}\right)}{2a^2n}$$

Mathematica [A]
time = 0.20, size = 90, normalized size = 0.92

$$\frac{-2ax^{-n}+\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2b\log\left(x^n\right)+b\log\left(a+x^n(b+cx^n)\right)}{2a^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((-2*a)/x^n + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sq
rt[-b^2 + 4*a*c] - 2*b*Log[x^n] + b*Log[a + x^n*(b + c*x^n)])/(2*a^2*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 657 vs. 2(92) = 184.
time = 0.14, size = 658, normalized size = 6.71

method	result
risch	$-\frac{x^{-n}}{an}-\frac{4n^2\ln(x)abc}{4a^3cn^2-a^2b^2n^2}+\frac{n^2\ln(x)b^3}{4a^3cn^2-a^2b^2n^2}+\frac{2\ln\left(x^n-\frac{-2abc+b^3+\sqrt{-16a^3c^3+20a^2b^2c^2-8a\,b^4c+b^6}}{2c\left(2ac-b^2\right)}\right)bc}{a(4ac-b^2)n}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/n/(x^n)-4/(4*a^3*c*n^2-a^2*b^2*n^2)*n^2*ln(x)*a*b*c+1/(4*a^3*c*n^2-a^2
*b^2*n^2)*n^2*ln(x)*b^3+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3
*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b*c-1/2/a^2/(4*a*c
-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)
^(1/2))/c/(2*a*c-b^2))*b^3+1/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a*b*c+b^3+(
-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*(-16*a^3*c^
3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c
-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b*c-1
/2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*
a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*b^3-1/2/a^2/(4*a*c-b^2)/n*ln(x^n+1/2*(2*
a*b*c-b^3+(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2))/c/(2*a*c-b^2))*
(-16*a^3*c^3+20*a^2*b^2*c^2-8*a*b^4*c+b^6)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -1/(a*n*x^n) - integrate((c*x^n + b)/(a*c*x*x^(2*n) + a*b*x*x^n + a^2*x), x
)
```

Fricas [A]
time = 0.38, size = 333, normalized size = 3.40

$$\left[-\frac{2\,(b^3-4\,abc)nx^n\log(x)+(b^2-2\,ac)\sqrt{b^2-4\,ac}\,x^n\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)x^n+\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x^n\log\left(cx^{2\,n}+bx^n+a\right)}{2\,(a^2b^2-4\,a^3c)nx^n},\ -\frac{2\,(b^3-4\,abc)nx^n\log(x)+2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\,x^n\arctan\left(-\frac{2\sqrt{-b^2+4\,ac}\,cx^n+\sqrt{-b^2+4\,ac}\,b}{b^2-4\,ac}\right)+2\,ab^2-8\,a^2c-(b^3-4\,abc)x^n\log\left(cx^{2\,n}+bx^n+a\right)}{2\,(a^2b^2-4\,a^3c)nx^n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^3 - 4*a*b*c)*n*x^n*log(x) + (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^n
*log((2*c^2*x^(2*n) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqr
t(b^2 - 4*a*c)*b)/(c*x^(2*n) + b*x^n + a)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4*a
*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a^2*b^2 - 4*a^3*c)*n*x^n), -1/2*(2*
(b^3 - 4*a*b*c)*n*x^n*log(x) + 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^n*arcta
n(-(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + 2*a
*b^2 - 8*a^2*c - (b^3 - 4*a*b*c)*x^n*log(c*x^(2*n) + b*x^n + a))/((a^2*b^2
- 4*a^3*c)*n*x^n)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.554 $\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=126

$$-\frac{x^{-2n}}{2an}+\frac{bx^{-n}}{a^2n}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\,n}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}$$

```
[Out] -1/2/a/n/(x^(2*n))+b/a^2/n/(x^n)+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(a+b
*x^n+c*x^(2*n))/a^3/n+b*(-3*a*c+b^2)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2)
)/a^3/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1371, 723, 814, 648, 632, 212, 642}

$$\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3n\sqrt{b^2-4ac}}-\frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{bx^{-n}}{a^2n}-\frac{x^{-2n}}{2an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/2*1/(a*n*x^(2*n)) + b/(a^2*n*x^n) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^
n)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*n) + ((b^2 - a*c)*Log[x])/a^3
 - ((b^2 - a*c)*Log[a + b*x^n + c*x^(2*n)])/(2*a^3*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1-2n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx+cx^2)}\,dx, x, x^n\right)}{n}$$
$$= -\frac{x^{-2n}}{2an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x^2(a+bx+cx^2)}\,dx, x, x^n\right)}{an}$$
$$= -\frac{x^{-2n}}{2an} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax^2} + \frac{b^2-ac}{a^2x} + \frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)}\right)dx, x, x^n\right)}{an}$$
$$= -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{(b^2-ac)\log(x)}{a^3} + \frac{\text{Subst}\left(\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2}\,dx, x, x^n\right)}{a^3n}$$
$$= -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^n\right)}{2a^3n} - \frac{(b^2}{}$$
$$= -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^3n} + \frac{(b(b^2-3ac))\,\text{S}}{}$$
$$= -\frac{x^{-2n}}{2an} + \frac{bx^{-n}}{a^2n} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\,n} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)}{}$$

Mathematica [A]
time = 0.34, size = 115, normalized size = 0.91

$$\frac{ax^{-2n}(-a+2bx^n) - \frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + 2(b^2-ac)\log(x^n) - (b^2-ac)\log(a+x^n(b+cx^n))}{2a^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 2*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((a*(-a + 2*b*x^n))/x^(2*n) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^n)/Sqrt[
-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x^n] - (b^2 - a*c)*L
og[a + x^n*(b + c*x^n)])/(2*a^3*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(120) = 240.
time = 0.12, size = 958, normalized size = 7.60

method	result
risch	$\frac{b\,x^{-n}}{a^2n} - \frac{x^{-2n}}{2an} - \frac{4n^2\ln(x)a^2c^2}{4a^4cn^2-a^3b^2n^2} + \frac{5n^2\ln(x)a\,b^2c}{4a^4cn^2-a^3b^2n^2} - \frac{n^2\ln(x)b^4}{4a^4cn^2-a^3b^2n^2} + \frac{2\ln\left(x^n+\frac{3a\,b^2c-b^4+\sqrt{-36a^3b^2c^3+33a^2b}}{2cb\left(3ac-b^2\right)}\right.}{a(4ac-b^2)n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] b/a^2/n/(x^n)-1/2/a/n/(x^n)^2-4/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a^2*c^2
+5/(4*a^4*c*n^2-a^3*b^2*n^2)*n^2*ln(x)*a*b^2*c-1/(4*a^4*c*n^2-a^3*b^2*n^2)*
n^2*ln(x)*b^4+2/a/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+
33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*c^2-5/2/a^2/(4*a*c-b
^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+
b^8)^(1/2))/c/b/(3*a*c-b^2))*b^2*c+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^
2*c-b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b
^2))*b^4+1/2/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(3*a*b^2*c-b^4+(-36*a^3*b^2*c^3+3
3*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*(-36*a^3*b^2*c^3+33*a
^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)+2/a/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b
^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*
c^2-5/2/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^
2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2))*b^2*c+1/2/a^3/(4*a*c-b^2)
/n*ln(x^n-1/2*(-3*a*b^2*c+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^
8)^(1/2))/c/b/(3*a*c-b^2))*b^4-1/2/a^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-3*a*b^2*c
+b^4+(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2))/c/b/(3*a*c-b^2)
)*(-36*a^3*b^2*c^3+33*a^2*b^4*c^2-10*a*b^6*c+b^8)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x^n - a)/(a^2*n*x^(2*n)) + integrate((b*c*x^n + b^2 - a*c)/(a^2*c*
x*x^(2*n) + a^2*b*x*x^n + a^3*x), x)
```

Fricas [A]
time = 0.39, size = 429, normalized size = 3.40

$$\left[-\frac{a^{2} b^{2} - 4 a^{3} c - 2 (b^{4} - 5 a b^{2} c + 4 a^{2} c^{2}) n x^{2n} \log(x) + (b^{3} - 3 abc)\sqrt{b^{2} - 4ac}\, x^{2n} \log\left(\frac{2c^{2}x^{2n} + b^{2} - 2ac + 2\left(bc - \sqrt{b^{2} - 4ac}\, c\right)x^{n} - \sqrt{b^{2} - 4ac}\, b}{cx^{2n} + bx^{n} + a}\right) + (b^{4} - 5ab^{2}c + 4a^{2}c^{2})x^{2n}\log(cx^{2n} + bx^{n} + a) - 2(ab^{3} - 4a^{2}bc)x^{n}}{2(a^{3}b^{2} - 4a^{4}c)nx^{2n}}, -\frac{a^{2}b^{2} - 4a^{3}c - 2(b^{4} - 5ab^{2}c + 4a^{2}c^{2})nx^{2n}\log(x) - 2(b^{3} - 3abc)\sqrt{-b^{2} + 4ac}\, x^{2n}\arctan\left(-\frac{2\sqrt{-b^{2} + 4ac}\, cx^{n} + \sqrt{-b^{2} + 4ac}\, b}{b^{2} - 4ac}\right) + (b^{4} - 5ab^{2}c + 4a^{2}c^{2})x^{2n}\log(cx^{2n} + bx^{n} + a) - 2(ab^{3} - 4a^{2}bc)x^{n}}{2(a^{3}b^{2} - 4a^{4}c)nx^{2n}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x)
 + (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^(2*n)*log((2*c^2*x^(2*n) + b^2 - 2*a
*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a*c)*b)/(c*x^(2*n) +
```

```
b*x^n + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*n)*log(c*x^(2*n) + b*x^n +
 a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b^2 - 4*a^4*c)*n*x^(2*n)), -1/2*(a^2
*b^2 - 4*a^3*c - 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*n*x^(2*n)*log(x) - 2*(b^3
- 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^(2*n)*arctan(-(2*sqrt(-b^2 + 4*a*c)*c*x^n +
 sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^(2*
n)*log(c*x^(2*n) + b*x^n + a) - 2*(a*b^3 - 4*a^2*b*c)*x^n)/((a^3*b^2 - 4*a^
4*c)*n*x^(2*n))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-2*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(-2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2\,n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(2*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.555 $\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=164

$$-\frac{x^{-3n}}{3an}+\frac{bx^{-2n}}{2a^2n}-\frac{(b^2-ac)\,x^{-n}}{a^3n}-\frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}\,n}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2a}{}$$

```
[Out] -1/3/a/n/(x^(3*n))+1/2*b/a^2/n/(x^(2*n))+(a*c-b^2)/a^3/n/(x^n)-b*(-2*a*c+b^
2)*ln(x)/a^4+1/2*b*(-2*a*c+b^2)*ln(a+b*x^n+c*x^(2*n))/a^4/n-(2*a^2*c^2-4*a*
b^2*c+b^4)*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1/2))/a^4/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1371, 723, 814, 648, 632, 212, 642}

$$\frac{b(b^2-2ac)\log\left(a+bx^n+cx^{2n}\right)}{2a^4n}-\frac{b\log(x)\left(b^2-2ac\right)}{a^4}-\frac{x^{-n}\left(b^2-ac\right)}{a^3n}+\frac{bx^{-2n}}{2a^2n}-\frac{\left(2a^2c^2-4ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4n\sqrt{b^2-4ac}}-\frac{x^{-3n}}{3an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/3*1/(a*n*x^(3*n)) + b/(2*a^2*n*x^(2*n)) - (b^2 - a*c)/(a^3*n*x^n) - ((b^
4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a^4*S
qrt[b^2 - 4*a*c]*n) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a
 + b*x^n + c*x^(2*n)])/(2*a^4*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1-3n}}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx+cx^2)}\,dx, x, x^n\right)}{n}$$

$$= -\frac{x^{-3n}}{3an} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x^3(a+bx+cx^2)}\,dx, x, x^n\right)}{an}$$

$$= -\frac{x^{-3n}}{3an} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax^3} + \frac{b^2-ac}{a^2x^2} + \frac{-b^3+2abc}{a^3x} + \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a^3(a+bx+cx^2)}\right)\,dx, x, x^n\right)}{an}$$

$$= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)\,x^{-n}}{a^3n} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{\text{Subst}\left(\int \frac{b^4-3ab^2c+a^2c^2+bc(b^2}{a+bx+cx^2}\right.}{a^4n}$$

$$= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)\,x^{-n}}{a^3n} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{(b(b^2-2ac))\,\text{Subst}\left(\int \frac{b+}{a+bx}\right.}{2a^4n}$$

$$= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)\,x^{-n}}{a^3n} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{b(b^2-2ac)\log\left(a+bx^n+\right.}{2a^4n}$$

$$= -\frac{x^{-3n}}{3an} + \frac{bx^{-2n}}{2a^2n} - \frac{(b^2-ac)\,x^{-n}}{a^3n} - \frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}\,n} - $$

Mathematica [A]
time = 0.29, size = 147, normalized size = 0.90

$$\frac{ax^{-3n}(-2a^2-6b^2x^{2n}+3ax^n(b+2cx^n)) + \frac{6(b^4-4ab^2c+2a^2c^2)\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 6(b^3-2abc)\log(x^n) + 3(b^3-2abc)\log(a+x^n(b+cx^n))}{6a^4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((a*(-2*a^2 - 6*b^2*x^(2*n) + 3*a*x^n*(b + 2*c*x^n)))/x^(3*n) + (6*(b^4 - 4
*a*b^2*c + 2*a^2*c^2)*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
 4*a*c] - 6*(b^3 - 2*a*b*c)*Log[x^n] + 3*(b^3 - 2*a*b*c)*Log[a + x^n*(b + c
*x^n)])/(6*a^4*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. 2(158) = 316.
time = 0.14, size = 1300, normalized size = 7.93

method	result
risch	$\frac{x^{-n}c}{a^2n} - \frac{x^{-n}b^2}{a^3n} + \frac{b\,x^{-2n}}{2a^2n} - \frac{x^{-3n}}{3an} + \frac{8n^2\ln(x)a^2b\,c^2}{4a^5c\,n^2-a^4b^2n^2} - \frac{6n^2\ln(x)a\,b^3c}{4a^5c\,n^2-a^4b^2n^2} + \frac{n^2\ln(x)b^5}{4a^5c\,n^2-a^4b^2n^2} - \frac{4\ln\left(x^n+\frac{2a^2b\,c^2-4a\,b^3c+}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/n/(x^n)*c-1/a^3/n/(x^n)*b^2+1/2*b/a^2/n/(x^n)^2-1/3/a/n/(x^n)^3+8/(4*
a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*a^2*b*c^2-6/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*
ln(x)*a*b^3*c+1/(4*a^5*c*n^2-a^4*b^2*n^2)*n^2*ln(x)*b^5-4/a^2/(4*a*c-b^2)/n
*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b
^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*
b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^
5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2
*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-b^2)/n*ln(x^n+1/2*(2*a^2*b*c^
2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-1
2*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^5+1/2/a^4/(4*a*c-b^2)
/n*ln(x^n+1/2*(2*a^2*b*c^2-4*a*b^3*c+b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3
*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4)
)*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10
)^(1/2)-4/a^2/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5
*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c
/(2*a^2*c^2-4*a*b^2*c+b^4))*b*c^2+3/a^3/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*
c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2
-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*b^3*c-1/2/a^4/(4*a*c-
b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-16*a^5*c^5+68*a^4*b^2*c^4-9
6*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2))/c/(2*a^2*c^2-4*a*b^2*c
+b^4))*b^5-1/2/a^4/(4*a*c-b^2)/n*ln(x^n-1/2*(-2*a^2*b*c^2+4*a*b^3*c-b^5+(-1
6*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/
2))/c/(2*a^2*c^2-4*a*b^2*c+b^4))*(-16*a^5*c^5+68*a^4*b^2*c^4-96*a^3*b^4*c^3
+52*a^2*b^6*c^2-12*a*b^8*c+b^10)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a*b*x^n - 2*a^2 - 6*(b^2 - a*c)*x^(2*n))/(a^3*n*x^(3*n)) + integrate
(-(b^3 - 2*a*b*c + (b^2*c - a*c^2)*x^n)/(a^3*c*x*x^(2*n) + a^3*b*x*x^n + a^
4*x), x)
```

Fricas [A]
time = 0.36, size = 522, normalized size = 3.18

$$\left[-\frac{2a^3b^2 - 8a^4c + 6(b^5 - 6ab^3c + 8a^2bc^2)nx^{3n}\log(x) - 3(b^4 - 4ab^2c + 2a^2c^2)\sqrt{b^2 - 4ac}\,x^{3n}\log\left(\frac{2c^2x^{2n} + b^2 - 2ac + 2\left(bc - \sqrt{b^2 - 4ac}\,c\right)x^n - \sqrt{b^2 - 4ac}\,b}{cx^{2n} + bx^n + a}\right) - 3(b^5 - 6ab^3c + 8a^2bc^2)x^{3n}\log(cx^{2n} + bx^n + a) + 6(ab^4 - 5a^2b^2c + 4a^3c^2)x^{2n} - 3(a^2b^3 - 4a^3bc)x^n}{6(a^4b^2 - 4a^5c)nx^{3n}}, -\frac{2a^3b^2 - 8a^4c + 6(b^5 - 6ab^3c + 8a^2bc^2)nx^{3n}\log(x) + 6(b^4 - 4ab^2c + 2a^2c^2)\sqrt{-b^2 + 4ac}\,x^{3n}\arctan\left(-\frac{2\sqrt{-b^2 + 4ac}\,cx^n + \sqrt{-b^2 + 4ac}\,b}{b^2 - 4ac}\right) - 3(b^5 - 6ab^3c + 8a^2bc^2)x^{3n}\log(cx^{2n} + bx^n + a) + 6(ab^4 - 5a^2b^2c + 4a^3c^2)x^{2n} - 3(a^2b^3 - 4a^3bc)x^n}{6(a^4b^2 - 4a^5c)nx^{3n}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*a^3*b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*lo
g(x) - 3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^(3*n)*log((2*c^2
*x^(2*n) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*x^n - sqrt(b^2 - 4*a
*c)*b)/(c*x^(2*n) + b*x^n + a)) - 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^(3*n)
*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^(2*n) -
 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4*a^5*c)*n*x^(3*n)), -1/6*(2*a^3*
b^2 - 8*a^4*c + 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n*x^(3*n)*log(x) + 6*(b^4
 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^(3*n)*arctan(-(2*sqrt(-b^2 +
 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - 3*(b^5 - 6*a*b^3*c +
 8*a^2*b*c^2)*x^(3*n)*log(c*x^(2*n) + b*x^n + a) + 6*(a*b^4 - 5*a^2*b^2*c +
 4*a^3*c^2)*x^(2*n) - 3*(a^2*b^3 - 4*a^3*b*c)*x^n)/((a^4*b^2 - 4*a^5*c)*n*x
^(3*n))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3\,n+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(3*n + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.556 $\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=353

$$\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n}-\frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n}+\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\cdots\right.}{\sqrt{b^2-4ac}\,\left(-b-\cdots\right.}$$

```
[Out] 2*2^(3/4)*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x^(1/4*n)/(-b-(-4*a*c+b^2)^(1/2))^
(1/4))/n/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)+2*2^(3/4)*c^(3/4)
*arctanh(2^(1/4)*c^(1/4)*x^(1/4*n)/(-b-(-4*a*c+b^2)^(1/2))^(1/4))/n/(-b-(-4
*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)-2*2^(3/4)*c^(3/4)*arctan(2^(1/4)*
c^(1/4)*x^(1/4*n)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/n/(-4*a*c+b^2)^(1/2)/(-b+(
-4*a*c+b^2)^(1/2))^(3/4)-2*2^(3/4)*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x^(1/4*n
)/(-b+(-4*a*c+b^2)^(1/2))^(1/4))/n/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^2)^(1/2
))^(3/4)
```

Rubi [A]
time = 0.35, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1395, 1361, 218, 214, 211}

$$\frac{2\,2^{3/4}c^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\,\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\,\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\,\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2\,2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{n\sqrt{b^2-4ac}\,\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)
*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) + (2*2^(3/4)*c^(3/4)*A
rcTanh[(2^(1/4)*c^(1/4)*x^(n/4))/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2
 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2*2^(3/4)*c^(3/4)*ArcTanh[(2
^(1/4)*c^(1/4)*x^(n/4))/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]
*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1395

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[2
*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = \frac{4\text{Subst}\left(\int \frac{1}{a+bx^4+cx^8}\,dx, x, x^{n/4}\right)}{n}$$

$$= \frac{(4c)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\,n} - \frac{(4c)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,d\right.}{\sqrt{b^2-4ac}\,n}$$

$$= \frac{(4c)\text{Subst}\left(\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx, x, x^{n/4}\right)}{\sqrt{b^2-4ac}\sqrt{-b-\sqrt{b^2-4ac}}\,n} + \frac{(4c)\text{Subst}\left(\int \frac{}{\sqrt{-b-}}\right.}{\sqrt{b^2-4}}$$

$$= \frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2\,2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x^{n/4}}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}n}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 62, normalized size = 0.18

$$\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{-n\log(x)+4\log\left(x^{n/4}-\#1\right)}{b\#1^3+2c\#1^7}\&\right]}{4n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (-(n*Log[x]) + 4*Log[x^(n/4) - #1])/(b*#1^3
 + 2*c*#1^7) & ]/(4*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.49, size = 280, normalized size = 0.79

method	result
risch	$\sum_{\_R=\text{RootOf}\left(\left(256a^7c^4n^8-256a^6b^2c^3n^8+96a^5b^4c^2n^8-16a^4b^6c\,n^8+a^3b^8n^8\right)\_Z^8+\left(-48a^3b\,c^3n^4+40a^2b^3c^2n^4-11a\,b^5c\,n^4+b^7n^4\right)\_Z^4\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] sum(_R*ln(x^(1/4*n)+(16/(a*c^2-b^2*c)*n^5*b*a^5*c^2-8/(a*c^2-b^2*c)*n^5*b^3
*a^4*c+1/(a*c^2-b^2*c)*n^5*b^5*a^3)*_R^5+(2/(a*c^2-b^2*c)*n*a^2*c^2-4/(a*c^
2-b^2*c)*n*b^2*a*c+1/(a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((256*a^7*c^4*n^8-25
6*a^6*b^2*c^3*n^8+96*a^5*b^4*c^2*n^8-16*a^4*b^6*c*n^8+a^3*b^8*n^8)*_Z^8+(-4
8*a^3*b*c^3*n^4+40*a^2*b^3*c^2*n^4-11*a*b^5*c*n^4+b^7*n^4)*_Z^4+c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4426 vs. 2(273) = 546.
time = 0.63, size = 4426, normalized size = 12.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] -2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*arctan(1/16*sqrt(2)*(2*sqrt(2)*((a^3*b^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5 - 128*a^8*c^6)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5)*n^3*x)*x^(1/4*n - 1)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)) + sqrt(2)*((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3)*n^3*x)*sqrt((4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2*x^(1/2*n - 2) - sqrt(2)*((a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*c^4)*n^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - (b^8 - 8*a*b^6*c + 21*a^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4)*n^2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))/x^2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))/(b^4*c^3 - 2*a*b^2*c^4 + a^2*c^5)) + 2*sqrt(2)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*arctan(1/8*(2*((a^3*b^10*c - 15*a^4*b^8*c^2 + 86*a^5*b^6*c^3 - 232*a^6*b^4*c^4 + 288*a^7*b^2*c^5 - 128*a^8*c^6)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^9*c - 10*a*b^7*c^2 + 33*a^2*b^5*c^3 - 40*a^3*b^3*c^4 + 16*a^4*b*c^5)*n^3*x)*x^(1/4*n - 1)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)) + ((a^3*b^8 - 14*a^4*b^6*c + 72*a^5*b^4*c^2 - 160*a^6*b^2*c^3 + 128*a^7*c^4)*n^7*x*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*

```
a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^7 - 9*a*b^5*c + 24*a^2*
b^3*c^2 - 16*a^3*b*c^3)*n^3*x)*sqrt(sqrt(2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*
c + 16*a^5*c^2)*n^4)))*sqrt((4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x^2*x^(1/2
*n - 2) + sqrt(2)*((a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c
^3 + 64*a^7*b*c^4)*n^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + (b^8 - 8*a*b^6*c + 21*a^2*b^4*
c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4)*n^2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2)*n^4)))/x^2)*sqrt(((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt(
(b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*n^8)) - b^3 + 3*a*b*c)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))
)/(b^4*c^3 - 2*a*b^2*c^4 + a^2*c^5)) + 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((a^3
*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6
*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/(
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4)))*log(-(4*(b^2*c - a*c^2)*x*x^(1/
4*n - 1) + sqrt(2)*((a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*n^5*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)*n^8)) - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(sqrt(2)*sqrt(-((a^3*b^4 - 8
*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)*n^8)) + b^3 - 3*a*b*c)/((a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4))))/x) - 1/2*sqrt(2)*sqrt(sqrt(2)*sqrt(-((
a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*n^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((
a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/4*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4377 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{\frac{n}{4}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/4 - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/4 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.557 $\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=610

$$-\frac{2^{2/3}\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}+\frac{2^{2/3}\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}+\frac{2^{2/3}c^{2/3}\log\left(\sqrt[]{\ldots}\right.}{\sqrt{b^2-\ldots}}$$

```
[Out] 2^(2/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x^(1/3*n)+(b-(-4*a*c+b^2)^(1/2))^(1/3))/
n/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^2)^(1/2)-1/2*c^(2/3)*ln(2^(2/3)*c^
(2/3)*x^(2/3*n)-2^(1/3)*c^(1/3)*x^(1/3*n)*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(
-4*a*c+b^2)^(1/2))^(2/3))*2^(2/3)/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a*c+b^
2)^(1/2)-2^(2/3)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x^(1/3*n)/(b-(-4*a
*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)/(-4*a
*c+b^2)^(1/2)-2^(2/3)*c^(2/3)*ln(2^(1/3)*c^(1/3)*x^(1/3*n)+(b+(-4*a*c+b^2)^
(1/2))^(1/3))/n/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/2*c^(2/3)
*ln(2^(2/3)*c^(2/3)*x^(2/3*n)-2^(1/3)*c^(1/3)*x^(1/3*n)*(b+(-4*a*c+b^2)^(1/
2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*2^(2/3)/n/(-4*a*c+b^2)^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(2/3)+2^(2/3)*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x^(1
/3*n)/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*3^(1/2)/n/(-4*a*c+b^2)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A]
time = 0.71, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1395, 1361, 206, 31, 648, 631, 210, 642}

$$-\frac{2^{2/3}\sqrt{3}\,c^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{2^{2/3}\sqrt{3}\,c^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\right)}{n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}\,n\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x^{n/3}\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^{2n/3}\right)}{\sqrt[3]{2}\,n\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3))/(b - Sqr
t[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*n)) + (2^(2/3)*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x^(n/3
))/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^
2 - 4*a*c])^(2/3)*n) + (2^(2/3)*c^(2/3)*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) +
 2^(1/3)*c^(1/3)*x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])^(2/3)
*n) - (2^(2/3)*c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*
x^(n/3)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n) - (c^(2/3)*Lo
```

```
g[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(
1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b
- Sqrt[b^2 - 4*a*c])^(2/3)*n) + (c^(2/3)*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3)
- 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3) + 2^(2/3)*c^(2/3)*x
^((2*n)/3)])/(2^(1/3)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])^(2/3)*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
```

```
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1395

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[2
*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = \frac{3\mathrm{Subst}\left(\int \frac{1}{a+bx^3+cx^6}\,dx, x, x^{n/3}\right)}{n}$$

$$= \frac{(3c)\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\,n} - \frac{(3c)\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\, d\right.}{\sqrt{b^2-4ac}\,n}$$

$$= \frac{(2^{2/3}c)\,\mathrm{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx, x, x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{(2^{2/3}c)\,\mathrm{Subst}\left(\int \frac{}{\left(b-\sqrt{b\right.}}\right.}{\sqrt{}}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\right.}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-\right.}}$$

$$= \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x^{n/3}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \frac{2^{2/3}c^{2/3}\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}\right.}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-\right.}}$$

$$= -\frac{2^{2/3}\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x^{n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \frac{2^{2/3}\sqrt{3}\,c^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}}{\sqrt[3]{b+\sqrt{b^2}}}}{\sqrt{3}}\right.}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.12, size = 62, normalized size = 0.10

$$\frac{\mathrm{RootSum}\left[a+b\#1^3+c\#1^6\&, \frac{-n\log(x)+3\log\left(x^{n/3}-\#1\right)}{b\#1^2+2c\#1^5}\&\right]}{3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (-(n*Log[x]) + 3*Log[x^(n/3) - #1])/(b*#1^2
 + 2*c*#1^5) & ]/(3*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.31, size = 260, normalized size = 0.43

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3n^6-48a^4b^2c^2n^6+12a^3b^4c\,n^6-a^2b^6n^6\right)\_Z^6+\left(16a^2b\,c^2n^3-8a\,b^3c\,n^3+b^5n^3\right)\_Z^3+c^2\right)} \_R\ln\left(x^{\frac{n}{3}}+\left(-\frac{16n}{2c^2}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] sum(_R*ln(x^(1/3*n)+(-16/(2*a*c^2-b^2*c)*n^4*b*a^4*c^2+8/(2*a*c^2-b^2*c)*n^
4*b^3*a^3*c-1/(2*a*c^2-b^2*c)*n^4*b^5*a^2)*_R^4+(4/(2*a*c^2-b^2*c)*n*a^2*c^
2-5/(2*a*c^2-b^2*c)*n*b^2*a*c+1/(2*a*c^2-b^2*c)*n*b^4)*_R),_R=RootOf((64*a^
5*c^3*n^6-48*a^4*b^2*c^2*n^6+12*a^3*b^4*c*n^6-a^2*b^6*n^6)*_Z^6+(16*a^2*b*c
^2*n^3-8*a*b^3*c*n^3+b^5*n^3)*_Z^3+c^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4699 vs. 2(465) = 930.
time = 0.61, size = 4699, normalized size = 7.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*(1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a
^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/
((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b
^8*c - 14*a^3*b^6*c^2 + 72*a^4*b^4*c^3 - 160*a^5*b^2*c^4 + 128*a^6*c^5)*n^5
*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2 - 64*a^7*c^3)*n^6)) - sqrt(3)*(b^7*c - 8*a*b^5*c^2 + 20*a^2*b^3*c^3 -
```

16*a^3*b*c^4)*n^2*x)*x^(1/3*n - 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3) + sqrt(2)*(1/2)^(2/3)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*n^5*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n^2*x)*sqrt((2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) - (1/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3) - (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3))/x^2)*(((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3) + 2*sqrt(3)*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4))/(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)) - 2*sqrt(3)*(1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^2*b^8*c - 14*a^3*b^6*c^2 + 72*a^4*b^4*c^3 - 160*a^5*b^2*c^4 + 128*a^6*c^5)*n^5*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + sqrt(3)*(b^7*c - 8*a*b^5*c^2 + 20*a^2*b^3*c^3 - 16*a^3*b*c^4)*n^2*x)*x^(1/3*n - 1)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2/3) + sqrt(2)*(1/2)^(2/3)*(sqrt(3)*(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*n^5*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + sqrt(3)*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*n^2*x)*sqrt((2*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*x^2*x^(2/3*n - 2) + (1/2)^(1/3)*((a^2*b^7*c - 10*a^3*b^5*c^2 + 32*a^4*b^3*c^3 - 32*a^5*b*c^4)*n^4*x*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*n*x)*x^(1/3*n - 1)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3) + (1/2)^(2/3)*((a^2*b^9 - 14*a^3*b^7*c + 72*a^4*b^5*c^2 - 160*a^5*b^3*c^3 + 128*a^6*b*c^4)*n^5*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + (b^8 - 10*a*b^6*c + 36*a^2*b^4*c^2 - 56*a^3*b^2*c^3 + 32*a^4*c^4)*n^2)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 - 4*a^3*c)*n^3))^(2

```
/3))/x^2)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a
^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((a^2*b^2 -
 4*a^3*c)*n^3))^(2/3) - 2*sqrt(3)*(b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4))/(b^4
*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)) + (1/2)^(1/3)*(((a^2*b^2 - 4*a^3*c)*n^3*sq
rt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2
- 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(2*(b^2*c -
2*a*c^2)*x*x^(1/3*n - 1) + (1/2)^(1/3)*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c
^2)*n^4*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^
6*b^2*c^2 - 64*a^7*c^3)*n^6)) - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*n)*(((a^2*b^2
 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c
 + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) + b)/((a^2*b^2 - 4*a^3*c)*n^3))^(1/3)
)/x) + (1/2)^(1/3)*(-((a^2*b^2 - 4*a^3*c)*n^3*sqrt((b^4 - 4*a*b^2*c + 4*a^2
*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a^6*b^2*c^2 - 64*a^7*c^3)*n^6)) - b)/((
a^2*b^2 - 4*a^3*c)*n^3))^(1/3)*log(-(2*(b^2*c - 2*a*c^2)*x*x^(1/3*n - 1) -
(1/2)^(1/3)*((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*n^4*sqrt((b^4 - 4*a*b^2
*c + 4*a^2*c^2)/((a^4*b^6 - 12*a^5*b^4*c + 48*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/3*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{\frac{n}{3}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/3 - 1)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(x^(n/3 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.558 $\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=169

$$\frac{2\sqrt{2}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,n}-\frac{2\sqrt{2}\,\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}\,n}$$

```
[Out] 2*arctan(x^(1/2*n)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^
(1/2)/n/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2*arctan(x^(1/2*n)*
2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/n/(-4*a*c+b^2
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1395, 1107, 211}

$$\frac{2\sqrt{2}\,\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{n\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{2\sqrt{2}\,\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x^{n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{n\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*n) - (2*Sqrt[2]*Sqrt[
c]*ArcTan[(Sqrt[2]*Sqrt[c]*x^(n/2))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*n)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1395

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[2
*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\int \frac{x^{-1+\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx = \frac{2\mathrm{Subst}\left(\int \frac{1}{a+bx^2+cx^4}\,dx, x, x^{n/2}\right)}{n}$$

$$= \frac{(2c)\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^{n/2}\right)}{\sqrt{b^2-4ac}\,n} - \frac{(2c)\mathrm{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx$$

$$= \frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^{n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,n} - \frac{2\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^{n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,n}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.11, size = 60, normalized size = 0.36

$$\frac{\mathrm{RootSum}\left[a+b\#1^2+c\#1^4\&, \frac{-n\log(x)+2\log\left(x^{n/2}-\#1\right)}{b\#1+2c\#1^3}\&\right]}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] RootSum[a + b*#1^2 + c*#1^4 & , (-(n*Log[x]) + 2*Log[x^(n/2) - #1])/(b*#1 +
 2*c*#1^3) & ]/(2*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 114, normalized size = 0.67

method	result
risch	$\sum_{\_R=\mathrm{RootOf}\left((16a^3c^2n^4-8a^2b^2cn^4+ab^4n^4)\_Z^4+(-4abcn^2+b^3n^2)\_Z^2+c\right)} \_R\ln\left(x^{\frac{n}{2}}+\left(4n^3b\,a^2-\frac{n^3b^3a}{c}\right)\_R^3+\left(\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] sum(_R*ln(x^(1/2*n)+(4*n^3*b*a^2-1/c*n^3*b^3*a)*_R^3+(2*a*n-1/c*n*b^2)*_R),
_R=RootOf((16*a^3*c^2*n^4-8*a^2*b^2*c*n^4+a*b^4*n^4)*_Z^4+(-4*a*b*c*n^2+b^3
*n^2)*_Z^2+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^(1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 801 vs. 2(129) = 258.
time = 0.36, size = 801, normalized size = 4.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4))
+ b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) + sqrt(2)*((a*b^3 -
4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 - 4*a*c)*n)*sqrt(-(
(a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^
2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2
- 4*a^3*c)*n^4)) + b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x^(1/2*n - 1) - s
qrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - (b^2 -
4*a*c)*n)*sqrt(-((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) +
b)/((a*b^2 - 4*a^2*c)*n^2)))/x) - 1/2*sqrt(2)*sqrt(((a*b^2 - 4*a^2*c)*n^2*s
qrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2))*log((4*c*x*x
^(1/2*n - 1) + sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2*b^2 - 4*a^3*c)
*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*
a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x) + 1/2*sqrt(2)*sqrt(((a*b^2 -
 4*a^2*c)*n^2*sqrt(1/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2
))*log((4*c*x*x^(1/2*n - 1) - sqrt(2)*((a*b^3 - 4*a^2*b*c)*n^3*sqrt(1/((a^2
*b^2 - 4*a^3*c)*n^4)) + (b^2 - 4*a*c)*n)*sqrt(((a*b^2 - 4*a^2*c)*n^2*sqrt(1
/((a^2*b^2 - 4*a^3*c)*n^4)) - b)/((a*b^2 - 4*a^2*c)*n^2)))/x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1035 vs. 2(129) = 258.
time = 6.50, size = 1035, normalized size = 6.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] 1/2*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)
*b*c^2)*arctan(2*sqrt(1/2)*sqrt(x^n)/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b
^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2
*c^3)*abs(c)) + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2
 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b
^2 - 4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*sqrt(x^n)/sqrt((b - sqrt(b^2 - 4*a*c)
)/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*
c^2 - 4*a^2*c^3)*abs(c)))/n
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{n}{2}-1}}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^(n/2 - 1)/(a + b*x^n + c*x^(2*n)), x)
```

3.559 $\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=205

$$-\frac{2x^{-n/2}}{an}+\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,n}+\frac{\sqrt{2}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,n}$$

```
[Out] -2/a/n/(x^(1/2*n))+arctan(2^(1/2)*a^(1/2)/(x^(1/2*n))/(b-(-4*a*c+b^2)^(1/2)
)^(1/2))*2^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^(3/2)/n/(b-(-4*a*c+b^
2)^(1/2))^(1/2)+arctan(2^(1/2)*a^(1/2)/(x^(1/2*n))/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*2^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^(3/2)/n/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1395, 1354, 1136, 1180, 211}

$$\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}n\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a^{3/2}n\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2x^{-n/2}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -2/(a*n*x^(n/2)) + (Sqrt[2]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[a])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]*n) + (Sqrt[2]*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[a])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*x^(n/2))])/(a^(3/2)*Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*n)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
```

```
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1395

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[2
*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1-\frac{n}{2}}}{a+bx^n+cx^{2n}}\,dx &= -\frac{2\text{Subst}\left(\int \frac{1}{a+\frac{c}{x^4}+\frac{b}{x^2}}\,dx,x,x^{-n/2}\right)}{n}\\
&= -\frac{2\text{Subst}\left(\int \frac{x^4}{c+bx^2+ax^4}\,dx,x,x^{-n/2}\right)}{n}\\
&= -\frac{2x^{-n/2}}{an}+\frac{2\text{Subst}\left(\int \frac{c+bx^2}{c+bx^2+ax^4}\,dx,x,x^{-n/2}\right)}{an}\\
&= -\frac{2x^{-n/2}}{an}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^2}\,dx,x,x^{-n/2}\right)}{an}+\frac{\left(b+\right.}{}\\
&= -\frac{2x^{-n/2}}{an}+\frac{\sqrt{2}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,x^{-n/2}}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,n}+\frac{\sqrt{2}\left(b+\frac{b^2}{\sqrt{b^2}}\right.}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 105, normalized size = 0.51

$$-\frac{4x^{-n/2}-\text{RootSum}\left[c+b\#1^2+a\#1^4\&,\frac{cn\log(x)+2c\log\left(x^{-n/2}-\#1\right)+bn\log(x)\#1^2+2b\log\left(x^{-n/2}-\#1\right)\#1^2}{b\#1+2a\#1^3}\&\right]}{2an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/2)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/2*(4/x^(n/2) - RootSum[c + b*#1^2 + a*#1^4 & , (c*n*Log[x] + 2*c*Log[x^(
-1/2*n) - #1] + b*n*Log[x]*#1^2 + 2*b*Log[x^(-1/2*n) - #1]*#1^2)/(b*#1 + 2*
a*#1^3) & ])/(a*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 268, normalized size = 1.31

method	result
risch	$-\frac{2x^{-\frac{n}{2}}}{an}+\left(\sum_{\_R=\text{RootOf}\left(\left(16a^5c^2n^4-8a^4b^2c\,n^4+a^3b^4n^4\right)\_Z^4+\left(12a^2b\,c^2n^2-7a\,b^3c\,n^2+b^5n^2\right)\_Z^2+c^3\right)}\_R\ln\left(x^{\frac{n}{2}}+\left(-\frac{8}{a\,c}\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/a/n/(x^(1/2*n))+sum(_R*ln(x^(1/2*n)+(-8/(a*c^3-b^2*c^2)*n^3*a^5*c^2+6/(a
*c^3-b^2*c^2)*n^3*b^2*a^4*c-1/(a*c^3-b^2*c^2)*n^3*b^4*a^3)*_R^3+(-5/(a*c^3-
b^2*c^2)*n*b*a^2*c^2+5/(a*c^3-b^2*c^2)*n*b^3*a*c-1/(a*c^3-b^2*c^2)*n*b^5)*_
R),_R=RootOf((16*a^5*c^2*n^4-8*a^4*b^2*c*n^4+a^3*b^4*n^4)*_Z^4+(12*a^2*b*c^
2*n^2-7*a*b^3*c*n^2+b^5*n^2)*_Z^2+c^3))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -2/(a*n*x^(1/2*n)) - integrate((c*x^(3/2*n) + b*x^(1/2*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. 2(169) = 338.
time = 0.41, size = 1229, normalized size = 6.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)
)*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*((a^3*b^3 - 4*a^4*b*c)
*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - (b^4 - 5
*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*a^4
*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*b^2 - 4*
a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) - sqrt(2)*((a^3*b^3 -
 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4))
 - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(-((a^3*b^2 - 4*a^4*c)*n^2*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) + b^3 - 3*a*b*c)/((a^3*
b^2 - 4*a^4*c)*n^2)))/x) - sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n^2*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*c)/((a^
3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) + sqrt(2)*(
(a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^
7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(((a^3*b^2 - 4*a^4*c)*n^2
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*b*
c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) + sqrt(2)*a*n*sqrt(((a^3*b^2 - 4*a^4*c)*n
^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3 + 3*a*
b*c)/((a^3*b^2 - 4*a^4*c)*n^2))*log(-(4*(b^2*c - a*c^2)*x*x^(-1/2*n - 1) -
sqrt(2)*((a^3*b^3 - 4*a^4*b*c)*n^3*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b
^2 - 4*a^7*c)*n^4)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*n)*sqrt(((a^3*b^2 - 4*a
^4*c)*n^2*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*n^4)) - b^3
 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*n^2)))/x) - 4*x*x^(-1/2*n - 1))/(a*n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(-1/2*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{2}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/2 + 1)*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(n/2 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.560 $\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=699

$$-\frac{3x^{-n/3}}{an}-\frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n}-\frac{\sqrt{3}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,a^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}n}$$

```
[Out] -3/a/n/(x^(1/3*n))+1/2*ln(2^(1/3)*a^(1/3)/(x^(1/3*n))+(b-(-4*a*c+b^2)^(1/2)
)^(1/3))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b-(-4*a*c+b^
2)^(1/2))^(2/3)-1/4*ln(2^(2/3)*a^(2/3)/(x^(2/3*n))-2^(1/3)*a^(1/3)*(b-(-4*a
*c+b^2)^(1/2))^(1/3)/(x^(1/3*n))+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/2*a
rctan(1/3*(1-2*2^(1/3)*a^(1/3)/(x^(1/3*n))/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^
(1/2))*3^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b-(-4*
a*c+b^2)^(1/2))^(2/3)+1/2*ln(2^(1/3)*a^(1/3)/(x^(1/3*n))+(b+(-4*a*c+b^2)^(1
/2))^(1/3))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b+(-4*a*
c+b^2)^(1/2))^(2/3)-1/4*ln(2^(2/3)*a^(2/3)/(x^(2/3*n))-2^(1/3)*a^(1/3)*(b+(
-4*a*c+b^2)^(1/2))^(1/3)/(x^(1/3*n))+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(b+(-2*a
*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(b+(-4*a*c+b^2)^(1/2))^(2/3)-
1/2*arctan(1/3*(1-2*2^(1/3)*a^(1/3)/(x^(1/3*n))/(b+(-4*a*c+b^2)^(1/2))^(1/3
))*3^(1/2))*3^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a^(4/3)/n/(
b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A]
time = 0.93, antiderivative size = 699, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1395, 1354, 1381, 1436, 206, 31, 648, 631, 210, 642}

$$-\frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\sqrt{3}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}\right)}{2\sqrt[3]{2}\,a^{4/3}n\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\log\left(2^{2/3}a^{2/3}x^{-2n/3}-\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}\right)}{2\sqrt[3]{2}\,a^{4/3}n\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{3x^{-n/3}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -3/(a*n*x^(n/3)) - (Sqrt[3]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1
 - (2*2^(1/3)*a^(1/3))/((b - Sqrt[b^2 - 4*a*c])^(1/3)*x^(n/3)))/Sqrt[3]])/(
2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)*n) - (Sqrt[3]*(b + (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*a^(1/3))/((b + Sqrt[b^2 - 4*a
*c])^(1/3)*x^(n/3)))/Sqrt[3]])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/
3)*n) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^
```

```
(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c]
)^(2/3)*n) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a
*c])^(1/3) + (2^(1/3)*a^(1/3))/x^(n/3)])/(2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4
*a*c])^(2/3)*n) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2
- 4*a*c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b - Sqr
t[b^2 - 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b - Sqrt[b^2 - 4*a*c])
^(2/3)*n) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*
c])^(2/3) + (2^(2/3)*a^(2/3))/x^((2*n)/3) - (2^(1/3)*a^(1/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3))/x^(n/3)])/(2*2^(1/3)*a^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)
*n)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1395

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[2
*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{3}}}{a+bx^n+cx^{2n}}\,dx = -\frac{3\text{Subst}\left(\int \frac{1}{a+\frac{c}{x^6}+\frac{b}{x^3}}\,dx, x, x^{-n/3}\right)}{n}$$

$$= -\frac{3\text{Subst}\left(\int \frac{x^6}{c+bx^3+ax^6}\,dx, x, x^{-n/3}\right)}{n}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{3\text{Subst}\left(\int \frac{c+bx^3}{c+bx^3+ax^6}\,dx, x, x^{-n/3}\right)}{an}$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(3\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^3}\,dx, x, x^{-n/3}\right)}{2an} + \cdots$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{a}\,x}\,dx, x, x^{-n/3}\right)}{\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \cdots$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \cdots$$

$$= -\frac{3x^{-n/3}}{an} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}\right)}{\sqrt[3]{2}\,a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} + \cdots$$

$$= -\frac{3x^{-n/3}}{an} - \frac{\sqrt{3}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{a}\,x^{-n/3}}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,a^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}n} - \cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.14, size = 107, normalized size = 0.15

$$-\frac{9x^{-n/3}-\text{RootSum}\left[c+b\#1^3+a\#1^6\&,\frac{cn\log(x)+3c\log\left(x^{-n/3}-\#1\right)+bn\log(x)\#1^3+3b\log\left(x^{-n/3}-\#1\right)\#1^3}{b\#1^2+2a\#1^5}\&\right]}{3an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/3)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -1/3*(9/x^(n/3) - RootSum[c + b*#1^3 + a*#1^6 & , (c*n*Log[x] + 3*c*Log[x^(
-1/3*n) - #1] + b*n*Log[x]*#1^3 + 3*b*Log[x^(-1/3*n) - #1]*#1^3)/(b*#1^2 +
2*a*#1^5) & ])/(a*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.50, size = 534, normalized size = 0.76

method	result
risch	$-\frac{3x^{-\frac{n}{3}}}{an}+\left(\sum_{\_R=\text{RootOf}\left(\left(64a^7c^3n^6-48a^6b^2c^2n^6+12a^5b^4c\,n^6-a^4b^6n^6\right)\_Z^6+\left(-32a^3b\,c^3n^3+32a^2b^3c^2n^3-10a\,b^5c\,n^3+b^7n^3\right)\_Z\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/a/n/(x^(1/3*n))+sum(_R*ln(x^(1/3*n)+(-64/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)
*n^5*a^8*c^4+112/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^2*a^7*c^3-60/(2*a^2*
c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^4*a^6*c^2+13/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)
*n^5*b^6*a^5*c-1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^5*b^8*a^4)*_R^5+(28/(2*a
^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b*a^4*c^4-63/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3
)*n^2*b^3*a^3*c^3+42/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^5*a^2*c^2-11/(2*
a^2*c^5-4*a*b^2*c^4+b^4*c^3)*n^2*b^7*a*c+1/(2*a^2*c^5-4*a*b^2*c^4+b^4*c^3)*
n^2*b^9)*_R^2),_R=RootOf((64*a^7*c^3*n^6-48*a^6*b^2*c^2*n^6+12*a^5*b^4*c*n^
6-a^4*b^6*n^6)*_Z^6+(-32*a^3*b*c^3*n^3+32*a^2*b^3*c^2*n^3-10*a*b^5*c*n^3+b^
7*n^3)*_Z^3+c^4))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -3/(a*n*x^(1/3*n)) - integrate((c*x^(5/3*n) + b*x^(2/3*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6279 vs. $2(567) = 1134$.
time = 1.74, size = 6279, normalized size = 8.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(3)*(1/2)^(1/3)*a*n*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^
6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c
 + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*
c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^4*b^12*c - 17*a^5*b^1
0*c^2 + 114*a^6*b^8*c^3 - 378*a^7*b^6*c^4 + 632*a^8*b^4*c^5 - 480*a^9*b^2*c
^6 + 128*a^10*c^7)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^
2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3
)*n^6)) - sqrt(3)*(b^13*c - 15*a*b^11*c^2 + 88*a^2*b^9*c^3 - 252*a^3*b^7*c^
4 + 356*a^4*b^5*c^5 - 220*a^5*b^3*c^6 + 48*a^6*b*c^7)*n^2*x)*x^(-1/3*n - 1)
*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*
b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c
^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(2/3) - sqrt(2)*(1/2)
^(2/3)*(sqrt(3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*c^2 - 112*a^7*b^2*c^3
+ 64*a^8*c^4)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3
 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6
)) - sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*a^3*b^3*c^3 + 24*a^4*b
*c^4)*n^2*x)*sqrt((2*(b^8*c^2 - 8*a*b^6*c^3 + 20*a^2*b^4*c^4 - 16*a^3*b^2*c
^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) - (1/2)^(1/3)*((a^4*b^9*c - 12*a^5*b^7*c
^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c^5)*n^4*x*sqrt((b^8 - 8*a*
b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4
*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - (b^10*c - 12*a*b^8*c^2 + 52*a^2
*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a^5*c^6)*n*x)*x^(-1/3*n - 1
)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3
*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*
c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3) - (1/2)^(2/3)*
((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*a^7*b^5*c^3 + 352*a^8*b^3*
c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^
2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3
)*n^6)) - (b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 - 200*a^3*b^6*c^3 + 260*a^4*
b^4*c^4 - 152*a^5*b^2*c^5 + 32*a^6*c^6)*n^2)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt
((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6
- 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^
4*b^2 - 4*a^5*c)*n^3))^(2/3))/x^2)*(((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*
a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b
^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + b^3 - 2*a*b*c)/((a^4*b^2 - 4*
a^5*c)*n^3))^(2/3) + 2*sqrt(3)*(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - 16
```

```
*a^3*b^2*c^7 + 4*a^4*c^8))/(b^8*c^4 - 8*a*b^6*c^5 + 20*a^2*b^4*c^6 - 16*a^3
*b^2*c^7 + 4*a^4*c^8)) - 4*sqrt(3)*(1/2)^(1/3)*a*n*(-((a^4*b^2 - 4*a^5*c)*n
^3*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a
^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*
c)/((a^4*b^2 - 4*a^5*c)*n^3))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*(a^
4*b^12*c - 17*a^5*b^10*c^2 + 114*a^6*b^8*c^3 - 378*a^7*b^6*c^4 + 632*a^8*b^
4*c^5 - 480*a^9*b^2*c^6 + 128*a^10*c^7)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^
2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*
b^2*c^2 - 64*a^11*c^3)*n^6)) + sqrt(3)*(b^13*c - 15*a*b^11*c^2 + 88*a^2*b^9
*c^3 - 252*a^3*b^7*c^4 + 356*a^4*b^5*c^5 - 220*a^5*b^3*c^6 + 48*a^6*b*c^7)*
n^2*x)*x^(-1/3*n - 1)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c + 20
*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^
10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3))
^(2/3) - sqrt(2)*(1/2)^(2/3)*(sqrt(3)*(a^4*b^8 - 13*a^5*b^6*c + 60*a^6*b^4*
c^2 - 112*a^7*b^2*c^3 + 64*a^8*c^4)*n^5*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^
4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*
c^2 - 64*a^11*c^3)*n^6)) + sqrt(3)*(b^9 - 11*a*b^7*c + 42*a^2*b^5*c^2 - 62*
a^3*b^3*c^3 + 24*a^4*b*c^4)*n^2*x)*sqrt((2*(b^8*c^2 - 8*a*b^6*c^3 + 20*a^2*
b^4*c^4 - 16*a^3*b^2*c^5 + 4*a^4*c^6)*x^2*x^(-2/3*n - 2) + (1/2)^(1/3)*((a^
4*b^9*c - 12*a^5*b^7*c^2 + 50*a^6*b^5*c^3 - 80*a^7*b^3*c^4 + 32*a^8*b*c^5)*
n^4*x*sqrt((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/
((a^8*b^6 - 12*a^9*b^4*c + 48*a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^10*c -
 12*a*b^8*c^2 + 52*a^2*b^6*c^3 - 96*a^3*b^4*c^4 + 68*a^4*b^2*c^5 - 16*a^5*c
^6)*n*x)*x^(-1/3*n - 1)*(-((a^4*b^2 - 4*a^5*c)*n^3*sqrt((b^8 - 8*a*b^6*c +
20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*
a^10*b^2*c^2 - 64*a^11*c^3)*n^6)) - b^3 + 2*a*b*c)/((a^4*b^2 - 4*a^5*c)*n^3
))^(1/3) + (1/2)^(2/3)*((a^4*b^11 - 16*a^5*b^9*c + 98*a^6*b^7*c^2 - 280*a^7
*b^5*c^3 + 352*a^8*b^3*c^4 - 128*a^9*b*c^5)*n^5*sqrt((b^8 - 8*a*b^6*c + 20*
a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 4*a^4*c^4)/((a^8*b^6 - 12*a^9*b^4*c + 48*a^1
0*b^2*c^2 - 64*a^11*c^3)*n^6)) + (b^12 - 14*a*b^10*c + 76*a^2*b^8*c^2 - 200
*a^3*b^6*c^3 + 260*a^4*b^4*c^4 - 152*a^5*b^2*c^...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/3*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/3*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(x^(-1/3*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{3}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^(n/3 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.561 $\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=414

$$-\frac{4x^{-n/4}}{an}-\frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\cdots}{\sqrt[4]{-b+\cdots}}\right)}{a^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}\cdots}$$

```
[Out] -4/a/n/(x^(1/4*n))-2^(3/4)*arctan(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b-(-4*a*c+b
^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b-(-4*a*c
+b^2)^(1/2))^(3/4)-2^(3/4)*arctanh(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b-(-4*a*c+
b^2)^(1/2))^(1/4))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b-(-4*a*
c+b^2)^(1/2))^(3/4)-2^(3/4)*arctan(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b+(-4*a*c+
b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b+(-4*a*c
+b^2)^(1/2))^(3/4)-2^(3/4)*arctanh(2^(1/4)*a^(1/4)/(x^(1/4*n))/(-b+(-4*a*c+
b^2)^(1/2))^(1/4))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^(5/4)/n/(-b+(-4*a*c
+b^2)^(1/2))^(3/4)
```

Rubi [A]
time = 0.52, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1395, 1354, 1381, 1436, 218, 214, 211}

$$-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{a^{5/4}n\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{4x^{-n/4}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -4/(a*n*x^(n/4)) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2
^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/4)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])
*ArcTan[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4))])/(a^(5/
4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2
 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*x^(n/4
))])/(a^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)*n) - (2^(3/4)*(b - (b^2 - 2*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*a^(1/4))/((-b + Sqrt[b^2 - 4*a*c])^(
1/4)*x^(n/4))])/(a^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)*n)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1354

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^
(2*n*p)*(c + b/x^n + a/x^(2*n))^p, x] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n
] && LtQ[n, 0] && IntegerQ[p]
```

Rule 1381

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[d^(2*n - 1)*(d*x)^(m - 2*n + 1)*((a + b*x^n + c*x^(2*n))^(
p + 1)/(c*(m + 2*n*p + 1))), x] - Dist[d^(2*n)/(c*(m + 2*n*p + 1)), Int[(d*
x)^(m - 2*n)*Simp[a*(m - 2*n + 1) + b*(m + n*(p - 1) + 1)*x^n, x]*(a + b*x^
n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1, 0]
 && IntegerQ[p]
```

Rule 1395

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
 :> Dist[1/(m + 1), Subst[Int[(a + b*x^Simplify[n/(m + 1)] + c*x^Simplify[2
*(n/(m + 1))])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&
EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[n/(m + 1)]] &&  !I
ntegerQ[n]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{x^{-1-\frac{n}{4}}}{a+bx^n+cx^{2n}}\,dx = -\frac{4\text{Subst}\left(\int \frac{1}{a+\frac{c}{x^8}+\frac{b}{x^4}}\,dx, x, x^{-n/4}\right)}{n}$$

$$= -\frac{4\text{Subst}\left(\int \frac{x^8}{c+bx^4+ax^8}\,dx, x, x^{-n/4}\right)}{n}$$

$$= -\frac{4x^{-n/4}}{an} + \frac{4\text{Subst}\left(\int \frac{c+bx^4}{c+bx^4+ax^8}\,dx, x, x^{-n/4}\right)}{an}$$

$$= -\frac{4x^{-n/4}}{an} + \frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+ax^4}\,dx, x, x^{-n/4}\right)}{an} + \cdot$$

$$= -\frac{4x^{-n/4}}{an} - \frac{\left(2\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{a}\,x^2}\,dx, \cdot\right.}{a\sqrt{-b+\sqrt{b^2-4ac}}\,n}$$

$$= -\frac{4x^{-n/4}}{an} - \frac{2^{3/4}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{a}\,x^{-n/4}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{a^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}n} - \frac{2^{3/4}\left(b-\frac{}{\sqrt{b}}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.14, size = 105, normalized size = 0.25

$$\frac{-16x^{-n/4}+\text{RootSum}\left[c+b\#1^4+a\#1^8\&, \frac{cn\log(x)+4c\log\left(x^{-n/4}-\#1\right)+bn\log(x)\#1^4+4b\log\left(x^{-n/4}-\#1\right)\#1^4}{b\#1^3+2a\#1^7}\&\right]}{4an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n/4)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-16/x^(n/4) + RootSum[c + b*#1^4 + a*#1^8 & , (c*n*Log[x] + 4*c*Log[x^(-1/
4*n) - #1] + b*n*Log[x]*#1^4 + 4*b*Log[x^(-1/4*n) - #1]*#1^4)/(b*#1^3 + 2*a
*#1^7) & ])/(4*a*n)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.77, size = 630, normalized size = 1.52

method	result

risch	$-\frac{4x^{-\frac{n}{4}}}{an}+\Bigg(\sum_{\_R=\mathrm{RootOf}((256a^9c^4n^8-256a^8b^2c^3n^8+96a^7b^4c^2n^8-16a^6b^6cn^8+a^5b^8n^8)\_Z^8+(80a^4bc^4n^4-120a^3b^3c^3n^4+61a^2b^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] -4/a/n/(x^(1/4*n))+sum(_R*ln(x^(1/4*n)+(-128/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*
n^7*a^10*c^5+352/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^2*a^9*c^4-280/(a^2*c^6
-3*a*b^2*c^5+b^4*c^4)*n^7*b^4*a^8*c^3+98/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*
b^6*a^7*c^2-16/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^7*b^8*a^6*c+1/(a^2*c^6-3*a*b
^2*c^5+b^4*c^4)*n^7*b^10*a^5)*_R^7+(-36/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b
*a^5*c^5+129/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^3*a^4*c^4-138/(a^2*c^6-3*a
*b^2*c^5+b^4*c^4)*n^3*b^5*a^3*c^3+63/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^7*
a^2*c^2-13/(a^2*c^6-3*a*b^2*c^5+b^4*c^4)*n^3*b^9*a*c+1/(a^2*c^6-3*a*b^2*c^5
+b^4*c^4)*n^3*b^11)*_R^3),_R=RootOf((256*a^9*c^4*n^8-256*a^8*b^2*c^3*n^8+96
*a^7*b^4*c^2*n^8-16*a^6*b^6*c*n^8+a^5*b^8*n^8)*_Z^8+(80*a^4*b*c^4*n^4-120*a
^3*b^3*c^3*n^4+61*a^2*b^5*c^2*n^4-13*a*b^7*c*n^4+b^9*n^4)*_Z^4+c^5))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -4/(a*n*x^(1/4*n)) - integrate((c*x^(7/4*n) + b*x^(3/4*n))/(a*c*x*x^(2*n) +
 a*b*x*x^n + a^2*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5712 vs. $2(342) = 684$.
time = 0.98, size = 5712, normalized size = 13.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^1
0*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^
3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*arctan(1/16
*sqrt(2)*(2*sqrt(2)*((a^5*b^14*c - 19*a^6*b^12*c^2 + 147*a^7*b^10*c^3 - 590
```

```
*a^8*b^8*c^4 + 1290*a^9*b^6*c^5 - 1464*a^10*b^4*c^6 + 736*a^11*b^2*c^7 - 12
8*a^12*c^8)*n^7*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8))
+ (b^15*c - 16*a*b^13*c^2 + 103*a^2*b^11*c^3 - 340*a^3*b^9*c^4 + 605*a^4*b^
7*c^5 - 554*a^5*b^5*c^6 + 224*a^6*b^3*c^7 - 32*a^7*b*c^8)*n^3*x)*x^(-1/4*n
- 1)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)) - sqrt(2)*((a^5*b^10 - 16*a^6*b^8*c + 9
8*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*n^7*x*sqr
t((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^11 - 13*a*b^9*c
+ 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5)*n^3*x)
*sqrt((4*(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)
*x^2*x^(-1/2*n - 2) + sqrt(2)*((a^5*b^11 - 15*a^6*b^9*c + 85*a^7*b^7*c^2 -
220*a^8*b^5*c^3 + 240*a^9*b^3*c^4 - 64*a^10*b*c^5)*n^6*sqrt((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c +
48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + (b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2
 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6)*n^2)*sqr
t(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2
*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^
6*b^2*c + 16*a^7*c^2)*n^4)))/x^2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*
b^3*c - 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*sqrt(sqrt
(2)*sqrt(((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))/(b^8*c^5 - 6*a*b^6*c^6 + 11*a^2*b^4*c^7
 - 6*a^3*b^2*c^8 + a^4*c^9)) - 4*sqrt(2)*a*n*sqrt(sqrt(2)*sqrt(-((a^5*b^4 -
 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2)*n^4)))*arctan(1/8*(2*((a^5*b^14*c - 19*a^6*b^12*c^2 + 147*a^7*b^10
*c^3 - 590*a^8*b^8*c^4 + 1290*a^9*b^6*c^5 - 1464*a^10*b^4*c^6 + 736*a^11*b^
2*c^7 - 128*a^12*c^8)*n^7*x*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c
^3)*n^8)) - (b^15*c - 16*a*b^13*c^2 + 103*a^2*b^11*c^3 - 340*a^3*b^9*c^4 +
605*a^4*b^7*c^5 - 554*a^5*b^5*c^6 + 224*a^6*b^3*c^7 - 32*a^7*b*c^8)*n^3*x)*
x^(-1/4*n - 1)*sqrt(sqrt(2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4)))*sqrt(-((a^5*b^
4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
```

```
6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c +
16*a^7*c^2)*n^4)) - ((a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^
4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*n^7*x*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b
^2*c^2 - 64*a^13*c^3)*n^8)) - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3
*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5)*n^3*x)*sqrt(sqrt(2)*sqrt(-((a^5*
b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*n^4*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)*n^8)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)/((a^5*b^4 - 8*a^6*b^2*c
+ 16*a^7*c^2)*n^4)))*sqrt((4*(b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^
3*b^2*c^5 + a^4*c^6)*x^2*x^(-1/2*n - 2) - sqrt(2)*((a^5*b^11 - 15*a^6*b^9*c
 + 85*a^7*b^7*c^2 - 220*a^8*b^5*c^3 + 240*a^9*b^3*c^4 - 64*a^10*b*c^5)*n^6*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*n^8)) - (b^12 - 12*a*b^1
0*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 +
 8*a^6*c^6)*n^2)*sqrt(-((a^5*b^4 - 8*a^6*b^2*c ...
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/4*n)/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/4*n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^(-1/4*n - 1)/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{\frac{n}{4}+1}\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))),x)
[Out] int(1/(x^(n/4 + 1)*(a + b*x^n + c*x^(2*n))), x)
```

3.562 $\int \frac{x^2}{a+bx^n+cx^{2n}} \, dx$

Optimal. Leaf size=140

$$-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)}-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)}$$

```
[Out] -2/3*c*x^3*hypergeom([1, 3/n],[(3+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b
^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2/3*c*x^3*hypergeom([1, 3/n],[(3+n)/n],-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1397, 371}

$$-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4
*a*c])])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (2*c*x^3*Hypergeometric2
F1[1, 3/n, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*(b^2 - 4*a*c
+ b*Sqrt[b^2 - 4*a*c]))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1397

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[2*(c/q), Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{x^2}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x^2}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)} - \frac{2cx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)}$$

Mathematica [A]
time = 0.50, size = 265, normalized size = 1.89

$$-\frac{2}{3}cx^3\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-3/n}\,{}_2F_1\left(-\frac{3}{n},-\frac{3}{n};\frac{-3+n}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{1-8^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-3/n}\,{}_2F_1\left(-\frac{3}{n},-\frac{3}{n};\frac{-3+n}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x^3*((1 - Hypergeometric2F1[-3/n, -3/n, (-3 + n)/n, (b - Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c
])/c + x^n))^(3/n))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeometr
ic2F1[-3/n, -3/n, (-3 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c
] + 2*c*x^n)]/(8^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(3/n)))
/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(x^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + bx^n + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x**2/(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{a + b\,x^n + c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x^2/(a + b*x^n + c*x^(2*n)), x)
```

3.563 $\int \frac{x}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=136

$$-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

```
[Out] -c*x^2*hypergeom([1, 2/n],[(2+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4
*a*c-b*(-4*a*c+b^2)^(1/2))-c*x^2*hypergeom([1, 2/n],[(2+n)/n],-2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1397, 371}

$$-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])) - (c*x^2*Hypergeometric2F1[1,
2/n, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b*Sqrt[
b^2 - 4*a*c])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1397

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[2*(c/q), Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{x}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$
$$= -\frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{cx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

Mathematica [A]
time = 0.42, size = 263, normalized size = 1.93

$$-cx^2\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-2/n}{}_2F_1\left(-\frac{2}{n},-\frac{2}{n};\frac{-2+n}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{1-4^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-2/n}{}_2F_1\left(-\frac{2}{n},-\frac{2}{n};\frac{-2+n}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -(c*x^2*((1 - Hypergeometric2F1[-2/n, -2/n, (-2 + n)/n, (b - Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*a*c])
/c + x^n))^(2/n))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeometric
2F1[-2/n, -2/n, (-2 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)]/(4^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(2/n)))/(
Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(x/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral(x/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + bx^n + cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n+c*x**(2*n)),x)
[Out] Integral(x/(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate(x/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{a + b\,x^n + c\,x^{2\,n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^n + c*x^(2*n)),x)
[Out] int(x/(a + b*x^n + c*x^(2*n)), x)
```

3.564 $\int \frac{1}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=124

$$-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

```
[Out] -2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*a
*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*hypergeom([1, 1/n],[1+1/n],-2*c*x^n/(b+(-4*a
*c+b^2)^(1/2)))/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1361, 251}

$$-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(-1),x]
```

```
[Out] (-2*c*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) - (2*c*x*Hypergeometric2F1[1
, n^(-1), 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(b^2 - 4*a*c + b
*Sqrt[b^2 - 4*a*c])
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{a+bx^n+cx^{2n}}\,dx = \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^n}\,dx}{\sqrt{b^2-4ac}}$$
$$= -\frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 261 vs. 2(124) = 248.
time = 0.39, size = 261, normalized size = 2.10

$$-2cx\left(\frac{1-\left(\frac{x^n}{-\frac{-b+\sqrt{b^2-4ac}}{2c}+x^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{1-2^{-1/n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-1/n}{}_2F_1\left(-\frac{1}{n},-\frac{1}{n};\frac{-1+n}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(-1),x]
```

```
[Out] -2*c*x*((1 - Hypergeometric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b - Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(x^n/(-1/2*(-b + Sqrt[b^2 - 4*
a*c])/c + x^n))^n^(-1))/(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c]) + (1 - Hypergeo
metric2F1[-n^(-1), -n^(-1), (-1 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n)]/(2^n^(-1)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
))^n^(-1)))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + bx^n + cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(1/(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{a + b\,x^n + c\,x^{2\,n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(1/(a + b*x^n + c*x^(2*n)), x)
```

3.565 $\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}\,dx$

Optimal. Leaf size=74

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,n}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}$$

```
[Out] ln(x)/a-1/2*ln(a+b*x^n+c*x^(2*n))/a/n+b*arctanh((b+2*c*x^n)/(-4*a*c+b^2)^(1
/2))/a/n/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1371, 719, 29, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{an\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^n)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]*n) + Log[
x]/a - Log[a + b*x^n + c*x^(2*n)]/(2*a*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^n\right)}{an}+\frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,x^n\right)}{an}\\
&= \frac{\log(x)}{a}-\frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^n\right)}{2an}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^n\right)}{2an}\\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}+\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^n\right)}{an}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^n}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,n}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^n+cx^{2n}\right)}{2an}
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 72, normalized size = 0.97

$$-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx^n}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2\log\left(x^n\right)+\log\left(a+x^n(b+cx^n)\right)}{2an}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] -1/2*((2*b*ArcTan[(b + 2*c*x^n)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2
*Log[x^n] + Log[a + x^n*(b + c*x^n)])/(a*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(68) = 136$.
time = 0.07, size = 397, normalized size = 5.36

method	result
risch	$\frac{4n^2\ln(x)ac}{4a^2cn^2-a\,b^2n^2}-\frac{n^2\ln(x)b^2}{4a^2cn^2-a\,b^2n^2}-\frac{2\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)c}{(4ac-b^2)n}+\frac{\ln\left(x^n-\frac{-b^2+\sqrt{-4a\,b^2c+b^4}}{2bc}\right)b^2}{2a(4ac-b^2)n}+\frac{\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)*a*c-1/(4*a^2*c*n^2-a*b^2*n^2)*n^2*ln(x)
*b^2-2/(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/
(4*a*c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2+1/2/a/(4*a*
c-b^2)/n*ln(x^n-1/2*(-b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/
2)-2/(4*a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/(4*
a*c-b^2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/a/(4*a*c-b^
2)/n*ln(x^n+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

Fricas [A]
time = 0.38, size = 259, normalized size = 3.50

$$\left[\frac{2\,(b^2-4\,ac)n\log\left(x\right)+\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^{2\,n}+b^2-2\,ac+2\,\left(bc+\sqrt{b^2-4\,ac}\,c\right)x^n+\sqrt{b^2-4\,ac}\,b}{cx^{2\,n}+bx^n+a}\right)-(b^2-4\,ac)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,(ab^2-4\,a^2c)n},\ \frac{2\,(b^2-4\,ac)n\log\left(x\right)+2\,\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{2\,\sqrt{-b^2+4\,ac}\,cx^n+\sqrt{-b^2+4\,ac}\,b}{b^2-4\,ac}\right)-(b^2-4\,ac)\log\left(cx^{2\,n}+bx^n+a\right)}{2\,(ab^2-4\,a^2c)n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2 - 4*a*c)*n*log(x) + sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^(2*n) + b
^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*x^n + sqrt(b^2 - 4*a*c)*b)/(c*x^
(2*n) + b*x^n + a)) - (b^2 - 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4
*a^2*c)*n), 1/2*(2*(b^2 - 4*a*c)*n*log(x) + 2*sqrt(-b^2 + 4*a*c)*b*arctan(-
(2*sqrt(-b^2 + 4*a*c)*c*x^n + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - (b^2 -
 4*a*c)*log(c*x^(2*n) + b*x^n + a))/((a*b^2 - 4*a^2*c)*n)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(63) = 126$.
time = 28.26, size = 362, normalized size = 4.89

$$\begin{cases} \frac{4bcn\log(x)}{b^3n+2b^2cnx^n} - \frac{4bc\log\left(\frac{b}{2c}+x^n\right)}{b^3n+2b^2cnx^n} + \frac{4bc}{b^3n+2b^2cnx^n} + \frac{8c^2nx^n\log(x)}{b^3n+2b^2cnx^n} - \frac{8c^2x^n\log\left(\frac{b}{2c}+x^n\right)}{b^3n+2b^2cnx^n} & \text{for}\ a = \frac{b^2}{4c} \\ -\frac{x^{-n}}{bn} - \frac{c\log(x^n)}{b^2n} + \frac{c\log\left(\frac{b}{c}+x^n\right)}{b^2n} & \text{for}\ a = 0 \\ \frac{\log(x)}{a+b+c} & \text{for}\ n = 0 \\ \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^n\right)}{an} & \text{for}\ c = 0 \\ -\frac{b\log\left(\frac{b}{2c}+x^n-\frac{\sqrt{-4ac+b^2}}{2c}\right)}{2an\sqrt{-4ac+b^2}} + \frac{b\log\left(\frac{b}{2c}+x^n+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{2an\sqrt{-4ac+b^2}} + \frac{\log(x)}{a} - \frac{\log\left(\frac{b}{2c}+x^n-\frac{\sqrt{-4ac+b^2}}{2c}\right)}{2an} - \frac{\log\left(\frac{b}{2c}+x^n+\frac{\sqrt{-4ac+b^2}}{2c}\right)}{2an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((4*b*c*n*log(x)/(b**3*n + 2*b**2*c*n*x**n) - 4*b*c*log(b/(2*c) +
x**n)/(b**3*n + 2*b**2*c*n*x**n) + 4*b*c/(b**3*n + 2*b**2*c*n*x**n) + 8*c**
2*n*x**n*log(x)/(b**3*n + 2*b**2*c*n*x**n) - 8*c**2*x**n*log(b/(2*c) + x**n
)/(b**3*n + 2*b**2*c*n*x**n), Eq(a, b**2/(4*c))), (-1/(b*n*x**n) - c*log(x*
*n)/(b**2*n) + c*log(b/c + x**n)/(b**2*n), Eq(a, 0)), (log(x)/(a + b + c),
Eq(n, 0)), (log(x)/a - log(a/b + x**n)/(a*n), Eq(c, 0)), (-b*log(b/(2*c) +
x**n - sqrt(-4*a*c + b**2)/(2*c))/(2*a*n*sqrt(-4*a*c + b**2)) + b*log(b/(2*
c) + x**n + sqrt(-4*a*c + b**2)/(2*c))/(2*a*n*sqrt(-4*a*c + b**2)) + log(x)
/a - log(b/(2*c) + x**n - sqrt(-4*a*c + b**2)/(2*c))/(2*a*n) - log(b/(2*c)
+ x**n + sqrt(-4*a*c + b**2)/(2*c))/(2*a*n), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x), x)
```

Mupad [B]

time = 1.61, size = 224, normalized size = 3.03

$$\frac{\ln\left(-\frac{1}{c\,x}-\frac{(2\,a\,n+b\,n\,x^n)\,\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{2\,c\,x\,(a\,b^2\,n-4\,a^2\,c\,n)}\right)\,\left(4\,a\,c+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{2\,(a\,b^2\,n-4\,a^2\,c\,n)}-\frac{\ln\left(\frac{(2\,a\,n+b\,n\,x^n)\,\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2\right)}{2\,c\,x\,(a\,b^2\,n-4\,a^2\,c\,n)}-\frac{1}{c\,x}\right)\,\left(b\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b^2\right)}{2\,(a\,b^2\,n-4\,a^2\,c\,n)}+\frac{\ln(x)\,(n-1)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] (log(- 1/(c*x) - ((2*a*n + b*n*x^n)*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))/
(2*c*x*(a*b^2*n - 4*a^2*c*n)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) - b^2))/(2*(a
*b^2*n - 4*a^2*c*n)) - (log(((2*a*n + b*n*x^n)*(b*(b^2 - 4*a*c)^(1/2) - 4*a
*c + b^2))/(2*c*x*(a*b^2*n - 4*a^2*c*n)) - 1/(c*x))*(b*(b^2 - 4*a*c)^(1/2)
- 4*a*c + b^2))/(2*(a*b^2*n - 4*a^2*c*n)) + (log(x)*(n - 1))/(a*n)
```

3.566 $\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=142

$$\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x}+\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x}$$

```
[Out] 2*c*hypergeom([1, -1/n],[(-1+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/x/(b^2-
4*a*c-b*(-4*a*c+b^2)^(1/2))+2*c*hypergeom([1, -1/n],[(-1+n)/n],-2*c*x^n/(b+
(-4*a*c+b^2)^(1/2)))/x/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1397, 371}

$$\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2*c*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x) + (2*c*Hypergeometric2F1
[1, -n^(-1), -((1 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a
*c + b*Sqrt[b^2 - 4*a*c])*x)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1397

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[2*(c/q), Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx^n+cx^{2n})}\,dx = \frac{(2c)\int \frac{1}{x^2\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{x^2\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x} + \frac{2c\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x}$$

Mathematica [A]
time = 0.31, size = 240, normalized size = 1.69

$$\frac{2^{1+\frac{1}{n}}c\left(\frac{\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{\frac{1}{n}}{}_2F_1\left(1+\frac{1}{n},1+\frac{1}{n};2+\frac{1}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{-b+\sqrt{b^2-4ac}-2cx^n}+\frac{x^{-n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{1+\frac{1}{n}}{}_2F_1\left(1+\frac{1}{n},1+\frac{1}{n};2+\frac{1}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{c}\right)}{\sqrt{b^2-4ac}\,(1+n)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2^(1 + n^(-1))*c*((((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^n^(-1)*Hype
rgeometric2F1[1 + n^(-1), 1 + n^(-1), 2 + n^(-1), (b - Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n) + (((
c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(1 + n^(-1))*Hypergeometric2F1[1
+ n^(-1), 1 + n^(-1), 2 + n^(-1), (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^n)])/(c*x^n)))/(Sqrt[b^2 - 4*a*c]*(1 + n)*x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+b\,x^n+c\,x^{2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/x^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**n + c*x**(2*n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^2*(a + b*x^n + c*x^(2*n))), x)
```

3.567 $\int \frac{1}{x^3(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=140

$$\frac{c\,_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2}+\frac{c\,_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x^2}$$

```
[Out] c*hypergeom([1, -2/n],[(-2+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))/x^2/(b^2-
4*a*c-b*(-4*a*c+b^2)^(1/2))+c*hypergeom([1, -2/n],[(-2+n)/n],-2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))/x^2/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.03, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1397, 371}

$$\frac{c\,_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\,_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*Hypergeometric2F1[1, -2/n, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*
c])])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x^2) + (c*Hypergeometric2F1[1, -
2/n, -((2 - n)/n), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c + b*S
qrt[b^2 - 4*a*c])*x^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1397

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[2*(c/q), Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx^n+cx^{2n})}\,dx = \frac{(2c)\int \frac{1}{x^3\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{x^3\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2} + \frac{c\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac+b\sqrt{b^2-4ac}\right)x^2}$$

Mathematica [A]
time = 0.34, size = 258, normalized size = 1.84

$$\frac{2^{\frac{2+n}{n}}c\left(\frac{\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{2/n}{}_2F_1\left(\frac{2+n}{n},\frac{2+n}{n};2+\frac{2}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)}{-b+\sqrt{b^2-4ac}-2cx^n}+\frac{x^{-n}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{\frac{2+n}{n}}{}_2F_1\left(\frac{2+n}{n},\frac{2+n}{n};2+\frac{2}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)}{c}\right)}{\sqrt{b^2-4ac}\,(2+n)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2^((2 + n)/n)*c*((((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^(2/n)*Hyperg
eometric2F1[(2 + n)/n, (2 + n)/n, 2 + 2/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqr
t[b^2 - 4*a*c] + 2*c*x^n)])/(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^n) + (((c*x^n)/
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((2 + n)/n)*Hypergeometric2F1[(2 + n)/n,
 (2 + n)/n, 2 + 2/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)])/(c*x^n)))/(Sqrt[b^2 - 4*a*c]*(2 + n)*x^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+b\,x^n+c\,x^{2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(1/x^3/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral(1/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n + c*x^(2*n))),x)
```

```
[Out] int(1/(x^3*(a + b*x^n + c*x^(2*n))), x)
```

3.568 $\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=148

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{4}{n};-\frac{1}{2},-\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/4*x^4*AppellF1(4/n,-1/2,-1/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{x^4\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{4}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -1/2, -1/2, (4 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
```

```
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}\int x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{x^4\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{4}{n};-\frac{1}{2},-\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(148) = 296.
time = 0.56, size = 365, normalized size = 2.47

$$\frac{x^4\left(4(4+n)\left(a+x^n(b+cx^n)\right)+an(4+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+2bnx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4+n}{n};\frac{1}{2},\frac{1}{2};2+\frac{4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{4(4+n)^2\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*(4*(4 + n)*(a + x^n*(b + c*x^n)) + a*n*(4 + n)*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*b*n*x^n*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(4 + n)/n, 1/2
, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^
2 - 4*a*c])]))/(4*(4 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(x**3*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int(x^3*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.569 $\int x^2 \sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=148

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{3}{n};-\frac{1}{2},-\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/3*x^3*AppellF1(3/n,-1/2,-1/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{x^3\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{3}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -1/2, -1/2, (3 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
```

```
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}\int x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{x^3\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{3}{n};-\frac{1}{2},-\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 366 vs. 2(148) = 296.
time = 0.52, size = 366, normalized size = 2.47

$$\frac{x^3\left(6(3+n)\left(a+x^n(b+cx^n)\right)+2an(3+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+3bnx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3+n}{n};\frac{1}{2},\frac{1}{2};2+\frac{3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{6(3+n)^2\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*(6*(3 + n)*(a + x^n*(b + c*x^n)) + 2*a*n*(3 + n)*Sqrt[(b - Sqrt[b^2 -
4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 3*b*n*x^n
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 + n)/n, 1
/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])]))/(6*(3 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(x**2*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int(x^2*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.570 $\int x\sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=148

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{2}{n};-\frac{1}{2},-\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/2*x^2*AppellF1(2/n,-1/2,-1/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c
*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$\frac{x^2\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{2}{n};-\frac{1}{2},-\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -1/2, -1/2, (2 + n)/n, (-2*c
*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
```

```
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx^n+cx^{2n}}\,dx&=\frac{\sqrt{a+bx^n+cx^{2n}}\int x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\&=\frac{x^2\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{2}{n};-\frac{1}{2},-\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 364 vs. 2(148) = 296.
time = 0.47, size = 364, normalized size = 2.46

$$\frac{x^2\left(2(2+n)\left(a+x^n(b+cx^n)\right)+an(2+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+bnx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2+n}{n};\frac{1}{2},\frac{1}{2};2+\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{2(2+n)^2\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*(2*(2 + n)*(a + x^n*(b + c*x^n)) + a*n*(2 + n)*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n*Sqr
t[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(2 + n)/n, 1/2,
1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])]))/(2*(2 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a + bx^n + cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(x*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int(x*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.571 $\int \sqrt{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=139

$$\frac{x\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] x*AppellF1(1/n,-1/2,-1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+
(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1362, 440}

$$\frac{x\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -1/2, -1/2, 1 + n^(-1), (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt
[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
```

```
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+bx^n+cx^{2n}}\,dx &= \frac{\sqrt{a+bx^n+cx^{2n}}\int \sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{x\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{1}{2},-\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 351 vs. 2(139) = 278.
time = 0.46, size = 351, normalized size = 2.53

$$\frac{x\left(bnx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+2(1+n)\left(a+x^n(b+cx^n)+an\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{2(1+n)^2\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1
 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 + n)*(a + x^n*(b + c*x^n) + a*n*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1
 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4
*a*c])])))/(2*(1 + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + bx^n + cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x^n+c\,x^{2\,n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int((a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.572 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\,\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\,\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}\,n}$$

```
[Out] -arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))*a^(1/2)/n+1/2*b
*arctanh(1/2*(b+2*c*x^n)/c^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/n/c^(1/2)+(a+b*
x^n+c*x^(2*n))^(1/2)/n
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1371, 748, 857, 635, 212, 738}

$$\frac{\sqrt{a+bx^n+cx^{2n}}}{n}-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\,\sqrt{a+bx^n+cx^{2n}}}\right)}{n}+\frac{b\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\,\sqrt{a+bx^n+cx^{2n}}}\right)}{2\sqrt{c}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] Sqrt[a + b*x^n + c*x^(2*n)]/n - (Sqrt[a]*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*S
qrt[a + b*x^n + c*x^(2*n)])])/n + (b*ArcTanh[(b + 2*c*x^n)/(2*Sqrt[c]*Sqrt[
a + b*x^n + c*x^(2*n)])])/(2*Sqrt[c]*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{\text{Subst}\left(\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{2n}$$

$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{n} + \frac{b\text{Subst}\left(\int \frac{}{\sqrt{a+}}\right.}{}$$

$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{(2a)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b\text{Subst}\left(\int\right.}{}$$

$$= \frac{\sqrt{a+bx^n+cx^{2n}}}{n} - \frac{\sqrt{a}\,\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\,\sqrt{a+bx^n+cx^{2n}}}\right)}{n} + \frac{b\tanh^{-1}\left(\frac{}{2\sqrt{c}}\right.}{2}$$

Mathematica [A]
time = 0.26, size = 113, normalized size = 0.95

$$\frac{2\sqrt{a+x^n(b+cx^n)}+4\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^n-\sqrt{a+x^n(b+cx^n)}}{\sqrt{a}}\right)-\frac{b\log\left(n\left(b+2cx^n-2\sqrt{c}\,\sqrt{a+x^n(b+cx^n)}\right)\right)}{\sqrt{c}}}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x,x]
```

```
[Out] (2*Sqrt[a + x^n*(b + c*x^n)] + 4*Sqrt[a]*ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^
n*(b + c*x^n)])/Sqrt[a]] - (b*Log[n*(b + 2*c*x^n - 2*Sqrt[c]*Sqrt[a + x^n*(
b + c*x^n)])])/Sqrt[c])/(2*n)
```

Maple [A]
time = 0.10, size = 125, normalized size = 1.05

method	result
risch	$\frac{\sqrt{a+b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}}}{n}+\frac{b\ln\left(\frac{\frac{b}{2}+c\,e^{n\ln(x)}}{\sqrt{c}}+\sqrt{a+b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}}\right)}{2n\sqrt{c}}-\frac{\sqrt{a}\,\ln\left(\left(2a+b\,e^{n\ln(x)}+2\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

[Out] 1/n*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2)+1/2/n*b*ln((1/2*b+c*exp(n*ln(x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/c^(1/2)-1/n*a^(1/2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))/exp(n*ln(x)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)

Fricas [A]
time = 0.41, size = 658, normalized size = 5.53

$$\left[\frac{b\sqrt{c}\log\left(-8\,c^2x^{2n}-8\,bcx^n-b^2-4\,ac-4\left(2\,c^{\frac{3}{2}}x^n+b\sqrt{c}\right)\sqrt{cx^{2n}+bx^n+a}\right)+2\sqrt{a}\,c\log\left(-\frac{8\,abx^n+8\,a^2+(b^2+4\,ac)x^{2n}-4\left(\sqrt{a}\,bx^n+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2n}+bx^n+a}}{x^{2n}}\right)+4\sqrt{cx^{2n}+bx^n+a}\,c}{4\,cn}, -\frac{b\sqrt{-c}\arctan\left(\frac{\left(2\sqrt{-c}\,cx^n+b\sqrt{-c}\right)\sqrt{cx^{2n}+bx^n+a}}{2\,(c^2x^{2n}+bcx^n+ac)}\right)-\sqrt{a}\,c\log\left(-\frac{8\,abx^n+8\,a^2+(b^2+4\,ac)x^{2n}-4\left(\sqrt{a}\,bx^n+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2n}+bx^n+a}}{x^{2n}}\right)-2\sqrt{cx^{2n}+bx^n+a}\,c}{2\,cn}, \frac{4\sqrt{-a}\,c\arctan\left(\frac{\left(\sqrt{-a}\,bx^n+2\sqrt{-a}\,a\right)\sqrt{cx^{2n}+bx^n+a}}{2\,(acx^{2n}+abx^n+a^2)}\right)+b\sqrt{c}\log\left(-8\,c^2x^{2n}-8\,bcx^n-b^2-4\,ac-4\left(2\,c^{\frac{3}{2}}x^n+b\sqrt{c}\right)\sqrt{cx^{2n}+bx^n+a}\right)+4\sqrt{cx^{2n}+bx^n+a}\,c}{4\,cn}, \frac{2\sqrt{-a}\,c\arctan\left(\frac{\left(\sqrt{-a}\,bx^n+2\sqrt{-a}\,a\right)\sqrt{cx^{2n}+bx^n+a}}{2\,(acx^{2n}+abx^n+a^2)}\right)-b\sqrt{-c}\arctan\left(\frac{\left(2\sqrt{-c}\,cx^n+b\sqrt{-c}\right)\sqrt{cx^{2n}+bx^n+a}}{2\,(c^2x^{2n}+bcx^n+ac)}\right)+2\sqrt{cx^{2n}+bx^n+a}\,c}{2\,cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="fricas")

[Out] [1/4*(b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 2*sqrt(a)*c*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), -1/2*(b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) - sqrt(a)*c*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) - 2*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/4*(4*sqrt(-a)*c*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) + b*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 4*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n), 1/2*(2*sqrt(-a)*c*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) - b*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*sqrt(c*x^(2*n) + b*x^n + a)*c)/(c*n)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^{n} + cx^{2n}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,x^n+c\,x^{2\,n}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2)/x,x)
```

```
[Out] int((a + b*x^n + c*x^(2*n))^(1/2)/x, x)
```

3.573 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{1}{2},-\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -AppellF1(-1/n,-1/2,-1/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/
(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x/(1+2*c*x^n/(b-(-4*a*c+b
^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{1}{2},-\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

```
[Out] -((Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -1/2, -1/2, -((1 - n)/n),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x
*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
```

```
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^2}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}\int \frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}{x^2}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{1}{2},-\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(149) = 298.
time = 0.48, size = 365, normalized size = 2.45

$$\frac{2(-1+n)\left(a+x^n(b+cx^n)\right)-2a(-1+n)n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};\frac{-1+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+bnx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{-1+n}{n};\frac{1}{2},\frac{1}{2};2-\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{2(-1+n)^2x\sqrt{a+x^n\left(b+cx^n\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^2,x]
```

```
[Out] (2*(-1 + n)*(a + x^n*(b + c*x^n)) - 2*a*(-1 + n)*n*Sqrt[(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(-1 + n)/n,
1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + S
qrt[b^2 - 4*a*c])])/(2*(-1 + n)^2*x*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,x^n+c\,x^{2n}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
[Out] int((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**2,x)
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\,x^n+c\,x^{2\,n}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2)/x^2,x)
[Out] int((a + b*x^n + c*x^(2*n))^(1/2)/x^2, x)
```

3.574 $\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx$

Optimal. Leaf size=151

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{2}{n};-\frac{1}{2},-\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -1/2*AppellF1(-2/n,-1/2,-1/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x^2/(1+2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$-\frac{\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{2}{n};-\frac{1}{2},-\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

```
[Out] -1/2*(Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -1/2, -1/2, -((2 - n)/n),
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x
^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
```

```
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^n+cx^{2n}}}{x^3}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}\int \frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}{x^3}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{2}{n};-\frac{1}{2},-\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(151) = 302.
time = 0.44, size = 365, normalized size = 2.42

$$\frac{2(-2+n)\left(a+x^n(b+cx^n)\right)-a(-2+n)n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{-2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+bnx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{-2+n}{n};\frac{1}{2},\frac{1}{2};2-\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{2(-2+n)^2x^2\sqrt{a+x^n\left(b+cx^n\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n + c*x^(2*n)]/x^3,x]
```

```
[Out] (2*(-2 + n)*(a + x^n*(b + c*x^n)) - a*(-2 + n)*n*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + b*n*x^n*Sqrt
[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(-2 + n)/n, 1/2,
1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2
- 4*a*c])])/(2*(-2 + n)^2*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,x^n+c\,x^{2n}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + bx^n + cx^{2n}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**n + c*x**(2*n))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\,x^n + c\,x^{2\,n}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(1/2)/x^3,x)
[Out] int((a + b*x^n + c*x^(2*n))^(1/2)/x^3, x)
```

3.575 $\int x^3(a+bx^n+cx^{2n})^{3/2}\,dx$

Optimal. Leaf size=149

$$\frac{ax^4\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{4}{n};-\frac{3}{2},-\frac{3}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/4*a*x^4*AppellF1(4/n,-3/2,-3/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{ax^4\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{4}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^4*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[4/n, -3/2, -3/2, (4 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
```

```
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx&=\frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int x^3\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\&=\frac{ax^4\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{4}{n};-\frac{3}{2},-\frac{3}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 518 vs. 2(149) = 298.
time = 1.02, size = 518, normalized size = 3.48

$$\frac{x^4\left(2(4+n)\left(32a^2c(2+3n+n^2)+x^n(b+cx^n)\left(3b^2n^2+2bc(32+36n+7n^2)x^n+8c^2(8+6n+n^2)x^{2n}\right)+a\left(3b^2n^2+2bc(64+84n+23n^2)x^n+8c^2(16+18n+5n^2)x^{2n}\right)\right)-6an^2(4+n)\left(b^2-2ac(2+n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)-3bn^2\left(b^2(8+n)-4ac(8+3n)\right)x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4+n}{n};\frac{1}{2},\frac{1}{2};2+\frac{4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{16c(2+n)(4+n)^2(4+3n)\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*(2*(4 + n)*(32*a^2*c*(2 + 3*n + n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*
b*c*(32 + 36*n + 7*n^2)*x^n + 8*c^2*(8 + 6*n + n^2)*x^(2*n)) + a*(3*b^2*n^2
 + 2*b*c*(64 + 84*n + 23*n^2)*x^n + 8*c^2*(16 + 18*n + 5*n^2)*x^(2*n))) - 6
*a*n^2*(4 + n)*(b^2 - 2*a*c*(2 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(b^2*(8 + n) - 4*a
*c*(8 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appe
llF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(16*c*(2 + n)*(4 + n)^2*(4 + 3*n)*Sqrt[a
+ x^n*(b + c*x^n)])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+bx^{n}+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(x^3*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.576 $\int x^2(a+bx^n+cx^{2n})^{3/2}\,dx$

Optimal. Leaf size=149

$$\frac{ax^3\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{3}{n};-\frac{3}{2},-\frac{3}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/3*a*x^3*AppellF1(3/n,-3/2,-3/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{ax^3\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{3}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^3*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[3/n, -3/2, -3/2, (3 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
```

```
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int x^2\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int x^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$=\frac{ax^3\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{3}{n};-\frac{3}{2},-\frac{3}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 524 vs. 2(149) = 298.
time = 1.03, size = 524, normalized size = 3.52

$$\frac{x^3\left(2(3+n)\left(4a^2c(9+18n+8n^2)+x^n(b+cx^n)\left(3b^2n^2+2bc(18+27n+7n^2)x^n+4c^2(9+9n+2n^2)x^{2n}\right)+a\left(3b^2n^2+2bc(36+63n+23n^2)x^n+4c^2(18+27n+10n^2)x^{2n}\right)\right)+2an^2(3+n)\left(-3b^2+4ac(3+2n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)-3bn^2\left(-12ac(2+n)+b^2(6+n)\right)x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3+n}{n};\frac{1}{2},\frac{1}{2};2+\frac{3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{24c(1+n)(3+n)^2(3+2n)\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(2*(3 + n)*(4*a^2*c*(9 + 18*n + 8*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 +
2*b*c*(18 + 27*n + 7*n^2)*x^n + 4*c^2*(9 + 9*n + 2*n^2)*x^(2*n)) + a*(3*b^2
*n^2 + 2*b*c*(36 + 63*n + 23*n^2)*x^n + 4*c^2*(18 + 27*n + 10*n^2)*x^(2*n))
) + 2*a*n^2*(3 + n)*(-3*b^2 + 4*a*c*(3 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(-12*a*c
*(2 + n) + b^2*(6 + n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(24*c*(1 + n)*(3 + n)^2*(3 + 2*
n)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(a+bx^{n}+cx^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(x^2*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.577 $\int x(a+bx^n+cx^{2n})^{3/2}\,dx$

Optimal. Leaf size=149

$$\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{2}{n};-\frac{3}{2},-\frac{3}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] 1/2*a*x^2*AppellF1(2/n,-3/2,-3/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{2}{n};-\frac{3}{2},-\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x^2*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[2/n, -3/2, -3/2, (2 + n)/n, (-2
*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
```

```
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int x\left(a+bx^n+cx^{2n}\right)^{3/2}\,dx=\frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int x\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$=\frac{ax^2\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{2}{n};-\frac{3}{2},-\frac{3}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 520 vs. 2(149) = 298.
time = 1.02, size = 520, normalized size = 3.49

$$\frac{x^2\left(2(2+n)\left(16a^2c(1+3n+2n^2)+x^n(b+cx^n)\left(3b^2n^2+2bc(8+18n+7n^2)x^n+8c^2(2+3n+n^2)x^{2n}\right)+a\left(3b^2n^2+2bc(16+42n+23n^2)x^n+8c^2(4+9n+5n^2)x^{2n}\right)\right)-6an^2(2+n)\left(b^2-4ac(1+n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)-3bn^2\left(b^2(4+n)-4ac(4+3n)\right)x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2+n}{n};\frac{1}{2},\frac{1}{2};2+\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{16c(1+n)(2+n)^2(2+3n)\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^n + c*x^(2*n))^(3/2),x]

[Out] (x^2*(2*(2 + n)*(16*a^2*c*(1 + 3*n + 2*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 +
2*b*c*(8 + 18*n + 7*n^2)*x^n + 8*c^2*(2 + 3*n + n^2)*x^(2*n)) + a*(3*b^2*n^
2 + 2*b*c*(16 + 42*n + 23*n^2)*x^n + 8*c^2*(4 + 9*n + 5*n^2)*x^(2*n))) - 6*
a*n^2*(2 + n)*(b^2 - 4*a*c*(1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*n^2*(b^2*(4 + n) - 4*a*
c*(4 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(16*c*(1 + n)*(2 + n)^2*(2 + 3*n)*Sqrt[a +
 x^n*(b + c*x^n)])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral(x*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*x, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^n + c*x^(2*n))^(3/2),x)
```

```
[Out] int(x*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.578 $\int (a+bx^n+cx^{2n})^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{ax\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] a*x*AppellF1(1/n,-3/2,-3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/(1+2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1362, 440}

$$\frac{ax\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*x*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[n^(-1), -3/2, -3/2, 1 + n^(-1), (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sq
rt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
```

```
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^n+cx^{2n}\right)^{3/2}\,dx &= \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\\ &= \frac{ax\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1}{n};-\frac{3}{2},-\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 513 vs. 2(140) = 280.
time = 1.05, size = 513, normalized size = 3.66

$$\frac{x\left(-3bn^2(b^2(2+n)-4ac(2+3n))x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+2(1+n)\left(4a^2c(1+6n+8n^2)+x^n(b+cx^n)\left(3b^2n^2+2bc(2+9n+7n^2)x^n+4c^2(1+3n+2n^2)x^{2n}\right)+a\left(3b^2n^2+2bc(4+21n+23n^2)x^n+4c^2(2+9n+10n^2)x^{2n}\right)-3an^2(b^2-4ac(1+2n))\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{8c(1+n)^2(1+2n)(1+3n)\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2),x]
[Out] (x*(-3*b*n^2*(b^2*(2 + n) - 4*a*c*(2 + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + 2*(1 +
 n)*(4*a^2*c*(1 + 6*n + 8*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*b*c*(2 + 9*
n + 7*n^2)*x^n + 4*c^2*(1 + 3*n + 2*n^2)*x^(2*n)) + a*(3*b^2*n^2 + 2*b*c*(4
 + 21*n + 23*n^2)*x^n + 4*c^2*(2 + 9*n + 10*n^2)*x^(2*n)) - 3*a*n^2*(b^2 -
4*a*c*(1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(8*c*(1 + n)^2*(1 + 2*n)*(1 + 3*n)*Sqrt[a
 + x^n*(b + c*x^n)])
```

Maple [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int((a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.579 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx$

Optimal. Leaf size=173

$$\frac{(b^2+8ac+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\frac{(a+bx^n+cx^{2n})^{3/2}}{3n}-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}-\frac{b(b^2-}{}$$

```
[Out] 1/3*(a+b*x^n+c*x^(2*n))^(3/2)/n-a^(3/2)*arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+
b*x^n+c*x^(2*n))^(1/2))/n-1/16*b*(-12*a*c+b^2)*arctanh(1/2*(b+2*c*x^n)/c^(1
/2)/(a+b*x^n+c*x^(2*n))^(1/2))/c^(3/2)/n+1/8*(b^2+8*a*c+2*b*c*x^n)*(a+b*x^n
+c*x^(2*n))^(1/2)/c/n
```

Rubi [A]
time = 0.11, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1371, 748, 828, 857, 635, 212, 738}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{n}-\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^n}{2\sqrt{c}\sqrt{a+bx^n+cx^{2n}}}\right)}{16c^{3/2}n}+\frac{(8ac+b^2+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn}+\frac{(a+bx^n+cx^{2n})^{3/2}}{3n}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^n)*Sqrt[a + b*x^n + c*x^(2*n)])/(8*c*n) + (a + b*x^
n + c*x^(2*n))^(3/2)/(3*n) - (a^(3/2)*ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt
[a + b*x^n + c*x^(2*n)]])])/n - (b*(b^2 - 12*a*c)*ArcTanh[(b + 2*c*x^n)/(2*S
qrt[c]*Sqrt[a + b*x^n + c*x^(2*n)]])])/(16*c^(3/2)*n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x}\,dx, x, x^n\right)}{n}$$

$$= \frac{(a+bx^n+cx^{2n})^{3/2}}{3n} - \frac{\text{Subst}\left(\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^n\right)}{2n}$$

$$= \frac{(b^2+8ac+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{(a+bx^n+cx^{2n})^{3/2}}{3n} + \frac{\text{Subst}\left(\int \frac{8a^2c-\frac{1}{2}b}{x\sqrt{a+}}\right.}{}$$

$$= \frac{(b^2+8ac+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{(a+bx^n+cx^{2n})^{3/2}}{3n} + \frac{a^2\text{Subst}\left(\int \frac{}{x\sqrt{a}}\right.}{}$$

$$= \frac{(b^2+8ac+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{(a+bx^n+cx^{2n})^{3/2}}{3n} - \frac{(2a^2)\,\text{Subst}\left(\int \frac{}{4c}\right.}{}$$

$$= \frac{(b^2+8ac+2bcx^n)\sqrt{a+bx^n+cx^{2n}}}{8cn} + \frac{(a+bx^n+cx^{2n})^{3/2}}{3n} - \frac{a^{3/2}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.}{}$$

Mathematica [A]
time = 0.63, size = 159, normalized size = 0.92

$$\frac{2\sqrt{c}\,\sqrt{a+x^n(b+cx^n)}\,(3b^2+14bcx^n+8c(4a+cx^{2n}))+96a^{3/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^n-\sqrt{a+x^n(b+cx^n)}}{\sqrt{a}}\right)+3(b^3-12abc)\log\left(cn\left(b+2cx^n-2\sqrt{c}\,\sqrt{a+x^n(b+cx^n)}\right)\right)}{48c^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x,x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x^n*(b + c*x^n)]*(3*b^2 + 14*b*c*x^n + 8*c*(4*a + c*x^(
2*n))) + 96*a^(3/2)*c^(3/2)*ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^n*(b + c*x^n)
])/Sqrt[a]] + 3*(b^3 - 12*a*b*c)*Log[c*n*(b + 2*c*x^n - 2*Sqrt[c]*Sqrt[a +
x^n*(b + c*x^n)])])/(48*c^(3/2)*n)
```

Maple [A]
time = 0.10, size = 209, normalized size = 1.21

method	result
risch	$\frac{(8c^2e^{2n\ln(x)}+14bc\,e^{n\ln(x)}+32ac+3b^2)\sqrt{a+b\,e^{n\ln(x)}+c\,e^{2n\ln(x)}}}{24cn} + \frac{3ab\ln\left(\frac{\frac{b}{2}+c\,e^{n\ln(x)}}{\sqrt{c}}+\sqrt{a+b\,e^{n\ln(x)}+c\,e^{2n\ln(}}\right.}{4\sqrt{c}\,n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(8*c^2*exp(n*ln(x))^2+14*b*c*exp(n*ln(x))+32*a*c+3*b^2)*(a+b*exp(n*ln(
x))+c*exp(n*ln(x))^2)^(1/2)/c/n+3/4/c^(1/2)/n*a*b*ln((1/2*b+c*exp(n*ln(x)))
/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))-1/16/c^(3/2)/n*b^3*ln((
1/2*b+c*exp(n*ln(x)))/c^(1/2)+(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)^(1/2))-1/
n*a^(3/2)*ln((2*a+b*exp(n*ln(x))+2*a^(1/2)*(a+b*exp(n*ln(x))+c*exp(n*ln(x))
^2)^(1/2))/exp(n*ln(x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

Fricas [A]
time = 0.45, size = 827, normalized size = 4.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(
sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) - 3*(b^3 -
 12*a*b*c)*sqrt(c)*log(-8*c^2*x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3
/2)*x^n + b*sqrt(c))*sqrt(c*x^(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b
*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(
24*a^(3/2)*c^2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)
*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 3*(b^3 - 12*a*b
*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n + b*sqrt(-c))*sqrt(c*x^(2*n) + b*
x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2*(8*c^3*x^(2*n) + 14*b*c^2*x^n +
 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/96*(96*sqrt(-a
)*a*c^2*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n +
 a)/(a*c*x^(2*n) + a*b*x^n + a^2)) - 3*(b^3 - 12*a*b*c)*sqrt(c)*log(-8*c^2*
x^(2*n) - 8*b*c*x^n - b^2 - 4*a*c - 4*(2*c^(3/2)*x^n + b*sqrt(c))*sqrt(c*x^
(2*n) + b*x^n + a)) + 4*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)
*sqrt(c*x^(2*n) + b*x^n + a))/(c^2*n), 1/48*(48*sqrt(-a)*a*c^2*arctan(1/2*(
sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2*n) + a
*b*x^n + a^2)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*arctan(1/2*(2*sqrt(-c)*c*x^n +
 b*sqrt(-c))*sqrt(c*x^(2*n) + b*x^n + a)/(c^2*x^(2*n) + b*c*x^n + a*c)) + 2
*(8*c^3*x^(2*n) + 14*b*c^2*x^n + 3*b^2*c + 32*a*c^2)*sqrt(c*x^(2*n) + b*x^n
 + a))/(c^2*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(3/2)/x,x)
```

```
[Out] int((a + b*x^n + c*x^(2*n))^(3/2)/x, x)
```

3.580 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx$

Optimal. Leaf size=150

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{3}{2},-\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -a*AppellF1(-1/n,-3/2,-3/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x/(1+2*c*x^n/(b-(-4*a*c
+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{3}{2},-\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] -((a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-n^(-1), -3/2, -3/2, -((1 - n)/n)
, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
(x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
```

```
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^2}\,dx = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int \frac{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^2}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{1}{n};-\frac{3}{2},-\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 526 vs. 2(150) = 300.
time = 1.01, size = 526, normalized size = 3.51

$$\frac{2(-1+n)\left(4a^2c(1-6n+8n^2)+x^n(b+cx^n)\left(3b^2n^2+2bc(2-9n+7n^2)x^n+4c^2(1-3n+2n^2)x^{2n}\right)+a(3b^2n^2+2bc(4-21n+23n^2)x^n+4c^2(2-9n+10n^2)x^{2n})\right)-6a(-1+n)n^2(b^2+4ac(-1+2n))\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};\frac{-1+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)-3b(4ac(2-3n)+b^2(-2+n))n^2x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{-1+n}{n};\frac{1}{2},\frac{1}{2};2-\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{8c(-1+n)^2(-1+2n)(-1+3n)x\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^2,x]
```

```
[Out] (2*(-1 + n)*(4*a^2*c*(1 - 6*n + 8*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*b*c
*(2 - 9*n + 7*n^2)*x^n + 4*c^2*(1 - 3*n + 2*n^2)*x^(2*n)) + a*(3*b^2*n^2 +
2*b*c*(4 - 21*n + 23*n^2)*x^n + 4*c^2*(2 - 9*n + 10*n^2)*x^(2*n))) - 6*a*(-
1 + n)*n^2*(b^2 + 4*a*c*(-1 + 2*n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/
(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*(4*a*c*(2 - 3*n) +
 b^2*(-2 + n))*n^2*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]
*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(8*c*(-1 + n)^2*(-1 + 2*n)*(-1 +
3*n)*x*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**2,x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(3/2)/x^2,x)
[Out] int((a + b*x^n + c*x^(2*n))^(3/2)/x^2, x)
```

3.581 $\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx$

Optimal. Leaf size=152

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{2}{n};-\frac{3}{2},-\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] -1/2*a*AppellF1(-2/n,-3/2,-3/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*
c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/x^2/(1+2*c*x^n/(b-(
-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$-\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{2}{n};-\frac{3}{2},-\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] -1/2*(a*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[-2/n, -3/2, -3/2, -((2 - n)/n)
, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/
(x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
```

```
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(a+bx^n+cx^{2n})^{3/2}}{x^3}\,dx = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int \frac{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{x^3}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= -\frac{a\sqrt{a+bx^n+cx^{2n}}\,F_1\left(-\frac{2}{n};-\frac{3}{2},-\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 520 vs. 2(152) = 304.
time = 1.03, size = 520, normalized size = 3.42

$$\frac{2(-2+n)\left(16a^2c(1-3n+2n^2)+x^n(b+cx^n)\left(3b^2n^2+2bc(8-18n+7n^2)x^n+8c^2(2-3n+n^2)x^{2n}\right)+a\left(3b^2n^2+2bc(16-42n+23n^2)x^n+8c^2(4-9n+5n^2)x^{2n}\right)\right)-6a\left(b^2+4ac(-1+n)\right)(-2+n)n^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{-2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)-3b\left(4ac(4-3n)+b^2(-4+n)\right)n^2x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{-2+n}{n};\frac{1}{2},\frac{1}{2};2-\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{16c(-2+n)^2(-1+n)(-2+3n)x^2\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(3/2)/x^3,x]
```

```
[Out] (2*(-2 + n)*(16*a^2*c*(1 - 3*n + 2*n^2) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*b*
c*(8 - 18*n + 7*n^2)*x^n + 8*c^2*(2 - 3*n + n^2)*x^(2*n)) + a*(3*b^2*n^2 +
2*b*c*(16 - 42*n + 23*n^2)*x^n + 8*c^2*(4 - 9*n + 5*n^2)*x^(2*n))) - 6*a*(b
^2 + 4*a*c*(-1 + n))*(-2 + n)*n^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 3*b*(4*a*c*(4 - 3*n) + b^2*
(-4 + n))*n^2*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*
a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*Appe
llF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])/(16*c*(-2 + n)^2*(-1 + n)*(-2 + 3*n)*x^2*
Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
```

```
[Out] int((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n+c*x**(2*n))**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(3/2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n+c*x^(2*n))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)/x^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n + c*x^(2*n))^(3/2)/x^3,x)
[Out] int((a + b*x^n + c*x^(2*n))^(3/2)/x^3, x)
```

3.582 $\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=148

$$\frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/4*x^4*AppellF1(4/n,1/2,1/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1399, 524}

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+}}}{\sqrt{a+bx^n+cx^{2n}}}$$
$$= \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},\right.}{4\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A]
time = 0.16, size = 175, normalized size = 1.18

$$\frac{x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{4\sqrt{a+x^n\left(b+cx^n\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n, 1/2
, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/(4*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+b\,x^n+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a + bx^n + cx^{2n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{a + b\,x^n + c\,x^{2\,n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^3/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.583 $\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=148

$$\frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/3*x^3*AppellF1(3/n,1/2,1/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^2}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1 ...}}}{\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}} ...\right.}{3\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A]
time = 0.14, size = 175, normalized size = 1.18

$$\frac{x^3\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{3\sqrt{a+x^n\left(b+cx^n\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^3*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n, 1/2
, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/(3*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+b\,x^n+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a + bx^n + cx^{2n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{a + b\,x^n + c\,x^{2\,n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

```
[Out] int(x^2/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.584 $\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=148

$$\frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/2*x^2*AppellF1(2/n,1/2,1/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*Sqrt[a + b*x^n + c*x^
(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+}}}{\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},\right.}{2\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A]
time = 0.14, size = 175, normalized size = 1.18

$$\frac{x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{2\sqrt{a+x^n\,(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x^n + c*x^(2*n)],x]
[Out] (x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n, 1/2
, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])])/(2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+b\,x^n+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int(x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx^n + cx^{2n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral(x/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a + b\,x^n + c\,x^{2\,n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int(x/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.585 $\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=139

$$\frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] x*AppellF1(1/n,1/2,1/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1362, 440}

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[a + b*x^n + c*x^(
2*n)]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1}}}{\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4a}}\right.}{\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A]
time = 0.10, size = 166, normalized size = 1.19

$$\frac{x\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{a+x^n\,(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[n^(-1), 1/
2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqr
t[b^2 - 4*a*c])])/Sqrt[a + x^n*(b + c*x^n)]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,x^n+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^n + cx^{2n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b\,x^n + c\,x^{2\,n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n + c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.586 $\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}\,n}$$

```
[Out] -arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/n/a^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1371, 738, 212}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(Sqrt[a]*n
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^n\right)}{n}$$
$$= -\frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{n}$$
$$= -\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{\sqrt{a}\,n}$$

Mathematica [A]
time = 0.12, size = 46, normalized size = 0.98

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x^n-\sqrt{a+x^n(b+cx^n)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^n + c*x^(2*n)]),x]
[Out] (2*ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^n*(b + c*x^n)])/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+b\,x^n+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

Fricas [A]
time = 0.35, size = 148, normalized size = 3.15

$$\left[\frac{\log\left(-\frac{8\,abx^n+8\,a^2+(b^2+4\,ac)x^{2\,n}-4\left(\sqrt{a}\,bx^n+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+bx^n+a}}{x^{2\,n}}\right)}{2\,\sqrt{a}\,n}, \frac{\sqrt{-a}\,\arctan\left(\frac{\left(\sqrt{-a}\,bx^n+2\,\sqrt{-a}\,a\right)\sqrt{cx^{2\,n}+bx^n+a}}{2\,(acx^{2\,n}+abx^n+a^2)}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) - 4*(sqrt(a)*b*x^n + 2
*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n))/(sqrt(a)*n), sqrt(-a)*arcta
n(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*n) + b*x^n + a)/(a*c*x^(2
*n) + a*b*x^n + a^2))/(a*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**n + c*x**(2*n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

3.587 $\int \frac{1}{x^2 \sqrt{a+bx^n+cx^{2n}}} \, dx$

Optimal. Leaf size=149

$$-\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -AppellF1(-1/n,1/2,1/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/x/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x*Sqrt[a + b*x^n +
 c*x^(2*n)]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\cdots}}{\sqrt{a+bx^n+cx^{2n}}}$$
$$= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-\cdots}}\cdots\right)}{x\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A]
time = 0.15, size = 173, normalized size = 1.16

$$-\frac{\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};\frac{-1+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{x\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -((Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1
/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(x*Sqrt[a + x^n*(b + c*x^n)]))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+b\,x^n+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a + bx^n + cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**n + c*x**(2*n))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n + c*x^(2*n))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

3.588 $\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=151

$$-\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -1/2*AppellF1(-2/n,1/2,1/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/x^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -1/2*(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, -((2 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(x^2*Sqrt[a + b*x^n
 + c*x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\ \sqrt{\ldots}}}{\sqrt{a+bx^n+cx^{2n}}}$$
$$= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-\ldots}}\ldots\right)}{2x^2\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [A]
time = 0.15, size = 175, normalized size = 1.16

$$-\frac{\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{-2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{2x^2\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^n + c*x^(2*n)]),x]
```

```
[Out] -1/2*(Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1
/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])])/(x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+b\,x^n+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{a+bx^n+cx^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**n + c*x**(2*n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^(2*n) + b*x^n + a)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{a+b\,x^n+c\,x^{2\,n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n + c*x^(2*n))^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^n + c*x^(2*n))^(1/2)), x)
```

3.589 $\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{n};\frac{3}{2},\frac{3}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/4*x^4*AppellF1(4/n,3/2,3/2,(4+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{x^4\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{4}{n};\frac{3}{2},\frac{3}{2};\frac{n+4}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{4a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[4/n, 3/2, 3/2, (4 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{x^3}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^3}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\cdots\right.}}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{x^4\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{4}{n};\frac{3}{2},\frac{3}{2};\frac{4+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4a\cdots}}\right.}{4a\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(151) = 302.
time = 0.68, size = 398, normalized size = 2.64

$$\frac{x^4\left(-8(4+n)\left(b^2-2ac+bcx^n\right)-\left(b^2(-8+n)-4ac(-4+n)\right)(4+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4}{n};\frac{1}{2},\frac{1}{2};\frac{4+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+32bcx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{4+n}{n};\frac{1}{2},\frac{1}{2};2+\frac{4}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{4a\left(-b^2+4ac\right)n(4+n)\sqrt{a+x^n\left(b+cx^n\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^4*(-8*(4 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-8 + n) - 4*a*c*(-4 + n))*
(4 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[4/n,
1/2, 1/2, (4 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + 32*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])]*AppellF1[(4 + n)/n, 1/2, 1/2, 2 + 4/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(4*a*(-b^2 + 4*a*c)*n*(4 + n
)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + bx^n + cx^{2n})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(x**3/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(x^3/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(a + b\,x^n + c\,x^{2\,n}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(x^3/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.590 $\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{n};\frac{3}{2},\frac{3}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/3*x^3*AppellF1(3/n,3/2,3/2,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{x^3\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{n};\frac{3}{2},\frac{3}{2};\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{3a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[3/n, 3/2, 3/2, (3 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x^2}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{}{b+}\right.}}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{x^3\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{n};\frac{3}{2},\frac{3}{2};\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right.}{3a\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(151) = 302.
time = 0.63, size = 398, normalized size = 2.64

$$\frac{x^3\left(-6(3+n)\left(b^2-2ac+bcx^n\right)-\left(b^2(-6+n)-4ac(-3+n)\right)(3+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{n};\frac{1}{2},\frac{1}{2};\frac{3+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+18bcx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3+n}{n};\frac{1}{2},\frac{1}{2};2+\frac{3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{3a\left(-b^2+4ac\right)n(3+n)\sqrt{a+x^n\left(b+cx^n\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^3*(-6*(3 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-6 + n) - 4*a*c*(-3 + n))*
(3 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/n,
1/2, 1/2, (3 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + 18*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
4*a*c])]*AppellF1[(3 + n)/n, 1/2, 1/2, 2 + 3/n, (-2*c*x^n)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(3*a*(-b^2 + 4*a*c)*n*(3 + n
)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + bx^n + cx^{2n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(x**2/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(x^2/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.591 $\int \frac{x}{\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=151

$$\frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{n};\frac{3}{2},\frac{3}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] 1/2*x^2*AppellF1(2/n,3/2,3/2,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*
x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1399, 524}

$$\frac{x^2\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{n};\frac{3}{2},\frac{3}{2};\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sq
rt[b^2 - 4*a*c])]*AppellF1[2/n, 3/2, 3/2, (2 + n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{x}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{x}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\cdots\right.}}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{x^2\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{2}{n};\frac{3}{2},\frac{3}{2};\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\cdots\right.}{2a\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 398 vs. 2(151) = 302.

time = 0.62, size = 398, normalized size = 2.64

$$\frac{x^2\left(-4(2+n)\left(b^2-2ac+bcx^n\right)-\left(b^2(-4+n)-4ac(-2+n)\right)(2+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+8bcx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{2+n}{n};\frac{1}{2},\frac{1}{2};2+\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{2a\left(-b^2+4ac\right)n(2+n)\sqrt{a+x^n\left(b+cx^n\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x^2*(-4*(2 + n)*(b^2 - 2*a*c + b*c*x^n) - (b^2*(-4 + n) - 4*a*c*(-2 + n))*
(2 + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/n,
1/2, 1/2, (2 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])] + 8*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])]*AppellF1[(2 + n)/n, 1/2, 1/2, 2 + 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(2*a*(-b^2 + 4*a*c)*n*(2 + n)
*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^n+cx^{2n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(x/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(x/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int(x/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.592 $\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] x*AppellF1(1/n,3/2,3/2,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-
4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/a/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1362, 440}

$$\frac{x\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{a\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[n^(-1), 3/2, 3/2, 1 + n^(-1), (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[a + b*x^n + c*
x^(2*n)])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{}{b+}\right.}}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{x\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{n};\frac{3}{2},\frac{3}{2};1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4a}}\right.}{a\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 384 vs. 2(142) = 284.
time = 0.68, size = 384, normalized size = 2.70

$$\frac{x\left(2bcx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(1+\frac{1}{n};\frac{1}{2},\frac{1}{2};2+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)-(1+n)\left(2(b^2-2ac+bcx^n)+(b^2(-2+n)-4ac(-1+n))\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{n};\frac{1}{2},\frac{1}{2};1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{a(-b^2+4ac)\,n(1+n)\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n + c*x^(2*n))^(-3/2),x]
```

```
[Out] (x*(2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]
)]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1
[1 + n^(-1), 1/2, 1/2, 2 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - (1 + n)*(2*(b^2 - 2*a*c + b*c*x^n) + (b^2
*(-2 + n) - 4*a*c*(-1 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[n^(-1), 1/2, 1/2, 1 + n^(-1), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*n*(1 + n)*Sq
rt[a + x^n*(b + c*x^n)])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx^{n} + cx^{2n}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral((a + b*x**n + c*x**(2*n))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x^n + c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.593 $\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{2(b^2-2ac+bcx^n)}{a\,(b^2-4ac)\,n\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\,\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

```
[Out] -arctanh(1/2*(2*a+b*x^n)/a^(1/2)/(a+b*x^n+c*x^(2*n))^(1/2))/a^(3/2)/n+2*(b^
2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/n/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1371, 754, 12, 738, 212}

$$\frac{2(-2ac+b^2+bcx^n)}{an\,(b^2-4ac)\,\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\,\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*n*Sqrt[a + b*x^n + c*x^(2*n)])
 - ArcTanh[(2*a + b*x^n)/(2*Sqrt[a]*Sqrt[a + b*x^n + c*x^(2*n)])]/(a^(3/2)*
n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1371

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^{3/2}}\,dx,x,x^n\right)}{n}\\
&= \frac{2(b^2-2ac+bcx^n)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}}-\frac{2\text{Subst}\left(\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx,x,x^n\right)}{a\left(b^2-4ac\right)n}\\
&= \frac{2(b^2-2ac+bcx^n)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}}+\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^n\right)}{an}\\
&= \frac{2(b^2-2ac+bcx^n)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}}-\frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^n}{\sqrt{a+bx^n+cx^{2n}}}\right)}{an}\\
&= \frac{2(b^2-2ac+bcx^n)}{a\left(b^2-4ac\right)n\sqrt{a+bx^n+cx^{2n}}}-\frac{\tanh^{-1}\left(\frac{2a+bx^n}{2\sqrt{a}\sqrt{a+bx^n+cx^{2n}}}\right)}{a^{3/2}n}
\end{aligned}$$

Mathematica [A]
time = 0.63, size = 98, normalized size = 1.00

$$\frac{2\left(-\frac{\sqrt{a}\,(-b^2+2ac-bcx^n)}{(b^2-4ac)\sqrt{a+x^n\left(b+cx^n\right)}}+\tanh^{-1}\left(\frac{\sqrt{c}\,x^n-\sqrt{a+x^n\left(b+cx^n\right)}}{\sqrt{a}}\right)\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] (2*(-((Sqrt[a]*(-b^2 + 2*a*c - b*c*x^n))/((b^2 - 4*a*c)*Sqrt[a + x^n*(b + c
*x^n)])) + ArcTanh[(Sqrt[c]*x^n - Sqrt[a + x^n*(b + c*x^n)])/Sqrt[a]]))/(a^
(3/2)*n)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,x^{n}+c\,x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(88) = 176.
time = 0.46, size = 449, normalized size = 4.58

$$\left[\frac{\left((b^2c-4\,ac^2)\sqrt{a}\,x^{2\,n}+(b^3-4\,abc)\sqrt{a}\,x^n+(ab^2-4\,a^2c)\sqrt{a}\right)\log\left(-\frac{8\,abx^n+8\,a^2+(b^2+4\,ac)x^{2\,n}-4\left(\sqrt{a}\,bx^n+2\,a^{\frac{3}{2}}\right)\sqrt{cx^{2\,n}+bx^n+a}}{x^{2\,n}}\right)+4\,(abcx^n+ab^2-2\,a^2c)\sqrt{cx^{2\,n}+bx^n+a}}{2\left((a^2b^2c-4\,a^3c^2)nx^{2\,n}+(a^2b^3-4\,a^3bc)nx^n+(a^3b^2-4\,a^4c)n\right)},\ \frac{\left((b^2c-4\,ac^2)\sqrt{-a}\,x^{2\,n}+(b^3-4\,abc)\sqrt{-a}\,x^n+(ab^2-4\,a^2c)\sqrt{-a}\right)\arctan\left(\frac{\left(\sqrt{-a}\,bx^n+2\,\sqrt{-a}\,a\right)\sqrt{cx^{2\,n}+bx^n+a}}{2\,(acx^{2\,n}+abx^n+a^2)}\right)+2\,(abcx^n+ab^2-2\,a^2c)\sqrt{cx^{2\,n}+bx^n+a}}{(a^2b^2c-4\,a^3c^2)nx^{2\,n}+(a^2b^3-4\,a^3bc)nx^n+(a^3b^2-4\,a^4c)n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*sqrt(a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(a)*x^n + (a
*b^2 - 4*a^2*c)*sqrt(a))*log(-(8*a*b*x^n + 8*a^2 + (b^2 + 4*a*c)*x^(2*n) -
4*(sqrt(a)*b*x^n + 2*a^(3/2))*sqrt(c*x^(2*n) + b*x^n + a))/x^(2*n)) + 4*(a*
b*c*x^n + a*b^2 - 2*a^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4*a^3
*c^2)*n*x^(2*n) + (a^2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n), (((
b^2*c - 4*a*c^2)*sqrt(-a)*x^(2*n) + (b^3 - 4*a*b*c)*sqrt(-a)*x^n + (a*b^2 -
 4*a^2*c)*sqrt(-a))*arctan(1/2*(sqrt(-a)*b*x^n + 2*sqrt(-a)*a)*sqrt(c*x^(2*
n) + b*x^n + a)/(a*c*x^(2*n) + a*b*x^n + a^2)) + 2*(a*b*c*x^n + a*b^2 - 2*a
```

```
^2*c)*sqrt(c*x^(2*n) + b*x^n + a))/((a^2*b^2*c - 4*a^3*c^2)*n*x^(2*n) + (a^
2*b^3 - 4*a^3*b*c)*n*x^n + (a^3*b^2 - 4*a^4*c)*n)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(1/(x*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)),x)
[Out] int(1/(x*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

3.594 $\int \frac{1}{x^2(a+bx^n+cx^{2n})^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{n};\frac{3}{2},\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -AppellF1(-1/n,3/2,3/2,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b
+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^n/(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/x/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{n};\frac{3}{2},\frac{3}{2};-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ax\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]*AppellF1[-n^(-1), 3/2, 3/2, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*x*Sqrt[a + b*x^n
 + c*x^(2*n)]))
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{1}{x^2 \left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^2\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}}}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{n};\frac{3}{2},\frac{3}{2};-\frac{1-n}{n};-\frac{}{b-\sqrt{b}}\right.}{ax\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 395 vs. 2(152) = 304.
time = 0.59, size = 395, normalized size = 2.60

$$\frac{(-1+n)\left(-4ac(1+n)+b^2(2+n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{1}{n};\frac{1}{2},\frac{1}{2};\frac{-1+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)-2\left((-1+n)\left(b^2-2ac+bcx^n\right)+bcx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{-1+n}{n};\frac{1}{2},\frac{1}{2};2-\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{a\left(-b^2+4ac\right)(-1+n)nx\sqrt{a+x^n\left(b+cx^n\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] ((-1 + n)*(-4*a*c*(1 + n) + b^2*(2 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-n^(-1), 1/2, 1/2, (-1 + n)/n, (-2*c*x^n)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 2*((-1 + n)*(b^2
- 2*a*c + b*c*x^n) + b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c])]*AppellF1[(-1 + n)/n, 1/2, 1/2, 2 - n^(-1), (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(a*(-b^2 + 4*a*c)*(-1 + n)
*n*x*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\left(a+bx^{n}+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(1/(x**2*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)),x)
[Out] int(1/(x^2*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

3.595 $\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx$

Optimal. Leaf size=154

$$-\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(-\frac{2}{n};\frac{3}{2},\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] -1/2*AppellF1(-2/n,3/2,3/2,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^
n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x
^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/x^2/(a+b*x^n+c*x^(2*n))^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$-\frac{\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{2}{n};\frac{3}{2},\frac{3}{2};-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] -1/2*(Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-2/n, 3/2, 3/2, -((2 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*x^2*Sqrt[a + b*x
^n + c*x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{1}{x^3 \left(a+bx^n+cx^{2n}\right)^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{1}{x^3\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(\right.}}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= -\frac{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{n};\frac{3}{2},\frac{3}{2};-\frac{2-n}{n};-\frac{2c}{b-\sqrt{b^2}}\right.}{2ax^2\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 399 vs. 2(154) = 308.
time = 0.60, size = 399, normalized size = 2.59

$$\frac{(-2+n)\left(-4ac(2+n)+b^2(4+n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{2}{n};\frac{1}{2},\frac{1}{2};\frac{-2+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)-4\left((-2+n)\left(b^2-2ac+bcx^n\right)+2bcx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{-2+n}{n};\frac{1}{2},\frac{1}{2};2-\frac{2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{2a\left(-b^2+4ac\right)(-2+n)nx^2\sqrt{a+x^n\left(b+cx^n\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x]
```

```
[Out] ((-2 + n)*(-4*a*c*(2 + n) + b^2*(4 + n))*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-2/n, 1/2, 1/2, (-2 + n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] - 4*((-2 + n)*(b^2 - 2
*a*c + b*c*x^n) + 2*b*c*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]*AppellF1[(-2 + n)/n, 1/2, 1/2, 2 - 2/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]))/(2*a*(-b^2 + 4*a*c)*(-2 + n)*n
*x^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^n+cx^{2n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral(1/(x**3*(a + b*x**n + c*x**(2*n))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate(1/((c*x^(2*n) + b*x^n + a)^(3/2)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)),x)
[Out] int(1/(x^3*(a + b*x^n + c*x^(2*n))^(3/2)), x)
```

3.596 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3\,dx$

Optimal. Leaf size=182

$$\frac{3a^2bx^{1+n}(dx)^m}{1+m+n}+\frac{3a(b^2+ac)\,x^{1+2n}(dx)^m}{1+m+2n}+\frac{b(b^2+6ac)\,x^{1+3n}(dx)^m}{1+m+3n}+\frac{3c(b^2+ac)\,x^{1+4n}(dx)^m}{1+m+4n}+\frac{3bc^2x^{1+5n}(dx)^m}{1+m+5n}$$

```
[Out] 3*a^2*b*x^(1+n)*(d*x)^m/(1+m+n)+3*a*(a*c+b^2)*x^(1+2*n)*(d*x)^m/(1+m+2*n)+b
*(6*a*c+b^2)*x^(1+3*n)*(d*x)^m/(1+m+3*n)+3*c*(a*c+b^2)*x^(1+4*n)*(d*x)^m/(1
+m+4*n)+3*b*c^2*x^(1+5*n)*(d*x)^m/(1+m+5*n)+c^3*x^(1+6*n)*(d*x)^m/(1+m+6*n)
+a^3*(d*x)^(1+m)/d/(1+m)
```

Rubi [A]
time = 0.11, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1367, 20, 30}

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2bx^{n+1}(dx)^m}{m+n+1}+\frac{3ax^{2n+1}(ac+b^2)\,(dx)^m}{m+2n+1}+\frac{bx^{3n+1}(6ac+b^2)\,(dx)^m}{m+3n+1}+\frac{3cx^{4n+1}(ac+b^2)\,(dx)^m}{m+4n+1}+\frac{3bc^2x^{5n+1}(dx)^m}{m+5n+1}+\frac{c^3x^{6n+1}(dx)^m}{m+6n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] (3*a^2*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(1 + 2*n)*(d*x
)^m)/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) +
(3*c*(b^2 + a*c)*x^(1 + 4*n)*(d*x)^m)/(1 + m + 4*n) + (3*b*c^2*x^(1 + 5*n)*
(d*x)^m)/(1 + m + 5*n) + (c^3*x^(1 + 6*n)*(d*x)^m)/(1 + m + 6*n) + (a^3*(d*
x)^(1 + m))/(d*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] &&  !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^3\,dx = \int \left(a^3(dx)^m+3a^2bx^n(dx)^m+3ab^2\left(1+\frac{ac}{b^2}\right)x^{2n}(dx)^m+b^3\left(1+\frac{6ac}{b^2}\right)x^3\right.$$
$$= \frac{a^3(dx)^{1+m}}{d(1+m)}+\left(3a^2b\right)\int x^n(dx)^m\,dx+\left(3bc^2\right)\int x^{5n}(dx)^m\,dx+c^3\int x^{6n}(dx$$
$$= \frac{a^3(dx)^{1+m}}{d(1+m)}+\left(3a^2bx^{-m}(dx)^m\right)\int x^{m+n}\,dx+\left(3bc^2x^{-m}(dx)^m\right)\int x^{m+5n}\,dx$$
$$= \frac{3a^2bx^{1+n}(dx)^m}{1+m+n}+\frac{3a(b^2+ac)\,x^{1+2n}(dx)^m}{1+m+2n}+\frac{b(b^2+6ac)\,x^{1+3n}(dx)^m}{1+m+3n}+\frac{3c}{}$$

Mathematica [A]
time = 0.56, size = 137, normalized size = 0.75

$$x(dx)^m\left(\frac{a^3}{1+m}+\frac{3a^2bx^n}{1+m+n}+\frac{3a(b^2+ac)\,x^{2n}}{1+m+2n}+\frac{b(b^2+6ac)\,x^{3n}}{1+m+3n}+\frac{3c(b^2+ac)\,x^{4n}}{1+m+4n}+\frac{3bc^2x^{5n}}{1+m+5n}+\frac{c^3x^{6n}}{1+m+6n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] x*(d*x)^m*(a^3/(1 + m) + (3*a^2*b*x^n)/(1 + m + n) + (3*a*(b^2 + a*c)*x^(2*
n))/(1 + m + 2*n) + (b*(b^2 + 6*a*c)*x^(3*n))/(1 + m + 3*n) + (3*c*(b^2 + a
*c)*x^(4*n))/(1 + m + 4*n) + (3*b*c^2*x^(5*n))/(1 + m + 5*n) + (c^3*x^(6*n)
)/(1 + m + 6*n))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.08, size = 3798, normalized size = 20.87

method	result	size
risch	Expression too large to display	3798

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x,method=_RETURNVERBOSE)
```

```
[Out] x*(2904*a*b*c*m*n^2*(x^n)^3+b^3*(x^n)^3+726*b^3*m^2*n^2*(x^n)^3+1080*a^2*c*
m*n^5*(x^n)^2+57*a*b^2*m^5*n*(x^n)^2+411*a*b^2*m^4*n^2*(x^n)^2+1383*a*b^2*m
^3*n^3*(x^n)^2+2106*a*b^2*m^2*n^4*(x^n)^2+c^3*m^6*(x^n)^6+6*c^3*m^5*(x^n)^6
+1383*a^2*c*n^3*(x^n)^2+1383*a*b^2*n^3*(x^n)^2+18*a*c^2*(x^n)^4*m+51*a*c^2*
(x^n)^4*n+18*b^2*c*(x^n)^4*m+51*b^2*c*(x^n)^4*n+1740*a^2*b*n^3*x^n+45*a^2*c
*m^2*(x^n)^2+508*b^3*n^4*(x^n)^3+240*b^3*n^5*(x^n)^3+15*c^3*m^2*(x^n)^6+85*
c^3*n^2*(x^n)^6+15*b^3*m^4*(x^n)^3+340*c^3*m*n^2*(x^n)^6+3*a^2*c*m^6*(x^n)^
2+3*a*b^2*m^6*(x^n)^2+45*a^2*b*m^2*x^n+45*a*c^2*m^4*(x^n)^4+1188*a*c^2*n^4*
```

(x^n)^4+90*b^3*m^4*n*(x^n)^3+484*b^3*m^3*n^2*(x^n)^3+1116*b^3*m^2*n^3*(x^n)
^3+1016*b^3*m*n^4*(x^n)^3+a^3+21*a^3*m^5*n+175*a^3*m^4*n^2+735*a^3*m^3*n^3+
1624*a^3*m^2*n^4+1764*a^3*m*n^5+105*a^3*m^4*n+700*a^3*m^3*n^2+2205*a^3*m^2*
n^3+3248*a^3*m*n^4+60*a^2*b*m^3*x^n+15*c^3*m^5*n*(x^n)^6+85*c^3*m^4*n^2*(x^
n)^6+225*c^3*m^3*n^3*(x^n)^6+274*c^3*m^2*n^4*(x^n)^6+120*c^3*m*n^5*(x^n)^6+
3*b*c^2*m^6*(x^n)^5+75*c^3*m^4*n*(x^n)^6+340*c^3*m^3*n^2*(x^n)^6+675*c^3*m^
2*n^3*(x^n)^6+548*c^3*m*n^4*(x^n)^6+180*b^3*m^2*n*(x^n)^3+a^3*m^6+6*a^3*m^5
+1764*a^3*n^5+15*a^3*m^4+1624*a^3*n^4+15*c^3*m^4*(x^n)^6+274*c^3*n^4*(x^n)^
6+b^3*m^6*(x^n)^3+540*a*c^2*n^5*(x^n)^4+720*a^3*n^6+570*a^2*c*m^2*n*(x^n)^2
+1644*a^2*c*m*n^2*(x^n)^2+90*a*b*c*m^2*(x^n)^3+726*a*b*c*n^2*(x^n)^3+285*a^
2*c*m*n*(x^n)^2+36*a*b*c*(x^n)^3*m+108*a*b*c*(x^n)^3*n+48*b*c^2*m^5*n*(x^n)
^5+6*m*c^3*(x^n)^6+411*a^2*c*n^2*(x^n)^2+18*a^2*c*(x^n)^2*m+57*a^2*c*(x^n)^
2*n+1080*a^2*c*n^5*(x^n)^2+18*a*b^2*m^5*(x^n)^2+1080*a*b^2*n^5*(x^n)^2+60*a
*c^2*m^3*(x^n)^4+921*a*c^2*n^3*(x^n)^4+225*c^3*n^3*(x^n)^6+18*b^3*m^5*n*(x^
n)^3+726*a*b*c*m^4*n^2*(x^n)^3+2232*a*b*c*m^3*n^3*(x^n)^3+6*a*b*c*(x^n)^3+2
0*c^3*m^3*(x^n)^6+18*m*a^2*b*x^n+45*a*c^2*m^2*(x^n)^4+321*a*c^2*n^2*(x^n)^4
+45*b^2*c*m^2*(x^n)^4+321*b^2*c*n^2*(x^n)^4+18*m*b*c^2*(x^n)^5+48*b*c^2*(x^
n)^5*n+45*a^2*b*m^4*x^n+60*a^2*b*n*x^n+3132*a^2*b*n^4*x^n+60*a^2*c*m^3*(x^n
)^2+180*b^3*m^3*n*(x^n)^3+c^3*(x^n)^6+120*c^3*n^5*(x^n)^6+20*b^3*m^3*(x^n)^
3+15*b^3*m^2*(x^n)^3+121*b^3*n^2*(x^n)^3+6*m*b^3*(x^n)^3+18*b^3*(x^n)^3*n+2
0*a^3*m^3+15*a^3*m^2+175*a^3*n^2+21*a^3*n+3*(x^n)^5*b*c^2+3*(x^n)^4*c^2*a+2
10*a^3*m^3*n+1050*a^3*m^2*n^2+2205*a^3*m*n^3+36*a*b*c*m^5*(x^n)^3+1440*a*b*
c*n^5*(x^n)^3+510*a*c^2*m^3*n*(x^n)^4+6696*a*b*c*m^2*n^3*(x^n)^3+6096*a*b*c
*m*n^4*(x^n)^3+1080*a*b*c*m^3*n*(x^n)^3+1116*b^3*m*n^3*(x^n)^3+60*b^2*c*m^3
*(x^n)^4+921*b^2*c*n^3*(x^n)^4+45*b*c^2*m^2*(x^n)^5+285*b*c^2*n^2*(x^n)^5+1
8*a^2*b*m^5*x^n+2160*a^2*b*n^5*x^n+45*a^2*c*m^4*(x^n)^2+2106*a^2*c*n^4*(x^n
)^2+45*a*b^2*m^4*(x^n)^2+2106*a*b^2*n^4*(x^n)^2+570*a*b^2*m^2*n*(x^n)^2+164
4*a*b^2*m*n^2*(x^n)^2+600*a^2*b*m^2*n*x^n+285*a*b^2*m*n*(x^n)^2+3*(x^n)^2*a
*b^2+540*a*b*c*m*n*(x^n)^3+6*m*a^3+3*a*c^2*m^6*(x^n)^4+3*b^2*c*m^6*(x^n)^4+
18*b*c^2*m^5*(x^n)^5+432*b*c^2*n^5*(x^n)^5+150*c^3*m^3*n*(x^n)^6+510*c^3*m^
2*n^2*(x^n)^6+675*c^3*m*n^3*(x^n)^6+18*a*c^2*m^5*(x^n)^4+45*a*b^2*m^2*(x^n)
^2+411*a*b^2*n^2*(x^n)^2+18*m*a*b^2*(x^n)^2+4356*a*b*c*m^2*n^2*(x^n)^3+6696
*a*b*c*m*n^3*(x^n)^3+1080*a*b*c*m^2*n*(x^n)^3+121*b^3*m^4*n^2*(x^n)^3+372*b
^3*m^3*n^3*(x^n)^3+508*b^3*m^2*n^4*(x^n)^3+240*b^3*m*n^5*(x^n)^3+18*b^2*c*m
^5*(x^n)^4+540*b^2*c*n^5*(x^n)^4+45*b*c^2*m^4*(x^n)^5+972*b*c^2*n^4*(x^n)^5
+150*c^3*m^2*n*(x^n)^6+210*a^3*m^2*n+700*a^3*m*n^2+105*a^3*m*n+6*b^3*m^5*(x
^n)^3+57*a*b^2*(x^n)^2*n+321*b^2*c*m^4*n^2*(x^n)^4+921*b^2*c*m^3*n^3*(x^n)^
4+1188*b^2*c*m^2*n^4*(x^n)^4+540*b^2*c*m*n^5*(x^n)^4+240*b*c^2*m^4*n*(x^n)^
5+1140*b*c^2*m^3*n^2*(x^n)^5+2340*b*c^2*m^2*n^3*(x^n)^5+1944*b*c^2*m*n^4*(x
^n)^5+6*a*b*c*m^6*(x^n)^3+255*a*c^2*m^4*n*(x^n)^4+1284*a*c^2*m^3*n^2*(x^n)^
4+2763*a*c^2*m^2*n^3*(x^n)^4+3048*a*b*c*m^2*n^4*(x^n)^3+1440*a*b*c*m*n^5*(x
^n)^3+540*a*b*c*m^4*n*(x^n)^3+2904*a*b*c*m^3*n^2*(x^n)^3+108*a*b*c*m^5*n*(x
^n)^3+1926*a*c^2*m^2*n^2*(x^n)^4+2763*a*c^2*m*n^3*(x^n)^4+510*b^2*c*m^3*n*(
x^n)^4+1926*b^2*c*m^2*n^2*(x^n)^4+2763*b^2*c*m*n^3*(x^n)^4+480*b*c^2*m^2*n*
(x^n)^5+1140*b*c^2*m*n^2*(x^n)^5+60*a^2*b*m^5*n*x^n+465*a^2*b*m^4*n^2*x^n+1

```
740*a^2*b*m^3*n^3*x^n+3132*a^2*b*m^2*n^4*x^n+2160*a^2*b*m*n^5*x^n+15*c^3*(x
^n)^6*n+372*b^3*n^3*(x^n)^3+3*a^2*b*x^n+3*(x^n)^4*b^2*c+3*(x^n)^2*a^2*c+45*
b^2*c*m^4*(x^n)^4+1188*b^2*c*n^4*(x^n)^4+60*b*c^2*m^3*(x^n)^5+780*b*c^2*n^3
*(x^n)^5+465*a^2*b*n^2*x^n+75*c^3*m*n*(x^n)^6+3*a^2*b*m^6*x^n+18*a^2*c*m^5*
(x^n)^2+735*a^3*n^3+484*b^3*m*n^2*(x^n)^3+60*a*b^2*m^3*(x^n)^2+90*b^3*m*n*(
x^n)^3+1080*a*b^2*m*n^5*(x^n)^2+285*a^2*c*m^4*n*(x^n)^2+1644*a^2*c*m^3*n^2*
(x^n)^2+4149*a^2*c*m^2*n^3*(x^n)^2+4212*a^2*c*m*n^4*(x^n)^2+285*a*b^2*m^4*n
*(x^n)^2+1644*a*b^2*m^3*n^2*(x^n)^2+4149*a*b^2*m^2*n^3*(x^n)^2+4212*a*b^2*m
*n^4*(x^n)^2+1860*a^2*b*m*n^2*x^n+90*a*b*c*m^4*(x^n)^3+3048*a*b*c*n^4*(x^n)
^3+300*a^2*b*m*n*x^n+510*a*c^2*m^2*n*(x^n)^4+1284*a*c^2*m*n^2*(x^n)^4+510*b
^2*c*m^2*n*(x^n)^4+1284*b^2*c*m*n^2*(x^n)^4+240*b*c^2*m*n*(x^n)^5+300*a^2*b
*m^4*n*x^n+1860*a^2*b*m^3*n^2*x^n+5220*a^2*b*m^2*n^3*x^n+6264*a^2*b*m*n^4*x
^n+570*a^2*c*m^3*n*(x^n)^2+2466*a^2*c*m^2*n^2*(x^n)^2+4149*a^2*c*m*n^3*(x^n
)^2+570*a*b^2*m^3*n*(x^n)^2+2466*a*b^2*m^2*n^2*...
```

<u>Maxima</u> [A]
time = 0.30, size = 273, normalized size = 1.50

$$\frac{c^3 d^m x e^{(m\log(x)+6n\log(x))}}{m+6n+1}+\frac{3bc^2 d^m x e^{(m\log(x)+5n\log(x))}}{m+5n+1}+\frac{3b^2 c d^m x e^{(m\log(x)+4n\log(x))}}{m+4n+1}+\frac{3ac^2 d^m x e^{(m\log(x)+4n\log(x))}}{m+4n+1}+\frac{b^3 d^m x e^{(m\log(x)+3n\log(x))}}{m+3n+1}+\frac{6abcd^m x e^{(m\log(x)+3n\log(x))}}{m+3n+1}+\frac{3ab^2 d^m x e^{(m\log(x)+2n\log(x))}}{m+2n+1}+\frac{3a^2 c d^m x e^{(m\log(x)+2n\log(x))}}{m+2n+1}+\frac{3a^2 b d^m x e^{(m\log(x)+n\log(x))}}{m+n+1}+\frac{(dx)^{m+1}a^3}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] c^3*d^m*x*e^(m*log(x) + 6*n*log(x))/(m + 6*n + 1) + 3*b*c^2*d^m*x*e^(m*log(
x) + 5*n*log(x))/(m + 5*n + 1) + 3*b^2*c*d^m*x*e^(m*log(x) + 4*n*log(x))/(m
 + 4*n + 1) + 3*a*c^2*d^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + b^3*d
^m*x*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 1) + 6*a*b*c*d^m*x*e^(m*log(x) +
3*n*log(x))/(m + 3*n + 1) + 3*a*b^2*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*
n + 1) + 3*a^2*c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 3*a^2*b*d^
m*x*e^(m*log(x) + n*log(x))/(m + n + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1))
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 2303 vs. $2(182) = 364$.
time = 0.50, size = 2303, normalized size = 12.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] ((c^3*m^6 + 6*c^3*m^5 + 15*c^3*m^4 + 20*c^3*m^3 + 120*(c^3*m + c^3)*n^5 + 1
5*c^3*m^2 + 274*(c^3*m^2 + 2*c^3*m + c^3)*n^4 + 6*c^3*m + 225*(c^3*m^3 + 3*
c^3*m^2 + 3*c^3*m + c^3)*n^3 + c^3 + 85*(c^3*m^4 + 4*c^3*m^3 + 6*c^3*m^2 +
4*c^3*m + c^3)*n^2 + 15*(c^3*m^5 + 5*c^3*m^4 + 10*c^3*m^3 + 10*c^3*m^2 + 5*
c^3*m + c^3)*n)*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 3*(b*c^2*m^6 + 6*b*c^2*
m^5 + 15*b*c^2*m^4 + 20*b*c^2*m^3 + 144*(b*c^2*m + b*c^2)*n^5 + 15*b*c^2*m^
```

2 + 324*(b*c^2*m^2 + 2*b*c^2*m + b*c^2)*n^4 + 6*b*c^2*m + 260*(b*c^2*m^3 + 3*b*c^2*m^2 + 3*b*c^2*m + b*c^2)*n^3 + b*c^2 + 95*(b*c^2*m^4 + 4*b*c^2*m^3 + 6*b*c^2*m^2 + 4*b*c^2*m + b*c^2)*n^2 + 16*(b*c^2*m^5 + 5*b*c^2*m^4 + 10*b *c^2*m^3 + 10*b*c^2*m^2 + 5*b*c^2*m + b*c^2)*n)*x*x^(5*n)*e^(m*log(d) + m*l og(x)) + 3*((b^2*c + a*c^2)*m^6 + 6*(b^2*c + a*c^2)*m^5 + 180*(b^2*c + a*c^ 2 + (b^2*c + a*c^2)*m)*n^5 + 15*(b^2*c + a*c^2)*m^4 + 396*(b^2*c + a*c^2 + (b^2*c + a*c^2)*m^2 + 2*(b^2*c + a*c^2)*m)*n^4 + 20*(b^2*c + a*c^2)*m^3 + 3 07*((b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 3*(b^2*c + a*c^2)*m^2 + 3*(b^2*c + a*c^2)*m)*n^3 + b^2*c + a*c^2 + 15*(b^2*c + a*c^2)*m^2 + 107*((b^2*c + a* c^2)*m^4 + 4*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 6*(b^2*c + a*c^2)*m^2 + 4*(b^2*c + a*c^2)*m)*n^2 + 6*(b^2*c + a*c^2)*m + 17*((b^2*c + a*c^2)*m^5 + 5*(b^2*c + a*c^2)*m^4 + 10*(b^2*c + a*c^2)*m^3 + b^2*c + a*c^2 + 10*(b^2*c + a*c^2)*m^2 + 5*(b^2*c + a*c^2)*m)*n)*x*x^(4*n)*e^(m*log(d) + m*log(x)) + ((b^3 + 6*a*b*c)*m^6 + 6*(b^3 + 6*a*b*c)*m^5 + 240*(b^3 + 6*a*b*c + (b^3 + 6*a*b*c)*m)*n^5 + 15*(b^3 + 6*a*b*c)*m^4 + 508*(b^3 + 6*a*b*c + (b^3 + 6*a* b*c)*m^2 + 2*(b^3 + 6*a*b*c)*m)*n^4 + 20*(b^3 + 6*a*b*c)*m^3 + 372*((b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 3*(b^3 + 6*a*b*c)*m^2 + 3*(b^3 + 6*a*b*c)*m) *n^3 + b^3 + 6*a*b*c + 15*(b^3 + 6*a*b*c)*m^2 + 121*((b^3 + 6*a*b*c)*m^4 + 4*(b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 6*(b^3 + 6*a*b*c)*m^2 + 4*(b^3 + 6* a*b*c)*m)*n^2 + 6*(b^3 + 6*a*b*c)*m + 18*((b^3 + 6*a*b*c)*m^5 + 5*(b^3 + 6* a*b*c)*m^4 + 10*(b^3 + 6*a*b*c)*m^3 + b^3 + 6*a*b*c + 10*(b^3 + 6*a*b*c)*m^ 2 + 5*(b^3 + 6*a*b*c)*m)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*((a*b^2 + a^2*c)*m^6 + 6*(a*b^2 + a^2*c)*m^5 + 360*(a*b^2 + a^2*c + (a*b^2 + a^2*c)* m)*n^5 + 15*(a*b^2 + a^2*c)*m^4 + 702*(a*b^2 + a^2*c + (a*b^2 + a^2*c)*m^2 + 2*(a*b^2 + a^2*c)*m)*n^4 + 20*(a*b^2 + a^2*c)*m^3 + 461*((a*b^2 + a^2*c)* m^3 + a*b^2 + a^2*c + 3*(a*b^2 + a^2*c)*m^2 + 3*(a*b^2 + a^2*c)*m)*n^3 + a* b^2 + a^2*c + 15*(a*b^2 + a^2*c)*m^2 + 137*((a*b^2 + a^2*c)*m^4 + 4*(a*b^2 + a^2*c)*m^3 + a*b^2 + a^2*c + 6*(a*b^2 + a^2*c)*m^2 + 4*(a*b^2 + a^2*c)*m) *n^2 + 6*(a*b^2 + a^2*c)*m + 19*((a*b^2 + a^2*c)*m^5 + 5*(a*b^2 + a^2*c)*m^ 4 + 10*(a*b^2 + a^2*c)*m^3 + a*b^2 + a^2*c + 10*(a*b^2 + a^2*c)*m^2 + 5*(a* b^2 + a^2*c)*m)*n)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*(a^2*b*m^6 + 6*a^2 *b*m^5 + 15*a^2*b*m^4 + 20*a^2*b*m^3 + 720*(a^2*b*m + a^2*b)*n^5 + 15*a^2*b *m^2 + 1044*(a^2*b*m^2 + 2*a^2*b*m + a^2*b)*n^4 + 6*a^2*b*m + 580*(a^2*b*m^ 3 + 3*a^2*b*m^2 + 3*a^2*b*m + a^2*b)*n^3 + a^2*b + 155*(a^2*b*m^4 + 4*a^2*b *m^3 + 6*a^2*b*m^2 + 4*a^2*b*m + a^2*b)*n^2 + 20*(a^2*b*m^5 + 5*a^2*b*m^4 + 10*a^2*b*m^3 + 10*a^2*b*m^2 + 5*a^2*b*m + a^2*b)*n)*x*x^n*e^(m*log(d) + m* log(x)) + (a^3*m^6 + 720*a^3*n^6 + 6*a^3*m^5 + 15*a^3*m^4 + 20*a^3*m^3 + 17 64*(a^3*m + a^3)*n^5 + 15*a^3*m^2 + 1624*(a^3*m^2 + 2*a^3*m + a^3)*n^4 + 6* a^3*m + 735*(a^3*m^3 + 3*a^3*m^2 + 3*a^3*m + a^3)*n^3 + a^3 + 175*(a^3*m^4 + 4*a^3*m^3 + 6*a^3*m^2 + 4*a^3*m + a^3)*n^2 + 21*(a^3*m^5 + 5*a^3*m^4 + 10 *a^3*m^3 + 10*a^3*m^2 + 5*a^3*m + a^3)*n)*x*e^(m*log(d) + m*log(x)))/(m^7 + 720*(m + 1)*n^6 + 7*m^6 + 1764*(m^2 + 2*m + 1)*n^5 + 21*m^5 + 1624*(m^3 + 3*m^2 + 3*m + 1)*n^4 + 35*m^4 + 735*(m^4 + 4*m^3 + 6*m^2 + 4*m + 1)*n^3 + 3 5*m^3 + 175*(m^5 + 5*m^4 + 10*m^3 + 10*m^2 + 5*m + 1)*n^2 + 21*m^2 + 21*(m^ 6 + 6*m^5 + 15*m^4 + 20*m^3 + 15*m^2 + 6*m + 1)*n + 7*m + 1)

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 25656 vs. 2(182) = 364.
time = 3.75, size = 25656, normalized size = 140.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] (c^3*m^6*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^5*n*x*x^(6*n)*e^(m*lo
g(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 225*c
^3*m^3*n^3*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 274*c^3*m^2*n^4*x*x^(6*n)*e^
(m*log(d) + m*log(x)) + 120*c^3*m*n^5*x*x^(6*n)*e^(m*log(d) + m*log(x)) + 3
*b*c^2*m^6*x*x^(5*n)*e^(m*log(d) + m*log(x)) + c^3*m^6*x*x^(5*n)*e^(m*log(d
) + m*log(x)) + 48*b*c^2*m^5*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m
^5*n*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 285*b*c^2*m^4*n^2*x*x^(5*n)*e^(m*l
og(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 780*
b*c^2*m^3*n^3*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 225*c^3*m^3*n^3*x*x^(5*n)
*e^(m*log(d) + m*log(x)) + 972*b*c^2*m^2*n^4*x*x^(5*n)*e^(m*log(d) + m*log(
x)) + 274*c^3*m^2*n^4*x*x^(5*n)*e^(m*log(d) + m*log(x)) + 432*b*c^2*m*n^5*x
*x^(5*n)*e^(m*log(d) + m*log(x)) + 120*c^3*m*n^5*x*x^(5*n)*e^(m*log(d) + m*
log(x)) + 3*b^2*c*m^6*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 3*a*c^2*m^6*x*x^(
4*n)*e^(m*log(d) + m*log(x)) + 3*b*c^2*m^6*x*x^(4*n)*e^(m*log(d) + m*log(x)
) + c^3*m^6*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 51*b^2*c*m^5*n*x*x^(4*n)*e^
(m*log(d) + m*log(x)) + 51*a*c^2*m^5*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) +
48*b*c^2*m^5*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^5*n*x*x^(4*n)*e
^(m*log(d) + m*log(x)) + 321*b^2*c*m^4*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)
) + 321*a*c^2*m^4*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 285*b*c^2*m^4*n^2
*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 85*c^3*m^4*n^2*x*x^(4*n)*e^(m*log(d) +
 m*log(x)) + 921*b^2*c*m^3*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 921*a*c^
2*m^3*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 780*b*c^2*m^3*n^3*x*x^(4*n)*e
^(m*log(d) + m*log(x)) + 225*c^3*m^3*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x))
+ 1188*b^2*c*m^2*n^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 1188*a*c^2*m^2*n^4
*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 972*b*c^2*m^2*n^4*x*x^(4*n)*e^(m*log(d
) + m*log(x)) + 274*c^3*m^2*n^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 540*b^2
```

```
*c*m*n^5*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 540*a*c^2*m*n^5*x*x^(4*n)*e^(m
*log(d) + m*log(x)) + 432*b*c^2*m*n^5*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 1
20*c^3*m*n^5*x*x^(4*n)*e^(m*log(d) + m*log(x)) + b^3*m^6*x*x^(3*n)*e^(m*log
(d) + m*log(x)) + 6*a*b*c*m^6*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*b^2*c*m
^6*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3*a*c^2*m^6*x*x^(3*n)*e^(m*log(d) +
m*log(x)) + 3*b*c^2*m^6*x*x^(3*n)*e^(m*log(d) + m*log(x)) + c^3*m^6*x*x^(3*
n)*e^(m*log(d) + m*log(x)) + 18*b^3*m^5*n*x*x^(3*n)*e^(m*log(d) + m*log(x))
 + 108*a*b*c*m^5*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 51*b^2*c*m^5*n*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 51*a*c^2*m^5*n*x*x^(3*n)*e^(m*log(d) + m*log
(x)) + 48*b*c^2*m^5*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 15*c^3*m^5*n*x*x^
(3*n)*e^(m*log(d) + m*log(x)) + 121*b^3*m^4*n^2*x*x^(3*n)*e^(m*log(d) + m*l
og(x)) + 726*a*b*c*m^4*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 321*b^2*c*m^
4*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 321*a*c^2*m^4*n^2*x*x^(3*n)*e^(m*
log(d) + m*log(x)) + 285*b*c^2*m^4*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
85*c^3*m^4*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 372*b^3*m^3*n^3*x*x^(3*n
)*e^(m*log(d) + m*log(x)) + 2232*a*b*c*m^3*n^3*x*x^(3*n)*e^(m*log(d) + m*lo
g(x)) + 921*b^2*c*m^3*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 921*a*c^2*m^3
*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 780*b*c^2*m^3*n^3*x*x^(3*n)*e^(m*l
og(d) + m*log(x)) + 225*c^3*m^3*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 508
*b^3*m^2*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 3048*a*b*c*m^2*n^4*x*x^(3*
n)*e^(m*log(d) + m*log(x)) + 1188*b^2*c*m^2*n^4*x*x^(3*n)*e^(m*log(d) + m*l
og(x)) + 1188*a*c^2*m^2*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 972*b*c^2*m
^2*n^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 274*c^3*m^2*n^4*x*x^(3*n)*e^(m*l
og(d) + m*log(x)) + 240*b^3*m*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 1440*
a*b*c*m*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 540*b^2*c*m*n^5*x*x^(3*n)*e
^(m*log(d) + m*log(x)) + 540*a*c^2*m*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x))
+ 432*b*c^2*m*n^5*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 120*c^3*m*n^5*x*x^(3*
n)*e^(m*log(d) + m*log(x)) + 3*a*b^2*m^6*x*x^(2*n)*e^(m*log(d) + m*log(x))
+ b^3*m^6*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*a^2*c*m^6*x*x^(2*n)*e^(m*lo
g(d) + m*log(x)) + 6*a*b*c*m^6*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*b^2*c*
m^6*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 3*a*c^2*m^6*x*x^(2*n)*e^(m*log(d) +
 m*log(x)) + 3*b*c^2*m^6*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c^3*m^6*x*x^(2
*n)*e^(m*log(d) + m*log(x)) + 57*a*b^2*m^5*n*x*x^(2*n)*e^(m*log(d) + m*log(
x)) + 18*b^3*m^5*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 57*a^2*c*m^5*n*x*x^(
2*n)*e^(m*log(d) + m*log(x)) + 108*a*b*c*m^5*n*x*x^(2*n)*e^(m*log(d) + m*lo
g(x)) + 51*b^2*c*m^5*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 51*a*c^2*m^5*n*x
*x^(2*n)*e^(m*log(d) + m*log(x)) + 48*b*c^2*m^5*n*x*x^(2*n)*e^(m*log(d) + m
*log(x)) + 15*c^3*m^5*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 411*a*b^2*m^4*n
^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 121*b^3*m^4*n^2*x*x^(2*n)*e^(m*log(d
) + m*log(x)) + 411*a^2*c*m^4*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 726*a
*b*c*m^4*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) ...
```

Mupad [B]

time = 2.16, size = 1734, normalized size = 9.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^3,x)
```

```
[Out] (a^3*x*(d*x)^m)/(m + 1) + (c^3*x*x^(6*n)*(d*x)^m*(5*m + 15*n + 60*m*n + 255
*m*n^2 + 90*m^2*n + 450*m*n^3 + 60*m^3*n + 274*m*n^4 + 15*m^4*n + 10*m^2 +
10*m^3 + 5*m^4 + m^5 + 85*n^2 + 225*n^3 + 274*n^4 + 120*n^5 + 255*m^2*n^2 +
 225*m^2*n^3 + 85*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2
*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*
n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 +
 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*
n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*a*x*x^(2*n)*(d*x)^m*(a*c + b^2)*(
5*m + 19*n + 76*m*n + 411*m*n^2 + 114*m^2*n + 922*m*n^3 + 76*m^3*n + 702*m*
n^4 + 19*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 137*n^2 + 461*n^3 + 702*n^
4 + 360*n^5 + 411*m^2*n^2 + 461*m^2*n^3 + 137*m^3*n^2 + 1))/(6*m + 21*n + 1
05*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*
m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 17
5*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n
^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (b*x*x^(
3*n)*(d*x)^m*(6*a*c + b^2)*(5*m + 18*n + 72*m*n + 363*m*n^2 + 108*m^2*n + 7
44*m*n^3 + 72*m^3*n + 508*m*n^4 + 18*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5
+ 121*n^2 + 372*n^3 + 508*n^4 + 240*n^5 + 363*m^2*n^2 + 372*m^2*n^3 + 121*m
^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 2
10*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3
 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6
 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 +
 175*m^4*n^2 + 1) + (3*c*x*x^(4*n)*(d*x)^m*(a*c + b^2)*(5*m + 17*n + 68*m*n
 + 321*m*n^2 + 102*m^2*n + 614*m*n^3 + 68*m^3*n + 396*m*n^4 + 17*m^4*n + 10
*m^2 + 10*m^3 + 5*m^4 + m^5 + 107*n^2 + 307*n^3 + 396*n^4 + 180*n^5 + 321*m
^2*n^2 + 307*m^2*n^3 + 107*m^3*n^2 + 1))/(6*m + 21*n + 105*m*n + 700*m*n^2
+ 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*m^4*n + 1764*m*n^5
+ 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1
624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 +
1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*a^2*b*x*x^n*(d*x)^m*(5*m
 + 20*n + 80*m*n + 465*m*n^2 + 120*m^2*n + 1160*m*n^3 + 80*m^3*n + 1044*m*n
^4 + 20*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 155*n^2 + 580*n^3 + 1044*n^
4 + 720*n^5 + 465*m^2*n^2 + 580*m^2*n^3 + 155*m^3*n^2 + 1))/(6*m + 21*n + 1
05*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n + 3248*m*n^4 + 105*
m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 + 6*m^5 + m^6 + 17
5*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2*n^2 + 2205*m^2*n
^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^2 + 1) + (3*b*c^2
*x*x^(5*n)*(d*x)^m*(5*m + 16*n + 64*m*n + 285*m*n^2 + 96*m^2*n + 520*m*n^3
+ 64*m^3*n + 324*m*n^4 + 16*m^4*n + 10*m^2 + 10*m^3 + 5*m^4 + m^5 + 95*n^2
+ 260*n^3 + 324*n^4 + 144*n^5 + 285*m^2*n^2 + 260*m^2*n^3 + 95*m^3*n^2 + 1)
)/(6*m + 21*n + 105*m*n + 700*m*n^2 + 210*m^2*n + 2205*m*n^3 + 210*m^3*n +
```

```
3248*m*n^4 + 105*m^4*n + 1764*m*n^5 + 21*m^5*n + 15*m^2 + 20*m^3 + 15*m^4 +
 6*m^5 + m^6 + 175*n^2 + 735*n^3 + 1624*n^4 + 1764*n^5 + 720*n^6 + 1050*m^2
*n^2 + 2205*m^2*n^3 + 700*m^3*n^2 + 1624*m^2*n^4 + 735*m^3*n^3 + 175*m^4*n^
2 + 1)
```

3.597 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2 \, dx$

Optimal. Leaf size=117

$$\frac{2abx^{1+n}(dx)^m}{1+m+n}+\frac{(b^2+2ac)\,x^{1+2n}(dx)^m}{1+m+2n}+\frac{2bcx^{1+3n}(dx)^m}{1+m+3n}+\frac{c^2x^{1+4n}(dx)^m}{1+m+4n}+\frac{a^2(dx)^{1+m}}{d(1+m)}$$

```
[Out] 2*a*b*x^(1+n)*(d*x)^m/(1+m+n)+(2*a*c+b^2)*x^(1+2*n)*(d*x)^m/(1+m+2*n)+2*b*c
*x^(1+3*n)*(d*x)^m/(1+m+3*n)+c^2*x^(1+4*n)*(d*x)^m/(1+m+4*n)+a^2*(d*x)^(1+m
)/d/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1367, 20, 30}

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{x^{2n+1}(2ac+b^2)\,(dx)^m}{m+2n+1}+\frac{2abx^{n+1}(dx)^m}{m+n+1}+\frac{2bcx^{3n+1}(dx)^m}{m+3n+1}+\frac{c^2x^{4n+1}(dx)^m}{m+4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (2*a*b*x^(1 + n)*(d*x)^m)/(1 + m + n) + ((b^2 + 2*a*c)*x^(1 + 2*n)*(d*x)^m)
/(1 + m + 2*n) + (2*b*c*x^(1 + 3*n)*(d*x)^m)/(1 + m + 3*n) + (c^2*x^(1 + 4*
n)*(d*x)^m)/(1 + m + 4*n) + (a^2*(d*x)^(1 + m))/(d*(1 + m))
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 1367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^n + c*x^(2*n))^p, x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && IGtQ[p, 0] && !IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^2\,dx = \int \left(a^2(dx)^m+2abx^n(dx)^m+b^2\left(1+\frac{2ac}{b^2}\right)x^{2n}(dx)^m+2bcx^{3n}(dx)^m+c^2x\right.$$
$$= \frac{a^2(dx)^{1+m}}{d(1+m)}+(2ab)\int x^n(dx)^m\,dx+(2bc)\int x^{3n}(dx)^m\,dx+c^2\int x^{4n}(dx)^m$$
$$= \frac{a^2(dx)^{1+m}}{d(1+m)}+\left(2abx^{-m}(dx)^m\right)\int x^{m+n}\,dx+\left(2bcx^{-m}(dx)^m\right)\int x^{m+3n}\,dx+$$
$$= \frac{2abx^{1+n}(dx)^m}{1+m+n}+\frac{\left(b^2+2ac\right)x^{1+2n}(dx)^m}{1+m+2n}+\frac{2bcx^{1+3n}(dx)^m}{1+m+3n}+\frac{c^2x^{1+4n}(dx)^m}{1+m+4n}$$

Mathematica [A]
time = 0.16, size = 86, normalized size = 0.74

$$x(dx)^m\left(\frac{a^2}{1+m}+\frac{2abx^n}{1+m+n}+\frac{\left(b^2+2ac\right)x^{2n}}{1+m+2n}+\frac{2bcx^{3n}}{1+m+3n}+\frac{c^2x^{4n}}{1+m+4n}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] x*(d*x)^m*(a^2/(1 + m) + (2*a*b*x^n)/(1 + m + n) + ((b^2 + 2*a*c)*x^(2*n))/
(1 + m + 2*n) + (2*b*c*x^(3*n))/(1 + m + 3*n) + (c^2*x^(4*n))/(1 + m + 4*n)
)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 1065, normalized size = 9.10

method	result
risch	$\frac{x\left(8ab\,x^n m+12ab\,m^2x^n+a^2+2ab\,m^4x^n+2bc\,m^4x^{3n}+8b^2m^3n\,x^{2n}+19b^2m^2n^2x^{2n}+12b^2m\,n^3x^{2n}+8bc\,m^3x^{3n}+16bc\,n^3x^{3n}+18c^2mn\,x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(8*a*b*x^n*m+2*b*c*m^4*(x^n)^3+12*a*b*m^2*x^n+b^2*(x^n)^2+8*b^2*m^3*n*(x^
n)^2+19*b^2*m^2*n^2*(x^n)^2+a^2+12*b^2*m*n^3*(x^n)^2+8*b*c*m^3*(x^n)^3+16*b
*c*n^3*(x^n)^3+18*c^2*m*n*(x^n)^4+2*a*b*m^4*x^n+8*a*c*m^3*(x^n)^2+24*a*c*n^
3*(x^n)^2+2*a*c*m^4*(x^n)^2+2*(x^n)^3*b*c+2*(x^n)^2*a*c+c^2*m^4*(x^n)^4+(x^
n)^4*c^2+6*c^2*m^2*(x^n)^4+11*c^2*n^2*(x^n)^4+4*b^2*m^3*(x^n)^2+12*b^2*n^3*
(x^n)^2+4*m*c^2*(x^n)^4+6*c^2*(x^n)^4*n+6*b^2*m^2*(x^n)^2+19*b^2*n^2*(x^n)^
2+24*b^2*m*n*(x^n)^2+8*m*b*c*(x^n)^3+14*b*c*(x^n)^3*n+24*b^2*m^2*n*(x^n)^2+
18*a*b*x^n*n+2*a*b*x^n+6*c^2*m^3*n*(x^n)^4+11*c^2*m^2*n^2*(x^n)^4+6*c^2*m*n
^3*(x^n)^4+38*b^2*m*n^2*(x^n)^2+12*b*c*m^2*(x^n)^3+28*b*c*n^2*(x^n)^3+4*b^2
```

*(x^n)^2*m+8*b^2*(x^n)^2*n+70*a^2*m*n^2+30*a^2*m*n+52*a*b*n^2*x^n+8*a*c*(x^
n)^2*m+16*a*c*(x^n)^2*n+4*a^2*m+10*a^2*n+8*a*b*m^3*x^n+48*a*b*n^3*x^n+12*a*
c*m^2*(x^n)^2+38*a*c*n^2*(x^n)^2+4*c^2*m^3*(x^n)^4+6*c^2*n^3*(x^n)^4+b^2*m^
4*(x^n)^2+18*c^2*m^2*n*(x^n)^4+22*c^2*m*n^2*(x^n)^4+24*a^2*n^4+10*a^2*m^3*n
+35*a^2*m^2*n^2+50*a^2*m*n^3+30*a^2*m^2*n+a^2*m^4+4*a^2*m^3+50*a^2*n^3+6*a^
2*m^2+35*a^2*n^2+48*a*b*m*n^3*x^n+48*a*c*m^2*n*(x^n)^2+76*a*c*m*n^2*(x^n)^2
+42*b*c*m*n*(x^n)^3+54*a*b*m^2*n*x^n+104*a*b*m*n^2*x^n+48*a*c*m*n*(x^n)^2+5
4*a*b*m*n*x^n+14*b*c*m^3*n*(x^n)^3+28*b*c*m^2*n^2*(x^n)^3+16*b*c*m*n^3*(x^n
)^3+16*a*c*m^3*n*(x^n)^2+38*a*c*m^2*n^2*(x^n)^2+24*a*c*m*n^3*(x^n)^2+42*b*c
*m^2*n*(x^n)^3+56*b*c*m*n^2*(x^n)^3+18*a*b*m^3*n*x^n+52*a*b*m^2*n^2*x^n)/(1
+m)/(1+m+n)/(1+m+2*n)/(1+m+3*n)/(1+m+4*n)*exp(1/2*m*(-I*csgn(I*d*x)^3*Pi+I*
csgn(I*d*x)^2*csgn(I*d)*Pi+I*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(
I*d)*csgn(I*x)*Pi+2*ln(x)+2*ln(d)))

Maxima [A]
time = 0.30, size = 152, normalized size = 1.30

$$\frac{c^2 d^m x e^{(m\log(x)+4\,n\log(x))}}{m+4\,n+1}+\frac{2\,bcd^m x e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1}+\frac{b^2 d^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1}+\frac{2\,acd^m x e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1}+\frac{2\,abd^m x e^{(m\log(x)+n\log(x))}}{m+n+1}+\frac{(dx)^{m+1}a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] c^2*d^m*x*e^(m*log(x) + 4*n*log(x))/(m + 4*n + 1) + 2*b*c*d^m*x*e^(m*log(x)
 + 3*n*log(x))/(m + 3*n + 1) + b^2*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n
 + 1) + 2*a*c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + 2*a*b*d^m*x*e
^(m*log(x) + n*log(x))/(m + n + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(117) = 234.
time = 0.37, size = 706, normalized size = 6.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 4*c^2*m^3 + 6*c^2*m^2 + 6*(c^2*m + c^2)*n^3 + 4*c^2*m + 11*(c^2
*m^2 + 2*c^2*m + c^2)*n^2 + c^2 + 6*(c^2*m^3 + 3*c^2*m^2 + 3*c^2*m + c^2)*n
)*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*(b*c*m^4 + 4*b*c*m^3 + 6*b*c*m^2 +
8*(b*c*m + b*c)*n^3 + 4*b*c*m + 14*(b*c*m^2 + 2*b*c*m + b*c)*n^2 + b*c + 7*
(b*c*m^3 + 3*b*c*m^2 + 3*b*c*m + b*c)*n)*x*x^(3*n)*e^(m*log(d) + m*log(x))
+ ((b^2 + 2*a*c)*m^4 + 4*(b^2 + 2*a*c)*m^3 + 12*(b^2 + 2*a*c + (b^2 + 2*a*c
)*m)*n^3 + 6*(b^2 + 2*a*c)*m^2 + 19*((b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 2*(b
^2 + 2*a*c)*m)*n^2 + b^2 + 2*a*c + 4*(b^2 + 2*a*c)*m + 8*((b^2 + 2*a*c)*m^3
 + 3*(b^2 + 2*a*c)*m^2 + b^2 + 2*a*c + 3*(b^2 + 2*a*c)*m)*n)*x*x^(2*n)*e^(m
```

```
*log(d) + m*log(x)) + 2*(a*b*m^4 + 4*a*b*m^3 + 6*a*b*m^2 + 24*(a*b*m + a*b)
*n^3 + 4*a*b*m + 26*(a*b*m^2 + 2*a*b*m + a*b)*n^2 + a*b + 9*(a*b*m^3 + 3*a*
b*m^2 + 3*a*b*m + a*b)*n)*x*x^n*e^(m*log(d) + m*log(x)) + (a^2*m^4 + 24*a^2
*n^4 + 4*a^2*m^3 + 6*a^2*m^2 + 50*(a^2*m + a^2)*n^3 + 4*a^2*m + 35*(a^2*m^2
 + 2*a^2*m + a^2)*n^2 + a^2 + 10*(a^2*m^3 + 3*a^2*m^2 + 3*a^2*m + a^2)*n)*x
*e^(m*log(d) + m*log(x)))/(m^5 + 24*(m + 1)*n^4 + 5*m^4 + 50*(m^2 + 2*m + 1
)*n^3 + 10*m^3 + 35*(m^3 + 3*m^2 + 3*m + 1)*n^2 + 10*m^2 + 10*(m^4 + 4*m^3
+ 6*m^2 + 4*m + 1)*n + 5*m + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 12124 vs. $2(107) = 214$.
time = 142.98, size = 12124, normalized size = 103.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Piecewise(((a + b + c)**2*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a**2*log(x) +
2*a*b*x**n/n + a*c*x**(2*n)/n + b**2*x**(2*n)/(2*n) + 2*b*c*x**(3*n)/(3*n)
+ c**2*x**(4*n)/(4*n))/d, Eq(m, -1)), (a**2*Piecewise((-1/(4*n*(d*x)**(4*n)
), Ne(n, 0)), (log(x), True))/d + 2*a*b*Piecewise((-x**n/(3*n*(d*x)**(4*n))
, Ne(n, 0)), (log(x), True))/d + 2*a*c*Piecewise((-x**(2*n)/(2*n*(d*x)**(4*
n)), Ne(n, 0)), (log(x), True))/d + b**2*Piecewise((-x**(2*n)/(2*n*(d*x)**(
4*n)), Ne(n, 0)), (log(x), True))/d + 2*b*c*Piecewise((-x**(3*n)/(n*(d*x)**
(4*n)), Ne(n, 0)), (log(x), True))/d + c**2*Piecewise((0, (Abs(x) < 1) & (1
/Abs(x) < 1)), (log(x)/d**(4*n), Abs(x) < 1), (-log(1/x)/d**(4*n), 1/Abs(x)
 < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)/d**(4*n) + meijerg(((1, 1),
 ()), ((), (0, 0)), x)/d**(4*n), True))/d, Eq(m, -4*n - 1)), (a**2*Piecewis
e((-1/(3*n*(d*x)**(3*n)), Ne(n, 0)), (log(x), True))/d + 2*a*b*Piecewise((-
x**n/(2*n*(d*x)**(3*n)), Ne(n, 0)), (log(x), True))/d + 2*a*c*Piecewise((-x
**(2*n)/(n*(d*x)**(3*n)), Ne(n, 0)), (log(x), True))/d + b**2*Piecewise((-x
**(2*n)/(n*(d*x)**(3*n)), Ne(n, 0)), (log(x), True))/d + 2*b*c*Piecewise((0
, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(x)/d**(3*n), Abs(x) < 1), (-log(1/x)
/d**(3*n), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), ()), x)/d**(3*n)
 + meijerg(((1, 1), ()), ((), (0, 0)), x)/d**(3*n), True))/d + c**2*Piecewi
se((x**(4*n)/(n*(d*x)**(3*n)), Ne(n, 0)), (log(x), True))/d, Eq(m, -3*n - 1
)), (a**2*Piecewise((-1/(2*n*(d*x)**(2*n)), Ne(n, 0)), (log(x), True))/d +
2*a*b*Piecewise((-x**n/(n*(d*x)**(2*n)), Ne(n, 0)), (log(x), True))/d + 2*a
*c*Piecewise((0, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(x)/d**(2*n), Abs(x) <
 1), (-log(1/x)/d**(2*n), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0, 0), (
)), x)/d**(2*n) + meijerg(((1, 1), ()), ((), (0, 0)), x)/d**(2*n), True))/d
 + b**2*Piecewise((0, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(x)/d**(2*n), Abs
(x) < 1), (-log(1/x)/d**(2*n), 1/Abs(x) < 1), (-meijerg(((), (1, 1)), ((0,
0), ()), x)/d**(2*n) + meijerg(((1, 1), ()), ((), (0, 0)), x)/d**(2*n), Tru
```

```
e))/d + 2*b*c*Piecewise((x**(3*n)/(n*(d*x)**(2*n)), Ne(n, 0)), (log(x), Tru
e))/d + c**2*Piecewise((x**(4*n)/(2*n*(d*x)**(2*n)), Ne(n, 0)), (log(x), Tr
ue))/d, Eq(m, -2*n - 1)), (a**2*Piecewise((-1/(n*(d*x)**n), Ne(n, 0)), (log
(x), True))/d + 2*a*b*Piecewise((0, (Abs(x) < 1) & (1/Abs(x) < 1)), (log(x)
/d**n, Abs(x) < 1), (-log(1/x)/d**n, 1/Abs(x) < 1), (-meijerg(((), (1, 1)),
 ((0, 0), ()), x)/d**n + meijerg(((1, 1), ()), ((), (0, 0)), x)/d**n, True)
)/d + 2*a*c*Piecewise((x**(2*n)/(n*(d*x)**n), Ne(n, 0)), (log(x), True))/d
+ b**2*Piecewise((x**(2*n)/(n*(d*x)**n), Ne(n, 0)), (log(x), True))/d + 2*b
*c*Piecewise((x**(3*n)/(2*n*(d*x)**n), Ne(n, 0)), (log(x), True))/d + c**2*
Piecewise((x**(4*n)/(3*n*(d*x)**n), Ne(n, 0)), (log(x), True))/d, Eq(m, -n
- 1)), (a**2*m**4*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40
*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24
*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n
**2 + 10*n + 1) + 10*a**2*m**3*n*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35
*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n
 + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 +
 50*n**3 + 35*n**2 + 10*n + 1) + 4*a**2*m**3*x*(d*x)**m/(m**5 + 10*m**4*n +
 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2
 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m
 + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 35*a**2*m**2*n**2*x*(d*x)**m/(
m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n*
*3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n
**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 30*a**2*m**2
*n*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m*
*3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m
*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + 6*a**2*m**2*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m*
*3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*
n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2
 + 10*n + 1) + 50*a**2*m*n**3*x*(d*x)**m/(m**5 + 10*m**4*n + 5*m**4 + 35*m*
*3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2 + 60*m**2*n +
10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m + 24*n**4 + 50
*n**3 + 35*n**2 + 10*n + 1) + 70*a**2*m*n**2*x*(d*x)**m/(m**5 + 10*m**4*n +
 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 105*m**2*n**2
 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 + 40*m*n + 5*m
 + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 30*a**2*m*n*x*(d*x)**m/(m**5 +
 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 40*m**3*n + 10*m**3 + 50*m**2*n**3 + 1
05*m**2*n**2 + 60*m**2*n + 10*m**2 + 24*m*n**4 + 100*m*n**3 + 105*m*n**2 +
40*m*n + 5*m + 24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 4*a**2*m*x*(d*x)**
m/(m**5 + 10*m**4*n + 5*m**4 + 35*m**3*n**2 + 4...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5454 vs. $2(117) = 234$.
time = 3.70, size = 5454, normalized size = 46.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(4*n)*e^(m*log
(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 6*c^2*
m*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*x^(3*n)*e^(m*log(d) +
 m*log(x)) + c^2*m^4*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*x^(3*n)*e^(m*log(d) + m*log(x)
) + 28*b*c*m^2*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(
3*n)*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x
)) + 6*c^2*m*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^(2*n)*e^(m
*log(d) + m*log(x)) + 2*a*c*m^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*c*m
^4*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c^2*m^4*x*x^(2*n)*e^(m*log(d) + m*lo
g(x)) + 8*b^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*a*c*m^3*n*x*x^(2
*n)*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)
) + 6*c^2*m^3*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 19*b^2*m^2*n^2*x*x^(2*n
)*e^(m*log(d) + m*log(x)) + 38*a*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)
) + 28*b*c*m^2*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^(
2*n)*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x
)) + 24*a*c*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^(2*n
)*e^(m*log(d) + m*log(x)) + 6*c^2*m*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 2*a*b*m^4*x*x^n*e^(m*log(d) + m*log(x)) + b^2*m^4*x*x^n*e^(m*log(d) + m*lo
g(x)) + 2*a*c*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*x^n*e^(m*log(
d) + m*log(x)) + c^2*m^4*x*x^n*e^(m*log(d) + m*log(x)) + 18*a*b*m^3*n*x*x^n
*e^(m*log(d) + m*log(x)) + 8*b^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 16*a
*c*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 14*b*c*m^3*n*x*x^n*e^(m*log(d) + m
*log(x)) + 6*c^2*m^3*n*x*x^n*e^(m*log(d) + m*log(x)) + 52*a*b*m^2*n^2*x*x^n
*e^(m*log(d) + m*log(x)) + 19*b^2*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 3
8*a*c*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 28*b*c*m^2*n^2*x*x^n*e^(m*log
(d) + m*log(x)) + 11*c^2*m^2*n^2*x*x^n*e^(m*log(d) + m*log(x)) + 48*a*b*m*n
^3*x*x^n*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*x^n*e^(m*log(d) + m*log(x
)) + 24*a*c*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*x^n*e^(m*l
og(d) + m*log(x)) + 6*c^2*m*n^3*x*x^n*e^(m*log(d) + m*log(x)) + a^2*m^4*x*e
^(m*log(d) + m*log(x)) + 2*a*b*m^4*x*e^(m*log(d) + m*log(x)) + b^2*m^4*x*e^
(m*log(d) + m*log(x)) + 2*a*c*m^4*x*e^(m*log(d) + m*log(x)) + 2*b*c*m^4*x*e
^(m*log(d) + m*log(x)) + c^2*m^4*x*e^(m*log(d) + m*log(x)) + 10*a^2*m^3*n*x
*e^(m*log(d) + m*log(x)) + 18*a*b*m^3*n*x*e^(m*log(d) + m*log(x)) + 8*b^2*m
^3*n*x*e^(m*log(d) + m*log(x)) + 16*a*c*m^3*n*x*e^(m*log(d) + m*log(x)) + 1
4*b*c*m^3*n*x*e^(m*log(d) + m*log(x)) + 6*c^2*m^3*n*x*e^(m*log(d) + m*log(x
)) + 35*a^2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 52*a*b*m^2*n^2*x*e^(m*log(d
) + m*log(x)) + 19*b^2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 38*a*c*m^2*n^2*x
*e^(m*log(d) + m*log(x)) + 28*b*c*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 11*c^
2*m^2*n^2*x*e^(m*log(d) + m*log(x)) + 50*a^2*m*n^3*x*e^(m*log(d) + m*log(x)
) + 48*a*b*m*n^3*x*e^(m*log(d) + m*log(x)) + 12*b^2*m*n^3*x*e^(m*log(d) + m
```

```
*log(x)) + 24*a*c*m*n^3*x*e^(m*log(d) + m*log(x)) + 16*b*c*m*n^3*x*e^(m*log
(d) + m*log(x)) + 6*c^2*m*n^3*x*e^(m*log(d) + m*log(x)) + 24*a^2*n^4*x*e^(m
*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 18*c^2*
m^2*n*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 22*c^2*m*n^2*x*x^(4*n)*e^(m*log(d
) + m*log(x)) + 6*c^2*n^3*x*x^(4*n)*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*x
^(3*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(3*n)*e^(m*log(d) + m*log(x)
) + 42*b*c*m^2*n*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^(3*n)
*e^(m*log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) +
 22*c^2*m*n^2*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x*x^(3*n)*e^(m
*log(d) + m*log(x)) + 6*c^2*n^3*x*x^(3*n)*e^(m*log(d) + m*log(x)) + 4*b^2*m
^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 8*a*c*m^3*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + 8*b*c*m^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^(2*n)
*e^(m*log(d) + m*log(x)) + 24*b^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
 48*a*c*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 42*b*c*m^2*n*x*x^(2*n)*e^
(m*log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 38
*b^2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 76*a*c*m*n^2*x*x^(2*n)*e^(m*
log(d) + m*log(x)) + 56*b*c*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 22*c^
2*m*n^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 12*b^2*n^3*x*x^(2*n)*e^(m*log(d
) + m*log(x)) + 24*a*c*n^3*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 16*b*c*n^3*x
*x^(2*n)*e^(m*log(d) + m*log(x)) + 6*c^2*n^3*x*x^(2*n)*e^(m*log(d) + m*log(
x)) + 8*a*b*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 4*b^2*m^3*x*x^n*e^(m*log(d)
 + m*log(x)) + 8*a*c*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 8*b*c*m^3*x*x^n*e^
(m*log(d) + m*log(x)) + 4*c^2*m^3*x*x^n*e^(m*log(d) + m*log(x)) + 54*a*b*m^
2*n*x*x^n*e^(m*log(d) + m*log(x)) + 24*b^2*m^2*n*x*x^n*e^(m*log(d) + m*log(
x)) + 48*a*c*m^2*n*x*x^n*e^(m*log(d) + m*log(x)) + 42*b*c*m^2*n*x*x^n*e^(m*
log(d) + m*log(x)) + 18*c^2*m^2*n*x*x^n*e^(m*lo...
```

Mupad [B]
time = 1.62, size = 543, normalized size = 4.64

$$\frac{a^2\,x\,{\left(d\,x\right)}^m}{m+1}+\frac{x\,x^{2\,n}\,{\left(d\,x\right)}^m\,\left(b^2+2\,a\,c\right)\,\left(m^3+8\,m^2\,n+3\,m^2+19\,m\,n^2+16\,m\,n+3\,m+12\,n^3+19\,n^2+8\,n+1\right)}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n^3+35\,n^2+10\,n+1}+\frac{c^2\,x\,x^{4\,n}\,{\left(d\,x\right)}^m\,\left(m^3+6\,m^2\,n+3\,m^2+11\,m\,n^2+12\,m\,n+3\,m+6\,n^3+11\,n^2+6\,n+1\right)}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n^3+35\,n^2+10\,n+1}+\frac{2\,a\,b\,x\,x^n\,{\left(d\,x\right)}^m\,\left(m^3+9\,m^2\,n+3\,m^2+26\,m\,n^2+18\,m\,n+3\,m+24\,n^3+26\,n^2+9\,n+1\right)}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n^3+35\,n^2+10\,n+1}+\frac{2\,b\,c\,x\,x^{3\,n}\,{\left(d\,x\right)}^m\,\left(m^3+7\,m^2\,n+3\,m^2+14\,m\,n^2+14\,m\,n+3\,m+8\,n^3+14\,n^2+7\,n+1\right)}{m^4+10\,m^3\,n+4\,m^3+35\,m^2\,n^2+30\,m^2\,n+6\,m^2+50\,m\,n^3+70\,m\,n^2+30\,m\,n+4\,m+24\,n^4+50\,n^3+35\,n^2+10\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] (a^2*x*(d*x)^m)/(m + 1) + (x*x^(2*n)*(d*x)^m*(2*a*c + b^2)*(3*m + 8*n + 16*
m*n + 19*m*n^2 + 8*m^2*n + 3*m^2 + m^3 + 19*n^2 + 12*n^3 + 1))/(4*m + 10*n
+ 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*m^3 + m^4
+ 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (c^2*x*x^(4*n)*(d*x)^m*(3*m
+ 6*n + 12*m*n + 11*m*n^2 + 6*m^2*n + 3*m^2 + m^3 + 11*n^2 + 6*n^3 + 1))/(4
*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6*m^2 + 4*
m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (2*a*b*x*x^n*(d*x)
^m*(3*m + 9*n + 18*m*n + 26*m*n^2 + 9*m^2*n + 3*m^2 + m^3 + 26*n^2 + 24*n^3
 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 + 10*m^3*n + 6
*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1) + (2*b*c*x*
```

```
x^(3*n)*(d*x)^m*(3*m + 7*n + 14*m*n + 14*m*n^2 + 7*m^2*n + 3*m^2 + m^3 + 14
*n^2 + 8*n^3 + 1))/(4*m + 10*n + 30*m*n + 70*m*n^2 + 30*m^2*n + 50*m*n^3 +
10*m^3*n + 6*m^2 + 4*m^3 + m^4 + 35*n^2 + 50*n^3 + 24*n^4 + 35*m^2*n^2 + 1)
```

3.598 $\int (dx)^m (a+bx^n+cx^{2n})\,dx$

Optimal. Leaf size=58

$$\frac{bx^{1+n}(dx)^m}{1+m+n}+\frac{cx^{1+2n}(dx)^m}{1+m+2n}+\frac{a(dx)^{1+m}}{d(1+m)}$$

```
[Out] b*x^(1+n)*(d*x)^m/(1+m+n)+c*x^(1+2*n)*(d*x)^m/(1+m+2*n)+a*(d*x)^(1+m)/d/(1+
m)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {14, 20, 30}

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{bx^{n+1}(dx)^m}{m+n+1}+\frac{cx^{2n+1}(dx)^m}{m+2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (b*x^(1 + n)*(d*x)^m)/(1 + m + n) + (c*x^(1 + 2*n)*(d*x)^m)/(1 + m + 2*n) +
 (a*(d*x)^(1 + m))/(d*(1 + m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx)^m \left(a+bx^n+cx^{2n}\right)\,dx &= \int \left(a(dx)^m+bx^n(dx)^m+cx^{2n}(dx)^m\right)\,dx\\ &= \frac{a(dx)^{1+m}}{d(1+m)}+b\int x^n(dx)^m\,dx+c\int x^{2n}(dx)^m\,dx\\ &= \frac{a(dx)^{1+m}}{d(1+m)}+\left(bx^{-m}(dx)^m\right)\int x^{m+n}\,dx+\left(cx^{-m}(dx)^m\right)\int x^{m+2n}\,dx\\ &= \frac{bx^{1+n}(dx)^m}{1+m+n}+\frac{cx^{1+2n}(dx)^m}{1+m+2n}+\frac{a(dx)^{1+m}}{d(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 41, normalized size = 0.71

$$x(dx)^m\left(\frac{a}{1+m}+x^n\left(\frac{b}{1+m+n}+\frac{cx^n}{1+m+2n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] x*(d*x)^m*(a/(1 + m) + x^n*(b/(1 + m + n) + (c*x^n)/(1 + m + 2*n)))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 205, normalized size = 3.53

method	result
risch	$\frac{x\left(c\,m^2x^{2n}+cmn\,x^{2n}+b\,m^2x^n+2bmn\,x^n+2x^{2n}cm+c\,x^{2n}n+a\,m^2+3amn+2a\,n^2+2x^n bm+2b\,x^n n+c\,x^{2n}+2am+3an+b\,x^n+a\right)\mathrm{e}^{\frac{m\left(i\pi\right.}{}}}{(1+m)(1+m+n)(1+m+2n)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] x*(c*m^2*(x^n)^2+c*m*n*(x^n)^2+b*m^2*x^n+2*b*m*n*x^n+2*m*c*(x^n)^2+c*(x^n)^
2*n+a*m^2+3*a*m*n+2*a*n^2+2*x^n*b*m+2*b*x^n*n+c*(x^n)^2+2*a*m+3*a*n+b*x^n+a
)/(1+m)/(1+m+n)/(1+m+2*n)*exp(1/2*m*(-I*csgn(I*d*x)^3*Pi+I*csgn(I*d*x)^2*cs
gn(I*d)*Pi+I*csgn(I*d*x)^2*csgn(I*x)*Pi-I*csgn(I*d*x)*csgn(I*d)*csgn(I*x)*P
i+2*ln(x)+2*ln(d)))
```

Maxima [A]
time = 0.29, size = 65, normalized size = 1.12

$$\frac{cd^m xe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1}+\frac{bd^m xe^{(m\log(x)+n\log(x))}}{m+n+1}+\frac{(dx)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] c*d^m*x*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1) + b*d^m*x*e^(m*log(x) + n*l
og(x))/(m + n + 1) + (d*x)^(m + 1)*a/(d*(m + 1))

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(58) = 116$.
time = 0.36, size = 142, normalized size = 2.45

$$\frac{(cm^2 + 2\,cm + (cm + c)n + c)xx^{2\,n}e^{(m\log(d)+m\log(x))} + (bm^2 + 2\,bm + 2\,(bm + b)n + b)xx^ne^{(m\log(d)+m\log(x))} + (am^2 + 2\,an^2 + 2\,am + 3\,(am + a)n + a)xe^{(m\log(d)+m\log(x))}}{m^3 + 2\,(m + 1)n^2 + 3\,m^2 + 3\,(m^2 + 2\,m + 1)n + 3\,m + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] ((c*m^2 + 2*c*m + (c*m + c)*n + c)*x*x^(2*n)*e^(m*log(d) + m*log(x)) + (b*m
^2 + 2*b*m + 2*(b*m + b)*n + b)*x*x^n*e^(m*log(d) + m*log(x)) + (a*m^2 + 2*
a*n^2 + 2*a*m + 3*(a*m + a)*n + a)*x*e^(m*log(d) + m*log(x)))/(m^3 + 2*(m +
 1)*n^2 + 3*m^2 + 3*(m^2 + 2*m + 1)*n + 3*m + 1)

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1091 vs. $2(49) = 98$.
time = 18.64, size = 1091, normalized size = 18.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n)),x)

[Out] Piecewise(((a + b + c)*log(x)/d, Eq(m, -1) & Eq(n, 0)), ((a*log(x) + b*x**n
/n + c*x**(2*n)/(2*n))/d, Eq(m, -1)), (a*Piecewise((-1/(2*n*(d*x)**(2*n)),
Ne(n, 0)), (log(x), True))/d + b*Piecewise((-x**n/(n*(d*x)**(2*n)), Ne(n, 0
)), (log(x), True))/d + c*Piecewise((0, (Abs(x) < 1) & (1/Abs(x) < 1)), (lo
g(x)/d**(2*n), Abs(x) < 1), (-log(1/x)/d**(2*n), 1/Abs(x) < 1), (-meijerg((
(), (1, 1)), ((0, 0), ()), x)/d**(2*n) + meijerg(((1, 1), ()), ((), (0, 0))
, x)/d**(2*n), True))/d, Eq(m, -2*n - 1)), (a*Piecewise((-1/(n*(d*x)**n), N
e(n, 0)), (log(x), True))/d + b*Piecewise((0, (Abs(x) < 1) & (1/Abs(x) < 1)
), (log(x)/d**n, Abs(x) < 1), (-log(1/x)/d**n, 1/Abs(x) < 1), (-meijerg(((
), (1, 1)), ((0, 0), ()), x)/d**n + meijerg(((1, 1), ()), ((), (0, 0)), x)/d
n, True))/d + c*Piecewise((x(2*n)/(n*(d*x)**n), Ne(n, 0)), (log(x), Tru
e))/d, Eq(m, -n - 1)), (a*m**2*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n
2 + 6*m*n + 3*m + 2*n2 + 3*n + 1) + 3*a*m*n*x*(d*x)**m/(m**3 + 3*m**2*n
 + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a*m*x*(d*x)**m/(

```
m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*a
*n**2*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**
2 + 3*n + 1) + 3*a*n*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*
n + 3*m + 2*n**2 + 3*n + 1) + a*x*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*
n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + b*m**2*x*x**n*(d*x)**m/(m**3 + 3*m
**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*b*m*n*x*x**
n*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*
n + 1) + 2*b*m*x*x**n*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n
 + 3*m + 2*n**2 + 3*n + 1) + 2*b*n*x*x**n*(d*x)**m/(m**3 + 3*m**2*n + 3*m**
2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + b*x*x**n*(d*x)**m/(m**3 +
3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + c*m**2*x*x
**(2*n)*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**
2 + 3*n + 1) + c*m*n*x*x**(2*n)*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n*
*2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + 2*c*m*x*x**(2*n)*(d*x)**m/(m**3 + 3*
m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3*n + 1) + c*n*x*x**(2*
n)*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*n + 3*m + 2*n**2 + 3
*n + 1) + c*x*x**(2*n)*(d*x)**m/(m**3 + 3*m**2*n + 3*m**2 + 2*m*n**2 + 6*m*
n + 3*m + 2*n**2 + 3*n + 1), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(58) = 116$.
time = 3.74, size = 557, normalized size = 9.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] (c*m^2*x*x^(2*n)*e^(m*log(d) + m*log(x)) + c*m*n*x*x^(2*n)*e^(m*log(d) + m*
log(x)) + b*m^2*x*x^n*e^(m*log(d) + m*log(x)) + c*m^2*x*x^n*e^(m*log(d) + m
*log(x)) + 2*b*m*n*x*x^n*e^(m*log(d) + m*log(x)) + c*m*n*x*x^n*e^(m*log(d)
+ m*log(x)) + a*m^2*x*e^(m*log(d) + m*log(x)) + b*m^2*x*e^(m*log(d) + m*log
(x)) + c*m^2*x*e^(m*log(d) + m*log(x)) + 3*a*m*n*x*e^(m*log(d) + m*log(x))
+ 2*b*m*n*x*e^(m*log(d) + m*log(x)) + c*m*n*x*e^(m*log(d) + m*log(x)) + 2*a
*n^2*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*x^(2*n)*e^(m*log(d) + m*log(x)) +
c*n*x*x^(2*n)*e^(m*log(d) + m*log(x)) + 2*b*m*x*x^n*e^(m*log(d) + m*log(x))
 + 2*c*m*x*x^n*e^(m*log(d) + m*log(x)) + 2*b*n*x*x^n*e^(m*log(d) + m*log(x)
) + c*n*x*x^n*e^(m*log(d) + m*log(x)) + 2*a*m*x*e^(m*log(d) + m*log(x)) + 2
*b*m*x*e^(m*log(d) + m*log(x)) + 2*c*m*x*e^(m*log(d) + m*log(x)) + 3*a*n*x*
e^(m*log(d) + m*log(x)) + 2*b*n*x*e^(m*log(d) + m*log(x)) + c*n*x*e^(m*log(
d) + m*log(x)) + c*x*x^(2*n)*e^(m*log(d) + m*log(x)) + b*x*x^n*e^(m*log(d)
+ m*log(x)) + c*x*x^n*e^(m*log(d) + m*log(x)) + a*x*e^(m*log(d) + m*log(x))
 + b*x*e^(m*log(d) + m*log(x)) + c*x*e^(m*log(d) + m*log(x)))/(m^3 + 3*m^2*
n + 2*m*n^2 + 3*m^2 + 6*m*n + 2*n^2 + 3*m + 3*n + 1)
```

Mupad [B]

time = 1.41, size = 83, normalized size = 1.43

$$(dx)^m \left(\frac{ax}{m+1} + \frac{b x x^n (m+2n+1)}{m^2+3mn+2m+2n^2+3n+1} + \frac{c x x^{2n} (m+n+1)}{m^2+3mn+2m+2n^2+3n+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] (d*x)^m*((a*x)/(m + 1) + (b*x*x^n*(m + 2*n + 1))/(2*m + 3*n + 3*m*n + m^2 +
 2*n^2 + 1) + (c*x*x^(2*n)*(m + n + 1))/(2*m + 3*n + 3*m*n + m^2 + 2*n^2 +
1))
```

3.599 $\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=175

$$\frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)}-\frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d(1+m)}$$

```
[Out] 2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)-2*c*(d*x)^(1+m)*
hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)
/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]

time = 0.13, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1397, 371}

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n
)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d*(1
 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n,
(-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a
*c])*d*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1397

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[(d*x)^m/(b - q + 2*
c*x^n), x], x] - Dist[2*(c/q), Int[(d*x)^m/(b + q + 2*c*x^n), x], x]] /; Fr
eeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(dx)^m}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{(dx)^m}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(dx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d(1+m)} - \frac{2c(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};\right.}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4a\right.}}$$

Mathematica [A]
time = 0.64, size = 307, normalized size = 1.75

$$-\frac{x(dx)^m\left(\frac{2c\left(1-2^{-\frac{1+m}{n}}\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-\frac{1+m}{n}}{}_2F_1\left(-\frac{1+m}{n},-\frac{1+m}{n};1-\frac{1+m}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{2c\left(1-2^{-\frac{1+m}{n}}\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-\frac{1+m}{n}}{}_2F_1\left(-\frac{1+m}{n},-\frac{1+m}{n};\frac{-1-m+n}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((x*(d*x)^m*((2*c*(1 - Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (
1 + m)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)]/(2^((1
 + m)/n)*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n))))/(b^2 -
4*a*c - b*Sqrt[b^2 - 4*a*c]) + (2*c*(1 - Hypergeometric2F1[-((1 + m)/n), -(
(1 + m)/n), (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^n)]/(2^((1 + m)/n)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1
+ m)/n))))/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))))/(1 + m))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + bx^n + cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(d\,x)}^m}{a + b\,x^n + c\,x^{2\,n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n)), x)
```

3.600 $\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=328

$$\frac{(dx)^{1+m}\,(b^2-2ac+bcx^n)}{a\,(b^2-4ac)\,dn\,(a+bx^n+cx^{2n})}+\frac{c\left(\frac{4ac(1+m-2n)-b^2(1+m-n)}{\sqrt{b^2-4ac}}-b(1+m-n)\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\right.}{a\,(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)d(1+m)n}$$

```
[Out] (d*x)^(1+m)*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(a+b*x^n+c*x^(2*n))+c*(d
*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)
))*(-b*(1+m-n)+(4*a*c*(1+m-2*n)-b^2*(1+m-n))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+
b^2)/d/(1+m)/n/(b-(-4*a*c+b^2)^(1/2))-c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],
[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(4*a*c*(1+m-2*n)-b^2*(1+m-n)+b
*(1+m-n)*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/n/(b+(-4*a*c+b^2)
^(1/2))
```

Rubi [A]
time = 0.68, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1398, 1574, 371}

$$\frac{c(dx)^{m+1}\left(\frac{4ac(m-2n+1)-b^2(m-n+1)}{\sqrt{b^2-4ac}}-b(m-n+1)\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{ad(m+1)n\,(b^2-4ac)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(b(m-n+1)\sqrt{b^2-4ac}+4ac(m-2n+1)-(b^2(m-n+1))\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)n\,(b^2-4ac)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(dx)^{m+1}\,(-2ac+b^2+bcx^n)}{adn\,(b^2-4ac)\,(a+bx^n+cx^{2n})}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(a*(b^2 - 4*a*c)*d*n*(a + b*x^n + c
*x^(2*n))) + (c*((4*a*c*(1 + m - 2*n) - b^2*(1 + m - n))/Sqrt[b^2 - 4*a*c]
- b*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/
n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*
a*c])*d*(1 + m)*n) - (c*(4*a*c*(1 + m - 2*n) - b^2*(1 + m - n) + b*Sqrt[b^2
 - 4*a*c]*(1 + m - n))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m
 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)^(3/2)*(b + S
qrt[b^2 - 4*a*c])*d*(1 + m)*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1398

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b
^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p +
1) + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[p + 1, 0]
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^2}\,dx = \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{a(b^2-4ac)dn(a+bx^n+cx^{2n})} - \frac{\int \frac{(dx)^m(-2ac(1+m-2n)+b^2(1+m-n)+bc(1+m-n)x^n)}{a+bx^n+cx^{2n}}\,dx}{a(b^2-4ac)n}$$

$$= \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{a(b^2-4ac)dn(a+bx^n+cx^{2n})} - \frac{\int \left(\frac{\left(bc(1+m-n)+\frac{c\left(b^2-4ac+b^2m-4acm-b^2n+8acn\right)}{\sqrt{b^2-4ac}}\right)(dx)^m}{b-\sqrt{b^2-4ac}+2cx^n}\right.}{a(b^2}$$

$$= \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{a(b^2-4ac)dn(a+bx^n+cx^{2n})} + \frac{\left(c\left(4ac(1+m-2n)-b^2(1+m-n)-b\sqrt{b^2\right.\right.}{a(b^2-}$$

$$= \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{a(b^2-4ac)dn(a+bx^n+cx^{2n})} + \frac{c\left(4ac(1+m-2n)-b^2(1+m-n)-b\sqrt{b^2-\right.}{a(b^2-4ac)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1890 vs. $2(328) = 656$.
time = 4.36, size = 1890, normalized size = 5.76

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] (x*(d*x)^m*((2*b^2*c)/(a*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m))
 - (8*c^2)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)) + (2*b^2*c)/
(a*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + m)) + (8*c^2)/((-b^2 + 4*a*c +
b*Sqrt[b^2 - 4*a*c])*(1 + m)) - (2*b^2*c)/(a*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^
2 - 4*a*c])*(1 + m)*n) + (4*c^2)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])
*(1 + m)*n) + (4*c^2)/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + m)*n) + (2*
b^2*c)/(a*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + m)*n) - (2*b^2*c*m)/(a*
Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*n) + (4*c^2*m)/(Sqrt[b^2
- 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*n) + (4*c^2*m)/((b^2 - 4*a*c - b*S
qrt[b^2 - 4*a*c])*(1 + m)*n) + (2*b^2*c*m)/(a*(-b^2 + 4*a*c + b*Sqrt[b^2 -
4*a*c])*(1 + m)*n) - b^2/(a*n*(a + x^n*(b + c*x^n))) + (2*c)/(n*(a + x^n*(b
 + c*x^n))) - (b*c*x^n)/(a*n*(a + x^n*(b + c*x^n))) + (c*(4*a*c*Sqrt[b^2 -
4*a*c]*(1 + m - 2*n) + 4*a*b*c*(1 + m - n) - b^2*Sqrt[b^2 - 4*a*c]*(1 + m -
 n) + b^3*(-1 - m + n))*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (
1 + m)/n, (b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^((
1 + m)/n)*a*Sqrt[b^2 - 4*a*c]*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + m)*
n*((c*x^n)/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n)) + (b*c*(-1 - m +
 n)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, (b + Sqrt[
b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(2^((1 + m)/n)*a*Sqrt[b^2
 - 4*a*c]*(1 + m)*n*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n)
) - (2^((-1 - m + n)/n)*b^2*c*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n),
 (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)]
)/(a*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*((c*x^n)/(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n)) + (c^2*Hypergeometric2F1[-((1 + m)/n),
 -((1 + m)/n), (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^n)])/(2^((1 + m - 3*n)/n)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*
c])*(1 + m)*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n)) + (2^(
(-1 - m + n)/n)*b^2*c*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), (-1 - m
 + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(a*Sqr
t[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*n*((c*x^n)/(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^n))^((1 + m)/n)) - (c^2*Hypergeometric2F1[-((1 + m)/n), -((1
+ m)/n), (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^n)])/(2^((1 + m - 2*n)/n)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1
 + m)*n*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n)) + (2^((-1
- m + n)/n)*b^2*c*m*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), (-1 - m +
 n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/(a*Sqrt[
b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1 + m)*n*((c*x^n)/(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^n))^((1 + m)/n)) - (c^2*m*Hypergeometric2F1[-((1 + m)/n), -((1
+ m)/n), (-1 - m + n)/n, (b + Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^n)])/(2^((1 + m - 2*n)/n)*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*(1
 + m)*n*((c*x^n)/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n))))/(-b^2 +
4*a*c)
```

Maple [F]

time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a+b\,x^n+c\,x^{2n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] (b*c*d^m*x*e^(m*log(x) + n*log(x)) + (b^2*d^m - 2*a*c*d^m)*x*x^m)/(a^2*b^2*
n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)
*x^n) + integrate(-(b*c*d^m*(m - n + 1)*e^(m*log(x) + n*log(x)) + (b^2*d^m*
(m - n + 1) - 2*a*c*d^m*(m - 2*n + 1))*x^m)/(a^2*b^2*n - 4*a^3*c*n + (a*b^2
*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 + 2*(b*c*x^n
+ a*c)*x^(2*n)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^m}{\left(a+b\,x^n+c\,x^{2\,n}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n))^2,x)
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n))^2, x)
```

3.601 $\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^3} dx$

Optimal. Leaf size=615

$$\frac{(dx)^{1+m}\left(b^2-2ac+bcx^n\right)}{2a\left(b^2-4ac\right)dn\left(a+bx^n+cx^{2n}\right)^2}-\frac{(dx)^{1+m}\left(4a^2c^2(1+m-4n)-5ab^2c(1+m-3n)+b^4(1+m-2n)-b\right.}{2a^2\left(b^2-4ac\right)^2dn^2\left(a+bx^n+cx\right.}$$

```
[Out] 1/2*(d*x)^(1+m)*(b^2-2*a*c+b*c*x^n)/a/(-4*a*c+b^2)/d/n/(a+b*x^n+c*x^(2*n))^
2-1/2*(d*x)^(1+m)*(4*a^2*c^2*(1+m-4*n)-5*a*b^2*c*(1+m-3*n)+b^4*(1+m-2*n)-b*
c*(2*a*c*(2+2*m-7*n)-b^2*(1+m-2*n))*x^n)/a^2/(-4*a*c+b^2)^2/d/n^2/(a+b*x^n+
c*x^(2*n))-1/2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b
-(-4*a*c+b^2)^(1/2)))*(-b^4*(1+m^2+m*(2-3*n)-3*n+2*n^2)+6*a*b^2*c*(1+m^2+m*
(2-4*n)-4*n+3*n^2)-8*a^2*c^2*(1+m^2+m*(2-6*n)-6*n+8*n^2)+b*(2*a*c*(2+2*m-7*
n)-b^2*(1+m-2*n))*(1+m-n)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(5/2)/d/(1+m
)/n^2/(b-(-4*a*c+b^2)^(1/2))-1/2*c*(d*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m
+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(b^4*(1+m^2+m*(2-3*n)-3*n+2*n^2)-6*
a*b^2*c*(1+m^2+m*(2-4*n)-4*n+3*n^2)+8*a^2*c^2*(1+m^2+m*(2-6*n)-6*n+8*n^2)+b
*(2*a*c*(2+2*m-7*n)-b^2*(1+m-2*n))*(1+m-n)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+
b^2)^(5/2)/d/(1+m)/n^2/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]

time = 7.04, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1398, 1572, 1574, 371}

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^n))/(2*a*(b^2 - 4*a*c)*d*n*(a + b*x^n +
 c*x^(2*n))^2) - ((d*x)^(1 + m)*(4*a^2*c^2*(1 + m - 4*n) - 5*a*b^2*c*(1 + m
 - 3*n) + b^4*(1 + m - 2*n) - b*c*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n
))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*d*n^2*(a + b*x^n + c*x^(2*n))) - (c*(b*Sqrt
[b^2 - 4*a*c]*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n))*(1 + m - n) - b^4
*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*(1 + m^2 + m*(2 - 4*n) -
 4*n + 3*n^2) - 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))*(d*x)^(1 +
 m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)*n
^2) - (c*(b*Sqrt[b^2 - 4*a*c]*(2*a*c*(2 + 2*m - 7*n) - b^2*(1 + m - 2*n))*(
1 + m - n) + b^4*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) - 6*a*b^2*c*(1 + m^2
 + m*(2 - 4*n) - 4*n + 3*n^2) + 8*a^2*c^2*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*
```

```
n^2))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^(5/2)*(b + Sqrt[b^2 - 4*a
*c])*d*(1 + m)*n^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1398

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^n)*((a + b*x^n + c*x
^(2*n))^(p + 1)/(a*d*n*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b
^2 - 4*a*c)), Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(p + 1)*Simp[b^2*(n*(p +
1) + m + 1) - 2*a*c*(m + 2*n*(p + 1) + 1) + b*c*(2*n*p + 3*n + m + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a
*c, 0] && ILtQ[p + 1, 0]
```

Rule 1572

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m +
 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*
x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && ILtQ[p + 1, 0]
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^3}\,dx = \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{2a(b^2-4ac)\,dn\,(a+bx^n+cx^{2n})^2} - \frac{\int \frac{(dx)^m(-2ac(1+m-4n)+b^2(1+m-2n)+bc(1+m-3n)x^n}{(a+bx^n+cx^{2n})^2}}{2a(b^2-4ac)\,n}$$

$$= \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{2a(b^2-4ac)\,dn\,(a+bx^n+cx^{2n})^2} - \frac{(dx)^{1+m}(4a^2c^2(1+m-4n)-5ab^2c(1+m}{2a^2(}$$

$$= \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{2a(b^2-4ac)\,dn\,(a+bx^n+cx^{2n})^2} - \frac{(dx)^{1+m}(4a^2c^2(1+m-4n)-5ab^2c(1+m}{2a^2(}$$

$$= \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{2a(b^2-4ac)\,dn\,(a+bx^n+cx^{2n})^2} - \frac{(dx)^{1+m}(4a^2c^2(1+m-4n)-5ab^2c(1+m}{2a^2(}$$

$$= \frac{(dx)^{1+m}(b^2-2ac+bcx^n)}{2a(b^2-4ac)\,dn\,(a+bx^n+cx^{2n})^2} - \frac{(dx)^{1+m}(4a^2c^2(1+m-4n)-5ab^2c(1+m}{2a^2(}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 12289 vs. 2(615) = 1230.
time = 6.89, size = 12289, normalized size = 19.98

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^3,x]
[Out] Result too large to show
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a+b\,x^n+c\,x^{2n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x)
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*b^2*c*d^m*(5*m - 21*n + 5) - a*b^4*d^m*(m - 3*n + 1) - 4*a^3*c^2*
d^m*(m - 6*n + 1))*x*x^m + (2*a*b*c^3*d^m*(2*m - 7*n + 2) - b^3*c^2*d^m*(m
- 2*n + 1))*x*e^(m*log(x) + 3*n*log(x)) + (a*b^2*c^2*d^m*(9*m - 29*n + 9) -
 2*b^4*c*d^m*(m - 2*n + 1) - 4*a^2*c^3*d^m*(m - 4*n + 1))*x*e^(m*log(x) + 2
*n*log(x)) - (b^5*d^m*(m - 2*n + 1) - 4*a*b^3*c*d^m*(m - 3*n + 1) + 2*a^2*b
*c^2*d^m*n)*x*e^(m*log(x) + n*log(x)))/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*
a^6*c^2*n^2 + (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n
) + 2*(a^2*b^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2
*b^6*n^2 - 6*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a
^4*b^3*c*n^2 + 16*a^5*b*c^2*n^2)*x^n) - integrate(-1/2*(((m^2 - m*(3*n - 2)
 + 2*n^2 - 3*n + 1)*b^4*d^m - (5*m^2 - m*(21*n - 10) + 16*n^2 - 21*n + 5)*a
*b^2*c*d^m + 4*(m^2 - 2*m*(3*n - 1) + 8*n^2 - 6*n + 1)*a^2*c^2*d^m)*x^m + (
(m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c*d^m - 2*(2*m^2 - m*(9*n - 4) +
7*n^2 - 9*n + 2)*a*b*c^2*d^m)*e^(m*log(x) + n*log(x)))/(a^3*b^4*n^2 - 8*a^4
*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c
^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n),
x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x^n
 + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a*b*c*x^n + a
^2*c)*x^(2*n)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^m}{\left(a+b\,x^n+c\,x^{2\,n}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n))^3,x)
```

```
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n))^3, x)
```

3.602 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx$

Optimal. Leaf size=161

$$\frac{a(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1+m}{n};-\frac{3}{2},-\frac{3}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] a*(d*x)^(1+m)*AppellF1((1+m)/n,-3/2,-3/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/d/(1+m)/
(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))
^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1399, 524}

$$\frac{a(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{m+1}{n};-\frac{3}{2},-\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -3/2, -3/2
, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
```

```
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^{3/2} dx = \frac{\left(a\sqrt{a+bx^n+cx^{2n}}\right)\int (dx)^m \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2} \left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right.}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{a(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}\, F_1\left(\frac{1+m}{n};-\frac{3}{2},-\frac{3}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{}{b+}\right.}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 618 vs. $2(161) = 322$.
time = 2.22, size = 618, normalized size = 3.84

$$\frac{x(dx)^m\left(-6an^2(1+m+n)\left(b^2(1+m)-4ac(1+m+2n)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{n};\frac{1}{2},\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+(1+m)\left(2(1+m+n)\left(4a^2c(1+m^2+6n+8n^2+m(2+6n))+x^n(b+cx^n)\left(3b^2n^2+2bc(2+2m^2+9n+7n^2+m(4+9n))x^n+4c^2(1+m^2+3n+2n^2+m(2+3n))x^{2n}\right)+a\left(3b^2n^2+2bc(4+4m^2+21n+23n^2+m(8+21n))x^n+4c^2(2+2m^2+9n+10n^2+m(4+9n))x^{2n}\right)\right)-3bn^2\left(b^2(2+2m+n)-4ac(2+2m+3n)\right)x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m+n}{n};\frac{1}{2},\frac{1}{2};\frac{1+m+2n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{8c(1+m)(1+m+n)^2(1+m+2n)(1+m+3n)\sqrt{a+x^n(b+cx^n)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*(-6*a*n^2*(1 + m + n)*(b^2*(1 + m) - 4*a*c*(1 + m + 2*n))*Sqrt[(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2
, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b
^2 - 4*a*c])] + (1 + m)*(2*(1 + m + n)*(4*a^2*c*(1 + m^2 + 6*n + 8*n^2 + m*
(2 + 6*n)) + x^n*(b + c*x^n)*(3*b^2*n^2 + 2*b*c*(2 + 2*m^2 + 9*n + 7*n^2 +
m*(4 + 9*n))*x^n + 4*c^2*(1 + m^2 + 3*n + 2*n^2 + m*(2 + 3*n))*x^(2*n)) + a
*(3*b^2*n^2 + 2*b*c*(4 + 4*m^2 + 21*n + 23*n^2 + m*(8 + 21*n))*x^n + 4*c^2*
(2 + 2*m^2 + 9*n + 10*n^2 + m*(4 + 9*n))*x^(2*n))) - 3*b*n^2*(b^2*(2 + 2*m
+ n) - 4*a*c*(2 + 2*m + 3*n))*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])]*AppellF1[(1 + m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(8*c*(1 + m)
*(1 + m + n)^2*(1 + m + 2*n)*(1 + m + 3*n)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,x^n+c\,x^{2n}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + bx^n + cx^{2n}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(3/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^(3/2)*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(a+b\,x^n+c\,x^{2\,n}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int((d*x)^m*(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.603 $\int (dx)^m \sqrt{a+bx^n+cx^{2n}}\, dx$

Optimal. Leaf size=160

$$\frac{(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}\, F_1\left(\frac{1+m}{n};-\frac{1}{2},-\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] (d*x)^(1+m)*AppellF1((1+m)/n,-1/2,-1/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(
1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(a+b*x^n+c*x^(2*n))^(1/2)/d/(1+m)/(1
+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1399, 524}

$$\frac{(dx)^{m+1}\sqrt{a+bx^n+cx^{2n}}\, F_1\left(\frac{m+1}{n};-\frac{1}{2},-\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)]*AppellF1[(1 + m)/n, -1/2, -1/2,
(1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2
 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
```

```
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int (dx)^m\sqrt{a+bx^n+cx^{2n}}\,dx = \frac{\sqrt{a+bx^n+cx^{2n}}\int (dx)^m\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,dx}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{(dx)^{1+m}\sqrt{a+bx^n+cx^{2n}}\,F_1\left(\frac{1+m}{n};-\frac{1}{2},-\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2\ldots}{b+\sqrt{b^2\ldots}}\right.}{d(1+m)\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 388 vs. $2(160) = 320$.
time = 0.52, size = 388, normalized size = 2.42

$$\frac{x(dx)^m\left(2an(1+m+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{n};\frac{1}{2},\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+(1+m)\left(2(1+m+n)\left(a+x^n(b+cx^n)\right)+bnx^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m+n}{n};\frac{1}{2},\frac{1}{2};\frac{1+m+2n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{2(1+m)(1+m+n)^2\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*(2*a*n*(1 + m + n)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - S
qrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*(2*(1 + m + n)*
(a + x^n*(b + c*x^n)) + b*n*x^n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b -
 Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[(1 + m + n)/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/(2*(1 + m)*(1
+ m + n)^2*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)
[Out] int((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{a + bx^n + cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral((d*x)**m*sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^(2*n) + b*x^n + a)*(d*x)^m, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\sqrt{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int((d*x)^m*(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.604 $\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx$

Optimal. Leaf size=160

$$\frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{n};\frac{1}{2},\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (d*x)^(1+m)*AppellF1((1+m)/n,1/2,1/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/
2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/d/(1+m)/(a+b*x^n+c*x^(2*n))^(1/
2)
```

Rubi [A]
time = 0.11, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1399, 524}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{n};\frac{1}{2},\frac{1}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(d*(
1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(dx)^m}{\sqrt{a+bx^n+cx^{2n}}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+ \cdots}}}{\sqrt{a+bx^n+cx^{2n}}}$$
$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{n};\frac{1}{2},\frac{1}{2};\frac{1+m+n}{n};-\frac{\cdots}{b-\cdots}\right.}{d(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [**A**]
time = 0.15, size = 183, normalized size = 1.14

$$\frac{x(dx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{n};\frac{1}{2},\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{(1+m)\sqrt{a+x^n(b+cx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m/Sqrt[a + b*x^n + c*x^(2*n)],x]
```

```
[Out] (x*(d*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1
 + m)/n, 1/2, 1/2, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])])/((1 + m)*Sqrt[a + x^n*(b + c*x^n)])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a+b\,x^n+c\,x^{2n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\sqrt{a + bx^n + cx^{2n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(1/2),x)
[Out] Integral((d*x)**m/sqrt(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(1/2),x, algorithm="giac")
[Out] integrate((d*x)^m/sqrt(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d\,x)^m}{\sqrt{a + b\,x^n + c\,x^{2\,n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n))^(1/2),x)
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n))^(1/2), x)
```

3.605 $\int \frac{(dx)^m}{\left(a+bx^n+cx^{2n}\right)^{3/2}} \, dx$

Optimal. Leaf size=163

$$\frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{n};\frac{3}{2},\frac{3}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

```
[Out] (d*x)^(1+m)*AppellF1((1+m)/n,3/2,3/2,(1+m+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^(1/
2)*(1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/d/(1+m)/(a+b*x^n+c*x^(2*n))^(
1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1399, 524}

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{n};\frac{3}{2},\frac{3}{2};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^n+cx^{2n}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 3/2, 3/2, (1 + m + n)/n, (-
2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*d
*(1 + m)*Sqrt[a + b*x^n + c*x^(2*n)])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
```

```
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int \frac{(dx)^m}{(a+bx^n+cx^{2n})^{3/2}}\,dx = \frac{\left(\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(dx)^m}{\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\ldots\right.}}{a\sqrt{a+bx^n+cx^{2n}}}$$

$$= \frac{(dx)^{1+m}\sqrt{1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{n};\frac{3}{2},\frac{3}{2};\frac{1+m+n}{n};-\ldots\right.}{ad(1+m)\sqrt{a+bx^n+cx^{2n}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 428 vs. 2(163) = 326.
time = 1.02, size = 428, normalized size = 2.63

$$\frac{x(dx)^m\left(\left(-4ac(1+m-n)+b^2(2+2m-n)\right)(1+m+n)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m}{n};\frac{1}{2},\frac{1}{2};\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)-2(1+m)\left((1+m+n)(b^2-2ac+bcx^n)-bc(1+m)x^n\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1+m+n}{n};\frac{1}{2},\frac{1}{2};\frac{1+m+2n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{a(-b^2+4ac)(1+m)n(1+m+n)\sqrt{a+x^n(b+cx^n)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x]
```

```
[Out] (x*(d*x)^m*((-4*a*c*(1 + m - n) + b^2*(2 + 2*m - n))*(1 + m + n)*Sqrt[(b -
Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/n, 1/2, 1/2, (1
 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 -
 4*a*c])] - 2*(1 + m)*((1 + m + n)*(b^2 - 2*a*c + b*c*x^n) - b*c*(1 + m)*x^
n*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m + n)
/n, 1/2, 1/2, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n
)/(-b + Sqrt[b^2 - 4*a*c])])))/(a*(-b^2 + 4*a*c)*(1 + m)*n*(1 + m + n)*Sqrt
[a + x^n*(b + c*x^n)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a+b\,x^n+c\,x^{2n})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

```
[Out] int((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="maxima")
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + bx^n + cx^{2n})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*x**n+c*x**(2*n))**(3/2),x)
[Out] Integral((d*x)**m/(a + b*x**n + c*x**(2*n))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*x^n+c*x^(2*n))^(3/2),x, algorithm="giac")
[Out] integrate((d*x)^m/(c*x^(2*n) + b*x^n + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(d\,x\right)}^m}{{\left(a+b\,x^n+c\,x^{2\,n}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2),x)
[Out] int((d*x)^m/(a + b*x^n + c*x^(2*n))^(3/2), x)
```

3.606 $\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=158

$$\frac{(dx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{2c}{b-\sqrt{b^2}}\right.}{d(1+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*x^n
/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/d/(1+m)/((1+2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]

time = 0.09, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{(dx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m
 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(d*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int (dx)^m \left(a+bx^n+cx^{2n}\right)^p\,dx = \left(\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)$$

$$= \frac{(dx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^{p}}{d(1+m)}$$

Mathematica [A]
time = 0.31, size = 181, normalized size = 1.15

$$\frac{x(dx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^pF_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(d*x)^m*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,x^n+c\,x^{2n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,{\left(a+b\,x^n+c\,x^{2\,n}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] int((d*x)^m*(a + b*x^n + c*x^(2*n))^p, x)
```

3.607 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx$

Optimal. Leaf size=46

$$\frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

```
[Out] 1/4*a*(e*x+d)^4/e+1/6*b*(e*x+d)^6/e+1/8*c*(e*x+d)^8/e
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1156, 14}

$$\frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (a*(d + e*x)^4)/(4*e) + (b*(d + e*x)^6)/(6*e) + (c*(d + e*x)^8)/(8*e)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) \, dx &= \frac{\text{Subst}\left(\int x^3\left(a+bx^2+cx^4\right) \, dx,x,d+ex\right)}{e}\\
&= \frac{\text{Subst}\left(\int \left(ax^3+bx^5+cx^7\right) \, dx,x,d+ex\right)}{e}\\
&= \frac{a(d+ex)^4}{4e}+\frac{b(d+ex)^6}{6e}+\frac{c(d+ex)^8}{8e}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 150 vs. 2(46) = 92.
time = 0.03, size = 150, normalized size = 3.26

$$d^3 (a + bd^2 + cd^4)\, x + \frac{1}{2} d^2 (3a + 5bd^2 + 7cd^4)\, ex^2 + \frac{1}{3} d(3a + 10bd^2 + 21cd^4)\, e^2x^3 + \frac{1}{4}(a + 10bd^2 + 35cd^4)\, e^3x^4 + d(b + 7cd^2)\, e^4x^5 + \frac{1}{6}(b + 21cd^2)\, e^5x^6 + cde^6x^7 + \frac{1}{8} ce^7x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 + (d*(3
*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*x^4)/
4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7 + (c
*e^7*x^8)/8
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(40) = 80.
time = 0.24, size = 298, normalized size = 6.48

method	result
norman	$\frac{e^7 c x^8}{8} + d e^6 c x^7 + \left(\frac{7}{2} d^2 e^5 c + \frac{1}{6} b e^5\right) x^6 + \left(7 d^3 e^4 c + b d e^4\right) x^5 + \left(\frac{35}{4} d^4 e^3 c + \frac{5}{2} b d^2 e^3 + \frac{1}{4} a e^3\right) x^4 + (7$
gosper	$\frac{x(3e^7 c x^7+24d e^6 c x^6+84x^5 d^2 e^5 c+168c d^3 e^4 x^4+4x^5 b e^5+210x^3 d^4 e^3 c+24bd e^4 x^4+168x^2 c d^5 e^2+60x^3 b d^2 e^3+84xc d^6 e+80x^2 b d^3 e^2+}{24}$
risch	$\frac{1}{8} e^7 c x^8 + d e^6 c x^7 + \frac{7}{2} x^6 d^2 e^5 c + \frac{1}{6} x^6 b e^5 + 7c d^3 e^4 x^5 + bd e^4 x^5 + \frac{35}{4} x^4 d^4 e^3 c + \frac{5}{2} x^4 b d^2 e^3 + \frac{1}{4} x^4 a e^3$
default	$\frac{e^7 c x^8}{8} + d e^6 c x^7 + \frac{(15d^2 e^5 c+e^3(6d^2 e^2 c+e^2 b))x^6}{6} + \frac{(13d^3 e^4 c+3d e^2(6d^2 e^2 c+e^2 b)+e^3(4d^3 ec+2deb))x^5}{5} + \frac{(4d^4 e^3 c+3d^2 e(6}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*e^7*c*x^8+d*e^6*c*x^7+1/6*(15*d^2*e^5*c+e^3*(6*c*d^2*e^2+b*e^2))*x^6+1/
5*(13*d^3*e^4*c+3*d*e^2*(6*c*d^2*e^2+b*e^2)+e^3*(4*c*d^3*e+2*b*d*e))*x^5+1/
4*(4*d^4*e^3*c+3*d^2*e*(6*c*d^2*e^2+b*e^2)+3*d*e^2*(4*c*d^3*e+2*b*d*e)+e^3*
(c*d^4+b*d^2+a))*x^4+1/3*(d^3*(6*c*d^2*e^2+b*e^2)+3*d^2*e*(4*c*d^3*e+2*b*d*
e)+3*d*e^2*(c*d^4+b*d^2+a))*x^3+1/2*(d^3*(4*c*d^3*e+2*b*d*e)+3*d^2*e*(c*d^4
+b*d^2+a))*x^2+d^3*(c*d^4+b*d^2+a)*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(40) = 80.
time = 0.29, size = 155, normalized size = 3.37

$$\frac{1}{8} cx^8e^7 + cdx^7e^6 + \frac{1}{6}(21\, cd^2e^5 + be^5)x^6 + (7\, cd^3e^4 + bde^4)x^5 + \frac{1}{4}(35\, cd^4e^3 + 10\, bd^2e^3 + ae^3)x^4 + \frac{1}{3}(21\, cd^5e^2 + 10\, bd^3e^2 + 3\, ade^2)x^3 + \frac{1}{2}(7\, cd^6e + 5\, bd^4e + 3\, ad^2e)x^2 + (cd^7 + bd^5 + ad^3)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] 1/8*c*x^8*e^7 + c*d*x^7*e^6 + 1/6*(21*c*d^2*e^5 + b*e^5)*x^6 + (7*c*d^3*e^4
 + b*d*e^4)*x^5 + 1/4*(35*c*d^4*e^3 + 10*b*d^2*e^3 + a*e^3)*x^4 + 1/3*(21*c
*d^5*e^2 + 10*b*d^3*e^2 + 3*a*d*e^2)*x^3 + 1/2*(7*c*d^6*e + 5*b*d^4*e + 3*a
*d^2*e)*x^2 + (c*d^7 + b*d^5 + a*d^3)*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(40) = 80$.
time = 0.32, size = 137, normalized size = 2.98

$$\frac{1}{8}\,cx^{8}e^{7} + cdx^{7}e^{6} + \frac{1}{6}\,(21\,cd^{2} + b)x^{6}e^{5} + (7\,cd^{3} + bd)x^{5}e^{4} + \frac{1}{4}\,(35\,cd^{4} + 10\,bd^{2} + a)x^{4}e^{3} + \frac{1}{3}\,(21\,cd^{5} + 10\,bd^{3} + 3\,ad)x^{3}e^{2} + \frac{1}{2}\,(7\,cd^{6} + 5\,bd^{4} + 3\,ad^{2})x^{2}e + (cd^{7} + bd^{5} + ad^{3})x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/8*c*x^8*e^7 + c*d*x^7*e^6 + 1/6*(21*c*d^2 + b)*x^6*e^5 + (7*c*d^3 + b*d)*
x^5*e^4 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*x^4*e^3 + 1/3*(21*c*d^5 + 10*b*d^3
+ 3*a*d)*x^3*e^2 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2)*x^2*e + (c*d^7 + b*d^5
 + a*d^3)*x
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(34) = 68$.
time = 0.03, size = 178, normalized size = 3.87

$$cde^{6}x^{7} + \frac{ce^{7}x^{8}}{8} + x^{6}\left(\frac{be^{5}}{6} + \frac{7cd^{2}e^{5}}{2}\right) + x^{5}\left(bde^{4} + 7cd^{3}e^{4}\right) + x^{4}\left(\frac{ae^{3}}{4} + \frac{5bd^{2}e^{3}}{2} + \frac{35cd^{4}e^{3}}{4}\right) + x^{3}\left(ade^{2} + \frac{10bd^{3}e^{2}}{3} + 7cd^{5}e^{2}\right) + x^{2}\cdot\left(\frac{3ad^{2}e}{2} + \frac{5bd^{4}e}{2} + \frac{7cd^{6}e}{2}\right) + x\left(ad^{3} + bd^{5} + cd^{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*x**7 + c*e**7*x**8/8 + x**6*(b*e**5/6 + 7*c*d**2*e**5/2) + x**5*(b
*d*e**4 + 7*c*d**3*e**4) + x**4*(a*e**3/4 + 5*b*d**2*e**3/2 + 35*c*d**4*e**
3/4) + x**3*(a*d*e**2 + 10*b*d**3*e**2/3 + 7*c*d**5*e**2) + x**2*(3*a*d**2*
e/2 + 5*b*d**4*e/2 + 7*c*d**6*e/2) + x*(a*d**3 + b*d**5 + c*d**7)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(40) = 80$.
time = 5.40, size = 169, normalized size = 3.67

$$\frac{1}{2}\,(x^{2}e + 2\,dx)cd^{6} + \frac{3}{4}\,(x^{2}e + 2\,dx)^{2}cd^{4}e + \frac{1}{2}\,(x^{2}e + 2\,dx)^{3}cd^{2}e^{2} + \frac{1}{2}\,(x^{2}e + 2\,dx)bd^{4} + \frac{1}{8}\,(x^{2}e + 2\,dx)^{4}ce^{3} + \frac{1}{2}\,(x^{2}e + 2\,dx)^{2}bd^{2}e + \frac{1}{6}\,(x^{2}e + 2\,dx)^{3}be^{2} + \frac{1}{2}\,(x^{2}e + 2\,dx)ad^{2} + \frac{1}{4}\,(x^{2}e + 2\,dx)^{2}ae$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e + 2*d*x)*c*d^6 + 3/4*(x^2*e + 2*d*x)^2*c*d^4*e + 1/2*(x^2*e + 2*
d*x)^3*c*d^2*e^2 + 1/2*(x^2*e + 2*d*x)*b*d^4 + 1/8*(x^2*e + 2*d*x)^4*c*e^3
+ 1/2*(x^2*e + 2*d*x)^2*b*d^2*e + 1/6*(x^2*e + 2*d*x)^3*b*e^2 + 1/2*(x^2*e
+ 2*d*x)*a*d^2 + 1/4*(x^2*e + 2*d*x)^2*a*e
```

Mupad [B]
time = 0.08, size = 141, normalized size = 3.07

$$x\,\left(c\,d^7+b\,d^5+a\,d^3\right)+\frac{e^5\,x^6\,\left(21\,c\,d^2+b\right)}{6}+\frac{c\,e^7\,x^8}{8}+\frac{e^3\,x^4\,\left(35\,c\,d^4+10\,b\,d^2+a\right)}{4}+\frac{d^2\,e\,x^2\,\left(7\,c\,d^4+5\,b\,d^2+3\,a\right)}{2}+\frac{d\,e^2\,x^3\,\left(21\,c\,d^4+10\,b\,d^2+3\,a\right)}{3}+d\,e^4\,x^5\,\left(7\,c\,d^2+b\right)+c\,d\,e^6\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] x*(a*d^3 + b*d^5 + c*d^7) + (e^5*x^6*(b + 21*c*d^2))/6 + (c*e^7*x^8)/8 + (e
^3*x^4*(a + 10*b*d^2 + 35*c*d^4))/4 + (d^2*e*x^2*(3*a + 5*b*d^2 + 7*c*d^4))
/2 + (d*e^2*x^3*(3*a + 10*b*d^2 + 21*c*d^4))/3 + d*e^4*x^5*(b + 7*c*d^2) +
c*d*e^6*x^7
```

3.608 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

Optimal. Leaf size=89

$$\frac{a^2(d+ex)^4}{4e}+\frac{ab(d+ex)^6}{3e}+\frac{(b^2+2ac)\,(d+ex)^8}{8e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

```
[Out] 1/4*a^2*(e*x+d)^4/e+1/3*a*b*(e*x+d)^6/e+1/8*(2*a*c+b^2)*(e*x+d)^8/e+1/5*b*c
*(e*x+d)^10/e+1/12*c^2*(e*x+d)^12/e
```

Rubi [A]
time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1156, 1128, 645}

$$\frac{a^2(d+ex)^4}{4e}+\frac{(2ac+b^2)\,(d+ex)^8}{8e}+\frac{ab(d+ex)^6}{3e}+\frac{bc(d+ex)^{10}}{5e}+\frac{c^2(d+ex)^{12}}{12e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (a^2*(d + e*x)^4)/(4*e) + (a*b*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*(d + e*x
)^8)/(8*e) + (b*c*(d + e*x)^10)/(5*e) + (c^2*(d + e*x)^12)/(12*e)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 dx = \frac{\text{Subst}\left(\int x^3(a+bx^2+cx^4)^2\, dx, x, d+ex\right)}{e}$$
$$= \frac{\text{Subst}\left(\int x(a+bx+cx^2)^2\, dx, x, (d+ex)^2\right)}{2e}$$
$$= \frac{\text{Subst}\left(\int (a^2x+2abx^2+(b^2+2ac)\,x^3+2bcx^4+c^2x^5)\, dx, \right.}{2e}$$
$$= \frac{a^2(d+ex)^4}{4e} + \frac{ab(d+ex)^6}{3e} + \frac{(b^2+2ac)\,(d+ex)^8}{8e} + \frac{bc(d+}{5}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 401 vs. 2(89) = 178.
time = 0.08, size = 401, normalized size = 4.51

$$d^3\left(a+bd^2+cd^4\right)^2 x+\frac{1}{2}d^2\left(3a^2+10abd^2+7b^2d^4+14acd^4+18bcd^6+11c^2d^8\right)ex^2+\frac{1}{3}d\left(3a^2+20abd^2+21b^2d^4+42acd^4+72bcd^6+55c^2d^8\right)e^2x^3+\frac{1}{4}\left(a^2+20abd^2+35b^2d^4+70acd^4+168bcd^6+165c^2d^8\right)e^3x^4+\frac{1}{5}d\left(10ab+35b^2d^2+70acd^2+252bcd^4+330c^2d^6\right)e^4x^5+\frac{1}{6}\left(2ab+21b^2d^2+42acd^2+252bcd^4+462c^2d^6\right)e^5x^6+d\left(b^2+2ac+24bcd^2+66c^2d^4\right)e^6x^7+\frac{1}{8}\left(b^2+2ac+72bcd^2+330c^2d^4\right)e^7x^8+\frac{1}{3}cd\left(6b+55cd^2\right)e^8x^9+\frac{1}{10}c\left(2b+55cd^2\right)e^9x^{10}+c^2de^{10}x^{11}+\frac{1}{12}c^2e^{11}x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 14*a*c
*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21*b^2*
d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20*a*b*d^2
 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (d*(10
*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5 + ((
2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)/6 + d
*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 72*b*c*d
^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/3 + (c*(2*b +
 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1313 vs. 2(79) = 158.
time = 0.23, size = 1314, normalized size = 14.76

method	result
norman	$\frac{e^{11}c^2x^{12}}{12}+d\,e^{10}c^2x^{11}+\left(\frac{11}{2}d^2e^9c^2+\frac{1}{5}bc\,e^9\right)x^{10}+\left(\frac{55}{3}d^3e^8c^2+2bcd\,e^8\right)x^9+\left(\frac{165}{4}c^2d^4e^7+9bc\,d^2e^7+\right.$
gosper	$\frac{x(10e^{11}c^2x^{11}+120d\,e^{10}c^2x^{10}+660x^9d^2e^9c^2+2200x^8d^3e^8c^2+24x^9bc\,e^9+4950x^7c^2d^4e^7+240x^8bcd\,e^8+7920c^2d^5e^6x^6+1080x^7bc\,d^2e^7+}{}$
risch	$9x^2bc\,d^8e+7x^2ac\,d^6e+5x^2ab\,d^4e+2acd\,e^6x^7+x^3d\,e^2a^2+\frac{11}{2}x^2c^2d^{10}e+\frac{7}{2}x^2b^2d^6e+\frac{3}{2}x^2a^2d^2e+$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*e^11*c^2*x^12+d*e^10*c^2*x^11+1/10*(27*d^2*e^9*c^2+e^3*(2*(6*c*d^2*e^2
+b*e^2)*e^4*c+16*d^2*e^6*c^2))*x^10+1/9*(25*d^3*e^8*c^2+3*d*e^2*(2*(6*c*d^2
*e^2+b*e^2)*e^4*c+16*d^2*e^6*c^2)+e^3*(2*(4*c*d^3*e+2*b*d*e)*e^4*c+8*(6*c*d
^2*e^2+b*e^2)*d*e^3*c))*x^9+1/8*(8*c^2*d^4*e^7+3*d^2*e*(2*(6*c*d^2*e^2+b*e^
2)*e^4*c+16*d^2*e^6*c^2)+3*d*e^2*(2*(4*c*d^3*e+2*b*d*e)*e^4*c+8*(6*c*d^2*e^
2+b*e^2)*d*e^3*c)+e^3*(2*(c*d^4+b*d^2+a)*e^4*c+8*(4*c*d^3*e+2*b*d*e)*d*e^3*
c+(6*c*d^2*e^2+b*e^2)^2))*x^8+1/7*(d^3*(2*(6*c*d^2*e^2+b*e^2)*e^4*c+16*d^2*
e^6*c^2)+3*d^2*e*(2*(4*c*d^3*e+2*b*d*e)*e^4*c+8*(6*c*d^2*e^2+b*e^2)*d*e^3*c
)+3*d*e^2*(2*(c*d^4+b*d^2+a)*e^4*c+8*(4*c*d^3*e+2*b*d*e)*d*e^3*c+(6*c*d^2*e
^2+b*e^2)^2)+e^3*(8*(c*d^4+b*d^2+a)*d*e^3*c+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*
e^2+b*e^2)))*x^7+1/6*(d^3*(2*(4*c*d^3*e+2*b*d*e)*e^4*c+8*(6*c*d^2*e^2+b*e^2
)*d*e^3*c)+3*d^2*e*(2*(c*d^4+b*d^2+a)*e^4*c+8*(4*c*d^3*e+2*b*d*e)*d*e^3*c+(
6*c*d^2*e^2+b*e^2)^2)+3*d*e^2*(8*(c*d^4+b*d^2+a)*d*e^3*c+2*(4*c*d^3*e+2*b*d
*e)*(6*c*d^2*e^2+b*e^2))+e^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^
3*e+2*b*d*e)^2))*x^6+1/5*(d^3*(2*(c*d^4+b*d^2+a)*e^4*c+8*(4*c*d^3*e+2*b*d*e
)*d*e^3*c+(6*c*d^2*e^2+b*e^2)^2)+3*d^2*e*(8*(c*d^4+b*d^2+a)*d*e^3*c+2*(4*c*
d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d*e^2*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2
+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+2*e^3*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e))*x
^5+1/4*(d^3*(8*(c*d^4+b*d^2+a)*d*e^3*c+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b
*e^2))+3*d^2*e*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2
)+6*d*e^2*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+e^3*(c*d^4+b*d^2+a)^2)*x^4+1/
3*(d^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d^2*
e*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+3*d*e^2*(c*d^4+b*d^2+a)^2)*x^3+1/2*(2
*d^3*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+3*d^2*e*(c*d^4+b*d^2+a)^2)*x^2+d^3
*(c*d^4+b*d^2+a)^2*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(79) = 158$.
time = 0.49, size = 464, normalized size = 5.21

$$\frac{1}{12}c^2x^{12}e^{11} + c^2dx^{11}e^{10} + \frac{1}{10}(55c^2d^2e^9 + 2bce^9)x^{10} + \frac{1}{3}(55c^2d^3e^8 + 6bcde^8)x^9 + \frac{1}{8}(330c^2d^4e^7 + 72bcd^2e^7 + b^2e^7 + 2ace^7)x^8 + (66c^2d^5e^6 + 24bcd^3e^6 + (b^2e^6 + 2ace^6)d)x^7 + \frac{1}{6}(462c^2d^6e^5 + 252bcd^4e^5 + 21(b^2e^5 + 2ace^5)d^2 + 2abe^5)x^6 + \frac{1}{5}(330c^2d^7e^4 + 252bcd^5e^4 + 35(b^2e^4 + 2ace^4)d^3 + 10abde^4)x^5 + \frac{1}{4}(165c^2d^8e^3 + 168bcd^6e^3 + 35(b^2e^3 + 2ace^3)d^4 + 20abd^2e^3 + a^2e^3)x^4 + \frac{1}{3}(55c^2d^9e^2 + 72bcd^7e^2 + 21(b^2e^2 + 2ace^2)d^5 + 20abd^3e^2 + 3a^2de^2)x^3 + \frac{1}{2}(11c^2d^{10}e + 18bcd^8e + 7(b^2e + 2ace)d^6 + 10abd^4e + 3a^2d^2e)x^2 + (c^2d^{11} + 2bcd^9 + (b^2 + 2ac)d^7 + 2abd^5 + a^2d^3)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/12*c^2*x^12*e^11 + c^2*d*x^11*e^10 + 1/10*(55*c^2*d^2*e^9 + 2*b*c*e^9)*x^
10 + 1/3*(55*c^2*d^3*e^8 + 6*b*c*d*e^8)*x^9 + 1/8*(330*c^2*d^4*e^7 + 72*b*c
*d^2*e^7 + b^2*e^7 + 2*a*c*e^7)*x^8 + (66*c^2*d^5*e^6 + 24*b*c*d^3*e^6 + (b
^2*e^6 + 2*a*c*e^6)*d)*x^7 + 1/6*(462*c^2*d^6*e^5 + 252*b*c*d^4*e^5 + 21*(b
^2*e^5 + 2*a*c*e^5)*d^2 + 2*a*b*e^5)*x^6 + 1/5*(330*c^2*d^7*e^4 + 252*b*c*d
^5*e^4 + 35*(b^2*e^4 + 2*a*c*e^4)*d^3 + 10*a*b*d*e^4)*x^5 + 1/4*(165*c^2*d^
8*e^3 + 168*b*c*d^6*e^3 + 35*(b^2*e^3 + 2*a*c*e^3)*d^4 + 20*a*b*d^2*e^3 + a
^2*e^3)*x^4 + 1/3*(55*c^2*d^9*e^2 + 72*b*c*d^7*e^2 + 21*(b^2*e^2 + 2*a*c*e^
2)*d^5 + 20*a*b*d^3*e^2 + 3*a^2*d*e^2)*x^3 + 1/2*(11*c^2*d^10*e + 18*b*c*d^
```

```
8*e + 7*(b^2*e + 2*a*c*e)*d^6 + 10*a*b*d^4*e + 3*a^2*d^2*e)*x^2 + (c^2*d^11
 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(79) = 158$.
time = 0.33, size = 394, normalized size = 4.43

$$\frac{1}{12}\,c^2x^{12}e^{11} + c^2dx^{11}e^{10} + \frac{1}{10}\,(55\,c^2d^2 + 2\,bc)x^{10}e^9 + \frac{1}{3}\,(55\,c^2d^3 + 6\,bcd)x^9e^8 + \frac{1}{8}\,(330\,c^2d^4 + 72\,bcd^2 + b^2 + 2\,ac)x^8e^7 + (66\,c^2d^5 + 24\,bcd^3 + (b^2 + 2\,ac)d)x^7e^6 + \frac{1}{6}\,(462\,c^2d^6 + 252\,bcd^4 + 21\,(b^2 + 2\,ac)d^2 + 2\,ab)x^6e^5 + \frac{1}{5}\,(330\,c^2d^7 + 252\,bcd^5 + 35\,(b^2 + 2\,ac)d^3 + 10\,abd)x^5e^4 + \frac{1}{4}\,(165\,c^2d^8 + 168\,bcd^6 + 35\,(b^2 + 2\,ac)d^4 + 20\,abd^2 + a^2)x^4e^3 + \frac{1}{3}\,(55\,c^2d^9 + 72\,bcd^7 + 21\,(b^2 + 2\,ac)d^5 + 20\,abd^3 + 3\,a^2d)x^3e^2 + \frac{1}{2}\,(11\,c^2d^{10} + 18\,bcd^8 + 7\,(b^2 + 2\,ac)d^6 + 10\,abd^4 + 3\,a^2d^2)x^2e + (c^2d^{11} + 2\,bcd^9 + (b^2 + 2\,ac)d^7 + 2\,abd^5 + a^2d^3)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/12*c^2*x^12*e^11 + c^2*d*x^11*e^10 + 1/10*(55*c^2*d^2 + 2*b*c)*x^10*e^9 +
 1/3*(55*c^2*d^3 + 6*b*c*d)*x^9*e^8 + 1/8*(330*c^2*d^4 + 72*b*c*d^2 + b^2 +
 2*a*c)*x^8*e^7 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a*c)*d)*x^7*e^6 + 1/6
*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2 + 2*a*b)*x^6*e^5 + 1/5*(
330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^3 + 10*a*b*d)*x^5*e^4 + 1/4*
(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*c)*d^4 + 20*a*b*d^2 + a^2)*x^4*e
^3 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a
^2*d)*x^3*e^2 + 1/2*(11*c^2*d^10 + 18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*
b*d^4 + 3*a^2*d^2)*x^2*e + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*
b*d^5 + a^2*d^3)*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(71) = 142$.
time = 0.06, size = 559, normalized size = 6.28

$$c^2de^{10}x^{11} + \frac{c^2e^{11}x^{12}}{12} + x^{10}\left(\frac{bce^9}{5} + \frac{11c^2d^2e^9}{2}\right) + x^9\cdot\left(2bcde^8 + \frac{55c^2d^3e^8}{3}\right) + x^8\left(\frac{ace^7}{4} + \frac{b^2e^7}{8} + 9bcd^2e^7 + \frac{165c^2d^4e^7}{4}\right) + x^7\cdot\left(2acde^6 + b^2de^6 + 24bcd^3e^6 + 66c^2d^5e^6\right) + x^6\left(\frac{abe^5}{3} + 7acd^2e^5 + \frac{7b^2d^2e^5}{2} + 42bcd^4e^5 + 77c^2d^6e^5\right) + x^5\cdot\left(2abde^4 + 14acd^3e^4 + 7b^2d^3e^4 + \frac{252bcd^5e^4}{5} + 66c^2d^7e^4\right) + x^4\left(\frac{a^2e^3}{4} + 5abd^2e^3 + \frac{35acd^4e^3}{2} + \frac{35b^2d^4e^3}{4} + 42bcd^6e^3 + \frac{165c^2d^8e^3}{4}\right) + x^3\left(a^2de^2 + \frac{20abd^3e^2}{3} + 14acd^5e^2 + 7b^2d^5e^2 + 24bcd^7e^2 + \frac{55c^2d^9e^2}{3}\right) + x^2\cdot\left(\frac{3a^2d^2e}{2} + 5abd^4e + 7acd^6e + \frac{7b^2d^6e}{2} + 9bcd^8e + \frac{11c^2d^{10}e}{2}\right) + x\left(a^2d^3 + 2abd^5 + 2acd^7 + b^2d^7 + 2bcd^9 + c^2d^{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] c**2*d*e**10*x**11 + c**2*e**11*x**12/12 + x**10*(b*c*e**9/5 + 11*c**2*d**2
*e**9/2) + x**9*(2*b*c*d*e**8 + 55*c**2*d**3*e**8/3) + x**8*(a*c*e**7/4 + b
**2*e**7/8 + 9*b*c*d**2*e**7 + 165*c**2*d**4*e**7/4) + x**7*(2*a*c*d*e**6 +
 b**2*d*e**6 + 24*b*c*d**3*e**6 + 66*c**2*d**5*e**6) + x**6*(a*b*e**5/3 + 7
*a*c*d**2*e**5 + 7*b**2*d**2*e**5/2 + 42*b*c*d**4*e**5 + 77*c**2*d**6*e**5)
 + x**5*(2*a*b*d*e**4 + 14*a*c*d**3*e**4 + 7*b**2*d**3*e**4 + 252*b*c*d**5*
e**4/5 + 66*c**2*d**7*e**4) + x**4*(a**2*e**3/4 + 5*a*b*d**2*e**3 + 35*a*c*
d**4*e**3/2 + 35*b**2*d**4*e**3/4 + 42*b*c*d**6*e**3 + 165*c**2*d**8*e**3/4
) + x**3*(a**2*d*e**2 + 20*a*b*d**3*e**2/3 + 14*a*c*d**5*e**2 + 7*b**2*d**5
*e**2 + 24*b*c*d**7*e**2 + 55*c**2*d**9*e**2/3) + x**2*(3*a**2*d**2*e/2 + 5
*a*b*d**4*e + 7*a*c*d**6*e + 7*b**2*d**6*e/2 + 9*b*c*d**8*e + 11*c**2*d**10
*e/2) + x*(a**2*d**3 + 2*a*b*d**5 + 2*a*c*d**7 + b**2*d**7 + 2*b*c*d**9 + c
**2*d**11)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(79) = 158.

time = 4.41, size = 493, normalized size = 5.54

$$\frac{1}{2}\left(x^{2}e+2\,dx\right)c^{2}d^{10}+\frac{5}{4}\left(x^{2}e+2\,dx\right)^{2}c^{2}d^{8}e+\frac{5}{3}\left(x^{2}e+2\,dx\right)^{3}c^{2}d^{6}e^{2}+\left(x^{2}e+2\,dx\right)bcd^{8}+\frac{5}{4}\left(x^{2}e+2\,dx\right)^{4}c^{2}d^{4}e^{3}+2\left(x^{2}e+2\,dx\right)^{2}bcd^{6}e+\frac{1}{2}\left(x^{2}e+2\,dx\right)^{5}c^{2}d^{2}e^{4}+2\left(x^{2}e+2\,dx\right)^{3}bcd^{4}e^{2}+\frac{1}{2}\left(x^{2}e+2\,dx\right)b^{2}d^{6}+\left(x^{2}e+2\,dx\right)acd^{6}+\frac{1}{12}\left(x^{2}e+2\,dx\right)^{6}c^{2}e^{5}+\left(x^{2}e+2\,dx\right)^{4}bcd^{2}e^{3}+\frac{3}{4}\left(x^{2}e+2\,dx\right)^{2}b^{2}d^{4}e+\frac{3}{2}\left(x^{2}e+2\,dx\right)^{2}acd^{4}e+\frac{1}{5}\left(x^{2}e+2\,dx\right)^{5}bce^{4}+\frac{1}{2}\left(x^{2}e+2\,dx\right)^{3}b^{2}d^{2}e^{2}+\left(x^{2}e+2\,dx\right)^{3}acd^{2}e^{2}+\left(x^{2}e+2\,dx\right)abd^{4}+\frac{1}{8}\left(x^{2}e+2\,dx\right)^{4}b^{2}e^{3}+\frac{1}{4}\left(x^{2}e+2\,dx\right)^{4}ace^{3}+\left(x^{2}e+2\,dx\right)^{2}abd^{2}e+\frac{1}{3}\left(x^{2}e+2\,dx\right)^{3}abe^{2}+\frac{1}{2}\left(x^{2}e+2\,dx\right)a^{2}d^{2}+\frac{1}{4}\left(x^{2}e+2\,dx\right)^{2}a^{2}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e + 2*d*x)*c^2*d^10 + 5/4*(x^2*e + 2*d*x)^2*c^2*d^8*e + 5/3*(x^2*e
 + 2*d*x)^3*c^2*d^6*e^2 + (x^2*e + 2*d*x)*b*c*d^8 + 5/4*(x^2*e + 2*d*x)^4*c
^2*d^4*e^3 + 2*(x^2*e + 2*d*x)^2*b*c*d^6*e + 1/2*(x^2*e + 2*d*x)^5*c^2*d^2*
e^4 + 2*(x^2*e + 2*d*x)^3*b*c*d^4*e^2 + 1/2*(x^2*e + 2*d*x)*b^2*d^6 + (x^2*
e + 2*d*x)*a*c*d^6 + 1/12*(x^2*e + 2*d*x)^6*c^2*e^5 + (x^2*e + 2*d*x)^4*b*c
*d^2*e^3 + 3/4*(x^2*e + 2*d*x)^2*b^2*d^4*e + 3/2*(x^2*e + 2*d*x)^2*a*c*d^4*
e + 1/5*(x^2*e + 2*d*x)^5*b*c*e^4 + 1/2*(x^2*e + 2*d*x)^3*b^2*d^2*e^2 + (x^
2*e + 2*d*x)^3*a*c*d^2*e^2 + (x^2*e + 2*d*x)*a*b*d^4 + 1/8*(x^2*e + 2*d*x)^
4*b^2*e^3 + 1/4*(x^2*e + 2*d*x)^4*a*c*e^3 + (x^2*e + 2*d*x)^2*a*b*d^2*e + 1
/3*(x^2*e + 2*d*x)^3*a*b*e^2 + 1/2*(x^2*e + 2*d*x)*a^2*d^2 + 1/4*(x^2*e + 2
*d*x)^2*a^2*e
```

Mupad [B]

time = 1.48, size = 383, normalized size = 4.30

$$\frac{e^{7}\,x^{8}\,\left(b^{2}+72\,b\,c\,d^{2}+330\,c^{2}\,d^{4}+2\,a\,c\right)}{8}+\frac{e^{5}\,x^{6}\,\left(21\,b^{2}\,d^{2}+252\,b\,c\,d^{4}+2\,a\,b+462\,c^{2}\,d^{6}+42\,a\,c\,d^{2}\right)}{6}+\frac{e^{3}\,x^{4}\,\left(a^{2}+20\,a\,b\,d^{2}+70\,a\,c\,d^{4}+35\,b^{2}\,d^{4}+168\,b\,c\,d^{6}+165\,c^{2}\,d^{8}\right)}{4}+\frac{c^{2}\,e^{11}\,x^{12}}{12}+d^{3}\,x\,{\left(c\,d^{4}+b\,d^{2}+a\right)}^{2}+\frac{c\,e^{9}\,x^{10}\,\left(55\,c\,d^{2}+2\,b\right)}{10}+c^{2}\,d\,e^{10}\,x^{11}+\frac{d^{2}\,e\,x^{2}\,\left(3\,a^{2}+10\,a\,b\,d^{2}+14\,a\,c\,d^{4}+7\,b^{2}\,d^{4}+18\,b\,c\,d^{6}+11\,c^{2}\,d^{8}\right)}{2}+\frac{d\,e^{2}\,x^{3}\,\left(3\,a^{2}+20\,a\,b\,d^{2}+42\,a\,c\,d^{4}+21\,b^{2}\,d^{4}+72\,b\,c\,d^{6}+55\,c^{2}\,d^{8}\right)}{3}+d\,e^{6}\,x^{7}\,\left(b^{2}+24\,b\,c\,d^{2}+66\,c^{2}\,d^{4}+2\,a\,c\right)+\frac{d\,e^{4}\,x^{5}\,\left(35\,b^{2}\,d^{2}+252\,b\,c\,d^{4}+10\,a\,b+330\,c^{2}\,d^{6}+70\,a\,c\,d^{2}\right)}{5}+\frac{c\,d\,e^{8}\,x^{9}\,\left(55\,c\,d^{2}+6\,b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (e^7*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*x^6*(2*a*b + 21
*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (e^3*x^4*(a^2 + 35*
b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 168*b*c*d^6))/4 + (c^2*e^
11*x^12)/12 + d^3*x*(a + b*d^2 + c*d^4)^2 + (c*e^9*x^10*(2*b + 55*c*d^2))/1
0 + c^2*d*e^10*x^11 + (d^2*e*x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*a*b*d
^2 + 14*a*c*d^4 + 18*b*c*d^6))/2 + (d*e^2*x^3*(3*a^2 + 21*b^2*d^4 + 55*c^2*
d^8 + 20*a*b*d^2 + 42*a*c*d^4 + 72*b*c*d^6))/3 + d*e^6*x^7*(2*a*c + b^2 + 6
6*c^2*d^4 + 24*b*c*d^2) + (d*e^4*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2*d^6 + 7
0*a*c*d^2 + 252*b*c*d^4))/5 + (c*d*e^8*x^9*(6*b + 55*c*d^2))/3
```

3.609 $\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

Optimal. Leaf size=138

$$\frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{3a(b^2+ac)\,(d+ex)^8}{8e}+\frac{b(b^2+6ac)\,(d+ex)^{10}}{10e}+\frac{c(b^2+ac)\,(d+ex)^{12}}{4e}+\frac{3bc^2(d+e$$

```
[Out] 1/4*a^3*(e*x+d)^4/e+1/2*a^2*b*(e*x+d)^6/e+3/8*a*(a*c+b^2)*(e*x+d)^8/e+1/10*
b*(6*a*c+b^2)*(e*x+d)^10/e+1/4*c*(a*c+b^2)*(e*x+d)^12/e+3/14*b*c^2*(e*x+d)^
14/e+1/16*c^3*(e*x+d)^16/e
```

Rubi [A]
time = 0.25, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1156, 1128, 645}

$$\frac{a^3(d+ex)^4}{4e}+\frac{a^2b(d+ex)^6}{2e}+\frac{c(ac+b^2)\,(d+ex)^{12}}{4e}+\frac{b(6ac+b^2)\,(d+ex)^{10}}{10e}+\frac{3a(ac+b^2)\,(d+ex)^8}{8e}+\frac{3bc^2(d+ex)^{14}}{14e}+\frac{c^3(d+ex)^{16}}{16e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (a^3*(d + e*x)^4)/(4*e) + (a^2*b*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a*c)*(d +
 e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*(d + e*x)^10)/(10*e) + (c*(b^2 + a*c)*(d
+ e*x)^12)/(4*e) + (3*b*c^2*(d + e*x)^14)/(14*e) + (c^3*(d + e*x)^16)/(16*e
)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int (d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 dx = \frac{\text{Subst}\left(\int x^3(a+bx^2+cx^4)^3 \, dx, x, d+ex\right)}{e}$$
$$= \frac{\text{Subst}\left(\int x(a+bx+cx^2)^3 \, dx, x, (d+ex)^2\right)}{2e}$$
$$= \frac{\text{Subst}\left(\int \left(a^3x+3a^2bx^2+3a(b^2+ac)\,x^3+b(b^2+6ac)\,x^4 + \right.\right.}{2e}$$
$$= \frac{a^3(d+ex)^4}{4e} + \frac{a^2b(d+ex)^6}{2e} + \frac{3a(b^2+ac)\,(d+ex)^8}{8e} + \frac{b(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 797 vs. 2(138) = 276.
time = 0.19, size = 797, normalized size = 5.78

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5
*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 12
*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^10 + 35*
c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 +
84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2*d^10
 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*d^2 +
 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^2*d^8
 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 42*b^
3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2*d^8 + 10
01*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a*b*c*
d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)*e^6*
x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4 + 33
0*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a*b*c
+ 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*x^9 + ((b^
3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4 + 5005*c^3*d^6
)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4)*e^10*x^11 + (c
*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35*c*d^
2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 + (c^3
*e^15*x^16)/16
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7549 vs. 2(124) = 248.
time = 0.24, size = 7550, normalized size = 54.71 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(124) = 248$.
time = 0.30, size = 1019, normalized size = 7.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*c^3*x^16*e^15 + c^3*d*x^15*e^14 + 3/14*(35*c^3*d^2*e^13 + b*c^2*e^13)*
x^14 + (35*c^3*d^3*e^12 + 3*b*c^2*d*e^12)*x^13 + 1/4*(455*c^3*d^4*e^11 + 78
*b*c^2*d^2*e^11 + b^2*c*e^11 + a*c^2*e^11)*x^12 + 3*(91*c^3*d^5*e^10 + 26*b
*c^2*d^3*e^10 + (b^2*c*e^10 + a*c^2*e^10)*d)*x^11 + 1/10*(5005*c^3*d^6*e^9
+ 2145*b*c^2*d^4*e^9 + b^3*e^9 + 6*a*b*c*e^9 + 165*(b^2*c*e^9 + a*c^2*e^9)*
d^2)*x^10 + (715*c^3*d^7*e^8 + 429*b*c^2*d^5*e^8 + 55*(b^2*c*e^8 + a*c^2*e^
8)*d^3 + (b^3*e^8 + 6*a*b*c*e^8)*d)*x^9 + 3/8*(2145*c^3*d^8*e^7 + 1716*b*c^
2*d^6*e^7 + 330*(b^2*c*e^7 + a*c^2*e^7)*d^4 + a*b^2*e^7 + a^2*c*e^7 + 12*(b
^3*e^7 + 6*a*b*c*e^7)*d^2)*x^8 + 1/7*(5005*c^3*d^9*e^6 + 5148*b*c^2*d^7*e^6
 + 1386*(b^2*c*e^6 + a*c^2*e^6)*d^5 + 84*(b^3*e^6 + 6*a*b*c*e^6)*d^3 + 21*(
a*b^2*e^6 + a^2*c*e^6)*d)*x^7 + 1/2*(1001*c^3*d^10*e^5 + 1287*b*c^2*d^8*e^5
 + 462*(b^2*c*e^5 + a*c^2*e^5)*d^6 + 42*(b^3*e^5 + 6*a*b*c*e^5)*d^4 + a^2*b
*e^5 + 21*(a*b^2*e^5 + a^2*c*e^5)*d^2)*x^6 + 3/5*(455*c^3*d^11*e^4 + 715*b*
c^2*d^9*e^4 + 330*(b^2*c*e^4 + a*c^2*e^4)*d^7 + 42*(b^3*e^4 + 6*a*b*c*e^4)*
d^5 + 5*a^2*b*d*e^4 + 35*(a*b^2*e^4 + a^2*c*e^4)*d^3)*x^5 + 1/4*(455*c^3*d^
12*e^3 + 858*b*c^2*d^10*e^3 + 495*(b^2*c*e^3 + a*c^2*e^3)*d^8 + 84*(b^3*e^3
 + 6*a*b*c*e^3)*d^6 + 30*a^2*b*d^2*e^3 + 105*(a*b^2*e^3 + a^2*c*e^3)*d^4 +
a^3*e^3)*x^4 + (35*c^3*d^13*e^2 + 78*b*c^2*d^11*e^2 + 55*(b^2*c*e^2 + a*c^2
*e^2)*d^9 + 12*(b^3*e^2 + 6*a*b*c*e^2)*d^7 + 10*a^2*b*d^3*e^2 + 21*(a*b^2*e
^2 + a^2*c*e^2)*d^5 + a^3*d*e^2)*x^3 + 3/2*(5*c^3*d^14*e + 13*b*c^2*d^12*e
+ 11*(b^2*c*e + a*c^2*e)*d^10 + 3*(b^3*e + 6*a*b*c*e)*d^8 + 5*a^2*b*d^4*e +
 7*(a*b^2*e + a^2*c*e)*d^6 + a^3*d^2*e)*x^2 + (c^3*d^15 + 3*b*c^2*d^13 + 3*
(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b^2 + a^2*c
)*d^7 + a^3*d^3)*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(124) = 248$.
time = 0.34, size = 859, normalized size = 6.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*c^3*x^16*e^15 + c^3*d*x^15*e^14 + 3/14*(35*c^3*d^2 + b*c^2)*x^14*e^13
+ (35*c^3*d^3 + 3*b*c^2*d)*x^13*e^12 + 1/4*(455*c^3*d^4 + 78*b*c^2*d^2 + b^
2*c + a*c^2)*x^12*e^11 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 + (b^2*c + a*c^2)*d)*
x^11*e^10 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b^3 + 6*a*b*c + 165*(b^2*
c + a*c^2)*d^2)*x^10*e^9 + (715*c^3*d^7 + 429*b*c^2*d^5 + 55*(b^2*c + a*c^2
)*d^3 + (b^3 + 6*a*b*c)*d)*x^9*e^8 + 3/8*(2145*c^3*d^8 + 1716*b*c^2*d^6 + 3
30*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12*(b^3 + 6*a*b*c)*d^2)*x^8*e^7 +
1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 1386*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6
*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*x^7*e^6 + 1/2*(1001*c^3*d^10 + 1287*b*c
^2*d^8 + 462*(b^2*c + a*c^2)*d^6 + 42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b
^2 + a^2*c)*d^2)*x^6*e^5 + 3/5*(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c +
 a*c^2)*d^7 + 42*(b^3 + 6*a*b*c)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*
x^5*e^4 + 1/4*(455*c^3*d^12 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84
*(b^3 + 6*a*b*c)*d^6 + 30*a^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*x^4*e^
3 + (35*c^3*d^13 + 78*b*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b
*c)*d^7 + 10*a^2*b*d^3 + 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*x^3*e^2 + 3/2*(5*c
^3*d^14 + 13*b*c^2*d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 +
 5*a^2*b*d^4 + 7*(a*b^2 + a^2*c)*d^6 + a^3*d^2)*x^2*e + (c^3*d^15 + 3*b*c^2
*d^13 + 3*(b^2*c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b
^2 + a^2*c)*d^7 + a^3*d^3)*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1314 vs. $2(117) = 234$.
time = 0.13, size = 1314, normalized size = 9.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] c**3*d*e**14*x**15 + c**3*e**15*x**16/16 + x**14*(3*b*c**2*e**13/14 + 15*c*
*3*d**2*e**13/2) + x**13*(3*b*c**2*d*e**12 + 35*c**3*d**3*e**12) + x**12*(a
*c**2*e**11/4 + b**2*c*e**11/4 + 39*b*c**2*d**2*e**11/2 + 455*c**3*d**4*e**
11/4) + x**11*(3*a*c**2*d*e**10 + 3*b**2*c*d*e**10 + 78*b*c**2*d**3*e**10 +
 273*c**3*d**5*e**10) + x**10*(3*a*b*c*e**9/5 + 33*a*c**2*d**2*e**9/2 + b**
3*e**9/10 + 33*b**2*c*d**2*e**9/2 + 429*b*c**2*d**4*e**9/2 + 1001*c**3*d**6
*e**9/2) + x**9*(6*a*b*c*d*e**8 + 55*a*c**2*d**3*e**8 + b**3*d*e**8 + 55*b*
*2*c*d**3*e**8 + 429*b*c**2*d**5*e**8 + 715*c**3*d**7*e**8) + x**8*(3*a**2*
c*e**7/8 + 3*a*b**2*e**7/8 + 27*a*b*c*d**2*e**7 + 495*a*c**2*d**4*e**7/4 +
9*b**3*d**2*e**7/2 + 495*b**2*c*d**4*e**7/4 + 1287*b*c**2*d**6*e**7/2 + 643
5*c**3*d**8*e**7/8) + x**7*(3*a**2*c*d*e**6 + 3*a*b**2*d*e**6 + 72*a*b*c*d*
*3*e**6 + 198*a*c**2*d**5*e**6 + 12*b**3*d**3*e**6 + 198*b**2*c*d**5*e**6 +
 5148*b*c**2*d**7*e**6/7 + 715*c**3*d**9*e**6) + x**6*(a**2*b*e**5/2 + 21*a
**2*c*d**2*e**5/2 + 21*a*b**2*d**2*e**5/2 + 126*a*b*c*d**4*e**5 + 231*a*c**
```

```
2*d**6*e**5 + 21*b**3*d**4*e**5 + 231*b**2*c*d**6*e**5 + 1287*b*c**2*d**8*e
**5/2 + 1001*c**3*d**10*e**5/2) + x**5*(3*a**2*b*d*e**4 + 21*a**2*c*d**3*e*
*4 + 21*a*b**2*d**3*e**4 + 756*a*b*c*d**5*e**4/5 + 198*a*c**2*d**7*e**4 + 1
26*b**3*d**5*e**4/5 + 198*b**2*c*d**7*e**4 + 429*b*c**2*d**9*e**4 + 273*c**
3*d**11*e**4) + x**4*(a**3*e**3/4 + 15*a**2*b*d**2*e**3/2 + 105*a**2*c*d**4
*e**3/4 + 105*a*b**2*d**4*e**3/4 + 126*a*b*c*d**6*e**3 + 495*a*c**2*d**8*e*
*3/4 + 21*b**3*d**6*e**3 + 495*b**2*c*d**8*e**3/4 + 429*b*c**2*d**10*e**3/2
 + 455*c**3*d**12*e**3/4) + x**3*(a**3*d*e**2 + 10*a**2*b*d**3*e**2 + 21*a*
*2*c*d**5*e**2 + 21*a*b**2*d**5*e**2 + 72*a*b*c*d**7*e**2 + 55*a*c**2*d**9*
e**2 + 12*b**3*d**7*e**2 + 55*b**2*c*d**9*e**2 + 78*b*c**2*d**11*e**2 + 35*
c**3*d**13*e**2) + x**2*(3*a**3*d**2*e/2 + 15*a**2*b*d**4*e/2 + 21*a**2*c*d
**6*e/2 + 21*a*b**2*d**6*e/2 + 27*a*b*c*d**8*e + 33*a*c**2*d**10*e/2 + 9*b*
*3*d**8*e/2 + 33*b**2*c*d**10*e/2 + 39*b*c**2*d**12*e/2 + 15*c**3*d**14*e/2
) + x*(a**3*d**3 + 3*a**2*b*d**5 + 3*a**2*c*d**7 + 3*a*b**2*d**7 + 6*a*b*c*
d**9 + 3*a*c**2*d**11 + b**3*d**9 + 3*b**2*c*d**11 + 3*b*c**2*d**13 + c**3*
d**15)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1109 vs. $2(124) = 248$.
time = 4.05, size = 1109, normalized size = 8.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e + 2*d*x)*c^3*d^14 + 7/4*(x^2*e + 2*d*x)^2*c^3*d^12*e + 7/2*(x^2*
e + 2*d*x)^3*c^3*d^10*e^2 + 3/2*(x^2*e + 2*d*x)*b*c^2*d^12 + 35/8*(x^2*e +
2*d*x)^4*c^3*d^8*e^3 + 9/2*(x^2*e + 2*d*x)^2*b*c^2*d^10*e + 7/2*(x^2*e + 2*
d*x)^5*c^3*d^6*e^4 + 15/2*(x^2*e + 2*d*x)^3*b*c^2*d^8*e^2 + 3/2*(x^2*e + 2*
d*x)*b^2*c*d^10 + 3/2*(x^2*e + 2*d*x)*a*c^2*d^10 + 7/4*(x^2*e + 2*d*x)^6*c^
3*d^4*e^5 + 15/2*(x^2*e + 2*d*x)^4*b*c^2*d^6*e^3 + 15/4*(x^2*e + 2*d*x)^2*b
^2*c*d^8*e + 15/4*(x^2*e + 2*d*x)^2*a*c^2*d^8*e + 1/2*(x^2*e + 2*d*x)^7*c^3
*d^2*e^6 + 9/2*(x^2*e + 2*d*x)^5*b*c^2*d^4*e^4 + 5*(x^2*e + 2*d*x)^3*b^2*c*
d^6*e^2 + 5*(x^2*e + 2*d*x)^3*a*c^2*d^6*e^2 + 1/2*(x^2*e + 2*d*x)*b^3*d^8 +
 3*(x^2*e + 2*d*x)*a*b*c*d^8 + 1/16*(x^2*e + 2*d*x)^8*c^3*e^7 + 3/2*(x^2*e
+ 2*d*x)^6*b*c^2*d^2*e^5 + 15/4*(x^2*e + 2*d*x)^4*b^2*c*d^4*e^3 + 15/4*(x^2
*e + 2*d*x)^4*a*c^2*d^4*e^3 + (x^2*e + 2*d*x)^2*b^3*d^6*e + 6*(x^2*e + 2*d*
x)^2*a*b*c*d^6*e + 3/14*(x^2*e + 2*d*x)^7*b*c^2*e^6 + 3/2*(x^2*e + 2*d*x)^5
*b^2*c*d^2*e^4 + 3/2*(x^2*e + 2*d*x)^5*a*c^2*d^2*e^4 + (x^2*e + 2*d*x)^3*b^
3*d^4*e^2 + 6*(x^2*e + 2*d*x)^3*a*b*c*d^4*e^2 + 3/2*(x^2*e + 2*d*x)*a*b^2*d
^6 + 3/2*(x^2*e + 2*d*x)*a^2*c*d^6 + 1/4*(x^2*e + 2*d*x)^6*b^2*c*e^5 + 1/4*
(x^2*e + 2*d*x)^6*a*c^2*e^5 + 1/2*(x^2*e + 2*d*x)^4*b^3*d^2*e^3 + 3*(x^2*e
+ 2*d*x)^4*a*b*c*d^2*e^3 + 9/4*(x^2*e + 2*d*x)^2*a*b^2*d^4*e + 9/4*(x^2*e +
 2*d*x)^2*a^2*c*d^4*e + 1/10*(x^2*e + 2*d*x)^5*b^3*e^4 + 3/5*(x^2*e + 2*d*x
```

```
)^5*a*b*c*e^4 + 3/2*(x^2*e + 2*d*x)^3*a*b^2*d^2*e^2 + 3/2*(x^2*e + 2*d*x)^3
*a^2*c*d^2*e^2 + 3/2*(x^2*e + 2*d*x)*a^2*b*d^4 + 3/8*(x^2*e + 2*d*x)^4*a*b^
2*e^3 + 3/8*(x^2*e + 2*d*x)^4*a^2*c*e^3 + 3/2*(x^2*e + 2*d*x)^2*a^2*b*d^2*e
 + 1/2*(x^2*e + 2*d*x)^3*a^2*b*e^2 + 1/2*(x^2*e + 2*d*x)*a^3*d^2 + 1/4*(x^2
*e + 2*d*x)^2*a^3*e
```

Mupad [B]
time = 1.66, size = 777, normalized size = 5.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] (3*e^7*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4 + 330
*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*x^6*(a^2*b + 42*b^3*d
^4 + 1001*c^3*d^10 + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6 + 462*b^2*
c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4))/2 + (e^9*x^10*(b^3 + 5005*c^3*d^6
+ 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c))/10 + (c^3*e^15
*x^16)/16 + d^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*x^4*(a^3 + 84*b^3*d^6 + 455*
c^3*d^12 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d^4 + 495*a*c^2*d^8 + 4
95*b^2*c*d^8 + 858*b*c^2*d^10 + 504*a*b*c*d^6))/4 + (3*c^2*e^13*x^14*(b + 3
5*c*d^2))/14 + c^3*d*e^14*x^15 + d*e^2*x^3*(a^3 + 12*b^3*d^6 + 35*c^3*d^12
+ 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4 + 55*a*c^2*d^8 + 55*b^2*c*d^8
+ 78*b*c^2*d^10 + 72*a*b*c*d^6) + (c*e^11*x^12*(a*c + b^2 + 455*c^2*d^4 + 7
8*b*c*d^2))/4 + (d*e^6*x^7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 5005*c^3*d^8
 + 1386*a*c^2*d^4 + 1386*b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*c*d^2))/7 + (
3*d*e^4*x^5*(5*a^2*b + 42*b^3*d^4 + 455*c^3*d^10 + 35*a*b^2*d^2 + 35*a^2*c*
d^2 + 330*a*c^2*d^6 + 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*b*c*d^4))/5 + d
*e^8*x^9*(b^3 + 715*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 + 429*b*c^2*d^4 +
 6*a*b*c) + (3*d^2*e*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4))/2 +
 c^2*d*e^12*x^13*(3*b + 35*c*d^2) + 3*c*d*e^10*x^11*(a*c + b^2 + 91*c^2*d^4
 + 26*b*c*d^2)
```

3.610 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx$

Optimal. Leaf size=55

$$\frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

```
[Out] 1/4*a*f^3*(e*x+d)^4/e+1/6*b*f^3*(e*x+d)^6/e+1/8*c*f^3*(e*x+d)^8/e
```

Rubi [A]
time = 0.04, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {1156, 14}

$$\frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (a*f^3*(d + e*x)^4)/(4*e) + (b*f^3*(d + e*x)^6)/(6*e) + (c*f^3*(d + e*x)^8)
/(8*e)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right) dx &= \frac{f^3\text{Subst}\left(\int x^3(a+bx^2+cx^4)\, dx, x, d+ex\right)}{e}\\ &= \frac{f^3\text{Subst}\left(\int (ax^3+bx^5+cx^7)\, dx, x, d+ex\right)}{e}\\ &= \frac{af^3(d+ex)^4}{4e}+\frac{bf^3(d+ex)^6}{6e}+\frac{cf^3(d+ex)^8}{8e}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(55) = 110.
time = 0.01, size = 154, normalized size = 2.80

$$f^3\left(d^3\left(a+bd^2+cd^4\right)x+\frac{1}{2}d^2\left(3a+5bd^2+7cd^4\right)ex^2+\frac{1}{3}d\left(3a+10bd^2+21cd^4\right)e^2x^3+\frac{1}{4}\left(a+10bd^2+35cd^4\right)e^3x^4+d\left(b+7cd^2\right)e^4x^5+\frac{1}{6}\left(b+21cd^2\right)e^5x^6+cde^6x^7+\frac{1}{8}ce^7x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)*x + (d^2*(3*a + 5*b*d^2 + 7*c*d^4)*e*x^2)/2 +
(d*(3*a + 10*b*d^2 + 21*c*d^4)*e^2*x^3)/3 + ((a + 10*b*d^2 + 35*c*d^4)*e^3*
x^4)/4 + d*(b + 7*c*d^2)*e^4*x^5 + ((b + 21*c*d^2)*e^5*x^6)/6 + c*d*e^6*x^7
 + (c*e^7*x^8)/8)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(49) = 98.
time = 0.20, size = 349, normalized size = 6.35

method	result
gosper	$\frac{f^3x\left(3e^7cx^7+24de^6cx^6+84x^5d^2e^5c+168cd^3e^4x^4+4x^5be^5+210x^3d^4e^3c+24bde^4x^4+168x^2cd^5e^2+60x^3bd^2e^3+84xcd^6e+80x^2bd^3\right.}{24}$
norman	$\left(\frac{7}{2}d^2f^3e^5c+\frac{1}{6}be^5f^3\right)x^6+\left(7cd^5e^2f^3+\frac{10}{3}bd^3e^2f^3+ade^2f^3\right)x^3+\left(\frac{7}{2}cd^6ef^3+\frac{5}{2}bd^4ef^3+\frac{3}{2}ad\right.$
risch	$\frac{1}{8}e^7f^3cx^8+df^3e^6cx^7+\frac{7}{2}f^3x^6d^2e^5c+\frac{1}{6}f^3x^6be^5+7f^3cd^3e^4x^5+f^3bde^4x^5+\frac{35}{4}f^3x^4d^4e^3c+\frac{5}{2}f$
default	$\frac{e^7f^3cx^8}{8}+df^3e^6cx^7+\frac{\left(15d^2f^3e^5c+e^3f^3\left(6d^2e^2c+e^2b\right)\right)x^6}{6}+\frac{\left(13d^3f^3e^4c+3df^3e^2\left(6d^2e^2c+e^2b\right)+e^3f^3\left(4d^3ec+2deb\right)\right)x^5}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*e^7*f^3*c*x^8+d*f^3*e^6*c*x^7+1/6*(15*d^2*f^3*e^5*c+e^3*f^3*(6*c*d^2*e^
2+b*e^2))*x^6+1/5*(13*d^3*f^3*e^4*c+3*d*f^3*e^2*(6*c*d^2*e^2+b*e^2)+e^3*f^3
*(4*c*d^3*e+2*b*d*e))*x^5+1/4*(4*d^4*f^3*e^3*c+3*d^2*f^3*e*(6*c*d^2*e^2+b*e
^2)+3*d*f^3*e^2*(4*c*d^3*e+2*b*d*e)+e^3*f^3*(c*d^4+b*d^2+a))*x^4+1/3*(d^3*f
^3*(6*c*d^2*e^2+b*e^2)+3*d^2*f^3*e*(4*c*d^3*e+2*b*d*e)+3*d*f^3*e^2*(c*d^4+b
*d^2+a))*x^3+1/2*(d^3*f^3*(4*c*d^3*e+2*b*d*e)+3*d^2*f^3*e*(c*d^4+b*d^2+a))*
x^2+d^3*f^3*(c*d^4+b*d^2+a)*x
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(49) = 98.
time = 0.29, size = 179, normalized size = 3.25

$$\frac{1}{8}cf^3x^8e^7+cdf^3x^7e^6+\frac{1}{6}\left(21cd^2e^5+be^5\right)f^3x^6+\left(7cd^3e^4+bde^4\right)f^3x^5+\frac{1}{4}\left(35cd^4e^3+10bd^2e^3+ae^3\right)f^3x^4+\frac{1}{3}\left(21cd^5e^2+10bd^3e^2+3ade^2\right)f^3x^3+\frac{1}{2}\left(7cd^6e+5bd^4e+3ad^2e\right)f^3x^2+\left(cd^7+bd^5+ad^3\right)f^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] 1/8*c*f^3*x^8*e^7 + c*d*f^3*x^7*e^6 + 1/6*(21*c*d^2*e^5 + b*e^5)*f^3*x^6 +
(7*c*d^3*e^4 + b*d*e^4)*f^3*x^5 + 1/4*(35*c*d^4*e^3 + 10*b*d^2*e^3 + a*e^3)
*f^3*x^4 + 1/3*(21*c*d^5*e^2 + 10*b*d^3*e^2 + 3*a*d*e^2)*f^3*x^3 + 1/2*(7*c
*d^6*e + 5*b*d^4*e + 3*a*d^2*e)*f^3*x^2 + (c*d^7 + b*d^5 + a*d^3)*f^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(49) = 98$.
time = 0.40, size = 161, normalized size = 2.93

$$\frac{1}{8}\,cf^3x^8e^7 + cdf^3x^7e^6 + \frac{1}{6}\,(21\,cd^2 + b)\,f^3x^6e^5 + (7\,cd^3 + bd)\,f^3x^5e^4 + \frac{1}{4}\,(35\,cd^4 + 10\,bd^2 + a)\,f^3x^4e^3 + \frac{1}{3}\,(21\,cd^5 + 10\,bd^3 + 3\,ad)\,f^3x^3e^2 + \frac{1}{2}\,(7\,cd^6 + 5\,bd^4 + 3\,ad^2)\,f^3x^2e + (cd^7 + bd^5 + ad^3)\,f^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/8*c*f^3*x^8*e^7 + c*d*f^3*x^7*e^6 + 1/6*(21*c*d^2 + b)*f^3*x^6*e^5 + (7*c
*d^3 + b*d)*f^3*x^5*e^4 + 1/4*(35*c*d^4 + 10*b*d^2 + a)*f^3*x^4*e^3 + 1/3*(
21*c*d^5 + 10*b*d^3 + 3*a*d)*f^3*x^3*e^2 + 1/2*(7*c*d^6 + 5*b*d^4 + 3*a*d^2
)*f^3*x^2*e + (c*d^7 + b*d^5 + a*d^3)*f^3*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(44) = 88$.
time = 0.03, size = 240, normalized size = 4.36

$$cde^6f^3x^7 + \frac{ce^7f^3x^8}{8} + x^6\left(\frac{be^5f^3}{6} + \frac{7cd^2e^5f^3}{2}\right) + x^5\left(bde^4f^3 + 7cd^3e^4f^3\right) + x^4\left(\frac{ae^3f^3}{4} + \frac{5bd^2e^3f^3}{2} + \frac{35cd^4e^3f^3}{4}\right) + x^3\left(ade^2f^3 + \frac{10bd^3e^2f^3}{3} + 7cd^5e^2f^3\right) + x^2\cdot\left(\frac{3ad^2ef^3}{2} + \frac{5bd^4ef^3}{2} + \frac{7cd^6ef^3}{2}\right) + x\left(ad^3f^3 + bd^5f^3 + cd^7f^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] c*d*e**6*f**3*x**7 + c*e**7*f**3*x**8/8 + x**6*(b*e**5*f**3/6 + 7*c*d**2*e*
*5*f**3/2) + x**5*(b*d*e**4*f**3 + 7*c*d**3*e**4*f**3) + x**4*(a*e**3*f**3/
4 + 5*b*d**2*e**3*f**3/2 + 35*c*d**4*e**3*f**3/4) + x**3*(a*d*e**2*f**3 + 1
0*b*d**3*e**2*f**3/3 + 7*c*d**5*e**2*f**3) + x**2*(3*a*d**2*e*f**3/2 + 5*b*
d**4*e*f**3/2 + 7*c*d**6*e*f**3/2) + x*(a*d**3*f**3 + b*d**5*f**3 + c*d**7*
f**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(49) = 98$.
time = 3.76, size = 213, normalized size = 3.87

$$\frac{1}{2}\,(fx^2e + 2\,dfx)cd^6f^2 + \frac{1}{2}\,(fx^2e + 2\,dfx)bd^4f^2 + \frac{1}{2}\,(fx^2e + 2\,dfx)ad^2f^2 + \frac{18\,(fx^2e + 2\,dfx)^2cd^4f^2e + 12\,(fx^2e + 2\,dfx)^3cd^2fe^2 + 12\,(fx^2e + 2\,dfx)^2bd^2f^2e + 3\,(fx^2e + 2\,dfx)^4ce^3 + 4\,(fx^2e + 2\,dfx)^3bfe^2 + 6\,(fx^2e + 2\,dfx)^2af^2e}{24\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*(f*x^2*e + 2*d*f*x)*c*d^6*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*b*d^4*f^2 + 1/2
*(f*x^2*e + 2*d*f*x)*a*d^2*f^2 + 1/24*(18*(f*x^2*e + 2*d*f*x)^2*c*d^4*f^2*e
```

```
 + 12*(f*x^2*e + 2*d*f*x)^3*c*d^2*f*e^2 + 12*(f*x^2*e + 2*d*f*x)^2*b*d^2*f^
2*e + 3*(f*x^2*e + 2*d*f*x)^4*c*e^3 + 4*(f*x^2*e + 2*d*f*x)^3*b*f*e^2 + 6*(
f*x^2*e + 2*d*f*x)^2*a*f^2*e)/f
```

Mupad [B]
time = 0.08, size = 164, normalized size = 2.98

$$\frac{e^5\,f^3\,x^6\,\left(21\,c\,d^2+b\right)}{6}+\frac{c\,e^7\,f^3\,x^8}{8}+d^3\,f^3\,x\,\left(c\,d^4+b\,d^2+a\right)+\frac{e^3\,f^3\,x^4\,\left(35\,c\,d^4+10\,b\,d^2+a\right)}{4}+\frac{d^2\,e\,f^3\,x^2\,\left(7\,c\,d^4+5\,b\,d^2+3\,a\right)}{2}+\frac{d\,e^2\,f^3\,x^3\,\left(21\,c\,d^4+10\,b\,d^2+3\,a\right)}{3}+d\,e^4\,f^3\,x^5\,\left(7\,c\,d^2+b\right)+c\,d\,e^6\,f^3\,x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (e^5*f^3*x^6*(b + 21*c*d^2))/6 + (c*e^7*f^3*x^8)/8 + d^3*f^3*x*(a + b*d^2 +
 c*d^4) + (e^3*f^3*x^4*(a + 10*b*d^2 + 35*c*d^4))/4 + (d^2*e*f^3*x^2*(3*a +
 5*b*d^2 + 7*c*d^4))/2 + (d*e^2*f^3*x^3*(3*a + 10*b*d^2 + 21*c*d^4))/3 + d*
e^4*f^3*x^5*(b + 7*c*d^2) + c*d*e^6*f^3*x^7
```

3.611 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx$

Optimal. Leaf size=104

$$\frac{a^2f^3(d+ex)^4}{4e}+\frac{abf^3(d+ex)^6}{3e}+\frac{(b^2+2ac)\,f^3(d+ex)^8}{8e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}$$

```
[Out] 1/4*a^2*f^3*(e*x+d)^4/e+1/3*a*b*f^3*(e*x+d)^6/e+1/8*(2*a*c+b^2)*f^3*(e*x+d)
^8/e+1/5*b*c*f^3*(e*x+d)^10/e+1/12*c^2*f^3*(e*x+d)^12/e
```

Rubi [A]
time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1156, 1128, 645}

$$\frac{a^2f^3(d+ex)^4}{4e}+\frac{f^3(2ac+b^2)\,(d+ex)^8}{8e}+\frac{abf^3(d+ex)^6}{3e}+\frac{bcf^3(d+ex)^{10}}{5e}+\frac{c^2f^3(d+ex)^{12}}{12e}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (a^2*f^3*(d + e*x)^4)/(4*e) + (a*b*f^3*(d + e*x)^6)/(3*e) + ((b^2 + 2*a*c)*
f^3*(d + e*x)^8)/(8*e) + (b*c*f^3*(d + e*x)^10)/(5*e) + (c^2*f^3*(d + e*x)^
12)/(12*e)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2 \, dx = \frac{f^3 \text{Subst}\left(\int x^3 (a+bx^2+cx^4)^2 \, dx, x, d+ex\right)}{e}$$
$$= \frac{f^3 \text{Subst}\left(\int x (a+bx+cx^2)^2 \, dx, x, (d+ex)^2\right)}{2e}$$
$$= \frac{f^3 \text{Subst}\left(\int \left(a^2x+2abx^2+(b^2+2ac)\, x^3+2bcx^4+c^2x^5\right.}{2e}$$
$$= \frac{a^2f^3(d+ex)^4}{4e}+\frac{abf^3(d+ex)^6}{3e}+\frac{(b^2+2ac)\, f^3(d+ex}{8e}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 405 vs. 2(104) = 208.
time = 0.06, size = 405, normalized size = 3.89

$$f^3 \left(d^3(a+bd^2+cd^4)^2 x+\frac{1}{2}d^2\left(3a^2+10abd^2+7b^2d^4+14acd^4+18bcd^6+11c^2d^8\right) ex^2+\frac{1}{3}d\left(3a^2+20abd^2+21b^2d^4+42acd^4+72bcd^6+55c^2d^8\right) e^2x^3+\frac{1}{4}\left(a^2+20abd^2+35b^2d^4+70acd^4+168bcd^6+165c^2d^8\right) e^3x^4+\frac{1}{5}d\left(10ab+35b^2d^2+70acd^2+252bcd^4+330c^2d^6\right) e^4x^5+\frac{1}{6}\left(2ab+21b^2d^2+42acd^2+252bcd^4+462c^2d^6\right) e^5x^6+d\left(b^2+2ac+24bcd^2+66c^2d^4\right) e^6x^7+\frac{1}{8}\left(b^2+2ac+72bcd^2+330c^2d^4\right) e^7x^8+\frac{1}{3}cd\left(6b+55cd^2\right) e^8x^9+\frac{1}{10}c\left(2b+55cd^2\right) e^9x^{10}+c^2de^{10}x^{11}+\frac{1}{12}c^2e^{11}x^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)^2*x + (d^2*(3*a^2 + 10*a*b*d^2 + 7*b^2*d^4 + 1
4*a*c*d^4 + 18*b*c*d^6 + 11*c^2*d^8)*e*x^2)/2 + (d*(3*a^2 + 20*a*b*d^2 + 21
*b^2*d^4 + 42*a*c*d^4 + 72*b*c*d^6 + 55*c^2*d^8)*e^2*x^3)/3 + ((a^2 + 20*a*
b*d^2 + 35*b^2*d^4 + 70*a*c*d^4 + 168*b*c*d^6 + 165*c^2*d^8)*e^3*x^4)/4 + (
d*(10*a*b + 35*b^2*d^2 + 70*a*c*d^2 + 252*b*c*d^4 + 330*c^2*d^6)*e^4*x^5)/5
 + ((2*a*b + 21*b^2*d^2 + 42*a*c*d^2 + 252*b*c*d^4 + 462*c^2*d^6)*e^5*x^6)/
6 + d*(b^2 + 2*a*c + 24*b*c*d^2 + 66*c^2*d^4)*e^6*x^7 + ((b^2 + 2*a*c + 72*
b*c*d^2 + 330*c^2*d^4)*e^7*x^8)/8 + (c*d*(6*b + 55*c*d^2)*e^8*x^9)/3 + (c*(
2*b + 55*c*d^2)*e^9*x^10)/10 + c^2*d*e^10*x^11 + (c^2*e^11*x^12)/12)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1412 vs. 2(94) = 188.
time = 0.24, size = 1413, normalized size = 13.59

method	result
gosper	$\frac{f^3x\left(10e^{11}c^2x^{11}+120d\,e^{10}c^2x^{10}+660x^9d^2e^9c^2+2200x^8d^3e^8c^2+24x^9bc\,e^9+4950x^7c^2d^4e^7+240x^8bcd\,e^8+7920c^2d^5e^6x^6+1080x^7bc\,d\right.}{}$
norman	$\left(\frac{11}{2}d^2f^3e^9c^2+\frac{1}{5}bc\,e^9f^3\right)x^{10}+\left(\frac{55}{3}d^3f^3e^8c^2+2bcd\,e^8f^3\right)x^9+\left(\frac{165}{4}d^4f^3e^7c^2+9bc\,d^2e^7f^3+\frac{1}{4}ac\,e^7\right.$
risch	$d\,f^3e^{10}c^2x^{11}+f^3a^2d^3x+f^3c^2d^{11}x+f^3b^2d^7x+\frac{1}{8}f^3x^8b^2e^7+\frac{1}{4}f^3x^4a^2e^3+14f^3x^5ac\,d^3e^4+2f^3x$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*e^11*f^3*c^2*x^12+d*f^3*e^10*c^2*x^11+1/10*(27*d^2*f^3*e^9*c^2+e^3*f^3
*(2*(6*c*d^2*e^2+b*e^2)*e^4*c+16*d^2*e^6*c^2))*x^10+1/9*(25*d^3*f^3*e^8*c^2
+3*d*f^3*e^2*(2*(6*c*d^2*e^2+b*e^2)*e^4*c+16*d^2*e^6*c^2)+e^3*f^3*(2*(4*c*d
^3*e+2*b*d*e)*e^4*c+8*(6*c*d^2*e^2+b*e^2)*d*e^3*c))*x^9+1/8*(8*d^4*f^3*e^7*
c^2+3*d^2*f^3*e*(2*(6*c*d^2*e^2+b*e^2)*e^4*c+16*d^2*e^6*c^2)+3*d*f^3*e^2*(2
*(4*c*d^3*e+2*b*d*e)*e^4*c+8*(6*c*d^2*e^2+b*e^2)*d*e^3*c)+e^3*f^3*(2*(c*d^4
+b*d^2+a)*e^4*c+8*(4*c*d^3*e+2*b*d*e)*d*e^3*c+(6*c*d^2*e^2+b*e^2)^2))*x^8+1
/7*(d^3*f^3*(2*(6*c*d^2*e^2+b*e^2)*e^4*c+16*d^2*e^6*c^2)+3*d^2*f^3*e*(2*(4*
c*d^3*e+2*b*d*e)*e^4*c+8*(6*c*d^2*e^2+b*e^2)*d*e^3*c)+3*d*f^3*e^2*(2*(c*d^4
+b*d^2+a)*e^4*c+8*(4*c*d^3*e+2*b*d*e)*d*e^3*c+(6*c*d^2*e^2+b*e^2)^2)+e^3*f^
3*(8*(c*d^4+b*d^2+a)*d*e^3*c+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2)))*x^
7+1/6*(d^3*f^3*(2*(4*c*d^3*e+2*b*d*e)*e^4*c+8*(6*c*d^2*e^2+b*e^2)*d*e^3*c)+
3*d^2*f^3*e*(2*(c*d^4+b*d^2+a)*e^4*c+8*(4*c*d^3*e+2*b*d*e)*d*e^3*c+(6*c*d^2
*e^2+b*e^2)^2)+3*d*f^3*e^2*(8*(c*d^4+b*d^2+a)*d*e^3*c+2*(4*c*d^3*e+2*b*d*e)
*(6*c*d^2*e^2+b*e^2))+e^3*f^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e^2)+(4*c*d
^3*e+2*b*d*e)^2))*x^6+1/5*(d^3*f^3*(2*(c*d^4+b*d^2+a)*e^4*c+8*(4*c*d^3*e+2*
b*d*e)*d*e^3*c+(6*c*d^2*e^2+b*e^2)^2)+3*d^2*f^3*e*(8*(c*d^4+b*d^2+a)*d*e^3*
c+2*(4*c*d^3*e+2*b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d*f^3*e^2*(2*(c*d^4+b*d^2+a)
*(6*c*d^2*e^2+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+2*e^3*f^3*(c*d^4+b*d^2+a)*(4*c*
d^3*e+2*b*d*e))*x^5+1/4*(d^3*f^3*(8*(c*d^4+b*d^2+a)*d*e^3*c+2*(4*c*d^3*e+2*
b*d*e)*(6*c*d^2*e^2+b*e^2))+3*d^2*f^3*e*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2+b*e
^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d*f^3*e^2*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d*e)+
e^3*f^3*(c*d^4+b*d^2+a)^2)*x^4+1/3*(d^3*f^3*(2*(c*d^4+b*d^2+a)*(6*c*d^2*e^2
+b*e^2)+(4*c*d^3*e+2*b*d*e)^2)+6*d^2*f^3*e*(c*d^4+b*d^2+a)*(4*c*d^3*e+2*b*d
*e)+3*d*f^3*e^2*(c*d^4+b*d^2+a)^2)*x^3+1/2*(2*d^3*f^3*(c*d^4+b*d^2+a)*(4*c*
d^3*e+2*b*d*e)+3*d^2*f^3*e*(c*d^4+b*d^2+a)^2)*x^2+d^3*f^3*(c*d^4+b*d^2+a)^2
*x
```

<u>Maxima</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(94) = 188$.
time = 0.53, size = 500, normalized size = 4.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] 1/12*c^2*f^3*x^12*e^11 + c^2*d*f^3*x^11*e^10 + 1/10*(55*c^2*d^2*e^9 + 2*b*c
*e^9)*f^3*x^10 + 1/3*(55*c^2*d^3*e^8 + 6*b*c*d*e^8)*f^3*x^9 + 1/8*(330*c^2*
d^4*e^7 + 72*b*c*d^2*e^7 + b^2*e^7 + 2*a*c*e^7)*f^3*x^8 + (66*c^2*d^5*e^6 +
 24*b*c*d^3*e^6 + (b^2*e^6 + 2*a*c*e^6)*d)*f^3*x^7 + 1/6*(462*c^2*d^6*e^5 +
 252*b*c*d^4*e^5 + 21*(b^2*e^5 + 2*a*c*e^5)*d^2 + 2*a*b*e^5)*f^3*x^6 + 1/5*
(330*c^2*d^7*e^4 + 252*b*c*d^5*e^4 + 35*(b^2*e^4 + 2*a*c*e^4)*d^3 + 10*a*b*
```

```
d*e^4)*f^3*x^5 + 1/4*(165*c^2*d^8*e^3 + 168*b*c*d^6*e^3 + 35*(b^2*e^3 + 2*a
*c*e^3)*d^4 + 20*a*b*d^2*e^3 + a^2*e^3)*f^3*x^4 + 1/3*(55*c^2*d^9*e^2 + 72*
b*c*d^7*e^2 + 21*(b^2*e^2 + 2*a*c*e^2)*d^5 + 20*a*b*d^3*e^2 + 3*a^2*d*e^2)*
f^3*x^3 + 1/2*(11*c^2*d^10*e + 18*b*c*d^8*e + 7*(b^2*e + 2*a*c*e)*d^6 + 10*
a*b*d^4*e + 3*a^2*d^2*e)*f^3*x^2 + (c^2*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^
7 + 2*a*b*d^5 + a^2*d^3)*f^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(94) = 188$.
time = 0.34, size = 430, normalized size = 4.13

$$\frac{1}{12}\,c^{2}f^{3}x^{12}e^{11} + c^{2}df^{3}x^{11}e^{10} + \frac{1}{10}\,(55\,c^{2}d^{2} + 2\,bc)f^{3}x^{10}e^{9} + \frac{1}{3}\,(55\,c^{2}d^{3} + 6\,bcd)f^{3}x^{9}e^{8} + \frac{1}{8}\,(330\,c^{2}d^{4} + 72\,bcd^{2} + b^{2} + 2\,ac)f^{3}x^{8}e^{7} + (66\,c^{2}d^{5} + 24\,bcd^{3} + (b^{2} + 2\,ac)d)f^{3}x^{7}e^{6} + \frac{1}{6}\,(462\,c^{2}d^{6} + 252\,bcd^{4} + 21\,(b^{2} + 2\,ac)d^{2} + 2\,ab)f^{3}x^{6}e^{5} + \frac{1}{5}\,(330\,c^{2}d^{7} + 252\,bcd^{5} + 35\,(b^{2} + 2\,ac)d^{3} + 10\,abd)f^{3}x^{5}e^{4} + \frac{1}{4}\,(165\,c^{2}d^{8} + 168\,bcd^{6} + 35\,(b^{2} + 2\,ac)d^{4} + 20\,abd^{2} + a^{2})f^{3}x^{4}e^{3} + \frac{1}{3}\,(55\,c^{2}d^{9} + 72\,bcd^{7} + 21\,(b^{2} + 2\,ac)d^{5} + 20\,abd^{3} + 3\,a^{2}d)f^{3}x^{3}e^{2} + \frac{1}{2}\,(11\,c^{2}d^{10} + 18\,bcd^{8} + 7\,(b^{2} + 2\,ac)d^{6} + 10\,abd^{4} + 3\,a^{2}d^{2})f^{3}x^{2}e + (c^{2}d^{11} + 2\,bcd^{9} + (b^{2} + 2\,ac)d^{7} + 2\,abd^{5} + a^{2}d^{3})f^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] 1/12*c^2*f^3*x^12*e^11 + c^2*d*f^3*x^11*e^10 + 1/10*(55*c^2*d^2 + 2*b*c)*f^
3*x^10*e^9 + 1/3*(55*c^2*d^3 + 6*b*c*d)*f^3*x^9*e^8 + 1/8*(330*c^2*d^4 + 72
*b*c*d^2 + b^2 + 2*a*c)*f^3*x^8*e^7 + (66*c^2*d^5 + 24*b*c*d^3 + (b^2 + 2*a
*c)*d)*f^3*x^7*e^6 + 1/6*(462*c^2*d^6 + 252*b*c*d^4 + 21*(b^2 + 2*a*c)*d^2
+ 2*a*b)*f^3*x^6*e^5 + 1/5*(330*c^2*d^7 + 252*b*c*d^5 + 35*(b^2 + 2*a*c)*d^
3 + 10*a*b*d)*f^3*x^5*e^4 + 1/4*(165*c^2*d^8 + 168*b*c*d^6 + 35*(b^2 + 2*a*
c)*d^4 + 20*a*b*d^2 + a^2)*f^3*x^4*e^3 + 1/3*(55*c^2*d^9 + 72*b*c*d^7 + 21*
(b^2 + 2*a*c)*d^5 + 20*a*b*d^3 + 3*a^2*d)*f^3*x^3*e^2 + 1/2*(11*c^2*d^10 +
18*b*c*d^8 + 7*(b^2 + 2*a*c)*d^6 + 10*a*b*d^4 + 3*a^2*d^2)*f^3*x^2*e + (c^2
*d^11 + 2*b*c*d^9 + (b^2 + 2*a*c)*d^7 + 2*a*b*d^5 + a^2*d^3)*f^3*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(88) = 176$.
time = 0.07, size = 722, normalized size = 6.94

$$c^{2}de^{10}f^{3}x^{11} + \frac{c^{2}e^{11}f^{3}x^{12}}{12} + x^{10}\left(\frac{bce^{9}f^{3}}{5} + \frac{11c^{2}d^{2}e^{9}f^{3}}{2}\right) + x^{9}\cdot\left(2bcde^{8}f^{3} + \frac{55c^{2}d^{3}e^{8}f^{3}}{3}\right) + x^{8}\left(\frac{ace^{7}f^{3}}{4} + \frac{b^{2}e^{7}f^{3}}{8} + 9bcd^{2}e^{7}f^{3} + \frac{165c^{2}d^{4}e^{7}f^{3}}{4}\right) + x^{7}\cdot\left(2acde^{6}f^{3} + b^{2}de^{6}f^{3} + 24bcd^{3}e^{6}f^{3} + 66c^{2}d^{5}e^{6}f^{3}\right) + x^{6}\left(\frac{abe^{5}f^{3}}{3} + 7acd^{2}e^{5}f^{3} + \frac{7b^{2}d^{2}e^{5}f^{3}}{2} + 42bcd^{4}e^{5}f^{3} + 77c^{2}d^{6}e^{5}f^{3}\right) + x^{5}\cdot\left(2abde^{4}f^{3} + 14acd^{3}e^{4}f^{3} + 7b^{2}d^{3}e^{4}f^{3} + \frac{252bcd^{5}e^{4}f^{3}}{5} + 66c^{2}d^{7}e^{4}f^{3}\right) + x^{4}\left(\frac{a^{2}e^{3}f^{3}}{4} + 5abd^{2}e^{3}f^{3} + \frac{35acd^{4}e^{3}f^{3}}{2} + \frac{35b^{2}d^{4}e^{3}f^{3}}{4} + 42bcd^{6}e^{3}f^{3} + \frac{165c^{2}d^{8}e^{3}f^{3}}{4}\right) + x^{3}\left(a^{2}de^{2}f^{3} + \frac{20abd^{3}e^{2}f^{3}}{3} + 14acd^{5}e^{2}f^{3} + 7b^{2}d^{5}e^{2}f^{3} + 24bcd^{7}e^{2}f^{3} + \frac{55c^{2}d^{9}e^{2}f^{3}}{3}\right) + x^{2}\cdot\left(\frac{3a^{2}d^{2}ef^{3}}{2} + 5abd^{4}ef^{3} + 7acd^{6}ef^{3} + \frac{7b^{2}d^{6}ef^{3}}{2} + 9bcd^{8}ef^{3} + \frac{11c^{2}d^{10}ef^{3}}{2}\right) + x\left(a^{2}d^{3}f^{3} + 2abd^{5}f^{3} + 2acd^{7}f^{3} + b^{2}d^{7}f^{3} + 2bcd^{9}f^{3} + c^{2}d^{11}f^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] c**2*d*e**10*f**3*x**11 + c**2*e**11*f**3*x**12/12 + x**10*(b*c*e**9*f**3/5
 + 11*c**2*d**2*e**9*f**3/2) + x**9*(2*b*c*d*e**8*f**3 + 55*c**2*d**3*e**8*
f**3/3) + x**8*(a*c*e**7*f**3/4 + b**2*e**7*f**3/8 + 9*b*c*d**2*e**7*f**3 +
 165*c**2*d**4*e**7*f**3/4) + x**7*(2*a*c*d*e**6*f**3 + b**2*d*e**6*f**3 +
24*b*c*d**3*e**6*f**3 + 66*c**2*d**5*e**6*f**3) + x**6*(a*b*e**5*f**3/3 + 7
*a*c*d**2*e**5*f**3 + 7*b**2*d**2*e**5*f**3/2 + 42*b*c*d**4*e**5*f**3 + 77*
c**2*d**6*e**5*f**3) + x**5*(2*a*b*d*e**4*f**3 + 14*a*c*d**3*e**4*f**3 + 7*
b**2*d**3*e**4*f**3 + 252*b*c*d**5*e**4*f**3/5 + 66*c**2*d**7*e**4*f**3) +
x**4*(a**2*e**3*f**3/4 + 5*a*b*d**2*e**3*f**3 + 35*a*c*d**4*e**3*f**3/2 + 3
```

```
5*b**2*d**4*e**3*f**3/4 + 42*b*c*d**6*e**3*f**3 + 165*c**2*d**8*e**3*f**3/4
) + x**3*(a**2*d*e**2*f**3 + 20*a*b*d**3*e**2*f**3/3 + 14*a*c*d**5*e**2*f**
3 + 7*b**2*d**5*e**2*f**3 + 24*b*c*d**7*e**2*f**3 + 55*c**2*d**9*e**2*f**3/
3) + x**2*(3*a**2*d**2*e*f**3/2 + 5*a*b*d**4*e*f**3 + 7*a*c*d**6*e*f**3 + 7
*b**2*d**6*e*f**3/2 + 9*b*c*d**8*e*f**3 + 11*c**2*d**10*e*f**3/2) + x*(a**2
*d**3*f**3 + 2*a*b*d**5*f**3 + 2*a*c*d**7*f**3 + b**2*d**7*f**3 + 2*b*c*d**
9*f**3 + c**2*d**11*f**3)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(94) = 188.
time = 3.24, size = 615, normalized size = 5.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(f*x^2*e + 2*d*f*x)*c^2*d^10*f^2 + (f*x^2*e + 2*d*f*x)*b*c*d^8*f^2 + 1/
2*(f*x^2*e + 2*d*f*x)*b^2*d^6*f^2 + (f*x^2*e + 2*d*f*x)*a*c*d^6*f^2 + (f*x^
2*e + 2*d*f*x)*a*b*d^4*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*a^2*d^2*f^2 + 1/120*(1
50*(f*x^2*e + 2*d*f*x)^2*c^2*d^8*f^4*e + 200*(f*x^2*e + 2*d*f*x)^3*c^2*d^6*
f^3*e^2 + 240*(f*x^2*e + 2*d*f*x)^2*b*c*d^6*f^4*e + 150*(f*x^2*e + 2*d*f*x)
^4*c^2*d^4*f^2*e^3 + 240*(f*x^2*e + 2*d*f*x)^3*b*c*d^4*f^3*e^2 + 90*(f*x^2*
e + 2*d*f*x)^2*b^2*d^4*f^4*e + 180*(f*x^2*e + 2*d*f*x)^2*a*c*d^4*f^4*e + 60
*(f*x^2*e + 2*d*f*x)^5*c^2*d^2*f*e^4 + 120*(f*x^2*e + 2*d*f*x)^4*b*c*d^2*f^
2*e^3 + 60*(f*x^2*e + 2*d*f*x)^3*b^2*d^2*f^3*e^2 + 120*(f*x^2*e + 2*d*f*x)^
3*a*c*d^2*f^3*e^2 + 120*(f*x^2*e + 2*d*f*x)^2*a*b*d^2*f^4*e + 10*(f*x^2*e +
 2*d*f*x)^6*c^2*e^5 + 24*(f*x^2*e + 2*d*f*x)^5*b*c*f*e^4 + 15*(f*x^2*e + 2*
d*f*x)^4*b^2*f^2*e^3 + 30*(f*x^2*e + 2*d*f*x)^4*a*c*f^2*e^3 + 40*(f*x^2*e +
 2*d*f*x)^3*a*b*f^3*e^2 + 30*(f*x^2*e + 2*d*f*x)^2*a^2*f^4*e)/f^3
```

Mupad [B]
time = 1.47, size = 419, normalized size = 4.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (e^3*f^3*x^4*(a^2 + 35*b^2*d^4 + 165*c^2*d^8 + 20*a*b*d^2 + 70*a*c*d^4 + 16
8*b*c*d^6))/4 + (c^2*e^11*f^3*x^12)/12 + d^3*f^3*x*(a + b*d^2 + c*d^4)^2 +
(e^7*f^3*x^8*(2*a*c + b^2 + 330*c^2*d^4 + 72*b*c*d^2))/8 + (e^5*f^3*x^6*(2*
a*b + 21*b^2*d^2 + 462*c^2*d^6 + 42*a*c*d^2 + 252*b*c*d^4))/6 + (d^2*e*f^3*
x^2*(3*a^2 + 7*b^2*d^4 + 11*c^2*d^8 + 10*a*b*d^2 + 14*a*c*d^4 + 18*b*c*d^6)
)/2 + (d*e^2*f^3*x^3*(3*a^2 + 21*b^2*d^4 + 55*c^2*d^8 + 20*a*b*d^2 + 42*a*c
*d^4 + 72*b*c*d^6))/3 + d*e^6*f^3*x^7*(2*a*c + b^2 + 66*c^2*d^4 + 24*b*c*d^
```

```
2) + (d*e^4*f^3*x^5*(10*a*b + 35*b^2*d^2 + 330*c^2*d^6 + 70*a*c*d^2 + 252*b
*c*d^4))/5 + (c*e^9*f^3*x^10*(2*b + 55*c*d^2))/10 + c^2*d*e^10*f^3*x^11 + (
c*d*e^8*f^3*x^9*(6*b + 55*c*d^2))/3
```

3.612 $\int (df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3 \, dx$

Optimal. Leaf size=159

$$\frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{3a(b^2+ac)\,f^3(d+ex)^8}{8e}+\frac{b(b^2+6ac)\,f^3(d+ex)^{10}}{10e}+\frac{c(b^2+ac)\,f^3(d+ex)^{12}}{4e}$$

```
[Out] 1/4*a^3*f^3*(e*x+d)^4/e+1/2*a^2*b*f^3*(e*x+d)^6/e+3/8*a*(a*c+b^2)*f^3*(e*x+
d)^8/e+1/10*b*(6*a*c+b^2)*f^3*(e*x+d)^10/e+1/4*c*(a*c+b^2)*f^3*(e*x+d)^12/e
+3/14*b*c^2*f^3*(e*x+d)^14/e+1/16*c^3*f^3*(e*x+d)^16/e
```

Rubi [A]
time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1156, 1128, 645}

$$\frac{a^3f^3(d+ex)^4}{4e}+\frac{a^2bf^3(d+ex)^6}{2e}+\frac{cf^3(ac+b^2)\,(d+ex)^{12}}{4e}+\frac{bf^3(6ac+b^2)\,(d+ex)^{10}}{10e}+\frac{3af^3(ac+b^2)\,(d+ex)^8}{8e}+\frac{3bc^2f^3(d+ex)^{14}}{14e}+\frac{c^3f^3(d+ex)^{16}}{16e}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (a^3*f^3*(d + e*x)^4)/(4*e) + (a^2*b*f^3*(d + e*x)^6)/(2*e) + (3*a*(b^2 + a
*c)*f^3*(d + e*x)^8)/(8*e) + (b*(b^2 + 6*a*c)*f^3*(d + e*x)^10)/(10*e) + (c
*(b^2 + a*c)*f^3*(d + e*x)^12)/(4*e) + (3*b*c^2*f^3*(d + e*x)^14)/(14*e) +
(c^3*f^3*(d + e*x)^16)/(16*e)
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int (df + efx)^3 \left(a + b(d+ex)^2 + c(d+ex)^4\right)^3 \, dx = \frac{f^3 \text{Subst}\left(\int x^3 (a + bx^2 + cx^4)^3 \, dx, x, d+ex\right)}{e}$$
$$= \frac{f^3 \text{Subst}\left(\int x (a + bx + cx^2)^3 \, dx, x, (d+ex)^2\right)}{2e}$$
$$= \frac{f^3 \text{Subst}\left(\int (a^3 x + 3a^2 b x^2 + 3a(b^2+ac)\, x^3 + b(b^2 + 6ac$$
$$= \frac{a^3 f^3 (d+ex)^4}{4e} + \frac{a^2 b f^3 (d+ex)^6}{2e} + \frac{3a(b^2+ac)\, f^3 (d+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 801 vs. 2(159) = 318.
time = 0.03, size = 801, normalized size = 5.04

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] f^3*(d^3*(a + b*d^2 + c*d^4)^3*x + (3*d^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^
2 + 5*c*d^4)*e*x^2)/2 + d*(a^3 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4
 + 12*b^3*d^6 + 72*a*b*c*d^6 + 55*b^2*c*d^8 + 55*a*c^2*d^8 + 78*b*c^2*d^10
+ 35*c^3*d^12)*e^2*x^3 + ((a^3 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c*d
^4 + 84*b^3*d^6 + 504*a*b*c*d^6 + 495*b^2*c*d^8 + 495*a*c^2*d^8 + 858*b*c^2
*d^10 + 455*c^3*d^12)*e^3*x^4)/4 + (3*d*(5*a^2*b + 35*a*b^2*d^2 + 35*a^2*c*
d^2 + 42*b^3*d^4 + 252*a*b*c*d^4 + 330*b^2*c*d^6 + 330*a*c^2*d^6 + 715*b*c^
2*d^8 + 455*c^3*d^10)*e^4*x^5)/5 + ((a^2*b + 21*a*b^2*d^2 + 21*a^2*c*d^2 +
42*b^3*d^4 + 252*a*b*c*d^4 + 462*b^2*c*d^6 + 462*a*c^2*d^6 + 1287*b*c^2*d^8
 + 1001*c^3*d^10)*e^5*x^6)/2 + (d*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 504*a
*b*c*d^2 + 1386*b^2*c*d^4 + 1386*a*c^2*d^4 + 5148*b*c^2*d^6 + 5005*c^3*d^8)
*e^6*x^7)/7 + (3*(a*b^2 + a^2*c + 12*b^3*d^2 + 72*a*b*c*d^2 + 330*b^2*c*d^4
 + 330*a*c^2*d^4 + 1716*b*c^2*d^6 + 2145*c^3*d^8)*e^7*x^8)/8 + d*(b^3 + 6*a
*b*c + 55*b^2*c*d^2 + 55*a*c^2*d^2 + 429*b*c^2*d^4 + 715*c^3*d^6)*e^8*x^9 +
 ((b^3 + 6*a*b*c + 165*b^2*c*d^2 + 165*a*c^2*d^2 + 2145*b*c^2*d^4 + 5005*c^
3*d^6)*e^9*x^10)/10 + 3*c*d*(b^2 + a*c + 26*b*c*d^2 + 91*c^2*d^4)*e^10*x^11
 + (c*(b^2 + a*c + 78*b*c*d^2 + 455*c^2*d^4)*e^11*x^12)/4 + c^2*d*(3*b + 35
*c*d^2)*e^12*x^13 + (3*c^2*(b + 35*c*d^2)*e^13*x^14)/14 + c^3*d*e^14*x^15 +
 (c^3*e^15*x^16)/16)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 7696 vs. 2(145) = 290.
time = 0.24, size = 7697, normalized size = 48.41

method	result
gosper	$\frac{f^3x(35e^{15}c^3x^{15}+560d\,e^{14}c^3x^{14}+4200x^{13}d^2e^{13}c^3+19600c^3d^3e^{12}x^{12}+120x^{13}b\,c^2e^{13}+63700x^{11}d^4e^{11}c^3+1680b\,c^2d\,e^{12}x^{12}+152880c^3}{}$
norman	Expression too large to display
risch	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(145) = 290$.
time = 0.28, size = 1067, normalized size = 6.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] 1/16*c^3*f^3*x^16*e^15 + c^3*d*f^3*x^15*e^14 + 3/14*(35*c^3*d^2*e^13 + b*c^
2*e^13)*f^3*x^14 + (35*c^3*d^3*e^12 + 3*b*c^2*d*e^12)*f^3*x^13 + 1/4*(455*c
^3*d^4*e^11 + 78*b*c^2*d^2*e^11 + b^2*c*e^11 + a*c^2*e^11)*f^3*x^12 + 3*(91
*c^3*d^5*e^10 + 26*b*c^2*d^3*e^10 + (b^2*c*e^10 + a*c^2*e^10)*d)*f^3*x^11 +
 1/10*(5005*c^3*d^6*e^9 + 2145*b*c^2*d^4*e^9 + b^3*e^9 + 6*a*b*c*e^9 + 165*
(b^2*c*e^9 + a*c^2*e^9)*d^2)*f^3*x^10 + (715*c^3*d^7*e^8 + 429*b*c^2*d^5*e^
8 + 55*(b^2*c*e^8 + a*c^2*e^8)*d^3 + (b^3*e^8 + 6*a*b*c*e^8)*d)*f^3*x^9 + 3
/8*(2145*c^3*d^8*e^7 + 1716*b*c^2*d^6*e^7 + 330*(b^2*c*e^7 + a*c^2*e^7)*d^4
 + a*b^2*e^7 + a^2*c*e^7 + 12*(b^3*e^7 + 6*a*b*c*e^7)*d^2)*f^3*x^8 + 1/7*(5
005*c^3*d^9*e^6 + 5148*b*c^2*d^7*e^6 + 1386*(b^2*c*e^6 + a*c^2*e^6)*d^5 + 8
4*(b^3*e^6 + 6*a*b*c*e^6)*d^3 + 21*(a*b^2*e^6 + a^2*c*e^6)*d)*f^3*x^7 + 1/2
*(1001*c^3*d^10*e^5 + 1287*b*c^2*d^8*e^5 + 462*(b^2*c*e^5 + a*c^2*e^5)*d^6
+ 42*(b^3*e^5 + 6*a*b*c*e^5)*d^4 + a^2*b*e^5 + 21*(a*b^2*e^5 + a^2*c*e^5)*d
^2)*f^3*x^6 + 3/5*(455*c^3*d^11*e^4 + 715*b*c^2*d^9*e^4 + 330*(b^2*c*e^4 +
a*c^2*e^4)*d^7 + 42*(b^3*e^4 + 6*a*b*c*e^4)*d^5 + 5*a^2*b*d*e^4 + 35*(a*b^2
*e^4 + a^2*c*e^4)*d^3)*f^3*x^5 + 1/4*(455*c^3*d^12*e^3 + 858*b*c^2*d^10*e^3
 + 495*(b^2*c*e^3 + a*c^2*e^3)*d^8 + 84*(b^3*e^3 + 6*a*b*c*e^3)*d^6 + 30*a^
2*b*d^2*e^3 + 105*(a*b^2*e^3 + a^2*c*e^3)*d^4 + a^3*e^3)*f^3*x^4 + (35*c^3*
d^13*e^2 + 78*b*c^2*d^11*e^2 + 55*(b^2*c*e^2 + a*c^2*e^2)*d^9 + 12*(b^3*e^2
 + 6*a*b*c*e^2)*d^7 + 10*a^2*b*d^3*e^2 + 21*(a*b^2*e^2 + a^2*c*e^2)*d^5 + a
^3*d*e^2)*f^3*x^3 + 3/2*(5*c^3*d^14*e + 13*b*c^2*d^12*e + 11*(b^2*c*e + a*c
^2*e)*d^10 + 3*(b^3*e + 6*a*b*c*e)*d^8 + 5*a^2*b*d^4*e + 7*(a*b^2*e + a^2*c
```

```
*e)*d^6 + a^3*d^2*e)*f^3*x^2 + (c^3*d^15 + 3*b*c^2*d^13 + 3*(b^2*c + a*c^2)
*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b^2 + a^2*c)*d^7 + a^3*d^3
)*f^3*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 907 vs. $2(145) = 290$.
time = 0.36, size = 907, normalized size = 5.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] 1/16*c^3*f^3*x^16*e^15 + c^3*d*f^3*x^15*e^14 + 3/14*(35*c^3*d^2 + b*c^2)*f^
3*x^14*e^13 + (35*c^3*d^3 + 3*b*c^2*d)*f^3*x^13*e^12 + 1/4*(455*c^3*d^4 + 7
8*b*c^2*d^2 + b^2*c + a*c^2)*f^3*x^12*e^11 + 3*(91*c^3*d^5 + 26*b*c^2*d^3 +
 (b^2*c + a*c^2)*d)*f^3*x^11*e^10 + 1/10*(5005*c^3*d^6 + 2145*b*c^2*d^4 + b
^3 + 6*a*b*c + 165*(b^2*c + a*c^2)*d^2)*f^3*x^10*e^9 + (715*c^3*d^7 + 429*b
*c^2*d^5 + 55*(b^2*c + a*c^2)*d^3 + (b^3 + 6*a*b*c)*d)*f^3*x^9*e^8 + 3/8*(2
145*c^3*d^8 + 1716*b*c^2*d^6 + 330*(b^2*c + a*c^2)*d^4 + a*b^2 + a^2*c + 12
*(b^3 + 6*a*b*c)*d^2)*f^3*x^8*e^7 + 1/7*(5005*c^3*d^9 + 5148*b*c^2*d^7 + 13
86*(b^2*c + a*c^2)*d^5 + 84*(b^3 + 6*a*b*c)*d^3 + 21*(a*b^2 + a^2*c)*d)*f^3
*x^7*e^6 + 1/2*(1001*c^3*d^10 + 1287*b*c^2*d^8 + 462*(b^2*c + a*c^2)*d^6 +
42*(b^3 + 6*a*b*c)*d^4 + a^2*b + 21*(a*b^2 + a^2*c)*d^2)*f^3*x^6*e^5 + 3/5*
(455*c^3*d^11 + 715*b*c^2*d^9 + 330*(b^2*c + a*c^2)*d^7 + 42*(b^3 + 6*a*b*c
)*d^5 + 5*a^2*b*d + 35*(a*b^2 + a^2*c)*d^3)*f^3*x^5*e^4 + 1/4*(455*c^3*d^12
 + 858*b*c^2*d^10 + 495*(b^2*c + a*c^2)*d^8 + 84*(b^3 + 6*a*b*c)*d^6 + 30*a
^2*b*d^2 + 105*(a*b^2 + a^2*c)*d^4 + a^3)*f^3*x^4*e^3 + (35*c^3*d^13 + 78*b
*c^2*d^11 + 55*(b^2*c + a*c^2)*d^9 + 12*(b^3 + 6*a*b*c)*d^7 + 10*a^2*b*d^3
+ 21*(a*b^2 + a^2*c)*d^5 + a^3*d)*f^3*x^3*e^2 + 3/2*(5*c^3*d^14 + 13*b*c^2*
d^12 + 11*(b^2*c + a*c^2)*d^10 + 3*(b^3 + 6*a*b*c)*d^8 + 5*a^2*b*d^4 + 7*(a
*b^2 + a^2*c)*d^6 + a^3*d^2)*f^3*x^2*e + (c^3*d^15 + 3*b*c^2*d^13 + 3*(b^2*
c + a*c^2)*d^11 + (b^3 + 6*a*b*c)*d^9 + 3*a^2*b*d^5 + 3*(a*b^2 + a^2*c)*d^7
 + a^3*d^3)*f^3*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(141) = 282$.
time = 0.15, size = 1654, normalized size = 10.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3*(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] c**3*d*e**14*f**3*x**15 + c**3*e**15*f**3*x**16/16 + x**14*(3*b*c**2*e**13*
f**3/14 + 15*c**3*d**2*e**13*f**3/2) + x**13*(3*b*c**2*d*e**12*f**3 + 35*c*
*3*d**3*e**12*f**3) + x**12*(a*c**2*e**11*f**3/4 + b**2*c*e**11*f**3/4 + 39
*b*c**2*d**2*e**11*f**3/2 + 455*c**3*d**4*e**11*f**3/4) + x**11*(3*a*c**2*d
*e**10*f**3 + 3*b**2*c*d*e**10*f**3 + 78*b*c**2*d**3*e**10*f**3 + 273*c**3*
d**5*e**10*f**3) + x**10*(3*a*b*c*e**9*f**3/5 + 33*a*c**2*d**2*e**9*f**3/2
+ b**3*e**9*f**3/10 + 33*b**2*c*d**2*e**9*f**3/2 + 429*b*c**2*d**4*e**9*f**
3/2 + 1001*c**3*d**6*e**9*f**3/2) + x**9*(6*a*b*c*d*e**8*f**3 + 55*a*c**2*d
**3*e**8*f**3 + b**3*d*e**8*f**3 + 55*b**2*c*d**3*e**8*f**3 + 429*b*c**2*d*
*5*e**8*f**3 + 715*c**3*d**7*e**8*f**3) + x**8*(3*a**2*c*e**7*f**3/8 + 3*a*
b**2*e**7*f**3/8 + 27*a*b*c*d**2*e**7*f**3 + 495*a*c**2*d**4*e**7*f**3/4 +
9*b**3*d**2*e**7*f**3/2 + 495*b**2*c*d**4*e**7*f**3/4 + 1287*b*c**2*d**6*e*
*7*f**3/2 + 6435*c**3*d**8*e**7*f**3/8) + x**7*(3*a**2*c*d*e**6*f**3 + 3*a*
b**2*d*e**6*f**3 + 72*a*b*c*d**3*e**6*f**3 + 198*a*c**2*d**5*e**6*f**3 + 12
*b**3*d**3*e**6*f**3 + 198*b**2*c*d**5*e**6*f**3 + 5148*b*c**2*d**7*e**6*f*
*3/7 + 715*c**3*d**9*e**6*f**3) + x**6*(a**2*b*e**5*f**3/2 + 21*a**2*c*d**2
*e**5*f**3/2 + 21*a*b**2*d**2*e**5*f**3/2 + 126*a*b*c*d**4*e**5*f**3 + 231*
a*c**2*d**6*e**5*f**3 + 21*b**3*d**4*e**5*f**3 + 231*b**2*c*d**6*e**5*f**3
+ 1287*b*c**2*d**8*e**5*f**3/2 + 1001*c**3*d**10*e**5*f**3/2) + x**5*(3*a**
2*b*d*e**4*f**3 + 21*a**2*c*d**3*e**4*f**3 + 21*a*b**2*d**3*e**4*f**3 + 756
*a*b*c*d**5*e**4*f**3/5 + 198*a*c**2*d**7*e**4*f**3 + 126*b**3*d**5*e**4*f*
*3/5 + 198*b**2*c*d**7*e**4*f**3 + 429*b*c**2*d**9*e**4*f**3 + 273*c**3*d**
11*e**4*f**3) + x**4*(a**3*e**3*f**3/4 + 15*a**2*b*d**2*e**3*f**3/2 + 105*a
**2*c*d**4*e**3*f**3/4 + 105*a*b**2*d**4*e**3*f**3/4 + 126*a*b*c*d**6*e**3*
f**3 + 495*a*c**2*d**8*e**3*f**3/4 + 21*b**3*d**6*e**3*f**3 + 495*b**2*c*d*
*8*e**3*f**3/4 + 429*b*c**2*d**10*e**3*f**3/2 + 455*c**3*d**12*e**3*f**3/4)
 + x**3*(a**3*d*e**2*f**3 + 10*a**2*b*d**3*e**2*f**3 + 21*a**2*c*d**5*e**2*
f**3 + 21*a*b**2*d**5*e**2*f**3 + 72*a*b*c*d**7*e**2*f**3 + 55*a*c**2*d**9*
e**2*f**3 + 12*b**3*d**7*e**2*f**3 + 55*b**2*c*d**9*e**2*f**3 + 78*b*c**2*d
**11*e**2*f**3 + 35*c**3*d**13*e**2*f**3) + x**2*(3*a**3*d**2*e*f**3/2 + 15
*a**2*b*d**4*e*f**3/2 + 21*a**2*c*d**6*e*f**3/2 + 21*a*b**2*d**6*e*f**3/2 +
 27*a*b*c*d**8*e*f**3 + 33*a*c**2*d**10*e*f**3/2 + 9*b**3*d**8*e*f**3/2 + 3
3*b**2*c*d**10*e*f**3/2 + 39*b*c**2*d**12*e*f**3/2 + 15*c**3*d**14*e*f**3/2
) + x*(a**3*d**3*f**3 + 3*a**2*b*d**5*f**3 + 3*a**2*c*d**7*f**3 + 3*a*b**2*
d**7*f**3 + 6*a*b*c*d**9*f**3 + 3*a*c**2*d**11*f**3 + b**3*d**9*f**3 + 3*b*
*2*c*d**11*f**3 + 3*b*c**2*d**13*f**3 + c**3*d**15*f**3)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(145) = 290$.
time = 5.03, size = 1360, normalized size = 8.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3*(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2*(f*x^2*e + 2*d*f*x)*c^3*d^14*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*b*c^2*d^12*f
^2 + 3/2*(f*x^2*e + 2*d*f*x)*b^2*c*d^10*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a*c^2
*d^10*f^2 + 1/2*(f*x^2*e + 2*d*f*x)*b^3*d^8*f^2 + 3*(f*x^2*e + 2*d*f*x)*a*b
*c*d^8*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a*b^2*d^6*f^2 + 3/2*(f*x^2*e + 2*d*f*x
)*a^2*c*d^6*f^2 + 3/2*(f*x^2*e + 2*d*f*x)*a^2*b*d^4*f^2 + 1/2*(f*x^2*e + 2*
d*f*x)*a^3*d^2*f^2 + 1/560*(980*(f*x^2*e + 2*d*f*x)^2*c^3*d^12*f^6*e + 1960
*(f*x^2*e + 2*d*f*x)^3*c^3*d^10*f^5*e^2 + 2520*(f*x^2*e + 2*d*f*x)^2*b*c^2*
d^10*f^6*e + 2450*(f*x^2*e + 2*d*f*x)^4*c^3*d^8*f^4*e^3 + 4200*(f*x^2*e + 2
*d*f*x)^3*b*c^2*d^8*f^5*e^2 + 2100*(f*x^2*e + 2*d*f*x)^2*b^2*c*d^8*f^6*e +
2100*(f*x^2*e + 2*d*f*x)^2*a*c^2*d^8*f^6*e + 1960*(f*x^2*e + 2*d*f*x)^5*c^3
*d^6*f^3*e^4 + 4200*(f*x^2*e + 2*d*f*x)^4*b*c^2*d^6*f^4*e^3 + 2800*(f*x^2*e
 + 2*d*f*x)^3*b^2*c*d^6*f^5*e^2 + 2800*(f*x^2*e + 2*d*f*x)^3*a*c^2*d^6*f^5*
e^2 + 560*(f*x^2*e + 2*d*f*x)^2*b^3*d^6*f^6*e + 3360*(f*x^2*e + 2*d*f*x)^2*
a*b*c*d^6*f^6*e + 980*(f*x^2*e + 2*d*f*x)^6*c^3*d^4*f^2*e^5 + 2520*(f*x^2*e
 + 2*d*f*x)^5*b*c^2*d^4*f^3*e^4 + 2100*(f*x^2*e + 2*d*f*x)^4*b^2*c*d^4*f^4*
e^3 + 2100*(f*x^2*e + 2*d*f*x)^4*a*c^2*d^4*f^4*e^3 + 560*(f*x^2*e + 2*d*f*x
)^3*b^3*d^4*f^5*e^2 + 3360*(f*x^2*e + 2*d*f*x)^3*a*b*c*d^4*f^5*e^2 + 1260*(
f*x^2*e + 2*d*f*x)^2*a*b^2*d^4*f^6*e + 1260*(f*x^2*e + 2*d*f*x)^2*a^2*c*d^4
*f^6*e + 280*(f*x^2*e + 2*d*f*x)^7*c^3*d^2*f*e^6 + 840*(f*x^2*e + 2*d*f*x)^
6*b*c^2*d^2*f^2*e^5 + 840*(f*x^2*e + 2*d*f*x)^5*b^2*c*d^2*f^3*e^4 + 840*(f*
x^2*e + 2*d*f*x)^5*a*c^2*d^2*f^3*e^4 + 280*(f*x^2*e + 2*d*f*x)^4*b^3*d^2*f^
4*e^3 + 1680*(f*x^2*e + 2*d*f*x)^4*a*b*c*d^2*f^4*e^3 + 840*(f*x^2*e + 2*d*f
*x)^3*a*b^2*d^2*f^5*e^2 + 840*(f*x^2*e + 2*d*f*x)^3*a^2*c*d^2*f^5*e^2 + 840
*(f*x^2*e + 2*d*f*x)^2*a^2*b*d^2*f^6*e + 35*(f*x^2*e + 2*d*f*x)^8*c^3*e^7 +
 120*(f*x^2*e + 2*d*f*x)^7*b*c^2*f*e^6 + 140*(f*x^2*e + 2*d*f*x)^6*b^2*c*f^
2*e^5 + 140*(f*x^2*e + 2*d*f*x)^6*a*c^2*f^2*e^5 + 56*(f*x^2*e + 2*d*f*x)^5*
b^3*f^3*e^4 + 336*(f*x^2*e + 2*d*f*x)^5*a*b*c*f^3*e^4 + 210*(f*x^2*e + 2*d*
f*x)^4*a*b^2*f^4*e^3 + 210*(f*x^2*e + 2*d*f*x)^4*a^2*c*f^4*e^3 + 280*(f*x^2
*e + 2*d*f*x)^3*a^2*b*f^5*e^2 + 140*(f*x^2*e + 2*d*f*x)^2*a^3*f^6*e)/f^5
```

Mupad [B]
time = 1.65, size = 825, normalized size = 5.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] (3*e^7*f^3*x^8*(a*b^2 + a^2*c + 12*b^3*d^2 + 2145*c^3*d^8 + 330*a*c^2*d^4 +
 330*b^2*c*d^4 + 1716*b*c^2*d^6 + 72*a*b*c*d^2))/8 + (e^5*f^3*x^6*(a^2*b +
42*b^3*d^4 + 1001*c^3*d^10 + 21*a*b^2*d^2 + 21*a^2*c*d^2 + 462*a*c^2*d^6 +
462*b^2*c*d^6 + 1287*b*c^2*d^8 + 252*a*b*c*d^4))/2 + (e^9*f^3*x^10*(b^3 + 5
005*c^3*d^6 + 165*a*c^2*d^2 + 165*b^2*c*d^2 + 2145*b*c^2*d^4 + 6*a*b*c))/10
 + (c^3*e^15*f^3*x^16)/16 + d^3*f^3*x*(a + b*d^2 + c*d^4)^3 + (e^3*f^3*x^4*
(a^3 + 84*b^3*d^6 + 455*c^3*d^12 + 30*a^2*b*d^2 + 105*a*b^2*d^4 + 105*a^2*c
```

```
*d^4 + 495*a*c^2*d^8 + 495*b^2*c*d^8 + 858*b*c^2*d^10 + 504*a*b*c*d^6))/4 +
 (c*e^11*f^3*x^12*(a*c + b^2 + 455*c^2*d^4 + 78*b*c*d^2))/4 + (d*e^6*f^3*x^
7*(21*a*b^2 + 21*a^2*c + 84*b^3*d^2 + 5005*c^3*d^8 + 1386*a*c^2*d^4 + 1386*
b^2*c*d^4 + 5148*b*c^2*d^6 + 504*a*b*c*d^2))/7 + (3*d*e^4*f^3*x^5*(5*a^2*b
+ 42*b^3*d^4 + 455*c^3*d^10 + 35*a*b^2*d^2 + 35*a^2*c*d^2 + 330*a*c^2*d^6 +
 330*b^2*c*d^6 + 715*b*c^2*d^8 + 252*a*b*c*d^4))/5 + d*e^8*f^3*x^9*(b^3 + 7
15*c^3*d^6 + 55*a*c^2*d^2 + 55*b^2*c*d^2 + 429*b*c^2*d^4 + 6*a*b*c) + (3*c^
2*e^13*f^3*x^14*(b + 35*c*d^2))/14 + c^3*d*e^14*f^3*x^15 + d*e^2*f^3*x^3*(a
^3 + 12*b^3*d^6 + 35*c^3*d^12 + 10*a^2*b*d^2 + 21*a*b^2*d^4 + 21*a^2*c*d^4
+ 55*a*c^2*d^8 + 55*b^2*c*d^8 + 78*b*c^2*d^10 + 72*a*b*c*d^6) + (3*d^2*e*f^
3*x^2*(a + b*d^2 + c*d^4)^2*(a + 3*b*d^2 + 5*c*d^4))/2 + c^2*d*e^12*f^3*x^1
3*(3*b + 35*c*d^2) + 3*c*d*e^10*f^3*x^11*(a*c + b^2 + 91*c^2*d^4 + 26*b*c*d
^2)
```

3.613 $\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=193

$$\frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

```
[Out] x/c-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*
a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*
c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1156, 1136, 1180, 211}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e
*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d
 + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
```

```
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{e}\\ &= \frac{x}{c}-\frac{\text{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{ce}\\ &= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{2ce}-\\ &= \frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4}}\right.}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 219, normalized size = 1.13

$$\frac{2\sqrt{c}\,(d+ex)-\frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}}{2c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*
c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2)*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 158, normalized size = 0.82

method	result
default	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^{4}c\_Z^{4}+4d\,e^{3}c\_Z^{3}+\left(6d^{2}e^{2}c+e^{2}b\right)\_Z^{2}+\left(4d^{3}ec+2deb\right)\_Z+d^{4}c+d^{2}b+a\right)}\frac{\left(-\_R^{2}b\,e^{2}-2\_Rbde-d^{2}b-a\right)\ln\left(x-\right.}{2e^{3}c\_R^{3}+6d\,e^{2}c\_R^{2}+6c\,d^{2}e\_R+2c\,d^{3}+}}{2ce}$
risch	$\frac{x}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^{4}c\_Z^{4}+4d\,e^{3}c\_Z^{3}+\left(6d^{2}e^{2}c+e^{2}b\right)\_Z^{2}+\left(4d^{3}ec+2deb\right)\_Z+d^{4}c+d^{2}b+a\right)}\frac{\left(-\_R^{2}b\,e^{2}-2\_Rbde-d^{2}b-a\right)\ln\left(x-\right.}{2e^{3}c\_R^{3}+6d\,e^{2}c\_R^{2}+6c\,d^{2}e\_R+2c\,d^{3}+}}{2ce}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/2/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)/(c*x^4*e^4 + 4*c*d*x^3*
e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x +
 a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1167 vs. $2(157) = 314$.
time = 0.38, size = 1167, normalized size = 6.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*e^(-2)/(b^2*c^3 - 4*a*c^4))*log(-2*
(a*b^2 - a^2*c)*x*e - 2*(a*b^2 - a^2*c)*d + sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))*e - (b^4 - 5*a*b^2*
c + 4*a^2*c^2)*e)*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*e^(-2)/(b^2*c^3 - 4*a*c^4))) - sqr
t(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c +
 a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*e^(-2)/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 -
 a^2*c)*x*e - 2*(a*b^2 - a^2*c)*d - sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*sqrt((
b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))*e - (b^4 - 5*a*b^2*c + 4*a^
2*c^2)*e)*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*e^(-2)/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c
*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2
)/(b^2*c^6 - 4*a*c^7)))*e^(-2)/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*
x*e - 2*(a*b^2 - a^2*c)*d + sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e
)*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^2*c^6 - 4*a*c^7)))*e^(-2)/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(
b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c
^6 - 4*a*c^7)))*e^(-2)/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x*e - 2*
(a*b^2 - a^2*c)*d - sqrt(1/2)*((b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(b^2*c^6 - 4*a*c^7))*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*sqrt(-
(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*
c^6 - 4*a*c^7)))*e^(-2)/(b^2*c^3 - 4*a*c^4))) + 2*x)/c
```

Sympy [A]
time = 1.73, size = 178, normalized size = 0.92

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{2}c^{5}e^{4}-128ab^{2}c^{4}e^{4}+16b^{4}c^{3}e^{4}\right)+t^{2}\cdot\left(48a^{2}bc^{2}e^{2}-28ab^{3}ce^{2}+4b^{5}e^{2}\right)+a^{3},\left(t\mapsto t\log\left(x+\frac{32t^{3}abc^{4}e^{3}-8t^{3}b^{3}c^{3}e^{3}-4ta^{2}c^{2}e+8tab^{2}ce-2tb^{4}e+a^{2}cd-ab^{2}d}{a^{2}ce-ab^{2}e}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e**
4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + a**3, L
ambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t**3*b**3*c**3*e**3 - 4*_
t*a**2*c**2*e + 8*_t*a*b**2*c*e - 2*_t*b**4*e + a**2*c*d - a*b**2*d)/(a**2*
c*e - a*b**2*e)))) + x/c
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1194 vs. $2(157) = 314$.
time = 4.14, size = 1194, normalized size = 6.19

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*b*e^6 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e
^(-4)/c))*b*d*e^5 + b*d^2*e^4 + a*e^4)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(
b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*
d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))^2*b*e^6 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*e^5 + b*d^2*e^4 + a*e^4)*log(d*e^(-1) +
 x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1
) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*
(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*
d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqr
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqr
t(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*e^6 - 2*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*e^5 + b*d^2*e^4 +
 a*e^4)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e
^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*
e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*
e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*e^6
 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
*b*d*e^5 + b*d^2*e^4 + a*e^4)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*
c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))))*e^(-4)/c + x/c
```

Mupad [B]
time = 2.32, size = 2500, normalized size = 12.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*
e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13
- 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5
 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
```

b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)
)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)
))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b
^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i + (-(b^5 + b^2*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^11 +
4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e
^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 -
16*a*b*c^4*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^
2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^
4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*
a*b^2*c*e^12))/c)*1i)/((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e
^2 - 8*a*b^2*c^4*e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((
8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4
e^14))/c)(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c
- a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2
*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 -
8*a*b^2*c^4*e^2)))^(1/2) + (2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*
e^11)/c + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c) - (-(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((
2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 -
4*a*b^2*c^2*e^12)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*
b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^
2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2
*c^2*e^12 - 4*a*b^2*c*e^12))/c) + (2*a^2*b*e^10)/c))*(-(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*2i + atan(((-(b
^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
- b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/
2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c
^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) +
(2*b^4*d*e^11 + 4*a^2*c^2*d*e^11 - 8*a*b^2*c*d*e^11)/c + (2*x*(b^4*e^12 +

```
2*a^2*c^2*e^12 - 4*a*b^2*c*e^12))/c)*1i + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*((2*b^4*d*e^11 + 4*a^2*c^2
*d*e^11 - 8*a*b^2*c*d*e^11)/c - ((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c - (
(8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^
4*e^14))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*
c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^
2*c^4*e^2)))^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2
- 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12 + 2*a^2*c^2*e^12 - 4*a*b^2*c*e^
12))/c)*1i)/((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3
*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b
^2*c^4*e^2)))^(1/2)*(((16*a^2*c^3*e^12 - 4*a*b^2*c^2*e^12)/c + ((8*b^3*c^3*
d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2))
)^(1/2))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) ...
```

3.614 $\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=81

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,e}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[Out] 1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/c/e+1/2*b*arctanh((b+2*c*(e*x+d)^2)/(-4*a
*c+b^2)^(1/2))/c/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1156, 1128, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]
*e) + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*c*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d+ex\right.}{4ce} \\
&= \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2c(d\right.}{2ce} \\
&= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,e} + \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 77, normalized size = 0.95

$$\frac{-\frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.18, size = 151, normalized size = 1.86

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(e^{4}c\_Z^{4}+4d\,e^{3}c\_Z^{3}+\left(6d^{2}e^{2}c+e^{2}b\right)\_Z^{2}+\left(4d^{3}ec+2deb\right)\_Z+d^{4}c+d^{2}b+a\right)}\frac{\left(\_R^{3}e^{3}+3\_R^{2}d\,e^{2}+3\_R\,d^{2}e+d^{3}\right)\ln\left(x-\_R\right.}{2e^{3}c\_R^{3}+6d\,e^{2}c\_R^{2}+6c\,d^{2}e\_R+2c\,d^{3}+eb\_R}}{2e}$
risch	$\frac{\ln\left(\left(\left(-4abc\,e^{2}+b^{3}e^{2}+\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,b\,e^{2}\right)x^{2}+\left(-8abcde+2b^{3}de+2\sqrt{-b^{2}\left(4ac-b^{2}\right)}\,bde\right)x-4abc\,d^{2}+b^{3}d^{2}+\sqrt{-b\right.\right.}{\left(4ac-b^{2}\right)e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^3/((x*e + d)^4*c + (x*e + d)^2*b + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(74) = 148.
time = 0.36, size = 424, normalized size = 5.23

$$\left[\frac{\left(\sqrt{b^{2}-4ac}\,b\log\left(\frac{2c^{2}x^{4}e^{4}+8c^{2}dx^{3}e^{3}+2c^{2}d^{4}+2bcd^{2}+2\left(6c^{2}d^{2}+bc\right)x^{2}e^{2}+4\left(2c^{2}d^{3}+bcd\right)xe+b^{2}-2ac+\left(2cx^{2}e^{2}+4cdxe+2cd^{2}+b\right)\sqrt{b^{2}-4ac}}{cx^{4}e^{4}+4cdx^{3}e^{3}+cd^{4}+\left(6cd^{2}+b\right)x^{2}e^{2}+bd^{2}+2\left(2cd^{3}+bd\right)xe+a}\right)+\left(b^{2}-4ac\right)\log\left(cx^{4}e^{4}+4cdx^{3}e^{3}+cd^{4}+\left(6cd^{2}+b\right)x^{2}e^{2}+bd^{2}+2\left(2cd^{3}+bd\right)xe+a\right)\right)e^{(-1)}}{4\left(b^{2}c-4ac^{2}\right)},\ \frac{\left(2\sqrt{-b^{2}+4ac}\,b\arctan\left(-\frac{\left(2cx^{2}e^{2}+4cdxe+2cd^{2}+b\right)\sqrt{-b^{2}+4ac}}{b^{2}-4ac}\right)+\left(b^{2}-4ac\right)\log\left(cx^{4}e^{4}+4cdx^{3}e^{3}+cd^{4}+\left(6cd^{2}+b\right)x^{2}e^{2}+bd^{2}+2\left(2cd^{3}+bd\right)xe+a\right)\right)e^{(-1)}}{4\left(b^{2}c-4ac^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4
+ 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2
 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^
4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*x*e + a)) + (b^2 - 4*a*c)*log(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6
*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a))*e^(-1)/(b^2*c - 4
```

```
*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^2*e^2 + 4*c*d*x*e + 2*c
*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^4*e^4 +
 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*
x*e + a))*e^(-1)/(b^2*c - 4*a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(68) = 136$.
time = 0.92, size = 280, normalized size = 3.46

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)}+\frac{1}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+2a+2b^2e\left(-\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+bd^2}{be^2}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4ce\,(4ac-b^2)}+\frac{1}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+2a+2b^2e\left(\frac{b\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{1}{4ce}\right)+bd^2}{be^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))*log(2*d*x/e + x
**2 + (-8*a*c*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e))
 + 2*a + 2*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/(4*c*e
)) + b*d**2)/(b*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + 1/
(4*c*e))*log(2*d*x/e + x**2 + (-8*a*c*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*
c - b**2)) + 1/(4*c*e)) + 2*a + 2*b**2*e*(b*sqrt(-4*a*c + b**2)/(4*c*e*(4*a
*c - b**2)) + 1/(4*c*e)) + b*d**2)/(b*e**2))
```

Giac [A]
time = 3.94, size = 130, normalized size = 1.60

$$-\frac{b\arctan\left(\frac{2\,cd^2+2\,(x^2e+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{2\,\sqrt{-b^2+4\,ac}\,c}+\frac{e^{(-1)}\log\left(cd^4+2\,(x^2e+2\,dx)cd^2e+(x^2e+2\,dx)^2ce^2+bd^2+(x^2e+2\,dx)be+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(
-1)/(sqrt(-b^2 + 4*a*c)*c) + 1/4*e^(-1)*log(c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2
*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e + a)/c
```

Mupad [B]
time = 1.76, size = 278, normalized size = 3.43

$$\frac{4\,a\,c\,e\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}-\frac{b^2\,e\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,d^2}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,e^2\,x^2}{\sqrt{4\,a\,c-b^2}}+\frac{4\,c\,d\,e\,x}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,e\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (4*a*c*e*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^
2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b
```

```
^2*e*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*c*d^2*e^
2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2) - (b*ata
n(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2*c*e^2*x^2)/(4*
a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e*(4*a*c - b^2)^(
1/2))
```

3.615 $\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4} \, dx$

Optimal. Leaf size=164

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,\sqrt{c}\,\sqrt{b^2-4ac}\,e}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,\sqrt{c}\,\sqrt{b^2-4ac}\,e}$$

```
[Out] -1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*
c+b^2)^(1/2))^(1/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*arctan((e*x+d)
*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)
/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1156, 1144, 211}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,\sqrt{c}\,e\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,\sqrt{c}\,e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[b + Sqr
t[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d+ex\right)}{2e} + \frac{\left(1+\right.}{}$$
$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e} + \frac{\sqrt{b+\sqrt{b^2-4ac}}}{\sqrt{2}}$$

Mathematica [A]
time = 0.07, size = 175, normalized size = 1.07

$$\frac{\left(-b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[
b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Ar
cTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 140, normalized size = 0.85

method	result
default	$\frac{\sum_{\_R=\text{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6d\,e^2c\_R^2+6c\,d^2e\_R+2c\,d^3+eb\_R}}{2e}$

risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)} \frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6d\,e^2c\_R^2+6c\,d^2e\_R+2c\,d^3+eb\_R}}{2e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e
+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*
e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)^2/((x*e + d)^4*c + (x*e + d)^2*b + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(129) = 258.
time = 0.45, size = 599, normalized size = 3.65

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}\right)e^{(-2)}}{b^2c-4ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{\left(b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}\right)e^{(-2)}}{b^2c-4ac^2}}\,e}{\sqrt{b^2c^2-4ac^3}}+xe+d\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}\right)e^{(-2)}}{b^2c-4ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{\left(b+\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}\right)e^{(-2)}}{b^2c-4ac^2}}\,e}{\sqrt{b^2c^2-4ac^3}}+xe+d\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}\right)e^{(-2)}}{b^2c-4ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{\left(b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}\right)e^{(-2)}}{b^2c-4ac^2}}\,e}{\sqrt{b^2c^2-4ac^3}}+xe+d\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}\right)e^{(-2)}}{b^2c-4ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4ac^2\right)\sqrt{-\frac{\left(b-\frac{b^2c-4ac^2}{\sqrt{b^2c^2-4ac^3}}\right)e^{(-2)}}{b^2c-4ac^2}}\,e}{\sqrt{b^2c^2-4ac^3}}+xe+d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))*e^(-2)/
(b^2*c - 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*
c^2)/sqrt(b^2*c^2 - 4*a*c^3))*e^(-2)/(b^2*c - 4*a*c^2))*e/sqrt(b^2*c^2 - 4*
a*c^3) + x*e + d) - 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2
 - 4*a*c^3))*e^(-2)/(b^2*c - 4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqr
t(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))*e^(-2)/(b^2*c - 4*a*c^2)
)*e/sqrt(b^2*c^2 - 4*a*c^3) + x*e + d) - 1/2*sqrt(1/2)*sqrt(-(b - (b^2*c -
4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))*e^(-2)/(b^2*c - 4*a*c^2))*log(sqrt(1/2)*(
b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))*e^(-
2)/(b^2*c - 4*a*c^2))*e/sqrt(b^2*c^2 - 4*a*c^3) + x*e + d) + 1/2*sqrt(1/2)*
sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))*e^(-2)/(b^2*c - 4*a*c
^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2
*c^2 - 4*a*c^3))*e^(-2)/(b^2*c - 4*a*c^2))*e/sqrt(b^2*c^2 - 4*a*c^3) + x*e
+ d)
```

Sympy [A]
time = 0.74, size = 104, normalized size = 0.63

$$\operatorname{RootSum}\left(t^4\cdot\left(256a^2c^3e^4-128ab^2c^2e^4+16b^4ce^4\right)+t^2\left(-16abce^2+4b^3e^2\right)+a,\left(t\mapsto t\log\left(x+\frac{64t^3ac^2e^3-16t^3b^2ce^3-2tbe+d}{e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
+ _t**2*(-16*a*b*c*e**2 + 4*b**3*e**2) + a, Lambda(_t, _t*log(x + (64*_t**3
*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e + d)/e)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1285 vs. $2(129) = 258$.
time = 4.27, size = 1285, normalized size = 7.84

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*d*e + d^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) -
 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*d*e + d^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) -
 1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*d*e + d^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) -
```

```
1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*d*e + d^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e)
```

Mupad [B]
time = 1.74, size = 590, normalized size = 3.60

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{-\frac{b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}\,\left(x\,\left(4\,a\,c^2\,e^{12}-2\,b^2\,c\,e^{12}\right)+\frac{\left(x\,\left(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14}\right)+8\,b^3\,c^2\,d\,e^{13}-32\,a\,b\,c^3\,d\,e^{13}\right)\,\left(b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\right)}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}+4\,a\,c^2\,d\,e^{11}-2\,b^2\,c\,d\,e^{11}\right)}{a\,c\,e^{10}}\right)\,\sqrt{-\frac{b^3+\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}-2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}\,\left(x\,\left(4\,a\,c^2\,e^{12}-2\,b^2\,c\,e^{12}\right)-\frac{\left(x\,\left(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14}\right)+8\,b^3\,c^2\,d\,e^{13}-32\,a\,b\,c^3\,d\,e^{13}\right)\,\left(\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c\right)}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}+4\,a\,c^2\,d\,e^{11}-2\,b^2\,c\,d\,e^{11}\right)}{a\,c\,e^{10}}\right)\,\sqrt{\frac{\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3+4\,a\,b\,c}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] - 2*atanh(((-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c*e^2 + 16*
a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12 - 2*b^2*c*e^12) + (
(x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*e^13 - 32*a*b*c^3*d*e^1
3)*(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c*e^2 + 16*a^2*c^3*e
^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11 - 2*b^2*c*d*e^11))/(a*c*e^10))*(-(b
^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8
*a*b^2*c^2*e^2)))^(1/2) - 2*atanh(((((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b
*c)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e
^12 - 2*b^2*c*e^12) - ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*
e^13 - 32*a*b*c^3*d*e^13)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(b
^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11 - 2*b^2*c*d*
e^11))/(a*c*e^10))*(((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c*e^
2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)
```

3.616 $\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,e}$$

```
[Out] -arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1156, 1121, 632, 212}

$$-\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -(ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]/(Sqrt[b^2 - 4*a*c]*e))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{d+ex}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{e}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{2e}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c(d+ex)^2\right)}{e}$$
$$= -\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,e}$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.07

$$\frac{\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]]/(Sqrt[-b^2 + 4*a*c]*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 129, normalized size = 3.00

method	result
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(e^4c\_Z^4+4de^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)}\frac{\left(\_Re+d\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6de^2c\_R^2+6cd^2e\_R+2cd^3+eb\_R}}{2e}$
risch	$-\frac{\ln\left(\left(\sqrt{-4ac+b^2}\,e^2-e^2b\right)x^2+\left(2de\sqrt{-4ac+b^2}-2deb\right)x+\sqrt{-4ac+b^2}\,d^2-d^2b-2a\right)}{2\sqrt{-4ac+b^2}\,e}+\frac{\ln\left(\left(\sqrt{-4ac+b\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e
+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2
+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((x*e + d)/((x*e + d)^4*c + (x*e + d)^2*b + a), x)
```

Fricas [A]
time = 0.34, size = 266, normalized size = 6.19

$$\left[\frac{e^{(-1)}\log\left(\frac{2\,c^2x^4e^4+8\,c^2dx^3e^3+2\,c^2d^4+2\,bcd^2+2\,(6\,c^2d^2+bc)x^2e^2+4\,(2\,c^2d^3+bcd)xe+b^2-2\,ac-(2\,cx^2e^2+4\,cdxe+2\,cd^2+b)\sqrt{b^2-4\,ac}}{cx^4e^4+4\,cdx^3e^3+cd^4+(6\,cd^2+b)x^2e^2+bd^2+2\,(2\,cd^3+bd)xe+a}\right)}{2\,\sqrt{b^2-4\,ac}},\ -\frac{\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2e^2+4\,cdxe+2\,cd^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)e^{(-1)}}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/2*e^(-1)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4 + 2*b*c*d^2 +
2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2 - 2*a*c - (2*
c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4*e^4 + 4*c*d*
x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a
))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 + 4*c*d*x*e
+ 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))*e^(-1)/(b^2 - 4*a*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(39) = 78$.
time = 0.56, size = 168, normalized size = 3.91

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b+2cd^2}{2ce^2}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2
)) + b**2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e) + sqrt(
-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b*
*2*sqrt(-1/(4*a*c - b**2)) + b + 2*c*d**2)/(2*c*e**2))/(2*e)
```

Giac [A]
time = 4.50, size = 53, normalized size = 1.23

$$\frac{\arctan\left(\frac{2\,cd^2+2\,(x^2e+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-1)/sqr
t(-b^2 + 4*a*c)
```

Mupad [B]
time = 0.09, size = 61, normalized size = 1.42

$$\frac{\mathrm{atan}\left(\frac{2\,a\,c\,d^2+4\,a\,c\,d\,e\,x+2\,a\,c\,e^2\,x^2+a\,b}{a\,\sqrt{4\,a\,c-b^2}}\right)}{e\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] atan((a*b + 2*a*c*d^2 + 2*a*c*e^2*x^2 + 4*a*c*d*e*x)/(a*(4*a*c - b^2)^(1/2)
))/(e*(4*a*c - b^2)^(1/2))
```

3.617 $\int \frac{1}{(d+ex)(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal. Leaf size=94

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,e}+\frac{\log(d+ex)}{ae}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}$$

```
[Out] ln(e*x+d)/a/e-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a/e+1/2*b*arctanh((b+2*c*(e
*x+d)^2)/(-4*a*c+b^2)^(1/2))/a/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1156, 1128, 719, 29, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ae\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae}+\frac{\log(d+ex)}{ae}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]
*e) + Log[d + e*x]/(a*e) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*e)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)}\,dx,x,d+ex\right)}{e}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)}\,dx,x,(d+ex)^2\right)}{2e}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,(d+ex)^2\right)}{2ae} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx,x,(d+e\right.}{2ae}$$
$$= \frac{\log(d+ex)}{ae} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{4ae} - \frac{b\text{Subst}(\int}{}$$
$$= \frac{\log(d+ex)}{ae} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ae} + \frac{b\text{Subst}(\int}{}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,e} + \frac{\log(d+ex)}{ae} - \frac{\log\left(a+b(d+ex)^2\right.}{4ae}$$

Mathematica [A]
time = 0.06, size = 128, normalized size = 1.36

$$\frac{4\sqrt{b^2-4ac}\log(d+ex)-\left(b+\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{4a\sqrt{b^2-4ac}\,e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 -
4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.18, size = 184, normalized size = 1.96

method	result
risch	$\frac{\ln(ex+d)}{ae} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^2ce^2-a\,b^2e^2\right)\_Z^2+\left(4ace-b^2e\right)\_Z+c\right)} \_R\ln\left(\left(\left(10e^3ac-3b^2e^3\right)\_R+5ce^2\right)x^2+\left(\left(20acde^2-6b^2de^2\right)\right.\right.\right.}{2}$
default	$\frac{\ln(ex+d)}{ae} + \frac{\sum_{\_R=\text{RootOf}\left(e^4c\_Z^4+4de^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)} \frac{\left(-e^3c\_R^3-3de^2c\_R^2+e\left(-3cd^2\right.\right.}{2e^3c\_R^3+6de^2c\_R^2+6cd^2}}{2ae}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)/a/e+1/2/a/e*sum((-e^3*c*_R^3-3*d*e^2*c*_R^2+e*(-3*c*d^2-b)*_R-c*d
^3-b*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-
_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e
+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] e^(-1)*log(x*e + d)/a - integrate((c*x^3*e^3 + 3*c*d*x^2*e^2 + c*d^3 + b*d
+ (3*c*d^2*e + b*e)*x)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^
2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/a
```

Fricas [A]
time = 0.37, size = 460, normalized size = 4.89

$$\left[\frac{\left(\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4e^4+8\,c^2dx^3e^3+2\,c^2d^4+2\,bcd^2+2\,(6\,c^2d^2+bc)x^2e^2+4\,(2\,c^2d^3+bcd)xe+b^2-2\,ac+(2\,cx^2e^2+4\,cdxe+2\,cd^2+b)\sqrt{b^2-4\,ac}}{cx^4e^4+4\,cdx^3e^3+cd^4+(6\,cd^2+b)x^2e^2+bd^2+2\,(2\,cd^3+bd)xe+a}\right)-(b^2-4\,ac)\log\left(cx^4e^4+4\,cdx^3e^3+cd^4+(6\,cd^2+b)x^2e^2+bd^2+2\,(2\,cd^3+bd)xe+a\right)+4\,(b^2-4\,ac)\log\left(xe+d\right)\right)e^{(-1)}}{4\,(ab^2-4\,a^2c)}, \frac{\left(2\,\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{(2\,cx^2e^2+4\,cdxe+2\,cd^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^2-4\,ac)\log\left(cx^4e^4+4\,cdx^3e^3+cd^4+(6\,cd^2+b)x^2e^2+bd^2+2\,(2\,cd^3+bd)xe+a\right)+4\,(b^2-4\,ac)\log\left(xe+d\right)\right)e^{(-1)}}{4\,(ab^2-4\,a^2c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4
+ 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2
 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^
4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3
+ b*d)*x*e + a)) - (b^2 - 4*a*c)*log(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6
*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a) + 4*(b^2 - 4*a*c)*
log(x*e + d))*e^(-1)/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(
-(2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))
- (b^2 - 4*a*c)*log(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e
^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a) + 4*(b^2 - 4*a*c)*log(x*e + d))*e^(
-1)/(a*b^2 - 4*a^2*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(78) = 156$.
time = 13.55, size = 320, normalized size = 3.40

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4ae\left(4ac-b^2\right)}-\frac{1}{4ae}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2ce\left(-\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)+2ab^2e\left(-\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)-2ac+b^2+bcd^2}{bce^2}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4ae\left(4ac-b^2\right)}-\frac{1}{4ae}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2ce\left(\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)+2ab^2e\left(\frac{b\sqrt{-4ac+b^2}}{4ae(4ac-b^2)}-\frac{1}{4ae}\right)-2ac+b^2+bcd^2}{bce^2}\right)+\frac{\log\left(\frac{d}{e}+x\right)}{ae}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e))*log(2*d*x/e + x
**2 + (-8*a**2*c*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*
e)) + 2*a*b**2*e*(-b*sqrt(-4*a*c + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)
) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a*c + b**2)/(4*a*e*(4
*a*c - b**2)) - 1/(4*a*e))*log(2*d*x/e + x**2 + (-8*a**2*c*e*(b*sqrt(-4*a*c
 + b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) + 2*a*b**2*e*(b*sqrt(-4*a*c +
b**2)/(4*a*e*(4*a*c - b**2)) - 1/(4*a*e)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e
**2)) + log(d/e + x)/(a*e)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(87) = 174.
time = 4.83, size = 274, normalized size = 2.91

$$-\frac{e^{(-1)} \log\left(|cx^4e^4 + 4\,cdx^3e^3 + 6\,cd^2x^2e^2 + 4\,cd^3xe + cd^4 + bx^2e^2 + 2\,bdxe + bd^2 + a|\right)}{4\,a} + \frac{e^{(-1)} \log\left(|xe + d|\right)}{a} - \frac{\left(\frac{abce^3 \log\left(\left|bx^2e^2 + 2\,bdxe + \sqrt{b^2 - 4\,ac}\,x^2e^2 + 2\,\sqrt{b^2 - 4\,ac}\,dxe + bd^2 + \sqrt{b^2 - 4\,ac}\,d^2 + 2\,a\right|\right)}{\sqrt{b^2 - 4\,ac}} - \frac{abce^3 \log\left(\left|-bx^2e^2 - 2\,bdxe + \sqrt{b^2 - 4\,ac}\,x^2e^2 + 2\,\sqrt{b^2 - 4\,ac}\,dxe - bd^2 + \sqrt{b^2 - 4\,ac}\,d^2 - 2\,a\right|\right)}{\sqrt{b^2 - 4\,ac}}\right)e^{(-4)}}{4\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x
*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/a + e^(-1)*log(abs(x*e + d
))/a - 1/4*(a*b*c*e^3*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2
*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a))/sq
rt(b^2 - 4*a*c) - a*b*c*e^3*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a
*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2
*a))/sqrt(b^2 - 4*a*c))*e^(-4)/(a^2*c)
```

Mupad [B]
time = 2.50, size = 2173, normalized size = 23.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] log(d + e*x)/(a*e) - (log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d
*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3)*(2*b^2*e - 8*a*c*e))/
(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*
((b^2*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 40*a*b*c^3*e
^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b
^2*e - 8*a*c*e)^2*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 -
40*a*b*c^3*e^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(4*(4*a*b^2*e^2 - 16*a
^2*c*e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 40*a*b*c^3*e^19)
)/(16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(8*a^3*c^2*(25*
a*c - 6*b^2)) - (((b*(2*b^2*e - 8*a*c*e)^2*(12*b^3*c^2*e^19 - 40*a*b*c^3*e^
```

19))/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2) - (b^3*(12*b^3*c^2*e^19 - 40*a*b*c^3*e^19))/(64*a^3*e^3*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e - 8*a*c*e)*(10*b*c^3*e^18 + ((2*b^2*e - 8*a*c*e)*(12*b^3*c^2*e^19 - 40*a*b*c^3*e^19))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(4*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)))*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^2)^(3/2))/(b^2*c^2*e^14) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*((b^2*(((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b^2*e - 8*a*c*e)^2*(((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(4*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(b^2*c^4*e^14*(25*a*c - 6*b^2)) - (2*(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b*(2*b^2*e - 8*a*c*e)*(((2*b^2*e - 8*a*c*e)*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 4*b^2*c^2*e^16 + 10*b*c^3*d^2*e^16))/(4*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b^3*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(64*a^3*e^3*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e - 8*a*c*e)^2*(4*a*b^2*c^2*e^17 + 12*b^3*c^2*d^2*e^17 - 40*a*b*c^3*d^2*e^17))/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2)))/(b^2*c^4*e^14*(25*a*c - 6*b^2)) + (16*a^3*x*(((3*b^3 - 8*a*b*c)*((b^2*(((2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(16*a^2*e^2*(4*a*c - b^2)) - ((2*b^2*e - 8*a*c*e)^2*(((2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(4*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2) + (b^2*(2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(16*a^2*e^2*(4*a*c - b^2)*(4*a*b^2*e^2 - 16*a^2*c*e^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b*(2*b^2*e - 8*a*c*e)*(((2*b^2*e - 8*a*c*e)*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(2*(4*a*b^2*e^2 - 16*a^2*c*e^2)) + 20*b*c^3*d*e^17))/(4*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)) - (b^3*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(64*a^3*e^3*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e - 8*a*c*e)^2*(24*b^3*c^2*d*e^18 - 80*a*b*c^3*d*e^18))/(16*a*e*(4*a*c - b^2)^(1/2)*(4*a*b^2*e^2 - 16*a^2*c*e^2)^2)))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^2)^(3/2))/(b^2*c^2*e^14)))/(2*a*e*(4*a*c - b^2)^(1/2))

3.618 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=195

$$-\frac{1}{ae(d+ex)}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,e}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\cdots}{\sqrt{b+\cdots}}\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}\cdots}$$

```
[Out] -1/a/e/(e*x+d)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2))*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2
)*(1-b/(-4*a*c+b^2)^(1/2))/a/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1156, 1137, 1180, 211}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,ae\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,ae\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ae(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(1/(a*e*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*e) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx,x,d+ex\right)}{e}\\ &= -\frac{1}{ae(d+ex)}+\frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{ae}\\ &= -\frac{1}{ae(d+ex)}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2}}\right.}{2ae}\\ &= -\frac{1}{ae(d+ex)}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{b^2}}}\right.}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,e}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 206, normalized size = 1.06

$$-\frac{\frac{2}{d+ex}+\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.19, size = 168, normalized size = 0.86

method	result
default	$-\frac{1}{ae(ex+d)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)}\frac{\left(-\_R^2c\,e^2-2\_Rcde-c\,d^2-\right.}{2e^3c\_R^3+6d\,e^2c\_R^2+6c\,d^2e\_}}{2ae}$
risch	$-\frac{1}{ae(ex+d)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^5c^2e^4-8e^4b^2c\,a^4+b^4e^4a^3\right)\_Z^4+\left(12a^2b\,c^2e^2-7a\,b^3c\,e^2+b^5e^2\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(40a^5c^2e^5-22a^4b^2c\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a/e/(e*x+d)+1/2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*c*e^3+6
*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^
4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*
b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2*e^2 + 2*c*d*x*e + c*d^2 + b)/(c*x^4*e^4 + 4*c*d*x^3*e^3 +
 c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a),
x)/a - 1/(a*x*e^2 + a*d*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(158) = 316$.
time = 0.44, size = 1275, normalized size = 6.54

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*(a*x*e^2 + a*d*e)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*e^(-2)/(a^3*b^2 - 4
*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x*e - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*(
(a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b
^2 - 4*a^7*c))*e - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(-(b^3 - 3*a*b*c
+ (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))
)*e^(-2)/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*(a*x*e^2 + a*d*e)*sqrt(-(b^3 - 3
*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*
a^7*c)))*e^(-2)/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x*e - 2*(b^2*
c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 -
 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))*e - (b^5 - 5*a*b^3*c + 4*a^2*b*c
^2)*e)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(a^6*b^2 - 4*a^7*c)))*e^(-2)/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*(a*x
*e^2 + a*d*e)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^
2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*e^(-2)/(a^3*b^2 - 4*a^4*c))*log(-2*(b^
2*c^2 - a*c^3)*x*e - 2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^
2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))*e +
(b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*
c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*e^(-2)/(a^3*b^2 -
 4*a^4*c))) + sqrt(1/2)*(a*x*e^2 + a*d*e)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 -
 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*e^(-2)/(a^
3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x*e - 2*(b^2*c^2 - a*c^3)*d - sq
rt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(a^6*b^2 - 4*a^7*c))*e + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e)*sqrt(-(b^3 -
 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 -
4*a^7*c)))*e^(-2)/(a^3*b^2 - 4*a^4*c))) - 2)/(a*x*e^2 + a*d*e)
```

Sympy [A]
time = 3.01, size = 211, normalized size = 1.08

$$\operatorname{RootSum}\left(t^4\cdot\left(256a^5c^2e^4-128a^4b^2ce^4+16a^3b^4e^4\right)+t^2\cdot\left(48a^2bc^2e^2-28ab^3ce^2+4b^5e^2\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^2e^3+48t^3a^4b^2ce^3-8t^3a^3b^4e^3-10ta^2bc^2e+10tab^3ce-2tb^5e+ac^3d-b^2c^2d}{ac^3e-b^2c^2e}\right)\right)\right)-\frac{1}{ade+ae^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2*e**4 - 128*a**4*b**2*c*e**4 + 16*a**3*b**4*e**
4) + _t**2*(48*a**2*b*c**2*e**2 - 28*a*b**3*c*e**2 + 4*b**5*e**2) + c**3, L
ambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**3 + 48*_t**3*a**4*b**2*c*e**3
- 8*_t**3*a**3*b**4*e**3 - 10*_t*a**2*b*c**2*e + 10*_t*a*b**3*c*e - 2*_t*b*
*5*e + a*c**3*d - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e + a*e
**2*x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error index.cc index_gcd Error: Bad A
rgument ValueError index.cc index_gcd Error: Bad Argument ValueDone
```

Mupad [B]
time = 2.39, size = 2500, normalized size = 12.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] - atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*
c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3
*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^
(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 -
 2*a^3*b^2*c^3*d*e^11)*1i + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*
c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12)
 + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a
^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7
*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 -
 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b^3*c^2*e^12)*(-(b^5
+ b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)
+ 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11)*1i)/((-(b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 -
 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^
6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b
^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3
*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4
*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11) - (-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*
```

(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11) + 2*a^3*c^4*e^10))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*2i - atan(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) - 16*a^5*b*c^3*e^12 + 4*a^4*b^3*c^2*e^12)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11)*1i + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b^3*c^2*e^12)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 4*a^4*c^4*d*e^11 - 2*a^3*b^2*c^3*d*e^11)*1i)/((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2)*(x*(4*a^4*c^4*e^12 - 2*a^3*b^2*c^3*e^12) + ((x*(32*a^6*b*c^3*e^14 - 8*a^5*b^3*c^2*e^14) + 32*a^6*b*c^3*d*e^13 - 8*a^5*b^3*c^2*d*e^13)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))^(1/2) + 16*a^5*b*c^3*e^12 - 4*a^4*b^3*c^2*e^12)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2 + 16*a^5*c^2*e^2 - 8*a^4*b^2*c*e^2)))...

3.619 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=121

$$-\frac{1}{2ae(d+ex)^2}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,e}-\frac{b\log(d+ex)}{a^2e}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}$$

```
[Out] -1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/a^2/e+1/4*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^
2/e-1/2*(-2*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/e/(-
4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1156, 1128, 723, 814, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}-\frac{b\log(d+ex)}{a^2e}-\frac{1}{2ae(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/2*1/(a*e*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqr
t[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e) - (b*Log[d + e*x])/(a^2*e) + (
b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3 (a+b(d+ex)^2+c(d+ex)^4)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{1}{2ae(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2ae}$$

$$= -\frac{1}{2ae(d+ex)^2} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx, x, (d+$$

$$= -\frac{1}{2ae(d+ex)^2} - \frac{b\log(d+ex)}{a^2e} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, (\right.}{2a^2e}$$

$$= -\frac{1}{2ae(d+ex)^2} - \frac{b\log(d+ex)}{a^2e} + \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (\right.}{4a^2e}$$

$$= -\frac{1}{2ae(d+ex)^2} - \frac{b\log(d+ex)}{a^2e} + \frac{b\log\left(a+b(d+ex)^2+c(\right.}{4a^2e}$$

$$= -\frac{1}{2ae(d+ex)^2} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,e} - \frac{b\log(}{}$$

Mathematica [A]
time = 0.09, size = 154, normalized size = 1.27

$$\frac{-\frac{2a}{(d+ex)^2} - 4b\log(d+ex) + \frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c
])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b^2
 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 213, normalized size = 1.76

method	result

default	$-\frac{1}{2ae(ex+d)^2}-\frac{b\ln(ex+d)}{e\,a^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)}\frac{\left(bc\,e^3\_R^3+3\right.}{2e}}{2a^2e}$
risch	$-\frac{1}{2ae(ex+d)^2}-\frac{b\ln(ex+d)}{e\,a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c\,e^2-a^2b^2e^2\right)\_Z^2+\left(-4abce+b^3e\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(10a^3c\,e^4-3a^2b^2e^4\right)\_R^2-\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/e/a^2+1/2/a^2/e*sum((b*c*e^3*_R^3+3*b*c*d*e^
2*_R^2+e*(3*b*c*d^2-a*c+b^2)*_R+b*c*d^3-a*c*d+b^2*d)/(2*_R^3*c*e^3+6*_R^2*c
*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e
^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] -b*e^(-1)*log(x*e + d)/a^2 - 1/2/(a*x^2*e^3 + 2*a*d*x*e^2 + a*d^2*e) + inte
grate((b*c*x^3*e^3 + 3*b*c*d*x^2*e^2 + b*c*d^3 + (b^2 - a*c)*d + (3*b*c*d^2
*e + b^2*e - a*c*e)*x)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^
2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/a^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(112) = 224$.
time = 0.41, size = 802, normalized size = 6.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*x^2*e^2 + 2*(b^2 - 2*a*c)*d*x*e +
 (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3
+ 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*
d)*x*e + b^2 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4
*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 +
 2*(2*c*d^3 + b*d)*x*e + a)) - ((b^3 - 4*a*b*c)*x^2*e^2 + 2*(b^3 - 4*a*b*c)
```

```
*d*x*e + (b^3 - 4*a*b*c)*d^2)*log(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*
d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a) + 4*((b^3 - 4*a*b*c)*
x^2*e^2 + 2*(b^3 - 4*a*b*c)*d*x*e + (b^3 - 4*a*b*c)*d^2)*log(x*e + d))/((a^
2*b^2 - 4*a^3*c)*x^2*e^3 + 2*(a^2*b^2 - 4*a^3*c)*d*x*e^2 + (a^2*b^2 - 4*a^3
*c)*d^2*e), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*x^2*e^2 + 2*(b^2 - 2
*a*c)*d*x*e + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 +
4*c*d*x*e + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*
c)*x^2*e^2 + 2*(b^3 - 4*a*b*c)*d*x*e + (b^3 - 4*a*b*c)*d^2)*log(c*x^4*e^4 +
 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*
x*e + a) + 4*((b^3 - 4*a*b*c)*x^2*e^2 + 2*(b^3 - 4*a*b*c)*d*x*e + (b^3 - 4*
a*b*c)*d^2)*log(x*e + d))/((a^2*b^2 - 4*a^3*c)*x^2*e^3 + 2*(a^2*b^2 - 4*a^3
*c)*d*x*e^2 + (a^2*b^2 - 4*a^3*c)*d^2*e)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.83, size = 102, normalized size = 0.84

$$\frac{be^{(-1)}\log\left(c+\frac{b}{(xe+d)^2}+\frac{a}{(xe+d)^4}\right)}{4\,a^2}+\frac{(b^2-2\,ac)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^2}}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{2\,\sqrt{-b^2+4\,ac}\,a^2}-\frac{e^{(-1)}}{2\,(xe+d)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/4*b*e^(-1)*log(c + b/(x*e + d)^2 + a/(x*e + d)^4)/a^2 + 1/2*(b^2 - 2*a*c)
*arctan(-(b + 2*a/(x*e + d)^2)/sqrt(-b^2 + 4*a*c))*e^(-1)/(sqrt(-b^2 + 4*a*
c)*a^2) - 1/2*e^(-1)/((x*e + d)^2*a)
```

Mupad [B]
time = 5.86, size = 2500, normalized size = 20.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] (atan((16*a^6*x^2*(4*a*c - b^2)^(3/2)*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((
(20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(40*a^4
```

```
*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))
*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (6*b*c^4*e^17)
/a^2)*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (c^5*e^16
)/a^3 - (((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*
c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^
2*b^2*e^2)))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*
(2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(8*a^5*e*(
16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*a*c - b^2))/(4*a^2*e
*(4*a*c - b^2)^(1/2)) - ((2*a*c - b^2)^2*(2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^
3*e^19 - 12*a^3*b^3*c^2*e^19))/(32*a^7*e^2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(
4*a*c - b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + (((((((20*a^3*
c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e - 8*a*b*c*e)*(40*a^4*b*c^3*e
^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*a*c
- b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e
)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(8*a^5*e*(16*a^3*c*e^2 - 4*a^2
*b^2*e^2)*(4*a*c - b^2)^(1/2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4
*a^2*b^2*e^2)) + (((((20*a^3*c^4*e^18 + 2*a^2*b^2*c^3*e^18)/a^3 + ((2*b^3*e
 - 8*a*b*c*e)*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(2*a^3*(16*a^3*c*e
^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e
^2)) + (6*b*c^4*e^17)/a^2)*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - (
(2*a*c - b^2)^3*(40*a^4*b*c^3*e^19 - 12*a^3*b^3*c^2*e^19))/(64*a^9*e^3*(4*a
*c - b^2)^(3/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c -
b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c))))/(4*a^2*c^4*e^14 + b^4*c^2*e^14
 - 4*a*b^2*c^3*e^14) + (16*a^6*x*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((2*b^3
*e - 8*a*b*c*e)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d*e^17))/a^3 + ((40*
a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a*b*c*e))/(a^3*(16*a
^3*c*e^2 - 4*a^2*b^2*e^2))))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (12*b*c^4
*d*e^16)/a^2)*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (
2*c^5*d*e^15)/a^3 - ((((2*a*c - b^2)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3
*d*e^17))/a^3 + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8
*a*b*c*e))/(a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(1
/2)) + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)*(2*b^3*
e - 8*a*b*c*e))/(4*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)
))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^18 - 1
2*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)^2*(2*b^3*e - 8*a*b*c*e))/(16*a^7*e^2*(1
6*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 +
 24*a*b^2*c)) + ((((((2*a*c - b^2)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d
*e^17))/a^3 + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a
*b*c*e))/(a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(1/2
)) + ((40*a^4*b*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)*(2*b^3*e
- 8*a*b*c*e))/(4*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*(4*a*c - b^2)^(1/2)))
*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + ((((2*b^3*e -
8*a*b*c*e)*((2*(20*a^3*c^4*d*e^17 + 2*a^2*b^2*c^3*d*e^17))/a^3 + ((40*a^4*b
*c^3*d*e^18 - 12*a^3*b^3*c^2*d*e^18)*(2*b^3*e - 8*a*b*c*e))/(a^3*(16*a^3*c*
e^2 - 4*a^2*b^2*e^2))))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)) + (12*b*c^4*d*e^
```

```
16)/a^2)*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^
18 - 12*a^3*b^3*c^2*d*e^18)*(2*a*c - b^2)^3)/(32*a^9*e^3*(4*a*c - b^2)^(3/2
)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^
2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^
2*e^14 - 4*a*b^2*c^3*e^14) + (2*a^3*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^2 -
9*a*b^2*c)*((b*c^4*e^14 + c^5*d^2*e^14)/a^3 + (((4*a*b^2*c^3*e^15 - a^2*c^4
*e^15 + 6*a*b*c^4*d^2*e^15)/a^3 + (((4*a^2*b^3*c^2*e^16 - 4*a^3*b*c^3*e^16
+ 20*a^3*c^4*d^2*e^16 + 2*a^2*b^2*c^3*d^2*e^16)/a^3 - ((2*b^3*e - 8*a*b*c*e
)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(
2*a^3*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*
e^2 - 4*a^2*b^2*e^2)))*(2*b^3*e - 8*a*b*c*e))/(2*(16*a^3*c*e^2 - 4*a^2*b^2*
e^2)) - ((2*a*c - b^2)*((((4*a^2*b^3*c^2*e^16 - 4*a^3*b*c^3*e^16 + 20*a^3*c
^4*d^2*e^16 + 2*a^2*b^2*c^3*d^2*e^16)/a^3 - ((2*b^3*e - 8*a*b*c*e)*(4*a^4*b
^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(2*a^3*(16*
a^3*c*e^2 - 4*a^2*b^2*e^2)))*(2*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(1/2)) -
 ((2*a*c - b^2)*(2*b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*
d^2*e^17 - 40*a^4*b*c^3*d^2*e^17))/(8*a^5*e*(16*a^3*c*e^2 - 4*a^2*b^2*e^2)*
(4*a*c - b^2)^(1/2))))/(4*a^2*e*(4*a*c - b^2)^(1/2)) + ((2*a*c - b^2)^2*(2*
b^3*e - 8*a*b*c*e)*(4*a^4*b^2*c^2*e^17 + 12*a^3*b^3*c^2*d^2*e^17 - 40*a^4*b
*c^3*d^2*e^17))/(32*a^7*e^2*(16*a^3*c*e^2 - 4*a...
```

3.620 $\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=224

$$-\frac{1}{3ae(d+ex)^3}+\frac{b}{a^2e(d+ex)}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}\,e}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}\,a^2\sqrt{}}$$

```
[Out] -1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d)+1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/e*
2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+
(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/e
*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1156, 1137, 1295, 1180, 211}

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2e\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2e(d+ex)}-\frac{1}{3ae(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/3*1/(a*e*(d + e*x)^3) + b/(a^2*e*(d + e*x)) + (Sqrt[c]*(b + (b^2 - 2*a*c
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b - (b^2
 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{1}{3ae(d+ex)^3} + \frac{\text{Subst}\left(\int \frac{-3b-3cx^2}{x^2(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{3ae}$$

$$= -\frac{1}{3ae(d+ex)^3} + \frac{b}{a^2e(d+ex)} - \frac{\text{Subst}\left(\int \frac{-3(b^2-ac)-3bcx^2}{a+bx^2+cx^4}\,dx \right.}{3a^2e}$$

$$= -\frac{1}{3ae(d+ex)^3} + \frac{b}{a^2e(d+ex)} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right)\text{Su}}{}$$

$$= -\frac{1}{3ae(d+ex)^3} + \frac{b}{a^2e(d+ex)} + \frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan}{\sqrt{2}\,a^2\sqrt{b-}}$$

Mathematica [A]
time = 0.14, size = 235, normalized size = 1.05

$$\frac{-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}+\frac{3\sqrt{2}\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt
[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(6*a^2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 188, normalized size = 0.84

method	result
default	$-\frac{1}{3ae(ex+d)^3}+\frac{b}{a^2e(ex+d)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)}\frac{\left(\_R^2bc\,e^2\cdots\right.}{2e^3c\_R^3+6d\cdots}}{2a^2e}$
risch	$\frac{\frac{be\,x^2}{a^2}+\frac{2bdx}{a^2}-\frac{-3d^2b+a}{3e\,a^2}}{(ex+d)^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16e^4c^2a^7-8a^6b^2c\,e^4+a^5b^4e^4\right)\_Z^4+\left(-20e^2b\,c^3a^3+25b^3e^2c^2a^2-9b^5e^2ca+b^7e^2\right)\_Z^2+c^5\right)}\_R\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d)+1/2/a^2/e*sum((_R^2*b*c*e^2+2*_R*b*c*d*e
+b*c*d^2-a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+
b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+
(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

[Out] 1/3*(3*b*x^2*e^2 + 6*b*d*x*e + 3*b*d^2 - a)/(a^2*x^3*e^4 + 3*a^2*d*x^2*e^3 + 3*a^2*d^2*x*e^2 + a^2*d^3*e) + integrate((b*c*x^2*e^2 + 2*b*c*d*x*e + b*c *d^2 + b^2 - a*c)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/a^2

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1970 vs. $2(188) = 376$.
time = 0.42, size = 1970, normalized size = 8.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] 1/6*(6*b*x^2*e^2 + 12*b*d*x*e + 6*b*d^2 + 3*sqrt(1/2)*(a^2*x^3*e^4 + 3*a^2*d*x^2*e^3 + 3*a^2*d^2*x*e^2 + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*e^(-2)/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x*e + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c))*e - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*e^(-2)/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*(a^2*x^3*e^4 + 3*a^2*d*x^2*e^3 + 3*a^2*d^2*x*e^2 + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*e^(-2)/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x*e + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c))*e - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*e^(-2)/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*(a^2*x^3*e^4 + 3*a^2*d*x^2*e^3 + 3*a^2*d^2*x*e^2 + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*e^(-2)/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x*e + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c))*e + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*e^(-2)/(a^5*b^2 - 4*a^6*c))) + 3*sqrt(1/2)*(a^2*x^3*e^4 + 3*a^2*d*x^2*e^3 + 3*a^2*d^2*x*e^2 + a^2*d^3*e)*sqrt(-(b^5 - 5*a*b^3*c + 5*a

```
^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a
^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*e^(-2)/(a^5*b^2 - 4*a^6*c))*l
og(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x*e + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2
*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8 - 6*a
*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c))*e
 + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e)*sqrt(
-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*e^(-2)
/(a^5*b^2 - 4*a^6*c))) - 2*a)/(a^2*x^3*e^4 + 3*a^2*d*x^2*e^3 + 3*a^2*d^2*x*
e^2 + a^2*d^3*e)
```

Sympy [**A**]
time = 101.09, size = 347, normalized size = 1.55

$$\frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3e+9a^2d^2e^2x+9a^2de^3x^2+3a^2e^4x^3}+\operatorname{RootSum}\left(t^4\cdot\left(256a^7c^2e^4-128a^6b^2ce^4+16a^5b^4e^4\right)+t^2\left(-80a^3bc^3e^2+100a^2b^3c^2e^2-36ab^5ce^2+4b^7e^2\right)+c^5,\left(t\mapsto t\log\left(x+\frac{-96t^3a^7bc^2e^3+56t^3a^6b^3ce^3-8t^3a^5b^5e^3-4ta^4c^4e+32ta^3b^2c^3e-40ta^2b^4c^2e+16tab^6ce-2tb^8e+a^2c^5d-3ab^2c^4d+b^4c^3d}{a^2c^5e-3ab^2c^4e+b^4c^3e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e + 9*a**2*d**2*e*
*2*x + 9*a**2*d*e**3*x**2 + 3*a**2*e**4*x**3) + RootSum(_t**4*(256*a**7*c**
2*e**4 - 128*a**6*b**2*c*e**4 + 16*a**5*b**4*e**4) + _t**2*(-80*a**3*b*c**3
*e**2 + 100*a**2*b**3*c**2*e**2 - 36*a*b**5*c*e**2 + 4*b**7*e**2) + c**5, L
ambda(_t, _t*log(x + (-96*_t**3*a**7*b*c**2*e**3 + 56*_t**3*a**6*b**3*c*e**
3 - 8*_t**3*a**5*b**5*e**3 - 4*_t*a**4*c**4*e + 32*_t*a**3*b**2*c**3*e - 40
*_t*a**2*b**4*c**2*e + 16*_t*a*b**6*c*e - 2*_t*b**8*e + a**2*c**5*d - 3*a*b
**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2*c**4*e + b**4*c**3*e))))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1243 vs. $2(188) = 376$.
time = 3.55, size = 1243, normalized size = 5.55

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*c*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqr
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e
 - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
```

```
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d
*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2
*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(
-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e +
b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x - sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/a^2 + 1/3*(3*b*x^2*e^2 + 6*b*d*x*e + 3*b*
d^2 - a)*e^(-1)/((x*e + d)^3*a^2)
```

Mupad [B]
time = 2.83, size = 2500, normalized size = 11.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] ((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3 + e^3*x^3 +
3*d*e^2*x^2 + 3*d^2*e*x) - atan((((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*
a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c -
 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a
^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c
^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3
*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*
((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a
^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 2
5*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)
))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12 + 20*a^9*b^2*c^3*e^12) + 4
```

```
*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11)*1i + ((b^4*(
-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(
4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12
 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2
) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
+ 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*
c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2
*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2
)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2
+ 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) + 16*a^10*c^4*e^12 + 4*a^8*b^4*
c^2*e^12 - 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 -
 8*a^7*b^2*c^4*d*e^11)*1i)/(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b
*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^
2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^
12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2
+ a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*((x*(
32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13 - 8*a^10*b
^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2
*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*
c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1
/2) + 16*a^10*c^4*e^12 + 4*a^8*b^4*c^2*e^12 - 20*a^9*b^2*c^3*e^12) + 4*a^8*
c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b^2*c^4*d*e^11) - ((b^4*(-(4*a*c
- b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c -
b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4
*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6
*b^4*c^3*e^12 - 8*a^7*b^2*c^4*e^12) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
+ 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^
5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2
- 8*a^6*b^2*c*e^2)))^(1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) +
 32*a^11*b*c^3*d*e^13 - 8*a^10*b^3*c^2*d*e^13)*((b^4*(-(4*a*c - b^2)^3)^(1/
2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2)
 + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7
*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12
 + 20*a^9*b^2*c^3*e^12) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - 8*a^7*b
^2*c^4*d*e^11) + 2*a^6*b*c^5*e^10))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 +
20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*
c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 -
8*a^6*b^2*c*e^2)))^(1/2)*2i - atan(((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) -
 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5
*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 -
 8*a^6*b^2*c*e^2)))^(1/2)*(x*(4*a^8*c^5*e^12 + 2*a^6*b^4*c^3*e^12 - 8*a^7*b
```

```
^2*c^4*e^12) - (-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^
2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a
*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*c*e^2)))^(
1/2)*((x*(32*a^11*b*c^3*e^14 - 8*a^10*b^3*c^2*e^14) + 32*a^11*b*c^3*d*e^13
- 8*a^10*b^3*c^2*d*e^13)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2 + 16*a^7*c^2*e^2 - 8*a^6*b^2*
c*e^2)))^(1/2) - 16*a^10*c^4*e^12 - 4*a^8*b^4*c^2*e^12 + 20*a^9*b^2*c^3*e^1
2) + 4*a^8*c^5*d*e^11 + 2*a^6*b^4*c^3*d*e^11 - ...
```

3.621 $\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=270

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}\,e}+\frac{\left(b^2+4ac\right.}{2\sqrt{\ }}$$

```
[Out] 1/2*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/
4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b
^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)/e*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(
b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1156, 1134, 1180, 211}

$$\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a
*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
```

```
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx,\right.}{2\left(b^2-4ac\right)}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4\right.}{}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4\right.}{2\sqrt{2}\sqrt{c}\left(b^2-\right.}$$

Mathematica [A]

time = 0.31, size = 263, normalized size = 0.97

$$\frac{-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]

[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*e)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.18, size = 323, normalized size = 1.20

method	result
default	$\frac{-\frac{e^2 b x^3}{2(4ac-b^2)}-\frac{3bde x^2}{2(4ac-b^2)}-\frac{(3d^2b+2a)x}{2(4ac-b^2)}-\frac{d(d^2b+2a)}{2e(4ac-b^2)}}{ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2debx+d^2b+a}+\frac{\sum_{\_R=\mathrm{RootOf}(e^4c\_Z^4+4de^3c\_Z^3+(6d^2e^2c+e^2b)\_Z^2+(4d^3ec+}}{}$
risch	$\frac{-\frac{e^2 b x^3}{2(4ac-b^2)}-\frac{3bde x^2}{2(4ac-b^2)}-\frac{(3d^2b+2a)x}{2(4ac-b^2)}-\frac{d(d^2b+2a)}{2e(4ac-b^2)}}{ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2debx+d^2b+a}+\frac{\sum_{\_R=\mathrm{RootOf}(e^4c\_Z^4+4de^3c\_Z^3+(6d^2e^2c+e^2b)\_Z^2+(4d^3ec+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)

[Out] (-1/2*e^2*b/(4*a*c-b^2)*x^3-3/2/(4*a*c-b^2)*b*d*e*x^2-1/2*(3*b*d^2+2*a)/(4*a*c-b^2)*x-1/2*d/e*(b*d^2+2*a)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")

[Out] 1/2*(b*x^3*e^3 + 3*b*d*x^2*e^2 + b*d^3 + 2*a*d + (3*b*d^2*e + 2*a*e)*x)/((b^2*c*e - 4*a*c^2*e)*d^4 + 4*(b^2*c*e^4 - 4*a*c^2*e^4)*d*x^3 + (b^2*c*e^5 - 4*a*c^2*e^5)*x^4 + a*b^2*e - 4*a^2*c*e + (b^3*e - 4*a*b*c*e)*d^2 + (b^3*e^3 - 4*a*b*c*e^3 + 6*(b^2*c*e^3 - 4*a*c^2*e^3)*d^2)*x^2 + 2*(2*(b^2*c*e^2 - 4*a*c^2*e^2)*d^3 + (b^3*e^2 - 4*a*b*c*e^2)*d)*x) + 1/2*integrate((b*x^2*e^2

```
+ 2*b*d*x*e + b*d^2 - 2*a)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*
c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(b^2 - 4*a*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2336 vs. $2(229) = 458$.
time = 0.41, size = 2336, normalized size = 8.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^3*e^3 + 6*b*d*x^2*e^2 + 2*b*d^3 + 2*(3*b*d^2 + 2*a)*x*e + sqrt(1
/2)*((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*c - 4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2)*e)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c
^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)
)*e^(-2)/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 +
 4*a*c)*x*e + (3*b^2 + 4*a*c)*d + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)
*e + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e/sqrt(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b
^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*e^(-2)/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b
^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*c -
4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3 + 2*
(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2)*d^
4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-(b^3 + 12*a*b*c + (b^6*
c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3
 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*e^(-2)/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*
c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x*e + (3*b^2 + 4*a*c)*d - sqrt(1/2)*
((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^
3 - 64*a^3*b*c^4)*e/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^
3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*e^(-2)/(
b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) + sqrt(1/2)*((b^2*c -
 4*a*c^2)*x^4*e^5 + 4*(b^2*c - 4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2
*c - 4*a*c^2)*d^2)*x^2*e^3 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d
)*x*e^2 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e
)*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c
^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*e^(-2)/(b^6
*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x*e +
 (3*b^2 + 4*a*c)*d + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*e - 2*(b^7*c
 - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e/sqrt(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b
```

```
^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5))*e^(-2)/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a
^3*c^4))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*c - 4*a*c^2)*d*x^
3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3 + 2*(2*(b^2*c - 4
*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4
*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*
c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5))*e^(-2)/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*
c^4))*log((3*b^2 + 4*a*c)*x*e + (3*b^2 + 4*a*c)*d - sqrt(1/2)*((b^4 - 8*a*b
^2*c + 16*a^2*c^2)*e - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*
c^4)*e/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b
^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*e^(-2)/(b^6*c - 12*a*
b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) + 4*a*d)/((b^2*c - 4*a*c^2)*x^4*e^
5 + 4*(b^2*c - 4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^
2)*x^2*e^3 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*
c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 573 vs. $2(243) = 486$.

time = 11.68, size = 573, normalized size = 2.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*d - b*d**3 - 3*b*d*e**2*x**2 - b*e**3*x**3 + x*(-2*a*e - 3*b*d**2*e))
/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*
e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**
2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*
b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2
 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + RootSum(_t**4*(1048576*a**6*c**7*
e**4 - 1572864*a**5*b**2*c**6*e**4 + 983040*a**4*b**4*c**5*e**4 - 327680*a*
*3*b**6*c**4*e**4 + 61440*a**2*b**8*c**3*e**4 - 6144*a*b**10*c**2*e**4 + 25
6*b**12*c*e**4) + _t**2*(-12288*a**4*b*c**4*e**2 + 8192*a**3*b**3*c**3*e**2
 - 1536*a**2*b**5*c**2*e**2 + 16*b**9*e**2) + 16*a**3*c**2 + 24*a**2*b**2*c
 + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 12288*_t
**3*a**2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**7*c*e*
*3 + 64*_t*a**2*c**2*e - 128*_t*a*b**2*c*e - 4*_t*b**4*e + 4*a*c*d + 3*b**2
*d)/(4*a*c*e + 3*b**2*e))))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1304 vs. $2(229) = 458$.

time = 3.63, size = 1304, normalized size = 4.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))*b*d*e + b*d^2 - 2*a)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(
b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (
6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*b*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))*b*d*e + b*d^2 - 2*a)*log(d*e^(-1) + x - sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqr
t(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3
*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*e + b*d^2 - 2*a)*log(d*e^(-1) +
x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1)
 + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(
d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d
*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*e^2 - 2*(d*e^(-1) - sqrt(1
/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*e + b*d^2 - 2*a)*l
og(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^2 - 4*a*c)
+ 1/2*(b*x^3*e^3 + 3*b*d*x^2*e^2 + 3*b*d^2*x*e + b*d^3 + 2*a*x*e + 2*a*d)/(
(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*
e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

Mupad [B]
time = 4.76, size = 2500, normalized size = 9.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(-(((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^
13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c
```

^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^
14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*
a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c
^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^
2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 614
4*a^5*b^2*c^6*e^2)))^(1/2) - (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a
^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*
a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^
10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*
e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11
+ 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
+ (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c
^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2
- 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i + ((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3
*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*
c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b
^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^
14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2)
- 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*
a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^
2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + (2048*a^4*c^5*e^
12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^
9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^
2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b
^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^
3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c
^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(
1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 +
4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c
^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i)/(((((64*b
^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b
^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2
*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b
*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b
^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^
3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8
*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e
^2)))^(1/2) - (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12
 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*
c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 5

12*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11 - 4*b^6*c*d*e^11 + 8*a*b^4*c^2*d*e^11)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12 + 8*a^2*c^3*e^12 + 2*a*b^2*c^2*e^12))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - ((((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) + (2048*a^4*c^5*e^12 - 32*a*b^6*c^2*e^12 + 384*a^2*b^4*c^3*e^12 - 1536*a^3*b^2*c^4*e^12)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - ...

3.622 $\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=97

$$\frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

```
[Out] 1/2*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-b*arctanh(
(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1156, 1128, 652, 632, 212}

$$\frac{2a+b(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (2*a + b*(d + e*x)^2)/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) - (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3
/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}$$
$$= \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, \right.}{2\left(b^2-4ac\right.}$$
$$= \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx\right.}{(b^2-4}$$
$$= \frac{2a+b(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right.}{\left(b^2-4ac\right)^{3/2}e}$$

Mathematica [A]
time = 0.09, size = 100, normalized size = 1.03

$$\frac{\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} - \frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] ((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x)^2))
) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(
3/2))/(2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.19, size = 276, normalized size = 2.85

method	result
default	$\frac{-\frac{x^2be}{2(4ac-b^2)}-\frac{xbd}{4ac-b^2}-\frac{d^2b+2a}{2e(4ac-b^2)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3\right.\right.}\right.}{}$
risch	$\frac{-\frac{x^2be}{2(4ac-b^2)}-\frac{xbd}{4ac-b^2}-\frac{d^2b+2a}{2e(4ac-b^2)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a}+\frac{b\ln\left(\left(-\left(-4ac+b^2\right)^{\frac{3}{2}}e^2+4abc\,e^2-b^3e^2\right)x^2+\left(-2\left(-4ac+b^2\right)\right.\right.}{2(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2/(4*a*c-b^2)*x^2*b*e-1/(4*a*c-b^2)*x*b*d-1/2/e*(b*d^2+2*a)/(4*a*c-b^2)
)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*
d*e*x+b*d^2+a)+1/2*b/(4*a*c-b^2)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*c*d*e
^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c
*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] b*integrate((x*e + d)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2
*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(b^2 - 4*a*c) + 1/2*(b
*x^2*e^2 + 2*b*d*x*e + b*d^2 + 2*a)/((b^2*c*e - 4*a*c^2*e)*d^4 + 4*(b^2*c*e
^4 - 4*a*c^2*e^4)*d*x^3 + (b^2*c*e^5 - 4*a*c^2*e^5)*x^4 + a*b^2*e - 4*a^2*c
*e + (b^3*e - 4*a*b*c*e)*d^2 + (b^3*e^3 - 4*a*b*c*e^3 + 6*(b^2*c*e^3 - 4*a*
c^2*e^3)*d^2)*x^2 + 2*(2*(b^2*c*e^2 - 4*a*c^2*e^2)*d^3 + (b^3*e^2 - 4*a*b*c
*e^2)*d)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 440 vs. 2(93) = 186.
time = 0.41, size = 1007, normalized size = 10.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^3 - 4*a*b*c)*x^2*e^2 + 2*(b^3 - 4*a*b*c)*d*x*e + 2*a*b^2 - 8*a^2*c
 + (b^3 - 4*a*b*c)*d^2 - (b*c*x^4*e^4 + 4*b*c*d*x^3*e^3 + b*c*d^4 + b^2*d^2
 + (6*b*c*d^2 + b^2)*x^2*e^2 + 2*(2*b*c*d^3 + b^2*d)*x*e + a*b)*sqrt(b^2 -
4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*
c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2 - 2*a*c + (2*c*x^2
*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e
^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a)))/(
(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*e^5 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*d*x^3*e^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2
 + 16*a^2*c^3)*d^2)*x^2*e^3 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*e^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*
c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*x^2*e^2 + 2*(b^3 - 4*a*b*c)*d*x*e + 2*a*
b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*d^2 - 2*(b*c*x^4*e^4 + 4*b*c*d*x^3*e^3 + b*
c*d^4 + b^2*d^2 + (6*b*c*d^2 + b^2)*x^2*e^2 + 2*(2*b*c*d^3 + b^2*d)*x*e + a
*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*e^5
+ 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^3*e^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*x^2*e^3 + 2*(2*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*e^2
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^
4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(80) = 160$.
time = 2.77, size = 495, normalized size = 5.10

$$\frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2+2bcd^2}{2bce^2}\right)}{2e}-\frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2+2bcd^2}{2bce^2}\right)}{2e}+\frac{-2a-bd^2-2bdex-be^2x^2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2cde^4)+x^2\cdot(8abce^3+48ac^2d^2e^3-2b^3e^3-12b^2cd^2e^3)+x\left(16abcde^2+32ac^2d^3e^2-4b^3de^2-8b^2cd^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*sqrt(-1/
(4*a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(
4*a*c - b**2)**3) + b**2 + 2*b*c*d**2)/(2*b*c*e**2))/(2*e) - b*sqrt(-1/(4*a
*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)*
*3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c - b**2)**
3) + b**2 + 2*b*c*d**2)/(2*b*c*e**2))/(2*e) + (-2*a - b*d**2 - 2*b*d*e*x -
b*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e -
2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x*
*3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d*
*2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c*
*2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

Giac [**A**]

time = 4.75, size = 171, normalized size = 1.76

$$\frac{b \arctan\left(\frac{2\,cd^2+2\,(x^2e+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}+\frac{bd^2+(x^2e+2\,dx)be+2\,a}{2\,\left(cd^4+2\,(x^2e+2\,dx)cd^2e+(x^2e+2\,dx)^2ce^2+bd^2+(x^2e+2\,dx)be+a\right)(b^2e-4\,ace)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] b*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-1)/(
(b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) + 1/2*(b*d^2 + (x^2*e + 2*d*x)*b*e + 2*a)
/((c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x
^2*e + 2*d*x)*b*e + a)*(b^2*e - 4*a*c*e))
```

Mupad [B]

time = 1.77, size = 427, normalized size = 4.40

$$\frac{b\,\mathrm{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^4\,\left(x\,\left(\frac{b^3\,\left(2\,b^3\,c^2\,d\,e^9-8\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}-\frac{2\,b^2\,c^2\,d\,e^7}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}\right)+x^2\,\left(\frac{b^3\,\left(2\,b^3\,c^2\,e^{10}-8\,a\,b\,c^3\,e^{10}\right)}{2\,a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}-\frac{b^2\,c^2\,e^8}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}\right)-\frac{b^3\,\left(16\,a^2\,c^3\,e^8-4\,a\,b^2\,c^2\,e^8+8\,a\,b\,c^3\,d^2\,e^8-2\,b^3\,c^2\,d^2\,e^8\right)}{2\,a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}-\frac{b^2\,c^2\,d^2\,e^6}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}\right)}{2\,b^2\,c^2\,e^6}\right)}{e\,{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{\frac{b\,d^2+2\,a}{2\,e\,\left(4\,a\,c-b^2\right)}+\frac{b\,e\,x^2}{2\,\left(4\,a\,c-b^2\right)}+\frac{b\,d\,x}{4\,a\,c-b^2}}{a+x^2\,\left(6\,c\,d^2\,e^2+b\,e^2\right)+b\,d^2+c\,d^4+x\,\left(4\,c\,e\,d^3+2\,b\,e\,d\right)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (b*atan(((4*a*c - b^2)^4*(x*((b^3*(2*b^3*c^2*d*e^9 - 8*a*b*c^3*d*e^9))/(a*e
^2*(4*a*c - b^2)^(11/2)) - (2*b^2*c^2*d*e^7)/(a*(4*a*c - b^2)^(7/2))) + x^2
*((b^3*(2*b^3*c^2*e^10 - 8*a*b*c^3*e^10))/(2*a*e^2*(4*a*c - b^2)^(11/2)) -
(b^2*c^2*e^8)/(a*(4*a*c - b^2)^(7/2))) - (b^3*(16*a^2*c^3*e^8 - 4*a*b^2*c^2
*e^8 - 2*b^3*c^2*d^2*e^8 + 8*a*b*c^3*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(11/2
)) - (b^2*c^2*d^2*e^6)/(a*(4*a*c - b^2)^(7/2))))/(2*b^2*c^2*e^6)))/(e*(4*a*
c - b^2)^(3/2)) - ((2*a + b*d^2)/(2*e*(4*a*c - b^2)) + (b*e*x^2)/(2*(4*a*c
- b^2)) + (b*d*x)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c
*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

3.623 $\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} dx$

Optimal. Leaf size=254

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\;e}-\frac{\sqrt{c}\,(\cdots}{\cdots}$$

```
[Out] -1/2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1
/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*
b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(
1/2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/
2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1156, 1133, 1180, 211}

$$\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*((d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*S
qrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*
a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
```

```
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\text{Subst}\left(\int \frac{b-2cx^2}{a+bx^2+cx^4}\right.}{2\left(b^2-4a\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(c\left(2b-\sqrt{b^2-4a}\right.\right.}{}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4}\right.}{\sqrt{2}\left(b^2-\right.}$$

Mathematica [A]
time = 0.64, size = 247, normalized size = 0.97

$$-\frac{\frac{b(d+ex)+2c(d+ex)^3}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcT
an[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 319, normalized size = 1.26

method	result
default	$\frac{\frac{c e^2 x^3}{4ac-b^2}+\frac{3x^2 cde}{4ac-b^2}+\frac{(6c d^2+b)x}{8ac-2b^2}+\frac{d(2c d^2+b)}{2e(4ac-b^2)}}{c e^4x^4+4cd e^3x^3+6c d^2e^2x^2+4c d^3ex+b e^2x^2+d^4c+2debx+d^2b+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2d\right.\right.}}{}$
risch	$\frac{\frac{c e^2 x^3}{4ac-b^2}+\frac{3x^2 cde}{4ac-b^2}+\frac{(6c d^2+b)x}{8ac-2b^2}+\frac{d(2c d^2+b)}{2e(4ac-b^2)}}{c e^4x^4+4cd e^3x^3+6c d^2e^2x^2+4c d^3ex+b e^2x^2+d^4c+2debx+d^2b+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2d\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c*e^2/(4*a*c-b^2)*x^3+3/(4*a*c-b^2)*x^2*c*d*e+1/2*(6*c*d^2+b)/(4*a*c-b^2)*
x+1/2*d/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2
+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((2*_R
^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*
c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2
+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^3*e^3 + 6*c*d*x^2*e^2 + 2*c*d^3 + b*d + (6*c*d^2*e + b*e)*x)/((
b^2*c*e - 4*a*c^2*e)*d^4 + 4*(b^2*c*e^4 - 4*a*c^2*e^4)*d*x^3 + (b^2*c*e^5 -
 4*a*c^2*e^5)*x^4 + a*b^2*e - 4*a^2*c*e + (b^3*e - 4*a*b*c*e)*d^2 + (b^3*e^
3 - 4*a*b*c*e^3 + 6*(b^2*c*e^3 - 4*a*c^2*e^3)*d^2)*x^2 + 2*(2*(b^2*c*e^2 -
4*a*c^2*e^2)*d^3 + (b^3*e^2 - 4*a*b*c*e^2)*d)*x) - 1/2*integrate((2*c*x^2*e
```

```
^2 + 4*c*d*x*e + 2*c*d^2 - b)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 +
(6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(b^2 - 4*a*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2358 vs. $2(216) = 432$.
time = 0.43, size = 2358, normalized size = 9.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*x^3*e^3 + 12*c*d*x^2*e^2 + 4*c*d^3 + 2*(6*c*d^2 + b)*x*e + sqrt(1
/2)*((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*c - 4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a
*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3
 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*d^2)*e)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c
^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)*e^(-2)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c
 + 4*a*c^2)*x*e + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c +
 16*a^2*b*c^2)*e - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e/
sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12
*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*e^(-2)/(a*b^6 - 12*a^2*b^4*c
 + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4*e^5 +
4*(b^2*c - 4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x
^2*e^3 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c -
4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-(b^3 + 12*a*
b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*e^(-2)/(a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x*e + (3*b^2*c + 4*a*
c^2)*d - 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e - (a*b^8 - 8*a^2
*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
 64*a^5*c^3))*e^(-2)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)))
 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*c - 4*a*c^2)*d*x^3*e^4 + (
b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3 + 2*(2*(b^2*c - 4*a*c^2)*d
^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c +
(b^3 - 4*a*b*c)*d^2)*e)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*
a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64
*a^5*c^3))*e^(-2)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log
((3*b^2*c + 4*a*c^2)*x*e + (3*b^2*c + 4*a*c^2)*d + 1/2*sqrt(1/2)*((b^5 - 8*
a*b^3*c + 16*a^2*b*c^2)*e + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^
5*c^4)*e/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-
```

```
(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt
(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*e^(-2)/(a*b^6 - 12*
a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x
^4*e^5 + 4*(b^2*c - 4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^
2)*d^2)*x^2*e^3 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + (
(b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-(b^
3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*e^(-2)/(a*b^6 - 12*a^2
*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x*e + (3*b^2
*c + 4*a*c^2)*d - 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*e + (a*b^
8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*e/sqrt(a^2*b^6 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2
*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*
b^2*c^2 - 64*a^5*c^3))*e^(-2)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a
^4*c^3))) + 2*b*d)/((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*c - 4*a*c^2)*d*x^3*e
^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3 + 2*(2*(b^2*c - 4*a*
c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^
2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1312 vs. 2(216) = 432.
time = 4.14, size = 1312, normalized size = 5.17

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/4*((2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*c*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))*c*d*e + 2*c*d^2 - b)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-
(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e +
(6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
```

```
)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)*e^(-4)/c))^2*c*e^2 - 4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b
^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*e + 2*c*d^2 - b)*log(d*e^(-1) + x - sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*
d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*e + 2*c*d^2 - b)*log(d*e^(-
1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e
^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4
- 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^
2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) - sqrt(1/
2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*e^2 - 4*(d*e^(-1) -
 sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*e + 2*c*d^2
 - b)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^
(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^2 -
4*a*c) - 1/2*(2*c*x^3*e^3 + 6*c*d*x^2*e^2 + 6*c*d^2*x*e + 2*c*d^3 + b*x*e +
 b*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 +
 b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

Mupad [B]
time = 3.95, size = 2500, normalized size = 9.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(((((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 5
12*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 24
0*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*
b^2*c^5*e^2)))^(1/2)*((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4
*d*e^11)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c
^2*d*e^13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4
*d*e^13 - 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14
+ 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^
9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
```

*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (
8*b^7*c^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e
^12)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2
)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a
*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 12
80*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) -
(x*(4*a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i + (
((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*
b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b
^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5
e^2)))^(1/2)((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11
)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c^2*d*e^
13 - 512*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13
- 8192*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a
^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2
) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2
+ 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^
6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) + (8*b^7*c
^2*e^12 - 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4
*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2)^9)^(1
/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e
^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*
b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (x*(4*
a*c^4*e^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i)/((((-(4*a
*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3
)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*
e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))
^(1/2)*((64*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11)/(4*(b
^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c^2*d*e^13 - 51
2*a*b^7*c^3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192
*a^3*b^3*c^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) +
(x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*
c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9
+ 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096
*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e
^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) + (8*b^7*c^2*e^12
- 96*a*b^5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c^4*e^12)/(4*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b
^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 40
96*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3
*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (x*(4*a*c^4*e
^12 - 5*b^2*c^3*e^12))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (((-(4*a*c - b^2)^
9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280

*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2)*((6
4*a^2*c^5*d*e^11 + 20*b^4*c^3*d*e^11 - 96*a*b^2*c^4*d*e^11)/(4*(b^6 - 64*a^
3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((32*b^9*c^2*d*e^13 - 512*a*b^7*c^
3*d*e^13 + 8192*a^4*b*c^6*d*e^13 + 3072*a^2*b^5*c^4*d*e^13 - 8192*a^3*b^3*c
^5*d*e^13)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(8*b^7
*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))
/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4
*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12*e^2 + 4096*a^7*c^6*e
^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*
a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2)))^(1/2) - (8*b^7*c^2*e^12 - 96*a*b^
5*c^3*e^12 - 512*a^3*b*c^5*e^12 + 384*a^2*b^3*c...

3.624 $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=98

$$\frac{-b-2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

```
[Out] 1/2*(-b-2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+2*c*arcta
nh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Rubi [A]
time = 0.08, antiderivative size = 96, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1156, 1121, 628, 632, 212}

$$\frac{2c\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{b+2c(d+ex)^2}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)) + (2*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*
c)^(3/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{c\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\right.}{(b^2-4}$$

$$= -\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{(2c)\text{Subst}\left(\int \frac{1}{b^2-4ac}\right.}{(b}$$

$$= -\frac{b+2c(d+ex)^2}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{2c\tanh^{-1}\left(\frac{b+2c(d+}{\sqrt{b^2-}}\right.}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A]
time = 0.09, size = 98, normalized size = 1.00

$$-\frac{\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}+\frac{4c\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2\left(b^2-4ac\right)e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*ArcT
an[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/((b^2 - 4
*a*c)*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.08, size = 270, normalized size = 2.76

method	result
default	$\frac{\frac{c x^2 e}{4ac-b^2}+\frac{2xcd}{4ac-b^2}+\frac{2c d^2+b}{2e\left(4ac-b^2\right)}}{c e^4x^4+4cd e^3x^3+6c d^2e^2x^2+4c d^3ex+b e^2x^2+d^4c+2debx+d^2b+a}+\frac{c\left(\sum_{\_R=\text{RootOf}\left(e^4c\_Z^4+4d e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec\right.\right.}\right.}{}$
risch	$\frac{\frac{c x^2 e}{4ac-b^2}+\frac{2xcd}{4ac-b^2}+\frac{2c d^2+b}{2e\left(4ac-b^2\right)}}{c e^4x^4+4cd e^3x^3+6c d^2e^2x^2+4c d^3ex+b e^2x^2+d^4c+2debx+d^2b+a}+\frac{c\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}e^2+4abce^2-b^3e^2\right)x^2+\left(2\left(-4ac+b^2\right)^{\frac{3}{2}}de\right.\right.}{(-4a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (c/(4*a*c-b^2)*x^2*e+2/(4*a*c-b^2)*x*c*d+1/2/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*
e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x
+b*d^2+a)+c/(4*a*c-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c
*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -2*c*integrate((x*e + d)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*
d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(b^2 - 4*a*c) - 1/2
*(2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)/((b^2*c*e - 4*a*c^2*e)*d^4 + 4*(b^
2*c*e^4 - 4*a*c^2*e^4)*d*x^3 + (b^2*c*e^5 - 4*a*c^2*e^5)*x^4 + a*b^2*e - 4*
a^2*c*e + (b^3*e - 4*a*b*c*e)*d^2 + (b^3*e^3 - 4*a*b*c*e^3 + 6*(b^2*c*e^3 -
 4*a*c^2*e^3)*d^2)*x^2 + 2*(2*(b^2*c*e^2 - 4*a*c^2*e^2)*d^3 + (b^3*e^2 - 4*
a*b*c*e^2)*d)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. 2(92) = 184.
time = 0.36, size = 1028, normalized size = 10.49

$$\left[-\frac{2\left(b^2c-4ac^2\right)x^2e^2+4\left(b^2c-4ac^2\right)dxe+b^3-4abc+2\left(b^2c-4ac^2\right)d^2+2\left(c^2x^4e^4+4c^2dx^3e^3+c^2d^4+bcd^2+\left(6c^2d^2+bc\right)x^2e^2+2\left(2c^2d^3+bcd\right)xe+ac\right)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4e^4+8c^2dx^3e^3+2c^2d^4+2bcd^2+2\left(6c^2d^2+bc\right)x^2e^2+4\left(2c^2d^3+bcd\right)xe+b^2-2ac-\left(2cx^2e^2+4cdxe+2cd^2+b\right)\sqrt{b^2-4ac}}{cx^4e^4+4cdx^3e^3+cd^4+\left(6cd^2+b\right)x^2e^2+bd^2+2\left(2cd^3+bd\right)xe+a}\right)}{2\left(\left(b^4c-8ab^2c^2+16a^2c^3\right)x^4e^5+4\left(b^4c-8ab^2c^2+16a^2c^3\right)dx^3e^4+\left(b^5-8ab^3c+16a^2bc^2+6\left(b^4c-8ab^2c^2+16a^2c^3\right)d^2\right)x^2e^3+2\left(2\left(b^4c-8ab^2c^2+16a^2c^3\right)d^3+\left(b^5-8ab^3c+16a^2bc^2\right)d\right)xe^2+\left(ab^4-8a^2b^2c+16a^3c^2+\left(b^4c-8ab^2c^2+16a^2c^3\right)d^4+\left(b^5-8ab^3c+16a^2bc^2\right)d^2\right)e\right)}, -\frac{2\left(b^2c-4ac^2\right)x^2e^2+4\left(b^2c-4ac^2\right)dxe+b^3-4abc+2\left(b^2c-4ac^2\right)d^2-4\left(c^2x^4e^4+4c^2dx^3e^3+c^2d^4+bcd^2+\left(6c^2d^2+bc\right)x^2e^2+2\left(2c^2d^3+bcd\right)xe+ac\right)\sqrt{-b^2+4ac}\arctan\left(-\frac{\left(2cx^2e^2+4cdxe+2cd^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{2\left(\left(b^4c-8ab^2c^2+16a^2c^3\right)x^4e^5+4\left(b^4c-8ab^2c^2+16a^2c^3\right)dx^3e^4+\left(b^5-8ab^3c+16a^2bc^2+6\left(b^4c-8ab^2c^2+16a^2c^3\right)d^2\right)x^2e^3+2\left(2\left(b^4c-8ab^2c^2+16a^2c^3\right)d^3+\left(b^5-8ab^3c+16a^2bc^2\right)d\right)xe^2+\left(ab^4-8a^2b^2c+16a^3c^2+\left(b^4c-8ab^2c^2+16a^2c^3\right)d^4+\left(b^5-8ab^3c+16a^2bc^2\right)d^2\right)e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*x^2*e^2 + 4*(b^2*c - 4*a*c^2)*d*x*e + b^3 - 4*a*
b*c + 2*(b^2*c - 4*a*c^2)*d^2 + 2*(c^2*x^4*e^4 + 4*c^2*d*x^3*e^3 + c^2*d^4
+ b*c*d^2 + (6*c^2*d^2 + b*c)*x^2*e^2 + 2*(2*c^2*d^3 + b*c*d)*x*e + a*c)*sq
rt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4 + 2*b*c*d^
2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2 - 2*a*c -
 (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4*e^4 + 4*
c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e
 + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*e^5 + 4*(b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)*d*x^3*e^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^2)*x^2*e^3 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*e^2 + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c +
16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*x^2*e^2 + 4*(b^2*c - 4*a*c
^2)*d*x*e + b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*d^2 - 4*(c^2*x^4*e^4 + 4*c^
2*d*x^3*e^3 + c^2*d^4 + b*c*d^2 + (6*c^2*d^2 + b*c)*x^2*e^2 + 2*(2*c^2*d^3
+ b*c*d)*x*e + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 + 4*c*d*x*e + 2
*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*x^4*e^5 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^3*e^4 + (b^5 - 8
*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*x^2*e^3
 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*d)*x*e^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(83) = 166$.
time = 2.68, size = 495, normalized size = 5.05

$$-\frac{c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bc+2c^2d^2}{2c^2e^2}\right)}{e}+\frac{c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+bc+2c^2d^2}{2c^2e^2}\right)}{e}+\frac{b+2cd^2+4cdex+2ce^2x^2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot\left(8ac^2e^5-2b^2ce^5\right)+x^3\cdot\left(32ac^2de^4-8b^2cde^4\right)+x^2\cdot\left(8abce^3+48ac^2d^2e^3-2b^3e^3-12b^2cd^2e^3\right)+x\left(16abcde^2+32ac^2d^3e^2-4b^3de^2-8b^2cd^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*sqrt(-1/(
4*a*c - b**2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(
-1/(4*a*c - b**2)**3) + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + c*sqrt(-1/(4*
a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*sqrt(-1/(4*a*c - b**2)**
3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*sqrt(-1/(4*a*c - b**
2)**3) + b*c + 2*c**2*d**2)/(2*c**2*e**2))/e + (b + 2*c*d**2 + 4*c*d*e*x +
2*c*e**2*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e
- 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b**2*c*e**5) +
x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*
d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*
c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

Giac [**A**]

time = 3.97, size = 172, normalized size = 1.76

$$-\frac{2\,c\arctan\left(\frac{2\,cd^2+2\,(x^2e+2\,dx)ce+b}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,cd^2+2\,(x^2e+2\,dx)ce+b}{2\left(cd^4+2\,(x^2e+2\,dx)cd^2e+(x^2e+2\,dx)^2ce^2+bd^2+(x^2e+2\,dx)be+a\right)(b^2e-4\,ace)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -2*c*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-1
)/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e
 + b)/((c*d^4 + 2*(x^2*e + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2
 + (x^2*e + 2*d*x)*b*e + a)*(b^2*e - 4*a*c*e))
```

Mupad [B]
time = 1.72, size = 417, normalized size = 4.26

$$\frac{\frac{2\,c\,d^2+b}{2\,e\,(4\,a\,c-b^2)}+\frac{c\,e\,x^2}{4\,a\,c-b^2}+\frac{2\,c\,d\,x}{4\,a\,c-b^2}}{a+x^2\,\left(6\,c\,d^2\,e^2+b\,e^2\right)+b\,d^2+c\,d^4+x\,\left(4\,c\,e\,d^3+2\,b\,e\,d\right)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}+\frac{2\,c\,\mathrm{atan}\left(\frac{{(4\,a\,c-b^2)}^4\,\left(x\,\left(\frac{8\,c^4\,d\,e^7}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}-\frac{8\,b\,c^2\,\left(b^3\,c^2\,d\,e^9-4\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}\right)+x^2\,\left(\frac{4\,c^4\,e^8}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}-\frac{4\,b\,c^2\,\left(b^3\,c^2\,e^{10}-4\,a\,b\,c^3\,e^{10}\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}\right)+\frac{4\,c^4\,d^2\,e^6}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{4\,b\,c^2\,\left(8\,a^2\,c^3\,e^8-2\,a\,b^2\,c^2\,e^8+4\,a\,b\,c^3\,d^2\,e^8-b^3\,c^2\,d^2\,e^8\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}\right)}{8\,c^4\,e^6}\right)}{e\,{(4\,a\,c-b^2)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] ((b + 2*c*d^2)/(2*e*(4*a*c - b^2)) + (c*e*x^2)/(4*a*c - b^2) + (2*c*d*x)/(4
*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e +
4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*atan(((4*a*c - b^2)^4*(x*((8
*c^4*d*e^7)/(a*(4*a*c - b^2)^(7/2)) - (8*b*c^2*(b^3*c^2*d*e^9 - 4*a*b*c^3*d
*e^9))/(a*e^2*(4*a*c - b^2)^(11/2))) + x^2*((4*c^4*e^8)/(a*(4*a*c - b^2)^(7
/2)) - (4*b*c^2*(b^3*c^2*e^10 - 4*a*b*c^3*e^10))/(a*e^2*(4*a*c - b^2)^(11/2
))) + (4*c^4*d^2*e^6)/(a*(4*a*c - b^2)^(7/2)) + (4*b*c^2*(8*a^2*c^3*e^8 - 2
*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8 + 4*a*b*c^3*d^2*e^8))/(a*e^2*(4*a*c - b^2)
^(11/2))))/(8*c^4*e^6)))/(e*(4*a*c - b^2)^(3/2))
```

3.625 $\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=299

$$\frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e}$$

```
[Out] 1/2*(d/e+x)*(b^2-2*a*c+b*c*e^2*(d/e+x)^2)/a/(-4*a*c+b^2)/(a+b*e^2*(d/e+x)^2
+c*e^4*(d/e+x)^4)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))
^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)/e*2^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4*a*c+b^2)^(1/2))/a/(-4*a*
c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1120, 1106, 1180, 211}

$$\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,ae\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,ae\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(2*a*(b^2 - 4*a*c)*(a + b*e
^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*
(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
```

```
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(a+be^2x^2+ce^4x^4\right)^2}\,dx, x, \frac{d}{e}+x\right)$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} - \frac{\text{Subst}\left(\int \frac{b^2e^4-2ace}{\cdots}\right.}{\cdots}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} - \frac{\left(c\left(b^2-12ac-b\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{2a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+\cdots\right.}{2\sqrt{2}\,a\cdots}$$

Mathematica [A]

time = 0.58, size = 271, normalized size = 0.91

$$\frac{\frac{2(d+ex)\left(b^2-2ac+bc(d+ex)^2\right)}{\left(b^2-4ac\right)\left(a+(d+ex)^2\left(b+c(d+ex)^2\right)\right)}+\frac{\sqrt{2}\,\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\,\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-2),x]
```

```
[Out] ((2*(d + e*x)*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x
)^2*(b + c*(d + e*x)^2))) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b
^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 1
2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a*
e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 364, normalized size = 1.22

method	result
default	$\frac{-\frac{bc\,e^2x^3}{2a\left(4ac-b^2\right)}-\frac{3dbce\,x^2}{2a\left(4ac-b^2\right)}+\frac{\left(-3bc\,d^2+2ac-b^2\right)x}{2a\left(4ac-b^2\right)}+\frac{d\left(-bc\,d^2+2ac-b^2\right)}{2ea\left(4ac-b^2\right)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3e\right.\right.}}{}$
risch	$\frac{-\frac{bc\,e^2x^3}{2a\left(4ac-b^2\right)}-\frac{3dbce\,x^2}{2a\left(4ac-b^2\right)}+\frac{\left(-3bc\,d^2+2ac-b^2\right)x}{2a\left(4ac-b^2\right)}+\frac{d\left(-bc\,d^2+2ac-b^2\right)}{2ea\left(4ac-b^2\right)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3e\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*b*c*e^2/a/(4*a*c-b^2)*x^3-3/2*d*b*c*e/a/(4*a*c-b^2)*x^2+1/2*(-3*b*c*d
^2+2*a*c-b^2)/a/(4*a*c-b^2)*x+1/2*d/e*(-b*c*d^2+2*a*c-b^2)/a/(4*a*c-b^2))/(
c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e
*x+b*d^2+a)+1/4/a/(4*a*c-b^2)/e*sum((-_R^2*b*c*e^2-2*_R*b*c*d*e-b*c*d^2+6*a
*c-b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-
_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e
+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3*e^3 + 3*b*c*d*x^2*e^2 + b*c*d^3 + (b^2 - 2*a*c)*d + (3*b*c*d^2
*e + b^2*e - 2*a*c*e)*x)/((a*b^2*c*e - 4*a^2*c^2*e)*d^4 + 4*(a*b^2*c*e^4 -
4*a^2*c^2*e^4)*d*x^3 + (a*b^2*c*e^5 - 4*a^2*c^2*e^5)*x^4 + a^2*b^2*e - 4*a^
```

```
3*c*e + (a*b^3*e - 4*a^2*b*c*e)*d^2 + (a*b^3*e^3 - 4*a^2*b*c*e^3 + 6*(a*b^2
*c*e^3 - 4*a^2*c^2*e^3)*d^2)*x^2 + 2*(2*(a*b^2*c*e^2 - 4*a^2*c^2*e^2)*d^3 +
 (a*b^3*e^2 - 4*a^2*b*c*e^2)*d)*x) + 1/2*integrate((b*c*x^2*e^2 + 2*b*c*d*x
*e + b*c*d^2 + b^2 - 6*a*c)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6
*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(a*b^2 - 4*a^2*c
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3148 vs. $2(246) = 492$.

time = 0.43, size = 3148, normalized size = 10.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3*e^3 + 6*b*c*d*x^2*e^2 + 2*b*c*d^3 + 2*(3*b*c*d^2 + b^2 - 2*a
*c)*x*e - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4*e^5 + 4*(a*b^2*c - 4*a^2*c^2
)*d*x^3*e^4 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*x^2*e^3 + 2
*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*x*e^2 + ((a*b^2*c -
4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5
 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 6
4*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))*e^(-2)/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x*e + (5*b^4
*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d + 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))*e
 - (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*e^(-2)/(a^3*b^6 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4*e^5 + 4
*(a*b^2*c - 4*a^2*c^2)*d*x^3*e^4 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*
c^2)*d^2)*x^2*e^3 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)
*x*e^2 + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*
c)*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*e^(-2)/(a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*
a^2*c^4)*x*e + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*d - 1/2*sqrt(1/2)*(
(a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4
)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2
*c^2 - 64*a^9*c^3))*e - (b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c
^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12
```

```
*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c
^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*e^(-2)/(a^3*b^
6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4
*a^2*c^2)*x^4*e^5 + 4*(a*b^2*c - 4*a^2*c^2)*d*x^3*e^4 + (a*b^3 - 4*a^2*b*c
+ 6*(a*b^2*c - 4*a^2*c^2)*d^2)*x^2*e^3 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (
a*b^3 - 4*a^2*b*c)*d)*x*e^2 + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*
c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a
^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c
 + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*e^(
-2)/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2
- 81*a*b^2*c^3 + 324*a^2*c^4)*x*e + (5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4
)*d + 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^
3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))*e + (b^8 - 23*a*b^6*c + 190*a^2*b^
4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*
b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 -
18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*
c^3)))*e^(-2)/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sq
rt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4*e^5 + 4*(a*b^2*c - 4*a^2*c^2)*d*x^3*e^4
+ (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2)*d^2)*x^2*e^3 + 2*(2*(a*b^2*c
 - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*x*e^2 + ((a*b^2*c - 4*a^2*c^2)*d
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*d^2)*e)*sqrt(-(b^5 - 15*a*b^3*
c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*s
qrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))*e^(-2)/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x*e + (5*b^4*c^2 - 81*a*
b^2*c^3 + 324*a^2*c^4)*d - 1/2*sqrt(1/2)*((a^3*b^9 - 20*a^4*b^7*c + 144*a^5
*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2
*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))*e + (b^8 - 23
*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4)*e)*sqrt(-(b^5 -
 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*
a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*
a^8*b^2*c^2 - 64*a^9*c^3)))*e^(-2)/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2
 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*d)/((a*b^2*c - 4*a^2*c^2)*x^4*e^5 + 4*(a
*b^2*c - 4*a^2*c^2)*d*x^3*e^4 + (a*b^3 - 4*a^2*b*c + 6*(a*b^2*c - 4*a^2*c^2
)*d^2)*x^2*e^3 + 2*(2*(a*b^2*c - 4*a^2*c^2)*d^3 + (a*b^3 - 4*a^2*b*c)*d)*x*
e^2 + ((a*b^2*c - 4*a^2*c^2)*d^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*
d^2)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1357 vs. $2(246) = 492$.

time = 4.48, size = 1357, normalized size = 4.54

$$-\frac{\frac{\left(\left(de^{(-1)}+\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2+\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)^2bce^2-2\left(de^{(-1)}+\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2+\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)bcde+bcd^2+b^2-6ac\right)\log\left(de^{(-1)}+x+\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2+\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)}{2\left(de^{(-1)}+\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2+\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)^3ce^4-6\left(de^{(-1)}+\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2+\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)^2cde^3-2cd^3e-bde+\left(6cd^2e^2+be^2\right)\left(de^{(-1)}+\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2+\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)}+\frac{\left(\left(de^{(-1)}-\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2+\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)^2bce^2-2\left(de^{(-1)}-\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2+\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)bcde+bcd^2+b^2-6ac\right)\log\left(de^{(-1)}+x-\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2+\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)}{2\left(de^{(-1)}-\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2+\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)^3ce^4-6\left(de^{(-1)}-\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2+\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)^2cde^3-2cd^3e-bde+\left(6cd^2e^2+be^2\right)\left(de^{(-1)}-\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2+\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)}+\frac{\left(\left(de^{(-1)}+\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2-\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)^2bce^2-2\left(de^{(-1)}+\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2-\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)bcde+bcd^2+b^2-6ac\right)\log\left(de^{(-1)}+x+\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2-\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)}{2\left(de^{(-1)}+\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2-\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)^3ce^4-6\left(de^{(-1)}+\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2-\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)^2cde^3-2cd^3e-bde+\left(6cd^2e^2+be^2\right)\left(de^{(-1)}+\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2-\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)}+\frac{\left(\left(de^{(-1)}-\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2-\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)^2bce^2-2\left(de^{(-1)}-\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2-\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)bcde+bcd^2+b^2-6ac\right)\log\left(de^{(-1)}+x-\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2-\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)}{2\left(de^{(-1)}-\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2-\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)^3ce^4-6\left(de^{(-1)}-\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2-\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)^2cde^3-2cd^3e-bde+\left(6cd^2e^2+be^2\right)\left(de^{(-1)}-\sqrt{\frac{1}{2}}\sqrt{-\frac{\left(be^2-\sqrt{b^2-4ac}\,e^2\right)e^{(-4)}}{c}}\right)}}{4\left(ab^2-4a^2c\right)}+\frac{bcx^3e^3+3bcdx^2e^2+3bcd^2xe+bcd^3+b^2xe-2acxe+b^2d-2acd}{2\left(cx^4e^4+4cdx^3e^3+6cd^2x^2e^2+4cd^3xe+cd^4+bx^2e^2+2bdxe+bd^2+a\right)\left(ab^2e-4a^2ce\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*c*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*log(d*e^(-1) + x + sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqr
t(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3
*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*l
og(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) -
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d
*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*
e + b*c*d^2 + b^2 - 6*a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d
^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - 6*a*c)*log(d*e^(-1) + x - s
qrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(
-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3
- 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a*b^2 - 4*a^2*c) + 1/2*(b*c*x^3*
e^3 + 3*b*c*d*x^2*e^2 + 3*b*c*d^2*x*e + b*c*d^3 + b^2*x*e - 2*a*c*x*e + b^2
*d - 2*a*c*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e +
 c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(a*b^2*e - 4*a^2*c*e))
```

Mupad [B]
time = 4.85, size = 2500, normalized size = 8.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(((-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7
*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 2
40*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8
*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^
6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 6
4*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^13 + 64
*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 1
6384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*
e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^
8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*
e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288
*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c
*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 -
6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 7
2*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*
a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^
2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i - (-(b^11 + b^2*(
-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c
^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 -
1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)
*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3
*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^
4*c + 48*a^4*b^2*c^2)) - ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 -
 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^
13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a
^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^
4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c -
 b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840
*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e
^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*
```

b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) + (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(72*a^2*c^5*e^12 + b^4*c^3*e^12 - 14*a*b^2*c^4*e^12))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*1i)/((-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2)*(((6144*a^5*c^6*e^12 + 16*a*b^8*c^2*e^12 - 288*a^2*b^6*c^3*e^12 + 1920*a^3*b^4*c^4*e^12 - 5632*a^4*b^2*c^5*e^12)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + ((16384*a^6*b*c^6*d*e^13 + 64*a^2*b^9*c^2*d*e^13 - 1024*a^3*b^7*c^3*d*e^13 + 6144*a^4*b^5*c^4*d*e^13 - 16384*a^5*b^3*c^5*d*e^13)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(1024*a^5*b*c^5*e^14 - 16*a^2*b^7*c^2*e^14 + 192*a^3*b^5*c^3*e^14 - 768*a^4*b^3*c^4*e^14))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12*e^2 + 4096*a^9*c^6*e^2 - 24*a^4*b^10*c*e^2 + 240*a^5*b^8*c^2*e^2 - 1280*a^6*b^6*c^3*e^2 + 3840*a^7*b^4*c^4*e^2 - 6144*a^8*b^2*c^5*e^2)))^(1/2) - (1152*a^3*c^6*d*e^11 - 4*b^6*c^3*d*e^11 + 72*a*b^4*c^4*d*e^11 - 512*a^2*b^2*c^5*d*e^11)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48...

3.626 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=162

$$\frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}e}+\frac{\log(d+ex)}{a^2e}-\frac{\log\left(a+b(\ldots\right.}{}$$

```
[Out] 1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+
1/2*b*(-6*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*
c+b^2)^(3/2)/e+ln(e*x+d)/a^2/e-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^2/e
```

Rubi [A]
time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1156, 1128, 754, 814, 648, 632, 212, 642}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2e\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2e}+\frac{\log(d+ex)}{a^2e}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 -
4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e) + Log[d + e*x]/(a^2*e) - Log[a + b*(
d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \right.}{} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\left(\int \left(\right.\right.}{} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex}{a^2e} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex}{a^2e} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d+ex}{a^2e} \\
&= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b(b^2-6ac}{2a}
\end{aligned}$$

Mathematica [A]
time = 0.33, size = 235, normalized size = 1.45

$$\frac{\frac{2a(b^2-2ac+bc(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}+4\log(d+ex)-\frac{\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(b^3-6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b +
 c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c
] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/
(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.21, size = 399, normalized size = 2.46

method	result
default	$\frac{\ln(ex+d)}{e\,a^2}-\frac{\frac{abce\,x^2}{8ac-2b^2}+\frac{bcdax}{4ac-b^2}-\frac{a\left(-bc\,d^2+2ac-b^2\right)}{2e\left(4ac-b^2\right)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\,\_Z^4+4d\,e^3c\,\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3\right.\right.}}{}$
risch	$\frac{-\frac{c\,x^2be}{2a\left(4ac-b^2\right)}-\frac{xbcd}{\left(4ac-b^2\right)a}+\frac{-bc\,d^2+2ac-b^2}{2ea\left(4ac-b^2\right)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a}+\frac{\ln(ex+d)}{e\,a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3e^2-48a^4b^2c^2e^2+12a^3b^4c\,e^2-\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)/e/a^2-1/a^2*((1/2*a/(4*a*c-b^2)*b*c*e*x^2+b*c*d*a/(4*a*c-b^2)*x-1
/2*a/e*(-b*c*d^2+2*a*c-b^2)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e
^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/2/(4*a*c-b^2)/e*sum
((e^3*c*(4*a*c-b^2)*_R^3+3*d*e^2*c*(4*a*c-b^2)*_R^2+e*(12*a*c^2*d^2-3*b^2*c
*d^2+5*a*b*c-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d)/(2*_R^3*c*e^3+6
*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^
4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*
b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^2*e^2 + 2*b*c*d*x*e + b*c*d^2 + b^2 - 2*a*c)/((a*b^2*c*e - 4*a^2
*c^2*e)*d^4 + 4*(a*b^2*c*e^4 - 4*a^2*c^2*e^4)*d*x^3 + (a*b^2*c*e^5 - 4*a^2*
c^2*e^5)*x^4 + a^2*b^2*e - 4*a^3*c*e + (a*b^3*e - 4*a^2*b*c*e)*d^2 + (a*b^3
*e^3 - 4*a^2*b*c*e^3 + 6*(a*b^2*c*e^3 - 4*a^2*c^2*e^3)*d^2)*x^2 + 2*(2*(a*b
^2*c*e^2 - 4*a^2*c^2*e^2)*d^3 + (a*b^3*e^2 - 4*a^2*b*c*e^2)*d)*x) + integra
te(-((b^2*c - 4*a*c^2)*d^3 + 3*(b^2*c*e^2 - 4*a*c^2*e^2)*d*x^2 + (b^2*c*e^3
 - 4*a*c^2*e^3)*x^3 + (b^3 - 5*a*b*c)*d + (b^3*e - 5*a*b*c*e + 3*(b^2*c*e -
 4*a*c^2*e)*d^2)*x)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e
^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(a^2*b^2 - 4*a^3*c) + e^
(-1)*log(x*e + d)/a^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1163 vs. $2(155) = 310$.
time = 0.50, size = 2452, normalized size = 15.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2*e
^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*x*e + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((b^
3*c - 6*a*b*c^2)*x^4*e^4 + 4*(b^3*c - 6*a*b*c^2)*d*x^3*e^3 + (b^3*c - 6*a*b
*c^2)*d^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^
2)*x^2*e^2 + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 -
6*a*b^2*c)*d)*x*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 +
 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d
)*x*e + b^2 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 +
2*(2*c*d^3 + b*d)*x*e + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*e^4 +
 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^3*e^3 + a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*x^2*e^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*e)*log(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4
 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a) + 4*((b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*e^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d
*x^3*e^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^2)*x^2*e^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*e)*l
og(x*e + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4*e^5 + 4*(a^2*b^4
*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*x^3*e^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^
4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*x^2*e^3 + 2*(2*(a
^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^
4*b*c^2)*d)*x*e^2 + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^
3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e
), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2
*e^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*x*e + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + 2*
((b^3*c - 6*a*b*c^2)*x^4*e^4 + 4*(b^3*c - 6*a*b*c^2)*d*x^3*e^3 + (b^3*c - 6
*a*b*c^2)*d^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2
)*d^2)*x^2*e^2 + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^
4 - 6*a*b^2*c)*d)*x*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 + 4*c*d*x*e
+ 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*x^4*e^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^3*e^3 + a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*x^
2*e^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*e)*log(c*x^4*e^4 +
4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x
*e + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*e^4 + 4*(b^4*c - 8*a*b^
```

```
2*c^2 + 16*a^2*c^3)*d*x^3*e^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*x^2*e^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*d)*x*e)*log(x*e + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)
*x^4*e^5 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*x^3*e^4 + (a^2*b^5
- 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d
^2)*x^2*e^3 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5
- 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*x*e^2 + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c +
 16*a^4*b*c^2)*d^2)*e)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(155) = 310.
time = 3.65, size = 454, normalized size = 2.80

$$-\frac{(a^2b^3ce^3 - 6\,a^3bc^2e^3)\sqrt{b^2-4\,ac}\,\log\left(\left|bx^2e^2 + 2\,bdxe + \sqrt{b^2-4\,ac}\,x^2e^2 + 2\,\sqrt{b^2-4\,ac}\,dxe + bd^2 + \sqrt{b^2-4\,ac}\,d^2 + 2\,a\right|\right) - (a^2b^3ce^3 - 6\,a^3bc^2e^3)\sqrt{b^2-4\,ac}\,\log\left(\left|-bx^2e^2 - 2\,bdxe + \sqrt{b^2-4\,ac}\,x^2e^2 + 2\,\sqrt{b^2-4\,ac}\,dxe - bd^2 + \sqrt{b^2-4\,ac}\,d^2 - 2\,a\right|\right)}{4\,(a^4b^4ce^4 - 8\,a^5b^2c^2e^4 + 16\,a^6c^3e^4)} - \frac{e^{(-1)}\log\left(|cx^4e^4 + 4\,cdx^3e^3 + 6\,cd^2x^2e^2 + 4\,cd^3xe + cd^4 + bx^2e^2 + 2\,bdxe + bd^2 + a|\right)}{4\,a^2} + \frac{e^{(-1)}\log\left(|xe + d|\right)}{a^2} + \frac{(abcx^2e^2 + 2\,abcdxe + abcd^2 + ab^2 - 2\,a^2c)e^{(-1)}}{2\,(cx^4e^4 + 4\,cdx^3e^3 + 6\,cd^2x^2e^2 + 4\,cd^3xe + cd^4 + bx^2e^2 + 2\,bdxe + bd^2 + a)(b^2 - 4\,ac)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((a^2*b^3*c*e^3 - 6*a^3*b*c^2*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2*e^2
 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^
2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^2*b^3*c*e^3 - 6*a^3*b*c^2*e^3)*sqrt(
b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2
*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(a^4*b^4*
c*e^4 - 8*a^5*b^2*c^2*e^4 + 16*a^6*c^3*e^4) - 1/4*e^(-1)*log(abs(c*x^4*e^4
+ 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d
*x*e + b*d^2 + a))/a^2 + e^(-1)*log(abs(x*e + d))/a^2 + 1/2*(a*b*c*x^2*e^2
+ 2*a*b*c*d*x*e + a*b*c*d^2 + a*b^2 - 2*a^2*c)*e^(-1)/((c*x^4*e^4 + 4*c*d*x
^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*
d^2 + a)*(b^2 - 4*a*c)*a^2)
```

Mupad [B]
time = 11.35, size = 2500, normalized size = 15.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)

[Out] ((b^2 - 2*a*c + b*c*d^2)/(2*e*(a*b^2 - 4*a^2*c)) + (b*c*e*x^2)/(2*(a*b^2 - 4*a^2*c)) + (b*c*d*x)/(a*b^2 - 4*a^2*c))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + log(d + e*x)/(a^2*e) - (log((((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) - 1)*(((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) - 1)*((b*c^2*e^16*(a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^2 + (2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*(4*a*c - b^2)) - (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*(4*a*c - b^2)) - (4*b*c^3*d*e^17*x*(10*a*c - b^2))/(a*(4*a*c - b^2))))/(4*a^2*e) - (b*c^3*e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2)))/(4*a^2*e) + (b^3*c^5*e^16*x^2)/(a^3*(4*a*c - b^2)^3) + (b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*(4*a*c - b^2)^3))*((b^3*c^5*e^16*x^2)/(a^3*(4*a*c - b^2)^3) - ((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*(((a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*((b*c^2*e^16*(a^2*e*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*(4*a*c - b^2)^3))^(1/2) + 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^2 - (2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*(4*a*c - b^2)) + (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*(4*a*c - b^2)) + (4*b*c^3*d*e^17*x*(10*a*c - b^2))/(a*(4*a*c - b^2))))/(4*a^2*e) - (b*c^3*e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*(4*a*c - b^2)^2)))/(4*a^2*e) + (b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*(4*a*c - b^2)^3)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b*atan(((16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*(x^2*((((((b*((320*a^5*b*c^6*e^18 - 2*a^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(6*a*c - b^2))/(4*a^2*e*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(8*a^2*e*(4*a*c - b^2)^(3/2)*(a^3*b^6 -

```
64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e
^2 - 48*a^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 9
6*a^2*b^2*c^2*e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a
^3*b^4*c*e^2 + 192*a^4*b^2*c^2*e^2)) + (b*(6*a*c - b^2)*((6*a*b^5*c^4*e^17
+ 80*a^3*b*c^6*e^17 - 44*a^2*b^3*c^5*e^17)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6*e^18 - 2*a^2*b^7*c^3*e^18 + 36*a^
3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*e - 24*a*b^
4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 +
1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(2*(a^3*b^6 - 64*a^6*c^3 -
12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^
4*c*e^2 + 192*a^4*b^2*c^2*e^2)))*(2*b^6*e - 128*a^3*c^3*e + 96*a^2*b^2*c^2*
e - 24*a*b^4*c*e))/(2*(4*a^2*b^6*e^2 - 256*a^5*c^3*e^2 - 48*a^3*b^4*c*e^2 +
 192*a^4*b^2*c^2*e^2))))/(4*a^2*e*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)
^3*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e^19 - 184*a^4*b^7*c^3*e^19 + 1056
*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19))/(64*a^6*e^3*(4*a*c - b^2)^(9/2)
*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c
^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 -
400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a
*b^2*c)*((((6*a*b^5*c^4*e^17 + 80*a^3*b*c^6*e^17 - 44*a^2*b^3*c^5*e^17)/(a^
3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6*e^18
 - 2*a^2*b^7*c^3*e^18 + 36*a^3*b^5*c^4*e^18 - 192*a^4*b^3*c^5*e^18)/(a^3*b^
6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*e - 128*a^3*c^3*e
 + 96*a^2*b^2*c^2*e - 24*a*b^4*c*e)*(2560*a^7*b*c^6*e^19 + 12*a^3*b^9*c^2*e
^19 - 184*a^4*b^7*c^3*e^19 + 1056*a^5*b^5*c^4*e^19 - 2688*a^6*b^3*c^5*e^19)
)/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*...
```

3.627 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=348

$$-\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\sqrt{c}\left(3b^3-16abc+(3b^2\right.}{2\sqrt{2}\,a}$$

```
[Out] 1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/e/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)
/a/(-4*a*c+b^2)/e/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-1/4*arctan((e*x+d)*2^
(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*
c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a
*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.11, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1156, 1135, 1295, 1180, 211}

$$-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a^2e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{3b^2-10ac}{2a^2e\left(b^2-4ac\right)(d+ex)}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -1/2*(3*b^2 - 10*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*
(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a
^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(3*b^3 - 1
6*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
```

```
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \text{S}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc}{2a\left(b^2-4ac\right)e(d+ex)\,(a+}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc}{2a\left(b^2-4ac\right)e(d+ex)\,(a+}$$

$$= -\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)e(d+ex)} + \frac{b^2-2ac+bc}{2a\left(b^2-4ac\right)e(d+ex)\,(a+}$$

Mathematica [A]
time = 1.06, size = 339, normalized size = 0.97

$$\frac{-\frac{4}{d+ex}+\frac{2(d+ex)\left(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2\right)}{\left(-b^2+4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2
]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.21, size = 441, normalized size = 1.27

method	result

default	$-\frac{1}{a^2e(ex+d)}-\frac{\frac{\frac{c\,e^2\left(2ac-b^2\right)x^3}{8ac-2b^2}+\frac{3dce\left(2ac-b^2\right)x^2}{2\left(4ac-b^2\right)}+\frac{\left(6a\,c^2d^2-3b^2c\,d^2+3abc-b^3\right)x}{8ac-2b^2}+\frac{d\left(2a\,c^2d^2-b^2c\,d^2+3abc-b^3\right)}{2e\left(4ac-b^2\right)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a}+\frac{\_R=\text{RootOf}\left(e^4c\_Z^4-\right.}{}}{}$
risch	$\frac{-\frac{e^3c\left(10ac-3b^2\right)x^4}{2a^2\left(4ac-b^2\right)}-\frac{2d\,e^2c\left(10ac-3b^2\right)x^3}{a^2\left(4ac-b^2\right)}-\frac{\left(60a\,c^2d^2-18b^2c\,d^2+11abc-3b^3\right)e\,x^2}{2a^2\left(4ac-b^2\right)}-\frac{d\left(20a\,c^2d^2-6b^2c\,d^2+11abc-3b^3\right)x}{a^2\left(4ac-b^2\right)}-\frac{10a\,c^2d^4-3b^2c\,d^4+11a}{2e\,a^2\left(\right.}}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a\right)(ex+d)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/e/(e*x+d)-1/a^2*((1/2*c*e^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3+3/2*d*c*e*(2
*a*c-b^2)/(4*a*c-b^2)*x^2+1/2*(6*a*c^2*d^2-3*b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-
b^2)*x+1/2*d/e*(2*a*c^2*d^2-b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^4*x^4+
4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a
)+1/4/(4*a*c-b^2)/e*sum((c*e^2*(10*a*c-3*b^2)*_R^2+2*c*d*e*(10*a*c-3*b^2)*_
R+10*a*c^2*d^2-3*b^2*c*d^2+13*a*b*c-3*b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_
R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+
(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*d^4 + 4*(3*b^2*c*e^3 - 10*a*c^2*e^3)*d*x^3 + (3*
b^2*c*e^4 - 10*a*c^2*e^4)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*d^2
+ (3*b^3*e^2 - 11*a*b*c*e^2 + 6*(3*b^2*c*e^2 - 10*a*c^2*e^2)*d^2)*x^2 + 2*(
2*(3*b^2*c*e - 10*a*c^2*e)*d^3 + (3*b^3*e - 11*a*b*c*e)*d)*x)/((a^2*b^2*c*e
 - 4*a^3*c^2*e)*d^5 + 5*(a^2*b^2*c*e^5 - 4*a^3*c^2*e^5)*d*x^4 + (a^2*b^2*c*
e^6 - 4*a^3*c^2*e^6)*x^5 + (a^2*b^3*e - 4*a^3*b*c*e)*d^3 + (a^2*b^3*e^4 - 4
*a^3*b*c*e^4 + 10*(a^2*b^2*c*e^4 - 4*a^3*c^2*e^4)*d^2)*x^3 + (10*(a^2*b^2*c
*e^3 - 4*a^3*c^2*e^3)*d^3 + 3*(a^2*b^3*e^3 - 4*a^3*b*c*e^3)*d)*x^2 + (a^3*b
^2*e - 4*a^4*c*e)*d + (a^3*b^2*e^2 - 4*a^4*c*e^2 + 5*(a^2*b^2*c*e^2 - 4*a^3
*c^2*e^2)*d^4 + 3*(a^2*b^3*e^2 - 4*a^3*b*c*e^2)*d^2)*x) + 1/2*integrate(-(3
*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*d^2 + 2*(3*b^2*c*e - 10*a*c^2*e)*d*x
 + (3*b^2*c*e^2 - 10*a*c^2*e^2)*x^2)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b
*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(a^2*b^
2 - 4*a^3*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4244 vs. $2(303) = 606$.
time = 0.63, size = 4244, normalized size = 12.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*x^4*e^4 + 8*(3*b^2*c - 10*a*c^2)*d*x^3*e^3 + 2
*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^
2)*x^2*e^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2*c
- 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*x*e + sqrt(1/2)*((a^2*b^2*c - 4*a^3
*c^2)*x^5*e^6 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*x^4*e^5 + (a^2*b^3 - 4*a^3*b*c
+ 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*x^3*e^4 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3
 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*x^2*e^3 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c -
 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*x*e^2 + ((a^2*b^2*c - 4*a^3*
c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b
^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*
c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b
^2*c^2 - 64*a^13*c^3)))*e^(-2)/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 6
4*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^
3*c^6)*x*e - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^
6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a
^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c +
 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))*e - (27*b^11 - 486*a*b^9*c + 3330*a^2*
b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e)*sqrt(-
(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*
b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*
b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^
12*b^2*c^2 - 64*a^13*c^3)))*e^(-2)/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
 - 64*a^8*c^3))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5*e^6 + 5*(a^2*b^2*
c - 4*a^3*c^2)*d*x^4*e^5 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2
)*d^2)*x^3*e^4 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*
d)*x^2*e^3 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^
3 - 4*a^3*b*c)*d^2)*x*e^2 + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3
*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*
b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8
*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 62
5*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*e^(
-2)/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c
^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x*e - (189*b^6*c^3 -
 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^
```

```
5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c
^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a
^13*c^3))*e - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3
 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*
a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64
*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
+ 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))
*e^(-2)/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/2
)*((a^2*b^2*c - 4*a^3*c^2)*x^5*e^6 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*x^4*e^5 +
(a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*x^3*e^4 + (10*(a^2*b
^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*x^2*e^3 + (a^3*b^2 - 4*a
^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*x*e^2 +
 ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^
4*c)*d)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*
b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*e^(-2)/(a^5*b^6 - 12*a^6*b^4*c
 + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*
a^2*b^2*c^5 - 2500*a^3*c^6)*x*e - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*
b^2*c^5 - 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392
*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))*e + (27*b^11 - 48
6*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200
*a^5*b*c^5)*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3
 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*e^(-2)/(a^5*b^6 - 12*a^6*b
^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*
x^5*e^6 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*x^4*e^5 +...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 847 vs. 2(303) = 606.
time = 4.25, size = 847, normalized size = 2.43

$$\frac{\left(2\,(3\,a^3b^2c-10\,a^4c^2)\sqrt{2\,ab+2\,\sqrt{b^2-4\,ac}\,a}\,\sqrt{b^2-4\,ac}\,|a^2b^2e^2-4\,a^3ce^2|e^2-(a^2b^2e^2-4\,a^3ce^2)^2(3\,b^3-13\,abc)\sqrt{2\,ab+2\,\sqrt{b^2-4\,ac}\,a}+(3\,a^4b^7-31\,a^5b^5c+96\,a^6b^3c^2-80\,a^7bc^3)\sqrt{2\,ab+2\,\sqrt{b^2-4\,ac}\,a}\,e^4\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,e^{(-1)}}{(xe+d)\sqrt{\frac{a^2b^3e^2-4\,a^3bce^2+\sqrt{(a^2b^3e^2-4\,a^3bce^2)^2-4\,(a^3b^2e^4-4\,a^4ce^4)(a^2b^2c-4\,a^3c^2)}}{a^3b^2e^4-4\,a^4ce^4}}}\right)e^{(-3)}}{16\,(a^5b^2c-4\,a^6c^2)\sqrt{b^2-4\,ac}\,|a^2b^2e^2-4\,a^3ce^2||a|}+\frac{\left(2\,(3\,a^3b^2c-10\,a^4c^2)\sqrt{2\,ab-2\,\sqrt{b^2-4\,ac}\,a}\,\sqrt{b^2-4\,ac}\,|a^2b^2e^2-4\,a^3ce^2|e^2+(a^2b^2e^2-4\,a^3ce^2)^2(3\,b^3-13\,abc)\sqrt{2\,ab-2\,\sqrt{b^2-4\,ac}\,a}-(3\,a^4b^7-31\,a^5b^5c+96\,a^6b^3c^2-80\,a^7bc^3)\sqrt{2\,ab-2\,\sqrt{b^2-4\,ac}\,a}\,e^4\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,e^{(-1)}}{(xe+d)\sqrt{\frac{a^2b^3e^2-4\,a^3bce^2-\sqrt{(a^2b^3e^2-4\,a^3bce^2)^2-4\,(a^3b^2e^4-4\,a^4ce^4)(a^2b^2c-4\,a^3c^2)}}{a^3b^2e^4-4\,a^4ce^4}}}\right)e^{(-3)}}{16\,(a^5b^2c-4\,a^6c^2)\sqrt{b^2-4\,ac}\,|a^2b^2e^2-4\,a^3ce^2||a|}-\frac{\frac{b^2ce^{(-1)}}{xe+d}-\frac{2\,ac^2e^{(-1)}}{xe+d}+\frac{b^3e^{(-1)}}{(xe+d)^3}-\frac{3\,abce^{(-1)}}{(xe+d)^3}}{2\,(a^2b^2-4\,a^3c)\left(c+\frac{b}{(xe+d)^2}+\frac{a}{(xe+d)^4}\right)}-\frac{e^{(-1)}}{(xe+d)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*(3*a^3*b^2*c - 10*a^4*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt
(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*e^2 - (a^2*b^2*e^2 - 4*a^3*c*e
^2)^2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a) + (3*a^4*b^7 -
 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt(2*a*b + 2*sqrt(b^2 - 4*
a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-1)/((x*e + d)*sqrt((a^2*b^3*e^2 - 4*a^3
*b*c*e^2 + sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2)^2 - 4*(a^3*b^2*e^4 - 4*a^4*c*
e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*e^4 - 4*a^4*c*e^4))))*e^(-3)/((a^5*
b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*abs(a))
 + 1/16*(2*(3*a^3*b^2*c - 10*a^4*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*s
qrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*e^2 + (a^2*b^2*e^2 - 4*a^3*
c*e^2)^2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a) - (3*a^4*b^
7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*a^7*b*c^3)*sqrt(2*a*b - 2*sqrt(b^2 -
 4*a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-1)/((x*e + d)*sqrt((a^2*b^3*e^2 - 4*
a^3*b*c*e^2 - sqrt((a^2*b^3*e^2 - 4*a^3*b*c*e^2)^2 - 4*(a^3*b^2*e^4 - 4*a^4
*c*e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*e^4 - 4*a^4*c*e^4))))*e^(-3)/((a
^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*abs(a^2*b^2*e^2 - 4*a^3*c*e^2)*abs(
a)) - 1/2*(b^2*c*e^(-1)/(x*e + d) - 2*a*c^2*e^(-1)/(x*e + d) + b^3*e^(-1)/(
x*e + d)^3 - 3*a*b*c*e^(-1)/(x*e + d)^3)/((a^2*b^2 - 4*a^3*c)*(c + b/(x*e +
 d)^2 + a/(x*e + d)^4)) - e^(-1)/((x*e + d)*a^2)
```

Mupad [B]
time = 6.38, size = 2500, normalized size = 7.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] - atan(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 +
240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^
10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 -
3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 +
 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 - 9*b^4*(
-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^
7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^
9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*
a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*((-
(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c
```

^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1
/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^
8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5
e^2)))^(1/2)(x*(1048576*a^16*b*c^8*e^14 + 256*a^10*b^13*c^2*e^14 - 6144*a
^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 327680*a^13*b^7*c^5*e^14 + 98
3040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14) + 1048576*a^16*b*c^8*d*
e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^3*d*e^13 + 61440*a^12*b^
9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*a^14*b^5*c^6*d*e^13 - 15
72864*a^15*b^3*c^7*d*e^13) - 851968*a^14*b*c^8*e^12 - 192*a^8*b^13*c^2*e^12
 + 4672*a^9*b^11*c^3*e^12 - 47360*a^10*b^9*c^4*e^12 + 256000*a^11*b^7*c^5*e
^12 - 778240*a^12*b^5*c^6*e^12 + 1261568*a^13*b^3*c^7*e^12) + 204800*a^12*c
^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10*c^4*d*e^11 + 30112*a^8*
b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568*a^10*b^4*c^7*d*e^11 - 4
58752*a^11*b^2*c^8*d*e^11)*1i + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2)
+ 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^
4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c
+ 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^
2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a
^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(204800*a^12*c^9*e^12 +
144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30112*a^8*b^8*c^5*e^12 - 1
43360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 - 458752*a^11*b^2*c^8*e^1
2) + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^
2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*
a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2
)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240
*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*
b^2*c^5*e^2)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^
6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*
a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2
c(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2 + 4096*a^11*c^6*e^2 - 24*a^6
*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4
*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^16*b*c^8*e^14 + 256*a^1
0*b^13*c^2*e^14 - 6144*a^11*b^11*c^3*e^14 + 61440*a^12*b^9*c^4*e^14 - 32768
0*a^13*b^7*c^5*e^14 + 983040*a^14*b^5*c^6*e^14 - 1572864*a^15*b^3*c^7*e^14)
 + 1048576*a^16*b*c^8*d*e^13 + 256*a^10*b^13*c^2*d*e^13 - 6144*a^11*b^11*c^
3*d*e^13 + 61440*a^12*b^9*c^4*d*e^13 - 327680*a^13*b^7*c^5*d*e^13 + 983040*
a^14*b^5*c^6*d*e^13 - 1572864*a^15*b^3*c^7*d*e^13) + 851968*a^14*b*c^8*e^12
 + 192*a^8*b^13*c^2*e^12 - 4672*a^9*b^11*c^3*e^12 + 47360*a^10*b^9*c^4*e^12
 - 256000*a^11*b^7*c^5*e^12 + 778240*a^12*b^5*c^6*e^12 - 1261568*a^13*b^3*c
^7*e^12) + 204800*a^12*c^9*d*e^11 + 144*a^6*b^12*c^3*d*e^11 - 3264*a^7*b^10
*c^4*d*e^11 + 30112*a^8*b^8*c^5*d*e^11 - 143360*a^9*b^6*c^6*d*e^11 + 365568
*a^10*b^4*c^7*d*e^11 - 458752*a^11*b^2*c^8*d*e^11)*1i)/((-(9*b^13 - 9*b^4*(
-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^
7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^

```
9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^1
2*e^2 + 4096*a^11*c^6*e^2 - 24*a^6*b^10*c*e^2 + 240*a^7*b^8*c^2*e^2 - 1280*
a^8*b^6*c^3*e^2 + 3840*a^9*b^4*c^4*e^2 - 6144*a^10*b^2*c^5*e^2)))^(1/2)*(x*
(204800*a^12*c^9*e^12 + 144*a^6*b^12*c^3*e^12 - 3264*a^7*b^10*c^4*e^12 + 30
112*a^8*b^8*c^5*e^12 - 143360*a^9*b^6*c^6*e^12 + 365568*a^10*b^4*c^7*e^12 -
 458752*a^11*b^2*c^8*e^12) + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2
6880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 -
 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)...
```

3.628 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=213

$$-\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2}+\frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\left(b^4-6ab^2c+6a^2c^2\right)\tanh}{a^3\left(b^2-4ac\right.}$$

```
[Out] (3*a*c-b^2)/a^2/(-4*a*c+b^2)/e/(e*x+d)^2+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-
4*a*c+b^2)/e/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-(6*a^2*c^2-6*a*b^2*c+b^4
)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e-2*
b*ln(e*x+d)/a^3/e+1/2*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e
```

Rubi [A]
time = 0.24, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1156, 1128, 754, 814, 648, 632, 212, 642}

$$\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3e}-\frac{2b\log(d+ex)}{a^3e}-\frac{b^2-3ac}{a^2e\left(b^2-4ac\right)(d+ex)^2}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3e\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)^2)) + (b^2 - 2*a*c + b*c*(d
+ e*x)^2)/(2*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)
^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^
2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e) - (2*b*Log[d + e*x])/(a^3*e) + (b*
Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} -$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)} -$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2} + \frac{b^2-2ac+}{2a\left(b^2-4ac\right)e(d+ex)^2}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2} + \frac{b^2-2ac+}{2a\left(b^2-4ac\right)e(d+ex)^2}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2} + \frac{b^2-2ac+}{2a\left(b^2-4ac\right)e(d+ex)^2}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)e(d+ex)^2} + \frac{b^2-2ac+}{2a\left(b^2-4ac\right)e(d+ex)^2}$$

<u>Mathematica</u> [A]
time = 0.34, size = 284, normalized size = 1.33

$$\frac{-\frac{a}{(d+ex)^2}+\frac{a\left(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2\right)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)}-4b\log(d+ex)+\frac{\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(-b^4+6ab^2c-6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}}{2a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*
x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d + e
*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[
b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)
^(3/2))/(2*a^3*e)
```

<u>Maple</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 462, normalized size = 2.17

method	result
default	$-\frac{1}{2a^{2}e(ex+d)^{2}}-\frac{2b\ln(ex+d)}{a^{3}e}-\frac{\frac{\frac{ace\left(2ac-b^{2}\right)x^{2}}{8ac-2b^{2}}+\frac{cda\left(2ac-b^{2}\right)x}{4ac-b^{2}}+\frac{a\left(2a\,c^{2}d^{2}-b^{2}c\,d^{2}+3abc-b^{3}\right)}{2e\left(4ac-b^{2}\right)}}{c\,e^{4}x^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+d^{4}c+2debx+d^{2}b+a}+\frac{\_R=\text{RootOf}\left(e^{4}c\_Z^{4}+4d\,e^{3}c\_Z^{3}\right.}{}}{}$
risch	$\frac{-\frac{\left(3ac-b^{2}\right)e^{3}c\,x^{4}}{\left(4ac-b^{2}\right)a^{2}}-\frac{4\left(3ac-b^{2}\right)cd\,e^{2}x^{3}}{\left(4ac-b^{2}\right)a^{2}}-\frac{\left(36a\,c^{2}d^{2}-12b^{2}c\,d^{2}+7abc-2b^{3}\right)e\,x^{2}}{2a^{2}\left(4ac-b^{2}\right)}-\frac{d\left(12a\,c^{2}d^{2}-4b^{2}c\,d^{2}+7abc-2b^{3}\right)x}{a^{2}\left(4ac-b^{2}\right)}-\frac{6a\,c^{2}d^{4}-2b^{2}c\,d^{4}+7abc\,d^{2}-}{2e\,a^{2}\left(4ac-\right.}}{(ex+d)^{2}(c\,e^{4}x^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+d^{4}c+2debx+d^{2}b+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2/e/(e*x+d)^2-2*b*ln(e*x+d)/a^3/e-1/a^3*((1/2*a*c*e*(2*a*c-b^2)/(4*a
*c-b^2)*x^2+c*d*a*(2*a*c-b^2)/(4*a*c-b^2)*x+1/2*a/e*(2*a*c^2*d^2-b^2*c*d^2+
3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*
e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/(4*a*c-b^2)/e*sum((e^3*b*c*(-4*a*c
+b^2)*_R^3+3*d*e^2*b*c*(-4*a*c+b^2)*_R^2+e*(-12*a*b*c^2*d^2+3*b^3*c*d^2+3*a
^2*c^2-5*a*b^2*c+b^4)*_R-4*a*b*c^2*d^3+b^3*c*d^3+3*a^2*c^2*d-5*a*b^2*c*d+b^
4*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R)
,_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*
b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b^2*c - 3*a*c^2)*d^4 + 8*(b^2*c*e^3 - 3*a*c^2*e^3)*d*x^3 + 2*(b^2*
c*e^4 - 3*a*c^2*e^4)*x^4 + a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*d^2 + (2*b^3
*e^2 - 7*a*b*c*e^2 + 12*(b^2*c*e^2 - 3*a*c^2*e^2)*d^2)*x^2 + 2*(4*(b^2*c*e
- 3*a*c^2*e)*d^3 + (2*b^3*e - 7*a*b*c*e)*d)*x)/((a^2*b^2*c*e - 4*a^3*c^2*e)
*d^6 + 6*(a^2*b^2*c*e^6 - 4*a^3*c^2*e^6)*d*x^5 + (a^2*b^2*c*e^7 - 4*a^3*c^2
*e^7)*x^6 + (a^2*b^3*e - 4*a^3*b*c*e)*d^4 + (a^2*b^3*e^5 - 4*a^3*b*c*e^5 +
15*(a^2*b^2*c*e^5 - 4*a^3*c^2*e^5)*d^2)*x^4 + 4*(5*(a^2*b^2*c*e^4 - 4*a^3*c
^2*e^4)*d^3 + (a^2*b^3*e^4 - 4*a^3*b*c*e^4)*d)*x^3 + (a^3*b^2*e - 4*a^4*c*e
)*d^2 + (a^3*b^2*e^3 - 4*a^4*c*e^3 + 15*(a^2*b^2*c*e^3 - 4*a^3*c^2*e^3)*d^4
 + 6*(a^2*b^3*e^3 - 4*a^3*b*c*e^3)*d^2)*x^2 + 2*(3*(a^2*b^2*c*e^2 - 4*a^3*c
^2*e^2)*d^5 + 2*(a^2*b^3*e^2 - 4*a^3*b*c*e^2)*d^3 + (a^3*b^2*e^2 - 4*a^4*c*
e^2)*d)*x) - 2*integrate(-((b^3*c - 4*a*b*c^2)*d^3 + 3*(b^3*c*e^2 - 4*a*b*c
^2*e^2)*d*x^2 + (b^3*c*e^3 - 4*a*b*c^2*e^3)*x^3 + (b^4 - 5*a*b^2*c + 3*a^2*
```

```
c^2)*d + (b^4*e - 5*a*b^2*c*e + 3*a^2*c^2*e + 3*(b^3*c*e - 4*a*b*c^2*e)*d^2
)*x)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2
 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(a^3*b^2 - 4*a^4*c) - 2*b*e^(-1)*log(x*
e + d)/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2196 vs. $2(209) = 418$.
time = 0.63, size = 4518, normalized size = 21.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12
*a^3*c^3)*x^4*e^4 + 8*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*x^3*e^3 + 2*
(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a
^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*x^2*e^2 + (2*a*b^
5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a
^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*x*e + ((b^4*c - 6*
a*b^2*c^2 + 6*a^2*c^3)*x^6*e^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*x^5*
e^5 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^
2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*x^4*e^4 + (b^5 - 6*a*b^3*c +
6*a^2*b*c^2)*d^4 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*
b^3*c + 6*a^2*b*c^2)*d)*x^3*e^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)
*x^2*e^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^
2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2
*b^2*c + 6*a^3*c^2)*d)*x*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*
x^3*e^3 + 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^
3 + b*c*d)*x*e + b^2 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt
(b^2 - 4*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*x^6*e^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*x^5*e^5 + (b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*d^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*x^4*e^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^
2*c^2)*d^4 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4
*c + 16*a^2*b^2*c^2)*d)*x^3*e^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*d^2)*x^2*e^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^
3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*x*e)*log(c*x^4*e^4 + 4*c*d*x^3*
e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a) +
4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6*e^6 + 6*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d*x^5*e^5 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + (b^6 -
```

```
 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*
x^4*e^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + 4*(5*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*x^3*e^3 + (a*
b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*x^2*e^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2
*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2)*d)*x*e)*log(x*e + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6*e^7
 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*x^5*e^6 + (a^3*b^5 - 8*a^4*
b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^2)*x^4
*e^5 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^4
*b^3*c + 16*a^5*b*c^2)*d)*x^3*e^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 1
5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5 - 8*a^4*b^3*c +
 16*a^5*b*c^2)*d^2)*x^2*e^3 + 2*(3*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)
*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (a^4*b^4 - 8*a^5*b^2*
c + 16*a^6*c^2)*d)*x*e^2 + ((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^6 +
(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^
6*c^2)*d^2)*e), -1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a
^2*b^2*c^2 + 12*a^3*c^3)*x^4*e^4 + 8*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)
*d*x^3*e^3 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a
^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*x^
2*e^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2
*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*x*e
 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6*e^6 + 6*(b^4*c - 6*a*b^2*c^2 +
6*a^2*c^3)*d*x^5*e^5 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + (b^5 - 6*a*b
^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*x^4*e^4 + (b
^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*
d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*x^3*e^3 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c +
6*a^2*b*c^2)*d^2)*x^2*e^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b
^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3
 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*x*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6*e^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*d*x^5*e^5 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + (b^6 - 8*a
*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
[Out] Timed out
```

Giac [A]
time = 4.34, size = 224, normalized size = 1.05

$$\frac{(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^2}}{\sqrt{-b^2+4\,ac}}\right)e^{(-1)}}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}}+\frac{be^{(-1)}\log\left(c+\frac{b}{(xe+d)^2}+\frac{a}{(xe+d)^4}\right)}{2\,a^3}+\frac{\left(\frac{b^3c-3\,abc^2}{a}+\frac{(b^4e-4\,ab^2ce+2\,a^2c^2e)e^{(-1)}}{(xe+d)^2a}\right)e^{(-1)}}{2\,(b^2-4\,ac)a^2\left(c+\frac{b}{(xe+d)^2}+\frac{a}{(xe+d)^4}\right)}-\frac{e^{(-1)}}{2\,(xe+d)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan(-(b + 2*a/(x*e + d)^2)/sqrt(-b^2 + 4*a
*c))*e^(-1)/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) + 1/2*b*e^(-1)*log(c +
 b/(x*e + d)^2 + a/(x*e + d)^4)/a^3 + 1/2*((b^3*c - 3*a*b*c^2)/a + (b^4*e -
 4*a*b^2*c*e + 2*a^2*c^2*e)*e^(-1)/((x*e + d)^2*a))*e^(-1)/((b^2 - 4*a*c)*a
^2*(c + b/(x*e + d)^2 + a/(x*e + d)^4)) - 1/2*e^(-1)/((x*e + d)^2*a^2)
```

Mupad [B]
time = 12.32, size = 2500, normalized size = 11.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2)
 - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*d*
e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 - 6*
a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2*(2*
b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^2*b^2
)))/(x^4*(b*e^4 + 15*c*d^2*e^4) + a*d^2 + b*d^4 + c*d^6 + x*(2*a*d*e + 4*b*
d^3*e + 6*c*d^5*e) + x^2*(a*e^2 + 6*b*d^2*e^2 + 15*c*d^4*e^2) + x^3*(20*c*d
^3*e^3 + 4*b*d*e^3) + c*e^6*x^6 + 6*c*d*e^5*x^5) + (log((((b + a^3*e*(-(b^4
 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(((b + a^3*e*
(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*((4*c^2
*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*
b^2*c^2*d^2))/(a^2*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a
*b^2*c))/(a^2*(4*a*c - b^2)) - (2*b*c^2*e^16*(b + a^3*e*(-(b^4 + 6*a^2*c^2
- 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e
^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (
8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*(4*a*c - b^2))))/(2*a^3
*e) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2
*c - 23*a*b*c^2*d^2))/(a^4*(4*a*c - b^2)^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69
*a^2*c^2 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 +
69*a^2*c^2 - 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2)))/(2*a^3*e) - (8*c^5*e^16*x
^2*(3*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^
3 - 3*a*c^2*d^2 + b^2*c*d^2 - 4*a*b*c))/(a^6*(4*a*c - b^2)^3) - (16*c^5*d*e
```

^15*x*(3*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^3))*(((b - a^3*e*(-(b^4 + 6*a^2*c
^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(((b - a^3*e*(-(b^4 + 6
*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*((4*c^2*e^16*(2*b
^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*b^2*c^2*d^
2))/(a^2*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(
a^2*(4*a*c - b^2)) - (2*b*c^2*e^16*(b - a^3*e*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*
c)^2/(a^6*e^2*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 1
0*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (8*c^3*d*e^
17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*(4*a*c - b^2))))/(2*a^3*e) - (4*c
^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2*c - 23*a*
b*c^2*d^2))/(a^4*(4*a*c - b^2)^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 -
41*a*b^2*c))/(a^4*(4*a*c - b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2
- 41*a*b^2*c))/(a^4*(4*a*c - b^2)^2)))/(2*a^3*e) - (8*c^5*e^16*x^2*(3*a*c
- b^2)^3)/(a^6*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^
2*d^2 + b^2*c*d^2 - 4*a*b*c))/(a^6*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a
c - b^2)^3)/(a^6(4*a*c - b^2)^3)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c
*e - 64*a^3*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 +
48*a^5*b^2*c^2*e^2)) - (2*b*log(d + e*x))/(a^3*e) - (atan(((2*a^9*b^6*(4*a
*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4*a*c -
b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(x*((((8*(54*a^3*c^8*d*e
^15 - 2*b^6*c^5*d*e^15 + 18*a*b^4*c^6*d*e^15 - 54*a^2*b^2*c^7*d*e^15))/(a^6
*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((8*(276*a^5*b*c^7*d*
e^16 - 6*a^2*b^7*c^4*d*e^16 + 65*a^3*b^5*c^5*d*e^16 - 233*a^4*b^3*c^6*d*e^1
6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((8*(480*a^8*
c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6*a^5*b^6*c^4*d*e^17 + 30*a^6*b^4*c^5*d*e
^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^
8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*
(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264
*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/((a^6*b^6 - 64*a^9*c^3 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^
2 + 48*a^5*b^2*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3
*b*c^3*e))/(2*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2
*c^2*e^2)))*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e))/(2*
(a^3*b^6*e^2 - 64*a^6*c^3*e^2 - 12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)) - (
((((8*(480*a^8*c^7*d*e^17 - a^4*b^8*c^3*d*e^17 + 6*a^5*b^6*c^4*d*e^17 + 30*
a^6*b^4*c^5*d*e^17 - 272*a^7*b^2*c^6*d*e^17))/(a^6*b^6 - 64*a^9*c^3 - 12*a^
7*b^4*c + 48*a^8*b^2*c^2) - (4*(b^7*e + 48*a^2*b^3*c^2*e - 12*a*b^5*c*e - 6
4*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^6*b^9*c^2*d*e^18 - 46*a^7*b^7*c
^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9*b^3*c^5*d*e^18))/((a^6*b^6 - 6
4*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(a^3*b^6*e^2 - 64*a^6*c^3*e^2 -
12*a^4*b^4*c*e^2 + 48*a^5*b^2*c^2*e^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*
a^3*e*(4*a*c - b^2)^(3/2)) - (2*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7*e + 48*a
^2*b^3*c^2*e - 12*a*b^5*c*e - 64*a^3*b*c^3*e)*(640*a^10*b*c^6*d*e^18 + 3*a^
6*b^9*c^2*d*e^18 - 46*a^7*b^7*c^3*d*e^18 + 264*a^8*b^5*c^4*d*e^18 - 672*a^9
*b^3*c^5*d*e^18))/(a^3*e*(4*a*c - b^2)^(3/2)*(a...

3.629 $\int \frac{1}{(d+ex)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=408

$$-\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3}+\frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)e(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

```
[Out] 1/6*(14*a*c-5*b^2)/a^2/(-4*a*c+b^2)/e/(e*x+d)^3+1/2*b*(-19*a*c+5*b^2)/a^3/(
-4*a*c+b^2)/e/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(e*x+d
)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2+b*(-19*a*c+5*b^2
)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)-1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c
^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2-b*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))/a
^3/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 2.46, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1156, 1135, 1295, 1180, 211}

$$\frac{b(5b^2-19ac)}{2a^3e\left(b^2-4ac\right)(d+ex)}-\frac{5b^2-14ac}{6a^2e\left(b^2-4ac\right)(d+ex)^3}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{2ae\left(b^2-4ac\right)(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -1/6*(5*b^2 - 14*a*c)/(a^2*(b^2 - 4*a*c)*e*(d + e*x)^3) + (b*(5*b^2 - 19*a*
c))/(2*a^3*(b^2 - 4*a*c)*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*
a*(b^2 - 4*a*c)*e*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[
c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2
]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(5*b^4
- 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2
 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^4 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)e(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \text{S}\cdots$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b^2-2ac+bc\cdots}{2a\left(b^2-4ac\right)e(d+ex)^3(a+\cdots}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)e(d+ex)} + \frac{\cdots}{2a\cdots}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)e(d+ex)} + \frac{\cdots}{2a\cdots}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)e(d+ex)^3} + \frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)e(d+ex)} + \frac{\cdots}{2a\cdots}$$

Mathematica [A]
time = 1.93, size = 384, normalized size = 0.94

$$\frac{-\frac{4a}{(d+ex)^3}+\frac{24b}{d+ex}+\frac{6(d+ex)\left(b^4-4ab^2c+2a^2c^2+b^3c(d+ex)^2-3abc^2(d+ex)^2\right)}{(b^2-4ac)\left(a+(d+ex)^2(b+c(d+ex)^2)\right)}+\frac{3\sqrt{2}\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-5b^4+29ab^2c-28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{12a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2*
a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (
d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c +
 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c
- 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.21, size = 489, normalized size = 1.20

method	result
default	$-\frac{1}{3a^2e(ex+d)^3}+\frac{2b}{a^3e(ex+d)}-\frac{\frac{-\frac{bc\,e^2\left(3ac-b^2\right)x^3}{2\left(4ac-b^2\right)}-\frac{3dbce\left(3ac-b^2\right)x^2}{2\left(4ac-b^2\right)}+\frac{\left(-9ab\,c^2d^2+3b^3c\,d^2+2a^2c^2-4a\,b^2c+b^4\right)x}{8ac-2b^2}+\frac{d\left(-3ab\,c^2d^2+b^3c\right.}{2e\left(\right.}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a}}{}$
risch	$\frac{\frac{\left(19ac-5b^2\right)bc\,e^5x^6}{2\left(4ac-b^2\right)a^3}+\frac{3d\,e^4\left(19ac-5b^2\right)bc\,x^5}{\left(4ac-b^2\right)a^3}-\frac{\left(-855ab\,c^2d^2+225b^3c\,d^2+14a^2c^2-62a\,b^2c+15b^4\right)e^3x^4}{6\left(4ac-b^2\right)a^3}-\frac{2\left(-285ab\,c^2d^2+75b^3c\,d^2+14a^2c^2-62a\,b\right.}{3\left(4ac-b^2\right)a^3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^2/e/(e*x+d)^3+2/a^3*b/e/(e*x+d)-1/a^3*((-1/2*b*c*e^2*(3*a*c-b^2)/(4*
a*c-b^2)*x^3-3/2*d*b*c*e*(3*a*c-b^2)/(4*a*c-b^2)*x^2+1/2*(-9*a*b*c^2*d^2+3*
b^3*c*d^2+2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2)*x+1/2*d/e*(-3*a*b*c^2*d^2+b^
3*c*d^2+2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/
e*sum((b*c*e^2*(-19*a*c+5*b^2)*_R^2+2*b*c*d*e*(-19*a*c+5*b^2)*_R-19*a*b*c^2
*d^2+5*b^3*c*d^2+14*a^2*c^2-24*a*b^2*c+5*b^4)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+
6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z
^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*b^3*c - 19*a*b*c^2)*d^6 + 18*(5*b^3*c*e^5 - 19*a*b*c^2*e^5)*d*x^5
 + 3*(5*b^3*c*e^6 - 19*a*b*c^2*e^6)*x^6 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2
)*d^4 + (15*b^4*e^4 - 62*a*b^2*c*e^4 + 14*a^2*c^2*e^4 + 45*(5*b^3*c*e^4 - 1
9*a*b*c^2*e^4)*d^2)*x^4 - 2*a^2*b^2 + 8*a^3*c + 4*(15*(5*b^3*c*e^3 - 19*a*b
*c^2*e^3)*d^3 + (15*b^4*e^3 - 62*a*b^2*c*e^3 + 14*a^2*c^2*e^3)*d)*x^3 + 10*
(a*b^3 - 4*a^2*b*c)*d^2 + (45*(5*b^3*c*e^2 - 19*a*b*c^2*e^2)*d^4 + 10*a*b^3
*e^2 - 40*a^2*b*c*e^2 + 6*(15*b^4*e^2 - 62*a*b^2*c*e^2 + 14*a^2*c^2*e^2)*d^
2)*x^2 + 2*(9*(5*b^3*c*e - 19*a*b*c^2*e)*d^5 + 2*(15*b^4*e - 62*a*b^2*c*e +
 14*a^2*c^2*e)*d^3 + 10*(a*b^3*e - 4*a^2*b*c*e)*d)*x)/((a^3*b^2*c*e - 4*a^4
*c^2*e)*d^7 + 7*(a^3*b^2*c*e^7 - 4*a^4*c^2*e^7)*d*x^6 + (a^3*b^2*c*e^8 - 4*
a^4*c^2*e^8)*x^7 + (a^3*b^3*e - 4*a^4*b*c*e)*d^5 + (a^3*b^3*e^6 - 4*a^4*b*c
*e^6 + 21*(a^3*b^2*c*e^6 - 4*a^4*c^2*e^6)*d^2)*x^5 + 5*(7*(a^3*b^2*c*e^5 -
4*a^4*c^2*e^5)*d^3 + (a^3*b^3*e^5 - 4*a^4*b*c*e^5)*d)*x^4 + (a^4*b^2*e - 4*
```

```
a^5*c*e)*d^3 + (a^4*b^2*e^4 - 4*a^5*c*e^4 + 35*(a^3*b^2*c*e^4 - 4*a^4*c^2*e
^4)*d^4 + 10*(a^3*b^3*e^4 - 4*a^4*b*c*e^4)*d^2)*x^3 + (21*(a^3*b^2*c*e^3 -
4*a^4*c^2*e^3)*d^5 + 10*(a^3*b^3*e^3 - 4*a^4*b*c*e^3)*d^3 + 3*(a^4*b^2*e^3
- 4*a^5*c*e^3)*d)*x^2 + (7*(a^3*b^2*c*e^2 - 4*a^4*c^2*e^2)*d^6 + 5*(a^3*b^3
*e^2 - 4*a^4*b*c*e^2)*d^4 + 3*(a^4*b^2*e^2 - 4*a^5*c*e^2)*d^2)*x) - 1/2*int
egrate(-(5*b^4 - 24*a*b^2*c + 14*a^2*c^2 + (5*b^3*c - 19*a*b*c^2)*d^2 + 2*(
5*b^3*c*e - 19*a*b*c^2*e)*d*x + (5*b^3*c*e^2 - 19*a*b*c^2*e^2)*x^2)/(c*x^4*
e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^
3*e + b*d*e)*x + a), x)/(a^3*b^2 - 4*a^4*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5636 vs. $2(361) = 722$.
time = 0.73, size = 5636, normalized size = 13.81

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*x^6*e^6 + 36*(5*b^3*c - 19*a*b*c^2)*d*x^5*e^
5 + 6*(5*b^3*c - 19*a*b*c^2)*d^6 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45
*(5*b^3*c - 19*a*b*c^2)*d^2)*x^4*e^4 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)
*d^4 + 8*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2
)*d)*x^3*e^3 - 4*a^2*b^2 + 16*a^3*c + 2*(45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10
*a*b^3 - 40*a^2*b*c + 6*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*x^2*e^2 + 2
0*(a*b^3 - 4*a^2*b*c)*d^2 + 4*(9*(5*b^3*c - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 6
2*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 - 4*a^2*b*c)*d)*x*e - 3*sqrt(1/2)*(
(a^3*b^2*c - 4*a^4*c^2)*x^7*e^8 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*x^6*e^7 + (a^
3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*x^5*e^6 + 5*(7*(a^3*b^2
*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*x^4*e^5 + (a^4*b^2 - 4*a^5*c
 + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*x^3*e^4 +
 (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^
2 - 4*a^5*c)*d)*x^2*e^3 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a
^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*x*e^2 + ((a^3*b^2*c - 4*a^4*c^2)*d
^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)*sqrt(-(25*b^9
- 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7
*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a
*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108
*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 -
64*a^17*c^3)))*e^(-2)/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^
3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2
*c^7 + 9604*a^4*c^8)*x*e + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*
c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d + 1/2*sqrt(1/2)*((5*a^7*b^11 - 94
*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328
*a^12*b*c^5)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3
```

```
*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6
 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3))*e - (125*b^14 - 2425*a*b^
12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990
*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(-(25*b^9 - 315*a*
b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 1
2*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c
 + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2
*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*
c^3)))*e^(-2)/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) + 3
*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7*e^8 + 7*(a^3*b^2*c - 4*a^4*c^2)*d*x
^6*e^7 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*x^5*e^6 + 5
*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*x^4*e^5 + (a^4*b
^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^
2)*x^3*e^4 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3
 + 3*(a^4*b^2 - 4*a^5*c)*d)*x^2*e^3 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a
^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2)*x*e^2 + ((a^3*b^2*c -
4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*e)*sq
rt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*
b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b
^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4
*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^1
6*b^2*c^2 - 64*a^17*c^3)))*e^(-2)/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2
 - 64*a^10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 5
0421*a^3*b^2*c^7 + 9604*a^4*c^8)*x*e + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43
410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4*c^8)*d - 1/2*sqrt(1/2)*((5*a
^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^
3*c^4 - 3328*a^12*b*c^5)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2
 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6
)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3))*e - (125*b^14
 - 2425*a*b^12*c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*
c^4 - 172990*a^5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*e)*sqrt(-(25*
b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 +
(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 82
50*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 2
4108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^
2 - 64*a^17*c^3)))*e^(-2)/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^1
0*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7*e^8 + 7*(a^3*b^2*c - 4*
a^4*c^2)*d*x^6*e^7 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)
*x^5*e^6 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*x^4*
e^5 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3 - 4
*a^4*b*c)*d^2)*x^3*e^4 + (21*(a^3*b^2*c - 4*a^4...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1987 vs. $2(361) = 722$.
time = 3.37, size = 1987, normalized size = 4.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((5*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b^3*c*e^2 - 19*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) + sqrt(1/2)*sqrt(
-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*
a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (5*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 19*(d*e^(-1) - sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1)
 - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 3
8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*
b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)
*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (5*(d*e^(-1
) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2
- 19*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^2*a*b*c^2*e^2 - 10*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^
2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b
^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*
c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
```

```
^2)*e^(-4)/c))) + (5*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 19*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) - sqr
t(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3
*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x
 - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d
*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*
e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a^3*b^2 - 4*a^4*c) + 1/2*(b^
3*c*x^3*e^3 - 3*a*b*c^2*x^3*e^3 + 3*b^3*c*d*x^2*e^2 - 9*a*b*c^2*d*x^2*e^2 +
 3*b^3*c*d^2*x*e - 9*a*b*c^2*d^2*x*e + b^3*c*d^3 - 3*a*b*c^2*d^3 + b^4*x*e
- 4*a*b^2*c*x*e + 2*a^2*c^2*x*e + b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d)/((c*x^
4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 +
 2*b*d*x*e + b*d^2 + a)*(a^3*b^2*e - 4*a^4*c*e)) + 1/3*(6*b*x^2*e^2 + 12*b*
d*x*e + 6*b*d^2 - a)*e^(-1)/((x*e + d)^3*a^3)
```

Mupad [B]
time = 8.72, size = 2500, normalized size = 6.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] atan(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366
*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
- 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9
*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^
2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^
7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 2197
44*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) -
 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c
*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2
- 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1
/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*
b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b
^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 16
5*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 -
 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^1
1*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14
```

+ 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) - 917504*a^19*c^9*e^12 + 320*a^12*b^14*c^2*e^12 - 7936*a^13*b^12*c^3*e^12 + 82816*a^14*b^10*c^4*e^12 - 468480*a^15*b^8*c^5*e^12 + 1536000*a^16*b^6*c^6*e^12 - 2867200*a^17*b^4*c^7*e^12 + 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^9*d*e^11)*1i + (-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2 + 4096*a^13*c^6*e^2 - 24*a^8*b^10*c*e^2 + 240*a^9*b^8*c^2*e^2 - 1280*a^10*b^6*c^3*e^2 + 3840*a^11*b^4*c^4*e^2 - 6144*a^12*b^2*c^5*e^2)))^(1/2)*(x*(1048576*a^21*b*c^8*e^14 + 256*a^15*b^13*c^2*e^14 - 6144*a^16*b^11*c^3*e^14 + 61440*a^17*b^9*c^4*e^14 - 327680*a^18*b^7*c^5*e^14 + 983040*a^19*b^5*c^6*e^14 - 1572864*a^20*b^3*c^7*e^14) + 1048576*a^21*b*c^8*d*e^13 + 256*a^15*b^13*c^2*d*e^13 - 6144*a^16*b^11*c^3*d*e^13 + 61440*a^17*b^9*c^4*d*e^13 - 327680*a^18*b^7*c^5*d*e^13 + 983040*a^19*b^5*c^6*d*e^13 - 1572864*a^20*b^3*c^7*d*e^13) + 917504*a^19*c^9*e^12 - 320*a^12*b^14*c^2*e^12 + 7936*a^13*b^12*c^3*e^12 - 82816*a^14*b^10*c^4*e^12 + 468480*a^15*b^8*c^5*e^12 - 1536000*a^16*b^6*c^6*e^12 + 2867200*a^17*b^4*c^7*e^12 - 2719744*a^18*b^2*c^8*e^12) - x*(401408*a^16*c^10*e^12 - 400*a^9*b^14*c^3*e^12 + 9440*a^10*b^12*c^4*e^12 - 92816*a^11*b^10*c^5*e^12 + 488096*a^12*b^8*c^6*e^12 - 1458688*a^13*b^6*c^7*e^12 + 2401280*a^14*b^4*c^8*e^12 - 1871872*a^15*b^2*c^9*e^12) - 401408*a^16*c^10*d*e^11 + 400*a^9*b^14*c^3*d*e^11 - 9440*a^10*b^12*c^4*d*e^11 + 92816*a^11*b^10*c^5*d*e^11 - 488096*a^12*b^8*c^6*d*e^11 + 1458

```
688*a^13*b^6*c^7*d*e^11 - 2401280*a^14*b^4*c^8*d*e^11 + 1871872*a^15*b^2*c^
9*d*e^11)*1i)/((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c
^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 11...
```

3.630 $\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} dx$

Optimal. Leaf size=341

$$\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{(d+ex)\left(7b^2-4ac+12bc(d+ex)^2\right)}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(3b^2+4ac\ldots\right.}{4\sqrt{\ldots}}$$

[Out] 1/4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2-1/8*(e*x+d)*(7*b^2-4*a*c+12*b*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+3/8*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/8*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)

Rubi [A]

time = 0.64, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1156, 1134, 1192, 1180, 211}

$$\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]

[Out] ((d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) - ((d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
```

```
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \frac{2a-5bx^2}{(a+bx^2+cx^4)^2}\right.}{4\left(b^2-4a\right.}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4\right.}{8\left(b^2-4ac\right)^2e\left(a+b\right.}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4\right.}{8\left(b^2-4ac\right)^2e\left(a+b\right.}$$

$$= \frac{(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(d+ex)\left(7b^2-4\right.}{8\left(b^2-4ac\right)^2e\left(a+b\right.}$$

Mathematica [A]
time = 2.96, size = 328, normalized size = 0.96

$$\frac{-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{(d+ex)\left(-7b^2+4ac-12bc(d+ex)^2\right)}{(b^2-4ac)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 +
c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/((b^2
 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*(3*b^
2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
- (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqr
t[b + Sqrt[b^2 - 4*a*c]]))/(8*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 704, normalized size = 2.06

method	result

default	$\frac{-\frac{3c^{2}e^{6}b\,x^{7}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{21c^{2}d\,e^{5}b\,x^{6}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(-252bc\,d^{2}+4ac-19b^{2}\right)c\,e^{4}x^{5}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}+\frac{5cd\,e^{3}\left(-84bc\,d^{2}+4ac-19b^{2}\right)x^{4}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{e^{2}\left(420b\,c^{2}d^{4}-40a\,c^{2}d^{2}+\cdots\right.}{8\left(16a^{2}c^{2}-\cdots\right.}}{(c\,e\cdots}$
risch	$\frac{-\frac{3c^{2}e^{6}b\,x^{7}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{21c^{2}d\,e^{5}b\,x^{6}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(-252bc\,d^{2}+4ac-19b^{2}\right)c\,e^{4}x^{5}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}+\frac{5cd\,e^{3}\left(-84bc\,d^{2}+4ac-19b^{2}\right)x^{4}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{e^{2}\left(420b\,c^{2}d^{4}-40a\,c^{2}d^{2}+\cdots\right.}{8\left(16a^{2}c^{2}-\cdots\right.}}{(c\,e\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-21/2*c^2*d*e^5*b/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^6+1/8*(-252*b*c*d^2+4*a*c-19*b^2)*c*e^4/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^5+5/8*c*d*e^3*(-84*b*c*d^2+4*a*c-19*b^2)/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^4-1/8*e^2*(420*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+16*a*b*c+5*b^3)/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(252*b*c^2*d^4-40*a*c^2*d^2+190*b^2*
c*d^2+48*a*b*c+15*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(84*b*c^2*d^6-20*
a*c^2*d^4+95*b^2*c*d^4+48*a*b*c*d^2+15*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x-1/8*d/e*(12*b*c^2*d^6-4*a*c^2*d^4+19*b^2*c*d^4+16*a*b*c*
d^2+5*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+
3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*d*e-4*b*c*d
^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)
*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c
*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(12*b*c^2*x^7*e^7 + 84*b*c^2*d*x^6*e^6 + 12*b*c^2*d^7 + (19*b^2*c - 4*
a*c^2)*d^5 + (252*b*c^2*d^2*e^5 + 19*b^2*c*e^5 - 4*a*c^2*e^5)*x^5 + 5*(84*b
*c^2*d^3*e^4 + (19*b^2*c*e^4 - 4*a*c^2*e^4)*d)*x^4 + (5*b^3 + 16*a*b*c)*d^3
 + (420*b*c^2*d^4*e^3 + 5*b^3*e^3 + 16*a*b*c*e^3 + 10*(19*b^2*c*e^3 - 4*a*c
^2*e^3)*d^2)*x^3 + (252*b*c^2*d^5*e^2 + 10*(19*b^2*c*e^2 - 4*a*c^2*e^2)*d^3
 + 3*(5*b^3*e^2 + 16*a*b*c*e^2)*d)*x^2 + 3*(a*b^2 + 4*a^2*c)*d + (84*b*c^2*
d^6*e + 5*(19*b^2*c*e - 4*a*c^2*e)*d^4 + 3*a*b^2*e + 12*a^2*c*e + 3*(5*b^3*
e + 16*a*b*c*e)*d^2)*x)/((b^4*c^2*e - 8*a*b^2*c^3*e + 16*a^2*c^4*e)*d^8 + 8
*(b^4*c^2*e^8 - 8*a*b^2*c^3*e^8 + 16*a^2*c^4*e^8)*d*x^7 + (b^4*c^2*e^9 - 8*
```

```
a*b^2*c^3*e^9 + 16*a^2*c^4*e^9)*x^8 + 2*(b^5*c*e - 8*a*b^3*c^2*e + 16*a^2*b
*c^3*e)*d^6 + 2*(b^5*c*e^7 - 8*a*b^3*c^2*e^7 + 16*a^2*b*c^3*e^7 + 14*(b^4*c
^2*e^7 - 8*a*b^2*c^3*e^7 + 16*a^2*c^4*e^7)*d^2)*x^6 + a^2*b^4*e - 8*a^3*b^2
*c*e + 16*a^4*c^2*e + 4*(14*(b^4*c^2*e^6 - 8*a*b^2*c^3*e^6 + 16*a^2*c^4*e^6
)*d^3 + 3*(b^5*c*e^6 - 8*a*b^3*c^2*e^6 + 16*a^2*b*c^3*e^6)*d)*x^5 + (b^6*e
- 6*a*b^4*c*e + 32*a^3*c^3*e)*d^4 + (b^6*e^5 - 6*a*b^4*c*e^5 + 32*a^3*c^3*e
^5 + 70*(b^4*c^2*e^5 - 8*a*b^2*c^3*e^5 + 16*a^2*c^4*e^5)*d^4 + 30*(b^5*c*e^
5 - 8*a*b^3*c^2*e^5 + 16*a^2*b*c^3*e^5)*d^2)*x^4 + 4*(14*(b^4*c^2*e^4 - 8*a
*b^2*c^3*e^4 + 16*a^2*c^4*e^4)*d^5 + 10*(b^5*c*e^4 - 8*a*b^3*c^2*e^4 + 16*a
^2*b*c^3*e^4)*d^3 + (b^6*e^4 - 6*a*b^4*c*e^4 + 32*a^3*c^3*e^4)*d)*x^3 + 2*(
a*b^5*e - 8*a^2*b^3*c*e + 16*a^3*b*c^2*e)*d^2 + 2*(14*(b^4*c^2*e^3 - 8*a*b^
2*c^3*e^3 + 16*a^2*c^4*e^3)*d^6 + a*b^5*e^3 - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^
2*e^3 + 15*(b^5*c*e^3 - 8*a*b^3*c^2*e^3 + 16*a^2*b*c^3*e^3)*d^4 + 3*(b^6*e^
3 - 6*a*b^4*c*e^3 + 32*a^3*c^3*e^3)*d^2)*x^2 + 4*(2*(b^4*c^2*e^2 - 8*a*b^2*
c^3*e^2 + 16*a^2*c^4*e^2)*d^7 + 3*(b^5*c*e^2 - 8*a*b^3*c^2*e^2 + 16*a^2*b*c
^3*e^2)*d^5 + (b^6*e^2 - 6*a*b^4*c*e^2 + 32*a^3*c^3*e^2)*d^3 + (a*b^5*e^2 -
 8*a^2*b^3*c*e^2 + 16*a^3*b*c^2*e^2)*d)*x) - 3/8*integrate((4*b*c*x^2*e^2 +
 8*b*c*d*x*e + 4*b*c*d^2 - b^2 - 4*a*c)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4
+ b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(b^4
 - 8*a*b^2*c + 16*a^2*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6493 vs. $2(299) = 598$.
time = 0.50, size = 6493, normalized size = 19.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*b*c^2*x^7*e^7 + 168*b*c^2*d*x^6*e^6 + 24*b*c^2*d^7 + 2*(252*b*c^2
*d^2 + 19*b^2*c - 4*a*c^2)*x^5*e^5 + 2*(19*b^2*c - 4*a*c^2)*d^5 + 10*(84*b*
c^2*d^3 + (19*b^2*c - 4*a*c^2)*d)*x^4*e^4 + 2*(420*b*c^2*d^4 + 5*b^3 + 16*a
*b*c + 10*(19*b^2*c - 4*a*c^2)*d^2)*x^3*e^3 + 2*(5*b^3 + 16*a*b*c)*d^3 + 2*
(252*b*c^2*d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5*b^3 + 16*a*b*c)*d)*x^2*
e^2 + 2*(84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*d^4 + 3*a*b^2 + 12*a^2*c + 3
*(5*b^3 + 16*a*b*c)*d^2)*x*e + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2
*c^4)*x^8*e^9 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*x^7*e^8 + 2*(b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2
)*x^6*e^7 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*d)*x^5*e^6 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b
^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
*c^3)*d^2)*x^4*e^5 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*
x^3*e^4 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^
```

3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*x^2*e^3 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*x*e^2 + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*e^(-2)/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x*e + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d + 3/2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*e^(-2)/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8*e^9 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*x^7*e^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*x^6*e^7 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d)*x^5*e^6 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*x^4*e^5 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*x^3*e^4 + 2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^2)*x^2*e^3 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*x*e^2 + ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*e^(-2)/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x*e + 3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d - 3/2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*e - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*

```
c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*a
*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*
b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 1
60*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*e^(-2)
/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*
c^4 - 1024*a^6*c^5))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x
^8*e^9 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1688 vs. 2(299) = 598.
time = 3.54, size = 1688, normalized size = 4.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/16*((4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b*c*e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*log(d*e^(-1) + x + sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) +
 sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c
*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + 4*b*c*d^2 - b^2 -
4*a*c)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e
^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e
^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (4*(d
*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*
e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(
b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b
```

```
*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*
d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))) + (4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + 4*b*c*d^2 - b^2 - 4*a*c)*log(
d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(
2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*
c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqr
t(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^4 - 8*a*b^2*c
+ 16*a^2*c^2) - 1/8*(12*b*c^2*x^7*e^7 + 84*b*c^2*d*x^6*e^6 + 252*b*c^2*d^2*
x^5*e^5 + 420*b*c^2*d^3*x^4*e^4 + 420*b*c^2*d^4*x^3*e^3 + 252*b*c^2*d^5*x^2
*e^2 + 84*b*c^2*d^6*x*e + 12*b*c^2*d^7 + 19*b^2*c*x^5*e^5 - 4*a*c^2*x^5*e^5
 + 95*b^2*c*d*x^4*e^4 - 20*a*c^2*d*x^4*e^4 + 190*b^2*c*d^2*x^3*e^3 - 40*a*c
^2*d^2*x^3*e^3 + 190*b^2*c*d^3*x^2*e^2 - 40*a*c^2*d^3*x^2*e^2 + 95*b^2*c*d^
4*x*e - 20*a*c^2*d^4*x*e + 19*b^2*c*d^5 - 4*a*c^2*d^5 + 5*b^3*x^3*e^3 + 16*
a*b*c*x^3*e^3 + 15*b^3*d*x^2*e^2 + 48*a*b*c*d*x^2*e^2 + 15*b^3*d^2*x*e + 48
*a*b*c*d^2*x*e + 5*b^3*d^3 + 16*a*b*c*d^3 + 3*a*b^2*x*e + 12*a^2*c*x*e + 3*
a*b^2*d + 12*a^2*c*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d
^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^4*e - 8*a*b^2*c*e
+ 16*a^2*c^2*e))
```

Mupad [B]
time = 7.02, size = 2500, normalized size = 7.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] atan(((((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 - 1
5360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12)/
(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b
^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + ((1024*b^15*c^2*d*e^13 - 28672*
a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13
- 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^
5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240
*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*
a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7
*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^
6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b
^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b
^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a
^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^
```

2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*1i + ((18432*a^4*c^7*d*e^11 + 936*b^8*c^3*d*e^11 - 6912*a*b^6*c^4*d*e^11 + 11520*a^2*b^4*c^5*d*e^11)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((786432*a^6*c^8*e^12 - 192*b^12*c^2*e^12 + 3072*a*b^10*c^3*e^12 - 15360*a^2*b^8*c^4*e^12 + 245760*a^4*b^4*c^6*e^12 - 786432*a^5*b^2*c^7*e^12)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) - ((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2))*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*

```
e^2)))^(1/2) + (x*(144*a^2*c^5*e^12 + 117*b^4*c^3*e^12 + 72*a*b^2*c^4*e^12)
)/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))
*((9*((-(4*a*c - b^2)^15)^(1/2) - b^15 + 81920*a^7*b*c^7 + 560*a^2*b^11*c^2
 - 4160*a^3*b^9*c^3 + 11520*a^4*b^7*c^4 + 1024*a^5*b^5*c^5 - 61440*a^6*b^3*
c^6 - 20*a*b^13*c))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*
c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e
^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7
*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b...
```

3.631 $\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=150

$$\frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3b\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+}{\sqrt{}}\right.}{(b^2-4ac}$$

```
[Out] 1/4*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2-3/4*b*(b
+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+3*b*c*arctanh(
(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Rubi [A]
time = 0.13, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1156, 1128, 652, 628, 632, 212}

$$\frac{2a+b(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{3b\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3bc\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (2*a + b*(d + e*x)^2)/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)^2) - (3*b*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)) + (3*b*c*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]]
)/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(d+ex)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^3}{(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(3b)\text{Subst}\left(\int \frac{1}{(a+bx+\cdots}\right.}{4\left(b^2-\cdots\right.}$$

$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b(b+\cdots}{4\left(b^2-4ac\right)^2e\left(a+\cdots\right.}$$

$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b(b+\cdots}{4\left(b^2-4ac\right)^2e\left(a+\cdots\right.}$$

$$= \frac{2a+b(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3b(b+\cdots}{4\left(b^2-4ac\right)^2e\left(a+\cdots\right.}$$

Mathematica [A]

time = 0.15, size = 146, normalized size = 0.97

$$\frac{-\frac{3b(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4}+\frac{(b^2-4ac)(2a+b(d+ex)^2)}{(a+(d+ex)^2(b+c(d+ex)^2))^2}-\frac{12bc\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4\left(b^2-4ac\right)^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((b^2 -
 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2 - (1
2*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])
/(4*(b^2 - 4*a*c)^2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.21, size = 544, normalized size = 3.63

method	result
default	$\frac{-\frac{3c^2e^5b\,x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{9e^4b\,c^2d\,x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{9bc\,e^3\left(10c\,d^2+b\right)x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3d\,e^2cb\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{be\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{db\left(9c^2d^4+9b\cdots\right.}{16a^2c^2-\cdots}}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a\right)^2}$
risch	$\frac{-\frac{3c^2e^5b\,x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{9e^4b\,c^2d\,x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{9bc\,e^3\left(10c\,d^2+b\right)x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3d\,e^2cb\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{be\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{db\left(9c^2d^4+9b\cdots\right.}{16a^2c^2-\cdots}}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*e^4*b*c^2*d/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^5-9/4*b*c*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-3*
d*e^2*c*b*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*b*e*(45*c^2*d^4
+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-d*b*(9*c^2*d^4+9*b*c*
d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/4/e*(6*b*c^2*d^6+9*b^2*c*d^4+
10*a*b*c*d^2+2*b^3*d^2+8*a^2*c+a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^
4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2
+a)^2+3/2*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R
^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4
*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] -3*b*c*integrate((x*e + d)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*
c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(b^4 - 8*a*b^2*c
+ 16*a^2*c^2) - 1/4*(6*b*c^2*x^6*e^6 + 36*b*c^2*d*x^5*e^5 + 6*b*c^2*d^6 + 9
*b^2*c*d^4 + 9*(10*b*c^2*d^2*e^4 + b^2*c*e^4)*x^4 + 12*(10*b*c^2*d^3*e^3 +
3*b^2*c*d*e^3)*x^3 + a*b^2 + 8*a^2*c + 2*(b^3 + 5*a*b*c)*d^2 + 2*(45*b*c^2*
d^4*e^2 + 27*b^2*c*d^2*e^2 + b^3*e^2 + 5*a*b*c*e^2)*x^2 + 4*(9*b*c^2*d^5*e
+ 9*b^2*c*d^3*e + (b^3*e + 5*a*b*c*e)*d)*x)/((b^4*c^2*e - 8*a*b^2*c^3*e + 1
6*a^2*c^4*e)*d^8 + 8*(b^4*c^2*e^8 - 8*a*b^2*c^3*e^8 + 16*a^2*c^4*e^8)*d*x^7
 + (b^4*c^2*e^9 - 8*a*b^2*c^3*e^9 + 16*a^2*c^4*e^9)*x^8 + 2*(b^5*c*e - 8*a*
b^3*c^2*e + 16*a^2*b*c^3*e)*d^6 + 2*(b^5*c*e^7 - 8*a*b^3*c^2*e^7 + 16*a^2*b
*c^3*e^7 + 14*(b^4*c^2*e^7 - 8*a*b^2*c^3*e^7 + 16*a^2*c^4*e^7)*d^2)*x^6 + a
^2*b^4*e - 8*a^3*b^2*c*e + 16*a^4*c^2*e + 4*(14*(b^4*c^2*e^6 - 8*a*b^2*c^3*
e^6 + 16*a^2*c^4*e^6)*d^3 + 3*(b^5*c*e^6 - 8*a*b^3*c^2*e^6 + 16*a^2*b*c^3*e
^6)*d)*x^5 + (b^6*e - 6*a*b^4*c*e + 32*a^3*c^3*e)*d^4 + (b^6*e^5 - 6*a*b^4*
c*e^5 + 32*a^3*c^3*e^5 + 70*(b^4*c^2*e^5 - 8*a*b^2*c^3*e^5 + 16*a^2*c^4*e^5
)*d^4 + 30*(b^5*c*e^5 - 8*a*b^3*c^2*e^5 + 16*a^2*b*c^3*e^5)*d^2)*x^4 + 4*(1
4*(b^4*c^2*e^4 - 8*a*b^2*c^3*e^4 + 16*a^2*c^4*e^4)*d^5 + 10*(b^5*c*e^4 - 8*
a*b^3*c^2*e^4 + 16*a^2*b*c^3*e^4)*d^3 + (b^6*e^4 - 6*a*b^4*c*e^4 + 32*a^3*c
^3*e^4)*d)*x^3 + 2*(a*b^5*e - 8*a^2*b^3*c*e + 16*a^3*b*c^2*e)*d^2 + 2*(14*(
b^4*c^2*e^3 - 8*a*b^2*c^3*e^3 + 16*a^2*c^4*e^3)*d^6 + a*b^5*e^3 - 8*a^2*b^3
*c*e^3 + 16*a^3*b*c^2*e^3 + 15*(b^5*c*e^3 - 8*a*b^3*c^2*e^3 + 16*a^2*b*c^3*
e^3)*d^4 + 3*(b^6*e^3 - 6*a*b^4*c*e^3 + 32*a^3*c^3*e^3)*d^2)*x^2 + 4*(2*(b^
4*c^2*e^2 - 8*a*b^2*c^3*e^2 + 16*a^2*c^4*e^2)*d^7 + 3*(b^5*c*e^2 - 8*a*b^3*
c^2*e^2 + 16*a^2*b*c^3*e^2)*d^5 + (b^6*e^2 - 6*a*b^4*c*e^2 + 32*a^3*c^3*e^2
)*d^3 + (a*b^5*e^2 - 8*a^2*b^3*c*e^2 + 16*a^3*b*c^2*e^2)*d)*x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1787 vs. $2(146) = 292$.
time = 0.41, size = 3701, normalized size = 24.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*x^6*e^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*x^5*e^5
 + 6*(b^3*c^2 - 4*a*b*c^3)*d^6 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a
*b*c^3)*d^2)*x^4*e^4 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^
2*c^2)*d^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*
x^3*e^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 +
27*(b^4*c - 4*a*b^2*c^2)*d^2)*x^2*e^2 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^
2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a
```

*b^3*c - 20*a^2*b*c^2)*d)*x*e - 6*(b*c^3*x^8*e^8 + 8*b*c^3*d*x^7*e^7 + b*c^3*d^8 + 2*b^2*c^2*d^6 + 2*(14*b*c^3*d^2 + b^2*c^2)*x^6*e^6 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*x^5*e^5 + 2*a*b^2*c*d^2 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*x^4*e^4 + (b^3*c + 2*a*b*c^2)*d^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*x^3*e^3 + a^2*b*c + 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*x^2*e^2 + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*x*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8*e^9 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^7*e^8 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*x^6*e^7 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*x^5*e^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*x^4*e^5 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*x^3*e^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*x^2*e^3 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*x*e^2 + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e), -1/4*(6*(b^3*c^2 - 4*a*b*c^3)*x^6*e^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*x^5*e^5 + 6*(b^3*c^2 - 4*a*b*c^3)*d^6 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*x^4*e^4 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4*a*b^2*c^2)*d^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*x^3*e^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*x^2*e^2 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*x*e - 12*(b*c^3*x^8*e^8 + 8*b*c^3*d*x^7*e^7 + b*c^3*d^8 + 2*b^2*c^2*d^6 + 2*(14*b*c^3*d^2 + b^2*c^2)*x^6*e^6 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*x^5*e^5 + 2*a*b^2*c*d^2 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*x^4*e^4 + (b^3*c + 2*a*b*c^2)*d^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*x^3*e^3 +

```
 a^2*b*c + 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^
2)*d^2)*x^2*e^2 + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a
*b*c^2)*d^3)*x*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 + 4*c*d*x*e + 2*c
*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*
a^2*b^2*c^4 - 64*a^3*c^5)*x^8*e^9 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d*x^7*e^8 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64
*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)
*x^6*e^7 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3
 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*x^5*e^6 + (b
^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5
*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*x^4*e^5 + 4*(14*(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a
^3*b^2*c^3 - 128*a^4*c^4)*d)*x^3*e^4 + 2*(a*b^7...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1671 vs. $2(134) = 268$.
time = 7.81, size = 1671, normalized size = 11.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*b*c**4*sqr
t(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) - 3
6*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqrt(-1/(4*a*c - b**2)*
*5) + 3*b**2*c + 6*b*c**2*d**2)/(6*b*c**2*e**2))/(2*e) - 3*b*c*sqrt(-1/(4*a
*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*b*c**4*sqrt(-1/(4*a*c - b**2)
**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**2*sqrt(
-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c + 6*
b*c**2*d**2)/(6*b*c**2*e**2))/(2*e) + (-8*a**2*c - a*b**2 - 10*a*b*c*d**2 -
 2*b**3*d**2 - 9*b**2*c*d**4 - 6*b*c**2*d**6 - 36*b*c**2*d*e**5*x**5 - 6*b*
c**2*e**6*x**6 + x**4*(-9*b**2*c*e**4 - 90*b*c**2*d**2*e**4) + x**3*(-36*b*
*2*c*d*e**3 - 120*b*c**2*d**3*e**3) + x**2*(-10*a*b*c*e**2 - 2*b**3*e**2 -
54*b**2*c*d**2*e**2 - 90*b*c**2*d**4*e**2) + x*(-20*a*b*c*d*e - 4*b**3*d*e
- 36*b**2*c*d**3*e - 36*b*c**2*d**5*e))/(64*a**4*c**2*e - 32*a**3*b**2*c*e
+ 128*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b
**3*c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2
*e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4
*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e*
*9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 -
 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7
 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e*
```

```
*7 + 8*b**5*c*e**7 + 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**
6 + 3584*a**2*c**4*d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d*
*3*e**6 + 48*b**5*c*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3
*e**5 + 1920*a**2*b*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c
*e**5 - 960*a*b**3*c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**
5 + 120*b**5*c*d**2*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d
*e**4 + 2560*a**2*b*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c
*d*e**4 - 1280*a*b**3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6
*d*e**4 + 160*b**5*c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*
b*c**2*e**3 + 768*a**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c
**3*d**4*e**3 + 1792*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**
2*e**3 - 960*a*b**3*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d*
*2*e**3 + 120*b**5*c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c
**2*d*e**2 + 512*a**3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*
c**3*d**5*e**2 + 512*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d
**3*e**2 - 384*a*b**3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*
d**3*e**2 + 48*b**5*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(146) = 292$.
time = 4.02, size = 365, normalized size = 2.43

$$-\frac{3\,bc\arctan\left(\frac{2\,cd^{2}+2\,(x^{2}e+2\,dx)ce+b}{\sqrt{-b^{2}+4\,ac}}\right)e^{(-1)}}{(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2})\sqrt{-b^{2}+4\,ac}}-\frac{6\,bc^{2}d^{6}+18\,(x^{2}e+2\,dx)bc^{2}d^{4}e+18\,(x^{2}e+2\,dx)^{2}bc^{2}d^{2}e^{2}+9\,b^{2}cd^{4}+6\,(x^{2}e+2\,dx)^{3}bc^{2}e^{3}+18\,(x^{2}e+2\,dx)b^{2}cd^{2}e+9\,(x^{2}e+2\,dx)^{2}b^{2}ce^{2}+2\,b^{3}d^{2}+10\,abcd^{2}+2\,(x^{2}e+2\,dx)b^{3}e+10\,(x^{2}e+2\,dx)abce+ab^{2}+8\,a^{2}c}{4\,(cd^{4}+2\,(x^{2}e+2\,dx)cd^{2}e+(x^{2}e+2\,dx)^{2}ce^{2}+bd^{2}+(x^{2}e+2\,dx)be+a)^{2}(b^{4}e-8\,ab^{2}ce+16\,a^{2}c^{2}e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(
-1)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*b*c^2*d^6
+ 18*(x^2*e + 2*d*x)*b*c^2*d^4*e + 18*(x^2*e + 2*d*x)^2*b*c^2*d^2*e^2 + 9*b
^2*c*d^4 + 6*(x^2*e + 2*d*x)^3*b*c^2*e^3 + 18*(x^2*e + 2*d*x)*b^2*c*d^2*e +
 9*(x^2*e + 2*d*x)^2*b^2*c*e^2 + 2*b^3*d^2 + 10*a*b*c*d^2 + 2*(x^2*e + 2*d*
x)*b^3*e + 10*(x^2*e + 2*d*x)*a*b*c*e + a*b^2 + 8*a^2*c)/((c*d^4 + 2*(x^2*e
 + 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e +
 a)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

Mupad [**B**]
time = 3.85, size = 1182, normalized size = 7.88

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] - ((9*x^4*(b^2*c*e^3 + 10*b*c^2*d^2*e^3))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) + (a*b^2 + 8*a^2*c + 2*b^3*d^2 + 9*b^2*c*d^4 + 6*b*c^2*d^6 + 10*a*b*c*d^2
```

```
)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3*e + 27*b^2*c*d^2*e + 45*
b*c^2*d^4*e + 5*a*b*c*e))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3*(3*
b^2*c*e^2 + 10*b*c^2*d^2*e^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(b^3 +
 9*b^2*c*d^2 + 9*b*c^2*d^4 + 5*a*b*c))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*
b*c^2*e^5*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4*x^5)/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e
^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) +
x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^
3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c
^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30
*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d
^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*atan(((b^4*(4*a*c - b^2)^5 + 16*
a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x^2*((9*b^2*c^4*e^8)/
(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*(2*b^5*
c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*c^4*e^10))/(2*a*e^2*(4*a*c - b^2)^(
15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((18*b^2*c^4*d*e^7)/(a*(4*a*c -
b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*(2*b^5*c^2*d*e^9 -
16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4
+ 16*a^2*c^2 - 8*a*b^2*c))) + (9*b^3*c^2*(64*a^3*c^4*e^8 + 4*a*b^4*c^2*e^8
- 32*a^2*b^2*c^3*e^8 + 2*b^5*c^2*d^2*e^8 - 16*a*b^3*c^3*d^2*e^8 + 32*a^2*b*
c^4*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
 + (9*b^2*c^4*d^2*e^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
))))/(18*b^2*c^4*e^6)))/(e*(4*a*c - b^2)^(5/2))
```

3.632 $\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=363

$$-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{(d+ex)\left(b\left(b^2+8ac\right)+c\left(b^2+20ac\right)(d+ex)^2\right)}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(b^2+2\right.}{}$$

```
[Out] -1/4*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2
+1/8*(e*x+d)*(b*(8*a*c+b^2)+c*(20*a*c+b^2)*(e*x+d)^2)/a/(-4*a*c+b^2)^2/e/(a
+b*(e*x+d)^2+c*(e*x+d)^4)+1/16*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^
2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c+b*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a/
(-4*a*c+b^2)^2/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan((e*x+d)*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c-b*(-52*a*c
+b^2)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^2/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))
^(1/2)
```

Rubi [A]

time = 0.71, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1156, 1133, 1192, 1180, 211}

$$\frac{\sqrt{c}\left(\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}ae\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{(d+ex)\left(c\left(20ac+b^2\right)(d+ex)^2+b\left(8ac+b^2\right)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -1/4*((d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + ((d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*(d + e
*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt
[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^
2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*
c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(d+ex)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{\text{Subst}\left(\int \frac{b-10cx^2}{(a+bx^2+cx^4)}\right.}{4\left(b^2-4\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b(b^2+8a\right.}{8a\left(b^2-4ac\right)^2e\left(a\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b(b^2+8a\right.}{8a\left(b^2-4ac\right)^2e\left(a\right.}$$

$$= -\frac{(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{(d+ex)\left(b(b^2+8a\right.}{8a\left(b^2-4ac\right)^2e\left(a\right.}$$

Mathematica [A]
time = 3.21, size = 382, normalized size = 1.05

$$\frac{-\frac{4(b(d+ex)+2c(d+ex)^3)}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{2(d+ex)(b^3+8abc+b^2c(d+ex)^2+20ac^2(d+ex)^2)}{a(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b^3+52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}}{16e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] ((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 20*a*c
^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
(Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^
3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.25, size = 885, normalized size = 2.44

method	result

default	$\frac{\frac{c^2e^6\left(20ac+b^2\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{7c^2d\,e^5\left(20ac+b^2\right)x^6}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{\left(420a\,c^2d^2+21b^2c\,d^2+28abc+2b^3\right)c\,e^4x^5}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{5cd\,e^3\left(140a\,c^2d^2+7b^2c\,d^2+28abc+2b^3\right)x^4}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{e^2(}{}}{}$
risch	$\frac{\frac{c^2e^6\left(20ac+b^2\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{7c^2d\,e^5\left(20ac+b^2\right)x^6}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{\left(420a\,c^2d^2+21b^2c\,d^2+28abc+2b^3\right)c\,e^4x^5}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{5cd\,e^3\left(140a\,c^2d^2+7b^2c\,d^2+28abc+2b^3\right)x^4}{8\left(16a^2c^2-8a\,b^2c+b^4\right)a}+\frac{e^2(}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*c^2*e^6*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7+7/8*c^2*d*e^5*(2
0*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6+1/8*(420*a*c^2*d^2+21*b^2*c*d^2
+28*a*b*c+2*b^3)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5+5/8*c*d*e^3*(140*a*
c^2*d^2+7*b^2*c*d^2+28*a*b*c+2*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4+1/8*e^
2*(700*a*c^3*d^4+35*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+36*a^2*c^2+5*a
*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3+1/8*d*e*(420*a*c^3*d^4+21*b^2*
c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+108*a^2*c^2+15*a*b^2*c+3*b^4)/(16*a^2*
c^2-8*a*b^2*c+b^4)/a*x^2+1/8*(140*a*c^3*d^6+7*b^2*c^2*d^6+140*a*b*c^2*d^4+1
0*b^3*c*d^4+108*a^2*c^2*d^2+15*a*b^2*c*d^2+3*b^4*d^2+16*a^2*b*c-a*b^3)/(16*
a^2*c^2-8*a*b^2*c+b^4)/a*x+1/8*d/e*(20*a*c^3*d^6+b^2*c^2*d^6+28*a*b*c^2*d^4
+2*b^3*c*d^4+36*a^2*c^2*d^2+5*a*b^2*c*d^2+b^4*d^2+16*a^2*b*c-a*b^3)/(16*a^2
*c^2-8*a*b^2*c+b^4)/a)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x
+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/a/e*s
um((c*e^2*(20*a*c+b^2)*_R^2+2*c*d*e*(20*a*c+b^2)*_R+20*a*c^2*d^2+b^2*c*d^2-
16*a*b*c+b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)
*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c
*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^2*c^2 + 20*a*c^3)*d^7 + 7*(b^2*c^2*e^6 + 20*a*c^3*e^6)*d*x^6 + (b^2
*c^2*e^7 + 20*a*c^3*e^7)*x^7 + 2*(b^3*c + 14*a*b*c^2)*d^5 + (2*b^3*c*e^5 +
28*a*b*c^2*e^5 + 21*(b^2*c^2*e^5 + 20*a*c^3*e^5)*d^2)*x^5 + 5*(7*(b^2*c^2*e
^4 + 20*a*c^3*e^4)*d^3 + 2*(b^3*c*e^4 + 14*a*b*c^2*e^4)*d)*x^4 + (b^4 + 5*a
*b^2*c + 36*a^2*c^2)*d^3 + (35*(b^2*c^2*e^3 + 20*a*c^3*e^3)*d^4 + b^4*e^3 +
 5*a*b^2*c*e^3 + 36*a^2*c^2*e^3 + 20*(b^3*c*e^3 + 14*a*b*c^2*e^3)*d^2)*x^3
```

```
+ (21*(b^2*c^2*e^2 + 20*a*c^3*e^2)*d^5 + 20*(b^3*c*e^2 + 14*a*b*c^2*e^2)*d^
3 + 3*(b^4*e^2 + 5*a*b^2*c*e^2 + 36*a^2*c^2*e^2)*d)*x^2 - (a*b^3 - 16*a^2*b
*c)*d + (7*(b^2*c^2*e + 20*a*c^3*e)*d^6 + 10*(b^3*c*e + 14*a*b*c^2*e)*d^4 -
 a*b^3*e + 16*a^2*b*c*e + 3*(b^4*e + 5*a*b^2*c*e + 36*a^2*c^2*e)*d^2)*x)/((
a*b^4*c^2*e - 8*a^2*b^2*c^3*e + 16*a^3*c^4*e)*d^8 + 8*(a*b^4*c^2*e^8 - 8*a^
2*b^2*c^3*e^8 + 16*a^3*c^4*e^8)*d*x^7 + (a*b^4*c^2*e^9 - 8*a^2*b^2*c^3*e^9
+ 16*a^3*c^4*e^9)*x^8 + a^3*b^4*e - 8*a^4*b^2*c*e + 16*a^5*c^2*e + 2*(a*b^5
*c*e - 8*a^2*b^3*c^2*e + 16*a^3*b*c^3*e)*d^6 + 2*(a*b^5*c*e^7 - 8*a^2*b^3*c
^2*e^7 + 16*a^3*b*c^3*e^7 + 14*(a*b^4*c^2*e^7 - 8*a^2*b^2*c^3*e^7 + 16*a^3*
c^4*e^7)*d^2)*x^6 + 4*(14*(a*b^4*c^2*e^6 - 8*a^2*b^2*c^3*e^6 + 16*a^3*c^4*e
^6)*d^3 + 3*(a*b^5*c*e^6 - 8*a^2*b^3*c^2*e^6 + 16*a^3*b*c^3*e^6)*d)*x^5 + (
a*b^6*e - 6*a^2*b^4*c*e + 32*a^4*c^3*e)*d^4 + (a*b^6*e^5 - 6*a^2*b^4*c*e^5
+ 32*a^4*c^3*e^5 + 70*(a*b^4*c^2*e^5 - 8*a^2*b^2*c^3*e^5 + 16*a^3*c^4*e^5)*
d^4 + 30*(a*b^5*c*e^5 - 8*a^2*b^3*c^2*e^5 + 16*a^3*b*c^3*e^5)*d^2)*x^4 + 4*
(14*(a*b^4*c^2*e^4 - 8*a^2*b^2*c^3*e^4 + 16*a^3*c^4*e^4)*d^5 + 10*(a*b^5*c*
e^4 - 8*a^2*b^3*c^2*e^4 + 16*a^3*b*c^3*e^4)*d^3 + (a*b^6*e^4 - 6*a^2*b^4*c*
e^4 + 32*a^4*c^3*e^4)*d)*x^3 + 2*(a^2*b^5*e - 8*a^3*b^3*c*e + 16*a^4*b*c^2*
e)*d^2 + 2*(a^2*b^5*e^3 - 8*a^3*b^3*c*e^3 + 16*a^4*b*c^2*e^3 + 14*(a*b^4*c^
2*e^3 - 8*a^2*b^2*c^3*e^3 + 16*a^3*c^4*e^3)*d^6 + 15*(a*b^5*c*e^3 - 8*a^2*b
^3*c^2*e^3 + 16*a^3*b*c^3*e^3)*d^4 + 3*(a*b^6*e^3 - 6*a^2*b^4*c*e^3 + 32*a^
4*c^3*e^3)*d^2)*x^2 + 4*(2*(a*b^4*c^2*e^2 - 8*a^2*b^2*c^3*e^2 + 16*a^3*c^4*
e^2)*d^7 + 3*(a*b^5*c*e^2 - 8*a^2*b^3*c^2*e^2 + 16*a^3*b*c^3*e^2)*d^5 + (a*
b^6*e^2 - 6*a^2*b^4*c*e^2 + 32*a^4*c^3*e^2)*d^3 + (a^2*b^5*e^2 - 8*a^3*b^3*
c*e^2 + 16*a^4*b*c^2*e^2)*d)*x) + 1/8*integrate((b^3 - 16*a*b*c + (b^2*c +
20*a*c^2)*d^2 + 2*(b^2*c*e + 20*a*c^2*e)*d*x + (b^2*c*e^2 + 20*a*c^2*e^2)*x
^2)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2
+ 2*(2*c*d^3*e + b*d*e)*x + a), x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7597 vs. $2(325) = 650$.
time = 0.64, size = 7597, normalized size = 20.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*x^7*e^7 + 14*(b^2*c^2 + 20*a*c^3)*d*x^6*e^6 +
2*(b^2*c^2 + 20*a*c^3)*d^7 + 2*(2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c
^3)*d^2)*x^5*e^5 + 4*(b^3*c + 14*a*b*c^2)*d^5 + 10*(7*(b^2*c^2 + 20*a*c^3)*
d^3 + 2*(b^3*c + 14*a*b*c^2)*d)*x^4*e^4 + 2*(35*(b^2*c^2 + 20*a*c^3)*d^4 +
b^4 + 5*a*b^2*c + 36*a^2*c^2 + 20*(b^3*c + 14*a*b*c^2)*d^2)*x^3*e^3 + 2*(b^
4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 + 2*(21*(b^2*c^2 + 20*a*c^3)*d^5 + 20*(b^3*
c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d)*x^2*e^2 + 2*(7*(b
^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c +
```

3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*x*e - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*
b^2*c^3 + 16*a^3*c^4)*x^8*e^9 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*
d*x^7*e^8 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a
^2*b^2*c^3 + 16*a^3*c^4)*d^2)*x^6*e^7 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*x^5*e^6 + (
a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c
^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*x^4*e^5 + 4*(14*
(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2
+ 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*x^3*e^4 + 2*(a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3
*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*
a^2*b^4*c + 32*a^4*c^3)*d^2)*x^2*e^3 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16
*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6
*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*x*
e^2 + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6
*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2
)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10
 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 10
24*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c
+ 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*
e^(-2)/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*
a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b
^2*c^4 + 10000*a^3*c^5)*x*e + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*
c^4 + 10000*a^3*c^5)*d + 1/2*sqrt(1/2)*((a^3*b^14 - 38*a^4*b^12*c + 480*a^5
*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*
a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^
10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 -
 1024*a^11*c^5))*e - (b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^
3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a
^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 -
640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c +
 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3
+ 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*e^(-2)/(a^3*b^10 - 20*a^4*b^8*c + 16
0*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) + sqrt
(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8*e^9 + 8*(a*b^4*c^2 - 8*
a^2*b^2*c^3 + 16*a^3*c^4)*d*x^7*e^8 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b
c^3 + 14(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*x^6*e^7 + 4*(14*(a*
b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16
*a^3*b*c^3)*d)*x^5*e^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2
- 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*
c^3)*d^2)*x^4*e^5 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10
*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a
^4*c^3)*d)*x^3*e^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^
2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*

```
b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*x^2*e^3 + 4*(2*(a*b^
4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a
^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b
^3*c + 16*a^4*b*c^2)*d)*x*e^2 + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d
^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a
^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3
*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 +
1680*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c
^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)
/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b
^2*c^4 - 1024*a^11*c^5)))*e^(-2)/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2
 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 14
91*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x*e + (35*b^6*c^2 - 1491*
a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2295 vs. 2(325) = 650.
time = 3.38, size = 2295, normalized size = 6.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*b^2*c*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2*d^
2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4
*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
+ b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b^2*c*e^2 + 20*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
```

```
)*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) - sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2*d
^2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))^2*b^2*c*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*
c)*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) + sqrt(1/2)*sqrt(-(
b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2*
d^2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^
2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-
4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-
4)/c))^2*b^2*c*e^2 + 20*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a
*c)*e^2)*e^(-4)/c))^2*a*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*e - 40*(d*e^(-1) - sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*e + b^2*c*d^2 + 20*a*c^2
*d^2 + b^3 - 16*a*b*c)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e
^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + 1/8*(b^2*c^2*x^7*e^7 + 20*a*
c^3*x^7*e^7 + 7*b^2*c^2*d*x^6*e^6 + 140*a*c^3*d*x^6*e^6 + 21*b^2*c^2*d^2*x^
5*e^5 + 420*a*c^3*d^2*x^5*e^5 + 35*b^2*c^2*d^3*x^4*e^4 + 700*a*c^3*d^3*x^4*
e^4 + 35*b^2*c^2*d^4*x^3*e^3 + 700*a*c^3*d^4*x^3*e^3 + 21*b^2*c^2*d^5*x^2*e
^2 + 420*a*c^3*d^5*x^2*e^2 + 7*b^2*c^2*d^6*x*e + 140*a*c^3*d^6*x*e + b^2*c^
2*d^7 + 20*a*c^3*d^7 + 2*b^3*c*x^5*e^5 + 28*a*b*c^2*x^5*e^5 + 10*b^3*c*d*x^
4*e^4 + 140*a*b*c^2*d*x^4*e^4 + 20*b^3*c*d^2*x^3*e^3 + 280*a*b*c^2*d^2*x^3*
e^3 + 20*b^3*c*d^3*x^2*e^2 + 280*a*b*c^2*d^3*x^2*e^2 + 10*b^3*c*d^4*x*e + 1
40*a*b*c^2*d^4*x*e + 2*b^3*c*d^5 + 28*a*b*c^2*d^5 + b^4*x^3*e^3 + 5*a*b^2*c
*x^3*e^3 + 36*a^2*c^2*x^3*e^3 + 3*b^4*d*x^2*e^2 + 15*a*b^2*c*d*x^2*e^2 + 10
8*a^2*c^2*d*x^2*e^2 + 3*b^4*d^2*x*e + 15*a*b^2*c*d^2*x*e + 108*a^2*c^2*d^2*
x*e + b^4*d^3 + 5*a*b^2*c*d^3 + 36*a^2*c^2*d^3 - a*b^3*x*e + 16*a^2*b*c*x*e
 - a*b^3*d + 16*a^2*b*c*d)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 +
4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(a*b^4*e - 8*a^2
*b^2*c*e + 16*a^3*c^2*e))
```

Mupad [B]

time = 7.43, size = 2500, normalized size = 6.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((x^5*(2*b^3*c*e^4 + 420*a*c^3*d^2*e^4 + 21*b^2*c^2*d^2*e^4 + 28*a*b*c^2*e^
4))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(3*b^4*d*e + 21*b^2*c^2*d^5
*e + 108*a^2*c^2*d*e + 420*a*c^3*d^5*e + 20*b^3*c*d^3*e + 280*a*b*c^2*d^3*e
 + 15*a*b^2*c*d*e))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (7*x^6*(b^2*c^2*
d*e^5 + 20*a*c^3*d*e^5))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^7*(20*a*
c^3*e^6 + b^2*c^2*e^6))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(3*b^4*d^
2 - a*b^3 + 140*a*c^3*d^6 + 10*b^3*c*d^4 + 108*a^2*c^2*d^2 + 7*b^2*c^2*d^6
+ 16*a^2*b*c + 15*a*b^2*c*d^2 + 140*a*b*c^2*d^4))/(8*a*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)) + (x^3*(b^4*e^2 + 36*a^2*c^2*e^2 + 700*a*c^3*d^4*e^2 + 20*b^3*c
*d^2*e^2 + 35*b^2*c^2*d^4*e^2 + 5*a*b^2*c*e^2 + 280*a*b*c^2*d^2*e^2))/(8*a*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b^4*d^3 + 20*a*c^3*d^7 + 2*b^3*c*d^5 + 3
6*a^2*c^2*d^3 + b^2*c^2*d^7 - a*b^3*d + 16*a^2*b*c*d + 5*a*b^2*c*d^3 + 28*a
*b*c^2*d^5)/(8*a*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (5*x^4*(140*a*c^3*d^3*
e^3 + 7*b^2*c^2*d^3*e^3 + 2*b^3*c*d*e^3 + 28*a*b*c^2*d*e^3))/(8*a*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 +
12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*
b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*
b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^
3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*
d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 +
2*b*c*d^6 + 8*c^2*d*e^7*x^7) + atan(((((256*a*b^13*c^2*e^12 + 4194304*a^7*b
*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b
^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^
2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3
+ 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + ((67108864*a^9*b*c^9*d*e^13 - 409
6*a^2*b^15*c^2*d*e^13 + 114688*a^3*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d
*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384
*a^7*b^5*c^7*d*e^13 - 117440512*a^8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a
^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*
c^4 - 6144*a^7*b^2*c^5)) + (x*(262144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^1
4 + 5120*a^3*b^9*c^3*e^14 - 40960*a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^1
4 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 9
6*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2)
- 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^
9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a
*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^
13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*
e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6
```

*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12 - b^6*c^3*e^12 + 34*a*b^4*c^4*e^12 - 1472*a^2*b^2*c^5*e^12))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17 + b^2*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8 + 1140*a^2*b^13*c^2 - 10160*a^3*b^11*c^3 + 34880*a^4*b^9*c^4 + 43776*a^5*b^7*c^5 - 680960*a^6*b^5*c^6 + 1863680*a^7*b^3*c^7 - 55*a*b^15*c - 25*a*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)*1i + ((204800*a^5*c^8*d*e^11 - 16*b^10*c^3*d*e^11 + 672*a*b^8*c^4*d*e^11 - 28160*a^2*b^6*c^5*d*e^11 + 209920*a^3*b^4*c^6*d*e^11 - 479232*a^4*b^2*c^7*d*e^11)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((256*a*b^13*c^2*e^12 + 4194304*a^7*b*c^8*e^12 - 9216*a^2*b^11*c^3*e^12 + 122880*a^3*b^9*c^4*e^12 - 819200*a^4*b^7*c^5*e^12 + 2949120*a^5*b^5*c^6*e^12 - 5505024*a^6*b^3*c^7*e^12)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - ((671...

3.633 $\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} \, dx$

Optimal. Leaf size=152

$$\frac{-b-2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3c\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{6c^2\tanh^{-1}\left(\frac{b+}{\sqrt{b}}\right.}{\left(b^2-4ac\right.}$$

```
[Out] 1/4*(-b-2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+3/2*c*(
b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-6*c^2*arctanh
((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Rubi [A]

time = 0.13, antiderivative size = 150, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1156, 1121, 628, 632, 212}

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3c\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{b+2c(d+ex)^2}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -1/4*(b + 2*c*(d + e*x)^2)/((b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)^2) + (3*c*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) - (6*c^2*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*
c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{d+ex}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(3c)\text{Subst}\left(\int \frac{}{(a+bx}\right.}{2\left(b^2\right.}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c(b}{2\left(b^2-4ac\right)^2e(a}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c(b}{2\left(b^2-4ac\right)^2e(a}$$

$$= -\frac{b+2c(d+ex)^2}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3c(b}{2\left(b^2-4ac\right)^2e(a}$$

Mathematica [A]
time = 0.12, size = 147, normalized size = 0.97

$$\frac{\frac{(b^2-4ac)\left(-b-2c(d+ex)^2\right)}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{6c\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4}+\frac{24c^2\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4\left(b^2-4ac\right)^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4)
^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (24*
c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(
4*(b^2 - 4*a*c)^2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.18, size = 541, normalized size = 3.56

method	result
default	$\frac{\frac{3c^3e^5x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18e^4c^3d\,x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{ce\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cd\left(9c^2d^4+9bc\,d^2\cdots\right.}{16a^2c^2-8a\,b^2\cdots}}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a\right)^2}$
risch	$\frac{\frac{3c^3e^5x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18e^4c^3d\,x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{ce\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cd\left(9c^2d^4+9bc\,d^2\cdots\right.}{16a^2c^2-8a\,b^2\cdots}}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (3*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*e^4*c^3*d/(16*a^2*c^2-8*a*b^2*
c+b^4)*x^5+9/2*c^2*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+6*c^2*d*
e^2*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+c*e*(45*c^2*d^4+27*b*c*d^
2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+2*c*d*(9*c^2*d^4+9*b*c*d^2+5*a*
c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/4/e*(12*c^3*d^6+18*b*c^2*d^4+20*a*c^2
*d^2+4*b^2*c*d^2+10*a*b*c-b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d
*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3
*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2
+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_
Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 6*c^2*integrate((x*e + d)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c
*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(b^4 - 8*a*b^2*c +
 16*a^2*c^2) + 1/4*(12*c^3*x^6*e^6 + 72*c^3*d*x^5*e^5 + 12*c^3*d^6 + 18*b*c
^2*d^4 + 18*(10*c^3*d^2*e^4 + b*c^2*e^4)*x^4 + 24*(10*c^3*d^3*e^3 + 3*b*c^2
*d*e^3)*x^3 - b^3 + 10*a*b*c + 4*(b^2*c + 5*a*c^2)*d^2 + 4*(45*c^3*d^4*e^2
+ 27*b*c^2*d^2*e^2 + b^2*c*e^2 + 5*a*c^2*e^2)*x^2 + 8*(9*c^3*d^5*e + 9*b*c^
2*d^3*e + (b^2*c*e + 5*a*c^2*e)*d)*x)/((b^4*c^2*e - 8*a*b^2*c^3*e + 16*a^2*
```

```
c^4*e)*d^8 + 8*(b^4*c^2*e^8 - 8*a*b^2*c^3*e^8 + 16*a^2*c^4*e^8)*d*x^7 + (b^
4*c^2*e^9 - 8*a*b^2*c^3*e^9 + 16*a^2*c^4*e^9)*x^8 + 2*(b^5*c*e - 8*a*b^3*c^
2*e + 16*a^2*b*c^3*e)*d^6 + 2*(b^5*c*e^7 - 8*a*b^3*c^2*e^7 + 16*a^2*b*c^3*e
^7 + 14*(b^4*c^2*e^7 - 8*a*b^2*c^3*e^7 + 16*a^2*c^4*e^7)*d^2)*x^6 + a^2*b^4
*e - 8*a^3*b^2*c*e + 16*a^4*c^2*e + 4*(14*(b^4*c^2*e^6 - 8*a*b^2*c^3*e^6 +
16*a^2*c^4*e^6)*d^3 + 3*(b^5*c*e^6 - 8*a*b^3*c^2*e^6 + 16*a^2*b*c^3*e^6)*d)
*x^5 + (b^6*e - 6*a*b^4*c*e + 32*a^3*c^3*e)*d^4 + (b^6*e^5 - 6*a*b^4*c*e^5
+ 32*a^3*c^3*e^5 + 70*(b^4*c^2*e^5 - 8*a*b^2*c^3*e^5 + 16*a^2*c^4*e^5)*d^4
+ 30*(b^5*c*e^5 - 8*a*b^3*c^2*e^5 + 16*a^2*b*c^3*e^5)*d^2)*x^4 + 4*(14*(b^4
*c^2*e^4 - 8*a*b^2*c^3*e^4 + 16*a^2*c^4*e^4)*d^5 + 10*(b^5*c*e^4 - 8*a*b^3*
c^2*e^4 + 16*a^2*b*c^3*e^4)*d^3 + (b^6*e^4 - 6*a*b^4*c*e^4 + 32*a^3*c^3*e^4
)*d)*x^3 + 2*(a*b^5*e - 8*a^2*b^3*c*e + 16*a^3*b*c^2*e)*d^2 + 2*(14*(b^4*c^
2*e^3 - 8*a*b^2*c^3*e^3 + 16*a^2*c^4*e^3)*d^6 + a*b^5*e^3 - 8*a^2*b^3*c*e^3
 + 16*a^3*b*c^2*e^3 + 15*(b^5*c*e^3 - 8*a*b^3*c^2*e^3 + 16*a^2*b*c^3*e^3)*d
^4 + 3*(b^6*e^3 - 6*a*b^4*c*e^3 + 32*a^3*c^3*e^3)*d^2)*x^2 + 4*(2*(b^4*c^2*
e^2 - 8*a*b^2*c^3*e^2 + 16*a^2*c^4*e^2)*d^7 + 3*(b^5*c*e^2 - 8*a*b^3*c^2*e^
2 + 16*a^2*b*c^3*e^2)*d^5 + (b^6*e^2 - 6*a*b^4*c*e^2 + 32*a^3*c^3*e^2)*d^3
+ (a*b^5*e^2 - 8*a^2*b^3*c*e^2 + 16*a^3*b*c^2*e^2)*d)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1771 vs. $2(146) = 292$.
time = 0.44, size = 3670, normalized size = 24.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*x^6*e^6 + 72*(b^2*c^3 - 4*a*c^4)*d*x^5*e^5 + 1
2*(b^2*c^3 - 4*a*c^4)*d^6 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4
)*d^2)*x^4*e^4 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)
*d^4 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*x^3*e^3
+ 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*
c^2 - 4*a*b*c^3)*d^2)*x^2*e^2 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2 + 8*
(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c
^2 - 20*a^2*c^3)*d)*x*e + 12*(c^4*x^8*e^8 + 8*c^4*d*x^7*e^7 + c^4*d^8 + 2*b
*c^3*d^6 + 2*(14*c^4*d^2 + b*c^3)*x^6*e^6 + 4*(14*c^4*d^3 + 3*b*c^3*d)*x^5*
e^5 + 2*a*b*c^2*d^2 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*x^4*e
^4 + (b^2*c^2 + 2*a*c^3)*d^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*
a*c^3)*d)*x^3*e^3 + a^2*c^2 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b
^2*c^2 + 2*a*c^3)*d^2)*x^2*e^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (
b^2*c^2 + 2*a*c^3)*d^3)*x*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d
*x^3*e^3 + 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d
^3 + b*c*d)*x*e + b^2 - 2*a*c - (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqr
t(b^2 - 4*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2
```

+ b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8*e^9 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^7*e^8 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*x^6*e^7 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*x^5*e^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*x^4*e^5 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*x^3*e^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*x^2*e^3 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*x*e^2 + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e), 1/4*(12*(b^2*c^3 - 4*a*c^4)*x^6*e^6 + 72*(b^2*c^3 - 4*a*c^4)*d*x^5*e^5 + 12*(b^2*c^3 - 4*a*c^4)*d^6 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*x^4*e^4 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*x^3*e^3 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b*c^3)*d^2)*x^2*e^2 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*x*e - 24*(c^4*x^8*e^8 + 8*c^4*d*x^7*e^7 + c^4*d^8 + 2*b*c^3*d^6 + 2*(14*c^4*d^2 + b*c^3)*x^6*e^6 + 4*(14*c^4*d^3 + 3*b*c^3*d)*x^5*e^5 + 2*a*b*c^2*d^2 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*x^4*e^4 + (b^2*c^2 + 2*a*c^3)*d^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*x^3*e^3 + a^2*c^2 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*x^2*e^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*x*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8*e^9 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^7*e^8 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*x^6*e^7 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*x^5*e^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5

```
*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*x^4*e^5 + 4*(14*(b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a
^3*b^2*c^3 - 128*a^4*c^4)*d)*x^3*e^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3
*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1646 vs. $2(136) = 272$.
time = 7.48, size = 1646, normalized size = 10.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*sqr
t(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) - 3
6*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*sqrt(-1/(4*a*c - b**
2)**5) + 3*b*c**2 + 6*c**3*d**2)/(6*c**3*e**2))/e + 3*c**2*sqrt(-1/(4*a*c -
 b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*sqrt(-1/(4*a*c - b**2)**5) -
 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**4*c**3*sqrt(-1/(4*
a*c - b**2)**5) - 3*b**6*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b*c**2 + 6*c**
3*d**2)/(6*c**3*e**2))/e + (10*a*b*c + 20*a*c**2*d**2 - b**3 + 4*b**2*c*d**
2 + 18*b*c**2*d**4 + 12*c**3*d**6 + 72*c**3*d*e**5*x**5 + 12*c**3*e**6*x**6
 + x**4*(18*b*c**2*e**4 + 180*c**3*d**2*e**4) + x**3*(72*b*c**2*d*e**3 + 24
0*c**3*d**3*e**3) + x**2*(20*a*c**2*e**2 + 4*b**2*c*e**2 + 108*b*c**2*d**2*
e**2 + 180*c**3*d**4*e**2) + x*(40*a*c**2*d*e + 8*b**2*c*d*e + 72*b*c**2*d*
*3*e + 72*c**3*d**5*e))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**
2*d**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 1
28*a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c
*d**4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8
*b**5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c
**3*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3
*d*e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**
4*d**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e*
*7 + 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c*
*4*d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**
5*c*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**
2*b*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b*
*3*c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d
**2*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**
2*b*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a
*b**3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b*
*5*c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 76
8*a**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 +
```

```
1792*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b
**3*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b*
*5*c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512
*a**3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 +
 512*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a
*b**3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b
**5*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(146) = 292$.
time = 3.60, size = 365, normalized size = 2.40

$$\frac{6\,c^{2}\arctan\left(\frac{2\,cd^{2}+2\,(x^{2}e+2\,dx)ce+b}{\sqrt{-b^{2}+4\,ac}}\right)e^{(-1)}}{(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2})\sqrt{-b^{2}+4\,ac}}+\frac{12\,c^{3}d^{6}+36\,(x^{2}e+2\,dx)c^{3}d^{4}e+36\,(x^{2}e+2\,dx)^{2}c^{3}d^{2}e^{2}+18\,bc^{2}d^{4}+12\,(x^{2}e+2\,dx)^{3}c^{3}e^{3}+36\,(x^{2}e+2\,dx)bc^{2}d^{2}e+18\,(x^{2}e+2\,dx)^{2}bc^{2}e^{2}+4\,b^{2}cd^{2}+20\,ac^{2}d^{2}+4\,(x^{2}e+2\,dx)b^{2}ce+20\,(x^{2}e+2\,dx)ac^{2}e-b^{3}+10\,abc}{4\,(cd^{4}+2\,(x^{2}e+2\,dx)cd^{2}e+(x^{2}e+2\,dx)^{2}ce^{2}+bd^{2}+(x^{2}e+2\,dx)be+a)^{2}(b^{4}e-8\,ab^{2}ce+16\,a^{2}c^{2}e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*arctan((2*c*d^2 + 2*(x^2*e + 2*d*x)*c*e + b)/sqrt(-b^2 + 4*a*c))*e^(-
1)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(12*c^3*d^6 +
36*(x^2*e + 2*d*x)*c^3*d^4*e + 36*(x^2*e + 2*d*x)^2*c^3*d^2*e^2 + 18*b*c^2*
d^4 + 12*(x^2*e + 2*d*x)^3*c^3*e^3 + 36*(x^2*e + 2*d*x)*b*c^2*d^2*e + 18*(x
^2*e + 2*d*x)^2*b*c^2*e^2 + 4*b^2*c*d^2 + 20*a*c^2*d^2 + 4*(x^2*e + 2*d*x)*
b^2*c*e + 20*(x^2*e + 2*d*x)*a*c^2*e - b^3 + 10*a*b*c)/((c*d^4 + 2*(x^2*e +
 2*d*x)*c*d^2*e + (x^2*e + 2*d*x)^2*c*e^2 + b*d^2 + (x^2*e + 2*d*x)*b*e + a
)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

Mupad [B]
time = 3.80, size = 1157, normalized size = 7.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((12*c^3*d^6 - b^3 + 20*a*c^2*d^2 + 4*b^2*c*d^2 + 18*b*c^2*d^4 + 10*a*b*c)/
(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(45*c^3*d^4*e + 5*a*c^2*e + b^2
*c*e + 27*b*c^2*d^2*e))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*x^4*(b*c^2*e^3
+ 10*c^3*d^2*e^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*c^3*e^5*x^6)/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c) + (2*d*x*(5*a*c^2 + b^2*c + 9*c^3*d^4 + 9*b*c^2
*d^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(3*b*c^2*e^2 + 10*c^3*d^2*
e^2))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (18*c^3*d*e^4*x^5)/(b^4 + 16*a^2*c^2
 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^
2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e
 + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3
 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 1
2*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4)
```

+ a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6
 + 8*c^2*d*e^7*x^7) + (6*c^2*atan(((b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c
 - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x*((72*c^6*d*e^7)/(a*(4*a*c - b^2)^
(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (72*b*c^4*(b^5*c^2*d*e^9 - 8*a*b^3*
c^3*d*e^9 + 16*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*
c^2 - 8*a*b^2*c))) + x^2*((36*c^6*e^8)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c)) + (36*b*c^4*(b^5*c^2*e^10 - 8*a*b^3*c^3*e^10 + 16*a^2*b*
c^4*e^10))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (
36*c^6*d^2*e^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3
6*b*c^4*(32*a^3*c^4*e^8 + 2*a*b^4*c^2*e^8 - 16*a^2*b^2*c^3*e^8 + b^5*c^2*d^
2*e^8 - 8*a*b^3*c^3*d^2*e^8 + 16*a^2*b*c^4*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(
15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))/(72*c^6*e^6)))/(e*(4*a*c - b^2)^(5/
2))

3.634 $\int \frac{1}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=437

$$\frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2}+\frac{\left(\frac{d}{e}+x\right)\left(\left(b^2-7ac\right)\left(3b^2-4ac\right)+3bc(b^2-8ac)\,e^2\left(\frac{d}{e}+x\right.\right.}{8a^2\left(b^2-4ac\right)^2\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}$$

```
[Out] 1/4*(d/e+x)*(b^2-2*a*c+b*c*e^2*(d/e+x)^2)/a/(-4*a*c+b^2)/(a+b*e^2*(d/e+x)^2
+c*e^4*(d/e+x)^4)^2+1/8*(d/e+x)*((-7*a*c+b^2)*(-4*a*c+3*b^2)+3*b*c*(-8*a*c+
b^2)*e^2*(d/e+x)^2)/a^2/(-4*a*c+b^2)^2/(a+b*e^2*(d/e+x)^2+c*e^4*(d/e+x)^4)+
3/16*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
b^4-10*a*b^2*c+56*a^2*c^2+b*(-8*a*c+b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^
2)^(5/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan((e*x+d)*2^(1/2)
*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^4-10*a*b^2*c+56*a^2*c^2-b
*(-8*a*c+b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(5/2)/e*2^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 3.65, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1120, 1106, 1192, 1180, 211}

$$\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b(b^2-8ac)\sqrt{b^2-4ac}+b^4\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^2e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c-b(b^2-8ac)\sqrt{b^2-4ac}+b^4\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a^2e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\left(\frac{d}{e}+x\right)\left(3bce^2(b^2-8ac)\left(\frac{d}{e}+x\right)^2+(b^2-7ac)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)}+\frac{\left(\frac{d}{e}+x\right)\left(-2ac+b^2+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\left(b^2-4ac\right)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] ((d/e + x)*(b^2 - 2*a*c + b*c*e^2*(d/e + x)^2))/(4*a*(b^2 - 4*a*c)*(a + b*e
^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)^2) + ((d/e + x)*((b^2 - 7*a*c)*(3*b^2 -
 4*a*c) + 3*b*c*(b^2 - 8*a*c)*e^2*(d/e + x)^2))/(8*a^2*(b^2 - 4*a*c)^2*(a +
 b*e^2*(d/e + x)^2 + c*e^4*(d/e + x)^4)) + (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 5
6*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
 e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b
*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1120

```
Int[(P4_)^(p_), x_Symbol] :> With[{a = Coeff[P4, x, 0], b = Coeff[P4, x, 1]
, c = Coeff[P4, x, 2], d = Coeff[P4, x, 3], e = Coeff[P4, x, 4]}, Subst[Int
[SimplifyIntegrand[(a + d^4/(256*e^3) - b*(d/(8*e)) + (c - 3*(d^2/(8*e)))*x
^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] &
& NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[P4, x, 4] && NeQ[p, 2] && NeQ[p, 3]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{1}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx = \text{Subst}\left(\int \frac{1}{(a+be^2x^2+ce^4x^4)^3}\,dx, x, \frac{d}{e}+x\right)$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\,(b^2-4ac)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} - \text{Subst}\Big(\int \frac{b^2e^4-2ac}{\quad}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\,(b^2-4ac)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)\left((b^2-7\right.}{8a^2\left(b^2-4\right.}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\,(b^2-4ac)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)\left((b^2-7\right.}{8a^2\left(b^2-4\right.}$$

$$= \frac{\left(\frac{d}{e}+x\right)\left(b^2-2ac+bce^2\left(\frac{d}{e}+x\right)^2\right)}{4a\,(b^2-4ac)\left(a+be^2\left(\frac{d}{e}+x\right)^2+ce^4\left(\frac{d}{e}+x\right)^4\right)^2} + \frac{(d+ex)\left((b^2-7\right.}{8a^2\left(b^2-4\right.}$$

Mathematica [A]
time = 4.82, size = 424, normalized size = 0.97

$$\frac{\frac{4a(d+ex)(-b^2+2ac-bc(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{2(d+ex)(3b^4-25ab^2c+28a^2c^2+3b^3c(d+ex)^2-24abc^2(d+ex)^2)}{(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)}+\frac{3\sqrt{2}\sqrt{c}\left(b^4-10ab^2c+56a^2c^2+b^3\sqrt{b^2-4ac}-8abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(b^4-10ab^2c+56a^2c^2-b^3\sqrt{b^2-4ac}+8abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}}{16a^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*(d + e*x)^2 + c*(d + e*x)^4)^(-3),x]
```

```
[Out] ((4*a*(d + e*x)*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d
 + e*x)^2 + c*(d + e*x)^4)^2) + (2*(d + e*x)*(3*b^4 - 25*a*b^2*c + 28*a^2*c
^2 + 3*b^3*c*(d + e*x)^2 - 24*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + b
*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*
a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c
^2 - b^3*Sqrt[b^2 - 4*a*c] + 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]))/(16*a^2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 1010, normalized size = 2.31 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-3/8*c^2*e^6*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^7-21/8*c^2*d*e
^5*b*(8*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^6+1/8*(-504*a*b*c^2*d^2+6
3*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b^4)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a
^2*x^5+5/8*c*d*e^3*(-168*a*b*c^2*d^2+21*b^3*c*d^2+28*a^2*c^2-49*a*b^2*c+6*b
^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^4-1/8*e^2*(840*a*b*c^3*d^4-105*b^3*c^2
*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+4*a^2*b*c^2+20*a*b^3*c-
3*b^5)/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(504*a*b*c^3*d^4-63*b^3*c
^2*d^4-280*a^2*c^3*d^2+490*a*b^2*c^2*d^2-60*b^4*c*d^2+12*a^2*b*c^2+60*a*b^3
*c-9*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2*x^2+1/8*(-168*a*b*c^3*d^6+21*b^3*c
^2*d^6+140*a^2*c^3*d^4-245*a*b^2*c^2*d^4+30*b^4*c*d^4-12*a^2*b*c^2*d^2-60*a
*b^3*c*d^2+9*b^5*d^2+44*a^3*c^2-37*a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c
+b^4)/a^2*x+1/8*d/e*(-24*a*b*c^3*d^6+3*b^3*c^2*d^6+28*a^2*c^3*d^4-49*a*b^2*
c^2*d^4+6*b^4*c*d^4-4*a^2*b*c^2*d^2-20*a*b^3*c*d^2+3*b^5*d^2+44*a^3*c^2-37*
a^2*b^2*c+5*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a^2)/(c*e^4*x^4+4*c*d*e^3*x^3
+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/16/(16*
a^2*c^2-8*a*b^2*c+b^4)/a^2/e*sum((b*c*e^2*(-8*a*c+b^2)*_R^2+2*b*c*d*e*(-8*a
*c+b^2)*_R-8*a*b*c^2*d^2+b^3*c*d^2+28*a^2*c^2-9*a*b^2*c+b^4)/(2*_R^3*c*e^3+
6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z
^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2
*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^3*c^2 - 8*a*b*c^3)*d^7 + 21*(b^3*c^2*e^6 - 8*a*b*c^3*e^6)*d*x^6 +
 3*(b^3*c^2*e^7 - 8*a*b*c^3*e^7)*x^7 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3
)*d^5 + (6*b^4*c*e^5 - 49*a*b^2*c^2*e^5 + 28*a^2*c^3*e^5 + 63*(b^3*c^2*e^5
- 8*a*b*c^3*e^5)*d^2)*x^5 + 5*(21*(b^3*c^2*e^4 - 8*a*b*c^3*e^4)*d^3 + (6*b^
4*c*e^4 - 49*a*b^2*c^2*e^4 + 28*a^2*c^3*e^4)*d)*x^4 + (3*b^5 - 20*a*b^3*c -
 4*a^2*b*c^2)*d^3 + (3*b^5*e^3 - 20*a*b^3*c*e^3 - 4*a^2*b*c^2*e^3 + 105*(b^
3*c^2*e^3 - 8*a*b*c^3*e^3)*d^4 + 10*(6*b^4*c*e^3 - 49*a*b^2*c^2*e^3 + 28*a^
2*c^3*e^3)*d^2)*x^3 + (63*(b^3*c^2*e^2 - 8*a*b*c^3*e^2)*d^5 + 10*(6*b^4*c*e
^2 - 49*a*b^2*c^2*e^2 + 28*a^2*c^3*e^2)*d^3 + 3*(3*b^5*e^2 - 20*a*b^3*c*e^2
 - 4*a^2*b*c^2*e^2)*d)*x^2 + (5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*d + (21*
(b^3*c^2*e - 8*a*b*c^3*e)*d^6 + 5*a*b^4*e - 37*a^2*b^2*c*e + 44*a^3*c^2*e +
 5*(6*b^4*c*e - 49*a*b^2*c^2*e + 28*a^2*c^3*e)*d^4 + 3*(3*b^5*e - 20*a*b^3*
c*e - 4*a^2*b*c^2*e)*d^2)*x)/((a^2*b^4*c^2*e - 8*a^3*b^2*c^3*e + 16*a^4*c^4
```

```
*e)*d^8 + 8*(a^2*b^4*c^2*e^8 - 8*a^3*b^2*c^3*e^8 + 16*a^4*c^4*e^8)*d*x^7 +
(a^2*b^4*c^2*e^9 - 8*a^3*b^2*c^3*e^9 + 16*a^4*c^4*e^9)*x^8 + a^4*b^4*e - 8*
a^5*b^2*c*e + 16*a^6*c^2*e + 2*(a^2*b^5*c*e - 8*a^3*b^3*c^2*e + 16*a^4*b*c^
3*e)*d^6 + 2*(a^2*b^5*c*e^7 - 8*a^3*b^3*c^2*e^7 + 16*a^4*b*c^3*e^7 + 14*(a^
2*b^4*c^2*e^7 - 8*a^3*b^2*c^3*e^7 + 16*a^4*c^4*e^7)*d^2)*x^6 + 4*(14*(a^2*b
^4*c^2*e^6 - 8*a^3*b^2*c^3*e^6 + 16*a^4*c^4*e^6)*d^3 + 3*(a^2*b^5*c*e^6 - 8
*a^3*b^3*c^2*e^6 + 16*a^4*b*c^3*e^6)*d)*x^5 + (a^2*b^6*e - 6*a^3*b^4*c*e +
32*a^5*c^3*e)*d^4 + (a^2*b^6*e^5 - 6*a^3*b^4*c*e^5 + 32*a^5*c^3*e^5 + 70*(a
^2*b^4*c^2*e^5 - 8*a^3*b^2*c^3*e^5 + 16*a^4*c^4*e^5)*d^4 + 30*(a^2*b^5*c*e^
5 - 8*a^3*b^3*c^2*e^5 + 16*a^4*b*c^3*e^5)*d^2)*x^4 + 4*(14*(a^2*b^4*c^2*e^4
 - 8*a^3*b^2*c^3*e^4 + 16*a^4*c^4*e^4)*d^5 + 10*(a^2*b^5*c*e^4 - 8*a^3*b^3*
c^2*e^4 + 16*a^4*b*c^3*e^4)*d^3 + (a^2*b^6*e^4 - 6*a^3*b^4*c*e^4 + 32*a^5*c
^3*e^4)*d)*x^3 + 2*(a^3*b^5*e - 8*a^4*b^3*c*e + 16*a^5*b*c^2*e)*d^2 + 2*(a^
3*b^5*e^3 - 8*a^4*b^3*c*e^3 + 16*a^5*b*c^2*e^3 + 14*(a^2*b^4*c^2*e^3 - 8*a^
3*b^2*c^3*e^3 + 16*a^4*c^4*e^3)*d^6 + 15*(a^2*b^5*c*e^3 - 8*a^3*b^3*c^2*e^3
 + 16*a^4*b*c^3*e^3)*d^4 + 3*(a^2*b^6*e^3 - 6*a^3*b^4*c*e^3 + 32*a^5*c^3*e^
3)*d^2)*x^2 + 4*(2*(a^2*b^4*c^2*e^2 - 8*a^3*b^2*c^3*e^2 + 16*a^4*c^4*e^2)*d
^7 + 3*(a^2*b^5*c*e^2 - 8*a^3*b^3*c^2*e^2 + 16*a^4*b*c^3*e^2)*d^5 + (a^2*b^
6*e^2 - 6*a^3*b^4*c*e^2 + 32*a^5*c^3*e^2)*d^3 + (a^3*b^5*e^2 - 8*a^4*b^3*c*
e^2 + 16*a^5*b*c^2*e^2)*d)*x) - 3/8*integrate(-(b^4 - 9*a*b^2*c + 28*a^2*c^
2 + (b^3*c - 8*a*b*c^2)*d^2 + 2*(b^3*c*e - 8*a*b*c^2*e)*d*x + (b^3*c*e^2 -
8*a*b*c^2*e^2)*x^2)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e
^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(a^2*b^4 - 8*a^3*b^2*c +
 16*a^4*c^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8450 vs. $2(375) = 750$.
time = 0.80, size = 8450, normalized size = 19.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^3*c^2 - 8*a*b*c^3)*x^7*e^7 + 42*(b^3*c^2 - 8*a*b*c^3)*d*x^6*e^6
+ 6*(b^3*c^2 - 8*a*b*c^3)*d^7 + 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3 + 63
*(b^3*c^2 - 8*a*b*c^3)*d^2)*x^5*e^5 + 2*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^
3)*d^5 + 10*(21*(b^3*c^2 - 8*a*b*c^3)*d^3 + (6*b^4*c - 49*a*b^2*c^2 + 28*a^
2*c^3)*d)*x^4*e^4 + 2*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2 + 105*(b^3*c^2 - 8*
a*b*c^3)*d^4 + 10*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^2)*x^3*e^3 + 2*(3
*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*d^3 + 2*(63*(b^3*c^2 - 8*a*b*c^3)*d^5 + 10
*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^3 + 3*(3*b^5 - 20*a*b^3*c - 4*a^2*
b*c^2)*d)*x^2*e^2 + 2*(21*(b^3*c^2 - 8*a*b*c^3)*d^6 + 5*a*b^4 - 37*a^2*b^2*
c + 44*a^3*c^2 + 5*(6*b^4*c - 49*a*b^2*c^2 + 28*a^2*c^3)*d^4 + 3*(3*b^5 - 2
0*a*b^3*c - 4*a^2*b*c^2)*d^2)*x*e + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c
```

^3 + 16*a^4*c^4)*x^8*e^9 + 8*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d*x
^7*e^8 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3 + 14*(a^2*b^4*c^2 - 8*
a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*x^6*e^7 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3
 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d)*x^5*e^
6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 +
 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^2)*x^4*e
^5 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^5 + 10*(a^2*b^5*c -
 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d
)*x^3*e^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2 + 14*(a^2*b^4*c^2 - 8*a
^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^2)*x^2*e^3 + 4*(2*(a^2*b^4
*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*
a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^3 + (a^3*b^5 - 8*a^
4*b^3*c + 16*a^5*b*c^2)*d)*x*e^2 + ((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c
^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^4 + 2*(a^3*b^
5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b
^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*
a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8
 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^
10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^
4 - 1024*a^15*c^5)))*e^(-2)/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 64
0*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(21*b^8*c^3 - 447
*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*x*e + 27
*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416
*a^4*c^7)*d + 27/2*sqrt(1/2)*((a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2
- 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^
3*c^6 - 45056*a^12*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a
^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 -
640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5))*e - (b^14 - 32*a*b^1
2*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b
^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7)*e)*sqrt(-(b^9 - 21*a*b^7*c + 1
89*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*
c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*s
qrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/
(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^1
4*b^2*c^4 - 1024*a^15*c^5)))*e^(-2)/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*
c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) - 3*sqrt(1/2)*(
(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8*e^9 + 8*(a^2*b^4*c^2 - 8*a^3
*b^2*c^3 + 16*a^4*c^4)*d*x^7*e^8 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^2)*x^6*e^7 + 4*(14*(a
^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^3 + 3*(a^2*b^5*c - 8*a^3*b^3*c^2
 + 16*a^4*b*c^3)*d)*x^5*e^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3 + 70*(a^2
*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^4 + 30*(a^2*b^5*c - 8*a^3*b^3*c^2
+ 16*a^4*b*c^3)*d^2)*x^4*e^5 + 4*(14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*

```
c^4)*d^5 + 10*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^3 + (a^2*b^6 - 6
*a^3*b^4*c + 32*a^5*c^3)*d)*x^3*e^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c
^2 + 14*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^6 + 15*(a^2*b^5*c - 8*
a^3*b^3*c^2 + 16*a^4*b*c^3)*d^4 + 3*(a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*d^
2)*x^2*e^3 + 4*(2*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^7 + 3*(a^2*b
^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^5 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*
c^3)*d^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*x*e^2 + ((a^2*b^4*c^2
- 8*a^3*b^2*c^3 + 16*a^4*c^4)*d^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*
(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d^6 + (a^2*b^6 - 6*a^3*b^4*c + 3
2*a^5*c^3)*d^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*e)*sqrt(-(b^
9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2487 vs. 2(375) = 750.
time = 3.65, size = 2487, normalized size = 5.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
[Out] -3/16*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*b^3*c*e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 16*(d*e^(-1) + sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + b^3*c*d^2 - 8*a*b*c^
2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d
*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 +
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 16*(d*e^(-1) -
 sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + b
```

^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 16*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + b^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 16*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + b^3*c*d^2 - 8*a*b*c^2*d^2 + b^4 - 9*a*b^2*c + 28*a^2*c^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2) + 1/8*(3*b^3*c^2*x^7*e^7 - 24*a*b*c^3*x^7*e^7 + 21*b^3*c^2*d*x^6*e^6 - 168*a*b*c^3*d*x^6*e^6 + 63*b^3*c^2*d^2*x^5*e^5 - 504*a*b*c^3*d^2*x^5*e^5 + 105*b^3*c^2*d^3*x^4*e^4 - 840*a*b*c^3*d^3*x^4*e^4 + 105*b^3*c^2*d^4*x^3*e^3 - 840*a*b*c^3*d^4*x^3*e^3 + 63*b^3*c^2*d^5*x^2*e^2 - 504*a*b*c^3*d^5*x^2*e^2 + 21*b^3*c^2*d^6*x*e - 168*a*b*c^3*d^6*x*e + 3*b^3*c^2*d^7 - 24*a*b*c^3*d^7 + 6*b^4*c*x^5*e^5 - 49*a*b^2*c^2*x^5*e^5 + 28*a^2*c^3*x^5*e^5 + 30*b^4*c*d*x^4*e^4 - 245*a*b^2*c^2*d*x^4*e^4 + 140*a^2*c^3*d*x^4*e^4 + 60*b^4*c*d^2*x^3*e^3 - 490*a*b^2*c^2*d^2*x^3*e^3 + 280*a^2*c^3*d^2*x^3*e^3 + 60*b^4*c*d^3*x^2*e^2 - 490*a*b^2*c^2*d^3*x^2*e^2 + 280*a^2*c^3*d^3*x^2*e^2 + 30*b^4*c*d^4*x*e - 245*a*b^2*c^2*d^4*x*e + 140*a^2*c^3*d^4*x*e + 6*b^4*c*d^5 - 49*a*b^2*c^2*d^5 + 28*a^2*c^3*d^5 + 3*b^5*x^3*e^3 - 20*a*b^3*c*x^3*e^3 - 4*a^2*b*c^2*x^3*e^3 + 9*b^5*d*x^2*e^2 - 60*a*b^3*c*d*x^2*e^2 - 12*a^2*b*c^2*d*x^2*e^2 + 9*b^5*d^2*x*e - 60*a*b^3*c*d^2*x*e - 12*a^2*b*c^2*d^2*x*e + 3*b^5*d^3 - 20*a*b^3*c*d^3 - 4*a^2*b*c^2*d^3 + 5*a*b^4*x*e - 37*a^2*b^2*c*x*e + 44*a^3*c^2*x*e + 5*a*b^4*d - 37*a^2*b^2*c*d + 44*a^3*c^2*d)/((a^2*b^4*e - 8*a^3*b^2*c*e + 16*a^4*c^2*e)*(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2)

Mupad [B]
time = 7.80, size = 2500, normalized size = 5.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((3*b^5*d^3 + 44*a^3*c^2*d + 6*b^4*c*d^5 + 28*a^2*c^3*d^5 + 3*b^3*c^2*d^7 +
 5*a*b^4*d - 4*a^2*b*c^2*d^3 - 49*a*b^2*c^2*d^5 - 37*a^2*b^2*c*d - 20*a*b^3
*c*d^3 - 24*a*b*c^3*d^7)/(8*a^2*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^3*(3
*b^5*e^2 - 4*a^2*b*c^2*e^2 + 60*b^4*c*d^2*e^2 + 280*a^2*c^3*d^2*e^2 + 105*b
^3*c^2*d^4*e^2 - 20*a*b^3*c*e^2 - 840*a*b*c^3*d^4*e^2 - 490*a*b^2*c^2*d^2*e
^2))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(6*b^4*c*e^4 + 28*a^2*c^
3*e^4 - 49*a*b^2*c^2*e^4 + 63*b^3*c^2*d^2*e^4 - 504*a*b*c^3*d^2*e^4))/(8*a^
2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(9*b^5*d*e + 280*a^2*c^3*d^3*e + 6
3*b^3*c^2*d^5*e + 60*b^4*c*d^3*e - 12*a^2*b*c^2*d*e - 504*a*b*c^3*d^5*e - 4
90*a*b^2*c^2*d^3*e - 60*a*b^3*c*d*e))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
) + (5*x^4*(28*a^2*c^3*d*e^3 + 21*b^3*c^2*d^3*e^3 + 6*b^4*c*d*e^3 - 49*a*b^
2*c^2*d*e^3 - 168*a*b*c^3*d^3*e^3))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (21*x^6*(b^3*c^2*d*e^5 - 8*a*b*c^3*d*e^5))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (x*(5*a*b^4 + 44*a^3*c^2 + 9*b^5*d^2 - 37*a^2*b^2*c + 30*b^4*c*d
^4 + 140*a^2*c^3*d^4 + 21*b^3*c^2*d^6 - 12*a^2*b*c^2*d^2 - 245*a*b^2*c^2*d^
4 - 60*a*b^3*c*d^2 - 168*a*b*c^3*d^6))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (3*x^7*(b^3*c^2*e^6 - 8*a*b*c^3*e^6))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 +
 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^
2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c
^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d
*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 +
 b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^
2*d*e^7*x^7) - atan((((3612672*a^6*c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*
a*b^10*c^4*d*e^11 + 49824*a^2*b^8*c^5*d*e^11 - 340992*a^3*b^6*c^6*d*e^11 +
1410048*a^4*b^4*c^7*d*e^11 - 3391488*a^5*b^2*c^8*d*e^11)/(512*(a^4*b^12 + 4
096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^
8*b^4*c^4 - 6144*a^9*b^2*c^5)) - (((67108864*a^11*b*c^9*d*e^13 - 4096*a^4*b
^15*c^2*d*e^13 + 114688*a^5*b^13*c^3*d*e^13 - 1376256*a^6*b^11*c^4*d*e^13 +
 9175040*a^7*b^9*c^5*d*e^13 - 36700160*a^8*b^7*c^6*d*e^13 + 88080384*a^9*b^
5*c^7*d*e^13 - 117440512*a^10*b^3*c^8*d*e^13)/(512*(a^4*b^12 + 4096*a^10*c^
6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^4*c^4 -
 6144*a^9*b^2*c^5)) + (x*(262144*a^9*b*c^7*e^14 - 256*a^4*b^11*c^2*e^14 + 5
120*a^5*b^9*c^3*e^14 - 40960*a^6*b^7*c^4*e^14 + 163840*a^7*b^5*c^5*e^14 - 3
27680*a^8*b^3*c^6*e^14))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6
*b^4*c^2 - 256*a^7*b^2*c^3)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) -
1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c
```

^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 30105 60*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a* b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^ 2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 5376 0*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1 966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e ^2)))^(1/2) - (22020096*a^9*c^9*e^12 - 768*a^2*b^14*c^2*e^12 + 22272*a^3*b^ 12*c^3*e^12 - 282624*a^4*b^10*c^4*e^12 + 2027520*a^5*b^8*c^5*e^12 - 8847360 *a^6*b^6*c^6*e^12 + 23396352*a^7*b^4*c^7*e^12 - 34603008*a^8*b^2*c^8*e^12)/ (512*(a^4*b^12 + 4096*a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7 *b^6*c^3 + 3840*a^8*b^4*c^4 - 6144*a^9*b^2*c^5)))*(-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^15*c^2 - 8620*a^3*b^13*c^ 3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069824*a^6*b^7*c^6 - 2343936 *a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^7*b^16*c^2*e^2 - 7680*a^8 *b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^10*b^10*c^5*e^2 + 860160* a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949120*a^13*b^4*c^8*e^2 - 26 21440*a^14*b^2*c^9*e^2)))^(1/2) + (x*(14112*a^4*c^7*e^12 + 9*b^8*c^3*e^12 - 180*a*b^6*c^4*e^12 + 1530*a^2*b^4*c^5*e^12 - 6192*a^3*b^2*c^6*e^12))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))* (-(9*(b^19 + b^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^9*b*c^9 + 769*a^2*b^ 15*c^2 - 8620*a^3*b^13*c^3 + 63440*a^4*b^11*c^4 - 316864*a^5*b^9*c^5 + 1069 824*a^6*b^7*c^6 - 2343936*a^7*b^5*c^7 + 3010560*a^8*b^3*c^8 + 49*a^2*c^2*(- (4*a*c - b^2)^15)^(1/2) - 41*a*b^17*c - 11*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^5*b^20*e^2 + 1048576*a^15*c^10*e^2 - 40*a^6*b^18*c*e^2 + 720*a^ 7*b^16*c^2*e^2 - 7680*a^8*b^14*c^3*e^2 + 53760*a^9*b^12*c^4*e^2 - 258048*a^ 10*b^10*c^5*e^2 + 860160*a^11*b^8*c^6*e^2 - 1966080*a^12*b^6*c^7*e^2 + 2949 120*a^13*b^4*c^8*e^2 - 2621440*a^14*b^2*c^9*e^2)))^(1/2)*1i + ((3612672*a^6 *c^9*d*e^11 + 144*b^12*c^3*d*e^11 - 4032*a*b^10...

3.635 $\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=255

$$\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2}{4a^2\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b(b^4-10a}{}$$

```
[Out] 1/4*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^
2+1/4*(2*b^4-15*a*b^2*c+16*a^2*c^2+2*b*c*(-7*a*c+b^2)*(e*x+d)^2)/a^2/(-4*a*
c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/2*b*(30*a^2*c^2-10*a*b^2*c+b^4)*ar
ctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(5/2)/e+ln(e*x
+d)/a^3/e-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e
```

Rubi [A]

time = 0.34, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1156, 1128, 754, 836, 814, 648, 632, 212, 642}

$$-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3e}+\frac{\log(d+ex)}{a^3e}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3e\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c
*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*(
d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) +
 (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e) + Log[d + e*x]/(a^3*e) - Log[a + b
*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^{\cdots}}{4a^2\left(b^2-\cdots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^{\cdots}}{4a^2\left(b^2-\cdots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^{\cdots}}{4a^2\left(b^2-\cdots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^{\cdots}}{4a^2\left(b^2-\cdots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^{\cdots}}{4a^2\left(b^2-\cdots\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{2b^4-15ab^{\cdots}}{4a^2\left(b^2-\cdots\right.}$$

Mathematica [A]

time = 2.64, size = 391, normalized size = 1.53

$$\frac{\frac{a^2\left(-b^2+2ac-bc(d+ex)^2\right)}{(-b^2+4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{a\left(2b^4-15ab^2c+16a^2c^2+2b^3c(d+ex)^2-14abc^2(d+ex)^2\right)}{(b^2-4ac)^2\left(a+(d+ex)^2(b+c(d+ex)^2)\right)}+4\log(d+ex)-\frac{\left(b^5-10ab^3c+30a^2bc^2+b^4\sqrt{b^2-4ac}-8ab^2c\sqrt{b^2-4ac}+16a^2c^2\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{5/2}}+\frac{\left(b^5-10ab^3c+30a^2bc^2-b^4\sqrt{b^2-4ac}+8ab^2c\sqrt{b^2-4ac}-16a^2c^2\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{5/2}}}{4a^3e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] ((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2
+ c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d + e*
x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c*(d
 + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqr
t[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]
)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) + ((b^5
 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 -
 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d
+ e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.26, size = 966, normalized size = 3.79

method	result
default	$\frac{\ln(ex+d)}{a^3 e} - \frac{\frac{c^2 e^5 (7ac-b^2) a b x^6}{32a^2c^2-16ab^2c+2b^4} + \frac{3(7ac-b^2) a b c^2 d e^4 x^5}{16a^2c^2-8ab^2c+b^4} - \frac{e^3 a c(-210abc^2d^2+30b^3cd^2+16a^2c^2-29ab^2c+4b^4) x^4}{4(16a^2c^2-8ab^2c+b^4)} - \frac{c d e^2 a(-70abc^2d^2+10\cdots}{16a^2\cdots}}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] ln(e*x+d)/a^3/e-1/a^3*((1/2*c^2*e^5*(7*a*c-b^2)*a*b/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^6+3*(7*a*c-b^2)*a*b*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4*e^3*
a*c*(-210*a*b*c^2*d^2+30*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x^4-c*d*e^2*a*(-70*a*b*c^2*d^2+10*b^3*c*d^2+16*a^2*c^2-29*a
*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(105*a*b*c^3*d^4-15*b^
3*c^2*d^4-48*a^2*c^3*d^2+87*a*b^2*c^2*d^2-12*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-
b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+d*a*(21*a*b*c^3*d^4-3*b^3*c^2*d^4-16*a^
2*c^3*d^2+29*a*b^2*c^2*d^2-4*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)/(16*a^2*c^2
-8*a*b^2*c+b^4)*x-1/4/e*a*(-14*a*b*c^3*d^6+2*b^3*c^2*d^6+16*a^2*c^3*d^4-29*
a*b^2*c^2*d^4+4*b^4*c*d^4-2*a^2*b*c^2*d^2-12*a*b^3*c*d^2+2*b^5*d^2+24*a^3*c
^2-21*a^2*b^2*c+3*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x
^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/2/(16
*a^2*c^2-8*a*b^2*c+b^4)/e*sum((e^3*c*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*d*e^
2*c*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*c^2*d^2+3*b^
4*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*a*b^2*c^2*d^3+b^4*c
*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c
*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(b^3*c^2 - 7*a*b*c^3)*d^6 + 12*(b^3*c^2*e^5 - 7*a*b*c^3*e^5)*d*x^5 +
 2*(b^3*c^2*e^6 - 7*a*b*c^3*e^6)*x^6 + 3*a*b^4 - 21*a^2*b^2*c + 24*a^3*c^2
+ (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (4*b^4*c*e^4 - 29*a*b^2*c^2*e
^4 + 16*a^2*c^3*e^4 + 30*(b^3*c^2*e^4 - 7*a*b*c^3*e^4)*d^2)*x^4 + 4*(10*(b^
3*c^2*e^3 - 7*a*b*c^3*e^3)*d^3 + (4*b^4*c*e^3 - 29*a*b^2*c^2*e^3 + 16*a^2*c
^3*e^3)*d)*x^3 + 2*(b^5 - 6*a*b^3*c - a^2*b*c^2)*d^2 + 2*(b^5*e^2 - 6*a*b^3
*c*e^2 - a^2*b*c^2*e^2 + 15*(b^3*c^2*e^2 - 7*a*b*c^3*e^2)*d^4 + 3*(4*b^4*c*
e^2 - 29*a*b^2*c^2*e^2 + 16*a^2*c^3*e^2)*d^2)*x^2 + 4*(3*(b^3*c^2*e - 7*a*b
*c^3*e)*d^5 + (4*b^4*c*e - 29*a*b^2*c^2*e + 16*a^2*c^3*e)*d^3 + (b^5*e - 6*
a*b^3*c*e - a^2*b*c^2*e)*d)*x)/((a^2*b^4*c^2*e - 8*a^3*b^2*c^3*e + 16*a^4*c
^4*e)*d^8 + 8*(a^2*b^4*c^2*e^8 - 8*a^3*b^2*c^3*e^8 + 16*a^4*c^4*e^8)*d*x^7
+ (a^2*b^4*c^2*e^9 - 8*a^3*b^2*c^3*e^9 + 16*a^4*c^4*e^9)*x^8 + a^4*b^4*e -
8*a^5*b^2*c*e + 16*a^6*c^2*e + 2*(a^2*b^5*c*e - 8*a^3*b^3*c^2*e + 16*a^4*b*
c^3*e)*d^6 + 2*(a^2*b^5*c*e^7 - 8*a^3*b^3*c^2*e^7 + 16*a^4*b*c^3*e^7 + 14*(
a^2*b^4*c^2*e^7 - 8*a^3*b^2*c^3*e^7 + 16*a^4*c^4*e^7)*d^2)*x^6 + 4*(14*(a^2
*b^4*c^2*e^6 - 8*a^3*b^2*c^3*e^6 + 16*a^4*c^4*e^6)*d^3 + 3*(a^2*b^5*c*e^6 -
 8*a^3*b^3*c^2*e^6 + 16*a^4*b*c^3*e^6)*d)*x^5 + (a^2*b^6*e - 6*a^3*b^4*c*e
+ 32*a^5*c^3*e)*d^4 + (a^2*b^6*e^5 - 6*a^3*b^4*c*e^5 + 32*a^5*c^3*e^5 + 70*
(a^2*b^4*c^2*e^5 - 8*a^3*b^2*c^3*e^5 + 16*a^4*c^4*e^5)*d^4 + 30*(a^2*b^5*c*
e^5 - 8*a^3*b^3*c^2*e^5 + 16*a^4*b*c^3*e^5)*d^2)*x^4 + 4*(14*(a^2*b^4*c^2*e
^4 - 8*a^3*b^2*c^3*e^4 + 16*a^4*c^4*e^4)*d^5 + 10*(a^2*b^5*c*e^4 - 8*a^3*b^
3*c^2*e^4 + 16*a^4*b*c^3*e^4)*d^3 + (a^2*b^6*e^4 - 6*a^3*b^4*c*e^4 + 32*a^5
*c^3*e^4)*d)*x^3 + 2*(a^3*b^5*e - 8*a^4*b^3*c*e + 16*a^5*b*c^2*e)*d^2 + 2*(
a^3*b^5*e^3 - 8*a^4*b^3*c*e^3 + 16*a^5*b*c^2*e^3 + 14*(a^2*b^4*c^2*e^3 - 8*
a^3*b^2*c^3*e^3 + 16*a^4*c^4*e^3)*d^6 + 15*(a^2*b^5*c*e^3 - 8*a^3*b^3*c^2*e
^3 + 16*a^4*b*c^3*e^3)*d^4 + 3*(a^2*b^6*e^3 - 6*a^3*b^4*c*e^3 + 32*a^5*c^3*
e^3)*d^2)*x^2 + 4*(2*(a^2*b^4*c^2*e^2 - 8*a^3*b^2*c^3*e^2 + 16*a^4*c^4*e^2)
*d^7 + 3*(a^2*b^5*c*e^2 - 8*a^3*b^3*c^2*e^2 + 16*a^4*b*c^3*e^2)*d^5 + (a^2*
b^6*e^2 - 6*a^3*b^4*c*e^2 + 32*a^5*c^3*e^2)*d^3 + (a^3*b^5*e^2 - 8*a^4*b^3*
c*e^2 + 16*a^5*b*c^2*e^2)*d)*x) - integrate(((b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*d^3 + 3*(b^4*c*e^2 - 8*a*b^2*c^2*e^2 + 16*a^2*c^3*e^2)*d*x^2 + (b^4*c*
e^3 - 8*a*b^2*c^2*e^3 + 16*a^2*c^3*e^3)*x^3 + (b^5 - 9*a*b^3*c + 23*a^2*b*c
^2)*d + (b^5*e - 9*a*b^3*c*e + 23*a^2*b*c^2*e + 3*(b^4*c*e - 8*a*b^2*c^2*e
+ 16*a^2*c^3*e)*d^2)*x)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d
^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2) + e^(-1)*log(x*e + d)/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4859 vs. $2(248) = 496$.
time = 1.01, size = 9844, normalized size = 38.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^
2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^6*e^6 + 12*(a*b^5*c^2 - 11*a^2*b^3*c^3
 + 28*a^3*b*c^4)*d*x^5*e^5 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*
d^6 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(a*b^
5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^2)*x^4*e^4 + (4*a*b^6*c - 45*a^2*b
^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^4 + 4*(10*(a*b^5*c^2 - 11*a^2*b^3*
c^3 + 28*a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 6
4*a^4*c^4)*d)*x^3*e^3 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*
c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 4
5*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*x^2*e^2 + 2*(a*b^7 - 10*
a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^
3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 -
 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)
*x*e + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8*e^8 + 8*(b^5*c^2 - 10*a
*b^3*c^3 + 30*a^2*b*c^4)*d*x^7*e^7 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4
)*d^8 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 - 10*a*b^3*c
^3 + 30*a^2*b*c^4)*d^2)*x^6*e^6 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2
*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^
3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*x^5*e^
5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b
^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2
)*x^4*e^4 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 4*(14*(
b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*
a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)*x^3
*e^3 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^
3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*
(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*x^2*e^2 + 2*(a*b^6 -
 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2
*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^
5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b
^2*c^2)*d)*x*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*
c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x
*e + b^2 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c
))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*x*e + a)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*x^8*e^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
```

```
d*x^7*e^7 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^
2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*d^2)*x^6*e^6 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
- 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*x
^5*e^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4
+ 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c
 - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*x^4*e^4 + (b^8 - 10*a
*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 4*(14*(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c
^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*x^3*e^3 + 2*(a*b^7 - 12*a^2*b^5*c + 4
8*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^
4)*d^2)*x^2*e^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*
d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6
*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*x*e)*log(c*x^4*e^4 + 4*c*d*x^3*e^3
 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a) + 4*(
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8*e^8 + 8*(b^6*c^2
 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^7*e^7 + (b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*
b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*x^6*
e^6 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*x^5*e^5 + (b^8 - 10*a*b^6*c +
 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(248) = 496.
time = 4.04, size = 1012, normalized size = 3.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/4*((a^3*b^7*c*e^3 - 14*a^4*b^5*c^2*e^3 + 70*a^5*b^3*c^3*e^3 - 120*a^6*b*
c^4*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c
)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a
)) - (a^3*b^7*c*e^3 - 14*a^4*b^5*c^2*e^3 + 70*a^5*b^3*c^3*e^3 - 120*a^6*b*c
^4*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c
)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a
)))/(a^6*b^8*c*e^4 - 16*a^7*b^6*c^2*e^4 + 96*a^8*b^4*c^3*e^4 - 256*a^9*b^2*
c^4*e^4 + 256*a^10*c^5*e^4) - 1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3
+ 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a
))/a^3 + e^(-1)*log(abs(x*e + d))/a^3 + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3
*d^6 + 4*a*b^4*c*d^4 - 29*a^2*b^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 -
12*a^2*b^3*c*d^2 - 2*a^3*b*c^2*d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^
6 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c
^3*d*e^5)*x^5 + (30*a*b^3*c^2*d^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e
^4 - 29*a^2*b^2*c^2*e^4 + 16*a^3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 7
0*a^2*b*c^3*d^3*e^3 + 4*a*b^4*c*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d
*e^3)*x^3 + 2*(15*a*b^3*c^2*d^4*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^
2*e^2 - 87*a^2*b^2*c^2*d^2*e^2 + 48*a^3*c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3
*c*e^2 - a^3*b*c^2*e^2)*x^2 + 4*(3*a*b^3*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4
*a*b^4*c*d^3*e - 29*a^2*b^2*c^2*d^3*e + 16*a^3*c^3*d^3*e + a*b^5*d*e - 6*a^
2*b^3*c*d*e - a^3*b*c^2*d*e)*x)*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^
2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^2
 - 4*a*c)^2*a^3)
```

Mupad [B]
time = 17.98, size = 2500, normalized size = 9.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] ((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c^
2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*a*b
^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3))/(4*(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 + 4*b^4
*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2))/(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4))/(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5))/(2*(a^2*b^4 + 16
*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^3 + 3*b^3
*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*c^3*d^5))/
```

(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2*b^5*d^2 - 21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2*a^2*b*c^2*d ^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6)/(4*e*(a^2*b^4 + 16 *a^4*c^2 - 8*a^3*b^2*c)))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2* d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b* c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + log(d + e*x)/(a^3*e) - (log((((a^3*e*(-(b^ 2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) + 1)* (((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) + 1)*((2*b*c^2*e^16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3 *d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2))/(a^2*(4*a*c - b^2)^2) + (b*c^2*e^16*(a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^ (1/2) + 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10 *a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2 *a*b^2*c))/(a^2*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2* a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3*e) + (b*c^3*e^15*(7*a*c - b^2)*(4* b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*b^2*c ^2*d^2))/(a^4*(4*a*c - b^2)^4) - (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*c^3 + 409 *a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4) - (2*b*c^4*d*e^16*x*(6*b^ 6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4 *a^3*e) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6) + (b^2*c ^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c ^2*d^2))/(a^6*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^15*x*(7*a*c - b^2)^3)/(a^6* (4*a*c - b^2)^6))*(((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e ^2*(4*a*c - b^2)^5))^(1/2) - 1)*(((a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^ 2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) - 1)*((2*b*c^2*e^16*(2*b^5 + 46*a^ 2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2))/(a^2* (4*a*c - b^2)^2) - (b*c^2*e^16*(a^3*e*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*(4*a*c - b^2)^5))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/a^3 + (2*b*c^ 3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2) + (4*b*c^3 *d*e^17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*(4*a*c - b^2)^2)))/(4*a^3*e) - (b*c^3*e^15*(7*a*c - b^2)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c ^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2))/(a^4*(4*a*c - b^2)^4) + (b*c^4*e^1 7*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*(4*a*c - b ^2)^4) + (2*b*c^4*d*e^16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^ 4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3*e) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c^2*d^2))/(a^6*(4*a*c - b^2)^6) - (2*b^3*c^5 *d*e^15*x*(7*a*c - b^2)^3)/(a^6*(4*a*c - b^2)^6)))*(2*b^10*e - 2048*a^5*c^5 *e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3*e + 2560*a^4*b^2*c^4*e - 40*a*b^8 *c*e))/(2*(4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*a^4*b^8*c*e^2 + 640*a^5*b

```
^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c^4*e^2)) - (b*atan((x*(((
(((b*((2*(5120*a^10*b*c^9*d*e^17 + 2*a^4*b^13*c^3*d*e^17 - 36*a^5*b^11*c^4*
d*e^17 + 276*a^6*b^9*c^5*d*e^17 - 1216*a^7*b^7*c^6*d*e^17 + 3456*a^8*b^5*c^
7*d*e^17 - 6144*a^9*b^3*c^8*d*e^17))/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^1
0*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^
2*c^5) - ((2*b^10*e - 2048*a^5*c^5*e + 320*a^2*b^6*c^2*e - 1280*a^3*b^4*c^3
*e + 2560*a^4*b^2*c^4*e - 40*a*b^8*c*e)*(163840*a^13*b*c^9*d*e^18 - 12*a^6*
b^15*c^2*d*e^18 + 328*a^7*b^13*c^3*d*e^18 - 3840*a^8*b^11*c^4*d*e^18 + 2496
0*a^9*b^9*c^5*d*e^18 - 97280*a^10*b^7*c^6*d*e^18 + 227328*a^11*b^5*c^7*d*e^
18 - 294912*a^12*b^3*c^8*d*e^18))/((4*a^3*b^10*e^2 - 4096*a^8*c^5*e^2 - 80*
a^4*b^8*c*e^2 + 640*a^5*b^6*c^2*e^2 - 2560*a^6*b^4*c^3*e^2 + 5120*a^7*b^2*c
^4*e^2)*(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10...
```

3.636 $\int \frac{1}{(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=484

$$-\frac{3(5b^2-12ac)\,(b^2-5ac)}{8a^3\,(b^2-4ac)^2\,e(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{4a\,(b^2-4ac)\,e(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{5b^4-35ab^2c+36a^2c}{8a^2\,(b^2-4ac)^2\,e(d+ex}$$

[Out] -3/8*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/(e*x+d)+1/4*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+1/8*(5*b^4-35*a*b^2*c+36*a^2*c^2+b*c*(-32*a*c+5*b^2)*(e*x+d)^2)/a^2/(-4*a*c+b^2)^2/e/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/16*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b^2)+b*(124*a^2*c^2-47*a*b^2*c+5*b^4)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^2/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b^2)+(-124*a^2*b*c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^2/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)

Rubi [A]
time = 0.83, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1156, 1135, 1291, 1295, 1180, 211}

$$-\frac{3(5b^2-12ac)\,(b^2-5ac)}{8a^3e\,(b^2-4ac)^2\,(d+ex)}+\frac{36a^2c^2+bc(5b^2-32ac)\,(d+ex)^2-35ab^2c+5b^4}{8a^2e\,(b^2-4ac)^2\,(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)}-\frac{3\sqrt{c}\left(\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}+(5b^2-12ac)\,(b^2-5ac)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^3e\,(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left((5b^2-12ac)\,(b^2-5ac)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a^3e\,(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\,(b^2-4ac)\,(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]

[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
```

```
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(d+ex)^2 \left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \text{S}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{}{8a}$$

$$= -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2 e(d+ex)} + \frac{b^2-2ac+bc}{4a\left(b^2-4ac\right)e(d+ex)(a+b}$$

$$= -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2 e(d+ex)} + \frac{b^2-2ac+bc}{4a\left(b^2-4ac\right)e(d+ex)(a+b}$$

$$= -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2 e(d+ex)} + \frac{b^2-2ac+bc}{4a\left(b^2-4ac\right)e(d+ex)(a+b}$$

Mathematica [A]
time = 6.16, size = 560, normalized size = 1.16

$$-\frac{1}{a^3e(d+ex)}+\frac{b^3(d+ex)-3abc(d+ex)+b^2c(d+ex)^3-2ac^2(d+ex)^3}{4a^2\left(-b^2+4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{-7b^5(d+ex)+52ab^3c(d+ex)-84a^2bc^2(d+ex)-7b^4c(d+ex)^3+47ab^2c^2(d+ex)^3-52a^2c^3(d+ex)^3}{8a^3\left(-b^2+4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,e}-\frac{3\sqrt{c}\left(-5b^5+47ab^3c-124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -(1/(a^3*e*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d + e*
x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*a^2*b*c^2*(
d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*c^3*(d +
 e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (
3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 3
7*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a
*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(-5*b^5 + 47*a*b^3*c
```

```
- 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] +
60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 -
4*a*c]]*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.28, size = 1197, normalized size = 2.47

method	result
default	$-\frac{1}{a^{3} e \left(e x +d \right)}-\frac{\frac{c^{2} e^{6} \left(52 a^{2} c^{2}-47 a \,b^{2} c +7 b^{4}\right) x^{7}}{128 a^{2} c^{2}-64 a \,b^{2} c +8 b^{4}}+\frac{7 c^{2} d \,e^{5} \left(52 a^{2} c^{2}-47 a \,b^{2} c +7 b^{4}\right) x^{6}}{8 \left(16 a^{2} c^{2}-8 a \,b^{2} c +b^{4}\right)}+\frac{\left(1092 a^{2} c^{3} d^{2}-987 a \,b^{2} c^{2} d^{2}+147 b^{4} c \,d^{2}+136 a^{2} b \,c^{2}-99 a \,b \cdots}{128 a^{2} c^{2}-64 a \,b^{2} c +8 b^{4}}}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3/e/(e*x+d)-1/a^3*((1/8*c^2*e^6*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^7+7/8*c^2*d*e^5*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(16*a^2*
c^2-8*a*b^2*c+b^4)*x^6+1/8*(1092*a^2*c^3*d^2-987*a*b^2*c^2*d^2+147*b^4*c*d^
2+136*a^2*b*c^2-99*a*b^3*c+14*b^5)*e^4*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+5/8
*c*d*e^3*(364*a^2*c^3*d^2-329*a*b^2*c^2*d^2+49*b^4*c*d^2+136*a^2*b*c^2-99*a
*b^3*c+14*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+1/8*e^2*(1820*a^2*c^4*d^4-164
5*a*b^2*c^3*d^4+245*b^4*c^2*d^4+1360*a^2*b*c^3*d^2-990*a*b^3*c^2*d^2+140*b^
5*c*d^2+68*a^3*c^3+25*a^2*b^2*c^2-43*a*b^4*c+7*b^6)/(16*a^2*c^2-8*a*b^2*c+b
^4)*x^3+1/8*d*e*(1092*a^2*c^4*d^4-987*a*b^2*c^3*d^4+147*b^4*c^2*d^4+1360*a^
2*b*c^3*d^2-990*a*b^3*c^2*d^2+140*b^5*c*d^2+204*a^3*c^3+75*a^2*b^2*c^2-129*
a*b^4*c+21*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/8*(364*a^2*c^4*d^6-329*a*b
^2*c^3*d^6+49*b^4*c^2*d^6+680*a^2*b*c^3*d^4-495*a*b^3*c^2*d^4+70*b^5*c*d^4+
204*a^3*c^3*d^2+75*a^2*b^2*c^2*d^2-129*a*b^4*c*d^2+21*b^6*d^2+108*a^3*b*c^2
-66*a^2*b^3*c+9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/8*d/e*(52*a^2*c^4*d^6
-47*a*b^2*c^3*d^6+7*b^4*c^2*d^6+136*a^2*b*c^3*d^4-99*a*b^3*c^2*d^4+14*b^5*c
*d^4+68*a^3*c^3*d^2+25*a^2*b^2*c^2*d^2-43*a*b^4*c*d^2+7*b^6*d^2+108*a^3*b*c
^2-66*a^2*b^3*c+9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x
^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/16/(1
6*a^2*c^2-8*a*b^2*c+b^4)/e*sum((c*e^2*(60*a^2*c^2-37*a*b^2*c+5*b^4)*_R^2+2*
c*d*e*(60*a^2*c^2-37*a*b^2*c+5*b^4)*_R+60*a^2*c^3*d^2-37*a*b^2*c^2*d^2+5*b^
4*c*d^2+92*a^2*b*c^2-42*a*b^3*c+5*b^5)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*
d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c
*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 + 24*(5*b^4*c^2*e^7 - 3
7*a*b^2*c^3*e^7 + 60*a^2*c^4*e^7)*d*x^7 + 3*(5*b^4*c^2*e^8 - 37*a*b^2*c^3*e
^8 + 60*a^2*c^4*e^8)*x^8 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 +
 (30*b^5*c*e^6 - 227*a*b^3*c^2*e^6 + 392*a^2*b*c^3*e^6 + 84*(5*b^4*c^2*e^6
- 37*a*b^2*c^3*e^6 + 60*a^2*c^4*e^6)*d^2)*x^6 + 8*a^2*b^4 - 64*a^3*b^2*c +
128*a^4*c^2 + 6*(28*(5*b^4*c^2*e^5 - 37*a*b^2*c^3*e^5 + 60*a^2*c^4*e^5)*d^3
 + (30*b^5*c*e^5 - 227*a*b^3*c^2*e^5 + 392*a^2*b*c^3*e^5)*d)*x^5 + (15*b^6
- 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + (15*b^6*e^4 - 91*a*b^4*c
*e^4 + 25*a^2*b^2*c^2*e^4 + 324*a^3*c^3*e^4 + 210*(5*b^4*c^2*e^4 - 37*a*b^2
*c^3*e^4 + 60*a^2*c^4*e^4)*d^4 + 15*(30*b^5*c*e^4 - 227*a*b^3*c^2*e^4 + 392
*a^2*b*c^3*e^4)*d^2)*x^4 + 4*(42*(5*b^4*c^2*e^3 - 37*a*b^2*c^3*e^3 + 60*a^2
*c^4*e^3)*d^5 + 5*(30*b^5*c*e^3 - 227*a*b^3*c^2*e^3 + 392*a^2*b*c^3*e^3)*d^
3 + (15*b^6*e^3 - 91*a*b^4*c*e^3 + 25*a^2*b^2*c^2*e^3 + 324*a^3*c^3*e^3)*d)
*x^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d^2 + (84*(5*b^4*c^2*e^2
- 37*a*b^2*c^3*e^2 + 60*a^2*c^4*e^2)*d^6 + 25*a*b^5*e^2 - 194*a^2*b^3*c*e^2
 + 364*a^3*b*c^2*e^2 + 15*(30*b^5*c*e^2 - 227*a*b^3*c^2*e^2 + 392*a^2*b*c^3
*e^2)*d^4 + 6*(15*b^6*e^2 - 91*a*b^4*c*e^2 + 25*a^2*b^2*c^2*e^2 + 324*a^3*c
^3*e^2)*d^2)*x^2 + 2*(12*(5*b^4*c^2*e - 37*a*b^2*c^3*e + 60*a^2*c^4*e)*d^7
+ 3*(30*b^5*c*e - 227*a*b^3*c^2*e + 392*a^2*b*c^3*e)*d^5 + 2*(15*b^6*e - 91
*a*b^4*c*e + 25*a^2*b^2*c^2*e + 324*a^3*c^3*e)*d^3 + (25*a*b^5*e - 194*a^2*
b^3*c*e + 364*a^3*b*c^2*e)*d)*x)/((a^3*b^4*c^2*e - 8*a^4*b^2*c^3*e + 16*a^5
*c^4*e)*d^9 + 9*(a^3*b^4*c^2*e^9 - 8*a^4*b^2*c^3*e^9 + 16*a^5*c^4*e^9)*d*x^
8 + (a^3*b^4*c^2*e^10 - 8*a^4*b^2*c^3*e^10 + 16*a^5*c^4*e^10)*x^9 + 2*(a^3*
b^5*c*e - 8*a^4*b^3*c^2*e + 16*a^5*b*c^3*e)*d^7 + 2*(a^3*b^5*c*e^8 - 8*a^4*
b^3*c^2*e^8 + 16*a^5*b*c^3*e^8 + 18*(a^3*b^4*c^2*e^8 - 8*a^4*b^2*c^3*e^8 +
16*a^5*c^4*e^8)*d^2)*x^7 + 14*(6*(a^3*b^4*c^2*e^7 - 8*a^4*b^2*c^3*e^7 + 16*
a^5*c^4*e^7)*d^3 + (a^3*b^5*c*e^7 - 8*a^4*b^3*c^2*e^7 + 16*a^5*b*c^3*e^7)*d
)*x^6 + (a^3*b^6*e - 6*a^4*b^4*c*e + 32*a^6*c^3*e)*d^5 + (a^3*b^6*e^6 - 6*a
^4*b^4*c*e^6 + 32*a^6*c^3*e^6 + 126*(a^3*b^4*c^2*e^6 - 8*a^4*b^2*c^3*e^6 +
16*a^5*c^4*e^6)*d^4 + 42*(a^3*b^5*c*e^6 - 8*a^4*b^3*c^2*e^6 + 16*a^5*b*c^3*
e^6)*d^2)*x^5 + (126*(a^3*b^4*c^2*e^5 - 8*a^4*b^2*c^3*e^5 + 16*a^5*c^4*e^5)
*d^5 + 70*(a^3*b^5*c*e^5 - 8*a^4*b^3*c^2*e^5 + 16*a^5*b*c^3*e^5)*d^3 + 5*(a
^3*b^6*e^5 - 6*a^4*b^4*c*e^5 + 32*a^6*c^3*e^5)*d)*x^4 + 2*(a^4*b^5*e - 8*a^
5*b^3*c*e + 16*a^6*b*c^2*e)*d^3 + 2*(a^4*b^5*e^4 - 8*a^5*b^3*c*e^4 + 16*a^6
*b*c^2*e^4 + 42*(a^3*b^4*c^2*e^4 - 8*a^4*b^2*c^3*e^4 + 16*a^5*c^4*e^4)*d^6
+ 35*(a^3*b^5*c*e^4 - 8*a^4*b^3*c^2*e^4 + 16*a^5*b*c^3*e^4)*d^4 + 5*(a^3*b^
6*e^4 - 6*a^4*b^4*c*e^4 + 32*a^6*c^3*e^4)*d^2)*x^3 + 2*(18*(a^3*b^4*c^2*e^3
 - 8*a^4*b^2*c^3*e^3 + 16*a^5*c^4*e^3)*d^7 + 21*(a^3*b^5*c*e^3 - 8*a^4*b^3*
c^2*e^3 + 16*a^5*b*c^3*e^3)*d^5 + 5*(a^3*b^6*e^3 - 6*a^4*b^4*c*e^3 + 32*a^6
*c^3*e^3)*d^3 + 3*(a^4*b^5*e^3 - 8*a^5*b^3*c*e^3 + 16*a^6*b*c^2*e^3)*d)*x^2
 + (a^5*b^4*e - 8*a^6*b^2*c*e + 16*a^7*c^2*e)*d + (a^5*b^4*e^2 - 8*a^6*b^2*
```

```
c*e^2 + 16*a^7*c^2*e^2 + 9*(a^3*b^4*c^2*e^2 - 8*a^4*b^2*c^3*e^2 + 16*a^5*c^
4*e^2)*d^8 + 14*(a^3*b^5*c*e^2 - 8*a^4*b^3*c^2*e^2 + 16*a^5*b*c^3*e^2)*d^6
+ 5*(a^3*b^6*e^2 - 6*a^4*b^4*c*e^2 + 32*a^6*c^3*e^2)*d^4 + 6*(a^4*b^5*e^2 -
 8*a^5*b^3*c*e^2 + 16*a^6*b*c^2*e^2)*d^2)*x) - 3/8*integrate((5*b^5 - 42*a*
b^3*c + 92*a^2*b*c^2 + (5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*d^2 + 2*(5*b^4
*c*e - 37*a*b^2*c^2*e + 60*a^2*c^3*e)*d*x + (5*b^4*c*e^2 - 37*a*b^2*c^2*e^2
 + 60*a^2*c^3*e^2)*x^2)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d
^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 10150 vs. $2(444) = 888$.
time = 1.15, size = 10150, normalized size = 20.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8*e^8 + 48*(5*b^4*c^2 -
37*a*b^2*c^3 + 60*a^2*c^4)*d*x^7*e^7 + 6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2
*c^4)*d^8 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3 + 84*(5*b^4*c^2 - 3
7*a*b^2*c^3 + 60*a^2*c^4)*d^2)*x^6*e^6 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*
a^2*b*c^3)*d^6 + 12*(28*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b
^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d)*x^5*e^5 + 16*a^2*b^4 - 128*a^3*b^2
*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 +
210*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*
c^2 + 392*a^2*b*c^3)*d^2)*x^4*e^4 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2
 + 324*a^3*c^3)*d^4 + 8*(42*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5
*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25
*a^2*b^2*c^2 + 324*a^3*c^3)*d)*x^3*e^3 + 2*(84*(5*b^4*c^2 - 37*a*b^2*c^3 +
60*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^2)*x^2*e^2 + 2*(25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^
2)*d^2 + 4*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^
2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*x*e -
3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9*e^10 + 9*(a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*x^8*e^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c
^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^2)*x^7*
e^8 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a^3*b^5*c - 8
*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*x^6*e^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c
^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 42*(a^3*b^5*c - 8
*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*x^5*e^6 + (126*(a^3*b^4*c^2 - 8*a^4*b^2*c
^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3 +
```

```
5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*x^4*e^5 + 2*(a^4*b^5 - 8*a^5*b^3*
c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^6 + 35*(
a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(a^3*b^6 - 6*a^4*b^4*c +
32*a^6*c^3)*d^2)*x^3*e^4 + 2*(18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)
*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^5 + 5*(a^3*b^6 - 6*a
^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d)*x^
2*e^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^
3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5
*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 + 6*(a^4*b^5 - 8*a^5*b^3*c + 16*a
^6*b*c^2)*d^2)*x*e^2 + ((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*
(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 3
2*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 -
8*a^6*b^2*c + 16*a^7*c^2)*d)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7
*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10
- 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1
024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*
a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^1
4*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^
2*c^4 - 1024*a^19*c^5)))*e^(-2)/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2
- 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10
*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000
*a^4*b^2*c^8 - 810000*a^5*c^9)*x*e - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 +
571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5
*c^9)*d + 27/2*sqrt(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2
- 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a
^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250
*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 3
12300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^
6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5))*e - (125*b^1
7 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*
b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 +
 1324800*a^8*b*c^8)*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15
015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*
b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*
c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c
^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 -
20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1
024*a^19*c^5)))*e^(-2)/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^1
0*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))) + 3*sqrt(1/2)*((a^3*b^4*c^
2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9*e^10 + 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 +
 16*a^5*c^4)*d*x^8*e^9 + 2*(a^3*b^5*c - 8*a^4*b...
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1412 vs. $2(444) = 888$.
time = 3.63, size = 1412, normalized size = 2.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/64*(2*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt
(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*abs(a^3*b^4*e^2 - 8*a^4*b
^2*c*e^2 + 16*a^5*c^2*e^2)*e^2 - (a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^
2*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*
c)*a) + (5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c^3
+ 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a^12*b*c^6)*sqrt(2*a*b + 2
*sqrt(b^2 - 4*a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-1)/((x*e + d)*sqrt((a^3*b
^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2 + sqrt((a^3*b^5*e^2 - 8*a^4*b^3
*c*e^2 + 16*a^5*b*c^2*e^2)^2 - 4*(a^4*b^4*e^4 - 8*a^5*b^2*c*e^4 + 16*a^6*c^
2*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4*b^4*e^4 - 8*a^5*b^2*
c*e^4 + 16*a^6*c^2*e^4))))*e^(-3)/((a^7*b^6*c - 12*a^8*b^4*c^2 + 48*a^9*b^2
*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*abs(a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 1
6*a^5*c^2*e^2)*abs(a)) + 3/64*(2*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^
2*c^3 - 240*a^7*c^4)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*
abs(a^3*b^4*e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)*e^2 + (a^3*b^4*e^2 - 8*
a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(
2*a*b - 2*sqrt(b^2 - 4*a*c)*a) - (5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^
9*c^2 - 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a
^12*b*c^6)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*e^4)*arctan(2*sqrt(1/2)*e^(-
1)/((x*e + d)*sqrt((a^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2 - sqrt
((a^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c^2*e^2)^2 - 4*(a^4*b^4*e^4 - 8*
a^5*b^2*c*e^4 + 16*a^6*c^2*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/
(a^4*b^4*e^4 - 8*a^5*b^2*c*e^4 + 16*a^6*c^2*e^4))))*e^(-3)/((a^7*b^6*c - 12
*a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*abs(a^3*b^4*
e^2 - 8*a^4*b^2*c*e^2 + 16*a^5*c^2*e^2)*abs(a)) - 1/8*(7*b^4*c^2*e^(-1)/(x*
e + d) - 47*a*b^2*c^3*e^(-1)/(x*e + d) + 52*a^2*c^4*e^(-1)/(x*e + d) + 14*b
^5*c*e^(-1)/(x*e + d)^3 - 99*a*b^3*c^2*e^(-1)/(x*e + d)^3 + 136*a^2*b*c^3*e
^(-1)/(x*e + d)^3 + 7*b^6*e^(-1)/(x*e + d)^5 - 43*a*b^4*c*e^(-1)/(x*e + d)^
5 + 25*a^2*b^2*c^2*e^(-1)/(x*e + d)^5 + 68*a^3*c^3*e^(-1)/(x*e + d)^5 + 9*a
*b^5*e^(-1)/(x*e + d)^7 - 66*a^2*b^3*c*e^(-1)/(x*e + d)^7 + 108*a^3*b*c^2*e
^(-1)/(x*e + d)^7)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b/(x*e + d)^2
 + a/(x*e + d)^4)^2) - e^(-1)/((x*e + d)*a^3)
```

Mupad [B]
time = 14.38, size = 2500, normalized size = 5.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] - ((x^4*(15*b^6*e^3 + 324*a^3*c^3*e^3 + 450*b^5*c*d^2*e^3 + 25*a^2*b^2*c^2*
e^3 + 12600*a^2*c^4*d^4*e^3 + 1050*b^4*c^2*d^4*e^3 - 91*a*b^4*c*e^3 - 3405*
a*b^3*c^2*d^2*e^3 + 5880*a^2*b*c^3*d^2*e^3 - 7770*a*b^2*c^3*d^4*e^3))/(8*a*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^6*(30*b^5*c*e^5 - 227*a*b^3*c^2*
e^5 + 392*a^2*b*c^3*e^5 + 5040*a^2*c^4*d^2*e^5 + 420*b^4*c^2*d^2*e^5 - 3108
*a*b^2*c^3*d^2*e^5))/(8*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(30*b^
6*d^3 + 90*b^5*c*d^5 + 648*a^3*c^3*d^3 + 720*a^2*c^4*d^7 + 60*b^4*c^2*d^7 +
 25*a*b^5*d - 681*a*b^3*c^2*d^5 + 1176*a^2*b*c^3*d^5 - 444*a*b^2*c^3*d^7 +
50*a^2*b^2*c^2*d^3 - 194*a^2*b^3*c*d + 364*a^3*b*c^2*d - 182*a*b^4*c*d^3))/
(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (3*x^5*(1680*a^2*c^4*d^3*e^4 +
 140*b^4*c^2*d^3*e^4 + 30*b^5*c*d*e^4 - 227*a*b^3*c^2*d*e^4 + 392*a^2*b*c^3
*d*e^4 - 1036*a*b^2*c^3*d^3*e^4))/(4*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)
) + (3*x^8*(60*a^2*c^4*e^7 + 5*b^4*c^2*e^7 - 37*a*b^2*c^3*e^7))/(8*a*(a^2*b
^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^2*(90*b^6*d^2*e + 25*a*b^5*e + 1944*a^
3*c^3*d^2*e + 5040*a^2*c^4*d^6*e + 420*b^4*c^2*d^6*e - 194*a^2*b^3*c*e + 36
4*a^3*b*c^2*e + 450*b^5*c*d^4*e - 546*a*b^4*c*d^2*e - 3405*a*b^3*c^2*d^4*e
+ 5880*a^2*b*c^3*d^4*e - 3108*a*b^2*c^3*d^6*e + 150*a^2*b^2*c^2*d^2*e))/(8*
a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^3*(15*b^6*d*e^2 + 324*a^3*c^3*
d*e^2 + 150*b^5*c*d^3*e^2 + 2520*a^2*c^4*d^5*e^2 + 210*b^4*c^2*d^5*e^2 - 91
*a*b^4*c*d*e^2 + 25*a^2*b^2*c^2*d*e^2 - 1135*a*b^3*c^2*d^3*e^2 + 1960*a^2*b
*c^3*d^3*e^2 - 1554*a*b^2*c^3*d^5*e^2))/(2*a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*
b^2*c)) + (3*x^7*(60*a^2*c^4*d*e^6 + 5*b^4*c^2*d*e^6 - 37*a*b^2*c^3*d*e^6))
/(a*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (8*a^2*b^4 + 128*a^4*c^2 + 15*b
^6*d^4 - 64*a^3*b^2*c + 25*a*b^5*d^2 + 30*b^5*c*d^6 + 324*a^3*c^3*d^4 + 180
*a^2*c^4*d^8 + 15*b^4*c^2*d^8 - 194*a^2*b^3*c*d^2 + 364*a^3*b*c^2*d^2 - 227
*a*b^3*c^2*d^6 + 392*a^2*b*c^3*d^6 - 111*a*b^2*c^3*d^8 + 25*a^2*b^2*c^2*d^4
 - 91*a*b^4*c*d^4)/(8*a*e*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(10*b
^2*d^2*e^3 + 84*c^2*d^6*e^3 + 2*a*b*e^3 + 20*a*c*d^2*e^3 + 70*b*c*d^4*e^3)
+ x^7*(36*c^2*d^2*e^7 + 2*b*c*e^7) + x*(a^2*e + 5*b^2*d^4*e + 9*c^2*d^8*e +
 6*a*b*d^2*e + 10*a*c*d^4*e + 14*b*c*d^6*e) + x^4*(5*b^2*d*e^4 + 126*c^2*d^
5*e^4 + 10*a*c*d*e^4 + 70*b*c*d^3*e^4) + a^2*d + x^2*(10*b^2*d^3*e^2 + 36*c
^2*d^7*e^2 + 6*a*b*d*e^2 + 20*a*c*d^3*e^2 + 42*b*c*d^5*e^2) + x^6*(84*c^2*d
^3*e^6 + 14*b*c*d*e^6) + x^5*(b^2*e^5 + 126*c^2*d^4*e^5 + 2*a*c*e^5 + 42*b*
c*d^2*e^5) + b^2*d^5 + c^2*d^9 + c^2*e^9*x^9 + 2*a*b*d^3 + 2*a*c*d^5 + 2*b*
c*d^7 + 9*c^2*d*e^8*x^8) - atan(((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)
^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 +
1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 439042
```

56*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*
(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 10
48576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*
b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*
a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 26
21440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)
^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 +
1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 439042
56*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*
(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 10
48576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*
b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*
a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 26
21440*a^16*b^2*c^9*e^2)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)
^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 +
1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 439042
56*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*
(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^
15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2 + 10
48576*a^17*c^10*e^2 - 40*a^8*b^18*c*e^2 + 720*a^9*b^16*c^2*e^2 - 7680*a^10*
b^14*c^3*e^2 + 53760*a^11*b^12*c^4*e^2 - 258048*a^12*b^10*c^5*e^2 + 860160*
a^13*b^8*c^6*e^2 - 1966080*a^14*b^6*c^7*e^2 + 2949120*a^15*b^4*c^8*e^2 - 26
21440*a^16*b^2*c^9*e^2)))^(1/2)*(x*(1099511627776*a^26*b*c^13*e^14 - 262144
*a^15*b^23*c^2*e^14 + 11534336*a^16*b^21*c^3*e^14 - 230686720*a^17*b^19*c^4
*e^14 + 2768240640*a^18*b^17*c^5*e^14 - 22145925120*a^19*b^15*c^6*e^14 + 12
4017180672*a^20*b^13*c^7*e^14 - 496068722688*a^21*b^11*c^8*e^14 + 141733920
7680*a^22*b^9*c^9*e^14 - 2834678415360*a^23*b^7*c^10*e^14 + 3779571220480*a
^24*b^5*c^11*e^14 - 3023656976384*a^25*b^3*c^12...

3.637 $\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=325

$$-\frac{3(b^2-5ac)\,(b^2-2ac)}{2a^3\,(b^2-4ac)^2\,e(d+ex)^2}+\frac{b^2-2ac+bc(d+ex)^2}{4a\,(b^2-4ac)\,e(d+ex)^2\,\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3b^4-20ab^2c+20}{4a^2\,(b^2-4ac)^2\,e(d+}$$

```
[Out] -3/2*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/(e*x+d)^2+1/4*(b^2-2*a*
c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+1
/4*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*b*c*(-6*a*c+b^2)*(e*x+d)^2)/a^2/(-4*a*c+b
^2)^2/e/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/2*(-20*a^3*c^3+30*a^2*b^2*c
^2-10*a*b^4*c+b^6)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*
c+b^2)^(5/2)/e-3*b*ln(e*x+d)/a^4/e+3/4*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^4/
e
```

Rubi [A]
time = 0.39, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1156, 1128, 754, 836, 814, 648, 632, 212, 642}

$$\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4e}-\frac{3b\log(d+ex)}{a^4e}-\frac{3(b^2-5ac)\,(b^2-2ac)}{2a^3e\,(b^2-4ac)^2\,(d+ex)^2}+\frac{20a^2c^2+3bc(b^2-6ac)\,(d+ex)^2-20ab^2c+3b^4}{4a^2e\,(b^2-4ac)^2\,(d+ex)^2\,\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4e\,(b^2-4ac)^{5/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{4ae\,(b^2-4ac)\,(d+ex)^2\,\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*(d + e*x)^2) + (b
^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*(d + e*x)^2*(a + b*(d +
e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2
- 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*(d + e*x)^2*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*
ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2
)*e) - (3*b*Log[d + e*x])/(a^4*e) + (3*b*Log[a + b*(d + e*x)^2 + c*(d + e*x
)^4])/(4*a^4*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
```

```
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \cdots$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \cdots$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)e(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \cdots$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2e(d+ex)^2} + \frac{b^2-2ac+\cdots}{4a\left(b^2-4ac\right)e(d+ex)^2(a+\cdots}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2e(d+ex)^2} + \frac{b^2-2ac+\cdots}{4a\left(b^2-4ac\right)e(d+ex)^2(a+\cdots}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2e(d+ex)^2} + \frac{b^2-2ac+\cdots}{4a\left(b^2-4ac\right)e(d+ex)^2(a+\cdots}$$

$$= -\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2e(d+ex)^2} + \frac{b^2-2ac+\cdots}{4a\left(b^2-4ac\right)e(d+ex)^2(a+\cdots}$$

Mathematica [A]
time = 6.13, size = 491, normalized size = 1.51

$$-\frac{1}{2a^3e(d+ex)^2}+\frac{b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2}{4a^2(-b^2+4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{-4b^5+29ab^3c-46a^2bc^2-4b^4c(d+ex)^2+26ab^2c^2(d+ex)^2-28a^2c^3(d+ex)^2}{4a^3(-b^2+4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)}-\frac{3b\log(d+ex)}{a^4e}+\frac{3\left(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+16a^2bc^2\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{4a^4(b^2-4ac)^{5/2}e}+\frac{3\left(-b^6+10ab^4c-30a^2b^2c^2+20a^3c^3+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+16a^2bc^2\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{4a^4(b^2-4ac)^{5/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -1/2*1/(a^3*e*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2)
+ (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26*a*b^2*c^2*
(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*(a + b*(d +
 e*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e) + (3*(b^6 - 10*a*b^4
*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b
^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2
*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e) + (3*(-b^6 + 10*a*b^4*c - 30
*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*
a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d +
 e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.34, size = 1141, normalized size = 3.51

method	result
default	$-\frac{1}{2a^3e(ex+d)^2}-\frac{3b\ln(ex+d)}{a^4e}-\frac{\frac{c^2e^5\left(14a^2c^2-13a\,b^2c+2b^4\right)a\,x^6}{32a^2c^2-16a\,b^2c+2b^4}+\frac{3\left(14a^2c^2-13a\,b^2c+2b^4\right)a\,c^2d\,e^4x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{e^3ac\left(420a^2c^3d^2-390a\,b^2c^2d^2+\right.}{64a^2c^2-}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3/e/(e*x+d)^2-3*b*ln(e*x+d)/a^4/e-1/a^4*((1/2*c^2*e^5*(14*a^2*c^2-13
*a*b^2*c+2*b^4)*a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3*(14*a^2*c^2-13*a*b^2*c+2
*b^4)*a*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+1/4*e^3*a*c*(420*a^2*c^3*d
^2-390*a*b^2*c^2*d^2+60*b^4*c*d^2+74*a^2*b*c^2-55*a*b^3*c+8*b^5)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^4+c*d*e^2*a*(140*a^2*c^3*d^2-130*a*b^2*c^2*d^2+20*b^4*c*
d^2+74*a^2*b*c^2-55*a*b^3*c+8*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(
210*a^2*c^4*d^4-195*a*b^2*c^3*d^4+30*b^4*c^2*d^4+222*a^2*b*c^3*d^2-165*a*b^
3*c^2*d^2+24*b^5*c*d^2+18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^2+d*a*(42*a^2*c^4*d^4-39*a*b^2*c^3*d^4+6*b^4*c^2*d^4+74
*a^2*b*c^3*d^2-55*a*b^3*c^2*d^2+8*b^5*c*d^2+18*a^3*c^3+7*a^2*b^2*c^2-12*a*b
^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/4/e*a*(28*a^2*c^4*d^6-26*a*b^2*c
```

```
^3*d^6+4*b^4*c^2*d^6+74*a^2*b*c^3*d^4-55*a*b^3*c^2*d^4+8*b^5*c*d^4+36*a^3*c
^3*d^2+14*a^2*b^2*c^2*d^2-24*a*b^4*c*d^2+4*b^6*d^2+58*a^3*b*c^2-36*a^2*b^3*
c+5*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2
*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/2/(16*a^2*c^2-8*a*b
^2*c+b^4)/e*sum((e^3*b*c*(-16*a^2*c^2+8*a*b^2*c-b^4)*_R^3+3*d*e^2*b*c*(-16*
a^2*c^2+8*a*b^2*c-b^4)*_R^2+e*(-48*a^2*b*c^3*d^2+24*a*b^3*c^2*d^2-3*b^5*c*d
^2+10*a^3*c^3-23*a^2*b^2*c^2+9*a*b^4*c-b^6)*_R-16*a^2*b*c^3*d^3+8*a*b^3*c^2
*d^3-b^5*c*d^3+10*a^3*c^3*d-23*a^2*b^2*c^2*d+9*a*b^4*c*d-b^6*d)/(2*_R^3*c*e
^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c
*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+
d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^8 + 48*(b^4*c^2*e^7 - 7*a*b^
2*c^3*e^7 + 10*a^2*c^4*e^7)*d*x^7 + 6*(b^4*c^2*e^8 - 7*a*b^2*c^3*e^8 + 10*a
^2*c^4*e^8)*x^8 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^6 + 3*(4*b^5*
c*e^6 - 29*a*b^3*c^2*e^6 + 46*a^2*b*c^3*e^6 + 56*(b^4*c^2*e^6 - 7*a*b^2*c^3
*e^6 + 10*a^2*c^4*e^6)*d^2)*x^6 + 2*a^2*b^4 - 16*a^3*b^2*c + 32*a^4*c^2 + 6
*(56*(b^4*c^2*e^5 - 7*a*b^2*c^3*e^5 + 10*a^2*c^4*e^5)*d^3 + 3*(4*b^5*c*e^5
- 29*a*b^3*c^2*e^5 + 46*a^2*b*c^3*e^5)*d)*x^5 + 2*(3*b^6 - 18*a*b^4*c + 7*a
^2*b^2*c^2 + 50*a^3*c^3)*d^4 + (6*b^6*e^4 - 36*a*b^4*c*e^4 + 14*a^2*b^2*c^2
*e^4 + 100*a^3*c^3*e^4 + 420*(b^4*c^2*e^4 - 7*a*b^2*c^3*e^4 + 10*a^2*c^4*e^
4)*d^4 + 45*(4*b^5*c*e^4 - 29*a*b^3*c^2*e^4 + 46*a^2*b*c^3*e^4)*d^2)*x^4 +
4*(84*(b^4*c^2*e^3 - 7*a*b^2*c^3*e^3 + 10*a^2*c^4*e^3)*d^5 + 15*(4*b^5*c*e^
3 - 29*a*b^3*c^2*e^3 + 46*a^2*b*c^3*e^3)*d^3 + 2*(3*b^6*e^3 - 18*a*b^4*c*e^
3 + 7*a^2*b^2*c^2*e^3 + 50*a^3*c^3*e^3)*d)*x^3 + (9*a*b^5 - 68*a^2*b^3*c +
122*a^3*b*c^2)*d^2 + (168*(b^4*c^2*e^2 - 7*a*b^2*c^3*e^2 + 10*a^2*c^4*e^2)*
d^6 + 9*a*b^5*e^2 - 68*a^2*b^3*c*e^2 + 122*a^3*b*c^2*e^2 + 45*(4*b^5*c*e^2
- 29*a*b^3*c^2*e^2 + 46*a^2*b*c^3*e^2)*d^4 + 12*(3*b^6*e^2 - 18*a*b^4*c*e^2
 + 7*a^2*b^2*c^2*e^2 + 50*a^3*c^3*e^2)*d^2)*x^2 + 2*(24*(b^4*c^2*e - 7*a*b^
2*c^3*e + 10*a^2*c^4*e)*d^7 + 9*(4*b^5*c*e - 29*a*b^3*c^2*e + 46*a^2*b*c^3*
e)*d^5 + 4*(3*b^6*e - 18*a*b^4*c*e + 7*a^2*b^2*c^2*e + 50*a^3*c^3*e)*d^3 +
(9*a*b^5*e - 68*a^2*b^3*c*e + 122*a^3*b*c^2*e)*d)*x)/((a^3*b^4*c^2*e - 8*a^
4*b^2*c^3*e + 16*a^5*c^4*e)*d^10 + 10*(a^3*b^4*c^2*e^10 - 8*a^4*b^2*c^3*e^1
0 + 16*a^5*c^4*e^10)*d*x^9 + (a^3*b^4*c^2*e^11 - 8*a^4*b^2*c^3*e^11 + 16*a^
5*c^4*e^11)*x^10 + 2*(a^3*b^5*c*e - 8*a^4*b^3*c^2*e + 16*a^5*b*c^3*e)*d^8 +
 (2*a^3*b^5*c*e^9 - 16*a^4*b^3*c^2*e^9 + 32*a^5*b*c^3*e^9 + 45*(a^3*b^4*c^2
*e^9 - 8*a^4*b^2*c^3*e^9 + 16*a^5*c^4*e^9)*d^2)*x^8 + 8*(15*(a^3*b^4*c^2*e^
```

```
8 - 8*a^4*b^2*c^3*e^8 + 16*a^5*c^4*e^8)*d^3 + 2*(a^3*b^5*c*e^8 - 8*a^4*b^3*
c^2*e^8 + 16*a^5*b*c^3*e^8)*d)*x^7 + (a^3*b^6*e - 6*a^4*b^4*c*e + 32*a^6*c^
3*e)*d^6 + (a^3*b^6*e^7 - 6*a^4*b^4*c*e^7 + 32*a^6*c^3*e^7 + 210*(a^3*b^4*c
^2*e^7 - 8*a^4*b^2*c^3*e^7 + 16*a^5*c^4*e^7)*d^4 + 56*(a^3*b^5*c*e^7 - 8*a^
4*b^3*c^2*e^7 + 16*a^5*b*c^3*e^7)*d^2)*x^6 + 2*(126*(a^3*b^4*c^2*e^6 - 8*a^
4*b^2*c^3*e^6 + 16*a^5*c^4*e^6)*d^5 + 56*(a^3*b^5*c*e^6 - 8*a^4*b^3*c^2*e^6
 + 16*a^5*b*c^3*e^6)*d^3 + 3*(a^3*b^6*e^6 - 6*a^4*b^4*c*e^6 + 32*a^6*c^3*e^
6)*d)*x^5 + 2*(a^4*b^5*e - 8*a^5*b^3*c*e + 16*a^6*b*c^2*e)*d^4 + (2*a^4*b^5
*e^5 - 16*a^5*b^3*c*e^5 + 32*a^6*b*c^2*e^5 + 210*(a^3*b^4*c^2*e^5 - 8*a^4*b
^2*c^3*e^5 + 16*a^5*c^4*e^5)*d^6 + 140*(a^3*b^5*c*e^5 - 8*a^4*b^3*c^2*e^5 +
 16*a^5*b*c^3*e^5)*d^4 + 15*(a^3*b^6*e^5 - 6*a^4*b^4*c*e^5 + 32*a^6*c^3*e^5
)*d^2)*x^4 + 4*(30*(a^3*b^4*c^2*e^4 - 8*a^4*b^2*c^3*e^4 + 16*a^5*c^4*e^4)*d
^7 + 28*(a^3*b^5*c*e^4 - 8*a^4*b^3*c^2*e^4 + 16*a^5*b*c^3*e^4)*d^5 + 5*(a^3
*b^6*e^4 - 6*a^4*b^4*c*e^4 + 32*a^6*c^3*e^4)*d^3 + 2*(a^4*b^5*e^4 - 8*a^5*b
^3*c*e^4 + 16*a^6*b*c^2*e^4)*d)*x^3 + (a^5*b^4*e - 8*a^6*b^2*c*e + 16*a^7*c
^2*e)*d^2 + (a^5*b^4*e^3 - 8*a^6*b^2*c*e^3 + 16*a^7*c^2*e^3 + 45*(a^3*b^4*c
^2*e^3 - 8*a^4*b^2*c^3*e^3 + 16*a^5*c^4*e^3)*d^8 + 56*(a^3*b^5*c*e^3 - 8*a^
4*b^3*c^2*e^3 + 16*a^5*b*c^3*e^3)*d^6 + 15*(a^3*b^6*e^3 - 6*a^4*b^4*c*e^3 +
 32*a^6*c^3*e^3)*d^4 + 12*(a^4*b^5*e^3 - 8*a^5*b^3*c*e^3 + 16*a^6*b*c^2*e^3
)*d^2)*x^2 + 2*(5*(a^3*b^4*c^2*e^2 - 8*a^4*b^2*c^3*e^2 + 16*a^5*c^4*e^2)*d^
9 + 8*(a^3*b^5*c*e^2 - 8*a^4*b^3*c^2*e^2 + 16*a^5*b*c^3*e^2)*d^7 + 3*(a^3*b
^6*e^2 - 6*a^4*b^4*c*e^2 + 32*a^6*c^3*e^2)*d^5 + 4*(a^4*b^5*e^2 - 8*a^5*b^3
*c*e^2 + 16*a^6*b*c^2*e^2)*d^3 + (a^5*b^4*e^2 - 8*a^6*b^2*c*e^2 + 16*a^7*c^
2*e^2)*d)*x) + 3*integrate(((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + 3*(b
^5*c*e^2 - 8*a*b^3*c^2*e^2 + 16*a^2*b*c^3*e^2)*d*x^2 + (b^5*c*e^3 - 8*a*b^3
*c^2*e^3 + 16*a^2*b*c^3*e^3)*x^3 + (b^6 - 9*a*b^4*c + 23*a^2*b^2*c^2 - 10*a
^3*c^3)*d + (b^6*e - 9*a*b^4*c*e + 23*a^2*b^2*c^2*e - 10*a^3*c^3*e + 3*(b^5
*c*e - 8*a*b^3*c^2*e + 16*a^2*b*c^3*e)*d^2)*x)/(c*x^4*e^4 + 4*c*d*x^3*e^3 +
 c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a),
x)/(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2) - 3*b*e^(-1)*log(x*e + d)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7477 vs. $2(318) = 636$.
time = 1.59, size = 15081, normalized size = 46.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")

[Out] [-1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^8*e^8
 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*x^7*e^7
+ 2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 -
11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + 3*(4*a*b^7*c - 45*a^2*b
^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
```

38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*x^6*e^6 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2
+ 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 6*(56*(a*b^6*c^2 - 11*a^2*b^4*c^3
+ 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^
3*b^3*c^3 - 184*a^4*b*c^4)*d)*x^5*e^5 + (6*a*b^8 - 60*a^2*b^6*c + 158*a^3*b
^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 3
8*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*
b^3*c^3 - 184*a^4*b*c^4)*d^2)*x^4*e^4 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*
b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4
*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 +
162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b
^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*x^3*e^3 + (9*a^2*b^7 - 104*a^3*b^
5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 3
8*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*
b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*x^2*e^2 + (9*a^2*b^7 - 104*a^3*b^5*c +
394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3
*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*
b^3*c^2 - 488*a^5*b*c^3)*d)*x*e + 3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c
^4 - 20*a^3*c^5)*x^10*e^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 -
20*a^3*c^5)*d*x^9*e^9 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c
^5)*d^10 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b*c^4 + 45*(b^
6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*x^8*e^8 + 2*(b^7*c
- 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^8 + 8*(15*(b^6*c^2 - 10*
a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30
*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*x^7*e^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2
+ 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c
^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b
*c^4)*d^2)*x^6*e^6 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 4
0*a^4*c^4)*d^6 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c
^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^3 + 3
*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d)*x^5*e^
5 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*c^2
- 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b^5*c
^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*
c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*x^4*e^4 + 2*(a*b^7 - 10*a^2*b^5*c +
30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^4 + 4*(30*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^
2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 2
0*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 4
0*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d
)*x^3*e^3 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8
+ a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*
b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*

```
a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*x^2*e^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4
*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
- 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4
)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*
d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b
^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*x*e)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2
 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2 - 2*a*c + (2*c*x^2*e^2 +
4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*
d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a)) - 3*((b^7
*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^10*e^10 + 10*(b^7*c^
2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*x^9*e^9 + (b^7*c^2 - 12
*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + (2*b^8*c - 24*a*b^6*c^2
+ 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^
3*c^4 - 64*a^3*b*c^5)*d^2)*x^8*e^8 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c
^3 - 64*a^3*b^2*c^4)*d^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 -
 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.73, size = 377, normalized size = 1.16

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(-\frac{b+\frac{2\,a}{(xe+d)^{2}}}{\sqrt{-b^{2}+4\,ac}}\right)e^{(-1)}}{2\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{3\,be^{(-1)}\log\left(c+\frac{b}{(xe+d)^{2}}+\frac{a}{(xe+d)^{4}}\right)}{4\,a^{4}}-\frac{e^{(-1)}}{2\,(xe+d)^{2}a^{3}}+\frac{\left(5\,b^{5}c^{2}-36\,ab^{3}c^{3}+58\,a^{2}bc^{4}+\frac{2\left(5\,b^{6}ce-38\,ab^{4}c^{2}e+71\,a^{2}b^{2}c^{3}e-14\,a^{3}c^{4}e\right)e^{(-1)}}{(xe+d)^{2}}+\frac{\left(5\,b^{7}e^{2}-34\,ab^{5}ce^{2}+41\,a^{2}b^{3}c^{2}e^{2}+42\,a^{3}bc^{3}e^{2}\right)e^{(-2)}}{(xe+d)^{4}}+\frac{6\left(ab^{6}e^{3}-8\,a^{2}b^{4}ce^{3}+17\,a^{3}b^{2}c^{2}e^{3}-6\,a^{4}c^{3}e^{3}\right)e^{(-3)}}{(xe+d)^{6}}\right)e^{(-1)}}{4\left(b^{2}-4\,ac\right)^{2}a^{4}\left(c+\frac{b}{(xe+d)^{2}}+\frac{a}{(xe+d)^{4}}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan(-(b + 2*a/(x*e
+ d)^2)/sqrt(-b^2 + 4*a*c))*e^(-1)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sq
rt(-b^2 + 4*a*c)) + 3/4*b*e^(-1)*log(c + b/(x*e + d)^2 + a/(x*e + d)^4)/a^4
 - 1/2*e^(-1)/((x*e + d)^2*a^3) + 1/4*(5*b^5*c^2 - 36*a*b^3*c^3 + 58*a^2*b*
c^4 + 2*(5*b^6*c*e - 38*a*b^4*c^2*e + 71*a^2*b^2*c^3*e - 14*a^3*c^4*e)*e^(-
1)/(x*e + d)^2 + (5*b^7*e^2 - 34*a*b^5*c*e^2 + 41*a^2*b^3*c^2*e^2 + 42*a^3*
b*c^3*e^2)*e^(-2)/(x*e + d)^4 + 6*(a*b^6*e^3 - 8*a^2*b^4*c*e^3 + 17*a^3*b^2
*c^2*e^3 - 6*a^4*c^3*e^3)*e^(-3)/(x*e + d)^6)*e^(-1)/((b^2 - 4*a*c)^2*a^4*(
c + b/(x*e + d)^2 + a/(x*e + d)^4)^2)
```

Mupad [B]
time = 22.45, size = 2500, normalized size = 7.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] (log(((27*c^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b
^4*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*(4*a*c - b^2
)^6) - ((3*b - 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2
/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2
*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*
b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*(4*a*c - b^2)^4) - ((3*b - 3*a^4*e*(-
(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5
))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 +
 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/(a^3
*(4*a*c - b^2)^2) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2
*a*b^4*c))/(a^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b - 3*a^4*e*(-(b^6 - 20*a
^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(
a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2
 - 20*a*c*d*e*x))/a^4 + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^
2 - 2*a*b^4*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4*e) + (9*b*c^4*e^17*x^2*(6*b^
8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4
*a*c - b^2)^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2
- 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4*e) + (27*c
^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) + (54*c
^5*d*e^15*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6))*((27*c
^4*e^14*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 +
10*a^2*c^3*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*(4*a*c - b^2)^6) - ((3*
b + 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(
4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)*(4*b^6
- 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b^3*c^2*d^2
 + 90*a^2*b*c^3*d^2))/(a^6*(4*a*c - b^2)^4) - ((3*b + 3*a^4*e*(-(b^6 - 20*a
^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(
(6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^
4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3*d^2))/(a^3*(4*a*c - b
^2)^2) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/
(a^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b + 3*a^4*e*(-(b^6 - 20*a^3*c^3 + 30
*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2
*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d
*e*x))/a^4 + (12*c^3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4
*c))/(a^3*(4*a*c - b^2)^2)))/(4*a^4*e) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4
*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)
^4) + (18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*
```

```
b^2*c^3 - 89*a*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4*e) + (27*c^5*e^16*x^2
*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6) + (54*c^5*d*e^15*x
*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*(4*a*c - b^2)^6)))*(6*b^11*e + 960*
a^2*b^7*c^2*e - 3840*a^3*b^5*c^3*e + 7680*a^4*b^3*c^4*e - 120*a*b^9*c*e - 6
144*a^5*b*c^5*e))/(2*(4*a^4*b^10*e^2 - 4096*a^9*c^5*e^2 - 80*a^5*b^8*c*e^2
+ 640*a^6*b^6*c^2*e^2 - 2560*a^7*b^4*c^3*e^2 + 5120*a^8*b^2*c^4*e^2)) - ((x
^4*(6*b^6*e^3 + 100*a^3*c^3*e^3 + 180*b^5*c*d^2*e^3 + 14*a^2*b^2*c^2*e^3 +
4200*a^2*c^4*d^4*e^3 + 420*b^4*c^2*d^4*e^3 - 36*a*b^4*c*e^3 - 1305*a*b^3*c^
2*d^2*e^3 + 2070*a^2*b*c^3*d^2*e^3 - 2940*a*b^2*c^3*d^4*e^3))/(4*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^6*(4*b^5*c*e^5 - 29*a*b^3*c^2*e^5 + 46*a
^2*b*c^3*e^5 + 560*a^2*c^4*d^2*e^5 + 56*b^4*c^2*d^2*e^5 - 392*a*b^2*c^3*d^2
*e^5))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x*(12*b^6*d^3 + 36*b^5*c
*d^5 + 200*a^3*c^3*d^3 + 240*a^2*c^4*d^7 + 24*b^4*c^2*d^7 + 9*a*b^5*d - 261
*a*b^3*c^2*d^5 + 414*a^2*b*c^3*d^5 - 168*a*b^2*c^3*d^7 + 28*a^2*b^2*c^2*d^3
 - 68*a^2*b^3*c*d + 122*a^3*b*c^2*d - 72*a*b^4*c*d^3))/(2*(a^3*b^4 + 16*a^5
*c^2 - 8*a^4*b^2*c)) + (3*x^5*(560*a^2*c^4*d^3*e^4 + 56*b^4*c^2*d^3*e^4 + 1
2*b^5*c*d*e^4 - 87*a*b^3*c^2*d*e^4 + 138*a^2*b*c^3*d*e^4 - 392*a*b^2*c^3*d^
3*e^4))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^8*(10*a^2*c^4*e^7 +
 b^4*c^2*e^7 - 7*a*b^2*c^3*e^7))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) +
 (x^2*(36*b^6*d^2*e + 9*a*b^5*e + 600*a^3*c^3*d^2*e + 1680*a^2*c^4*d^6*e +
168*b^4*c^2*d^6*e - 68*a^2*b^3*c*e + 122*a^3*b*c^2*e + 180*b^5*c*d^4*e - 21
6*a*b^4*c*d^2*e - 1305*a*b^3*c^2*d^4*e + 2070*a^2*b*c^3*d^4*e - 1176*a*b^2*
c^3*d^6*e + 84*a^2*b^2*c^2*d^2*e))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c))
 + (x^3*(6*b^6*d*e^2 + 100*a^3*c^3*d*e^2 + 60*b^5*c*d^3*e^2 + 840*a^2*c^4*d
^5*e^2 + 84*b^4*c^2*d^5*e^2 - 36*a*b^4*c*d*e^2 + 14*a^2*b^2*c^2*d*e^2 - 435
*a*b^3*c^2*d^3*e^2 + 690*a^2*b*c^3*d^3*e^2 - 588*a*b^2*c^3*d^5*e^2))/(a^3*b
^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (12*x^7*(10*a^2*c^4*d*e^6 + b^4*c^2*d*e^6
- 7*a*b^2*c^3*d*e^6))/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (2*a^2*b^4 + 3
2*a^4*c^2 + 6*b^6*d^4 - 16*a^3*b^2*c + 9*a*b^5*d^2 + 12*b^5*c*d^6 + 100*a^3
*c^3*d^4 + 60*a^2*c^4*d^8 + 6*b^4*c^2*d^8 - 68*...
```

3.638 $\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4} dx$

Optimal. Leaf size=202

$$\frac{f^4x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}\,e}$$

```
[Out] f^4*x/c-1/2*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)-1/2*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2
))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1156, 1136, 1180, 211}

$$-\frac{f^4\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}e\sqrt{b-\sqrt{b^2-4ac}}}-\frac{f^4\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}e\sqrt{\sqrt{b^2-4ac}+b}}+\frac{f^4x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^4*x)/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt
[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*e) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt
[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
```

```
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(df+efx)^4}{a+b(d+ex)^2+c(d+ex)^4}\,dx &= \frac{f^4\text{Subst}\left(\int \frac{x^4}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e} \\
&= \frac{f^4x}{c} - \frac{f^4\text{Subst}\left(\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{ce} \\
&= \frac{f^4x}{c} - \frac{\left(\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, d\right.}{2ce} \\
&= \frac{f^4x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e} - \frac{\left(b+\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 222, normalized size = 1.10

$$
\frac{f^4\left(2\sqrt{c}\,(d+ex)-\frac{\sqrt{2}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2c^{3/2}e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^4*(2*Sqrt[c]*(d + e*x) - (Sqrt[2]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(b^2 - 2*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(2*c^(3/2)*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.20, size = 162, normalized size = 0.80

method	result
default	$f^4 \left(\frac{x}{c} + \frac{\sum_{\_R=\mathrm{RootOf}\left(e^4 c\_Z^4+4d\,e^3 c\_Z^3+\left(6d^2 e^2 c+e^2 b\right)\_Z^2+\left(4d^3 ec+2deb\right)\_Z+d^4 c+d^2 b+a\right)} \frac{\left(-\_R^2 b\,e^2-2\_R bde-d^2 b-a\right)\ln}{2e^3 c\_R^3+6d\,e^2 c\_R^2+6c\,d^2 e\_R+2} }{2ce} \right.$
risch	$\frac{f^4 x}{c} + \frac{f^4 \left(\sum_{\_R=\mathrm{RootOf}\left(e^4 c\_Z^4+4d\,e^3 c\_Z^3+\left(6d^2 e^2 c+e^2 b\right)\_Z^2+\left(4d^3 ec+2deb\right)\_Z+d^4 c+d^2 b+a\right)} \frac{\left(-\_R^2 b\,e^2-2\_R bde-d^2 b-a\right)\ln}{2e^3 c\_R^3+6d\,e^2 c\_R^2+6c\,d^2 e\_R+2} \right.}{2ce}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] f^4*(x/c+1/2/c/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2-a)/(2*_R^3*c*e^3+6*_R^2*
c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*
e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] f^4*x/c - f^4*integrate((b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)/(c*x^4*e^4 + 4*
c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*
d*e)*x + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1282 vs. 2(166) = 332.
time = 0.39, size = 1282, normalized size = 6.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(2*f^4*x - sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)*f^16/(b^2*c^6 - 4*a*c^7))*(b^2*c^3 - 4*a*c^4))*e^(-2)/(b^2*c^
3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*f^12*x*e - 2*(a*b^2 - a^2*c)*d*f^12 +
sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*f^8*e - sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)*f^16/(b^2*c^6 - 4*a*c^7))*(b^3*c^3 - 4*a*b*c^4)*e)*sqrt(-((b^3 - 3*a
*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/(b^2*c^6 - 4*a*c^7))*(b^2
*c^3 - 4*a*c^4))*e^(-2)/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-((b^3 - 3
*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/(b^2*c^6 - 4*a*c^7))*(b
^2*c^3 - 4*a*c^4))*e^(-2)/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*f^12*
x*e - 2*(a*b^2 - a^2*c)*d*f^12 - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*f
^8*e - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/(b^2*c^6 - 4*a*c^7))*(b^3*c^3
- 4*a*b*c^4)*e)*sqrt(-((b^3 - 3*a*b*c)*f^8 + sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)*f^16/(b^2*c^6 - 4*a*c^7))*(b^2*c^3 - 4*a*c^4))*e^(-2)/(b^2*c^3 - 4*a*c^4
))) - sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)*f^16/(b^2*c^6 - 4*a*c^7))*(b^2*c^3 - 4*a*c^4))*e^(-2)/(b^2*c^3 - 4*a*c
^4))*log(-2*(a*b^2 - a^2*c)*f^12*x*e - 2*(a*b^2 - a^2*c)*d*f^12 + sqrt(1/2)
*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*f^8*e + sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^
16/(b^2*c^6 - 4*a*c^7))*(b^3*c^3 - 4*a*b*c^4)*e)*sqrt(-((b^3 - 3*a*b*c)*f^8
 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/(b^2*c^6 - 4*a*c^7))*(b^2*c^3 - 4*
a*c^4))*e^(-2)/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*f
^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/(b^2*c^6 - 4*a*c^7))*(b^2*c^3 -
4*a*c^4))*e^(-2)/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*f^12*x*e - 2*(
a*b^2 - a^2*c)*d*f^12 - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*f^8*e + sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/(b^2*c^6 - 4*a*c^7))*(b^3*c^3 - 4*a*b*c
^4)*e)*sqrt(-((b^3 - 3*a*b*c)*f^8 - sqrt((b^4 - 2*a*b^2*c + a^2*c^2)*f^16/(
b^2*c^6 - 4*a*c^7))*(b^2*c^3 - 4*a*c^4))*e^(-2)/(b^2*c^3 - 4*a*c^4))))/c
```

Sympy [A]
time = 2.09, size = 219, normalized size = 1.08

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{2}c^{5}e^{4}-128ab^{2}c^{4}e^{4}+16b^{4}c^{3}e^{4}\right)+t^{2}\cdot\left(48a^{2}bc^{2}e^{2}f^{8}-28ab^{3}ce^{2}f^{8}+4b^{5}e^{2}f^{8}\right)+a^{3}f^{16},\left(t\mapsto t\log\left(x+\frac{32t^{3}abc^{4}e^{3}-8t^{3}b^{3}c^{3}e^{3}-4ta^{2}c^{2}ef^{8}+8tab^{2}cef^{8}-2tb^{4}ef^{8}+a^{2}cdf^{12}-ab^{2}df^{12}}{a^{2}cef^{12}-ab^{2}ef^{12}}\right)\right)\right)+\frac{f^{4}x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5*e**4 - 128*a*b**2*c**4*e**4 + 16*b**4*c**3*e**
4) + _t**2*(48*a**2*b*c**2*e**2*f**8 - 28*a*b**3*c*e**2*f**8 + 4*b**5*e**2*
f**8) + a**3*f**16, Lambda(_t, _t*log(x + (32*_t**3*a*b*c**4*e**3 - 8*_t**3
*b**3*c**3*e**3 - 4*_t*a**2*c**2*e*f**8 + 8*_t*a*b**2*c*e*f**8 - 2*_t*b**4*
e*f**8 + a**2*c*d*f**12 - a*b**2*d*f**12)/(a**2*c*e*f**12 - a*b**2*e*f**12)
))) + f**4*x/c
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1245 vs. 2(166) = 332.
time = 4.79, size = 1245, normalized size = 6.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] f^4*x/c + 1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))^2*b*f^4*e^6 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e^5 + b*d^2*f^4*e^4 + a*f^4*e^4)*log(d*e^(-
1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e
^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4
- 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^
2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*f^4*e^6 - 2*(d*e^(-1)
 - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e^5 +
 b*d^2*f^4*e^4 + a*f^4*e^4)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^
2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*
d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))) + ((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))^2*b*f^4*e^6 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e^5 + b*d^2*f^4*e^4 + a*f^4*e^4)*log(d*e^(
-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*
e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4
 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2
)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2
)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*f^4*e^6 - 2*(d*e^(-1
) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e^5
+ b*d^2*f^4*e^4 + a*f^4*e^4)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c
*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^
2)*e^(-4)/c))))*e^(-4)/c
```

Mupad [**B**]
time = 1.34, size = 2500, normalized size = 12.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)

[Out] atan((((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c + ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i + ((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i)/(((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c + ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) - ((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-

```
(b^5*f^8 + b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*
f^8 - a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 -
8*a*b^2*c^4*e^2)))^(1/2) + (2*a^2*b*e^10*f^12)/c))*(-(b^5*f^8 + b^2*f^8*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 - a*c*f^8*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/
2)*2i + atan((((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*
f^8)/c + ((16*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c + ((8*b^3*c^3*d*e^
13 - 32*a*b*c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(
b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f
^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8
*a*b^2*c^4*e^2)))^(1/2))*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 +
2*a^2*c^2*e^12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2)*1i
+ ((2*b^4*d*e^11*f^8 + 4*a^2*c^2*d*e^11*f^8 - 8*a*b^2*c*d*e^11*f^8)/c - ((1
6*a^2*c^3*e^12*f^4 - 4*a*b^2*c^2*e^12*f^4)/c - ((8*b^3*c^3*d*e^13 - 32*a*b*
c^4*d*e^13)/c + (2*x*(4*b^3*c^3*e^14 - 16*a*b*c^4*e^14))/c)*(-(b^5*f^8 - b^
2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^8
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 + b^4*c^3*e^2 - 8*a*b^2*c^4*e
^2)))^(1/2))*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*f
^8 - 7*a*b^3*c*f^8 + a*c*f^8*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*e^2 +
 b^4*c^3*e^2 - 8*a*b^2*c^4*e^2)))^(1/2) + (2*x*(b^4*e^12*f^8 + 2*a^2*c^2*e^
12*f^8 - 4*a*b^2*c*e^12*f^8))/c)*(-(b^5*f^8 - b^2*f^8*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2*f^8 - 7*a*b^3*c*f^8 + a*c*f^...
```

3.639 $\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=87

$$\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,e}+\frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

```
[Out] 1/4*f^3*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/c/e+1/2*b*f^3*arctanh((b+2*c*(e*x+d)^
2)/(-4*a*c+b^2)^(1/2))/c/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1156, 1128, 648, 632, 212, 642}

$$\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2ce\sqrt{b^2-4ac}}+\frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*
a*c]*e) + (f^3*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*c*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(df+efx)^3}{a+b(d+ex)^2+c(d+ex)^4}\,dx &= \frac{f^3\text{Subst}\left(\int \frac{x^3}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{e}\\
&= \frac{f^3\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{2e}\\
&= \frac{f^3\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4ce} - \frac{(bf^3)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, (d\right.}{4ce}\\
&= \frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce} + \frac{(bf^3)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b\,-\right.}{2ce}\\
&= \frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,e} + \frac{f^3\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4ce}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 80, normalized size = 0.92

$$
\frac{f^3\left(-\frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+b(d+ex)^2+c(d+ex)^4\right)\right)}{4ce}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f^3*((-2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]))/(4*c*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.27, size = 154, normalized size = 1.77

method	result
default	$\frac{f^3 \left(\sum_{\_R=\mathrm{RootOf}\left(e^4 c \_Z^4+4d e^3 c \_Z^3+\left(6d^2 e^2 c+e^2 b\right)\_Z^2+\left(4d^3 ec+2deb\right)\_Z+d^4 c+d^2 b+a\right)} \frac{\left(\_R^3 e^3+3\_R^2 d e^2+3\_R d^2 e+d^3\right)\ln\left(x\right.}{2e^3 c\_R^3+6d e^2 c\_R^2+6c d^2 e\_R+2c d^3+e}\right.}{2e}$
risch	$\frac{f^3 \ln\left(\left(\left(-4abc e^2+b^3 e^2+\sqrt{-b^2\left(4ac-b^2\right)}\, b e^2\right)x^2+\left(-8abcde+2b^3 de+2\sqrt{-b^2\left(4ac-b^2\right)}\, bde\right)x-4abc d^2+b^3 d^2+\sqrt{\right.\right.}{\left(4ac-b^2\right)e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f^3/e*sum((_R^3*e^3+3*_R^2*d*e^2+3*_R*d^2*e+d^3)/(2*_R^3*c*e^3+6*_R^2*c
*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e
^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((f*x*e + d*f)^3/((x*e + d)^4*c + (x*e + d)^2*b + a), x)
```

Fricas [A]
time = 0.36, size = 436, normalized size = 5.01

$$\left[\frac{\left(\sqrt{b^2-4ac}\, b f^3 \log\left(\frac{2c^2x^4e^4+8c^2dx^3e^3+2c^2d^4+2bcd^2+2\left(6c^2d^2+bc\right)x^2e^2+4\left(2c^2d^3+bcd\right)xe+b^2-2ac+\left(2cx^2e^2+4cdxe+2cd^2+b\right)\sqrt{b^2-4ac}}{cx^4e^4+4cdx^3e^3+cd^4+\left(6cd^2+b\right)x^2e^2+bd^2+2\left(2cd^3+bd\right)xe+a}\right)+\left(b^2-4ac\right)f^3\log\left(cx^4e^4+4cdx^3e^3+cd^4+\left(6cd^2+b\right)x^2e^2+bd^2+2\left(2cd^3+bd\right)xe+a\right)\right)e^{(-1)}}{4\left(b^2c-4ac^2\right)}, \frac{\left(2\sqrt{-b^2+4ac}\, b f^3 \arctan\left(-\frac{\left(2cx^2e^2+4cdxe+2cd^2+b\right)\sqrt{-b^2+4ac}}{b^2-4ac}\right)+\left(b^2-4ac\right)f^3\log\left(cx^4e^4+4cdx^3e^3+cd^4+\left(6cd^2+b\right)x^2e^2+bd^2+2\left(2cd^3+bd\right)xe+a\right)\right)e^{(-1)}}{4\left(b^2c-4ac^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*f^3*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*
d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e +
 b^2 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(
c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*
d^3 + b*d)*x*e + a)) + (b^2 - 4*a*c)*f^3*log(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*
```

```
d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a))*e^(-1)/(b
^2*c - 4*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*f^3*arctan(-(2*c*x^2*e^2 + 4*c
*d*x*e + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*f^3
*log(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*
(2*c*d^3 + b*d)*x*e + a))*e^(-1)/(b^2*c - 4*a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(75) = 150$.
time = 1.03, size = 332, normalized size = 3.82

$$\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(-\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right)+\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce\left(4ac-b^2\right)}+\frac{f^3}{4ce}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8ace\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+2af^3+2b^2e\left(\frac{bf^3\sqrt{-4ac+b^2}}{4ce(4ac-b^2)}+\frac{f^3}{4ce}\right)+bd^2f^3}{be^2f^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d
*x/e + x**2 + (-8*a*c*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2))
 + f**3/(4*c*e)) + 2*a*f**3 + 2*b**2*e*(-b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*
(4*a*c - b**2)) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3)) + (b*f**3*sqr
t(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*e))*log(2*d*x/e + x**2
+ (-8*a*c*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)) + f**3/(4*c*
e)) + 2*a*f**3 + 2*b**2*e*(b*f**3*sqrt(-4*a*c + b**2)/(4*c*e*(4*a*c - b**2)
) + f**3/(4*c*e)) + b*d**2*f**3)/(b*e**2*f**3))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(80) = 160$.
time = 4.13, size = 162, normalized size = 1.86

$$-\frac{bf^3\arctan\left(\frac{2\,cd^2f+2\,(fx^2e+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{2\,\sqrt{-b^2+4\,ac}\,c}+\frac{f^3e^{(-1)}\log\left(cd^4f^2+2\,(fx^2e+2\,dfx)cd^2fe+bd^2f^2+(fx^2e+2\,dfx)^2ce^2+(fx^2e+2\,dfx)bfe+af^2\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*b*f^3*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2
+ 4*a*c)*f))*e^(-1)/(sqrt(-b^2 + 4*a*c)*c) + 1/4*f^3*e^(-1)*log(c*d^4*f^2 +
 2*(f*x^2*e + 2*d*f*x)*c*d^2*f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f*x)^2*c*e^2
+ (f*x^2*e + 2*d*f*x)*b*f*e + a*f^2)/c
```

Mupad [B]
time = 0.44, size = 287, normalized size = 3.30

$$\frac{4\,a\,c\,e\,f^3\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}-\frac{b\,f^3\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,d^2}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,e^2\,x^2}{\sqrt{4\,a\,c-b^2}}+\frac{4\,c\,d\,e\,x}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,e\,\sqrt{4\,a\,c-b^2}}-\frac{b^2\,e\,f^3\,\ln\left(c\,d^4+4\,c\,d^3\,e\,x+6\,c\,d^2\,e^2\,x^2+b\,d^2+4\,c\,d\,e^3\,x^3+2\,b\,d\,e\,x+c\,e^4\,x^4+b\,e^2\,x^2+a\right)}{16\,a\,c^2\,e^2-4\,b^2\,c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (4*a*c*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b*d*e*x + 6*
c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e^2 - 4*b^2*c*e^2)
- (b*f^3*atan(b/(4*a*c - b^2)^(1/2) + (2*c*d^2)/(4*a*c - b^2)^(1/2) + (2*c*
e^2*x^2)/(4*a*c - b^2)^(1/2) + (4*c*d*e*x)/(4*a*c - b^2)^(1/2)))/(2*c*e*(4*
a*c - b^2)^(1/2)) - (b^2*e*f^3*log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^
4 + 2*b*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3))/(16*a*c^2*e
^2 - 4*b^2*c*e^2)
```

3.640 $\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=170

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,f^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,\sqrt{c}\,\sqrt{b^2-4ac}\,e}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,f^2\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,\sqrt{c}\,\sqrt{b^2-4ac}\,e}$$

```
[Out] -1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-
4*a*c+b^2)^(1/2))^(1/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*f^2*arctan
((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2
))^(1/2)/e*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1156, 1144, 211}

$$\frac{f^2\sqrt{\sqrt{b^2-4ac}+b}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,\sqrt{c}\,e\sqrt{b^2-4ac}}-\frac{f^2\sqrt{b-\sqrt{b^2-4ac}}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,\sqrt{c}\,e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)) + (Sqrt[b +
 Sqrt[b^2 - 4*a*c]]*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(df+efx)^2}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f^2\text{Subst}\left(\int \frac{x^2}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{e}$$
$$= \frac{\left(\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)f^2\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,d+ex\right)}{2e}$$
$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\,f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\,e}+\frac{\sqrt{b+\sqrt{b^2-4}}}{}$$

Mathematica [A]
time = 0.07, size = 178, normalized size = 1.05

$$\frac{f^2\left(\left(-b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)+\sqrt{b-\sqrt{b^2-4ac}}\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
[Out] (f^2*((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b -
Sqrt[b^2 - 4*a*c]]] + Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]]))/(Sqrt[
2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 143, normalized size = 0.84

method	result
default	$\frac{f^2\left(\sum_{\_R=\text{RootOf}\left(e^4c\_Z^4+4de^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)}\frac{\left(\_R^2e^2+2\_Rde+d^2\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6de^2c\_R^2+6cd^2e\_R+2cd^3+e}\right.}{2e}$

risch	$\frac{f^{2}\left(\sum_{\_R=\mathrm{RootOf}\left(e^{4}c\_Z^{4}+4d\,e^{3}c\_Z^{3}+\left(6d^{2}e^{2}c+e^{2}b\right)\_Z^{2}+\left(4d^{3}ec+2deb\right)\_Z+d^{4}c+d^{2}b+a\right)}\frac{\left(\_R^{2}e^{2}+2\_Rde+d^{2}\right)\ln\left(x-\_R\right)}{2e^{3}c\_R^{3}+6d\,e^{2}c\_R^{2}+6c\,d^{2}e\_R+2c\,d^{3}+eb\_}\right.}{2e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f^2/e*sum((_R^2*e^2+2*_R*d*e+d^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d
^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*
d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] integrate((f*x*e + d*f)^2/((x*e + d)^4*c + (x*e + d)^2*b + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 731 vs. 2(135) = 270.
time = 0.36, size = 731, normalized size = 4.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b*f^4 + sqrt(f^8/(b^2*c^2 - 4*a*c^3))*(b^2*c - 4*a*c^2
))*e^(-2)/(b^2*c - 4*a*c^2))*log(f^6*x*e + d*f^6 + sqrt(1/2)*sqrt(f^8/(b^2*
c^2 - 4*a*c^3))*(b^2*c - 4*a*c^2)*sqrt(-(b*f^4 + sqrt(f^8/(b^2*c^2 - 4*a*c^
3))*(b^2*c - 4*a*c^2))*e^(-2)/(b^2*c - 4*a*c^2))*e) - 1/2*sqrt(1/2)*sqrt(-(
b*f^4 + sqrt(f^8/(b^2*c^2 - 4*a*c^3))*(b^2*c - 4*a*c^2))*e^(-2)/(b^2*c - 4*
a*c^2))*log(f^6*x*e + d*f^6 - sqrt(1/2)*sqrt(f^8/(b^2*c^2 - 4*a*c^3))*(b^2*
c - 4*a*c^2)*sqrt(-(b*f^4 + sqrt(f^8/(b^2*c^2 - 4*a*c^3))*(b^2*c - 4*a*c^2)
)*e^(-2)/(b^2*c - 4*a*c^2))*e) - 1/2*sqrt(1/2)*sqrt(-(b*f^4 - sqrt(f^8/(b^2
*c^2 - 4*a*c^3))*(b^2*c - 4*a*c^2))*e^(-2)/(b^2*c - 4*a*c^2))*log(f^6*x*e +
 d*f^6 + sqrt(1/2)*sqrt(f^8/(b^2*c^2 - 4*a*c^3))*(b^2*c - 4*a*c^2)*sqrt(-(b
*f^4 - sqrt(f^8/(b^2*c^2 - 4*a*c^3))*(b^2*c - 4*a*c^2))*e^(-2)/(b^2*c - 4*a
*c^2))*e) + 1/2*sqrt(1/2)*sqrt(-(b*f^4 - sqrt(f^8/(b^2*c^2 - 4*a*c^3))*(b^2
*c - 4*a*c^2))*e^(-2)/(b^2*c - 4*a*c^2))*log(f^6*x*e + d*f^6 - sqrt(1/2)*sq
rt(f^8/(b^2*c^2 - 4*a*c^3))*(b^2*c - 4*a*c^2)*sqrt(-(b*f^4 - sqrt(f^8/(b^2*
c^2 - 4*a*c^3))*(b^2*c - 4*a*c^2))*e^(-2)/(b^2*c - 4*a*c^2))*e)
```

Sympy [A]
time = 0.86, size = 124, normalized size = 0.73

$$\operatorname{RootSum}\left(t^4\cdot\left(256a^2c^3e^4-128ab^2c^2e^4+16b^4ce^4\right)+t^2\left(-16abce^2f^4+4b^3e^2f^4\right)+af^8,\ \left(t\mapsto t\log\left(x+\frac{64t^3ac^2e^3-16t^3b^2ce^3-2tbef^4+df^6}{ef^6}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4)
+ _t**2*(-16*a*b*c*e**2*f**4 + 4*b**3*e**2*f**4) + a*f**8, Lambda(_t, _t*lo
g(x + (64*_t**3*a*c**2*e**3 - 16*_t**3*b**2*c*e**3 - 2*_t*b*e*f**4 + d*f**6
)/(e*f**6))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1325 vs. 2(135) = 270.
time = 4.02, size = 1325, normalized size = 7.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b
*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1) + sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e +
(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^
2 - b*d*e) - 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^
2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x - sqrt(1/2)*sqr
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1
/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 + 6*(d*e^(-1
) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d^2*e^2 -
2*c*d^3*e + (d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))*b*e^2 - b*d*e) - 1/2*((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
- sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^(-1) + x + s
qrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(
-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3
+ 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*
```

```
c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a
*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e) - 1/2*((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^
2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*f^2*e^2 - 2*(d*e^(-1) - sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*d*f^2*e + d^2*f^2)*log(d*e^
(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d
*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^
4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c)
)^2*c*d*e^3 + 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))*c*d^2*e^2 - 2*c*d^3*e + (d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*e^2 - b*d*e)
```

Mupad [B]
time = 1.79, size = 683, normalized size = 4.02

$$-2\,\mathrm{atanh}\left(\frac{\sqrt{-\frac{b^3\,f^4+f^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\,f^4}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}\,\left(x\,\left(4\,a\,c^2\,e^{12}\,f^4-2\,b^2\,c\,e^{12}\,f^4\right)+\frac{\left(x\,\left(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14}\right)+8\,b^3\,c^2\,d\,e^{13}-32\,a\,b\,c^3\,d\,e^{13}\right)\,\left(b^3\,f^4+f^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\,f^4\right)}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}+4\,a\,c^2\,d\,e^{11}\,f^4-2\,b^2\,c\,d\,e^{11}\,f^4\right)}{a\,c\,e^{10}\,f^6}\right)\,\sqrt{-\frac{b^3\,f^4+f^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-4\,a\,b\,c\,f^4}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}-2\,\mathrm{atanh}\left(\frac{\sqrt{\frac{f^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,f^4+4\,a\,b\,c\,f^4}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}\,\left(x\,\left(4\,a\,c^2\,e^{12}\,f^4-2\,b^2\,c\,e^{12}\,f^4\right)-\frac{\left(x\,\left(8\,b^3\,c^2\,e^{14}-32\,a\,b\,c^3\,e^{14}\right)+8\,b^3\,c^2\,d\,e^{13}-32\,a\,b\,c^3\,d\,e^{13}\right)\,\left(f^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,f^4+4\,a\,b\,c\,f^4\right)}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}+4\,a\,c^2\,d\,e^{11}\,f^4-2\,b^2\,c\,d\,e^{11}\,f^4\right)}{a\,c\,e^{10}\,f^6}\right)\,\sqrt{\frac{f^4\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,f^4+4\,a\,b\,c\,f^4}{8\,\left(16\,a^2\,c^3\,e^2-8\,a\,b^2\,c^2\,e^2+b^4\,c\,e^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] - 2*atanh(((-(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c*f^4)/(8*(b^4
*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12*f^4 - 2
*b^2*c*e^12*f^4) + ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d*e^1
3 - 32*a*b*c^3*d*e^13)*(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c*f^
4))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11*f^4
 - 2*b^2*c*d*e^11*f^4))/(a*c*e^10*f^6))*(-(b^3*f^4 + f^4*(-(4*a*c - b^2)^3)
^(1/2) - 4*a*b*c*f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(
1/2) - 2*atanh((((f^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*f^4 + 4*a*b*c*f^4)/(8*
(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)))^(1/2)*(x*(4*a*c^2*e^12*f^4
 - 2*b^2*c*e^12*f^4) - ((x*(8*b^3*c^2*e^14 - 32*a*b*c^3*e^14) + 8*b^3*c^2*d
*e^13 - 32*a*b*c^3*d*e^13)*(f^4*(-(4*a*c - b^2)^3)^(1/2) - b^3*f^4 + 4*a*b*
c*f^4))/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2)) + 4*a*c^2*d*e^11
*f^4 - 2*b^2*c*d*e^11*f^4))/(a*c*e^10*f^6))*((f^4*(-(4*a*c - b^2)^3)^(1/2)
- b^3*f^4 + 4*a*b*c*f^4)/(8*(b^4*c*e^2 + 16*a^2*c^3*e^2 - 8*a*b^2*c^2*e^2))
)^(1/2)
```

3.641 $\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx$

Optimal. Leaf size=44

$$-\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,e}$$

```
[Out] -f*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/e/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1156, 1121, 632, 212}

$$-\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] -((f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*e
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{df+efx}{a+b(d+ex)^2+c(d+ex)^4}\,dx = \frac{f\text{Subst}\left(\int \frac{x}{a+bx^2+cx^4}\,dx,x,d+ex\right)}{e}$$
$$= \frac{f\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,(d+ex)^2\right)}{2e}$$
$$= -\frac{f\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c(d+ex)^2\right)}{e}$$
$$= -\frac{f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,e}$$

Mathematica [A]
time = 0.01, size = 47, normalized size = 1.07

$$\frac{f\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}\,e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x]
```

```
[Out] (f*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 130, normalized size = 2.95

method	result
default	$\frac{f\left(\sum_{\_R=\text{RootOf}\left(e^4c\_Z^4+4de^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)}\frac{\left(\_Re+d\right)\ln\left(x-\_R\right)}{2e^3c\_R^3+6de^2c\_R^2+6cd^2e\_R+2cd^3+eb\_}\right.}{2e}$
risch	$-\frac{f\ln\left(\left(\sqrt{-4ac+b^2}\,e^2-e^2b\right)x^2+\left(2de\sqrt{-4ac+b^2}-2deb\right)x+\sqrt{-4ac+b^2}\,d^2-d^2b-2a\right)}{2\sqrt{-4ac+b^2}\,e}+\frac{f\ln\left(\left(\sqrt{-4ac+\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b
*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z
^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
[Out] integrate((f*x*e + d*f)/((x*e + d)^4*c + (x*e + d)^2*b + a), x)
```

Fricas [A]
time = 0.35, size = 268, normalized size = 6.09

$$\left[\frac{fe^{(-1)}\log\left(\frac{2\,c^2x^4e^4+8\,c^2dx^3e^3+2\,c^2d^4+2\,bcd^2+2\,(6\,c^2d^2+bc)x^2e^2+4\,(2\,c^2d^3+bcd)xe+b^2-2\,ac-(2\,cx^2e^2+4\,cdxe+2\,cd^2+b)\sqrt{b^2-4\,ac}}{cx^4e^4+4\,cdx^3e^3+cd^4+(6\,cd^2+b)x^2e^2+bd^2+2\,(2\,cd^3+bd)xe+a}\right)}{2\,\sqrt{b^2-4\,ac}}, -\frac{\sqrt{-b^2+4\,ac}\,f\arctan\left(-\frac{(2\,cx^2e^2+4\,cdxe+2\,cd^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)e^{(-1)}}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
[Out] [1/2*f*e^(-1)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4 + 2*b*c*d^2
+ 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2 - 2*a*c - (
2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4*e^4 + 4*c*
d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e +
 a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*f*arctan(-(2*c*x^2*e^2 + 4*c*d*
x*e + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))*e^(-1)/(b^2 - 4*a*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(41) = 82$.
time = 0.62, size = 189, normalized size = 4.30

$$-\frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{-4acf\sqrt{-\frac{1}{4ac-b^2}}+b^2f\sqrt{-\frac{1}{4ac-b^2}}+bf+2cd^2f}{2ce^2f}\right)}{2e}+\frac{f\sqrt{-\frac{1}{4ac-b^2}}\log\left(\frac{2dx}{e}+x^2+\frac{4acf\sqrt{-\frac{1}{4ac-b^2}}-b^2f\sqrt{-\frac{1}{4ac-b^2}}+bf+2cd^2f}{2ce^2f}\right)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
[Out] -f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (-4*a*c*f*sqrt(-1/(4*a*c -
b**2)) + b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*f))/(
2*e) + f*sqrt(-1/(4*a*c - b**2))*log(2*d*x/e + x**2 + (4*a*c*f*sqrt(-1/(4*a
*c - b**2)) - b**2*f*sqrt(-1/(4*a*c - b**2)) + b*f + 2*c*d**2*f)/(2*c*e**2*
f))/(2*e)
```

Giac [A]
time = 3.81, size = 62, normalized size = 1.41

$$\frac{f\arctan\left(\frac{2\,cd^2f+2\,(fx^2e+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] f*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a*c)*
f))*e^(-1)/sqrt(-b^2 + 4*a*c)
```

Mupad [B]
time = 1.62, size = 477, normalized size = 10.84

$$\frac{f\,\mathrm{atan}\left(\frac{\frac{f\left(4\,c^2\,d^2\,e^7\,f+4\,c^2\,e^9\,f\,x^2-\frac{f\left(8\,b\,c^2\,d^2\,e^8+16\,b\,c^2\,d\,e^9\,x+8\,b\,c^2\,e^{10}\,x^2+16\,a\,c^2\,e^8\right)}{2\,e\,\sqrt{b^2-4\,a\,c}}+8\,c^2\,d\,e^8\,f\,x\right)1{}\mathrm{i}}{2\,e\,\sqrt{b^2-4\,a\,c}}+\frac{f\left(4\,c^2\,d^2\,e^7\,f+4\,c^2\,e^9\,f\,x^2+\frac{f\left(8\,b\,c^2\,d^2\,e^8+16\,b\,c^2\,d\,e^9\,x+8\,b\,c^2\,e^{10}\,x^2+16\,a\,c^2\,e^8\right)}{2\,e\,\sqrt{b^2-4\,a\,c}}+8\,c^2\,d\,e^8\,f\,x\right)1{}\mathrm{i}}{2\,e\,\sqrt{b^2-4\,a\,c}}}{\frac{f\left(4\,c^2\,d^2\,e^7\,f+4\,c^2\,e^9\,f\,x^2-\frac{f\left(8\,b\,c^2\,d^2\,e^8+16\,b\,c^2\,d\,e^9\,x+8\,b\,c^2\,e^{10}\,x^2+16\,a\,c^2\,e^8\right)}{2\,e\,\sqrt{b^2-4\,a\,c}}+8\,c^2\,d\,e^8\,f\,x\right)}{2\,e\,\sqrt{b^2-4\,a\,c}}-\frac{f\left(4\,c^2\,d^2\,e^7\,f+4\,c^2\,e^9\,f\,x^2+\frac{f\left(8\,b\,c^2\,d^2\,e^8+16\,b\,c^2\,d\,e^9\,x+8\,b\,c^2\,e^{10}\,x^2+16\,a\,c^2\,e^8\right)}{2\,e\,\sqrt{b^2-4\,a\,c}}+8\,c^2\,d\,e^8\,f\,x\right)}{2\,e\,\sqrt{b^2-4\,a\,c}}}\right)1{}\mathrm{i}}{e\,\sqrt{b^2-4\,a\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4),x)
```

```
[Out] (f*atan(((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^2*e^8 + 8*b*c^2
*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2))
+ 8*c^2*d*e^8*f*x)*1i)/(2*e*(b^2 - 4*a*c)^(1/2)) + (f*(4*c^2*d^2*e^7*f + 4*
c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*
b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x)*1i)/(2*e*(b^2
- 4*a*c)^(1/2)))/((f*(4*c^2*d^2*e^7*f + 4*c^2*e^9*f*x^2 - (f*(16*a*c^2*e^8
+ 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c
)^(1/2)) + 8*c^2*d*e^8*f*x))/(2*e*(b^2 - 4*a*c)^(1/2)) - (f*(4*c^2*d^2*e^7*
f + 4*c^2*e^9*f*x^2 + (f*(16*a*c^2*e^8 + 8*b*c^2*d^2*e^8 + 8*b*c^2*e^10*x^2
 + 16*b*c^2*d*e^9*x))/(2*e*(b^2 - 4*a*c)^(1/2)) + 8*c^2*d*e^8*f*x))/(2*e*(b
^2 - 4*a*c)^(1/2))))*1i)/(e*(b^2 - 4*a*c)^(1/2))
```

3.642 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx$

Optimal. Leaf size=103

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,ef}+\frac{\log(d+ex)}{aef}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}$$

```
[Out] ln(e*x+d)/a/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a/e/f+1/2*b*arctanh((b+2*
c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a/e/f/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1156, 1128, 719, 29, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2aef\sqrt{b^2-4ac}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef}+\frac{\log(d+ex)}{aef}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]
*e*f) + Log[d + e*x]/(a*e*f) - Log[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a*
e*f)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{ef}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2ef}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, (d+ex)^2\right)}{2aef} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, (d$$
$$= \frac{\log(d+ex)}{aef} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, (d+ex)^2\right)}{4aef} - \frac{b\text{Sub}$$
$$= \frac{\log(d+ex)}{aef} - \frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4aef} + \frac{b\text{Subs}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,ef} + \frac{\log(d+ex)}{aef} - \frac{\log\left(a+b(d+\right.}{4}$$

Mathematica [A]
time = 0.05, size = 131, normalized size = 1.27

$$\frac{4\sqrt{b^2-4ac}\log(d+ex) - \left(b+\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right) + \left(b-\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{4a\sqrt{b^2-4ac}\,ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[d + e*x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^
2 - 4*a*c] + 2*c*(d + e*x)^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 -
4*a*c] + 2*c*(d + e*x)^2])/(4*a*Sqrt[b^2 - 4*a*c]*e*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 188, normalized size = 1.83

method	result
risch	$\frac{\ln(ex+d)}{aef} + \frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^2f^2e^2c-a\,b^2f^2e^2\right)\_Z^2+\left(4acef-b^2ef\right)\_Z+c\right)} \_R\ln\left(\left(\left(10ace^3f-3b^2e^3f\right)\_R+5ce^2\right)x^2+\left(\left(20acd\right.\right.\right.\right.}{2}$
default	$\frac{\frac{\ln(ex+d)}{ae}+\frac{\sum_{\_R=\text{RootOf}\left(e^4c\_Z^4+4de^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)} \frac{\left(-e^3c\_R^3-3de^2c\_R^2+e\left(-3cd^2-\right.\right.}{2e^3c\_R^3+6de^2c\_R^2+6cd^2e}}{2ae}}{f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(ln(e*x+d)/a/e+1/2/a/e*sum((-e^3*c*_R^3-3*d*e^2*c*_R^2+e*(-3*c*d^2-b)*_
R-c*d^3-b*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*
ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*
d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")
```

```
[Out] e^(-1)*log(x*e + d)/(a*f) - integrate((c*x^3*e^3 + 3*c*d*x^2*e^2 + c*d^3 +
b*d + (3*c*d^2*e + b*e)*x)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*
c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(a*f)
```

Fricas [A]
time = 0.37, size = 466, normalized size = 4.52

$$\left[\frac{\left(\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4e^4+8\,c^2dx^3e^3+2\,c^2d^4+2\,bcd^2+2\,(6\,c^2d^2+bc)x^2e^2+4\,(2\,c^2d^3+bcd)xe+b^2-2\,ac+(2\,cx^2e^2+4\,cdxe+2\,cd^2+b)\sqrt{b^2-4\,ac}}{cx^4e^4+4\,cdx^3e^3+cd^4+(6\,cd^2+b)x^2e^2+bd^2+2\,(2\,cd^3+bd)xe+a}\right)-(b^2-4\,ac)\log\left(cx^4e^4+4\,cdx^3e^3+cd^4+(6\,cd^2+b)x^2e^2+bd^2+2\,(2\,cd^3+bd)xe+a\right)+4\,(b^2-4\,ac)\log\left(xe+d\right)\right)e^{(-1)}}{4\,(ab^2-4\,a^2c)f}, \frac{\left(2\,\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{(2\,cx^2e^2+4\,cdxe+2\,cd^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^2-4\,ac)\log\left(cx^4e^4+4\,cdx^3e^3+cd^4+(6\,cd^2+b)x^2e^2+bd^2+2\,(2\,cd^3+bd)xe+a\right)+4\,(b^2-4\,ac)\log\left(xe+d\right)\right)e^{(-1)}}{4\,(ab^2-4\,a^2c)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4
 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2
 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^
4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3
 + b*d)*x*e + a)) - (b^2 - 4*a*c)*log(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6
*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a) + 4*(b^2 - 4*a*c)*
log(x*e + d))*e^(-1)/((a*b^2 - 4*a^2*c)*f), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arc
tan(-(2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*
c)) - (b^2 - 4*a*c)*log(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x
^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a) + 4*(b^2 - 4*a*c)*log(x*e + d))
*e^(-1)/((a*b^2 - 4*a^2*c)*f)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(83) = 166$.
time = 17.43, size = 348, normalized size = 3.38

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4aef\,(4ac-b^2)}-\frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)+2ab^2ef\left(-\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)-2ac+b^2+bcd^2}{bce^2}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4aef\,(4ac-b^2)}-\frac{1}{4aef}\right)\log\left(\frac{2dx}{e}+x^2+\frac{-8a^2cef\left(\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)+2ab^2ef\left(\frac{b\sqrt{-4ac+b^2}}{4aef(4ac-b^2)}-\frac{1}{4aef}\right)-2ac+b^2+bcd^2}{bce^2}\right)+\frac{\log\left(\frac{d}{e}+x\right)}{aef}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/e
 + x**2 + (-8*a**2*c*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) -
 1/(4*a*e*f)) + 2*a*b**2*e*f*(-b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2
)) - 1/(4*a*e*f)) - 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + (b*sqrt(-4*a*c +
 b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f))*log(2*d*x/e + x**2 + (-8*a**
2*c*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)) + 2*
a*b**2*e*f*(b*sqrt(-4*a*c + b**2)/(4*a*e*f*(4*a*c - b**2)) - 1/(4*a*e*f)) -
 2*a*c + b**2 + b*c*d**2)/(b*c*e**2)) + log(d/e + x)/(a*e*f)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(96) = 192.
time = 3.04, size = 285, normalized size = 2.77

$$-\frac{e^{(-1)}\log\left(|cx^4e^4 + 4\,cdx^3e^3 + 6\,cd^2x^2e^2 + 4\,cd^3xe + cd^4 + bx^2e^2 + 2\,bdxe + bd^2 + a|\right)}{4\,af} + \frac{e^{(-1)}\log\left(|xe + d|\right)}{af} - \frac{\left(\frac{abcfe^3\log\left(\left|bx^2e^2+2\,bdxe+\sqrt{b^2-4\,ac}\,x^2e^2+2\,\sqrt{b^2-4\,ac}\,dxe+bd^2+\sqrt{b^2-4\,ac}\,d^2+2\,a\right|\right)}{\sqrt{b^2-4\,ac}} - \frac{abcfe^3\log\left(\left|-bx^2e^2-2\,bdxe+\sqrt{b^2-4\,ac}\,x^2e^2+2\,\sqrt{b^2-4\,ac}\,dxe-bd^2+\sqrt{b^2-4\,ac}\,d^2-2\,a\right|\right)}{\sqrt{b^2-4\,ac}}\right)e^{(-4)}}{4\,a^2cf^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/4*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x
*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a*f) + e^(-1)*log(abs(x*e
 + d))/(a*f) - 1/4*(a*b*c*f*e^3*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 -
4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2
+ 2*a))/sqrt(b^2 - 4*a*c) - a*b*c*f*e^3*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sq
rt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*
a*c)*d^2 - 2*a))/sqrt(b^2 - 4*a*c))*e^(-4)/(a^2*c*f^2)
```

Mupad [B]
time = 3.46, size = 2520, normalized size = 24.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] log(d + e*x)/(a*e*f) - (log(a + b*d^2 + c*d^4 + b*e^2*x^2 + c*e^4*x^4 + 2*b
*d*e*x + 6*c*d^2*e^2*x^2 + 4*c*d^3*e*x + 4*c*d*e^3*x^3)*(2*b^2*e*f - 8*a*c*
e*f))/(2*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) - (b*atan((16*a^3*f^3*x*(4*a
*c - b^2)^(3/2)*(((3*b^3 - 8*a*b*c)*((b^2*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^
3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f
^2)) + (20*b*c^3*d*e^17)/f))/(16*a^2*e^2*f^2*(4*a*c - b^2)) - ((2*b^2*e*f -
 8*a*c*e*f)^2*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*
d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) + (20*b*c^3*d*e^17)/f))
```

/(4*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*f - 8*a*c*e*f)*
(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(4*a^2*e^2*f^3*(4*a*b^2*e^2*f^2
 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - (((b*(
2*b^2*e*f - 8*a*c*e*f)*((2*(2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*d*e^18*f - 20
*a*b*c^3*d*e^18*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)) + (20*b*c^3*d*
e^17)/f))/(4*a*e*f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2)^(1/2)
) - (b^3*(6*b^3*c^2*d*e^18*f - 20*a*b*c^3*d*e^18*f))/(16*a^3*e^3*f^4*(4*a*c
 - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f)^2*(6*b^3*c^2*d*e^18*f - 20*a*b*
c^3*d*e^18*f))/(4*a*e*f^2*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2*(4*a*c - b
^2)^(1/2)))*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/
2)*(25*a*c - 6*b^2))))/(b^2*c^2*e^14) + (2*f^3*(3*b^3 - 8*a*b*c)*(4*a*c - b
^2)^(3/2)*((b^2*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^2*e^16))/f + ((2*b^2*e*f -
8*a*c*e*f)*(2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17
f))/(f(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/(16*a^2*e^2*f^2*(4*a*c - b^
2)) - ((2*b^2*e*f - 8*a*c*e*f)^2*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^2*e^16))/f
 + ((2*b^2*e*f - 8*a*c*e*f)*(2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20
*a*b*c^3*d^2*e^17*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/(4*(4*a*b^
2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*f - 8*a*c*e*f)*(2*a*b^2*c^
2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17*f))/(8*a^2*e^2*f^3*(4
*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(b^2*c^4*e^14*(25*a*c -
 6*b^2)) + (16*a^3*f^3*x^2*(4*a*c - b^2)^(3/2)*(((3*b^3 - 8*a*b*c)*((b^2*((
10*b*c^3*e^18)/f + ((2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f - 20*a*b*c^3*
e^19*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/(16*a^2*e^2*f^2*(4*a*c
 - b^2)) - (((10*b*c^3*e^18)/f + ((2*b^2*e*f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f
 - 20*a*b*c^3*e^19*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)))*(2*b^2*e*f
 - 8*a*c*e*f)^2)/(4*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)^2) + (b^2*(2*b^2*e*
f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f - 20*a*b*c^3*e^19*f))/(8*a^2*e^2*f^3*(4*a*
b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2
)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b*((10*b*c^3*e^18)/f + ((2*b^2*e*
f - 8*a*c*e*f)*(6*b^3*c^2*e^19*f - 20*a*b*c^3*e^19*f))/(f*(4*a*b^2*e^2*f^2
 - 16*a^2*c*e^2*f^2)))*(2*b^2*e*f - 8*a*c*e*f))/(4*a*e*f*(4*a*b^2*e^2*f^2 -
16*a^2*c*e^2*f^2)*(4*a*c - b^2)^(1/2)) - (b^3*(6*b^3*c^2*e^19*f - 20*a*b*c^
3*e^19*f))/(32*a^3*e^3*f^4*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f
)^2*(6*b^3*c^2*e^19*f - 20*a*b*c^3*e^19*f))/(8*a*e*f^2*(4*a*b^2*e^2*f^2 - 1
6*a^2*c*e^2*f^2)^2*(4*a*c - b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(2
5*a*c - 6*b^2))))/(b^2*c^2*e^14) - (2*f^3*(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2
 - 14*a*b^2*c)*((b*(2*b^2*e*f - 8*a*c*e*f)*((2*(2*b^2*c^2*e^16 + 5*b*c^3*d^
2*e^16))/f + ((2*b^2*e*f - 8*a*c*e*f)*(2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e
^17*f - 20*a*b*c^3*d^2*e^17*f))/(f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2))))/
(4*a*e*f*(4*a*b^2*e^2*f^2 - 16*a^2*c*e^2*f^2)*(4*a*c - b^2)^(1/2)) - (b^3*(
2*a*b^2*c^2*e^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17*f))/(32*a^3
*e^3*f^4*(4*a*c - b^2)^(3/2)) + (b*(2*b^2*e*f - 8*a*c*e*f)^2*(2*a*b^2*c^2*e
^17*f + 6*b^3*c^2*d^2*e^17*f - 20*a*b*c^3*d^2*e^17*f))/(8*a*e*f^2*(4*a*b^2*
e^2*f^2 - 16*a^2*c*e^2*f^2)^2*(4*a*c - b^2)^(1/2))))/(b^2*c^4*e^14*(25*a*c
 - 6*b^2))))/(2*a*e*f*(4*a*c - b^2)^(1/2))

3.643 $\int \frac{1}{(df+efx)^2(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal. Leaf size=204

$$-\frac{1}{aef^2(d+ex)}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,ef^2}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}\,ef^2}$$

```
[Out] -1/a/e/f^2/(e*x+d)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2)
)^(1/2))*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a/e/f^2*2^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)-1/2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
)*c^(1/2)*(1-b/(-4*a*c+b^2)^(1/2))/a/e/f^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1156, 1137, 1180, 211}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,aef^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,aef^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{aef^2(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -(1/(a*e*f^2*(d + e*x))) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqr
t[b^2 - 4*a*c]]*e*f^2) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[
b^2 - 4*a*c]]*e*f^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
```

```
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(df+efx)^2\,(a+b(d+ex)^2+c(d+ex)^4)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{ef^2}$$

$$= -\frac{1}{aef^2(d+ex)} + \frac{\text{Subst}\left(\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx, x, d+ex\right)}{aef^2}$$

$$= -\frac{1}{aef^2(d+ex)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}+\frac{1}{2}\sqrt{}}\right.}{2aef^2}$$

$$= -\frac{1}{aef^2(d+ex)} - \frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{b-}}\right.}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,ef}$$

Mathematica [A]
time = 0.23, size = 209, normalized size = 1.02

$$-\frac{\frac{2}{d+ex}+\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2aef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/2*(2/(d + e*x) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(a*e*f^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.20, size = 172, normalized size = 0.84

method	result
default	$\frac{-\frac{1}{ae(ex+d)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4de^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)}\frac{\left(-\_R^2ce^2-2\_Rcde-cd^2-b\right.}{2e^3c\_R^3+6de^2c\_R^2+6cd^2e\_R}}{2ae}}{f^2}$
risch	$-\frac{1}{aef^2(ex+d)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16f^8e^4c^2a^5-8b^2f^8e^4ca^4+b^4f^8e^4a^3\right)\_Z^4+\left(12a^2bc^2e^2f^4-7ab^3ce^2f^4+b^5e^2f^4\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^2*(-1/a/e/(e*x+d)+1/2/a/e*sum((-_R^2*c*e^2-2*_R*c*d*e-c*d^2-b)/(2*_R^3*
c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^
4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4
*c+d^2*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] -1/(a*f^2*x*e^2 + a*d*f^2*e) - integrate((c*x^2*e^2 + 2*c*d*x*e + c*d^2 + b
)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 +
2*(2*c*d^3*e + b*d*e)*x + a), x)/(a*f^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1413 vs. 2(167) = 334.
time = 0.37, size = 1413, normalized size = 6.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] -1/2*(sqrt(1/2)*(a*f^2*x*e^2 + a*d*f^2*e)*sqrt(((a^3*b^2 - 4*a^4*c)*f^4*sqr
t((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*f^8)) - b^3 + 3*a*b*c)/(
(a^3*b^2 - 4*a^4*c)*f^4))*e^(-1)*log(-2*(b^2*c^2 - a*c^3)*x*e + sqrt(1/2)*(
(a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((
a^6*b^2 - 4*a^7*c)*f^8))*e + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*f^2*e)*sqrt(((
a^3*b^2 - 4*a^4*c)*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c
)*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*f^4))*e^(-1) - 2*(b^2*c^2 - a
*c^3)*d) - sqrt(1/2)*(a*f^2*x*e^2 + a*d*f^2*e)*sqrt(((a^3*b^2 - 4*a^4*c)*f^
4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*f^8)) - b^3 + 3*a*b
*c)/((a^3*b^2 - 4*a^4*c)*f^4))*e^(-1)*log(-2*(b^2*c^2 - a*c^3)*x*e - sqrt(1
/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/((a^6*b^2 - 4*a^7*c)*f^8))*e + (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*f^2*e)*sq
rt(((a^3*b^2 - 4*a^4*c)*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*
a^7*c)*f^8)) - b^3 + 3*a*b*c)/((a^3*b^2 - 4*a^4*c)*f^4))*e^(-1) - 2*(b^2*c^
2 - a*c^3)*d) - sqrt(1/2)*(a*f^2*x*e^2 + a*d*f^2*e)*sqrt(-((a^3*b^2 - 4*a^4
*c)*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*f^8)) + b^3 -
 3*a*b*c)*e^(-2)/((a^3*b^2 - 4*a^4*c)*f^4))*log(-2*(b^2*c^2 - a*c^3)*x*e -
2*(b^2*c^2 - a*c^3)*d + sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f^6*
sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*f^8))*e - (b^5 - 5*a*
b^3*c + 4*a^2*b*c^2)*f^2*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*f^4*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*f^8)) + b^3 - 3*a*b*c)*e^(-2)/((a^3*b
^2 - 4*a^4*c)*f^4))) + sqrt(1/2)*(a*f^2*x*e^2 + a*d*f^2*e)*sqrt(-((a^3*b^2
- 4*a^4*c)*f^4*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*f^8))
+ b^3 - 3*a*b*c)*e^(-2)/((a^3*b^2 - 4*a^4*c)*f^4))*log(-2*(b^2*c^2 - a*c^3)
*x*e - 2*(b^2*c^2 - a*c^3)*d - sqrt(1/2)*((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^
2)*f^6*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*f^8))*e - (b^5
 - 5*a*b^3*c + 4*a^2*b*c^2)*f^2*e)*sqrt(-((a^3*b^2 - 4*a^4*c)*f^4*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/((a^6*b^2 - 4*a^7*c)*f^8)) + b^3 - 3*a*b*c)*e^(-2)/
((a^3*b^2 - 4*a^4*c)*f^4))) + 2)/(a*f^2*x*e^2 + a*d*f^2*e)
```

Sympy [A]
time = 3.25, size = 258, normalized size = 1.26

$$\operatorname{RootSum}\left(t^{4}\cdot\left(256a^{5}c^{2}e^{4}f^{8}-128a^{4}b^{2}ce^{4}f^{8}+16a^{3}b^{4}e^{4}f^{8}\right)+t^{2}\cdot\left(48a^{2}bc^{2}e^{2}f^{4}-28ab^{3}ce^{2}f^{4}+4b^{5}e^{2}f^{4}\right)+c^{3},\left(t\mapsto t\log\left(x+\frac{-64t^{3}a^{5}c^{2}e^{3}f^{6}+48t^{3}a^{4}b^{2}ce^{3}f^{6}-8t^{3}a^{3}b^{4}e^{3}f^{6}-10ta^{2}bc^{2}ef^{2}+10tab^{3}cef^{2}-2tb^{5}ef^{2}+ac^{3}d-b^{2}c^{2}d}{ac^{3}e-b^{2}c^{2}e}\right)\right)\right)-\frac{1}{adef^{2}+ae^{2}f^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2*e**4*f**8 - 128*a**4*b**2*c*e**4*f**8 + 16*a**
3*b**4*e**4*f**8) + _t**2*(48*a**2*b*c**2*e**2*f**4 - 28*a*b**3*c*e**2*f**4
 + 4*b**5*e**2*f**4) + c**3, Lambda(_t, _t*log(x + (-64*_t**3*a**5*c**2*e**
```

```
3*f**6 + 48*_t**3*a**4*b**2*c*e**3*f**6 - 8*_t**3*a**3*b**4*e**3*f**6 - 10*
_t*a**2*b*c**2*e*f**2 + 10*_t*a*b**3*c*e*f**2 - 2*_t*b**5*e*f**2 + a*c**3*d
 - b**2*c**2*d)/(a*c**3*e - b**2*c**2*e)))) - 1/(a*d*e*f**2 + a*e**2*f**2*x
)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error index.cc index_gcd Error: Bad A
rgument ValueError index.cc index_gcd Error: Bad Argument ValueDone
```

Mupad [B]
time = 3.87, size = 2500, normalized size = 12.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] - atan(((-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*
a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6)
 - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*
e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2
) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^
2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e
^12*f^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4
+ 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6
- 2*a^3*b^2*c^3*d*e^11*f^6)*1i + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4
+ 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6
- 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^1
4*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4))
)^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2
) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6)*1i)/((-(b^5 + b^2*(-(4
```

```
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2
)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^1
4*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2
*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^
4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^
12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*
a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^
6) - (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4
*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) -
((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^1
3*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f
^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e^12
*f^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 1
6*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2
*a^3*b^2*c^3*d*e^11*f^6) + 2*a^3*c^4*e^10*f^4))*(-(b^5 + b^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*2i - ata
n(((-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b
^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a^3*b^2*c^3*e^12*f^6) - ((
x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^10) - 32*a^6*b*c^3*d*e^13*
f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4
 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) - 4*a^4*b^3*c^2*e^12*f
^8 + 16*a^5*b*c^3*e^12*f^8)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*
a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4*a^4*c^4*d*e^11*f^6 - 2*a
^3*b^2*c^3*d*e^11*f^6)*1i + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^2*f^4 + 16*
a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2)*(x*(4*a^4*c^4*e^12*f^6 - 2*a
^3*b^2*c^3*e^12*f^6) - ((x*(8*a^5*b^3*c^2*e^14*f^10 - 32*a^6*b*c^3*e^14*f^1
0) - 32*a^6*b*c^3*d*e^13*f^10 + 8*a^5*b^3*c^2*d*e^13*f^10)*(-(b^5 - b^2*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/
2) + 4*a^4*b^3*c^2*e^12*f^8 - 16*a^5*b*c^3*e^12*f^8)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^4*b^2*c*e^2*f^4)))^(1/2) + 4
*a^4*c^4*d*e^11*f^6 - 2*a^3*b^2*c^3*d*e^11*f^6)*1i)/((-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
```

```
(8*(a^3*b^4*e^2*f^4 + 16*a^5*c^2*e^2*f^4 - 8*a^...
```

3.644 $\int \frac{1}{(df+efx)^3(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal. Leaf size=133

$$-\frac{1}{2aef^3(d+ex)^2}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,ef^3}-\frac{b\log(d+ex)}{a^2ef^3}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}$$

```
[Out] -1/2/a/e/f^3/(e*x+d)^2-b*ln(e*x+d)/a^2/e/f^3+1/4*b*ln(a+b*(e*x+d)^2+c*(e*x+
d)^4)/a^2/e/f^3-1/2*(-2*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/
2))/a^2/e/f^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1156, 1128, 723, 814, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef^3\sqrt{b^2-4ac}}+\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef^3}-\frac{b\log(d+ex)}{a^2ef^3}-\frac{1}{2aef^3(d+ex)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/2*1/(a*e*f^3*(d + e*x)^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)
/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*e*f^3) - (b*Log[d + e*x])/(a^
2*e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(4*a^2*e*f^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{ef^3}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2ef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx, x, (d+ex)^2\right)}{2aef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx, x, \right.}{2aef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} - \frac{b\log(d+ex)}{a^2ef^3} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx\right.}{2a^2ef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} - \frac{b\log(d+ex)}{a^2ef^3} + \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx\right.}{4a^2ef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} - \frac{b\log(d+ex)}{a^2ef^3} + \frac{b\log\left(a+b(d+ex)^2\right.}{4a^2ef^3}$$

$$= -\frac{1}{2aef^3(d+ex)^2} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,ef^3} -$$

Mathematica [A]
time = 0.04, size = 157, normalized size = 1.18

$$\frac{-\frac{2a}{(d+ex)^2} - 4b\log(d+ex) + \frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{\sqrt{b^2-4ac}}}{4a^2ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^2 - 4*b*Log[d + e*x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c
])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/Sqrt[b^2 - 4*a*c] + ((-b^2
 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2*e*f^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 217, normalized size = 1.63

method	result

default	$\frac{-\frac{1}{2ae(ex+d)^2}-\frac{b\ln(ex+d)}{e\,a^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)}\frac{\left(bc\,e^3\_R^3+3bcd\,e^2\_\right.}{2e^3c\_R^{}}}{2a^2e}}{f^3}$
risch	$-\frac{1}{2ae\,f^3(ex+d)^2}-\frac{b\ln(ex+d)}{a^2e\,f^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c\,e^2f^6-a^2b^2e^2f^6\right)\_Z^2+\left(-4abce\,f^3+b^3e\,f^3\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(10a^3c\,e^4f^6-\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)

[Out] 1/f^3*(-1/2/a/e/(e*x+d)^2-b*ln(e*x+d)/e/a^2+1/2/a^2/e*sum((b*c*e^3*_R^3+3*b*c*d*e^2*_R^2+e*(3*b*c*d^2-a*c+b^2)*_R+b*c*d^3-a*c*d+b^2*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima")

[Out] -1/2/(a*f^3*x^2*e^3 + 2*a*d*f^3*x*e^2 + a*d^2*f^3*e) - b*e^(-1)*log(x*e + d)/(a^2*f^3) + integrate((b*c*x^3*e^3 + 3*b*c*d*x^2*e^2 + b*c*d^3 + (b^2 - a*c)*d + (3*b*c*d^2*e + b^2*e - a*c*e)*x)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(a^2*f^3)

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(124) = 248.
time = 0.49, size = 820, normalized size = 6.17

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas")

[Out] [-1/4*(2*a*b^2 - 8*a^2*c + ((b^2 - 2*a*c)*x^2*e^2 + 2*(b^2 - 2*a*c)*d*x*e + (b^2 - 2*a*c)*d^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3

```
+ 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*
d)*x*e + b^2 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4
*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 +
 2*(2*c*d^3 + b*d)*x*e + a)) - ((b^3 - 4*a*b*c)*x^2*e^2 + 2*(b^3 - 4*a*b*c)
*d*x*e + (b^3 - 4*a*b*c)*d^2)*log(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*
d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a) + 4*((b^3 - 4*a*b*c)*
x^2*e^2 + 2*(b^3 - 4*a*b*c)*d*x*e + (b^3 - 4*a*b*c)*d^2)*log(x*e + d))/((a^
2*b^2 - 4*a^3*c)*f^3*x^2*e^3 + 2*(a^2*b^2 - 4*a^3*c)*d*f^3*x*e^2 + (a^2*b^2
 - 4*a^3*c)*d^2*f^3*e), -1/4*(2*a*b^2 - 8*a^2*c + 2*((b^2 - 2*a*c)*x^2*e^2
+ 2*(b^2 - 2*a*c)*d*x*e + (b^2 - 2*a*c)*d^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((
b^3 - 4*a*b*c)*x^2*e^2 + 2*(b^3 - 4*a*b*c)*d*x*e + (b^3 - 4*a*b*c)*d^2)*log
(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c
*d^3 + b*d)*x*e + a) + 4*((b^3 - 4*a*b*c)*x^2*e^2 + 2*(b^3 - 4*a*b*c)*d*x*e
 + (b^3 - 4*a*b*c)*d^2)*log(x*e + d))/((a^2*b^2 - 4*a^3*c)*f^3*x^2*e^3 + 2*
(a^2*b^2 - 4*a^3*c)*d*f^3*x*e^2 + (a^2*b^2 - 4*a^3*c)*d^2*f^3*e)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(124) = 248.
time = 3.88, size = 348, normalized size = 2.62

$$\frac{be^{(-1)}\log\left(|cx^4e^4 + 4\,cdx^3e^3 + 6\,cd^2x^2e^2 + 4\,cd^3xe + cd^4 + bx^2e^2 + 2\,bdxe + bd^2 + a|\right)}{4\,a^2f^3} - \frac{be^{(-1)}\log\left(|xe+d|\right)}{a^2f^3} - \frac{e^{(-1)}}{2\,(xe+d)^2af^3} + \frac{\left(\frac{(a^2b^2cf^3e^3 - 2\,a^3c^2f^3e^3)\log\left(\left|bx^2e^2 + 2\,bdxe + \sqrt{b^2-4\,ac}\,x^2e^2 + 2\,\sqrt{b^2-4\,ac}\,dxe + bd^2 + \sqrt{b^2-4\,ac}\,d^2 + 2\,a\right|\right)}{\sqrt{b^2-4\,ac}} - \frac{(a^2b^2cf^3e^3 - 2\,a^3c^2f^3e^3)\log\left(\left|-bx^2e^2 - 2\,bdxe + \sqrt{b^2-4\,ac}\,x^2e^2 + 2\,\sqrt{b^2-4\,ac}\,dxe - bd^2 + \sqrt{b^2-4\,ac}\,d^2 - 2\,a\right|\right)}{\sqrt{b^2-4\,ac}}\right)e^{(-4)}}{4\,a^4cf^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] 1/4*b*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*
x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^2*f^3) - b*e^(-1)*log(
abs(x*e + d))/(a^2*f^3) - 1/2*e^(-1)/((x*e + d)^2*a*f^3) + 1/4*((a^2*b^2*c*
f^3*e^3 - 2*a^3*c^2*f^3*e^3)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a
*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2
*a))/sqrt(b^2 - 4*a*c) - (a^2*b^2*c*f^3*e^3 - 2*a^3*c^2*f^3*e^3)*log(abs(-b
*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*
e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a))/sqrt(b^2 - 4*a*c))*e^(-4)/(a^4*c*
f^6)
```

Mupad [B]
time = 6.98, size = 2500, normalized size = 18.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] (atan((16*a^6*f^9*x*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(((((2*b^3*e*f^3 - 8*a*
b*c*e*f^3)*((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6))/(a^3*f^9
) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8
*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(2*(16*a^
3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (12*b*c^4*d*e^16)/(a^2*f^6))*(2*b^3*e*f
^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (2*c^5*d*
e^15)/(a^3*f^9) - (((((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6)
)/(a^3*f^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3
*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*
(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3*d*e^18*f^
9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2))
/(4*a^5*e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))
*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e^18*f
^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2)
^2)/(16*a^7*e^2*f^15*(4*a*c - b^2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))
)/(8*a^3*c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((3*b^5 + 13*a^2*b*c^2 - 15*
a*b^3*c)*((((((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^17*f^6))/(a^3*f
^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 -
 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c -
 b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a
^3*b^3*c^2*d*e^18*f^9)*(2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(2*a*c - b^2))/(4*a^5*
e*f^12*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*
e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + ((((2*
b^3*e*f^3 - 8*a*b*c*e*f^3)*((2*(20*a^3*c^4*d*e^17*f^6 + 2*a^2*b^2*c^3*d*e^1
7*f^6))/(a^3*f^9) + ((40*a^4*b*c^3*d*e^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*
(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f
^6))))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (12*b*c^4*d*e^16)/(a^2*
f^6))*(2*a*c - b^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3*d*e
^18*f^9 - 12*a^3*b^3*c^2*d*e^18*f^9)*(2*a*c - b^2)^3)/(32*a^9*e^3*f^18*(4*a
*c - b^2)^(3/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*b^4 + 24*a*b
^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4*a*b^2*c^3*e
^14) + (16*a^6*f^9*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*((((((20*a^3*c^4*e^1
8*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*
(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)) + (6*b*c^4*e^17)/(a^2*f^6))*(2*b^3*e*f^3 - 8*a*b
*c*e*f^3))/(2*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)) + (c^5*e^16)/(a^3*f^9
```

```
) - ((2*a*c - b^2)*((((20*a^3*c^4*e^18*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f
^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^
3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b
^2))/(4*a^2*e*f^3*(4*a*c - b^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12
*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2))/(8*a^5*e*f^12
*(4*a*c - b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(4*a^2*e*f^3
*(4*a*c - b^2)^(1/2)) + ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19
*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2)^2)/(32*a^7*e^2*f^15*(4*a*c - b^
2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(8*a^3*c^2*(a^2*c^2 - 6*b^4 +
24*a*b^2*c)) + ((3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)*(((2*b^3*e*f^3 - 8*a*b*
c*e*f^3)*((((20*a^3*c^4*e^18*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*
b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9
))/(2*a^3*f^9*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*a*c - b^2))/(4*a^
2*e*f^3*(4*a*c - b^2)^(1/2)) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(12*a^3*b^3*c
^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(2*a*c - b^2))/(8*a^5*e*f^12*(4*a*c -
b^2)^(1/2)*(16*a^3*c*e^2*f^6 - 4*a^2*b^2*e^2*f^6))))/(2*(16*a^3*c*e^2*f^6 -
 4*a^2*b^2*e^2*f^6)) + ((12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9)*(
2*a*c - b^2)^3)/(64*a^9*e^3*f^18*(4*a*c - b^2)^(3/2)) + (((((20*a^3*c^4*e^1
8*f^6 + 2*a^2*b^2*c^3*e^18*f^6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*
(12*a^3*b^3*c^2*e^19*f^9 - 40*a^4*b*c^3*e^19*f^9))/(2*a^3*f^9*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - 8*a*b*c*e*f^3))/(2*(16*a^3*c*e^2
*f^6 - 4*a^2*b^2*e^2*f^6)) + (6*b*c^4*e^17)/(a^2*f^6))*(2*a*c - b^2))/(4*a^
2*e*f^3*(4*a*c - b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^2*c^2 - 6*
b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4*e^14 + b^4*c^2*e^14 - 4
*a*b^2*c^3*e^14) + (2*a^3*f^9*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^2 - 9*a*b^
2*c)*((b*c^4*e^14 + c^5*d^2*e^14)/(a^3*f^9) + (((((4*a^2*b^3*c^2*e^16*f^6 +
 20*a^3*c^4*d^2*e^16*f^6 - 4*a^3*b*c^3*e^16*f^6 + 2*a^2*b^2*c^3*d^2*e^16*f^
6)/(a^3*f^9) - ((2*b^3*e*f^3 - 8*a*b*c*e*f^3)*(4*a^4*b^2*c^2*e^17*f^9 - 40*
a^4*b*c^3*d^2*e^17*f^9 + 12*a^3*b^3*c^2*d^2*e^17*f^9))/(2*a^3*f^9*(16*a^3*c
*e^2*f^6 - 4*a^2*b^2*e^2*f^6)))*(2*b^3*e*f^3 - ...
```

3.645 $\int \frac{1}{(df+efx)^4(a+b(d+ex)^2+c(d+ex)^4)}\,dx$

Optimal. Leaf size=236

$$-\frac{1}{3aef^4(d+ex)^3}+\frac{b}{a^2ef^4(d+ex)}+\frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}\,ef^4}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\cdots}{\sqrt{2}\cdots}$$

```
[Out] -1/3/a/e/f^4/(e*x+d)^3+b/a^2/e/f^4/(e*x+d)+1/2*arctan((e*x+d)*2^(1/2)*c^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2)
)/a^2/e/f^4*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan((e*x+d)*2^(1/2)
*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^
(1/2))/a^2/e/f^4*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1156, 1137, 1295, 1180, 211}

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2ef^4\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2ef^4\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2ef^4(d+ex)}-\frac{1}{3aef^4(d+ex)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] -1/3*1/(a*e*f^4*(d + e*x)^3) + b/(a^2*e*f^4*(d + e*x)) + (Sqrt[c]*(b + (b^2
 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) + (Sqrt[
c]*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f
^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
```

```
4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{ef^4}$$

$$= -\frac{1}{3aef^4(d+ex)^3} + \frac{\text{Subst}\left(\int \frac{-3b-3cx^2}{x^2(a+bx^2+cx^4)}\,dx, x, d+ex\right)}{3aef^4}$$

$$= -\frac{1}{3aef^4(d+ex)^3} + \frac{b}{a^2ef^4(d+ex)} - \frac{\text{Subst}\left(\int \frac{-3(b^2-ac)-3\cdots}{a+bx^2+cx\cdots}\right.}{3a^2e\cdots}$$

$$= -\frac{1}{3aef^4(d+ex)^3} + \frac{b}{a^2ef^4(d+ex)} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right.\right.\cdots}{\cdots}$$

$$= -\frac{1}{3aef^4(d+ex)^3} + \frac{b}{a^2ef^4(d+ex)} + \frac{\sqrt{c}\left(b+\frac{b^2-2ac}{\sqrt{b^2-4a\cdots}}\right.\cdots}{\sqrt{2}\,a^2\cdots}$$

Mathematica [A]
time = 0.14, size = 238, normalized size = 1.01

$$\frac{-\frac{2a}{(d+ex)^3}+\frac{6b}{d+ex}+\frac{3\sqrt{2}\sqrt{c}\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6a^2ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x]
```

```
[Out] ((-2*a)/(d + e*x)^3 + (6*b)/(d + e*x) + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt
[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(6*a^2*e*f^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.20, size = 192, normalized size = 0.81

method	result
default	$\frac{-\frac{1}{3ae(ex+d)^3}+\frac{b}{a^2e(ex+d)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+2deb\right)\_Z+d^4c+d^2b+a\right)}\frac{\left(\_R^2bc\,e^2+2\_R\ldots\right.}{2e^3c\_R^3+6d\,e^2c\_\ldots}}{2a^2e}}{f^4}$
risch	$\frac{\frac{be\,x^2}{a^2}+\frac{2bdx}{a^2}-\frac{-3d^2b+a}{3e\,a^2}}{f^4(ex+d)^3}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16f^{16}e^4c^2a^7-8a^6b^2c\,e^4f^{16}+a^5b^4e^4f^{16}\right)\_Z^4+\left(-20b\,f^8e^2c^3a^3+25b^3f^8e^2c^2a^2-9b^5f^8e^2ca+b^7\ldots\right.\right.}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^4*(-1/3/a/e/(e*x+d)^3+b/a^2/e/(e*x+d)+1/2/a^2/e*sum((_R^2*b*c*e^2+2*_R*
b*c*d*e+b*c*d^2-a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+
_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2
)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="maxima"
)
```

```
[Out] 1/3*(3*b*x^2*e^2 + 6*b*d*x*e + 3*b*d^2 - a)/(a^2*f^4*x^3*e^4 + 3*a^2*d*f^4*
x^2*e^3 + 3*a^2*d^2*f^4*x*e^2 + a^2*d^3*f^4*e) + integrate((b*c*x^2*e^2 + 2
*b*c*d*x*e + b*c*d^2 + b^2 - a*c)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^
2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(a^2*f^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2138 vs. $2(200) = 400$.

time = 0.41, size = 2138, normalized size = 9.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="fricas"
)
```

```
[Out] 1/6*(6*b*x^2*e^2 + 12*b*d*x*e - 3*sqrt(1/2)*(a^2*f^4*x^3*e^4 + 3*a^2*d*f^4*
x^2*e^3 + 3*a^2*d^2*f^4*x*e^2 + a^2*d^3*f^4*e)*sqrt(((a^5*b^2 - 4*a^6*c)*f^
8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*
b^2 - 4*a^11*c)*f^16)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c
)*f^8))*e^(-1)*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x*e + sqrt(1/2)*((a^
5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*f^16))*e + (b^8 - 8*
a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*f^4*e)*sqrt(((a^5*b^
2 - 4*a^6*c)*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/((a^10*b^2 - 4*a^11*c)*f^16)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^
5*b^2 - 4*a^6*c)*f^8))*e^(-1) + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d) + 3*
sqrt(1/2)*(a^2*f^4*x^3*e^4 + 3*a^2*d*f^4*x^2*e^3 + 3*a^2*d^2*f^4*x*e^2 + a^
2*d^3*f^4*e)*sqrt(((a^5*b^2 - 4*a^6*c)*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*f^16)) - b^5 + 5*a
*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*f^8))*e^(-1)*log(2*(b^4*c^3 - 3*
a*b^2*c^4 + a^2*c^5)*x*e - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2
)*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((
a^10*b^2 - 4*a^11*c)*f^16))*e + (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*
b^2*c^3 + 4*a^4*c^4)*f^4*e)*sqrt(((a^5*b^2 - 4*a^6*c)*f^8*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*f^1
6)) - b^5 + 5*a*b^3*c - 5*a^2*b*c^2)/((a^5*b^2 - 4*a^6*c)*f^8))*e^(-1) + 2*
(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d) + 6*b*d^2 + 3*sqrt(1/2)*(a^2*f^4*x^3*e
^4 + 3*a^2*d*f^4*x^2*e^3 + 3*a^2*d^2*f^4*x*e^2 + a^2*d^3*f^4*e)*sqrt(-((a^5
*b^2 - 4*a^6*c)*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
+ a^4*c^4)/((a^10*b^2 - 4*a^11*c)*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e
^(-2)/((a^5*b^2 - 4*a^6*c)*f^8))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x*
e + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d + sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3
*c + 12*a^7*b*c^2)*f^12*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*
```

```
c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*f^16))*e - (b^8 - 8*a*b^6*c + 20*a^2*
b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*f^4*e)*sqrt(-((a^5*b^2 - 4*a^6*c)*f^8
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b
^2 - 4*a^11*c)*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^(-2)/((a^5*b^2 - 4
*a^6*c)*f^8))) - 3*sqrt(1/2)*(a^2*f^4*x^3*e^4 + 3*a^2*d*f^4*x^2*e^3 + 3*a^2
*d^2*f^4*x*e^2 + a^2*d^3*f^4*e)*sqrt(-((a^5*b^2 - 4*a^6*c)*f^8*sqrt((b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c
)*f^16)) + b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^(-2)/((a^5*b^2 - 4*a^6*c)*f^8))
*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x*e + 2*(b^4*c^3 - 3*a*b^2*c^4 + a
^2*c^5)*d - sqrt(1/2)*((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*f^12*sqrt((b^
8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^
11*c)*f^16))*e - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4
*c^4)*f^4*e)*sqrt(-((a^5*b^2 - 4*a^6*c)*f^8*sqrt((b^8 - 6*a*b^6*c + 11*a^2*
b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/((a^10*b^2 - 4*a^11*c)*f^16)) + b^5 - 5*
a*b^3*c + 5*a^2*b*c^2)*e^(-2)/((a^5*b^2 - 4*a^6*c)*f^8))) - 2*a)/(a^2*f^4*x
^3*e^4 + 3*a^2*d*f^4*x^2*e^3 + 3*a^2*d^2*f^4*x*e^2 + a^2*d^3*f^4*e)
```

Sympy [**A**]
time = 103.09, size = 411, normalized size = 1.74

$$\frac{-a+3bd^2+6bdex+3be^2x^2}{3a^2d^3ef^4+9a^2d^2e^2f^4x+9a^2de^3f^4x^2+3a^2e^4f^4x^3}+\operatorname{RootSum}\left(t^4\cdot\left(256a^7c^2e^4f^{16}-128a^6b^2ce^4f^{16}+16a^5b^4e^4f^{16}\right)+t^2\left(-80a^3bc^3e^2f^8+100a^2b^3c^2e^2f^8-36ab^5ce^2f^8+4b^7e^2f^8\right)+c^5,\left(t\mapsto t\log\left(x+\frac{-96t^3a^7bc^2e^3f^{12}+56t^3a^6b^3ce^3f^{12}-8t^3a^5b^5e^3f^{12}-4ta^4c^4ef^4+32ta^3b^2c^3ef^4-40ta^2b^4c^2ef^4+16tab^6cef^4-2tb^8ef^4+a^2c^5d-3ab^2c^4d+b^4c^3d}{a^2c^5e-3ab^2c^4e+b^4c^3e}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4),x)
```

```
[Out] (-a + 3*b*d**2 + 6*b*d*e*x + 3*b*e**2*x**2)/(3*a**2*d**3*e*f**4 + 9*a**2*d*
*2*e**2*f**4*x + 9*a**2*d*e**3*f**4*x**2 + 3*a**2*e**4*f**4*x**3) + RootSum
(_t**4*(256*a**7*c**2*e**4*f**16 - 128*a**6*b**2*c*e**4*f**16 + 16*a**5*b**
4*e**4*f**16) + _t**2*(-80*a**3*b*c**3*e**2*f**8 + 100*a**2*b**3*c**2*e**2*
f**8 - 36*a*b**5*c*e**2*f**8 + 4*b**7*e**2*f**8) + c**5, Lambda(_t, _t*log(
x + (-96*_t**3*a**7*b*c**2*e**3*f**12 + 56*_t**3*a**6*b**3*c*e**3*f**12 - 8
*_t**3*a**5*b**5*e**3*f**12 - 4*_t*a**4*c**4*e*f**4 + 32*_t*a**3*b**2*c**3*
e*f**4 - 40*_t*a**2*b**4*c**2*e*f**4 + 16*_t*a*b**6*c*e*f**4 - 2*_t*b**8*e*
f**4 + a**2*c**5*d - 3*a*b**2*c**4*d + b**4*c**3*d)/(a**2*c**5*e - 3*a*b**2
*c**4*e + b**4*c**3*e))))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1249 vs. $2(200) = 400$.
time = 4.33, size = 1249, normalized size = 5.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4),x, algorithm="giac")
```

```
[Out] -1/2*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*c*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqr
t(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e
 - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(
b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e
^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d
*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2
*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) + sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*e^2 - 2*(d*e^(
-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*e +
b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqr
t(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2
 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))^2*b*c*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*b*c*d*e + b*c*d^2 + b^2 - a*c)*log(d*e^(-1) + x - sqrt(1/2)
*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*
sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a^2*f^4) + 1/3*(3*b*x^2*e^2 + 6*b*d*x*e
+ 3*b*d^2 - a)*e^(-1)/((x*e + d)^3*a^2*f^4)
```

Mupad [B]
time = 3.17, size = 2500, normalized size = 10.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)),x)
```

```
[Out] ((2*b*d*x)/a^2 - (a - 3*b*d^2)/(3*a^2*e) + (b*e*x^2)/a^2)/(d^3*f^4 + e^3*f^
4*x^3 + 3*d*e^2*f^4*x^2 + 3*d^2*e*f^4*x) - atan((((b^4*(-(4*a*c - b^2)^3)^(
1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 +
16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)
```

^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(
1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8
+ 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^
3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)
^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^
8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10*b^3*c^2*e^
14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3
*c^2*d*e^13*f^20) - 16*a^10*c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12*f^16 + 20*a^
9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8
*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12
 - 8*a^7*b^2*c^4*d*e^11*f^12)*1i + ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 2
0*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c
 - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2
*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 +
 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5
*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e
^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7
 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b
^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2
*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a
^11*b*c^3*e^14*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^
20) + 16*a^10*c^4*e^12*f^16 + 4*a^8*b^4*c^2*e^12*f^16 - 20*a^9*b^2*c^3*e^12
f^16) + x(4*a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e
^12*f^12) + 4*a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c
^4*d*e^11*f^12)*1i)/(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 -
25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(
-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b
^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3
 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6
*b^2*c*e^2*f^8)))^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^
3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2
c(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a
^6*b^2*c*e^2*f^8)))^(1/2)*(x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14
*f^20) - 32*a^11*b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20) - 16*a^10*
c^4*e^12*f^16 - 4*a^8*b^4*c^2*e^12*f^16 + 20*a^9*b^2*c^3*e^12*f^16) + x*(4*
a^8*c^5*e^12*f^12 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*
a^8*c^5*d*e^11*f^12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12
) - ((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 +
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)
^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))^(
1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2
+ a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^
3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)))
^(1/2)*(((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^

```
2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^8)
))^(1/2)*(x*(8*a^10*b^3*c^2*e^14*f^20 - 32*a^11*b*c^3*e^14*f^20) - 32*a^11*
b*c^3*d*e^13*f^20 + 8*a^10*b^3*c^2*d*e^13*f^20) + 16*a^10*c^4*e^12*f^16 + 4
*a^8*b^4*c^2*e^12*f^16 - 20*a^9*b^2*c^3*e^12*f^16) + x*(4*a^8*c^5*e^12*f^12
 + 2*a^6*b^4*c^3*e^12*f^12 - 8*a^7*b^2*c^4*e^12*f^12) + 4*a^8*c^5*d*e^11*f^
12 + 2*a^6*b^4*c^3*d*e^11*f^12 - 8*a^7*b^2*c^4*d*e^11*f^12) + 2*a^6*b*c^5*e
^10*f^8))*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*
c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b
^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6*b^2*c*e^2*f^
8)))^(1/2)*2i - atan(((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3
+ 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*a^6
*b^2*c*e^2*f^8)))^(1/2)*((-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^
2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^2*f^8 + 16*a^7*c^2*e^2*f^8 - 8*
a^6*b^2*c*e^2*f^8)))^(1/2)*((x*(8*a^10*b^3*c^2*...
```

3.646 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=279

$$\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)f^4\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}\,e}+\frac{\left(b^2+4\right.}{}$$

```
[Out] 1/2*f^4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4
)+1/4*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(
-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)/e*2^(1/2)/c^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2)+1/4*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/c^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1156, 1134, 1180, 211}

$$\frac{f^4\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{f^4\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)) + ((b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b
^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4
*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
```

```
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^4\text{Subst}\left(\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{f^4\text{Subst}\left(\int \frac{2a-bx^2}{a+bx^2+cx^4}\,d\right.}{2\left(b^2-4ac\right.}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(\left(b^2+4ac-b\sqrt{b^2-\right.\right.}{}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4\right.}{2\sqrt{2}\sqrt{c}\left(b^2\right.}$$

Mathematica [A]
time = 0.30, size = 266, normalized size = 0.95

$$\frac{f^4\left(-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(4*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 327, normalized size = 1.17

method	result
default	$f^4 \left(\frac{-\frac{e^2 b x^3}{2(4ac-b^2)} - \frac{3bde\,x^2}{2(4ac-b^2)} - \frac{(3d^2b+2a)x}{2(4ac-b^2)} - \frac{d(d^2b+2a)}{2e(4ac-b^2)}}{c e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a} + \frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(\right.\right.}}{} \right.$
risch	$\frac{-\frac{b\,e^2f^4x^3}{2(4ac-b^2)} - \frac{3dbe\,f^4x^2}{2(4ac-b^2)} - \frac{f^4(3d^2b+2a)x}{2(4ac-b^2)} - \frac{d\,f^4(d^2b+2a)}{2e(4ac-b^2)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a} + \frac{f^4\left(\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f^4*((-1/2*e^2*b/(4*a*c-b^2)*x^3-3/2/(4*a*c-b^2)*b*d*e*x^2-1/2*(3*b*d^2+2*a
)/(4*a*c-b^2)*x-1/2*d/e*(b*d^2+2*a)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6
*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^
2)/e*sum((-_R^2*b*e^2-2*_R*b*d*e-b*d^2+2*a)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*
_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3
+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*f^4*integrate((b*x^2*e^2 + 2*b*d*x*e + b*d^2 - 2*a)/(c*x^4*e^4 + 4*c*d*
x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)
```

```
*x + a), x)/(b^2 - 4*a*c) + 1/2*(b*f^4*x^3*e^3 + 3*b*d*f^4*x^2*e^2 + (3*b*d
^2*e + 2*a*e)*f^4*x + (b*d^3 + 2*a*d)*f^4)/((b^2*c*e - 4*a*c^2*e)*d^4 + 4*(
b^2*c*e^4 - 4*a*c^2*e^4)*d*x^3 + (b^2*c*e^5 - 4*a*c^2*e^5)*x^4 + a*b^2*e -
4*a^2*c*e + (b^3*e - 4*a*b*c*e)*d^2 + (b^3*e^3 - 4*a*b*c*e^3 + 6*(b^2*c*e^3
 - 4*a*c^2*e^3)*d^2)*x^2 + 2*(2*(b^2*c*e^2 - 4*a*c^2*e^2)*d^3 + (b^3*e^2 -
4*a*b*c*e^2)*d)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. $2(238) = 476$.
time = 0.39, size = 2498, normalized size = 8.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] 1/4*(2*b*f^4*x^3*e^3 + 6*b*d*f^4*x^2*e^2 + 2*(3*b*d^2 + 2*a)*f^4*x*e + 2*(b
*d^3 + 2*a*d)*f^4 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*c - 4*a*c
^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3 + 2*(2*(b
^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2)*d^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt
(f^16/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(b^6*c - 12*a
*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*e^(-2)/(b^6*c - 12*a*b^4*c^2 + 48*
a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*f^12*x*e + (3*b^2 + 4*a*c)*d
*f^12 + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*f^8*e + 2*sqrt(f^16/(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(b^7*c - 12*a*b^5*c^2 +
48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^16/(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(b^6*c - 12*a*b^4*c^
2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*e^(-2)/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3 - 64*a^3*c^4))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*c - 4*
a*c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3 + 2*(2
*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2)*d^4
+ a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + s
qrt(f^16/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(b^6*c - 1
2*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*e^(-2)/(b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*f^12*x*e + (3*b^2 + 4*a*c
)*d*f^12 - sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*f^8*e + 2*sqrt(f^16/(b
^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(b^7*c - 12*a*b^5*c^2
 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 + sqrt(f^1
6/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(b^6*c - 12*a*b^4
*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*e^(-2)/(b^6*c - 12*a*b^4*c^2 + 48*a^2*
b^2*c^3 - 64*a^3*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*c -
 4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3 + 2
*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2)*d
```

```
^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8
- sqrt(f^16/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(b^6*c
- 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*e^(-2)/(b^6*c - 12*a*b^4*c^2
 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*f^12*x*e + (3*b^2 + 4*
a*c)*d*f^12 + sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*f^8*e - 2*sqrt(f^16
/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sqrt(
f^16/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(b^6*c - 12*a*
b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*e^(-2)/(b^6*c - 12*a*b^4*c^2 + 48*a
^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*
c - 4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3
+ 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2
)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f
^8 - sqrt(f^16/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(b^6
*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*e^(-2)/(b^6*c - 12*a*b^4*
c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*f^12*x*e + (3*b^2 +
 4*a*c)*d*f^12 - sqrt(1/2)*((b^4 - 8*a*b^2*c + 16*a^2*c^2)*f^8*e - 2*sqrt(f
^16/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(b^7*c - 12*a*b
^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*e)*sqrt(-((b^3 + 12*a*b*c)*f^8 - sq
rt(f^16/(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*(b^6*c - 12
*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*e^(-2)/(b^6*c - 12*a*b^4*c^2 + 4
8*a^2*b^2*c^3 - 64*a^3*c^4))))/((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*c - 4*a*
c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3 + 2*(2*(
b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2)*d^4 +
a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(252) = 504$.

time = 12.01, size = 641, normalized size = 2.30

$$\frac{-2adf^4 - bd^3f^4 - 3bde^2f^4x^2 - be^3f^4x^3 + x(-2aef^4 - 3bd^2ef^4)}{8a^2ce - 2ab^2e + 8abcd^2e + 8ac^2d^4e - 2b^3d^2e - 2b^2cd^4e + x^4\cdot(8ac^2e^5 - 2b^2ce^5) + x^3\cdot(32ac^2de^4 - 8b^2cde^4) + x^2\cdot(8abce^3 + 48ac^2d^2e^3 - 2b^3e^3 - 12b^2cd^2e^3) + x(16abcde^2 + 32ac^2d^3e^2 - 4b^3de^2 - 8b^2cd^3e^2)} + \operatorname{RootSum}\left(t^4\cdot(1048576a^6c^7e^4 - 1572864a^5b^2c^6e^4 + 983040a^4b^4c^5e^4 - 327680a^3b^6c^4e^4 + 61440a^2b^8c^3e^4 - 6144ab^{10}c^2e^4 + 256b^{12}ce^4) + t^2(-12288a^4bc^4e^2f^8 + 8192a^3b^3c^3e^2f^8 - 1536a^2b^5c^2e^2f^8 + 16b^7e^2f^8) + 16a^3c^2f^{16} + 24a^2b^2cf^{16} + 9ab^4f^{16}, \left(t \mapsto t\log\left(x + \frac{16384t^3a^3bc^4e^3 - 12288t^3a^2b^3c^3e^3 + 3072t^3ab^5c^2e^3 - 256t^3b^7ce^3 + 64ta^2c^2ef^8 - 128tab^2cef^8 - 4tb^4ef^8 + 4acdf^{12} + 3b^2df^{12}}{4acef^{12} + 3b^2ef^{12}}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

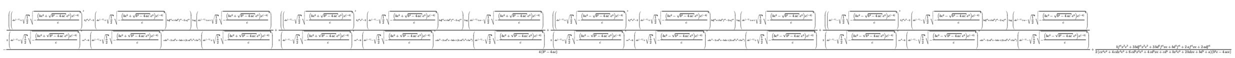
```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] (-2*a*d*f**4 - b*d**3*f**4 - 3*b*d*e**2*f**4*x**2 - b*e**3*f**4*x**3 + x*(-
2*a*e*f**4 - 3*b*d**2*e*f**4))/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e +
8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2
*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e
**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*
c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2)) + Roo
tSum(_t**4*(1048576*a**6*c**7*e**4 - 1572864*a**5*b**2*c**6*e**4 + 983040*a
**4*b**4*c**5*e**4 - 327680*a**3*b**6*c**4*e**4 + 61440*a**2*b**8*c**3*e**4
 - 6144*a*b**10*c**2*e**4 + 256*b**12*c*e**4) + _t**2*(-12288*a**4*b*c**4*e
**2*f**8 + 8192*a**3*b**3*c**3*e**2*f**8 - 1536*a**2*b**5*c**2*e**2*f**8 +
```

```
16*b**9*e**2*f**8) + 16*a**3*c**2*f**16 + 24*a**2*b**2*c*f**16 + 9*a*b**4*f
**16, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4*e**3 - 12288*_t**3*a**
2*b**3*c**3*e**3 + 3072*_t**3*a*b**5*c**2*e**3 - 256*_t**3*b**7*c*e**3 + 64
*_t*a**2*c**2*e*f**8 - 128*_t*a*b**2*c*e*f**8 - 4*_t*b**4*e*f**8 + 4*a*c*d*
f**12 + 3*b**2*d*f**12)/(4*a*c*e*f**12 + 3*b**2*e*f**12))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1370 vs. $2(238) = 476$.

time = 3.70, size = 1370, normalized size = 4.91



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^2*b*f^4*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))*b*d*f^4*e + b*d^2*f^4 - 2*a*f^4)*log(d*e^(-1) + x + sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + s
qrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d
^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*f^4*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e + b*d^2*f^4 - 2*a*f^
4)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4
)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1
) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1
) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*f^4*e^2
 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*
b*d*f^4*e + b*d^2*f^4 - 2*a*f^4)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2
 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 -
 sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e +
(6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*b*f^4*e^2 - 2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*d*f^4*e + b*d^2*f^4 - 2*a*f^4)*log(d*e^(-1)
 + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(
-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 -
6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*
c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^2 - 4*a*c) + 1/2*(b*f^
```

```
4*x^3*e^3 + 3*b*d*f^4*x^2*e^2 + 3*b*d^2*f^4*x*e + b*d^3*f^4 + 2*a*f^4*x*e +
 2*a*d*f^4)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c
*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

Mupad [B]
time = 5.09, size = 2500, normalized size = 8.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] atan(((((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^
4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c
^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 6
4*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5
*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4
*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6
*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 +
 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2))*(-(b^9*f^8 + f^8*(-(
4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^
3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2
*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c
^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^
2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^
4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2
*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c
^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a
^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2
 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i - ((128*a^3*c^4*
d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((2048*a^4*c^5*e^12*f^4 + 384*a^2*b^4
*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(b^6
- 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ((64*b^9*c^2*d*e^13 - 1024*a
*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16384*
a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) +
(x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^2*b^
3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*a*c
- b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*
f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c
^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2
)))^(1/2))*(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 -
96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^
```

2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a
^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (x*(b^4*c*e^12*f^8 + 8*a^2
*c^3*e^12*f^8 + 2*a*b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*
(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*
c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^
10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*
e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2)*1i)/((((2048*a^4*c^5*e^12*f^4 + 384*a^2
*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 - 32*a*b^6*c^2*e^12*f^4)/(8*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((64*b^9*c^2*d*e^13 - 10
24*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 + 6144*a^2*b^5*c^4*d*e^13 - 16
384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 - 1024*a^3*b*c^5*e^14 + 768*a^
2*b^3*c^4*e^14))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(4*
a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*
c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b
^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6
e^2)))^(1/2))(-(b^9*f^8 + f^8*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^
8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^
7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 38
40*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c^6*e^2)))^(1/2) - (128*a^3*c^4*d*e^11*f^
8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(b^4*c*e^12*f^8 + 8*a^2*c^3*e^12*f^8 + 2*a*
b^2*c^2*e^12*f^8))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9*f^8 + f^8*(-(
4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4*f^8 - 96*a^2*b^5*c^2*f^8 + 512*a^3*b^
3*c^3*f^8)/(32*(b^12*c*e^2 + 4096*a^6*c^7*e^2 - 24*a*b^10*c^2*e^2 + 240*a^2
*b^8*c^3*e^2 - 1280*a^3*b^6*c^4*e^2 + 3840*a^4*b^4*c^5*e^2 - 6144*a^5*b^2*c
^6*e^2)))^(1/2) + ((128*a^3*c^4*d*e^11*f^8 - 4*b^6*c*d*e^11*f^8 + 8*a*b^4*c
^2*d*e^11*f^8)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + ((204
8*a^4*c^5*e^12*f^4 + 384*a^2*b^4*c^3*e^12*f^4 - 1536*a^3*b^2*c^4*e^12*f^4 -
32*a*b^6*c^2*e^12*f^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)
) - ((64*b^9*c^2*d*e^13 - 1024*a*b^7*c^3*d*e^13 + 16384*a^4*b*c^6*d*e^13 +
6144*a^2*b^5*c^4*d*e^13 - 16384*a^3*b^3*c^5*d*e^13)/(8*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*b^7*c^2*e^14 - 192*a*b^5*c^3*e^14 -
1024*a^3*b*c^5*e^14 + 768*a^2*b^3*c^4*e^14))/(2...

3.647 $\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{f^3(2a+b(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

```
[Out] 1/2*f^3*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-b*f^3*
arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1156, 1128, 652, 632, 212}

$$\frac{f^3(2a+b(d+ex)^2)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(2*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)) - (b*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2 -
 4*a*c)^(3/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^3\text{Subst}\left(\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f^3\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= \frac{f^3(2a+b(d+ex)^2)}{2(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)} + \frac{(bf^3)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2} \cdots\right.}{2(b^2-4 \cdots}$$

$$= \frac{f^3(2a+b(d+ex)^2)}{2(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)} - \frac{(bf^3)\,\text{Subst}\left(\int \frac{1}{b^2-4ac- \cdots}\right.}{(b^2- \cdots}$$

$$= \frac{f^3(2a+b(d+ex)^2)}{2(b^2-4ac)\,e\,(a+b(d+ex)^2+c(d+ex)^4)} - \frac{bf^3\tanh^{-1}\left(\frac{b+2c(d+ex \cdots}{\sqrt{b^2-4 \cdots}}\right.}{(b^2-4ac)^{3/2}\,e}$$

Mathematica [A]
time = 0.09, size = 103, normalized size = 1.00

$$\frac{f^3\left(\frac{2a+b(d+ex)^2}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))} - \frac{2b\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] (f^3*((2*a + b*(d + e*x)^2)/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b + c*(d + e*x
)^2))) - (2*b*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a
*c)^(3/2)))/(2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.23, size = 280, normalized size = 2.72

method	result
default	$f^{3}\left(\frac{-\frac{x^{2} b e}{2\left(4 a c-b^{2}\right)}-\frac{x b d}{4 a c-b^{2}}-\frac{d^{2} b+2 a}{2 e\left(4 a c-b^{2}\right)}}{c\,e^{4} x^{4}+4 c d\,e^{3} x^{3}+6 c\,d^{2} e^{2} x^{2}+4 c\,d^{3} e x+b\,e^{2} x^{2}+d^{4} c+2 d e b x+d^{2} b+a}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(e^{4} c\,\_Z^{4}+4 d\,e^{3} c\,\_Z^{3}+\left(6 d^{2} e^{2} c+e^{2} b\right)\_Z^{2}\right.}\right.}{}\right.$
risch	$\frac{-\frac{b e\,f^{3} x^{2}}{2\left(4 a c-b^{2}\right)}-\frac{b d\,f^{3} x}{4 a c-b^{2}}-\frac{f^{3}\left(d^{2} b+2 a\right)}{2 e\left(4 a c-b^{2}\right)}}{c\,e^{4} x^{4}+4 c d\,e^{3} x^{3}+6 c\,d^{2} e^{2} x^{2}+4 c\,d^{3} e x+b\,e^{2} x^{2}+d^{4} c+2 d e b x+d^{2} b+a}+\frac{f^{3} b \ln\left(\left(\left(-\left(-4 a c+b^{2}\right)^{\frac{3}{2}} e^{2}+4 a b c\,e^{2}-b^{3} e^{2}\right) x^{2}+\left(-2\left(-4 a c+\right.\right.\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f^3*((-1/2/(4*a*c-b^2)*x^2*b*e-1/(4*a*c-b^2)*x*b*d-1/2/e*(b*d^2+2*a)/(4*a*c
-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4
+2*b*d*e*x+b*d^2+a)+1/2*b/(4*a*c-b^2)/e*sum((-_R*e-d)/(2*_R^3*c*e^3+6*_R^2*
c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*
e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] b*f^3*integrate((x*e + d)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c
*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(b^2 - 4*a*c) + 1/
2*(b*f^3*x^2*e^2 + 2*b*d*f^3*x*e + (b*d^2 + 2*a)*f^3)/((b^2*c*e - 4*a*c^2*e
)*d^4 + 4*(b^2*c*e^4 - 4*a*c^2*e^4)*d*x^3 + (b^2*c*e^5 - 4*a*c^2*e^5)*x^4 +
 a*b^2*e - 4*a^2*c*e + (b^3*e - 4*a*b*c*e)*d^2 + (b^3*e^3 - 4*a*b*c*e^3 + 6
*(b^2*c*e^3 - 4*a*c^2*e^3)*d^2)*x^2 + 2*(2*(b^2*c*e^2 - 4*a*c^2*e^2)*d^3 +
(b^3*e^2 - 4*a*b*c*e^2)*d)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 468 vs. 2(99) = 198.
time = 0.39, size = 1063, normalized size = 10.32

$$\left[\frac{(b^3-4abc)f^3x^2e^2+2(b^3-4abc)df^3xe+(2ab^2-8a^2c+(b^3-4abc)d^2)f^3-(bcf^3x^4e^4+4bcdf^3x^3e^3+(6bcd^2+b^2)f^3x^2e^2+2(2bcd^3+b^2d)f^3xe+(bcd^4+b^2d^2+ab)f^3)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4e^4+8c^2dx^3e^3+2c^2d^4+2bcd^2+2(6c^2d^2+bc)x^2e^2+4(2c^2d^3+bcd)xe+b^2-2ac+(2cx^2e^2+4cdxe+2cd^2+b)\sqrt{b^2-4ac}}{cx^4e^4+4cdx^3e^3+cd^4+(6cd^2+b)x^2e^2+bd^2+2(2cd^3+bd)xe+a}\right)}{2((b^4c-8ab^2c^2+16a^2c^3)x^4e^5+4(b^4c-8ab^2c^2+16a^2c^3)dx^3e^4+(b^5-8ab^3c+16a^2bc^2+6(b^4c-8ab^2c^2+16a^2c^3)d^2)x^2e^3+2(2(b^4c-8ab^2c^2+16a^2c^3)d^3+(b^5-8ab^3c+16a^2bc^2)d)xe^2+(ab^4-8a^2b^2c+16a^3c^2+(b^4c-8ab^2c^2+16a^2c^3)d^4+(b^5-8ab^3c+16a^2bc^2)d^2)e)}, \frac{(b^3-4abc)f^3x^2e^2+2(b^3-4abc)df^3xe+(2ab^2-8a^2c+(b^3-4abc)d^2)f^3-2(bcf^3x^4e^4+4bcdf^3x^3e^3+(6bcd^2+b^2)f^3x^2e^2+2(2bcd^3+b^2d)f^3xe+(bcd^4+b^2d^2+ab)f^3)\sqrt{-b^2+4ac}\arctan\left(-\frac{(2cx^2e^2+4cdxe+2cd^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)}{2((b^4c-8ab^2c^2+16a^2c^3)x^4e^5+4(b^4c-8ab^2c^2+16a^2c^3)dx^3e^4+(b^5-8ab^3c+16a^2bc^2+6(b^4c-8ab^2c^2+16a^2c^3)d^2)x^2e^3+2(2(b^4c-8ab^2c^2+16a^2c^3)d^3+(b^5-8ab^3c+16a^2bc^2)d)xe^2+(ab^4-8a^2b^2c+16a^3c^2+(b^4c-8ab^2c^2+16a^2c^3)d^4+(b^5-8ab^3c+16a^2bc^2)d^2)e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*((b^3 - 4*a*b*c)*f^3*x^2*e^2 + 2*(b^3 - 4*a*b*c)*d*f^3*x*e + (2*a*b^2
- 8*a^2*c + (b^3 - 4*a*b*c)*d^2)*f^3 - (b*c*f^3*x^4*e^4 + 4*b*c*d*f^3*x^3*e
^3 + (6*b*c*d^2 + b^2)*f^3*x^2*e^2 + 2*(2*b*c*d^3 + b^2*d)*f^3*x*e + (b*c*d
^4 + b^2*d^2 + a*b)*f^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3
*e^3 + 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 +
 b*c*d)*x*e + b^2 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^
2 - 4*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*
d^2 + 2*(2*c*d^3 + b*d)*x*e + a)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*
e^5 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^3*e^4 + (b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*x^2*e^3 + 2*(2*(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*
e^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3
)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), 1/2*((b^3 - 4*a*b*c)*f^3*
x^2*e^2 + 2*(b^3 - 4*a*b*c)*d*f^3*x*e + (2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c
)*d^2)*f^3 - 2*(b*c*f^3*x^4*e^4 + 4*b*c*d*f^3*x^3*e^3 + (6*b*c*d^2 + b^2)*f
^3*x^2*e^2 + 2*(2*b*c*d^3 + b^2*d)*f^3*x*e + (b*c*d^4 + b^2*d^2 + a*b)*f^3)
*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(-b
^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*e^5 + 4
*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^3*e^4 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*x^2*e^3 + 2*(2*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*e^2 + (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(88) = 176$.
time $= 2.94$, size $= 556$, normalized size $= 5.40$

$$\frac{bf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2bc^2f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3cf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e}-\frac{bf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2bc^2f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3cf^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5f^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2f^3+2bcd^2f^3}{2bce^2f^3}\right)}{2e}+\frac{-2af^3-bd^2f^3-2bdef^3x-be^2f^3x^2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2cde^4)+x^2\cdot(8abce^3+48ac^2d^2e^3-2b^3e^3-12b^2cd^2e^3)+x\left(16abcde^2+32ac^2d^3e^2-4b^3de^2-8b^2cd^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*b*c**2*f**
3*sqrt(-1/(4*a*c - b**2)**3) + 8*a*b**3*c*f**3*sqrt(-1/(4*a*c - b**2)**3) -
 b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b*c*d**2*f**3)/(2*b*c
*e**2*f**3))/(2*e) - b*f**3*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 +
 (16*a**2*b*c**2*f**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*f**3*sqrt(-1/
(4*a*c - b**2)**3) + b**5*f**3*sqrt(-1/(4*a*c - b**2)**3) + b**2*f**3 + 2*b
*c*d**2*f**3)/(2*b*c*e**2*f**3))/(2*e) + (-2*a*f**3 - b*d**2*f**3 - 2*b*d*e
*f**3*x - b*e**2*f**3*x**2)/(8*a**2*c*e - 2*a*b**2*e + 8*a*b*c*d**2*e + 8*a
```

```
*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**4*(8*a*c**2*e**5 - 2*b*
*2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**4) + x**2*(8*a*b*c*e**3
 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**2*e**3) + x*(16*a*b*c*d
*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2*c*d**3*e**2))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(99) = 198$.

time = 3.67, size = 211, normalized size = 2.05

$$\frac{bf^3\arctan\left(\frac{2\,cd^2f+2\,(fx^2e+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}+\frac{bd^2f^5+(fx^2e+2\,dfx)bf^4e+2\,af^5}{2\left(cd^4f^2+2\,(fx^2e+2\,dfx)cd^2fe+bd^2f^2+(fx^2e+2\,dfx)^2ce^2+(fx^2e+2\,dfx)bfe+af^2\right)(b^2e-4\,ace)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] b*f^3*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*a
*c)*f))*e^(-1)/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) + 1/2*(b*d^2*f^5 + (f*x^2
*e + 2*d*f*x)*b*f^4*e + 2*a*f^5)/((c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*
f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e +
 a*f^2)*(b^2*e - 4*a*c*e))
```

Mupad [**B**]

time = 1.90, size = 460, normalized size = 4.47

$$\frac{b\,f^3\,\mathrm{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^4\,\left(x\,\left(\frac{b^3\,f^6\,\left(2\,b^3\,c^2\,d\,e^9-8\,a\,b\,c^3\,d\,e^9\right)}{a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}-\frac{2\,b^2\,c^2\,d\,e^7\,f^6}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}\right)+x^2\,\left(\frac{b^3\,f^6\,\left(2\,b^3\,c^2\,e^{10}-8\,a\,b\,c^3\,e^{10}\right)}{2\,a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}-\frac{b^2\,c^2\,e^8\,f^6}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}\right)-\frac{b^3\,f^6\,\left(16\,a^2\,c^3\,e^8-4\,a\,b^2\,c^2\,e^8+8\,a\,b\,c^3\,d^2\,e^8-2\,b^3\,c^2\,d^2\,e^8\right)}{2\,a\,e^2\,{\left(4\,a\,c-b^2\right)}^{11/2}}-\frac{b^2\,c^2\,d^2\,e^6\,f^6}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}\right)}{2\,b^2\,c^2\,e^6\,f^6}\right)}{e\,{\left(4\,a\,c-b^2\right)}^{3/2}}-\frac{\frac{f^3\,\left(b\,d^2+2\,a\right)}{2\,e\,\left(4\,a\,c-b^2\right)}+\frac{b\,d\,f^3\,x}{4\,a\,c-b^2}+\frac{b\,e\,f^3\,x^2}{2\,\left(4\,a\,c-b^2\right)}}{a+x^2\,\left(6\,c\,d^2\,e^2+b\,e^2\right)+b\,d^2+c\,d^4+x\,\left(4\,c\,e\,d^3+2\,b\,e\,d\right)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] (b*f^3*atan(((4*a*c - b^2)^4*(x*((b^3*f^6*(2*b^3*c^2*d*e^9 - 8*a*b*c^3*d*e^
9))/(a*e^2*(4*a*c - b^2)^(11/2)) - (2*b^2*c^2*d*e^7*f^6)/(a*(4*a*c - b^2)^(
7/2))) + x^2*((b^3*f^6*(2*b^3*c^2*e^10 - 8*a*b*c^3*e^10))/(2*a*e^2*(4*a*c -
 b^2)^(11/2)) - (b^2*c^2*e^8*f^6)/(a*(4*a*c - b^2)^(7/2))) - (b^3*f^6*(16*a
^2*c^3*e^8 - 4*a*b^2*c^2*e^8 - 2*b^3*c^2*d^2*e^8 + 8*a*b*c^3*d^2*e^8))/(2*a
*e^2*(4*a*c - b^2)^(11/2)) - (b^2*c^2*d^2*e^6*f^6)/(a*(4*a*c - b^2)^(7/2)))
)/(2*b^2*c^2*e^6*f^6)))/(e*(4*a*c - b^2)^(3/2)) - ((f^3*(2*a + b*d^2))/(2*e
*(4*a*c - b^2)) + (b*d*f^3*x)/(4*a*c - b^2) + (b*e*f^3*x^2)/(2*(4*a*c - b^2
)))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d^3*e
) + c*e^4*x^4 + 4*c*d*e^3*x^3)
```

3.648 $\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=263

$$-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)f^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,e}-\frac{\sqrt{c}\ldots}{\ldots}$$

[Out] -1/2*f^2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)

Rubi [A]
time = 0.27, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1156, 1133, 1180, 211}

$$\frac{\sqrt{c}\,f^2\left(2b-\sqrt{b^2-4ac}\right)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,e\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\,f^2\left(\sqrt{b^2-4ac}+2b\right)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,e\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*e*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)) + (Sqrt[c]*(2*b - Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt
[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)
^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*
f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[
2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
```

```
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f^2\text{Subst}\left(\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx,x,d+ex\right)}{e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{f^2\text{Subst}\left(\int \frac{b-2cx^2}{a+bx^2+c}\right.}{2\left(b^2-\right.}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\left(c\left(2b-\sqrt{b^2-4a}\right.\right.}{}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4}\right.}{\sqrt{2}\left(b^2-\right.}$$

Mathematica [A]
time = 0.63, size = 250, normalized size = 0.95

$$-\frac{f^2\left(\frac{b(d+ex)+2c(d+ex)^3}{(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f^2*((b*(d + e*x) + 2*c*(d + e*x)^3)/((b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)) + (Sqrt[2]*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])
*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/e
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 323, normalized size = 1.23

method	result
default	$f^2 \left(\frac{\frac{c e^2 x^3}{4ac-b^2}+\frac{3x^2 cde}{4ac-b^2}+\frac{(6c d^2+b)x}{8ac-2b^2}+\frac{d(2c d^2+b)}{2e(4ac-b^2)}}{c e^4x^4+4cd e^3x^3+6c d^2e^2x^2+4c d^3ex+b e^2x^2+d^4c+2debx+d^2b+a} + \frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d\right.\right.}}{} \right.$
risch	$\frac{\frac{c e^2 f^2 x^3}{4ac-b^2}+\frac{3dce f^2x^2}{4ac-b^2}+\frac{f^2(6c d^2+b)x}{8ac-2b^2}+\frac{d f^2(2c d^2+b)}{2e(4ac-b^2)}}{c e^4x^4+4cd e^3x^3+6c d^2e^2x^2+4c d^3ex+b e^2x^2+d^4c+2debx+d^2b+a} + \frac{f^2\left(\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3e\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f^2*((c*e^2/(4*a*c-b^2)*x^3+3/(4*a*c-b^2)*x^2*c*d*e+1/2*(6*c*d^2+b)/(4*a*c-
b^2)*x+1/2*d/e*(2*c*d^2+b)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^
2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*c-b^2)/e*sum(
(2*_R^2*c*e^2+4*_R*c*d*e+2*c*d^2-b)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2
*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^
2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*f^2*integrate((2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 - b)/(c*x^4*e^4 + 4*c
*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d
```

```
*e)*x + a), x)/(b^2 - 4*a*c) - 1/2*(2*c*f^2*x^3*e^3 + 6*c*d*f^2*x^2*e^2 + (
6*c*d^2*e + b*e)*f^2*x + (2*c*d^3 + b*d)*f^2)/((b^2*c*e - 4*a*c^2*e)*d^4 +
4*(b^2*c*e^4 - 4*a*c^2*e^4)*d*x^3 + (b^2*c*e^5 - 4*a*c^2*e^5)*x^4 + a*b^2*e
 - 4*a^2*c*e + (b^3*e - 4*a*b*c*e)*d^2 + (b^3*e^3 - 4*a*b*c*e^3 + 6*(b^2*c*
e^3 - 4*a*c^2*e^3)*d^2)*x^2 + 2*(2*(b^2*c*e^2 - 4*a*c^2*e^2)*d^3 + (b^3*e^2
 - 4*a*b*c*e^2)*d)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2520 vs. $2(225) = 450$.
time = 0.40, size = 2520, normalized size = 9.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] -1/4*(4*c*f^2*x^3*e^3 + 12*c*d*f^2*x^2*e^2 + 2*(6*c*d^2 + b)*f^2*x*e + 2*(2
*c*d^3 + b*d)*f^2 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*c - 4*a*c
^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3 + 2*(2*(b
^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2)*d^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b
^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/(a^2*b^6 - 12*a^3
*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*e^(-2)/(a*b^6 - 12*a^2*b^4*c + 48*a
^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*f^6*x*e + (3*b^2*c + 4*a*
c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*f^4*e - (a*b^8
 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/(a^2*b^6 - 12*a^3*
b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*e)*sqrt(-((b^3 + 12*a*b*c)*f^4 + (a*b
^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/(a^2*b^6 - 12*a^3
*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*e^(-2)/(a*b^6 - 12*a^2*b^4*c + 48*a
^3*b^2*c^2 - 64*a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4*e^5 + 4*(b^2*
c - 4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)*x^2*e^3
+ 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c - 4*a*c^2
)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12*a*b*c)*f
^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*e^(-2)/(a*b^6 - 12*a^2*b^4
*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*f^6*x*e + (3*b^2
*c + 4*a*c^2)*d*f^6 - 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*f^4*e
 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*e)*sqrt(-((b^3 + 12*a*b*c)*f
^4 + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8/(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*e^(-2)/(a*b^6 - 12*a^2*b^4
*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4*e^5
+ 4*(b^2*c - 4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^2)*d^2)
*x^2*e^3 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + ((b^2*c
```

```
- 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-((b^3 + 12
*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8
/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*e^(-2)/(a*b^6 - 1
2*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*f^6*x*e
 + (3*b^2*c + 4*a*c^2)*d*f^6 + 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 16*a^2*b*c
^2)*f^4*e + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*sqrt(f^8/
(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*e)*sqrt(-((b^3 + 12
*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)*sqrt(f^8
/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*e^(-2)/(a*b^6 - 1
2*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)
*x^4*e^5 + 4*(b^2*c - 4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*
c^2)*d^2)*x^2*e^3 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 +
 ((b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)*sqrt(-(
(b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)
*sqrt(f^8/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*e^(-2)/(
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2
)*f^6*x*e + (3*b^2*c + 4*a*c^2)*d*f^6 - 1/2*sqrt(1/2)*((b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2)*f^4*e + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)*
sqrt(f^8/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*e)*sqrt(-(
(b^3 + 12*a*b*c)*f^4 - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)
*sqrt(f^8/(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)))*e^(-2)/(
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))))/((b^2*c - 4*a*c^2)*x
^4*e^5 + 4*(b^2*c - 4*a*c^2)*d*x^3*e^4 + (b^3 - 4*a*b*c + 6*(b^2*c - 4*a*c^
2)*d^2)*x^2*e^3 + 2*(2*(b^2*c - 4*a*c^2)*d^3 + (b^3 - 4*a*b*c)*d)*x*e^2 + (
(b^2*c - 4*a*c^2)*d^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2)*e)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. $2(225) = 450$.
time = 3.76, size = 1378, normalized size = 5.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] 1/4*((2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*c*f^2*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*c*d*f^2*e + 2*c*d^2*f^2 - b*f^2)*log(d*e^(-1) + x + sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2
)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) +
sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*
d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
+ sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*f^2*e^2 - 4*(d*e^(-1) - sqrt(1/2)*s
qrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*f^2*e + 2*c*d^2*f^2 - b
*f^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(
-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^
(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (2*(d*
e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*f^2
*e^2 - 4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))*c*d*f^2*e + 2*c*d^2*f^2 - b*f^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b
*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d
*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 -
4*a*c)*e^2)*e^(-4)/c))) + (2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))^2*c*f^2*e^2 - 4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*c*d*f^2*e + 2*c*d^2*f^2 - b*f^2)*log(d
*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2
*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt
(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^2 - 4*a*c) - 1/
2*(2*c*f^2*x^3*e^3 + 6*c*d*f^2*x^2*e^2 + 6*c*d^2*f^2*x*e + 2*c*d^3*f^2 + b*
f^2*x*e + b*d*f^2)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*
x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2*e - 4*a*c*e))
```

Mupad [B]
time = 4.40, size = 2500, normalized size = 9.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] ((x*(b*f^2 + 6*c*d^2*f^2))/(2*(4*a*c - b^2)) + (2*c*d^3*f^2 + b*d*f^2)/(2*e
*(4*a*c - b^2)) + (c*e^2*f^2*x^3)/(4*a*c - b^2) + (3*c*d*e*f^2*x^2)/(4*a*c
- b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*b*d*e + 4*c*d
```

^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + atan(((((f^4*(-(4*a*c - b^2)^9)^(1/2))
/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*
f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e
^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(
1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4
 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 -
 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*
b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*b^7*
c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*b^3*
c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^
2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^
4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*
e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4
*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) + (2*b^7
*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a^3*b
*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*a^2*
c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c^3*e
^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i + (((f^4*(-(4*a*c - b^2)^9)^(1
/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*
c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c
^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2
))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4
*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e
^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*
a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8*b^9*c^2*d*e^13 - 128*a*
b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^5*c^4*d*e^13 - 2048*a^3*
b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^
7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14)
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^
9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b
^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280
*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2) - (2
*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a*b^5*c^3*e^12*f^2 - 128*a
^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (16*
a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^2*c^4*d*e^11*f^4)/(b^6 -
 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4*a*c^4*e^12*f^4 - 5*b^2*c
^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*1i)/((2*a*c^4*e^10*f^6 + (3*b
^2*c^3*e^10*f^6)/2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (((f
^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b
^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^
10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^
2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^
9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b

^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280 *a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*(((8 *b^9*c^2*d*e^13 - 128*a*b^7*c^3*d*e^13 + 2048*a^4*b*c^6*d*e^13 + 768*a^2*b^ 5*c^4*d*e^13 - 2048*a^3*b^3*c^5*d*e^13)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) + (x*(8*b^7*c^2*e^14 - 96*a*b^5*c^3*e^14 - 512*a^3*b*c^5*e^14 + 384*a^2*b^3*c^4*e^14))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b^10*c*e^2 + 24 0*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e^2 - 6144*a^6* b^2*c^5*e^2))^(1/2) + (2*b^7*c^2*e^12*f^2 + 96*a^2*b^3*c^4*e^12*f^2 - 24*a* b^5*c^3*e^12*f^2 - 128*a^3*b*c^5*e^12*f^2)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c ^2 - 12*a*b^4*c)) + (16*a^2*c^5*d*e^11*f^4 + 5*b^4*c^3*d*e^11*f^4 - 24*a*b^ 2*c^4*d*e^11*f^4)/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (x*(4* a*c^4*e^12*f^4 - 5*b^2*c^3*e^12*f^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2* b^5*c^2*f^4 - 16*a^3*b^3*c^3*f^4)/(a*b^12*e^2 + 4096*a^7*c^6*e^2 - 24*a^2*b ^10*c*e^2 + 240*a^3*b^8*c^2*e^2 - 1280*a^4*b^6*c^3*e^2 + 3840*a^5*b^4*c^4*e ^2 - 6144*a^6*b^2*c^5*e^2))^(1/2)*((((f^4*(-(4*a*c - b^2)^9)^(1/2))/32 - (b ^9*f^4)/32 + 24*a^4*b*c^4*f^4 + 3*a^2*b^5*c^2*f...

3.649 $\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=98

$$-\frac{f(b+2c(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}e}$$

```
[Out] -1/2*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+2*c*f*a
rctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/e
```

Rubi [A]
time = 0.09, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1156, 1121, 628, 632, 212}

$$\frac{2cf\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{3/2}}-\frac{f(b+2c(d+ex)^2)}{2e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
 e*x)^4)) + (2*c*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/((b^2
- 4*a*c)^(3/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{f\text{Subst}\left(\int \frac{x}{(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{f(b+2c(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{(cf)\text{Subst}\left(\int \frac{1}{a+bx+\ldots}\right.}{(b^2-\ldots}$$

$$= -\frac{f(b+2c(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{(2cf)\text{Subst}\left(\int \frac{\ldots}{b^2-4\ldots}\right.}{(\ldots}$$

$$= -\frac{f(b+2c(d+ex)^2)}{2\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{2cf\tanh^{-1}\left(\frac{b+2c(\ldots}{\sqrt{b^2-\ldots}}\right.}{\left(b^2-4ac\right)^{3/2\ldots}}$$

Mathematica [A]
time = 0.08, size = 99, normalized size = 1.01

$$-\frac{f\left(\frac{b+2c(d+ex)^2}{a+b(d+ex)^2+c(d+ex)^4}+\frac{4c\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}\right)}{2\left(b^2-4ac\right)e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x]
```

```
[Out] -1/2*(f*((b + 2*c*(d + e*x)^2)/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (4*c*A
rcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]))/((b^2
 - 4*a*c)*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.08, size = 272, normalized size = 2.78

method	result
default	$f\left(\frac{\frac{c\,x^2 e}{4ac-b^2}+\frac{2xcd}{4ac-b^2}+\frac{2c\,d^2+b}{2e\left(4ac-b^2\right)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a}+\frac{c\left(\sum_{\_R=\text{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4\right.\right.}\right.}{}\right.$
risch	$\frac{\frac{fce\,x^2}{4ac-b^2}+\frac{2cdfx}{4ac-b^2}+\frac{f\left(2c\,d^2+b\right)}{2e\left(4ac-b^2\right)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a}+\frac{fc\ln\left(\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}e^2+4abce^2-b^3e^2\right)x^2+\left(2\left(-4ac+b^2\right)^{\frac{3}{2}}de\right.\right.\right.}{\left(-4a\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f*((c/(4*a*c-b^2)*x^2*e+2/(4*a*c-b^2)*x*c*d+1/2/e*(2*c*d^2+b)/(4*a*c-b^2))/
(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*
e*x+b*d^2+a)+c/(4*a*c-b^2)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R
*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(
6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima")
```

```
[Out] -2*c*f*integrate((x*e + d)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*
c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(b^2 - 4*a*c) - 1
/2*(2*c*f*x^2*e^2 + 4*c*d*f*x*e + (2*c*d^2 + b)*f)/((b^2*c*e - 4*a*c^2*e)*d
^4 + 4*(b^2*c*e^4 - 4*a*c^2*e^4)*d*x^3 + (b^2*c*e^5 - 4*a*c^2*e^5)*x^4 + a*
b^2*e - 4*a^2*c*e + (b^3*e - 4*a*b*c*e)*d^2 + (b^3*e^3 - 4*a*b*c*e^3 + 6*(b
^2*c*e^3 - 4*a*c^2*e^3)*d^2)*x^2 + 2*(2*(b^2*c*e^2 - 4*a*c^2*e^2)*d^3 + (b^
3*e^2 - 4*a*b*c*e^2)*d)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(94) = 188.
time = 0.39, size = 1052, normalized size = 10.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*f*x^2*e^2 + 4*(b^2*c - 4*a*c^2)*d*f*x*e + 2*(c^2
*f*x^4*e^4 + 4*c^2*d*f*x^3*e^3 + (6*c^2*d^2 + b*c)*f*x^2*e^2 + 2*(2*c^2*d^3
 + b*c*d)*f*x*e + (c^2*d^4 + b*c*d^2 + a*c)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2
*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^
2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2 - 2*a*c - (2*c*x^2*e^2 + 4*c*d*x*e
+ 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c
*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a)) + (b^3 - 4*a*b*c +
2*(b^2*c - 4*a*c^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*e^5 + 4
*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^3*e^4 + (b^5 - 8*a*b^3*c + 16*a^2*b
*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*x^2*e^3 + 2*(2*(b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*e^2 + (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 +
 (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2)*e), -1/2*(2*(b^2*c - 4*a*c^2)*f*x^2*
e^2 + 4*(b^2*c - 4*a*c^2)*d*f*x*e - 4*(c^2*f*x^4*e^4 + 4*c^2*d*f*x^3*e^3 +
(6*c^2*d^2 + b*c)*f*x^2*e^2 + 2*(2*c^2*d^3 + b*c*d)*f*x*e + (c^2*d^4 + b*c*
d^2 + a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2
 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c
^2)*d^2)*f)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*e^5 + 4*(b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*d*x^3*e^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c
- 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*x^2*e^3 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*e^2 + (a*b^4 - 8*a^2*b^
2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*
c + 16*a^2*b*c^2)*d^2)*e)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(87) = 174$.
time = 2.72, size = 525, normalized size = 5.36

$$-\frac{cf\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{-16a^2c^3f\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^2c^2f\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^4cf\sqrt{-\frac{1}{(4ac-b^2)^3}}+bcf+2c^2d^2f}{2c^2e^2f}\right)}{e}+\frac{cf\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(\frac{2dx}{e}+x^2+\frac{16a^2c^3f\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^2c^2f\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^4cf\sqrt{-\frac{1}{(4ac-b^2)^3}}+bcf+2c^2d^2f}{2c^2e^2f}\right)}{e}+\frac{bf+2cd^2f+4cdefx+2ce^2fx^2}{8a^2ce-2ab^2e+8abcd^2e+8ac^2d^4e-2b^3d^2e-2b^2cd^4e+x^4\cdot(8ac^2e^5-2b^2ce^5)+x^3\cdot(32ac^2de^4-8b^2cde^4)+x^2\cdot(8abce^3+48ac^2d^2e^3-2b^3e^3-12b^2cd^2e^3)+x\,(16abcde^2+32ac^2d^3e^2-4b^3de^2-8b^2cd^3e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] -c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (-16*a**2*c**3*f*sqrt(
-1/(4*a*c - b**2)**3) + 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) - b**4*c
*f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e +
 c*f*sqrt(-1/(4*a*c - b**2)**3)*log(2*d*x/e + x**2 + (16*a**2*c**3*f*sqrt(-
1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*f*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*
f*sqrt(-1/(4*a*c - b**2)**3) + b*c*f + 2*c**2*d**2*f)/(2*c**2*e**2*f))/e +
(b*f + 2*c*d**2*f + 4*c*d*e*f*x + 2*c*e**2*f*x**2)/(8*a**2*c*e - 2*a*b**2*e
 + 8*a*b*c*d**2*e + 8*a*c**2*d**4*e - 2*b**3*d**2*e - 2*b**2*c*d**4*e + x**
4*(8*a*c**2*e**5 - 2*b**2*c*e**5) + x**3*(32*a*c**2*d*e**4 - 8*b**2*c*d*e**
```

```
4) + x**2*(8*a*b*c*e**3 + 48*a*c**2*d**2*e**3 - 2*b**3*e**3 - 12*b**2*c*d**
2*e**3) + x*(16*a*b*c*d*e**2 + 32*a*c**2*d**3*e**2 - 4*b**3*d*e**2 - 8*b**2
*c*d**3*e**2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(94) = 188.
time = 3.78, size = 211, normalized size = 2.15

$$-\frac{2\,cf\arctan\left(\frac{2\,cd^2f+2\,(fx^2e+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,cd^2f^3+2\,(fx^2e+2\,dfx)cf^2e+bf^3}{2\,(cd^4f^2+2\,(fx^2e+2\,dfx)cd^2fe+bd^2f^2+(fx^2e+2\,dfx)^2ce^2+(fx^2e+2\,dfx)bfe+af^2)(b^2e-4\,ace)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -2*c*f*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4*
a*c)*f))*e^(-1)/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c*d^2*f^3 + 2*(
f*x^2*e + 2*d*f*x)*c*f^2*e + b*f^3)/((c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d
^2*f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*
e + a*f^2)*(b^2*e - 4*a*c*e))
```

Mupad [B]
time = 1.91, size = 442, normalized size = 4.51

$$\frac{\frac{f\,(2\,c\,d^2+b)}{2\,e\,(4\,a\,c-b^2)}+\frac{2\,c\,d\,f\,x}{4\,a\,c-b^2}+\frac{c\,e\,f\,x^2}{4\,a\,c-b^2}}{a+x^2\,(6\,c\,d^2\,e^2+b\,e^2)+b\,d^2+c\,d^4+x\,(4\,c\,e\,d^3+2\,b\,e\,d)+c\,e^4\,x^4+4\,c\,d\,e^3\,x^3}+\frac{2\,c\,f\,\mathrm{atan}\left(\frac{(4\,a\,c-b^2)^4\left(x\left(\frac{8\,c^4\,d\,e^7\,f^2}{a\,(4\,a\,c-b^2)^{7/2}}-\frac{8\,b\,c^2\,f^2\,(b^3\,c^2\,d\,e^9-4\,a\,b\,c^3\,d\,e^9)}{a\,e^2\,(4\,a\,c-b^2)^{11/2}}\right)+x^2\left(\frac{4\,c^4\,e^8\,f^2}{a\,(4\,a\,c-b^2)^{7/2}}-\frac{4\,b\,c^2\,f^2\,(b^3\,c^2\,e^{10}-4\,a\,b\,c^3\,e^{10})}{a\,e^2\,(4\,a\,c-b^2)^{11/2}}\right)+\frac{4\,c^4\,d^2\,e^6\,f^2}{a\,(4\,a\,c-b^2)^{7/2}}+\frac{4\,b\,c^2\,f^2\,(8\,a^2\,c^3\,e^8-2\,a\,b^2\,c^2\,e^8+4\,a\,b\,c^3\,d^2\,e^8-b^3\,c^2\,d^2\,e^8)}{a\,e^2\,(4\,a\,c-b^2)^{11/2}}\right)}{8\,c^4\,e^6\,f^2}\right)}{e\,(4\,a\,c-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2,x)
```

```
[Out] ((f*(b + 2*c*d^2))/(2*e*(4*a*c - b^2)) + (2*c*d*f*x)/(4*a*c - b^2) + (c*e*f
*x^2)/(4*a*c - b^2))/(a + x^2*(b*e^2 + 6*c*d^2*e^2) + b*d^2 + c*d^4 + x*(2*
b*d*e + 4*c*d^3*e) + c*e^4*x^4 + 4*c*d*e^3*x^3) + (2*c*f*atan(((4*a*c - b^2
)^4*(x*((8*c^4*d*e^7*f^2)/(a*(4*a*c - b^2)^(7/2)) - (8*b*c^2*f^2*(b^3*c^2*d
*e^9 - 4*a*b*c^3*d*e^9))/(a*e^2*(4*a*c - b^2)^(11/2))) + x^2*((4*c^4*e^8*f^
2)/(a*(4*a*c - b^2)^(7/2)) - (4*b*c^2*f^2*(b^3*c^2*e^10 - 4*a*b*c^3*e^10))/
(a*e^2*(4*a*c - b^2)^(11/2))) + (4*c^4*d^2*e^6*f^2)/(a*(4*a*c - b^2)^(7/2))
 + (4*b*c^2*f^2*(8*a^2*c^3*e^8 - 2*a*b^2*c^2*e^8 - b^3*c^2*d^2*e^8 + 4*a*b*
c^3*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(11/2))))/(8*c^4*e^6*f^2)))/(e*(4*a*c -
b^2)^(3/2))
```

3.650 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=174

$$\frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}ef}+\frac{\log(d+ex)}{a^2ef}-\frac{\log\left(a+\right.}{}$$

```
[Out] 1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^4
)+1/2*b*(-6*a*c+b^2)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^2/(-4*
a*c+b^2)^(3/2)/e/f+ln(e*x+d)/a^2/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^2/
e/f
```

Rubi [A]
time = 0.21, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {1156, 1128, 754, 814, 648, 632, 212, 642}

$$\frac{b\left(b^2-6ac\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^2ef\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^2ef}+\frac{\log(d+ex)}{a^2ef}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2
- 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)*e*f) + Log[d + e*x]/(a^2*e*f) - Log[a
 + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^2*e*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{ef}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^2}\,dx, x, (d+ex)^2\right)}{2ef}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\big(}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} - \frac{\text{Subst}\big(}{}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d\,-}{a^2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d\,-}{a^2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{\log(d\,-}{a^2e}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef\left(a+b(d+ex)^2+c(d+ex)^4\right)} + \frac{b(b^2\,-}{}$$

Mathematica [A]
time = 0.30, size = 238, normalized size = 1.37

$$\frac{\frac{2a(b^2-2ac+bc(d+ex)^2)}{(b^2-4ac)(a+(d+ex)^2(b+c(d+ex)^2))}+4\log(d+ex)-\frac{\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(b^3-6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}}{4a^2ef}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (d + e*x)^2*(b +
 c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c
] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/
(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^2*e*f)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.29, size = 403, normalized size = 2.32

method	result
default	$\frac{\frac{\ln(ex+d)}{e\,a^2}-\frac{\frac{abce\,x^2}{8ac-2b^2}+\frac{bcdax}{4ac-b^2}-\frac{a\left(-bc\,d^2+2ac-b^2\right)}{2e\left(4ac-b^2\right)}}{c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e^4c\_Z^4+4d\,e^3c\_Z^3+\left(6d^2e^2c+e^2b\right)\_Z^2+\left(4d^3ec+\right.\right.}}{}}{}$
risch	$\frac{-\frac{c\,x^2be}{2a\left(4ac-b^2\right)}-\frac{xbcd}{\left(4ac-b^2\right)a}+\frac{-bc\,d^2+2ac-b^2}{2ea\left(4ac-b^2\right)}}{f(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a)}+\frac{\ln(ex+d)}{a^2ef}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3e^2f^2-48a^4b^2c^2e^2f^2+12a\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(ln(e*x+d)/e/a^2-1/a^2*((1/2*a/(4*a*c-b^2)*b*c*e*x^2+b*c*d*a/(4*a*c-b^2
)*x-1/2*a/e*(-b*c*d^2+2*a*c-b^2)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/2/(4*a*c-b^2)/
e*sum((e^3*c*(4*a*c-b^2)*_R^3+3*d*e^2*c*(4*a*c-b^2)*_R^2+e*(12*a*c^2*d^2-3*
b^2*c*d^2+5*a*b*c-b^3)*_R+4*a*c^2*d^3-b^2*c*d^3+5*a*b*c*d-b^3*d)/(2*_R^3*c*
e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*
c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c
+d^2*b+a))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*(b*c*x^2*e^2 + 2*b*c*d*x*e + b*c*d^2 + b^2 - 2*a*c)/(4*(a*b^2*c*e^4 - 4
*a^2*c^2*e^4)*d*f*x^3 + (a*b^2*c*e^5 - 4*a^2*c^2*e^5)*f*x^4 + (a*b^3*e^3 -
4*a^2*b*c*e^3 + 6*(a*b^2*c*e^3 - 4*a^2*c^2*e^3)*d^2)*f*x^2 + 2*(2*(a*b^2*c*
e^2 - 4*a^2*c^2*e^2)*d^3 + (a*b^3*e^2 - 4*a^2*b*c*e^2)*d)*f*x + ((a*b^2*c*e
 - 4*a^2*c^2*e)*d^4 + a^2*b^2*e - 4*a^3*c*e + (a*b^3*e - 4*a^2*b*c*e)*d^2)*
f) + e^(-1)*log(x*e + d)/(a^2*f) - integrate(((b^2*c - 4*a*c^2)*d^3 + 3*(b^
2*c*e^2 - 4*a*c^2*e^2)*d*x^2 + (b^2*c*e^3 - 4*a*c^2*e^3)*x^3 + (b^3 - 5*a*b
*c)*d + (b^3*e - 5*a*b*c*e + 3*(b^2*c*e - 4*a*c^2*e)*d^2)*x)/((b^2*c - 4*a*
c^2)*d^4 + 4*(b^2*c*e^3 - 4*a*c^2*e^3)*d*x^3 + (b^2*c*e^4 - 4*a*c^2*e^4)*x^
4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + (b^3*e^2 - 4*a*b*c*e^2 + 6*(b^2
*c*e^2 - 4*a*c^2*e^2)*d^2)*x^2 + 2*(2*(b^2*c*e - 4*a*c^2*e)*d^3 + (b^3*e -
4*a*b*c*e)*d)*x), x)/(a^2*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1168 vs. $2(167) = 334$.
time = 0.60, size = 2462, normalized size = 14.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="fricas"
)
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2*e
^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*x*e + 2*(a*b^3*c - 4*a^2*b*c^2)*d^2 + ((b^
3*c - 6*a*b*c^2)*x^4*e^4 + 4*(b^3*c - 6*a*b*c^2)*d*x^3*e^3 + (b^3*c - 6*a*b
*c^2)*d^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c + 6*(b^3*c - 6*a*b*c^2)*d^
2)*x^2*e^2 + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)*d^3 + (b^4 -
6*a*b^2*c)*d)*x*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 +
 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d
)*x*e + b^2 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 +
2*(2*c*d^3 + b*d)*x*e + a)) - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*e^4 +
 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^3*e^3 + a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*x^2*e^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^
5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*e)*log(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4
 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a) + 4*((b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*e^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d
*x^3*e^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*d^2)*x^2*e^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c -
 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*e)*l
og(x*e + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f*x^4*e^5 + 4*(a^2*b
^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*f*x^3*e^4 + (a^2*b^5 - 8*a^3*b^3*c + 1
6*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^2)*f*x^2*e^3 + 2
*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c +
 16*a^4*b*c^2)*d)*f*x*e^2 + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*
c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2
)*d^2)*f*e), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*
b*c^2)*x^2*e^2 + 4*(a*b^3*c - 4*a^2*b*c^2)*d*x*e + 2*(a*b^3*c - 4*a^2*b*c^2
)*d^2 + 2*((b^3*c - 6*a*b*c^2)*x^4*e^4 + 4*(b^3*c - 6*a*b*c^2)*d*x^3*e^3 +
(b^3*c - 6*a*b*c^2)*d^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c + 6*(b^3*c -
 6*a*b*c^2)*d^2)*x^2*e^2 + (b^4 - 6*a*b^2*c)*d^2 + 2*(2*(b^3*c - 6*a*b*c^2)
*d^3 + (b^4 - 6*a*b^2*c)*d)*x*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 +
4*c*d*x*e + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*x^4*e^4 + 4*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^3*
```

```
e^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)
*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*d^2)*x^2*e^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d)*x*e)*log(c*
x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^
3 + b*d)*x*e + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4*e^4 + 4*(b^4*
c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^3*e^3 + a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2
+ 6*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^2)*x^2*e^2 + (b^5 - 8*a*b^3*c + 16
*a^2*b*c^2)*d^2 + 2*(2*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d^3 + (b^5 - 8*a*
b^3*c + 16*a^2*b*c^2)*d)*x*e)*log(x*e + d))/((a^2*b^4*c - 8*a^3*b^2*c^2 + 1
6*a^4*c^3)*f*x^4*e^5 + 4*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d*f*x^3*e
^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 6*(a^2*b^4*c - 8*a^3*b^2*c^2 +
 16*a^4*c^3)*d^2)*f*x^2*e^3 + 2*(2*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)
*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*f*x*e^2 + (a^3*b^4 - 8*a^4
*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*d^4 + (a^2*b
^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*f*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(167) = 334.
time = 3.98, size = 476, normalized size = 2.74

$$-\frac{(a^2b^3cfe^3-6a^3bc^2fe^3)\sqrt{b^2-4ac}\log\left(\left|bx^2e^2+2bdxe+\sqrt{b^2-4ac}\,x^2e^2+2\sqrt{b^2-4ac}\,dxe+bd^2+\sqrt{b^2-4ac}\,d^2+2a\right|\right)-(a^2b^3cfe^3-6a^3bc^2fe^3)\sqrt{b^2-4ac}\log\left(\left|-bx^2e^2-2bdxe+\sqrt{b^2-4ac}\,x^2e^2+2\sqrt{b^2-4ac}\,dxe-bd^2+\sqrt{b^2-4ac}\,d^2-2a\right|\right)}{4(a^4b^4cf^2e^4-8a^5b^2c^2f^2e^4+16a^6c^3f^2e^4)}-\frac{e^{(-1)}\log\left(|cx^4e^4+4cdx^3e^3+6cd^2x^2e^2+4cd^3xe+cd^4+bx^2e^2+2bdxe+bd^2+a|\right)}{4a^2f}+\frac{e^{(-1)}\log\left(|xe+d|\right)}{a^2f}+\frac{(abcx^2e^2+2abcdxe+abcd^2+ab^2-2a^2c)e^{(-1)}}{2(cx^4e^4+4cdx^3e^3+6cd^2x^2e^2+4cd^3xe+cd^4+bx^2e^2+2bdxe+bd^2+a)(b^2-4ac)a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*((a^2*b^3*c*f*e^3 - 6*a^3*b*c^2*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2
*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e +
b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^2*b^3*c*f*e^3 - 6*a^3*b*c^2*f*e^
3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2
*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(
a^4*b^4*c*f^2*e^4 - 8*a^5*b^2*c^2*f^2*e^4 + 16*a^6*c^3*f^2*e^4) - 1/4*e^(-1
)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4
 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^2*f) + e^(-1)*log(abs(x*e + d))/(
a^2*f) + 1/2*(a*b*c*x^2*e^2 + 2*a*b*c*d*x*e + a*b*c*d^2 + a*b^2 - 2*a^2*c)*
```

e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2 - 4*a*c)*a^2*f)

Mupad [B]
time = 11.69, size = 2500, normalized size = 14.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)

[Out] ((b^2 - 2*a*c + b*c*d^2)/(2*e*(a*b^2 - 4*a^2*c)) + (b*c*e*x^2)/(2*(a*b^2 - 4*a^2*c)) + (b*c*d*x)/(a*b^2 - 4*a^2*c))/(a*f + x^2*(b*e^2*f + 6*c*d^2*e^2*f) + x*(4*c*d^3*e*f + 2*b*d*e*f) + b*d^2*f + c*d^4*f + c*e^4*f*x^4 + 4*c*d*e^3*f*x^3) - (log((((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) - 1)*(((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) - 1)*((2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*f*(4*a*c - b^2)) + (b*c^2*e^16*(a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2*f) - (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*f*(4*a*c - b^2)) - (4*b*c^3*d*e^17*x*(10*a*c - b^2))/(a*f*(4*a*c - b^2))))/(4*a^2*e*f) - (b*c^3*e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*f^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2)))/(4*a^2*e*f) + (b^3*c^5*e^16*x^2)/(a^3*f^3*(4*a*c - b^2)^3) + (b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*f^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*f^3*(4*a*c - b^2)^3))*((b^3*c^5*e^16*x^2)/(a^3*f^3*(4*a*c - b^2)^3) - ((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*(((a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*((b*c^2*e^16*(a^2*e*f*(-(b^2*(6*a*c - b^2)^2)/(a^4*e^2*f^2*(4*a*c - b^2)^3))^(1/2) + 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^2*f) - (2*b*c^2*e^16*(2*b^3 - 10*a*c^2*d^2 + b^2*c*d^2 - 10*a*b*c))/(a*f*(4*a*c - b^2)) + (2*b*c^3*e^18*x^2*(10*a*c - b^2))/(a*f*(4*a*c - b^2)) + (4*b*c^3*d*e^17*x*(10*a*c - b^2))/(a*f*(4*a*c - b^2))))/(4*a^2*e*f) - (b*c^3*e^15*(4*b^3 - 20*a*c^2*d^2 + 6*b^2*c*d^2 - 17*a*b*c))/(a^2*f^2*(4*a*c - b^2)^2) + (2*b*c^4*e^17*x^2*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2) + (4*b*c^4*d*e^16*x*(10*a*c - 3*b^2))/(a^2*f^2*(4*a*c - b^2)^2)))/(4*a^2*e*f) + (b^2*c^4*e^14*(b^2 - 4*a*c + b*c*d^2))/(a^3*f^3*(4*a*c - b^2)^3) + (2*b^3*c^5*d*e^15*x)/(a^3*f^3*(4*a*c - b^2)^3)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) + log(d + e*x)/(a^2*e*f) + (b*atan((x^2*((((b*(6*a*c - b^2)*((6*a*b^5*c^4*e^17*f + 80*a^3*b*c^6*e^17*f - 44*a^2*b^3*c^5*e^17*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) - (((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^

18*f^2 + 192*a^4*b^3*c^5*e^18*f^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2))))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) - (((b*((2*a^2*b^7*c^3*e^18*f^2 - 36*a^3*b^5*c^4*e^18*f^2 + 192*a^4*b^3*c^5*e^18*f^2 - 320*a^5*b*c^6*e^18*f^2)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3) + ((2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(2*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(6*a*c - b^2))/(4*a^2*e*f*(4*a*c - b^2)^(3/2)) + (b*(6*a*c - b^2)*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f)*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(8*a^2*e*f*(4*a*c - b^2)^(3/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)))*(2*b^6*e*f - 128*a^3*c^3*e*f + 96*a^2*b^2*c^2*e*f - 24*a*b^4*c*e*f))/(2*(4*a^2*b^6*e^2*f^2 - 256*a^5*c^3*e^2*f^2 + 192*a^4*b^2*c^2*e^2*f^2 - 48*a^3*b^4*c*e^2*f^2)) + (b^3*(6*a*c - b^2)^3*(12*a^3*b^9*c^2*e^19*f^3 - 184*a^4*b^7*c^3*e^19*f^3 + 1056*a^5*b^5*c^4*e^19*f^3 - 2688*a^6*b^3*c^5*e^19*f^3 + 2560*a^7*b*c^6*e^19*f^3))/(64*a^6*e^3*f^3*(4*a*c - b^2)^(9/2)*(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f^3 + 48*a^5*b^2*c^2*f^3)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*((((6*a*b^5*c^4*e^17*f + 80*a^3*b*c^6*e^17*f - 44*a^2*b^3*c^5*e^17*f)/(a^3*b^6*f^3 - 64*a^6*c^3*f^3 - 12*a^4*b^4*c*f...

3.651 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=360

$$-\frac{3b^2-10ac}{2a^2\left(b^2-4ac\right)ef^2(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\sqrt{c}\left(3b^3-16abc+\right.}{2\sqrt{}}$$

```
[Out] 1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/e/f^2/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d
)^2)/a/(-4*a*c+b^2)/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-1/4*arctan((e
*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c
+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)/e/f^2*2^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2
))/a^2/(-4*a*c+b^2)^(3/2)/e/f^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.13, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1156, 1135, 1295, 1180, 211}

$$-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2ef^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a^2ef^2\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{3b^2-10ac}{2a^2ef^2\left(b^2-4ac\right)(d+ex)}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^2\left(b^2-4ac\right)(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -1/2*(3*b^2 - 10*a*c)/(a^2*(b^2 - 4*a*c)*e*f^2*(d + e*x)) + (b^2 - 2*a*c +
b*c*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a + b*(d + e*x)^2 + c*
(d + e*x)^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) + (Sqrt[c
]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
```

```
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{ef^2}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\,(b^2-4ac)\,ef^2(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)}$$

$$= -\frac{3b^2-10ac}{2a^2\,(b^2-4ac)\,ef^2(d+ex)} + \frac{b^2-2a}{2a\,(b^2-4ac)\,ef^2(d+e}$$

$$= -\frac{3b^2-10ac}{2a^2\,(b^2-4ac)\,ef^2(d+ex)} + \frac{b^2-2a}{2a\,(b^2-4ac)\,ef^2(d+e}$$

$$= -\frac{3b^2-10ac}{2a^2\,(b^2-4ac)\,ef^2(d+ex)} + \frac{b^2-2a}{2a\,(b^2-4ac)\,ef^2(d+e}$$

Mathematica [A]
time = 1.03, size = 342, normalized size = 0.95

$$\frac{-\frac{4}{d+ex}+\frac{2(d+ex)(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a^2ef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-4/(d + e*x) + (2*(d + e*x)*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(
d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (Sqrt[2
]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*
Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2*e*f^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.21, size = 445, normalized size = 1.24

method	result

default	$-\frac{1}{a^2 e(ex+d)}-\frac{\frac{c e^2\left(2ac-b^2\right)x^3}{8ac-2b^2}+\frac{3dce\left(2ac-b^2\right)x^2}{2\left(4ac-b^2\right)}+\frac{\left(6a c^2 d^2-3b^2 c d^2+3abc-b^3\right)x}{8ac-2b^2}+\frac{d\left(2a c^2 d^2-b^2 c d^2+3abc-b^3\right)}{2e\left(4ac-b^2\right)}}{c e^4 x^4+4cd e^3 x^3+6c d^2 e^2 x^2+4c d^3 ex+b e^2 x^2+d^4 c+2debx+d^2 b+a}+\frac{\_R=\mathrm{RootOf}\left(e^4 c\_Z^4+4d e \cdots\right.}{}$
risch	$\frac{-\frac{e^3 c\left(10ac-3b^2\right)x^4}{2a^2\left(4ac-b^2\right)}-\frac{2d e^2 c\left(10ac-3b^2\right)x^3}{a^2\left(4ac-b^2\right)}-\frac{\left(60a c^2 d^2-18b^2 c d^2+11abc-3b^3\right)e x^2}{2a^2\left(4ac-b^2\right)}-\frac{d\left(20a c^2 d^2-6b^2 c d^2+11abc-3b^3\right)x}{a^2\left(4ac-b^2\right)}-\frac{10a c^2 d^4-3b^2 c d^4+11a \cdots}{2e a^2 \left(\cdots\right.}}{f^2(ex+d)(c e^4 x^4+4cd e^3 x^3+6c d^2 e^2 x^2+4c d^3 ex+b e^2 x^2+d^4 c+2debx+d^2 b+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^2*(-1/a^2/e/(e*x+d)-1/a^2*((1/2*c*e^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3+3/2*d
*c*e*(2*a*c-b^2)/(4*a*c-b^2)*x^2+1/2*(6*a*c^2*d^2-3*b^2*c*d^2+3*a*b*c-b^3)/
(4*a*c-b^2)*x+1/2*d/e*(2*a*c^2*d^2-b^2*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)+1/4/(4*a*c-b^2)/e*sum((c*e^2*(10*a*c-3*b^2)*_R^2+2*c*d*e*(10*a*c-3
*b^2)*_R+10*a*c^2*d^2-3*b^2*c*d^2+13*a*b*c-3*b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*
e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*
c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*d^4 + 4*(3*b^2*c*e^3 - 10*a*c^2*e^3)*d*x^3 + (3*
b^2*c*e^4 - 10*a*c^2*e^4)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*d^2
+ (3*b^3*e^2 - 11*a*b*c*e^2 + 6*(3*b^2*c*e^2 - 10*a*c^2*e^2)*d^2)*x^2 + 2*(
2*(3*b^2*c*e - 10*a*c^2*e)*d^3 + (3*b^3*e - 11*a*b*c*e)*d)*x)/(5*(a^2*b^2*c
*e^5 - 4*a^3*c^2*e^5)*d*f^2*x^4 + (a^2*b^2*c*e^6 - 4*a^3*c^2*e^6)*f^2*x^5 +
 (a^2*b^3*e^4 - 4*a^3*b*c*e^4 + 10*(a^2*b^2*c*e^4 - 4*a^3*c^2*e^4)*d^2)*f^2
*x^3 + (10*(a^2*b^2*c*e^3 - 4*a^3*c^2*e^3)*d^3 + 3*(a^2*b^3*e^3 - 4*a^3*b*c
*e^3)*d)*f^2*x^2 + (a^3*b^2*e^2 - 4*a^4*c*e^2 + 5*(a^2*b^2*c*e^2 - 4*a^3*c^
2*e^2)*d^4 + 3*(a^2*b^3*e^2 - 4*a^3*b*c*e^2)*d^2)*f^2*x + ((a^2*b^2*c*e - 4
*a^3*c^2*e)*d^5 + (a^2*b^3*e - 4*a^3*b*c*e)*d^3 + (a^3*b^2*e - 4*a^4*c*e)*d
)*f^2) - 1/2*integrate((3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*d^2 + 2*(3*
b^2*c*e - 10*a*c^2*e)*d*x + (3*b^2*c*e^2 - 10*a*c^2*e^2)*x^2)/((b^2*c - 4*a
*c^2)*d^4 + 4*(b^2*c*e^3 - 4*a*c^2*e^3)*d*x^3 + (b^2*c*e^4 - 4*a*c^2*e^4)*x
^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + (b^3*e^2 - 4*a*b*c*e^2 + 6*(b^
```

```
2*c*e^2 - 4*a*c^2*e^2)*d^2)*x^2 + 2*(2*(b^2*c*e - 4*a*c^2*e)*d^3 + (b^3*e -
 4*a*b*c*e)*d)*x), x)/(a^2*f^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4442 vs. $2(315) = 630$.
time = 0.54, size = 4442, normalized size = 12.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*x^4*e^4 + 8*(3*b^2*c - 10*a*c^2)*d*x^3*e^3 + 2
*(3*b^2*c - 10*a*c^2)*d^4 + 2*(3*b^3 - 11*a*b*c + 6*(3*b^2*c - 10*a*c^2)*d^
2)*x^2*e^2 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c)*d^2 + 4*(2*(3*b^2*c
- 10*a*c^2)*d^3 + (3*b^3 - 11*a*b*c)*d)*x*e + sqrt(1/2)*((a^2*b^2*c - 4*a^3
*c^2)*f^2*x^5*e^6 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*f^2*x^4*e^5 + (a^2*b^3 - 4*
a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*f^2*x^3*e^4 + (10*(a^2*b^2*c - 4*
a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*f^2*x^2*e^3 + (a^3*b^2 - 4*a^4*c
+ 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*f^2*x*e^2 +
((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4
*c)*d)*f^2*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3
+ (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*f^4*sqrt((81*b^8 -
 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^
6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*f^8)))*e^(-2)/((a^5*b^6
- 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*f^4))*log(-(189*b^6*c^3 - 197
1*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x*e - (189*b^6*c^3 - 1971*a*
b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3*a^5*b^10 -
 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 128
0*a^10*c^5)*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^
2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13
*c^3)*f^8))*e - (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c
^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*f^2*e)*sqrt(-(9*b^7 - 105*a*b^5*c
+ 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^
2 - 64*a^8*c^3)*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*
a^13*c^3)*f^8)))*e^(-2)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*
c^3)*f^4))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*f^2*x^5*e^6 + 5*(a^2*b^2*c
 - 4*a^3*c^2)*d*f^2*x^4*e^5 + (a^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*
c^2)*d^2)*f^2*x^3*e^4 + (10*(a^2*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^
3*b*c)*d)*f^2*x^2*e^3 + (a^3*b^2 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4
+ 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*f^2*x*e^2 + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (
a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*b^2 - 4*a^4*c)*d)*f^2*e)*sqrt(-(9*b^7 - 105
*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a
```

```
^7*b^2*c^2 - 64*a^8*c^3)*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2
- 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*
c^2 - 64*a^13*c^3)*f^8)))*e^(-2)/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3)*f^4))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 -
 2500*a^3*c^6)*x*e - (189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 250
0*a^3*c^6)*d - 1/2*sqrt(1/2)*((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2
- 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*f^6*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6
- 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*f^8))*e - (27*b^11 - 486*a
*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^
5*b*c^5)*f^2*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^
3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*f^4*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*
b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*f^8)))*e^(-2)/((a^5*b^
6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*f^4))) - sqrt(1/2)*((a^2*b^
2*c - 4*a^3*c^2)*f^2*x^5*e^6 + 5*(a^2*b^2*c - 4*a^3*c^2)*d*f^2*x^4*e^5 + (a
^2*b^3 - 4*a^3*b*c + 10*(a^2*b^2*c - 4*a^3*c^2)*d^2)*f^2*x^3*e^4 + (10*(a^2
*b^2*c - 4*a^3*c^2)*d^3 + 3*(a^2*b^3 - 4*a^3*b*c)*d)*f^2*x^2*e^3 + (a^3*b^2
 - 4*a^4*c + 5*(a^2*b^2*c - 4*a^3*c^2)*d^4 + 3*(a^2*b^3 - 4*a^3*b*c)*d^2)*f
^2*x*e^2 + ((a^2*b^2*c - 4*a^3*c^2)*d^5 + (a^2*b^3 - 4*a^3*b*c)*d^3 + (a^3*
b^2 - 4*a^4*c)*d)*f^2*e)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420
*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*f^4*sqr
t((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4
)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)*f^8)))*e^(-2)
/((a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*f^4))*log(-(189*b^
6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x*e - (189*b^6*c^
3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d + 1/2*sqrt(1/2)*((3
*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^
2*c^4 - 1280*a^10*c^5)*f^6*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 -
2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^
2 - 64*a^13*c^3)*f^8))*e + (27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 1054
9*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*f^2*e)*sqrt(-(9*b^7 - 1
05*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48
*a^7*b^2*c^2 - 64*a^8*c^3)*f^4*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^
2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/((a^10*b^6 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 999 vs. 2(315) = 630.
time = 4.02, size = 999, normalized size = 2.78

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] -1/2*(b^2*c*e^(-1)/((f*x*e + d*f)*f) - 2*a*c^2*e^(-1)/((f*x*e + d*f)*f) + b
^3*f*e^(-1)/(f*x*e + d*f)^3 - 3*a*b*c*f*e^(-1)/(f*x*e + d*f)^3)/((a^2*b^2 -
 4*a^3*c)*(c + b*f^2/(f*x*e + d*f)^2 + a*f^4/(f*x*e + d*f)^4)) - e^(-1)/((f
*x*e + d*f)*a^2*f) + 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 80*
a^7*b*c^3)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*f^8*e^4 + 2*(3*a^3*b^2*c - 1
0*a^4*c^2)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*f^4*abs(a^
2*b^2*f^4*e^2 - 4*a^3*c*f^4*e^2)*e^2 - (a^2*b^2*f^4*e^2 - 4*a^3*c*f^4*e^2)^
2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2
)*e^(-1)/((f*x*e + d*f)*f*sqrt((a^2*b^3*f^4*e^2 - 4*a^3*b*c*f^4*e^2 + sqrt(
(a^2*b^3*f^4*e^2 - 4*a^3*b*c*f^4*e^2)^2 - 4*(a^3*b^2*f^8*e^4 - 4*a^4*c*f^8*
e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*f^8*e^4 - 4*a^4*c*f^8*e^4))))*e^(-3
)/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*f^6*abs(a^2*b^2*f^4*e^2 - 4*a^
3*c*f^4*e^2)*abs(a)) - 1/16*((3*a^4*b^7 - 31*a^5*b^5*c + 96*a^6*b^3*c^2 - 8
0*a^7*b*c^3)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*f^8*e^4 - 2*(3*a^3*b^2*c -
 10*a^4*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*f^4*abs(
a^2*b^2*f^4*e^2 - 4*a^3*c*f^4*e^2)*e^2 - (a^2*b^2*f^4*e^2 - 4*a^3*c*f^4*e^2
)^2*(3*b^3 - 13*a*b*c)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1
/2)*e^(-1)/((f*x*e + d*f)*f*sqrt((a^2*b^3*f^4*e^2 - 4*a^3*b*c*f^4*e^2 - sqr
t((a^2*b^3*f^4*e^2 - 4*a^3*b*c*f^4*e^2)^2 - 4*(a^3*b^2*f^8*e^4 - 4*a^4*c*f^
8*e^4)*(a^2*b^2*c - 4*a^3*c^2)))/(a^3*b^2*f^8*e^4 - 4*a^4*c*f^8*e^4))))*e^(
-3)/((a^5*b^2*c - 4*a^6*c^2)*sqrt(b^2 - 4*a*c)*f^6*abs(a^2*b^2*f^4*e^2 - 4*
a^3*c*f^4*e^2)*abs(a))
```

Mupad [B]
time = 7.29, size = 2500, normalized size = 6.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] - atan(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*
```

c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 1048576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10 - 1572864*a^15*b^3*c^7*d*e^13*f^10) - 192*a^8*b^13*c^2*e^12*f^8 + 4672*a^9*b^11*c^3*e^12*f^8 - 47360*a^10*b^9*c^4*e^12*f^8 + 256000*a^11*b^7*c^5*e^12*f^8 - 778240*a^12*b^5*c^6*e^12*f^8 + 1261568*a^13*b^3*c^7*e^12*f^8 - 851968*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^12*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^12*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6)*1i + (-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11*c^6*e^2*f^4 + 240*a^7*b^8*c^2*e^2*f^4 - 1280*a^8*b^6*c^3*e^2*f^4 + 3840*a^9*b^4*c^4*e^2*f^4 - 6144*a^10*b^2*c^5*e^2*f^4 - 24*a^6*b^10*c*e^2*f^4)))^(1/2)*(x*(256*a^10*b^13*c^2*e^14*f^10 - 6144*a^11*b^

```
11*c^3*e^14*f^10 + 61440*a^12*b^9*c^4*e^14*f^10 - 327680*a^13*b^7*c^5*e^14*
f^10 + 983040*a^14*b^5*c^6*e^14*f^10 - 1572864*a^15*b^3*c^7*e^14*f^10 + 104
8576*a^16*b*c^8*e^14*f^10) + 1048576*a^16*b*c^8*d*e^13*f^10 + 256*a^10*b^13
*c^2*d*e^13*f^10 - 6144*a^11*b^11*c^3*d*e^13*f^10 + 61440*a^12*b^9*c^4*d*e^
13*f^10 - 327680*a^13*b^7*c^5*d*e^13*f^10 + 983040*a^14*b^5*c^6*d*e^13*f^10
 - 1572864*a^15*b^3*c^7*d*e^13*f^10) + 192*a^8*b^13*c^2*e^12*f^8 - 4672*a^9
*b^11*c^3*e^12*f^8 + 47360*a^10*b^9*c^4*e^12*f^8 - 256000*a^11*b^7*c^5*e^12
*f^8 + 778240*a^12*b^5*c^6*e^12*f^8 - 1261568*a^13*b^3*c^7*e^12*f^8 + 85196
8*a^14*b*c^8*e^12*f^8) + x*(204800*a^12*c^9*e^12*f^6 + 144*a^6*b^12*c^3*e^1
2*f^6 - 3264*a^7*b^10*c^4*e^12*f^6 + 30112*a^8*b^8*c^5*e^12*f^6 - 143360*a^
9*b^6*c^6*e^12*f^6 + 365568*a^10*b^4*c^7*e^12*f^6 - 458752*a^11*b^2*c^8*e^1
2*f^6) + 204800*a^12*c^9*d*e^11*f^6 + 144*a^6*b^12*c^3*d*e^11*f^6 - 3264*a^
7*b^10*c^4*d*e^11*f^6 + 30112*a^8*b^8*c^5*d*e^11*f^6 - 143360*a^9*b^6*c^6*d
*e^11*f^6 + 365568*a^10*b^4*c^7*d*e^11*f^6 - 458752*a^11*b^2*c^8*d*e^11*f^6
)*1i)/((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*
a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 2
5*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^5*b^12*e^2*f^4 + 4096*a^11...
```

3.652 $\int \frac{1}{(df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=228

$$-\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2}+\frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{\left(b^4-6ab^2c+6a^2c^2\right)}{a^3\left(b^2-\right.}$$

```
[Out] (3*a*c-b^2)/a^2/(-4*a*c+b^2)/e/f^3/(e*x+d)^2+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/
a/(-4*a*c+b^2)/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-(6*a^2*c^2-6*a*b
^2*c+b^4)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3
/2)/e/f^3-2*b*ln(e*x+d)/a^3/e/f^3+1/2*b*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e
/f^3
```

Rubi [A]
time = 0.25, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {1156, 1128, 754, 814, 648, 632, 212, 642}

$$\frac{b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{2a^3ef^3}-\frac{2b\log(d+ex)}{a^3ef^3}-\frac{b^2-3ac}{a^2ef^3\left(b^2-4ac\right)(d+ex)^2}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{a^3ef^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^3\left(b^2-4ac\right)(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2)) + (b^2 - 2*a*c + b*c
*(d + e*x)^2)/(2*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a + b*(d + e*x)^2 + c*(
d + e*x)^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)
/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)*e*f^3) - (2*b*Log[d + e*x])/(
a^3*e*f^3) + (b*Log[a + b*(d + e*x)^2 + c*(d + e*x)^4])/(2*a^3*e*f^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)^3 \left(a+b(d+ex)^2+c(d+ex)^4\right)^2} \, dx = \frac{\text{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)^2} \, dx, x, d+ex\right)}{ef^3}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^2} \, dx, x, (d+ex)^2\right)}{2ef^3}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac}{2a\left(b^2-4ac\right)ef^3(d+ex)}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac}{2a\left(b^2-4ac\right)ef^3(d+ex)}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac}{2a\left(b^2-4ac\right)ef^3(d+ex)}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)ef^3(d+ex)^2} + \frac{b^2-2ac}{2a\left(b^2-4ac\right)ef^3(d+ex)}$$

Mathematica [A]
time = 0.34, size = 287, normalized size = 1.26

$$\frac{-\frac{a}{(d+ex)^2} + \frac{a\left(b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2\right)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)} - 4b\log(d+ex) + \frac{\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}} + \frac{\left(-b^4+6ab^2c-6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{3/2}}}{2a^3ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] (-(a/(d + e*x)^2) + (a*(b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c^2*(d + e*
x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - 4*b*Log[d + e
*x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[
b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(
3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqr
t[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)
^(3/2))/(2*a^3*e*f^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.25, size = 466, normalized size = 2.04

method	result
default	$\frac{-\frac{1}{2a^{2}e(ex+d)^{2}}-\frac{2b\ln(ex+d)}{a^{3}e}-\frac{\frac{ace\left(2ac-b^{2}\right)x^{2}}{8ac-2b^{2}}+\frac{cda\left(2ac-b^{2}\right)x}{4ac-b^{2}}+\frac{a\left(2a\,c^{2}d^{2}-b^{2}c\,d^{2}+3abc-b^{3}\right)}{2e\left(4ac-b^{2}\right)}}{c\,e^{4}x^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+d^{4}c+2debx+d^{2}b+a}+\frac{\_R=\mathrm{RootOf}\left(e^{4}c\_Z^{4}+4d\,e^{3}c\_Z^{3}+\left(6d^{2}e^{2}\right.\right.}{}}{}$
risch	$\frac{-\frac{\left(3ac-b^{2}\right)e^{3}c\,x^{4}}{\left(4ac-b^{2}\right)a^{2}}-\frac{4\left(3ac-b^{2}\right)cd\,e^{2}x^{3}}{\left(4ac-b^{2}\right)a^{2}}-\frac{\left(36a\,c^{2}d^{2}-12b^{2}c\,d^{2}+7abc-2b^{3}\right)e\,x^{2}}{2a^{2}\left(4ac-b^{2}\right)}-\frac{d\left(12a\,c^{2}d^{2}-4b^{2}c\,d^{2}+7abc-2b^{3}\right)x}{a^{2}\left(4ac-b^{2}\right)}-\frac{6a\,c^{2}d^{4}-2b^{2}c\,d^{4}+7abc\,d^{2}-}{2e\,a^{2}\left(4ac-\right.}}{f^{3}(ex+d)^{2}(c\,e^{4}x^{4}+4cd\,e^{3}x^{3}+6c\,d^{2}e^{2}x^{2}+4c\,d^{3}ex+b\,e^{2}x^{2}+d^{4}c+2debx+d^{2}b+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^3*(-1/2/a^2/e/(e*x+d)^2-2*b*ln(e*x+d)/a^3/e-1/a^3*((1/2*a*c*e*(2*a*c-b^
2)/(4*a*c-b^2)*x^2+c*d*a*(2*a*c-b^2)/(4*a*c-b^2)*x+1/2*a/e*(2*a*c^2*d^2-b^2
*c*d^2+3*a*b*c-b^3)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4
*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/(4*a*c-b^2)/e*sum((e^3*b*c*
(-4*a*c+b^2)*_R^3+3*d*e^2*b*c*(-4*a*c+b^2)*_R^2+e*(-12*a*b*c^2*d^2+3*b^3*c*
d^2+3*a^2*c^2-5*a*b^2*c+b^4)*_R-4*a*b*c^2*d^3+b^3*c*d^3+3*a^2*c^2*d-5*a*b^2
*c*d+b^4*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*l
n(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d
^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] -1/2*(2*(b^2*c - 3*a*c^2)*d^4 + 8*(b^2*c*e^3 - 3*a*c^2*e^3)*d*x^3 + 2*(b^2*
c*e^4 - 3*a*c^2*e^4)*x^4 + a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*d^2 + (2*b^3
*e^2 - 7*a*b*c*e^2 + 12*(b^2*c*e^2 - 3*a*c^2*e^2)*d^2)*x^2 + 2*(4*(b^2*c*e
- 3*a*c^2*e)*d^3 + (2*b^3*e - 7*a*b*c*e)*d)*x)/(6*(a^2*b^2*c*e^6 - 4*a^3*c^
2*e^6)*d*f^3*x^5 + (a^2*b^2*c*e^7 - 4*a^3*c^2*e^7)*f^3*x^6 + (a^2*b^3*e^5 -
 4*a^3*b*c*e^5 + 15*(a^2*b^2*c*e^5 - 4*a^3*c^2*e^5)*d^2)*f^3*x^4 + 4*(5*(a^
2*b^2*c*e^4 - 4*a^3*c^2*e^4)*d^3 + (a^2*b^3*e^4 - 4*a^3*b*c*e^4)*d)*f^3*x^3
 + (a^3*b^2*e^3 - 4*a^4*c*e^3 + 15*(a^2*b^2*c*e^3 - 4*a^3*c^2*e^3)*d^4 + 6*
(a^2*b^3*e^3 - 4*a^3*b*c*e^3)*d^2)*f^3*x^2 + 2*(3*(a^2*b^2*c*e^2 - 4*a^3*c^
2*e^2)*d^5 + 2*(a^2*b^3*e^2 - 4*a^3*b*c*e^2)*d^3 + (a^3*b^2*e^2 - 4*a^4*c*e
^2)*d)*f^3*x + ((a^2*b^2*c*e - 4*a^3*c^2*e)*d^6 + (a^2*b^3*e - 4*a^3*b*c*e)
*d^4 + (a^3*b^2*e - 4*a^4*c*e)*d^2)*f^3) - 2*b*e^(-1)*log(x*e + d)/(a^3*f^3
```

```
) + 2*integrate(((b^3*c - 4*a*b*c^2)*d^3 + 3*(b^3*c*e^2 - 4*a*b*c^2*e^2)*d*
x^2 + (b^3*c*e^3 - 4*a*b*c^2*e^3)*x^3 + (b^4 - 5*a*b^2*c + 3*a^2*c^2)*d + (
b^4*e - 5*a*b^2*c*e + 3*a^2*c^2*e + 3*(b^3*c*e - 4*a*b*c^2*e)*d^2)*x)/((b^2
*c - 4*a*c^2)*d^4 + 4*(b^2*c*e^3 - 4*a*c^2*e^3)*d*x^3 + (b^2*c*e^4 - 4*a*c^
2*e^4)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*d^2 + (b^3*e^2 - 4*a*b*c*e^2
 + 6*(b^2*c*e^2 - 4*a*c^2*e^2)*d^2)*x^2 + 2*(2*(b^2*c*e - 4*a*c^2*e)*d^3 +
(b^3*e - 4*a*b*c*e)*d)*x), x)/(a^3*f^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2217 vs. $2(224) = 448$.

time = 1.34, size = 4560, normalized size = 20.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] [-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12
*a^3*c^3)*x^4*e^4 + 8*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*x^3*e^3 + 2*
(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a
^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^2)*x^2*e^2 + (2*a*b^
5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a
^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*d)*x*e + ((b^4*c - 6*
a*b^2*c^2 + 6*a^2*c^3)*x^6*e^6 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*x^5*
e^5 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^
2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^2)*x^4*e^4 + (b^5 - 6*a*b^3*c +
6*a^2*b*c^2)*d^4 + 4*(5*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*
b^3*c + 6*a^2*b*c^2)*d)*x^3*e^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)
*x^2*e^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^
2 + 6*a^2*c^3)*d^5 + 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2
*b^2*c + 6*a^3*c^2)*d)*x*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*
x^3*e^3 + 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^
3 + b*c*d)*x*e + b^2 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt
(b^2 - 4*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 +
 b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3
)*x^6*e^6 + 6*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*x^5*e^5 + (b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*d^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*
c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*x^4*e^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^
2*c^2)*d^4 + 4*(5*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4
*c + 16*a^2*b^2*c^2)*d)*x^3*e^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*d^2)*x^2*e^2 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^
```

```
3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*x*e)*log(c*x^4*e^4 + 4*c*d*x^3*
e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a) +
4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6*e^6 + 6*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d*x^5*e^5 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + (b^6 -
 8*a*b^4*c + 16*a^2*b^2*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*
x^4*e^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^4 + 4*(5*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*d^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d)*x^3*e^3 + (a*
b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*
d^4 + 6*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^2)*x^2*e^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*d^2 + 2*(3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + 2
*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c
^2)*d)*x*e)*log(x*e + d))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*f^3*x^6
*e^7 + 6*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d*f^3*x^5*e^6 + (a^3*b^5
- 8*a^4*b^3*c + 16*a^5*b*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*
d^2)*f^3*x^4*e^5 + 4*(5*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^3 + (a^3
*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d)*f^3*x^3*e^4 + (a^4*b^4 - 8*a^5*b^2*c
+ 16*a^6*c^2 + 15*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*d^4 + 6*(a^3*b^5
 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^2)*f^3*x^2*e^3 + 2*(3*(a^3*b^4*c - 8*a^4*b
^2*c^2 + 16*a^5*c^3)*d^5 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^3 + (
a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d)*f^3*x*e^2 + ((a^3*b^4*c - 8*a^4*b^2*
c^2 + 16*a^5*c^3)*d^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*d^4 + (a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2)*d^2)*f^3*e), -1/2*(a^2*b^4 - 8*a^3*b^2*c + 1
6*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4*e^4 + 8*(a*b^4*c -
 7*a^2*b^2*c^2 + 12*a^3*c^3)*d*x^3*e^3 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^
3*c^3)*d^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2 + 12*(a*b^4*c - 7*a^2*b
^2*c^2 + 12*a^3*c^3)*d^2)*x^2*e^2 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)
*d^2 + 2*(4*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d^3 + (2*a*b^5 - 15*a^2*
b^3*c + 28*a^3*b*c^2)*d)*x*e + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6*e^6
 + 6*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d*x^5*e^5 + (b^4*c - 6*a*b^2*c^2 + 6
*a^2*c^3)*d^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2 + 15*(b^4*c - 6*a*b^2*c^2 +
6*a^2*c^3)*d^2)*x^4*e^4 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^4 + 4*(5*(b^4*c
 - 6*a*b^2*c^2 + 6*a^2*c^3)*d^3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d)*x^3*e^
3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + 15*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)
*d^4 + 6*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*d^2)*x^2*e^2 + (a*b^4 - 6*a^2*b^2*
c + 6*a^3*c^2)*d^2 + 2*(3*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d^5 + 2*(b^5 -
6*a*b^3*c + 6*a^2*b*c^2)*d^3 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*d)*x*e)*sq
rt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(-b^2
+ 4*a*c)/(b^2 - 4*a*c)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6*e^6 + 6
*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d*x^5*e^5 + (b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*d^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 687 vs. 2(224) = 448.
time = 3.06, size = 687, normalized size = 3.01

$$\frac{\left(a^3b^4cf^3e^3-6a^4b^2c^2f^3e^3+6a^5c^3f^3e^3\right)\sqrt{b^2-4ac}\log\left(\left|bx^2e^2+2bdxe+\sqrt{b^2-4ac}x^2e^2+2\sqrt{b^2-4ac}dxe+bd^2+\sqrt{b^2-4ac}d^2+2a\right|\right)-\left(a^3b^4cf^3e^3-6a^4b^2c^2f^3e^3+6a^5c^3f^3e^3\right)\sqrt{b^2-4ac}\log\left(\left|-bx^2e^2-2bdxe+\sqrt{b^2-4ac}x^2e^2+2\sqrt{b^2-4ac}dxe-bd^2+\sqrt{b^2-4ac}d^2-2a\right|\right)}{2\left(a^6b^4cf^6e^4-8a^7b^2c^2f^6e^4+16a^8c^3f^6e^4\right)}+\frac{be^{(-1)}\log\left(\left|cx^4e^4+4cdx^3e^3+6cd^2x^2e^2+4cd^3xe+cd^4+bx^2e^2+2bdxe+bd^2+a\right|\right)}{2a^3f^3}-\frac{2be^{(-1)}\log\left(\left|xe+d\right|\right)}{a^3f^3}-\frac{\left(2ab^2cd^4-6a^2c^2d^4+2ab^3d^2-7a^2bcd^2+2\left(ab^2ce^4-3a^2c^2e^4\right)x^4+a^2b^2-4a^3c+8\left(ab^2cde^3-3a^2c^2de^3\right)x^3+\left(12ab^2cd^2e^2-36a^2c^2d^2e^2+2ab^3e^2-7a^2bce^2\right)x^2+2\left(4ab^2cd^3e-12a^2c^2d^3e+2ab^3de-7a^2bcde\right)x\right)e^{(-1)}}{2\left(cx^4e^4+4cdx^3e^3+6cd^2x^2e^2+4cd^3xe+cd^4+bx^2e^2+2bdxe+bd^2+a\right)\left(b^2-4ac\right)\left(xe+d\right)^2a^3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] 1/2*((a^3*b^4*c*f^3*e^3 - 6*a^4*b^2*c^2*f^3*e^3 + 6*a^5*c^3*f^3*e^3)*sqrt(b
^2 - 4*a*c)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*s
qrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^3*b^4*c
*f^3*e^3 - 6*a^4*b^2*c^2*f^3*e^3 + 6*a^5*c^3*f^3*e^3)*sqrt(b^2 - 4*a*c)*log
(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*
c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2*a)))/(a^6*b^4*c*f^6*e^4 - 8*a^
7*b^2*c^2*f^6*e^4 + 16*a^8*c^3*f^6*e^4) + 1/2*b*e^(-1)*log(abs(c*x^4*e^4 +
4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x
*e + b*d^2 + a))/(a^3*f^3) - 2*b*e^(-1)*log(abs(x*e + d))/(a^3*f^3) - 1/2*(
2*a*b^2*c*d^4 - 6*a^2*c^2*d^4 + 2*a*b^3*d^2 - 7*a^2*b*c*d^2 + 2*(a*b^2*c*e^
4 - 3*a^2*c^2*e^4)*x^4 + a^2*b^2 - 4*a^3*c + 8*(a*b^2*c*d*e^3 - 3*a^2*c^2*d
*e^3)*x^3 + (12*a*b^2*c*d^2*e^2 - 36*a^2*c^2*d^2*e^2 + 2*a*b^3*e^2 - 7*a^2*
b*c*e^2)*x^2 + 2*(4*a*b^2*c*d^3*e - 12*a^2*c^2*d^3*e + 2*a*b^3*d*e - 7*a^2*
b*c*d*e)*x)*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*
x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)*(b^2 - 4*a*c)*(x*e + d)^2*
a^3*f^3)
```

Mupad [B]
time = 13.52, size = 2500, normalized size = 10.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] ((x*(2*b^3*d - 12*a*c^2*d^3 + 4*b^2*c*d^3 - 7*a*b*c*d))/(4*a^3*c - a^2*b^2)
 - (x^4*(3*a*c^2*e^3 - b^2*c*e^3))/(4*a^3*c - a^2*b^2) - (4*x^3*(3*a*c^2*d*
e^2 - b^2*c*d*e^2))/(4*a^3*c - a^2*b^2) + (a*b^2 - 4*a^2*c + 2*b^3*d^2 - 6*
a*c^2*d^4 + 2*b^2*c*d^4 - 7*a*b*c*d^2)/(2*e*(4*a^3*c - a^2*b^2)) + (x^2*(2*
b^3*e - 36*a*c^2*d^2*e + 12*b^2*c*d^2*e - 7*a*b*c*e))/(2*(4*a^3*c - a^2*b^2
```

)))/(x^3*(20*c*d^3*e^3*f^3 + 4*b*d*e^3*f^3) + x*(2*a*d*e*f^3 + 4*b*d^3*e*f^3 + 6*c*d^5*e*f^3) + x^4*(b*e^4*f^3 + 15*c*d^2*e^4*f^3) + x^2*(a*e^2*f^3 + 6*b*d^2*e^2*f^3 + 15*c*d^4*e^2*f^3) + a*d^2*f^3 + b*d^4*f^3 + c*d^6*f^3 + c*e^6*f^3*x^6 + 6*c*d*e^5*f^3*x^5) + (log((((b + a^3*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(((b + a^3*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*((4*c^2*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*b^2*c^2*d^2))/(a^2*f^3*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*f^3*(4*a*c - b^2)) - (2*b*c^2*e^16*(b + a^3*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^3*f^3) + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*f^3*(4*a*c - b^2))))/(2*a^3*e*f^3) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^4*f^6*(4*a*c - b^2)^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c - b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c - b^2)^2)))/(2*a^3*e*f^3) - (8*c^5*e^16*x^2*(3*a*c - b^2)^3)/(a^6*f^9*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^2*d^2 + b^2*c*d^2 - 4*a*b*c))/(a^6*f^9*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a*c - b^2)^3)/(a^6*f^9*(4*a*c - b^2)^3))*(((b - a^3*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(((b - a^3*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*((4*c^2*e^16*(2*b^5 + 6*a^2*b*c^2 + b^4*c*d^2 - 30*a^2*c^3*d^2 - 10*a*b^3*c + 2*a*b^2*c^2*d^2))/(a^2*f^3*(4*a*c - b^2)) + (4*c^3*e^18*x^2*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*f^3*(4*a*c - b^2)) - (2*b*c^2*e^16*(b - a^3*e*f^3*(-(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2/(a^6*e^2*f^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^3*f^3) + (8*c^3*d*e^17*x*(b^4 - 30*a^2*c^2 + 2*a*b^2*c))/(a^2*f^3*(4*a*c - b^2))))/(2*a^3*e*f^3) - (4*c^3*e^15*(3*a*c - b^2)*(4*b^4 + 3*a^2*c^2 + 6*b^3*c*d^2 - 17*a*b^2*c - 23*a*b*c^2*d^2))/(a^4*f^6*(4*a*c - b^2)^2) + (4*b*c^4*e^17*x^2*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c - b^2)^2) + (8*b*c^4*d*e^16*x*(6*b^4 + 69*a^2*c^2 - 41*a*b^2*c))/(a^4*f^6*(4*a*c - b^2)^2)))/(2*a^3*e*f^3) - (8*c^5*e^16*x^2*(3*a*c - b^2)^3)/(a^6*f^9*(4*a*c - b^2)^3) + (8*c^4*e^14*(3*a*c - b^2)^2*(b^3 - 3*a*c^2*d^2 + b^2*c*d^2 - 4*a*b*c))/(a^6*f^9*(4*a*c - b^2)^3) - (16*c^5*d*e^15*x*(3*a*c - b^2)^3)/(a^6*f^9*(4*a*c - b^2)^3)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)) - (2*b*log(d + e*x))/(a^3*e*f^3) - (atan(((2*a^9*b^6*f^9*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*f^9*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*f^9*(4*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*f^9*(4*a*c - b^2)^(9/2))*(x*((((8*(54*a^3*c^8*d*e^15 - 2*b^6*c^5*d*e^15 + 18*a*b^4*c^6*d*e^15 - 54*a^2*b^2*c^7*d*e^15))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (((8*(276*a^5*b*c^7*d*e^16*f^3 - 6*a^2*b^7*c^4*d*e^16*f^3 + 65*a^3*b^5*c^5*d*e^16*f^3 - 233*a^4*b^3*c^6*d*e^16*f^3))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (((8*(

```
480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^17*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6
+ 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 6
4*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12
*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6
*d*e^18*f^9 + 3*a^6*b^9*c^2*d*e^18*f^9 - 46*a^7*b^7*c^3*d*e^18*f^9 + 264*a^
8*b^5*c^4*d*e^18*f^9 - 672*a^9*b^3*c^5*d*e^18*f^9))/((a^6*b^6*f^9 - 64*a^9*
c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*b^2*c^2*f^9)*(a^3*b^6*e^2*f^6 - 64*a^6*
c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f^6)))*(b^7*e*f^3 -
 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e*f^3))/(2*(a^3*b^6
*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 - 12*a^4*b^4*c*e^2*f
^6)))*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a^2*b^3*c^2*e
*f^3))/(2*(a^3*b^6*e^2*f^6 - 64*a^6*c^3*e^2*f^6 + 48*a^5*b^2*c^2*e^2*f^6 -
12*a^4*b^4*c*e^2*f^6)) - (((((8*(480*a^8*c^7*d*e^17*f^6 - a^4*b^8*c^3*d*e^1
7*f^6 + 6*a^5*b^6*c^4*d*e^17*f^6 + 30*a^6*b^4*c^5*d*e^17*f^6 - 272*a^7*b^2*
c^6*d*e^17*f^6))/(a^6*b^6*f^9 - 64*a^9*c^3*f^9 - 12*a^7*b^4*c*f^9 + 48*a^8*
b^2*c^2*f^9) - (4*(b^7*e*f^3 - 12*a*b^5*c*e*f^3 - 64*a^3*b*c^3*e*f^3 + 48*a
^2*b^3*c^2*e*f^3)*(640*a^10*b*c^6*d*e^18*f^9 + ...
```

3.653 $\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx$

Optimal. Leaf size=423

$$-\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3}+\frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)ef^4(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^4(d+ex)^3\left(a+b(d+ex)^2+c(d\right.}$$

```
[Out] 1/6*(14*a*c-5*b^2)/a^2/(-4*a*c+b^2)/e/f^4/(e*x+d)^3+1/2*b*(-19*a*c+5*b^2)/a
^3/(-4*a*c+b^2)/e/f^4/(e*x+d)+1/2*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/
e/f^4/(e*x+d)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/4*arctan((e*x+d)*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2+b*(-
19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e/f^4*2^(1/2)/(b-(
-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2-b*(-19*a*c+5*b^2)*(-4*a
*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)/e/f^4*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A]
time = 2.38, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1156, 1135, 1295, 1180, 211}

$$\frac{b(5b^2-19ac)}{2a^3ef^4\left(b^2-4ac\right)(d+ex)}-\frac{5b^2-14ac}{6a^2ef^4\left(b^2-4ac\right)(d+ex)^3}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3ef^4\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3ef^4\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{2aef^4\left(b^2-4ac\right)(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] -1/6*(5*b^2 - 14*a*c)/(a^2*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3) + (b*(5*b^2 - 1
9*a*c))/(2*a^3*(b^2 - 4*a*c)*e*f^4*(d + e*x)) + (b^2 - 2*a*c + b*c*(d + e*x
)^2)/(2*a*(b^2 - 4*a*c)*e*f^4*(d + e*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^
4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*c)*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^4) -
(Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*
Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(df+efx)^4\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+bx^2+cx^4)^2}\,dx, x, d+ex\right)}{ef^4}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{2a\left(b^2-4ac\right)ef^4(d+ex)^3\left(a+b(d+ex)^2+c(d+ex)^4\right.}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b^2-2}{2a\left(b^2-4ac\right)ef^4(d+e}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)ef^4(d+}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)ef^4(d+}$$

$$= -\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)ef^4(d+ex)^3} + \frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)ef^4(d+}$$

Mathematica [A]
time = 1.91, size = 387, normalized size = 0.91

$$\frac{-\frac{4a}{(d+ex)^3}+\frac{24b}{d+ex}+\frac{6(d+ex)\left(b^4-4ab^2c+2a^2c^2+b^3c(d+ex)^2-3abc^2(d+ex)^2\right)}{(b^2-4ac)\left(a+(d+ex)^2(b+c(d+ex)^2)\right)}+\frac{3\sqrt{2}\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-5b^4+29ab^2c-28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{12a^3ef^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x]
```

```
[Out] ((-4*a)/(d + e*x)^3 + (24*b)/(d + e*x) + (6*(d + e*x)*(b^4 - 4*a*b^2*c + 2*
a^2*c^2 + b^3*c*(d + e*x)^2 - 3*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)*(a + (
d + e*x)^2*(b + c*(d + e*x)^2))) + (3*Sqrt[2]*Sqrt[c]*(5*b^4 - 29*a*b^2*c +
 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29*a*b^2*c
- 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3*e*f^4)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 493, normalized size = 1.17

method	result
default	$\frac{-\frac{1}{3a^2e(ex+d)^3}+\frac{2b}{a^3e(ex+d)}-\frac{-\frac{bce^2(3ac-b^2)x^3}{2(4ac-b^2)}-\frac{3dbce(3ac-b^2)x^2}{2(4ac-b^2)}+\frac{(-9abc^2d^2+3b^3cd^2+2a^2c^2-4ab^2c+b^4)x}{8ac-2b^2}+\frac{d(-3abc^2d^2+b^3cd^2+2a^2c^2}{2e(4ac-b^2)}}{ce^4x^4+4cde^3x^3+6cd^2e^2x^2+4cd^3ex+be^2x^2+d^4c+2debx+d^2b+a}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^4*(-1/3/a^2/e/(e*x+d)^3+2/a^3*b/e/(e*x+d)-1/a^3*((-1/2*b*c*e^2*(3*a*c-b
^2)/(4*a*c-b^2)*x^3-3/2*d*b*c*e*(3*a*c-b^2)/(4*a*c-b^2)*x^2+1/2*(-9*a*b*c^2
*d^2+3*b^3*c*d^2+2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2)*x+1/2*d/e*(-3*a*b*c^2
*d^2+b^3*c*d^2+2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2))/(c*e^4*x^4+4*c*d*e^3*x
^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)+1/4/(4*a*
c-b^2)/e*sum((b*c*e^2*(-19*a*c+5*b^2)*_R^2+2*b*c*d*e*(-19*a*c+5*b^2)*_R-19*
a*b*c^2*d^2+5*b^3*c*d^2+14*a^2*c^2-24*a*b^2*c+5*b^4)/(2*_R^3*c*e^3+6*_R^2*c
*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e
^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="maxim
a")
```

```
[Out] 1/6*(3*(5*b^3*c - 19*a*b*c^2)*d^6 + 18*(5*b^3*c*e^5 - 19*a*b*c^2*e^5)*d*x^5
 + 3*(5*b^3*c*e^6 - 19*a*b*c^2*e^6)*x^6 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2
)*d^4 + (15*b^4*e^4 - 62*a*b^2*c*e^4 + 14*a^2*c^2*e^4 + 45*(5*b^3*c*e^4 - 1
9*a*b*c^2*e^4)*d^2)*x^4 - 2*a^2*b^2 + 8*a^3*c + 4*(15*(5*b^3*c*e^3 - 19*a*b
*c^2*e^3)*d^3 + (15*b^4*e^3 - 62*a*b^2*c*e^3 + 14*a^2*c^2*e^3)*d)*x^3 + 10*
(a*b^3 - 4*a^2*b*c)*d^2 + (45*(5*b^3*c*e^2 - 19*a*b*c^2*e^2)*d^4 + 10*a*b^3
*e^2 - 40*a^2*b*c*e^2 + 6*(15*b^4*e^2 - 62*a*b^2*c*e^2 + 14*a^2*c^2*e^2)*d^
2)*x^2 + 2*(9*(5*b^3*c*e - 19*a*b*c^2*e)*d^5 + 2*(15*b^4*e - 62*a*b^2*c*e +
 14*a^2*c^2*e)*d^3 + 10*(a*b^3*e - 4*a^2*b*c*e)*d)*x)/(7*(a^3*b^2*c*e^7 - 4
*a^4*c^2*e^7)*d*f^4*x^6 + (a^3*b^2*c*e^8 - 4*a^4*c^2*e^8)*f^4*x^7 + (a^3*b^
3*e^6 - 4*a^4*b*c*e^6 + 21*(a^3*b^2*c*e^6 - 4*a^4*c^2*e^6)*d^2)*f^4*x^5 + 5
*(7*(a^3*b^2*c*e^5 - 4*a^4*c^2*e^5)*d^3 + (a^3*b^3*e^5 - 4*a^4*b*c*e^5)*d)*
f^4*x^4 + (a^4*b^2*e^4 - 4*a^5*c*e^4 + 35*(a^3*b^2*c*e^4 - 4*a^4*c^2*e^4)*d
^4 + 10*(a^3*b^3*e^4 - 4*a^4*b*c*e^4)*d^2)*f^4*x^3 + (21*(a^3*b^2*c*e^3 - 4
```

```
*a^4*c^2*e^3)*d^5 + 10*(a^3*b^3*e^3 - 4*a^4*b*c*e^3)*d^3 + 3*(a^4*b^2*e^3 -
 4*a^5*c*e^3)*d)*f^4*x^2 + (7*(a^3*b^2*c*e^2 - 4*a^4*c^2*e^2)*d^6 + 5*(a^3*
b^3*e^2 - 4*a^4*b*c*e^2)*d^4 + 3*(a^4*b^2*e^2 - 4*a^5*c*e^2)*d^2)*f^4*x + (
(a^3*b^2*c*e - 4*a^4*c^2*e)*d^7 + (a^3*b^3*e - 4*a^4*b*c*e)*d^5 + (a^4*b^2*
e - 4*a^5*c*e)*d^3)*f^4) + 1/2*integrate((5*b^4 - 24*a*b^2*c + 14*a^2*c^2 +
 (5*b^3*c - 19*a*b*c^2)*d^2 + 2*(5*b^3*c*e - 19*a*b*c^2*e)*d*x + (5*b^3*c*e
^2 - 19*a*b*c^2*e^2)*x^2)/((b^2*c - 4*a*c^2)*d^4 + 4*(b^2*c*e^3 - 4*a*c^2*e
^3)*d*x^3 + (b^2*c*e^4 - 4*a*c^2*e^4)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*
c)*d^2 + (b^3*e^2 - 4*a*b*c*e^2 + 6*(b^2*c*e^2 - 4*a*c^2*e^2)*d^2)*x^2 + 2*
(2*(b^2*c*e - 4*a*c^2*e)*d^3 + (b^3*e - 4*a*b*c*e)*d)*x), x)/(a^3*f^4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5856 vs. $2(376) = 752$.

time = 0.72, size = 5856, normalized size = 13.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="frica
s")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*x^6*e^6 + 36*(5*b^3*c - 19*a*b*c^2)*d*x^5*e^
5 + 6*(5*b^3*c - 19*a*b*c^2)*d^6 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2 + 45
*(5*b^3*c - 19*a*b*c^2)*d^2)*x^4*e^4 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)
*d^4 + 8*(15*(5*b^3*c - 19*a*b*c^2)*d^3 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2
)*d)*x^3*e^3 - 4*a^2*b^2 + 16*a^3*c + 2*(45*(5*b^3*c - 19*a*b*c^2)*d^4 + 10
*a*b^3 - 40*a^2*b*c + 6*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^2)*x^2*e^2 + 3
*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*f^4*x^7*e^8 + 7*(a^3*b^2*c - 4*a^4*c^2)
*d*f^4*x^6*e^7 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*a^4*c^2)*d^2)*f^4
*x^5*e^6 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4*a^4*b*c)*d)*f^4*
x^4*e^5 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*d^4 + 10*(a^3*b^3
 - 4*a^4*b*c)*d^2)*f^4*x^3*e^4 + (21*(a^3*b^2*c - 4*a^4*c^2)*d^5 + 10*(a^3*
b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*f^4*x^2*e^3 + (7*(a^3*b^2*c
 - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b^2 - 4*a^5*c)*d^2
)*f^4*x*e^2 + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*a^4*b*c)*d^5 + (a
^4*b^2 - 4*a^5*c)*d^3)*f^4*e)*sqrt(((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^
2 - 64*a^10*c^3)*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 8
3630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((
a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*f^16)) - 25*b^9 +
 315*a*b^7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3 - 1260*a^4*b*c^4)/((a^7*
b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*f^8))*e^(-1)*log((1125*b
^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 9604*a^4
*c^8)*x*e + 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^7*c^2 - 2
576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*f^12*sqrt((625*b^12
 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^
```

```
4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*
b^2*c^2 - 64*a^17*c^3)*f^16))*e + (125*b^14 - 2425*a*b^12*c + 18940*a^2*b^1
0*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 + 79408
*a^6*b^2*c^6 - 10976*a^7*c^7)*f^4*e)*sqrt(((a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3)*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*
c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*
c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*f^16)) - 2
5*b^9 + 315*a*b^7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3 - 1260*a^4*b*c^4)
/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*f^8))*e^(-1) + (1
125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7 + 960
4*a^4*c^8)*d) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*f^4*x^7*e^8 + 7*(a^3*b
^2*c - 4*a^4*c^2)*d*f^4*x^6*e^7 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c - 4*
a^4*c^2)*d^2)*f^4*x^5*e^6 + 5*(7*(a^3*b^2*c - 4*a^4*c^2)*d^3 + (a^3*b^3 - 4
*a^4*b*c)*d)*f^4*x^4*e^5 + (a^4*b^2 - 4*a^5*c + 35*(a^3*b^2*c - 4*a^4*c^2)*
d^4 + 10*(a^3*b^3 - 4*a^4*b*c)*d^2)*f^4*x^3*e^4 + (21*(a^3*b^2*c - 4*a^4*c^
2)*d^5 + 10*(a^3*b^3 - 4*a^4*b*c)*d^3 + 3*(a^4*b^2 - 4*a^5*c)*d)*f^4*x^2*e^
3 + (7*(a^3*b^2*c - 4*a^4*c^2)*d^6 + 5*(a^3*b^3 - 4*a^4*b*c)*d^4 + 3*(a^4*b
^2 - 4*a^5*c)*d^2)*f^4*x*e^2 + ((a^3*b^2*c - 4*a^4*c^2)*d^7 + (a^3*b^3 - 4*
a^4*b*c)*d^5 + (a^4*b^2 - 4*a^5*c)*d^3)*f^4*e)*sqrt(((a^7*b^6 - 12*a^8*b^4*
c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*f^8*sqrt((625*b^12 - 8250*a*b^10*c + 3952
5*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 +
 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*
f^16)) - 25*b^9 + 315*a*b^7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3 - 1260*
a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*f^8))*e
^(-1)*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b
^2*c^7 + 9604*a^4*c^8)*x*e - 1/2*sqrt(1/2)*((5*a^7*b^11 - 94*a^8*b^9*c + 70
0*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*f^
12*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 +
 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15
*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)*f^16))*e + (125*b^14 - 2425*a*b^12*
c + 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^
5*b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7)*f^4*e)*sqrt(((a^7*b^6 - 12*a
^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*f^8*sqrt((625*b^12 - 8250*a*b^10*c
 + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^
2*c^5 + 2401*a^6*c^6)/((a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^1
7*c^3)*f^16)) - 25*b^9 + 315*a*b^7*c - 1386*a^2*b^5*c^2 + 2415*a^3*b^3*c^3
 - 1260*a^4*b*c^4)/((a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*
f^8))*e^(-1) + (1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*
a^3*b^2*c^7 + 9604*a^4*c^8)*d) + 20*(a*b^3 - 4*a^2*b*c)*d^2 + 4*(9*(5*b^3*c
 - 19*a*b*c^2)*d^5 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d^3 + 10*(a*b^3 -
 4*a^2*b*c)*d)*x*e - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*f^4*x^7*e^8 + 7*(
a^3*b^2*c - 4*a^4*c^2)*d*f^4*x^6*e^7 + (a^3*b^3 - 4*a^4*b*c + 21*(a^3*b^2*c
 - 4*a^4*c^2)*d^2)*f^4*x^5*e^6 + 5*(7*(a^3*b^2*...
```

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2002 vs. $2(376) = 752$.
time = 3.22, size = 2002, normalized size = 4.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2,x, algorithm="giac"
)
```

```
[Out] -1/4*((5*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b^3*c*e^2 - 19*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c
)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) + sqrt(1/2)*sqrt(
-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*
a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (5*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 +
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 19*(d*e^(-1) - sqrt(1/2)*sq
rt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1)
 - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 3
8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*
b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)
*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (5*(d*e^(-1
) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^3*c*e^2
- 19*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
^2*a*b*c^2*e^2 - 10*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*
e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^
2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3*c*d^2 - 19*a*b*c^2*d^2 + 5*b
```

```
^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - s
qrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sq
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*
e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*
c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e
^2)*e^(-4)/c))) + (5*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)
*e^2)*e^(-4)/c))^2*b^3*c*e^2 - 19*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt
(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*b*c^2*e^2 - 10*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^3*c*d*e + 38*(d*e^(-1) - sqr
t(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*b*c^2*d*e + 5*b^3
*c*d^2 - 19*a*b*c^2*d^2 + 5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*log(d*e^(-1) + x
 - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d
*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*
e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(
-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a^3*b^2*f^4 - 4*a^4*c*f^4) +
 1/2*(b^3*c*x^3*e^3 - 3*a*b*c^2*x^3*e^3 + 3*b^3*c*d*x^2*e^2 - 9*a*b*c^2*d*x
^2*e^2 + 3*b^3*c*d^2*x*e - 9*a*b*c^2*d^2*x*e + b^3*c*d^3 - 3*a*b*c^2*d^3 +
b^4*x*e - 4*a*b^2*c*x*e + 2*a^2*c^2*x*e + b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d
)/((a^3*b^2*f^4*e - 4*a^4*c*f^4*e)*(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2
*e^2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)) + 1/3*(6*b
*x^2*e^2 + 12*b*d*x*e + 6*b*d^2 - a)*e^(-1)/((x*e + d)^3*a^3*f^4)
```

Mupad [B]
time = 10.45, size = 2500, normalized size = 5.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^4*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2),x)
```

```
[Out] atan(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366
*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5
 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
- 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2
*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^
2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*
c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3
 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^
3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)
^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4
096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8
+ 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2
*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c
```

^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^
5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*
a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^
8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 614
4*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*(256*a^15*b^13*c
^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^9*c^4*e^14*f^20
- 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*f^20 - 1572864*a
^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 1048576*a^21*b*c^8*
d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^11*c^3*d*e^13*f^2
0 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*e^13*f^20 + 9830
40*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f^20) - 917504*a^
19*c^9*e^12*f^16 + 320*a^12*b^14*c^2*e^12*f^16 - 7936*a^13*b^12*c^3*e^12*f^
16 + 82816*a^14*b^10*c^4*e^12*f^16 - 468480*a^15*b^8*c^5*e^12*f^16 + 153600
0*a^16*b^6*c^6*e^12*f^16 - 2867200*a^17*b^4*c^7*e^12*f^16 + 2719744*a^18*b^
2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*b^14*c^3*e^12*f^
12 + 9440*a^10*b^12*c^4*e^12*f^12 - 92816*a^11*b^10*c^5*e^12*f^12 + 488096*
a^12*b^8*c^6*e^12*f^12 - 1458688*a^13*b^6*c^7*e^12*f^12 + 2401280*a^14*b^4*
c^8*e^12*f^12 - 1871872*a^15*b^2*c^9*e^12*f^12) - 401408*a^16*c^10*d*e^11*f
^12 + 400*a^9*b^14*c^3*d*e^11*f^12 - 9440*a^10*b^12*c^4*d*e^11*f^12 + 92816
*a^11*b^10*c^5*d*e^11*f^12 - 488096*a^12*b^8*c^6*d*e^11*f^12 + 1458688*a^13
*b^6*c^7*d*e^11*f^12 - 2401280*a^14*b^4*c^8*d*e^11*f^12 + 1871872*a^15*b^2*
c^9*d*e^11*f^12)*1i + (-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*
a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 21
9744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2)
- 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240
*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^
8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15
- 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 3
5767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3
*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2
*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*
b^12*e^2*f^8 + 4096*a^13*c^6*e^2*f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*
b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 2
4*a^8*b^10*c*e^2*f^8)))^(1/2)*((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2)
- 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7
*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)
^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a
*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12*e^2*f^8 + 4096*a^13*c^6*e^2*
f^8 + 240*a^9*b^8*c^2*e^2*f^8 - 1280*a^10*b^6*c^3*e^2*f^8 + 3840*a^11*b^4*c
^4*e^2*f^8 - 6144*a^12*b^2*c^5*e^2*f^8 - 24*a^8*b^10*c*e^2*f^8)))^(1/2)*(x*
(256*a^15*b^13*c^2*e^14*f^20 - 6144*a^16*b^11*c^3*e^14*f^20 + 61440*a^17*b^
9*c^4*e^14*f^20 - 327680*a^18*b^7*c^5*e^14*f^20 + 983040*a^19*b^5*c^6*e^14*
f^20 - 1572864*a^20*b^3*c^7*e^14*f^20 + 1048576*a^21*b*c^8*e^14*f^20) + 104

```
8576*a^21*b*c^8*d*e^13*f^20 + 256*a^15*b^13*c^2*d*e^13*f^20 - 6144*a^16*b^1
1*c^3*d*e^13*f^20 + 61440*a^17*b^9*c^4*d*e^13*f^20 - 327680*a^18*b^7*c^5*d*
e^13*f^20 + 983040*a^19*b^5*c^6*d*e^13*f^20 - 1572864*a^20*b^3*c^7*d*e^13*f
^20) + 917504*a^19*c^9*e^12*f^16 - 320*a^12*b^14*c^2*e^12*f^16 + 7936*a^13*
b^12*c^3*e^12*f^16 - 82816*a^14*b^10*c^4*e^12*f^16 + 468480*a^15*b^8*c^5*e^
12*f^16 - 1536000*a^16*b^6*c^6*e^12*f^16 + 2867200*a^17*b^4*c^7*e^12*f^16 -
 2719744*a^18*b^2*c^8*e^12*f^16) - x*(401408*a^16*c^10*e^12*f^12 - 400*a^9*
b^14*c^3*e^12*f^12 + 9440*a^10*b^12*c^4*e^12*f^...
```

3.654 $\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3} \, dx$

Optimal. Leaf size=353

$$\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\frac{f^4(d+ex)\left(7b^2-4ac+12bc(d+ex)^2\right)}{8\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{c}\left(3b^2+4\right.}{}$$

```
[Out] 1/4*f^4*(e*x+d)*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4
)^2-1/8*f^4*(e*x+d)*(7*b^2-4*a*c+12*b*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e
*x+d)^2+c*(e*x+d)^4)+3/8*f^4*arctan((e*x+d)*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5
/2)/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/8*f^4*arctan((e*x+d)*2^(1/2)*c
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^
(1/2))/(-4*a*c+b^2)^(5/2)/e*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1156, 1134, 1192, 1180, 211}

$$\frac{3\sqrt{c}f^4\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}f^4\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}e\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{f^4(d+ex)\left(-4ac+7b^2+12bc(d+ex)^2\right)}{8e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^4*(d + e*x)*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2
 + c*(d + e*x)^4)^2) - (f^4*(d + e*x)*(7*b^2 - 4*a*c + 12*b*c*(d + e*x)^2))
/(8*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[c]*(3*
b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*f^4*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt
[b^2 - 4*a*c]]*e) - (3*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*f^4*
ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]
*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
```

```
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(df+efx)^4}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^4\text{Subst}\left(\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4\text{Subst}\left(\int \frac{2a-5bx}{(a+bx^2+c}\right.}{4\left(b^2-\right.}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-\right.}{8\left(b^2-4ac\right)^2e\left(a+\right.}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-\right.}{8\left(b^2-4ac\right)^2e\left(a+\right.}$$

$$= \frac{f^4(d+ex)\left(2a+b(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{f^4(d+ex)\left(7b^2-\right.}{8\left(b^2-4ac\right)^2e\left(a+\right.}$$

Mathematica [A]
time = 2.75, size = 331, normalized size = 0.94

$$\frac{f^4\left(-\frac{2\left(-2a(d+ex)-b(d+ex)^3\right)}{(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{(d+ex)\left(-7b^2+4ac-12bc(d+ex)^2\right)}{(b^2-4ac)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^4*((-2*(-2*a*(d + e*x) - b*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)
^2 + c*(d + e*x)^4)^2) + ((d + e*x)*(-7*b^2 + 4*a*c - 12*b*c*(d + e*x)^2))/
((b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) + (3*Sqrt[2]*Sqrt[c]*
(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(8*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.24, size = 708, normalized size = 2.01

method	result

default	$f^4\left(\frac{-\frac{3c^2e^6b\,x^7}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{21c^2d\,e^5b\,x^6}{2(16a^2c^2-8a\,b^2c+b^4)}+\frac{(-252bc\,d^2+4ac-19b^2)c\,e^4x^5}{128a^2c^2-64a\,b^2c+8b^4}+\frac{5cd\,e^3(-84bc\,d^2+4ac-19b^2)x^4}{8(16a^2c^2-8a\,b^2c+b^4)}-\frac{e^2(420b\,c^2d^4-40a\,c^2}{8(16a^2}}{}\right.$
risch	$\frac{-\frac{3c^2e^6b\,f^4x^7}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{21c^2d\,e^5b\,f^4x^6}{2(16a^2c^2-8a\,b^2c+b^4)}+\frac{(-252bc\,d^2+4ac-19b^2)c\,e^4f^4x^5}{128a^2c^2-64a\,b^2c+8b^4}+\frac{5cd\,e^3f^4(-84bc\,d^2+4ac-19b^2)x^4}{8(16a^2c^2-8a\,b^2c+b^4)}-\frac{e^2f^4(420b\,c^2d^4-40a\,c}{8(16a^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f^4*((-3/2*c^2*e^6*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-21/2*c^2*d*e^5*b/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^6+1/8*(-252*b*c*d^2+4*a*c-19*b^2)*c*e^4/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^5+5/8*c*d*e^3*(-84*b*c*d^2+4*a*c-19*b^2)/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^4-1/8*e^2*(420*b*c^2*d^4-40*a*c^2*d^2+190*b^2*c*d^2+16*a*b*c+5*
b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*d*e*(252*b*c^2*d^4-40*a*c^2*d^2+190
*b^2*c*d^2+48*a*b*c+15*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/8*(84*b*c^2*d^
6-20*a*c^2*d^4+95*b^2*c*d^4+48*a*b*c*d^2+15*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a
^2*c^2-8*a*b^2*c+b^4)*x-1/8*d/e*(12*b*c^2*d^6-4*a*c^2*d^4+19*b^2*c*d^4+16*a
*b*c*d^2+5*b^3*d^2+12*a^2*c+3*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4
+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+
a)^2+3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-4*_R^2*b*c*e^2-8*_R*b*c*d*e-4*
b*c*d^2+4*a*c+b^2)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e
+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2
+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] -3/8*f^4*integrate((4*b*c*x^2*e^2 + 8*b*c*d*x*e + 4*b*c*d^2 - b^2 - 4*a*c)/
(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*
(2*c*d^3*e + b*d*e)*x + a), x)/(b^4 - 8*a*b^2*c + 16*a^2*c^2) - 1/8*(12*b*c
^2*f^4*x^7*e^7 + 84*b*c^2*d*f^4*x^6*e^6 + (252*b*c^2*d^2*e^5 + 19*b^2*c*e^5
 - 4*a*c^2*e^5)*f^4*x^5 + 5*(84*b*c^2*d^3*e^4 + (19*b^2*c*e^4 - 4*a*c^2*e^4
)*d)*f^4*x^4 + (420*b*c^2*d^4*e^3 + 5*b^3*e^3 + 16*a*b*c*e^3 + 10*(19*b^2*c
```

```
*e^3 - 4*a*c^2*e^3)*d^2)*f^4*x^3 + (252*b*c^2*d^5*e^2 + 10*(19*b^2*c*e^2 -
4*a*c^2*e^2)*d^3 + 3*(5*b^3*e^2 + 16*a*b*c*e^2)*d)*f^4*x^2 + (84*b*c^2*d^6*
e + 5*(19*b^2*c*e - 4*a*c^2*e)*d^4 + 3*a*b^2*e + 12*a^2*c*e + 3*(5*b^3*e +
16*a*b*c*e)*d^2)*f^4*x + (12*b*c^2*d^7 + (19*b^2*c - 4*a*c^2)*d^5 + (5*b^3
+ 16*a*b*c)*d^3 + 3*(a*b^2 + 4*a^2*c)*d)*f^4)/((b^4*c^2*e - 8*a*b^2*c^3*e +
 16*a^2*c^4*e)*d^8 + 8*(b^4*c^2*e^8 - 8*a*b^2*c^3*e^8 + 16*a^2*c^4*e^8)*d*x
^7 + (b^4*c^2*e^9 - 8*a*b^2*c^3*e^9 + 16*a^2*c^4*e^9)*x^8 + 2*(b^5*c*e - 8*
a*b^3*c^2*e + 16*a^2*b*c^3*e)*d^6 + 2*(b^5*c*e^7 - 8*a*b^3*c^2*e^7 + 16*a^2
*b*c^3*e^7 + 14*(b^4*c^2*e^7 - 8*a*b^2*c^3*e^7 + 16*a^2*c^4*e^7)*d^2)*x^6 +
 a^2*b^4*e - 8*a^3*b^2*c*e + 16*a^4*c^2*e + 4*(14*(b^4*c^2*e^6 - 8*a*b^2*c^
3*e^6 + 16*a^2*c^4*e^6)*d^3 + 3*(b^5*c*e^6 - 8*a*b^3*c^2*e^6 + 16*a^2*b*c^3
*e^6)*d)*x^5 + (b^6*e - 6*a*b^4*c*e + 32*a^3*c^3*e)*d^4 + (b^6*e^5 - 6*a*b^
4*c*e^5 + 32*a^3*c^3*e^5 + 70*(b^4*c^2*e^5 - 8*a*b^2*c^3*e^5 + 16*a^2*c^4*e
^5)*d^4 + 30*(b^5*c*e^5 - 8*a*b^3*c^2*e^5 + 16*a^2*b*c^3*e^5)*d^2)*x^4 + 4*
(14*(b^4*c^2*e^4 - 8*a*b^2*c^3*e^4 + 16*a^2*c^4*e^4)*d^5 + 10*(b^5*c*e^4 -
8*a*b^3*c^2*e^4 + 16*a^2*b*c^3*e^4)*d^3 + (b^6*e^4 - 6*a*b^4*c*e^4 + 32*a^3
*c^3*e^4)*d)*x^3 + 2*(a*b^5*e - 8*a^2*b^3*c*e + 16*a^3*b*c^2*e)*d^2 + 2*(14
*(b^4*c^2*e^3 - 8*a*b^2*c^3*e^3 + 16*a^2*c^4*e^3)*d^6 + a*b^5*e^3 - 8*a^2*b
^3*c*e^3 + 16*a^3*b*c^2*e^3 + 15*(b^5*c*e^3 - 8*a*b^3*c^2*e^3 + 16*a^2*b*c^
3*e^3)*d^4 + 3*(b^6*e^3 - 6*a*b^4*c*e^3 + 32*a^3*c^3*e^3)*d^2)*x^2 + 4*(2*(
b^4*c^2*e^2 - 8*a*b^2*c^3*e^2 + 16*a^2*c^4*e^2)*d^7 + 3*(b^5*c*e^2 - 8*a*b^
3*c^2*e^2 + 16*a^2*b*c^3*e^2)*d^5 + (b^6*e^2 - 6*a*b^4*c*e^2 + 32*a^3*c^3*e
^2)*d^3 + (a*b^5*e^2 - 8*a^2*b^3*c*e^2 + 16*a^3*b*c^2*e^2)*d)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6666 vs. $2(311) = 622$.
time = 0.56, size = 6666, normalized size = 18.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] -1/16*(24*b*c^2*f^4*x^7*e^7 + 168*b*c^2*d*f^4*x^6*e^6 + 2*(252*b*c^2*d^2 +
19*b^2*c - 4*a*c^2)*f^4*x^5*e^5 + 10*(84*b*c^2*d^3 + (19*b^2*c - 4*a*c^2)*d
)*f^4*x^4*e^4 + 2*(420*b*c^2*d^4 + 5*b^3 + 16*a*b*c + 10*(19*b^2*c - 4*a*c^
2)*d^2)*f^4*x^3*e^3 + 2*(252*b*c^2*d^5 + 10*(19*b^2*c - 4*a*c^2)*d^3 + 3*(5
*b^3 + 16*a*b*c)*d)*f^4*x^2*e^2 + 2*(84*b*c^2*d^6 + 5*(19*b^2*c - 4*a*c^2)*
d^4 + 3*a*b^2 + 12*a^2*c + 3*(5*b^3 + 16*a*b*c)*d^2)*f^4*x*e + 2*(12*b*c^2*
d^7 + (19*b^2*c - 4*a*c^2)*d^5 + (5*b^3 + 16*a*b*c)*d^3 + 3*(a*b^2 + 4*a^2*
c)*d)*f^4 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8*e^9 + 8*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*x^7*e^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*x^6*e^7 + 4*(14*(
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b
```

*c^3)*d)*x^5*e^6 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)*x^4*e^5
+ 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a*b^3*c^2
+ 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*x^3*e^4 + 2*(14*(b^
4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2
+ 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c + 32*a^3
*c^3)*d^2)*x^2*e^3 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7 + 3*(b^5
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^3 +
(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*x*e^2 + ((b^4*c^2 - 8*a*b^2*c^3 +
16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2*b^4 - 8*
a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2*b*c^2)*
f^8 + sqrt(f^16/(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^
3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*
c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*e^(-2)/(a*b^10 -
20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*
a^6*c^5))*log(27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*f^12*x*e + 27*(5*b^4
*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*f^12 + 27/2*sqrt(1/2)*((b^8 - 8*a*b^6*c +
128*a^3*b^2*c^3 - 256*a^4*c^4)*f^8*e - sqrt(f^16/(a^2*b^10 - 20*a^3*b^8*c
+ 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*(a*
b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4
+ 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^2
*b*c^2)*f^8 + sqrt(f^16/(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^
5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*(a*b^10 - 20*a^2*b^8*c + 160*
a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*e^(-2)/(a
*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4
- 1024*a^6*c^5))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8*
e^9 + 8*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d*x^7*e^8 + 2*(b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3 + 14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^2)*x^6*e^7
+ 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^3 + 3*(b^5*c - 8*a*b^3*c^2 +
16*a^2*b*c^3)*d)*x^5*e^6 + (b^6 - 6*a*b^4*c + 32*a^3*c^3 + 70*(b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4)*d^4 + 30*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^2)
*x^4*e^5 + 4*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^5 + 10*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*d^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d)*x^3*e^4 +
2*(14*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^6 + a*b^5 - 8*a^2*b^3*c + 16*a
^3*b*c^2 + 15*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^4 + 3*(b^6 - 6*a*b^4*c
+ 32*a^3*c^3)*d^2)*x^2*e^3 + 4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d^7
+ 3*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 + (b^6 - 6*a*b^4*c + 32*a^3*c
^3)*d^3 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d)*x*e^2 + ((b^4*c^2 - 8*a*b
^2*c^3 + 16*a^2*c^4)*d^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^6 + a^2
*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*d^4 + 2*(a
*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d^2)*e)*sqrt(-((b^5 + 40*a*b^3*c + 80*a^
2*b*c^2)*f^8 + sqrt(f^16/(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a
^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*(a*b^10 - 20*a^2*b^8*c + 160
*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*e^(-2)/(

```
a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^
4 - 1024*a^6*c^5))*log(27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*f^12*x*e +
27*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*d*f^12 - 27/2*sqrt(1/2)*((b^8 - 8*
a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4)*f^8*e - sqrt(f^16/(a^2*b^10 - 20*a
^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*
c^5))*(a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5
*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)*e)*sqrt(-((b^5 + 40*a*b^3*c
 + 80*a^2*b*c^2)*f^8 + sqrt(f^16/(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2
 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*(a*b^10 - 20*a^2*b^8
*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*
e^(-2)/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**4/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1844 vs. $2(311) = 622$.
time = 3.26, size = 1844, normalized size = 5.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^4/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 3/16*((4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*b*c*f^4*e^2 - 8*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*
c)*e^2)*e^(-4)/c))*b*c*d*f^4*e + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*log(d
*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2
*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c
*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)
/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt
(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (4*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*f^4*e^2 - 8*
(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*
d*f^4*e + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)*log(d*e^(-1) + x - sqrt(1/2)
*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*
sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sq
rt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^
3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sq
```

```
rt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + (4*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 -
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b*c*f^4*e^2 - 8*(d*e^(-1) + sqrt(1/2)*s
qrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b*c*d*f^4*e + 4*b*c*d^2*f^4
 - b^2*f^4 - 4*a*c*f^4)*log(d*e^(-1) + x + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^
2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 -
sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*
e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^
(-4)/c))) + (4*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*b*c*f^4*e^2 - 8*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2
- 4*a*c)*e^2)*e^(-4)/c))*b*c*d*f^4*e + 4*b*c*d^2*f^4 - b^2*f^4 - 4*a*c*f^4)
*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(b^4 - 8*a*b
^2*c + 16*a^2*c^2) - 1/8*(12*b*c^2*f^4*x^7*e^7 + 84*b*c^2*d*f^4*x^6*e^6 + 2
52*b*c^2*d^2*f^4*x^5*e^5 + 420*b*c^2*d^3*f^4*x^4*e^4 + 420*b*c^2*d^4*f^4*x^
3*e^3 + 252*b*c^2*d^5*f^4*x^2*e^2 + 84*b*c^2*d^6*f^4*x*e + 12*b*c^2*d^7*f^4
 + 19*b^2*c*f^4*x^5*e^5 - 4*a*c^2*f^4*x^5*e^5 + 95*b^2*c*d*f^4*x^4*e^4 - 20
*a*c^2*d*f^4*x^4*e^4 + 190*b^2*c*d^2*f^4*x^3*e^3 - 40*a*c^2*d^2*f^4*x^3*e^3
 + 190*b^2*c*d^3*f^4*x^2*e^2 - 40*a*c^2*d^3*f^4*x^2*e^2 + 95*b^2*c*d^4*f^4*
x*e - 20*a*c^2*d^4*f^4*x*e + 19*b^2*c*d^5*f^4 - 4*a*c^2*d^5*f^4 + 5*b^3*f^4
*x^3*e^3 + 16*a*b*c*f^4*x^3*e^3 + 15*b^3*d*f^4*x^2*e^2 + 48*a*b*c*d*f^4*x^2
*e^2 + 15*b^3*d^2*f^4*x*e + 48*a*b*c*d^2*f^4*x*e + 5*b^3*d^3*f^4 + 16*a*b*c
*d^3*f^4 + 3*a*b^2*f^4*x*e + 12*a^2*c*f^4*x*e + 3*a*b^2*d*f^4 + 12*a^2*c*d*
f^4)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4 +
b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^4*e - 8*a*b^2*c*e + 16*a^2*c^2*e))
```

Mupad [B]
time = 7.52, size = 2500, normalized size = 7.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^4/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] atan(((-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8
- 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 102
4*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*
e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 76
80*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 86
0160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 -
2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c
^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 229376
```

0*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) - (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*b^8*c^3*d*e^11*f^8 - 6912*a*b^6*c^4*d*e^11*f^8 + 11520*a^2*b^4*c^5*d*e^11*f^8)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(144*a^2*c^5*e^12*f^8 + 117*b^4*c^3*e^12*f^8 + 72*a*b^2*c^4*e^12*f^8))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*1i + (-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2)*((((1024*b^15*c^2*d*e^13 - 28672*a*b^13*c^3*d*e^13 - 16777216*a^7*b*c^9*d*e^13 + 344064*a^2*b^11*c^4*d*e^13 - 2293760*a^3*b^9*c^5*d*e^13 + 9175040*a^4*b^7*c^6*d*e^13 - 22020096*a^5*b^5*c^7*d*e^13 + 29360128*a^6*b^3*c^8*d*e^13)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(128*b^11*c^2*e^14 - 2560*a*b^9*c^3*e^14 - 131072*a^5*b*c^7*e^14 + 20480*a^2*b^7*c^4*e^14 - 81920*a^3*b^5*c^5*e^14 + 163840*a^4*b^3*c^6*e^14))/(16*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^

2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (786432*a^6*c^8*e^12*f^4 - 192*b^12*c^2*e^12*f^4 - 15360*a^2*b^8*c^4*e^12*f^4 + 245760*a^4*b^4*c^6*e^12*f^4 - 786432*a^5*b^2*c^7*e^12*f^4 + 3072*a*b^10*c^3*e^12*f^4)/(128*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)))*(-(9*(b^15*f^8 + f^8*(-(4*a*c - b^2)^15)^(1/2) - 81920*a^7*b*c^7*f^8 - 560*a^2*b^11*c^2*f^8 + 4160*a^3*b^9*c^3*f^8 - 11520*a^4*b^7*c^4*f^8 - 1024*a^5*b^5*c^5*f^8 + 61440*a^6*b^3*c^6*f^8 + 20*a*b^13*c*f^8))/(512*(a*b^20*e^2 + 1048576*a^11*c^10*e^2 - 40*a^2*b^18*c*e^2 + 720*a^3*b^16*c^2*e^2 - 7680*a^4*b^14*c^3*e^2 + 53760*a^5*b^12*c^4*e^2 - 258048*a^6*b^10*c^5*e^2 + 860160*a^7*b^8*c^6*e^2 - 1966080*a^8*b^6*c^7*e^2 + 2949120*a^9*b^4*c^8*e^2 - 2621440*a^10*b^2*c^9*e^2)))^(1/2) + (18432*a^4*c^7*d*e^11*f^8 + 936*...

3.655 $\int \frac{(df+efx)^3}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=159

$$\frac{f^3(2a+b(d+ex)^2)}{4(b^2-4ac)e(a+b(d+ex)^2+c(d+ex)^4)^2}-\frac{3bf^3(b+2c(d+ex)^2)}{4(b^2-4ac)^2e(a+b(d+ex)^2+c(d+ex)^4)}+\frac{3bcf^3\tanh^{-1}(\ldots}{(b^2-4\ldots}$$

```
[Out] 1/4*f^3*(2*a+b*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2-3/4*
b*f^3*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+3*b*c*
f^3*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Rubi [A]

time = 0.14, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1156, 1128, 652, 628, 632, 212}

$$-\frac{3bf^3(b+2c(d+ex)^2)}{4e(b^2-4ac)^2(a+b(d+ex)^2+c(d+ex)^4)}+\frac{f^3(2a+b(d+ex)^2)}{4e(b^2-4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{3bcf^3\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e(b^2-4ac)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^3*(2*a + b*(d + e*x)^2))/(4*(b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)^2) - (3*b*f^3*(b + 2*c*(d + e*x)^2))/(4*(b^2 - 4*a*c)^2*e*(a + b*(d
 + e*x)^2 + c*(d + e*x)^4)) + (3*b*c*f^3*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt
[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{(df+efx)^3}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^3\text{Subst}\left(\int \frac{x^3}{(a+bx^2+cx^4)^3}\,dx,x,d+ex\right)}{e}$$

$$= \frac{f^3\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^3}\,dx,x,(d+ex)^2\right)}{2e}$$

$$= \frac{f^3(2a+b(d+ex)^2)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{\left(3bf^3\right)\text{Subst}\left(\int \frac{}{(a+bx}\right.}{4\left(b^2-\right.}$$

$$= \frac{f^3(2a+b(d+ex)^2)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3(b+2}{4\left(b^2-4ac\right)^2e\left(a+b\right.}$$

$$= \frac{f^3(2a+b(d+ex)^2)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3(b+2}{4\left(b^2-4ac\right)^2e\left(a+b\right.}$$

$$= \frac{f^3(2a+b(d+ex)^2)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{3bf^3(b+2}{4\left(b^2-4ac\right)^2e\left(a+b\right.}$$

Mathematica [A]

time = 0.14, size = 149, normalized size = 0.94

$$\frac{f^3\left(-\frac{3b(b+2c(d+ex)^2)}{a+b(d+ex)^2+c(d+ex)^4}+\frac{(b^2-4ac)\,(2a+b(d+ex)^2)}{(a+(d+ex)^2(b+c(d+ex)^2))^2}-\frac{12bc\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}\right)}{4\left(b^2-4ac\right)^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^3*((-3*b*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + ((
b^2 - 4*a*c)*(2*a + b*(d + e*x)^2))/(a + (d + e*x)^2*(b + c*(d + e*x)^2))^2
 - (12*b*c*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c]))/(4*(b^2 - 4*a*c)^2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.26, size = 548, normalized size = 3.45

method	result
default	$f^3\left(\frac{-\frac{3c^2e^5b\,x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{9e^4b\,c^2d\,x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{9bc\,e^3\left(10c\,d^2+b\right)x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3d\,e^2cb\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{be\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-db(9}{(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b}\right.$
risch	$\frac{-\frac{3c^2e^5b\,f^3x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{9f^3e^4b\,c^2d\,x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{9bc\,e^3f^3\left(10c\,d^2+b\right)x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3d\,e^2cb\,f^3\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{be\,f^3\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-db\,f^3}{(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f^3*((-3/2*c^2*e^5*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-9*e^4*b*c^2*d/(16*a^2*c
^2-8*a*b^2*c+b^4)*x^5-9/4*b*c*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^4-3*d*e^2*c*b*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/2*b*e*(45*c^
2*d^4+27*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-d*b*(9*c^2*d^4+9
*b*c*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/4/e*(6*b*c^2*d^6+9*b^2*c
*d^4+10*a*b*c*d^2+2*b^3*d^2+8*a^2*c+a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e
^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+
b*d^2+a)^2+3/2*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((-_R*e-d)/(2*_R^3*c*e^3
+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_
Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^
2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] -3*b*c*f^3*integrate((x*e + d)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 +
 (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(b^4 - 8*a*b^
2*c + 16*a^2*c^2) - 1/4*(6*b*c^2*f^3*x^6*e^6 + 36*b*c^2*d*f^3*x^5*e^5 + 9*(
10*b*c^2*d^2*e^4 + b^2*c*e^4)*f^3*x^4 + 12*(10*b*c^2*d^3*e^3 + 3*b^2*c*d*e^
3)*f^3*x^3 + 2*(45*b*c^2*d^4*e^2 + 27*b^2*c*d^2*e^2 + b^3*e^2 + 5*a*b*c*e^2
)*f^3*x^2 + 4*(9*b*c^2*d^5*e + 9*b^2*c*d^3*e + (b^3*e + 5*a*b*c*e)*d)*f^3*x
 + (6*b*c^2*d^6 + 9*b^2*c*d^4 + a*b^2 + 8*a^2*c + 2*(b^3 + 5*a*b*c)*d^2)*f^
3)/((b^4*c^2*e - 8*a*b^2*c^3*e + 16*a^2*c^4*e)*d^8 + 8*(b^4*c^2*e^8 - 8*a*b
^2*c^3*e^8 + 16*a^2*c^4*e^8)*d*x^7 + (b^4*c^2*e^9 - 8*a*b^2*c^3*e^9 + 16*a^
2*c^4*e^9)*x^8 + 2*(b^5*c*e - 8*a*b^3*c^2*e + 16*a^2*b*c^3*e)*d^6 + 2*(b^5*
c*e^7 - 8*a*b^3*c^2*e^7 + 16*a^2*b*c^3*e^7 + 14*(b^4*c^2*e^7 - 8*a*b^2*c^3*
e^7 + 16*a^2*c^4*e^7)*d^2)*x^6 + a^2*b^4*e - 8*a^3*b^2*c*e + 16*a^4*c^2*e +
 4*(14*(b^4*c^2*e^6 - 8*a*b^2*c^3*e^6 + 16*a^2*c^4*e^6)*d^3 + 3*(b^5*c*e^6
- 8*a*b^3*c^2*e^6 + 16*a^2*b*c^3*e^6)*d)*x^5 + (b^6*e - 6*a*b^4*c*e + 32*a^
3*c^3*e)*d^4 + (b^6*e^5 - 6*a*b^4*c*e^5 + 32*a^3*c^3*e^5 + 70*(b^4*c^2*e^5
- 8*a*b^2*c^3*e^5 + 16*a^2*c^4*e^5)*d^4 + 30*(b^5*c*e^5 - 8*a*b^3*c^2*e^5 +
 16*a^2*b*c^3*e^5)*d^2)*x^4 + 4*(14*(b^4*c^2*e^4 - 8*a*b^2*c^3*e^4 + 16*a^2
*c^4*e^4)*d^5 + 10*(b^5*c*e^4 - 8*a*b^3*c^2*e^4 + 16*a^2*b*c^3*e^4)*d^3 + (
b^6*e^4 - 6*a*b^4*c*e^4 + 32*a^3*c^3*e^4)*d)*x^3 + 2*(a*b^5*e - 8*a^2*b^3*c
*e + 16*a^3*b*c^2*e)*d^2 + 2*(14*(b^4*c^2*e^3 - 8*a*b^2*c^3*e^3 + 16*a^2*c^
4*e^3)*d^6 + a*b^5*e^3 - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + 15*(b^5*c*e^3
 - 8*a*b^3*c^2*e^3 + 16*a^2*b*c^3*e^3)*d^4 + 3*(b^6*e^3 - 6*a*b^4*c*e^3 + 3
2*a^3*c^3*e^3)*d^2)*x^2 + 4*(2*(b^4*c^2*e^2 - 8*a*b^2*c^3*e^2 + 16*a^2*c^4*
e^2)*d^7 + 3*(b^5*c*e^2 - 8*a*b^3*c^2*e^2 + 16*a^2*b*c^3*e^2)*d^5 + (b^6*e^
2 - 6*a*b^4*c*e^2 + 32*a^3*c^3*e^2)*d^3 + (a*b^5*e^2 - 8*a^2*b^3*c*e^2 + 16
*a^3*b*c^2*e^2)*d)*x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1839 vs. $2(155) = 310$.
time = 0.44, size = 3805, normalized size = 23.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] [-1/4*(6*(b^3*c^2 - 4*a*b*c^3)*f^3*x^6*e^6 + 36*(b^3*c^2 - 4*a*b*c^3)*d*f^3
*x^5*e^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*c^3)*d^2)*f^3*x^4*e
^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b^2*c^2)*d)*f^3*x^3*
e^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*a*b*c^3)*d^4 + 27*(
```

b^4*c - 4*a*b^2*c^2)*d^2)*f^3*x^2*e^2 + 4*(9*(b^3*c^2 - 4*a*b*c^3)*d^5 + 9*
(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^2)*d)*f^3*x*e + (6*
(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2 + 9*(b^4*c - 4
*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)*f^3 - 6*(b*c^3*f^3*
x^8*e^8 + 8*b*c^3*d*f^3*x^7*e^7 + 2*(14*b*c^3*d^2 + b^2*c^2)*f^3*x^6*e^6 +
4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*f^3*x^5*e^5 + (70*b*c^3*d^4 + 30*b^2*c^2*d^2
 + b^3*c + 2*a*b*c^2)*f^3*x^4*e^4 + 4*(14*b*c^3*d^5 + 10*b^2*c^2*d^3 + (b^3
*c + 2*a*b*c^2)*d)*f^3*x^3*e^3 + 2*(14*b*c^3*d^6 + 15*b^2*c^2*d^4 + a*b^2*c
 + 3*(b^3*c + 2*a*b*c^2)*d^2)*f^3*x^2*e^2 + 4*(2*b*c^3*d^7 + 3*b^2*c^2*d^5
+ a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*f^3*x*e + (b*c^3*d^8 + 2*b^2*c^2*d^6
 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f^3)*sqrt(b^2 - 4*a*c
)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d
^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2 - 2*a*c + (2*c*x^2*e^2
+ 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 +
c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a)))/((b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8*e^9 + 8*(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^7*e^8 + 2*(b^7*c - 12*a*b^5*c
^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5)*d^2)*x^6*e^7 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^
3*b*c^4)*d)*x^5*e^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 -
 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^
4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*x^4*e^5
+ 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*x^3*e^4 + 2*(a*b^7 - 1
2*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 +
48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^
3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c
^3 - 128*a^4*c^4)*d^2)*x^2*e^3 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b
*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c
^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*x*e^2 +
 ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12
*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*
b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*d^2)*e), -1/4*(6*(b^3*c^2 - 4*a*b*c^3)*f^3*x^6*e^6 + 36*(b^3*c^2
 - 4*a*b*c^3)*d*f^3*x^5*e^5 + 9*(b^4*c - 4*a*b^2*c^2 + 10*(b^3*c^2 - 4*a*b*
c^3)*d^2)*f^3*x^4*e^4 + 12*(10*(b^3*c^2 - 4*a*b*c^3)*d^3 + 3*(b^4*c - 4*a*b
^2*c^2)*d)*f^3*x^3*e^3 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2 + 45*(b^3*c^2 - 4*
a*b*c^3)*d^4 + 27*(b^4*c - 4*a*b^2*c^2)*d^2)*f^3*x^2*e^2 + 4*(9*(b^3*c^2 -
4*a*b*c^3)*d^5 + 9*(b^4*c - 4*a*b^2*c^2)*d^3 + (b^5 + a*b^3*c - 20*a^2*b*c^
2)*d)*f^3*x*e + (6*(b^3*c^2 - 4*a*b*c^3)*d^6 + a*b^4 + 4*a^2*b^2*c - 32*a^3
c^2 + 9(b^4*c - 4*a*b^2*c^2)*d^4 + 2*(b^5 + a*b^3*c - 20*a^2*b*c^2)*d^2)*

```
f^3 - 12*(b*c^3*f^3*x^8*e^8 + 8*b*c^3*d*f^3*x^7*e^7 + 2*(14*b*c^3*d^2 + b^2
*c^2)*f^3*x^6*e^6 + 4*(14*b*c^3*d^3 + 3*b^2*c^2*d)*f^3*x^5*e^5 + (70*b*c^3*
d^4 + 30*b^2*c^2*d^2 + b^3*c + 2*a*b*c^2)*f^3*x^4*e^4 + 4*(14*b*c^3*d^5 + 1
0*b^2*c^2*d^3 + (b^3*c + 2*a*b*c^2)*d)*f^3*x^3*e^3 + 2*(14*b*c^3*d^6 + 15*b
^2*c^2*d^4 + a*b^2*c + 3*(b^3*c + 2*a*b*c^2)*d^2)*f^3*x^2*e^2 + 4*(2*b*c^3*
d^7 + 3*b^2*c^2*d^5 + a*b^2*c*d + (b^3*c + 2*a*b*c^2)*d^3)*f^3*x*e + (b*c^3
*d^8 + 2*b^2*c^2*d^6 + 2*a*b^2*c*d^2 + (b^3*c + 2*a*b*c^2)*d^4 + a^2*b*c)*f
^3)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5)*x^8*e^9 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5)*d*x^7*e^8 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 1
4*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*x^6*e^7 + 4*(
14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c -
12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*x^5*e^6 + (b^8 - 10*a*b^6*
c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*
c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4)*d^2)*x^4*e^5 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1794 vs. $2(144) = 288$.
time = 7.93, size = 1794, normalized size = 11.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] 3*b*c*f**3*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*b*c**
4*f**3*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*f**3*sqrt(-1/(4*a*c
- b**2)**5) - 36*a*b**5*c**2*f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*f**
3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c*f**3 + 6*b*c**2*d**2*f**3)/(6*b*c**
2*e**2*f**3))/(2*e) - 3*b*c*f**3*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x
**2 + (192*a**3*b*c**4*f**3*sqrt(-1/(4*a*c - b**2)**5) - 144*a**2*b**3*c**3
*f**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b**5*c**2*f**3*sqrt(-1/(4*a*c - b**
2)**5) - 3*b**7*c*f**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**2*c*f**3 + 6*b*c**
2*d**2*f**3)/(6*b*c**2*e**2*f**3))/(2*e) + (-8*a**2*c*f**3 - a*b**2*f**3 -
10*a*b*c*d**2*f**3 - 2*b**3*d**2*f**3 - 9*b**2*c*d**4*f**3 - 6*b*c**2*d**6*
f**3 - 36*b*c**2*d*e**5*f**3*x**5 - 6*b*c**2*e**6*f**3*x**6 + x**4*(-9*b**2
*c*e**4*f**3 - 90*b*c**2*d**2*e**4*f**3) + x**3*(-36*b**2*c*d*e**3*f**3 - 1
20*b*c**2*d**3*e**3*f**3) + x**2*(-10*a*b*c*e**2*f**3 - 2*b**3*e**2*f**3 -
54*b**2*c*d**2*e**2*f**3 - 90*b*c**2*d**4*e**2*f**3) + x*(-20*a*b*c*d*e*f**
3 - 4*b**3*d*e*f**3 - 36*b**2*c*d**3*e*f**3 - 36*b*c**2*d**5*e*f**3))/(64*a
**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d**2*e + 128*a**3*c**3*d**4
*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*a**2*b*c**3*d**6*e + 64*a*
```

```
*2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d**4*e - 64*a*b**3*c**2*d**6
*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b**5*c*d**6*e + 4*b**4*c**2*
d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3*e**9 + 4*b**4*c**2*e**9)
+ x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3*d*e**8 + 32*b**4*c**2*d*e**8
) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**4*d**2*e**7 - 64*a*b**3*c**2*
e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e**7 + 112*b**4*c**2*d**2*e**7)
 + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c**4*d**3*e**6 - 384*a*b**3*c**
2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**5*c*d*e**6 + 224*b**4*c**2*d*
*3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**2*b*c**3*d**2*e**5 + 4480*a**
2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b**3*c**2*d**2*e**5 - 2240*a*b*
*2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d**2*e**5 + 280*b**4*c**2*d**4
*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**2*b*c**3*d**3*e**4 + 3584*a**
2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a*b**3*c**2*d**3*e**4 - 1792*a
*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b**5*c*d**3*e**4 + 224*b**4*c**
2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 768*a**3*c**3*d**2*e**3 - 64*a*
*2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 + 1792*a**2*c**4*d**6*e**3 + 8*
a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b**3*c**2*d**4*e**3 - 896*a*b*
*2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b**5*c*d**4*e**3 + 112*b**4*c**
2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512*a**3*c**3*d**3*e**2 - 128*a*
*2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 + 512*a**2*c**4*d**7*e**2 + 16
*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a*b**3*c**2*d**5*e**2 - 256*a*
b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b**5*c*d**5*e**2 + 32*b**4*c**
2*d**7*e**2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(155) = 310$.
time = 3.90, size = 447, normalized size = 2.81

$$-\frac{3\,bcf^3\arctan\left(\frac{2\,cd^2f+2\,(fx^2e+2\,dfx)ce+bf}{\sqrt{-b^2+4\,ac}\,f}\right)e^{(-1)}}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{6\,bc^2d^6f^7+18\,(fx^2e+2\,dfx)bc^2d^4f^6e+9\,b^2cd^4f^7+18\,(fx^2e+2\,dfx)^2bc^2d^2f^5e^2+18\,(fx^2e+2\,dfx)b^2cd^2f^6e+2\,b^3d^2f^7+10\,abcd^2f^7+6\,(fx^2e+2\,dfx)^3bc^2f^4e^3+9\,(fx^2e+2\,dfx)^2b^2cf^5e^2+2\,(fx^2e+2\,dfx)b^3f^6e+10\,(fx^2e+2\,dfx)abcf^6e+ab^2f^7+8\,a^2cf^7}{4\,\left(cd^4f^2+2\,(fx^2e+2\,dfx)cd^2fe+bd^2f^2+(fx^2e+2\,dfx)^2ce^2+(fx^2e+2\,dfx)bfe+af^2\right)^2(b^4e-8\,ab^2ce+16\,a^2c^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*f^3*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2
+ 4*a*c)*f))*e^(-1)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1
/4*(6*b*c^2*d^6*f^7 + 18*(f*x^2*e + 2*d*f*x)*b*c^2*d^4*f^6*e + 9*b^2*c*d^4*
f^7 + 18*(f*x^2*e + 2*d*f*x)^2*b*c^2*d^2*f^5*e^2 + 18*(f*x^2*e + 2*d*f*x)*b
^2*c*d^2*f^6*e + 2*b^3*d^2*f^7 + 10*a*b*c*d^2*f^7 + 6*(f*x^2*e + 2*d*f*x)^3
*b*c^2*f^4*e^3 + 9*(f*x^2*e + 2*d*f*x)^2*b^2*c*f^5*e^2 + 2*(f*x^2*e + 2*d*f
*x)*b^3*f^6*e + 10*(f*x^2*e + 2*d*f*x)*a*b*c*f^6*e + a*b^2*f^7 + 8*a^2*c*f^
7)/((c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*f*e + b*d^2*f^2 + (f*x^2*e + 2
*d*f*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e + a*f^2)^2*(b^4*e - 8*a*b^2*c*e
 + 16*a^2*c^2*e))
```

Mupad [B]

time = 4.02, size = 1267, normalized size = 7.97

$$-\frac{\frac{8a^2cf^3+ab^2f^3+10abcd^2f^3+2b^3d^2f^3+9b^2cd^4f^3+6bc^2d^6f^3}{4e\,(16a^2c^2-8ab^2c+b^4)}+\frac{x^2\,(eb^3f^3+27eb^2cd^2f^3+45ebc^2d^4f^3+5aebcf^3)}{2\,(16a^2c^2-8ab^2c+b^4)}+\frac{9x^4\,(b^2ce^3f^3+10bc^2d^2e^3f^3)}{4\,(16a^2c^2-8ab^2c+b^4)}+\frac{3dx^3\,(3b^2ce^2f^3+10bc^2d^2e^2f^3)}{16a^2c^2-8ab^2c+b^4}+\frac{dx\,(b^3f^3+9b^2cd^2f^3+9bc^2d^4f^3+5abcf^3)}{16a^2c^2-8ab^2c+b^4}+\frac{3bc^2e^5f^3x^6}{2\,(16a^2c^2-8ab^2c+b^4)}+\frac{9bc^2de^4f^3x^5}{16a^2c^2-8ab^2c+b^4}}{x^2\,(6b^2d^2e^2+30bcd^4e^2+2abe^2+28c^2d^6e^2+12acd^2e^2)+x^6\,(28c^2d^2e^6+2bce^6)+x\,(4eb^2d^3+12ebcd^5+4aebd+8ec^2d^7+8aecd^3)+x^3\,(4b^2de^3+40bcd^3e^3+56c^2d^5e^3+8acde^3)+x^5\,(56c^2d^3e^5+12bcde^5)+x^4\,(b^2e^4+30bcd^2e^4+70c^2d^4e^4+2ace^4)+a^2+b^2d^4+c^2d^8+c^2e^8x^8+2abd^2+2acd^4+2bcd^6+8c^2de^7x^7}-\frac{3bcf^3\,\mathrm{atan}\left(\frac{\left(b^4\,(4ac-b^2)^5+16a^2c^2\,(4ac-b^2)^5-8ab^2c\,(4ac-b^2)^5\right)\left(x^2\left(\frac{9b^2c^4e^8f^6}{a\,(4ac-b^2)^{9/2}\,(16a^2c^2-8ab^2c+b^4)}+\frac{9b^3c^2f^6\,(32a^2bc^4e^{10}-16ab^3c^3e^{10}+2b^5c^2e^{10})}{2ae^2\,(4ac-b^2)^{15/2}\,(16a^2c^2-8ab^2c+b^4)}\right)+x\left(\frac{9b^3c^2f^6\,(32da^2bc^4e^9-16dab^3c^3e^9+2db^5c^2e^9)}{ae^2\,(4ac-b^2)^{15/2}\,(16a^2c^2-8ab^2c+b^4)}+\frac{18b^2c^4de^7f^6}{a\,(4ac-b^2)^{9/2}\,(16a^2c^2-8ab^2c+b^4)}\right)+\frac{9b^2c^4d^2e^6f^6}{a\,(4ac-b^2)^{9/2}\,(16a^2c^2-8ab^2c+b^4)}+\frac{9b^3c^2f^6\,(64a^3c^4e^8-32a^2b^2c^3e^8+32a^2bc^4d^2e^8+4ab^4c^2e^8-16ab^3c^3d^2e^8+2b^5c^2d^2e^8)}{2ae^2\,(4ac-b^2)^{15/2}\,(16a^2c^2-8ab^2c+b^4)}\right)}{18b^2c^4e^6f^6}\right)}{e\,(4ac-b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^3/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] - ((a*b^2*f^3 + 8*a^2*c*f^3 + 2*b^3*d^2*f^3 + 9*b^2*c*d^4*f^3 + 6*b*c^2*d^6
*f^3 + 10*a*b*c*d^2*f^3)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(b^3*e
*f^3 + 27*b^2*c*d^2*e*f^3 + 45*b*c^2*d^4*e*f^3 + 5*a*b*c*e*f^3))/(2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (9*x^4*(b^2*c*e^3*f^3 + 10*b*c^2*d^2*e^3*f^3))/(
4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (3*d*x^3*(3*b^2*c*e^2*f^3 + 10*b*c^2*d^
2*e^2*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (d*x*(b^3*f^3 + 9*b^2*c*d^2*f^
3 + 9*b*c^2*d^4*f^3 + 5*a*b*c*f^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*b*c
^2*e^5*f^3*x^6)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*d*e^4*f^3*x^5
)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*
a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^
6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e)
 + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*
(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4
 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*
a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) - (3*b*c*f^3*atan(((b^4*(4*a*c - b^2
)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x^2*((9*b^2*
c^4*e^8*f^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^
3*c^2*f^6*(2*b^5*c^2*e^10 - 16*a*b^3*c^3*e^10 + 32*a^2*b*c^4*e^10))/(2*a*e^
2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((9*b^3*c^2*f^6
*(2*b^5*c^2*d*e^9 - 16*a*b^3*c^3*d*e^9 + 32*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c
 - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (18*b^2*c^4*d*e^7*f^6)/(a*
(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (9*b^2*c^4*d^2*e^6*f
^6)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b^3*c^2*f^6
*(64*a^3*c^4*e^8 + 4*a*b^4*c^2*e^8 - 32*a^2*b^2*c^3*e^8 + 2*b^5*c^2*d^2*e^8
 - 16*a*b^3*c^3*d^2*e^8 + 32*a^2*b*c^4*d^2*e^8))/(2*a*e^2*(4*a*c - b^2)^(15
/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))/(18*b^2*c^4*e^6*f^6)))/(e*(4*a*c - b^
2)^(5/2))
```

3.656 $\int \frac{(df+efx)^2}{(a+b(d+ex)^2+c(d+ex)^4)^3}\,dx$

Optimal. Leaf size=375

$$-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{f^2(d+ex)\left(b(b^2+8ac)+c(b^2+20ac)(d+ex)^2\right)}{8a\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{c}\left(b^2\right.}{}$$

```
[Out] -1/4*f^2*(e*x+d)*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^
4)^2+1/8*f^2*(e*x+d)*(b*(8*a*c+b^2)+c*(20*a*c+b^2)*(e*x+d)^2)/a/(-4*a*c+b^2
)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/16*f^2*arctan((e*x+d)*2^(1/2)*c^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+20*a*c+b*(-52*a*c+b^2)/(-4*a*c+b^
2)^(1/2))/a/(-4*a*c+b^2)^2/e*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*f^2*
arctan((e*x+d)*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2+2
0*a*c-b*(-52*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^2/e*2^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.67, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {1156, 1133, 1192, 1180, 211}

$$\frac{\sqrt{c}\,f^2\left(\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,ae\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\,f^2\left(-\frac{b(b^2-52ac)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,ae\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{f^2(d+ex)\left(c(20ac+b^2)(d+ex)^2+b(8ac+b^2)\right)}{8ae\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -1/4*(f^2*(d + e*x)*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*e*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)^2) + (f^2*(d + e*x)*(b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c
)*(d + e*x)^2))/(8*a*(b^2 - 4*a*c)^2*e*(a + b*(d + e*x)^2 + c*(d + e*x)^4))
 + (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b
^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e) + (Sqrt[c]*(b^2 + 20*a*c - (b*
(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c])*f^2*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2
 - 4*a*c]]*e)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{(df+efx)^2}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f^2\text{Subst}\left(\int \frac{x^2}{(a+bx^2+cx^4)^3}\,dx,x,d+ex\right)}{e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2\text{Subst}\left(\int \frac{b-10}{(a+bx^2-}\right.}{4\left(b^2-\right.}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b(b^2+\right.}{8a\left(b^2-4ac\right)^2e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b(b^2+\right.}{8a\left(b^2-4ac\right)^2e}$$

$$= -\frac{f^2(d+ex)\left(b+2c(d+ex)^2\right)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{f^2(d+ex)\left(b(b^2+\right.}{8a\left(b^2-4ac\right)^2e}$$

Mathematica [A]
time = 3.26, size = 385, normalized size = 1.03

$$\frac{f^2\left(-\frac{4\left(b(d+ex)+2c(d+ex)^3\right)}{(b^2-4ac)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2(d+ex)\left(b^3+8abc+b^2c(d+ex)^2+20ac^2(d+ex)^2\right)}{a(b^2-4ac)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b^3+52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)}{16e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f^2*((-4*(b*(d + e*x) + 2*c*(d + e*x)^3))/((b^2 - 4*a*c)*(a + b*(d + e*x)^
2 + c*(d + e*x)^4)^2) + (2*(d + e*x)*(b^3 + 8*a*b*c + b^2*c*(d + e*x)^2 + 2
0*a*c^2*(d + e*x)^2))/(a*(b^2 - 4*a*c)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]
*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c
)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])))/(16*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.20, size = 889, normalized size = 2.37

method	result
default	$f^2\left(\frac{\frac{c^2e^6(20ac+b^2)x^7}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{7c^2d\,e^5(20ac+b^2)x^6}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{(420a\,c^2d^2+21b^2c\,d^2+28abc+2b^3)c\,e^4x^5}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{5cd\,e^3(140a\,c^2d^2+7b^2c\,d^2+28abc+2b^3)x^4}{8(16a^2c^2-8a\,b^2c+b^4)a}+\cdots}{}\right.$
risch	$\frac{\frac{c^2e^6f^2(20ac+b^2)x^7}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{7c^2d\,e^5f^2(20ac+b^2)x^6}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{(420a\,c^2d^2+21b^2c\,d^2+28abc+2b^3)c\,e^4f^2x^5}{8(16a^2c^2-8a\,b^2c+b^4)a}+\frac{5cd\,e^3f^2(140a\,c^2d^2+7b^2c\,d^2+28abc+2b^3)x^4}{8(16a^2c^2-8a\,b^2c+b^4)a}+\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f^2*((1/8*c^2*e^6*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^7+7/8*c^2*d*e
^5*(20*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^6+1/8*(420*a*c^2*d^2+21*b^2*
c*d^2+28*a*b*c+2*b^3)*c*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^5+5/8*c*d*e^3*(1
40*a*c^2*d^2+7*b^2*c*d^2+28*a*b*c+2*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^4+1
/8*e^2*(700*a*c^3*d^4+35*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+36*a^2*c^
2+5*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^3+1/8*d*e*(420*a*c^3*d^4+21
*b^2*c^2*d^4+280*a*b*c^2*d^2+20*b^3*c*d^2+108*a^2*c^2+15*a*b^2*c+3*b^4)/(16
*a^2*c^2-8*a*b^2*c+b^4)/a*x^2+1/8*(140*a*c^3*d^6+7*b^2*c^2*d^6+140*a*b*c^2*
d^4+10*b^3*c*d^4+108*a^2*c^2*d^2+15*a*b^2*c*d^2+3*b^4*d^2+16*a^2*b*c-a*b^3)
/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x+1/8*d/e*(20*a*c^3*d^6+b^2*c^2*d^6+28*a*b*c^
2*d^4+2*b^3*c*d^4+36*a^2*c^2*d^2+5*a*b^2*c*d^2+b^4*d^2+16*a^2*b*c-a*b^3)/(1
6*a^2*c^2-8*a*b^2*c+b^4)/a)/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^
3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/16/(16*a^2*c^2-8*a*b^2*c+b^4)/
a/e*sum((c*e^2*(20*a*c+b^2)*_R^2+2*c*d*e*(20*a*c+b^2)*_R+20*a*c^2*d^2+b^2*c
*d^2-16*a*b*c+b^3)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e
+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2
+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] 1/8*f^2*integrate((b^3 - 16*a*b*c + (b^2*c + 20*a*c^2)*d^2 + 2*(b^2*c*e + 2
0*a*c^2*e)*d*x + (b^2*c*e^2 + 20*a*c^2*e^2)*x^2)/(c*x^4*e^4 + 4*c*d*x^3*e^3
```

```
 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a)
, x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + 1/8*(7*(b^2*c^2*e^6 + 20*a*c^3*e^
6)*d*f^2*x^6 + (b^2*c^2*e^7 + 20*a*c^3*e^7)*f^2*x^7 + (2*b^3*c*e^5 + 28*a*b
*c^2*e^5 + 21*(b^2*c^2*e^5 + 20*a*c^3*e^5)*d^2)*f^2*x^5 + 5*(7*(b^2*c^2*e^4
 + 20*a*c^3*e^4)*d^3 + 2*(b^3*c*e^4 + 14*a*b*c^2*e^4)*d)*f^2*x^4 + (35*(b^2
*c^2*e^3 + 20*a*c^3*e^3)*d^4 + b^4*e^3 + 5*a*b^2*c*e^3 + 36*a^2*c^2*e^3 + 2
0*(b^3*c*e^3 + 14*a*b*c^2*e^3)*d^2)*f^2*x^3 + (21*(b^2*c^2*e^2 + 20*a*c^3*e
^2)*d^5 + 20*(b^3*c*e^2 + 14*a*b*c^2*e^2)*d^3 + 3*(b^4*e^2 + 5*a*b^2*c*e^2
+ 36*a^2*c^2*e^2)*d)*f^2*x^2 + (7*(b^2*c^2*e + 20*a*c^3*e)*d^6 + 10*(b^3*c*
e + 14*a*b*c^2*e)*d^4 - a*b^3*e + 16*a^2*b*c*e + 3*(b^4*e + 5*a*b^2*c*e + 3
6*a^2*c^2*e)*d^2)*f^2*x + ((b^2*c^2 + 20*a*c^3)*d^7 + 2*(b^3*c + 14*a*b*c^2
)*d^5 + (b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 - (a*b^3 - 16*a^2*b*c)*d)*f^2)/(
(a*b^4*c^2*e - 8*a^2*b^2*c^3*e + 16*a^3*c^4*e)*d^8 + 8*(a*b^4*c^2*e^8 - 8*a
^2*b^2*c^3*e^8 + 16*a^3*c^4*e^8)*d*x^7 + (a*b^4*c^2*e^9 - 8*a^2*b^2*c^3*e^9
 + 16*a^3*c^4*e^9)*x^8 + a^3*b^4*e - 8*a^4*b^2*c*e + 16*a^5*c^2*e + 2*(a*b^
5*c*e - 8*a^2*b^3*c^2*e + 16*a^3*b*c^3*e)*d^6 + 2*(a*b^5*c*e^7 - 8*a^2*b^3*
c^2*e^7 + 16*a^3*b*c^3*e^7 + 14*(a*b^4*c^2*e^7 - 8*a^2*b^2*c^3*e^7 + 16*a^3
*c^4*e^7)*d^2)*x^6 + 4*(14*(a*b^4*c^2*e^6 - 8*a^2*b^2*c^3*e^6 + 16*a^3*c^4*
e^6)*d^3 + 3*(a*b^5*c*e^6 - 8*a^2*b^3*c^2*e^6 + 16*a^3*b*c^3*e^6)*d)*x^5 +
(a*b^6*e - 6*a^2*b^4*c*e + 32*a^4*c^3*e)*d^4 + (a*b^6*e^5 - 6*a^2*b^4*c*e^5
 + 32*a^4*c^3*e^5 + 70*(a*b^4*c^2*e^5 - 8*a^2*b^2*c^3*e^5 + 16*a^3*c^4*e^5)
*d^4 + 30*(a*b^5*c*e^5 - 8*a^2*b^3*c^2*e^5 + 16*a^3*b*c^3*e^5)*d^2)*x^4 + 4
*(14*(a*b^4*c^2*e^4 - 8*a^2*b^2*c^3*e^4 + 16*a^3*c^4*e^4)*d^5 + 10*(a*b^5*c
*e^4 - 8*a^2*b^3*c^2*e^4 + 16*a^3*b*c^3*e^4)*d^3 + (a*b^6*e^4 - 6*a^2*b^4*c
*e^4 + 32*a^4*c^3*e^4)*d)*x^3 + 2*(a^2*b^5*e - 8*a^3*b^3*c*e + 16*a^4*b*c^2
*e)*d^2 + 2*(a^2*b^5*e^3 - 8*a^3*b^3*c*e^3 + 16*a^4*b*c^2*e^3 + 14*(a*b^4*c
^2*e^3 - 8*a^2*b^2*c^3*e^3 + 16*a^3*c^4*e^3)*d^6 + 15*(a*b^5*c*e^3 - 8*a^2*
b^3*c^2*e^3 + 16*a^3*b*c^3*e^3)*d^4 + 3*(a*b^6*e^3 - 6*a^2*b^4*c*e^3 + 32*a
^4*c^3*e^3)*d^2)*x^2 + 4*(2*(a*b^4*c^2*e^2 - 8*a^2*b^2*c^3*e^2 + 16*a^3*c^4
*e^2)*d^7 + 3*(a*b^5*c*e^2 - 8*a^2*b^3*c^2*e^2 + 16*a^3*b*c^3*e^2)*d^5 + (a
*b^6*e^2 - 6*a^2*b^4*c*e^2 + 32*a^4*c^3*e^2)*d^3 + (a^2*b^5*e^2 - 8*a^3*b^3
*c*e^2 + 16*a^4*b*c^2*e^2)*d)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7734 vs. $2(337) = 674$.
time = 0.61, size = 7734, normalized size = 20.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*f^2*x^7*e^7 + 14*(b^2*c^2 + 20*a*c^3)*d*f^2*x^
6*e^6 + 2*(2*b^3*c + 28*a*b*c^2 + 21*(b^2*c^2 + 20*a*c^3)*d^2)*f^2*x^5*e^5
```

+ 10*(7*(b^2*c^2 + 20*a*c^3)*d^3 + 2*(b^3*c + 14*a*b*c^2)*d)*f^2*x^4*e^4 + 2*(35*(b^2*c^2 + 20*a*c^3)*d^4 + b^4 + 5*a*b^2*c + 36*a^2*c^2 + 20*(b^3*c + 14*a*b*c^2)*d^2)*f^2*x^3*e^3 + 2*(21*(b^2*c^2 + 20*a*c^3)*d^5 + 20*(b^3*c + 14*a*b*c^2)*d^3 + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d)*f^2*x^2*e^2 + 2*(7*(b^2*c^2 + 20*a*c^3)*d^6 + 10*(b^3*c + 14*a*b*c^2)*d^4 - a*b^3 + 16*a^2*b*c + 3*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^2)*f^2*x*e + 2*((b^2*c^2 + 20*a*c^3)*d^7 + 2*(b^3*c + 14*a*b*c^2)*d^5 + (b^4 + 5*a*b^2*c + 36*a^2*c^2)*d^3 - (a*b^3 - 16*a^2*b*c)*d)*f^2 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8*e^9 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*x^7*e^8 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*x^6*e^7 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*x^5*e^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^2)*x^4*e^5 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d)*x^3*e^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*x^2*e^3 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*x*e^2 + ((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*e^(-2)/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*f^6*x*e + (35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*d*f^6 + 1/2*sqrt(1/2)*((b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5)*f^4*e - (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5))*e)*sqrt(-((b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)*f^8/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*e^(-2)/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8*e^9 + 8*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d*x^7*e^8 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^2)*x

```
^6*e^7 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^3 + 3*(a*b^5*c -
8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d)*x^5*e^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^
3 + 70*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^4 + 30*(a*b^5*c - 8*a^2*b
^3*c^2 + 16*a^3*b*c^3)*d^2)*x^4*e^5 + 4*(14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16
*a^3*c^4)*d^5 + 10*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*d^3 + (a*b^6 -
6*a^2*b^4*c + 32*a^4*c^3)*d)*x^3*e^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*
c^2 + 14*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^6 + 15*(a*b^5*c - 8*a^2
*b^3*c^2 + 16*a^3*b*c^3)*d^4 + 3*(a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^2)*x^
2*e^3 + 4*(2*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d^7 + 3*(a*b^5*c - 8*
a^2*b^3*c^2 + 16*a^3*b*c^3)*d^5 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^3 +
(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d)*x*e^2 + ((a*b^4*c^2 - 8*a^2*b^2*c
^3 + 16*a^3*c^4)*d^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*
a^2*b^3*c^2 + 16*a^3*b*c^3)*d^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*d^4 +
2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d^2)*e)*sqrt(-((b^7 - 35*a*b^5*c +
 280*a^2*b^3*c^2 + 1680*a^3*b*c^3)*f^4 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5
*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 5
0*a*b^2*c + 625*a^2*c^2)*f^8/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 6
40*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*e^(-2)/(a^3*b^10 - 20
*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^
8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 1500...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. $2(337) = 674$.
time = 3.69, size = 2527, normalized size = 6.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(((d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))^2*b^2*c*f^2*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*
a*c)*e^2)*e^(-4)/c))^2*a*c^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*f^2*e - 40*(d*e^(-1) + sqrt(1/
2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*f^2*e + b^2*c*d
^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)*log(d*e^(-1) + x + sqrt
```

(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1)
 + sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2
*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2
 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^2*c*f^2*e^2 + 20*(d*e^(-1) - sqrt(
1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*c^2*f^2*e^2 - 2*(
d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c
*d*f^2*e - 40*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e
^(-4)/c))*a*c^2*d*f^2*e + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a
*b*c*f^2)*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2
)*e^(-4)/c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a
*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(
d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 + sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((
d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^2
*c*f^2*e^2 + 20*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)
*e^(-4)/c))^2*a*c^2*f^2*e^2 - 2*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b
^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*f^2*e - 40*(d*e^(-1) + sqrt(1/2)*sqrt(-
(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*a*c^2*d*f^2*e + b^2*c*d^2*f^2 +
20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)*log(d*e^(-1) + x + sqrt(1/2)*sqr
t(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))/(2*(d*e^(-1) + sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^3*c*e^4 - 6*(d*e^(-1) + sqrt(1
/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e
- b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1) + sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b
^2 - 4*a*c)*e^2)*e^(-4)/c))) + ((d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b
^2 - 4*a*c)*e^2)*e^(-4)/c))^2*b^2*c*f^2*e^2 + 20*(d*e^(-1) - sqrt(1/2)*sqrt
(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))^2*a*c^2*f^2*e^2 - 2*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))*b^2*c*d*f^2*e
- 40*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))
*a*c^2*d*f^2*e + b^2*c*d^2*f^2 + 20*a*c^2*d^2*f^2 + b^3*f^2 - 16*a*b*c*f^2)
*log(d*e^(-1) + x - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/
c))/(2*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c
))^3*c*e^4 - 6*(d*e^(-1) - sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*
e^(-4)/c))^2*c*d*e^3 - 2*c*d^3*e - b*d*e + (6*c*d^2*e^2 + b*e^2)*(d*e^(-1)
- sqrt(1/2)*sqrt(-(b*e^2 - sqrt(b^2 - 4*a*c)*e^2)*e^(-4)/c))))/(a*b^4 - 8*a
^2*b^2*c + 16*a^3*c^2) + 1/8*(b^2*c^2*f^2*x^7*e^7 + 20*a*c^3*f^2*x^7*e^7 +
7*b^2*c^2*d*f^2*x^6*e^6 + 140*a*c^3*d*f^2*x^6*e^6 + 21*b^2*c^2*d^2*f^2*x^5*
e^5 + 420*a*c^3*d^2*f^2*x^5*e^5 + 35*b^2*c^2*d^3*f^2*x^4*e^4 + 700*a*c^3*d^
3*f^2*x^4*e^4 + 35*b^2*c^2*d^4*f^2*x^3*e^3 + 700*a*c^3*d^4*f^2*x^3*e^3 + 21
*b^2*c^2*d^5*f^2*x^2*e^2 + 420*a*c^3*d^5*f^2*x^2*e^2 + 7*b^2*c^2*d^6*f^2*x*
e + 140*a*c^3*d^6*f^2*x*e + b^2*c^2*d^7*f^2 + 20*a*c^3*d^7*f^2 + 2*b^3*c*f^
2*x^5*e^5 + 28*a*b*c^2*f^2*x^5*e^5 + 10*b^3*c*d*f^2*x^4*e^4 + 140*a*b*c^2*d
*f^2*x^4*e^4 + 20*b^3*c*d^2*f^2*x^3*e^3 + 280*a*b*c^2*d^2*f^2*x^3*e^3 + 20*
b^3*c*d^3*f^2*x^2*e^2 + 280*a*b*c^2*d^3*f^2*x^2*e^2 + 10*b^3*c*d^4*f^2*x*e

```
+ 140*a*b*c^2*d^4*f^2*x*e + 2*b^3*c*d^5*f^2 + 28*a*b*c^2*d^5*f^2 + b^4*f^2*
x^3*e^3 + 5*a*b^2*c*f^2*x^3*e^3 + 36*a^2*c^2*f^2*x^3*e^3 + 3*b^4*d*f^2*x^2*
e^2 + 15*a*b^2*c*d*f^2*x^2*e^2 + 108*a^2*c^2*d*f^2*x^2*e^2 + 3*b^4*d^2*f^2*
x*e + 15*a*b^2*c*d^2*f^2*x*e + 108*a^2*c^2*d^2*f^2*x*e + b^4*d^3*f^2 + 5*a*
b^2*c*d^3*f^2 + 36*a^2*c^2*d^3*f^2 - a*b^3*f^2*x*e + 16*a^2*b*c*f^2*x*e - a
*b^3*d*f^2 + 16*a^2*b*c*d*f^2)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^
2 + 4*c*d^3*x*e + c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(a*b^4*e - 8
*a^2*b^2*c*e + 16*a^3*c^2*e))
```

Mupad [B]
time = 7.94, size = 2500, normalized size = 6.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)^2/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] atan((((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3
*b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13
 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^
8*b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b
^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(262
144*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*
a^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*
(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))
*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 +
1140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43
776*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55
*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 +
1048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6
*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a
^9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621
440*a^12*b^2*c^9*e^2)))^(1/2) - (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^1
1*c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2
 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c
^8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^
2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 +
 b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*
c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^
5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4
- 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c
^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2
+ 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2
 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c
^9*e^2)))^(1/2) + (204800*a^5*c^8*d*e^11*f^4 - 16*b^10*c^3*d*e^11*f^4 + 672
```

*a*b^8*c^4*d*e^11*f^4 - 28160*a^2*b^6*c^5*d*e^11*f^4 + 209920*a^3*b^4*c^6*d *e^11*f^4 - 479232*a^4*b^2*c^7*d*e^11*f^4)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 614 4*a^7*b^2*c^5)) + (x*(800*a^3*c^6*e^12*f^4 - b^6*c^3*e^12*f^4 - 1472*a^2*b^ 2*c^5*e^12*f^4 + 34*a*b^4*c^4*e^12*f^4))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^ 3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(b^17*f^4 + b^2*f^4*(-(4*a* c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c^2*f^4 - 10160* a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5*f^4 - 680960*a ^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^10*e^2 - 40*a^4 *b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12 *c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^9*b^8*c^6*e^2 - 1966080*a^10* b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 2621440*a^12*b^2*c^9*e^2)))^(1/2)* 1i + ((((67108864*a^9*b*c^9*d*e^13 - 4096*a^2*b^15*c^2*d*e^13 + 114688*a^3* b^13*c^3*d*e^13 - 1376256*a^4*b^11*c^4*d*e^13 + 9175040*a^5*b^9*c^5*d*e^13 - 36700160*a^6*b^7*c^6*d*e^13 + 88080384*a^7*b^5*c^7*d*e^13 - 117440512*a^8 *b^3*c^8*d*e^13)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^ 8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(2621 44*a^7*b*c^7*e^14 - 256*a^2*b^11*c^2*e^14 + 5120*a^3*b^9*c^3*e^14 - 40960*a ^4*b^7*c^4*e^14 + 163840*a^5*b^5*c^5*e^14 - 327680*a^6*b^3*c^6*e^14))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))* (-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1 140*a^2*b^13*c^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 437 76*a^5*b^7*c^5*f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55* a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1 048576*a^13*c^10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6* b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e^2 + 860160*a^ 9*b^8*c^6*e^2 - 1966080*a^10*b^6*c^7*e^2 + 2949120*a^11*b^4*c^8*e^2 - 26214 40*a^12*b^2*c^9*e^2)))^(1/2) + (122880*a^3*b^9*c^4*e^12*f^2 - 9216*a^2*b^11 *c^3*e^12*f^2 - 819200*a^4*b^7*c^5*e^12*f^2 + 2949120*a^5*b^5*c^6*e^12*f^2 - 5505024*a^6*b^3*c^7*e^12*f^2 + 256*a*b^13*c^2*e^12*f^2 + 4194304*a^7*b*c^ 8*e^12*f^2)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)))*(-(b^17*f^4 + b^2*f^4*(-(4*a*c - b^2)^15)^(1/2) - 1720320*a^8*b*c^8*f^4 + 1140*a^2*b^13*c ^2*f^4 - 10160*a^3*b^11*c^3*f^4 + 34880*a^4*b^9*c^4*f^4 + 43776*a^5*b^7*c^5 *f^4 - 680960*a^6*b^5*c^6*f^4 + 1863680*a^7*b^3*c^7*f^4 - 55*a*b^15*c*f^4 - 25*a*c*f^4*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^3*b^20*e^2 + 1048576*a^13*c^ 10*e^2 - 40*a^4*b^18*c*e^2 + 720*a^5*b^16*c^2*e^2 - 7680*a^6*b^14*c^3*e^2 + 53760*a^7*b^12*c^4*e^2 - 258048*a^8*b^10*c^5*e...

3.657 $\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{f(b+2c(d+ex)^2)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{3cf(b+2c(d+ex)^2)}{2\left(b^2-4ac\right)^2e\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{6c^2f\tanh^{-1}}{(b^2-}$$

```
[Out] -1/4*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)^2+3/2*c
*f*(b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^2/e/(a+b*(e*x+d)^2+c*(e*x+d)^4)-6*c^2*f*a
rctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/e
```

Rubi [A]
time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1156, 1121, 628, 632, 212}

$$-\frac{6c^2f\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{e\left(b^2-4ac\right)^{5/2}}+\frac{3cf(b+2c(d+ex)^2)}{2e\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{f(b+2c(d+ex)^2)}{4e\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] -1/4*(f*(b + 2*c*(d + e*x)^2))/((b^2 - 4*a*c)*e*(a + b*(d + e*x)^2 + c*(d +
 e*x)^4)^2) + (3*c*f*(b + 2*c*(d + e*x)^2))/(2*(b^2 - 4*a*c)^2*e*(a + b*(d
+ e*x)^2 + c*(d + e*x)^4)) - (6*c^2*f*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^
2 - 4*a*c]])/((b^2 - 4*a*c)^(5/2)*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{df+efx}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{f\text{Subst}\left(\int \frac{x}{(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{e}$$

$$= \frac{f\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2e}$$

$$= -\frac{f(b+2c(d+ex)^2)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} - \frac{(3cf)\text{Subst}\left(\int \frac{}{(a+bx}\right.}{2\left(b^2\right.}$$

$$= -\frac{f(b+2c(d+ex)^2)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf(b+}{2\left(b^2-4ac\right)^2e(a+}$$

$$= -\frac{f(b+2c(d+ex)^2)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf(b+}{2\left(b^2-4ac\right)^2e(a+}$$

$$= -\frac{f(b+2c(d+ex)^2)}{4\left(b^2-4ac\right)e\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{3cf(b+}{2\left(b^2-4ac\right)^2e(a+}$$

Mathematica [A]
time = 0.12, size = 148, normalized size = 0.97

$$\frac{f\left(\frac{(b^2-4ac)\left(-b-2c(d+ex)^2\right)}{\left(a+b(d+ex)^2+c(d+ex)^4\right)^2} + \frac{6c\left(b+2c(d+ex)^2\right)}{a+b(d+ex)^2+c(d+ex)^4} + \frac{24c^2\tan^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}\right)}{4\left(b^2-4ac\right)^2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x]
```

```
[Out] (f*(((b^2 - 4*a*c)*(-b - 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)
^4)^2 + (6*c*(b + 2*c*(d + e*x)^2))/(a + b*(d + e*x)^2 + c*(d + e*x)^4) + (
24*c^2*ArcTan[(b + 2*c*(d + e*x)^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
))/(4*(b^2 - 4*a*c)^2*e)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.19, size = 543, normalized size = 3.55

method	result
default	$f\left(\frac{\frac{3c^3e^5x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18e^4c^3d\,x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6c^2d\,e^2\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{ce\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2cd\left(9c^2d^4\right.}{16a^2c}}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+c\right.}\right.$
risch	$\frac{\frac{3c^3e^5f\,x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{18f\,e^4c^3d\,x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{9c^2e^3f\left(10c\,d^2+b\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{6d\,e^2c^2f\left(10c\,d^2+3b\right)x^3}{16a^2c^2-8a\,b^2c+b^4}+\frac{cef\left(45c^2d^4+27bc\,d^2+5ac+b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}+\frac{2dcf\left(9c^2d^4+\right.}{16a^2c^2-}}{\left(c\,e^4x^4+4cd\,e^3x^3+6c\,d^2e^2x^2+4c\,d^3ex+b\,e^2x^2+d^4c+2debx+d^2b+a\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f*((3*c^3*e^5/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+18*e^4*c^3*d/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^5+9/2*c^2*e^3*(10*c*d^2+b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+6*c^2
*d*e^2*(10*c*d^2+3*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+c*e*(45*c^2*d^4+27*b*c
*d^2+5*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+2*c*d*(9*c^2*d^4+9*b*c*d^2+5
*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/4/e*(12*c^3*d^6+18*b*c^2*d^4+20*a*
c^2*d^2+4*b^2*c*d^2+10*a*b*c-b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*
c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^
2+3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((_R*e+d)/(2*_R^3*c*e^3+6*_R^2*c*d*
e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*
c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima")
```

```
[Out] 6*c^2*f*integrate((x*e + d)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6
*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/(b^4 - 8*a*b^2*c
 + 16*a^2*c^2) + 1/4*(12*c^3*f*x^6*e^6 + 72*c^3*d*f*x^5*e^5 + 18*(10*c^3*d^
2*e^4 + b*c^2*e^4)*f*x^4 + 24*(10*c^3*d^3*e^3 + 3*b*c^2*d*e^3)*f*x^3 + 4*(4
```

```
5*c^3*d^4*e^2 + 27*b*c^2*d^2*e^2 + b^2*c*e^2 + 5*a*c^2*e^2)*f*x^2 + 8*(9*c^
3*d^5*e + 9*b*c^2*d^3*e + (b^2*c*e + 5*a*c^2*e)*d)*f*x + (12*c^3*d^6 + 18*b
*c^2*d^4 - b^3 + 10*a*b*c + 4*(b^2*c + 5*a*c^2)*d^2)*f)/((b^4*c^2*e - 8*a*b
^2*c^3*e + 16*a^2*c^4*e)*d^8 + 8*(b^4*c^2*e^8 - 8*a*b^2*c^3*e^8 + 16*a^2*c^
4*e^8)*d*x^7 + (b^4*c^2*e^9 - 8*a*b^2*c^3*e^9 + 16*a^2*c^4*e^9)*x^8 + 2*(b^
5*c*e - 8*a*b^3*c^2*e + 16*a^2*b*c^3*e)*d^6 + 2*(b^5*c*e^7 - 8*a*b^3*c^2*e^
7 + 16*a^2*b*c^3*e^7 + 14*(b^4*c^2*e^7 - 8*a*b^2*c^3*e^7 + 16*a^2*c^4*e^7)*
d^2)*x^6 + a^2*b^4*e - 8*a^3*b^2*c*e + 16*a^4*c^2*e + 4*(14*(b^4*c^2*e^6 -
8*a*b^2*c^3*e^6 + 16*a^2*c^4*e^6)*d^3 + 3*(b^5*c*e^6 - 8*a*b^3*c^2*e^6 + 16
*a^2*b*c^3*e^6)*d)*x^5 + (b^6*e - 6*a*b^4*c*e + 32*a^3*c^3*e)*d^4 + (b^6*e^
5 - 6*a*b^4*c*e^5 + 32*a^3*c^3*e^5 + 70*(b^4*c^2*e^5 - 8*a*b^2*c^3*e^5 + 16
*a^2*c^4*e^5)*d^4 + 30*(b^5*c*e^5 - 8*a*b^3*c^2*e^5 + 16*a^2*b*c^3*e^5)*d^2
)*x^4 + 4*(14*(b^4*c^2*e^4 - 8*a*b^2*c^3*e^4 + 16*a^2*c^4*e^4)*d^5 + 10*(b^
5*c*e^4 - 8*a*b^3*c^2*e^4 + 16*a^2*b*c^3*e^4)*d^3 + (b^6*e^4 - 6*a*b^4*c*e^
4 + 32*a^3*c^3*e^4)*d)*x^3 + 2*(a*b^5*e - 8*a^2*b^3*c*e + 16*a^3*b*c^2*e)*d
^2 + 2*(14*(b^4*c^2*e^3 - 8*a*b^2*c^3*e^3 + 16*a^2*c^4*e^3)*d^6 + a*b^5*e^3
 - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + 15*(b^5*c*e^3 - 8*a*b^3*c^2*e^3 + 1
6*a^2*b*c^3*e^3)*d^4 + 3*(b^6*e^3 - 6*a*b^4*c*e^3 + 32*a^3*c^3*e^3)*d^2)*x^
2 + 4*(2*(b^4*c^2*e^2 - 8*a*b^2*c^3*e^2 + 16*a^2*c^4*e^2)*d^7 + 3*(b^5*c*e^
2 - 8*a*b^3*c^2*e^2 + 16*a^2*b*c^3*e^2)*d^5 + (b^6*e^2 - 6*a*b^4*c*e^2 + 32
*a^3*c^3*e^2)*d^3 + (a*b^5*e^2 - 8*a^2*b^3*c*e^2 + 16*a^3*b*c^2*e^2)*d)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1791 vs. $2(149) = 298$.
time = 0.48, size = 3710, normalized size = 24.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(b^2*c^3 - 4*a*c^4)*f*x^6*e^6 + 72*(b^2*c^3 - 4*a*c^4)*d*f*x^5*e^5
 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*f*x^4*e^4 + 24*(10
*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*f*x^3*e^3 + 4*(b^4*c
+ a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b
*c^3)*d^2)*f*x^2*e^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^
3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*f*x*e + 12*(c^4*f*x^8*e^8 + 8*
c^4*d*f*x^7*e^7 + 2*(14*c^4*d^2 + b*c^3)*f*x^6*e^6 + 4*(14*c^4*d^3 + 3*b*c^
3*d)*f*x^5*e^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*f*x^4*e^4
+ 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*f*x^3*e^3 + 2*(14*c
^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*f*x^2*e^2 + 4*
(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*f*x*e + (c^
4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^4 + a^2*c^2)*f)
*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4 + 2*b*c
*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2 - 2*a*
```

c - (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a)) + (12*(b^2*c^3 - 4*a*c^4)*d^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2)*f)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8*e^9 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^7*e^8 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*x^6*e^7 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*x^5*e^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*x^4*e^5 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*x^3*e^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2)*x^2*e^3 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*x*e^2 + ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2)*e), 1/4*(12*(b^2*c^3 - 4*a*c^4)*f*x^6*e^6 + 72*(b^2*c^3 - 4*a*c^4)*d*f*x^5*e^5 + 18*(b^3*c^2 - 4*a*b*c^3 + 10*(b^2*c^3 - 4*a*c^4)*d^2)*f*x^4*e^4 + 24*(10*(b^2*c^3 - 4*a*c^4)*d^3 + 3*(b^3*c^2 - 4*a*b*c^3)*d)*f*x^3*e^3 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3 + 45*(b^2*c^3 - 4*a*c^4)*d^4 + 27*(b^3*c^2 - 4*a*b*c^3)*d^2)*f*x^2*e^2 + 8*(9*(b^2*c^3 - 4*a*c^4)*d^5 + 9*(b^3*c^2 - 4*a*b*c^3)*d^3 + (b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d)*f*x*e - 24*(c^4*f*x^8*e^8 + 8*c^4*d*f*x^7*e^7 + 2*(14*c^4*d^2 + b*c^3)*f*x^6*e^6 + 4*(14*c^4*d^3 + 3*b*c^3*d)*f*x^5*e^5 + (70*c^4*d^4 + 30*b*c^3*d^2 + b^2*c^2 + 2*a*c^3)*f*x^4*e^4 + 4*(14*c^4*d^5 + 10*b*c^3*d^3 + (b^2*c^2 + 2*a*c^3)*d)*f*x^3*e^3 + 2*(14*c^4*d^6 + 15*b*c^3*d^4 + a*b*c^2 + 3*(b^2*c^2 + 2*a*c^3)*d^2)*f*x^2*e^2 + 4*(2*c^4*d^7 + 3*b*c^3*d^5 + a*b*c^2*d + (b^2*c^2 + 2*a*c^3)*d^3)*f*x*e + (c^4*d^8 + 2*b*c^3*d^6 + 2*a*b*c^2*d^2 + (b^2*c^2 + 2*a*c^3)*d^4 + a^2*c^2)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (12*(b^2*c^3 - 4*a*c^4)*d^6 - b^5 + 14*a*b^3*c - 40*a^2*b*c^2 + 18*(b^3*c^2 - 4*a*b*c^3)*d^4 + 4*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*d^2)*f)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8*e^9 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^7*e^8 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*x^

```
6*e^7 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 +
3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*x^5*e^6 + (b^8
- 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 + 70*(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c - 12*a*b^5*c^
2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*x^4*e^5 + 4*(14*(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*c^2 + 48*a^2
*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*
b^2*c^3 - 128*a^4*c^4)*d)*x^3*e^4 + 2*(a*b^7 - ...
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1707 vs. $2(139) = 278$.

time = 7.70, size = 1707, normalized size = 11.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] -3*c**2*f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (-192*a**3*c**5*f
*sqrt(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)**
5) - 36*a*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*f*sqrt(-1/(4
*a*c - b**2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + 3*c**2*
f*sqrt(-1/(4*a*c - b**2)**5)*log(2*d*x/e + x**2 + (192*a**3*c**5*f*sqrt(-1/
(4*a*c - b**2)**5) - 144*a**2*b**2*c**4*f*sqrt(-1/(4*a*c - b**2)**5) + 36*a
*b**4*c**3*f*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*f*sqrt(-1/(4*a*c - b*
*2)**5) + 3*b*c**2*f + 6*c**3*d**2*f)/(6*c**3*e**2*f))/e + (10*a*b*c*f + 20
*a*c**2*d**2*f - b**3*f + 4*b**2*c*d**2*f + 18*b*c**2*d**4*f + 12*c**3*d**6
*f + 72*c**3*d*e**5*f*x**5 + 12*c**3*e**6*f*x**6 + x**4*(18*b*c**2*e**4*f +
 180*c**3*d**2*e**4*f) + x**3*(72*b*c**2*d*e**3*f + 240*c**3*d**3*e**3*f) +
 x**2*(20*a*c**2*e**2*f + 4*b**2*c*e**2*f + 108*b*c**2*d**2*e**2*f + 180*c*
*3*d**4*e**2*f) + x*(40*a*c**2*d*e*f + 8*b**2*c*d*e*f + 72*b*c**2*d**3*e*f
+ 72*c**3*d**5*e*f))/(64*a**4*c**2*e - 32*a**3*b**2*c*e + 128*a**3*b*c**2*d
**2*e + 128*a**3*c**3*d**4*e + 4*a**2*b**4*e - 64*a**2*b**3*c*d**2*e + 128*
a**2*b*c**3*d**6*e + 64*a**2*c**4*d**8*e + 8*a*b**5*d**2*e - 24*a*b**4*c*d*
*4*e - 64*a*b**3*c**2*d**6*e - 32*a*b**2*c**3*d**8*e + 4*b**6*d**4*e + 8*b*
*5*c*d**6*e + 4*b**4*c**2*d**8*e + x**8*(64*a**2*c**4*e**9 - 32*a*b**2*c**3
*e**9 + 4*b**4*c**2*e**9) + x**7*(512*a**2*c**4*d*e**8 - 256*a*b**2*c**3*d*
e**8 + 32*b**4*c**2*d*e**8) + x**6*(128*a**2*b*c**3*e**7 + 1792*a**2*c**4*d
**2*e**7 - 64*a*b**3*c**2*e**7 - 896*a*b**2*c**3*d**2*e**7 + 8*b**5*c*e**7
+ 112*b**4*c**2*d**2*e**7) + x**5*(768*a**2*b*c**3*d*e**6 + 3584*a**2*c**4*
d**3*e**6 - 384*a*b**3*c**2*d*e**6 - 1792*a*b**2*c**3*d**3*e**6 + 48*b**5*c
*d*e**6 + 224*b**4*c**2*d**3*e**6) + x**4*(128*a**3*c**3*e**5 + 1920*a**2*b
*c**3*d**2*e**5 + 4480*a**2*c**4*d**4*e**5 - 24*a*b**4*c*e**5 - 960*a*b**3*
c**2*d**2*e**5 - 2240*a*b**2*c**3*d**4*e**5 + 4*b**6*e**5 + 120*b**5*c*d**2
*e**5 + 280*b**4*c**2*d**4*e**5) + x**3*(512*a**3*c**3*d*e**4 + 2560*a**2*b
```

```
*c**3*d**3*e**4 + 3584*a**2*c**4*d**5*e**4 - 96*a*b**4*c*d*e**4 - 1280*a*b*
*3*c**2*d**3*e**4 - 1792*a*b**2*c**3*d**5*e**4 + 16*b**6*d*e**4 + 160*b**5*
c*d**3*e**4 + 224*b**4*c**2*d**5*e**4) + x**2*(128*a**3*b*c**2*e**3 + 768*a
**3*c**3*d**2*e**3 - 64*a**2*b**3*c*e**3 + 1920*a**2*b*c**3*d**4*e**3 + 179
2*a**2*c**4*d**6*e**3 + 8*a*b**5*e**3 - 144*a*b**4*c*d**2*e**3 - 960*a*b**3
*c**2*d**4*e**3 - 896*a*b**2*c**3*d**6*e**3 + 24*b**6*d**2*e**3 + 120*b**5*
c*d**4*e**3 + 112*b**4*c**2*d**6*e**3) + x*(256*a**3*b*c**2*d*e**2 + 512*a*
*3*c**3*d**3*e**2 - 128*a**2*b**3*c*d*e**2 + 768*a**2*b*c**3*d**5*e**2 + 51
2*a**2*c**4*d**7*e**2 + 16*a*b**5*d*e**2 - 96*a*b**4*c*d**3*e**2 - 384*a*b*
*3*c**2*d**5*e**2 - 256*a*b**2*c**3*d**7*e**2 + 16*b**6*d**3*e**2 + 48*b**5
*c*d**5*e**2 + 32*b**4*c**2*d**7*e**2))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 445 vs. 2(149) = 298.
time = 3.10, size = 445, normalized size = 2.91

$$\frac{6\,c^{2}f\arctan\left(\frac{2\,cd^{2}f+2\,(fx^{2}e+2\,dfx)ce+bf}{\sqrt{-b^{2}+4\,ac}\,f}\right)e^{(-1)}}{(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2})\sqrt{-b^{2}+4\,ac}}+\frac{12\,c^{3}d^{6}f^{5}+36\,(fx^{2}e+2\,dfx)c^{3}d^{4}f^{4}e+18\,bc^{2}d^{4}f^{5}+36\,(fx^{2}e+2\,dfx)^{2}c^{3}d^{2}f^{3}e^{2}+36\,(fx^{2}e+2\,dfx)bc^{2}d^{2}f^{4}e+4\,b^{2}cd^{2}f^{5}+20\,ac^{2}d^{2}f^{5}+12\,(fx^{2}e+2\,dfx)^{3}c^{3}f^{2}e^{3}+18\,(fx^{2}e+2\,dfx)^{2}bc^{2}f^{3}e^{2}+4\,(fx^{2}e+2\,dfx)b^{2}cf^{4}e+20\,(fx^{2}e+2\,dfx)ac^{2}f^{4}e-b^{3}f^{5}+10\,abcf^{5}}{4\,(cd^{4}f^{2}+2\,(fx^{2}e+2\,dfx)cd^{2}fe+bd^{2}f^{2}+(fx^{2}e+2\,dfx)^{2}ce^{2}+(fx^{2}e+2\,dfx)bfe+af^{2})^{2}(b^{4}e-8\,ab^{2}ce+16\,a^{2}c^{2}e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*f*arctan((2*c*d^2*f + 2*(f*x^2*e + 2*d*f*x)*c*e + b*f)/(sqrt(-b^2 + 4
*a*c)*f))*e^(-1)/((b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*
(12*c^3*d^6*f^5 + 36*(f*x^2*e + 2*d*f*x)*c^3*d^4*f^4*e + 18*b*c^2*d^4*f^5 +
 36*(f*x^2*e + 2*d*f*x)^2*c^3*d^2*f^3*e^2 + 36*(f*x^2*e + 2*d*f*x)*b*c^2*d^
2*f^4*e + 4*b^2*c*d^2*f^5 + 20*a*c^2*d^2*f^5 + 12*(f*x^2*e + 2*d*f*x)^3*c^3
*f^2*e^3 + 18*(f*x^2*e + 2*d*f*x)^2*b*c^2*f^3*e^2 + 4*(f*x^2*e + 2*d*f*x)*b
^2*c*f^4*e + 20*(f*x^2*e + 2*d*f*x)*a*c^2*f^4*e - b^3*f^5 + 10*a*b*c*f^5)/(
(c*d^4*f^2 + 2*(f*x^2*e + 2*d*f*x)*c*d^2*f*e + b*d^2*f^2 + (f*x^2*e + 2*d*f
*x)^2*c*e^2 + (f*x^2*e + 2*d*f*x)*b*f*e + a*f^2)^2*(b^4*e - 8*a*b^2*c*e + 1
6*a^2*c^2*e))
```

Mupad [B]
time = 3.99, size = 1199, normalized size = 7.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*f + e*f*x)/(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3,x)
```

```
[Out] ((x^2*(5*a*c^2*e*f + b^2*c*e*f + 45*c^3*d^4*e*f + 27*b*c^2*d^2*e*f))/(b^4 +
 16*a^2*c^2 - 8*a*b^2*c) + (12*c^3*d^6*f - b^3*f + 20*a*c^2*d^2*f + 4*b^2*c
*d^2*f + 18*b*c^2*d^4*f + 10*a*b*c*f)/(4*e*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
+ (9*x^4*(10*c^3*d^2*e^3*f + b*c^2*e^3*f))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (2*d*x*(9*c^3*d^4*f + 5*a*c^2*f + b^2*c*f + 9*b*c^2*d^2*f))/(b^4 + 16*
a^2*c^2 - 8*a*b^2*c) + (6*d*x^3*(10*c^3*d^2*e^2*f + 3*b*c^2*e^2*f))/(b^4 +
```

16*a^2*c^2 - 8*a*b^2*c) + (3*c^3*e^5*f*x^6)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (18*c^3*d*e^4*f*x^5)/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(x^2*(6*b^2*d^2*e^2 + 28*c^2*d^6*e^2 + 2*a*b*e^2 + 12*a*c*d^2*e^2 + 30*b*c*d^4*e^2) + x^6*(28*c^2*d^2*e^6 + 2*b*c*e^6) + x*(4*b^2*d^3*e + 8*c^2*d^7*e + 8*a*c*d^3*e + 12*b*c*d^5*e + 4*a*b*d*e) + x^3*(4*b^2*d*e^3 + 56*c^2*d^5*e^3 + 8*a*c*d*e^3 + 40*b*c*d^3*e^3) + x^5*(56*c^2*d^3*e^5 + 12*b*c*d*e^5) + x^4*(b^2*e^4 + 70*c^2*d^4*e^4 + 2*a*c*e^4 + 30*b*c*d^2*e^4) + a^2 + b^2*d^4 + c^2*d^8 + c^2*e^8*x^8 + 2*a*b*d^2 + 2*a*c*d^4 + 2*b*c*d^6 + 8*c^2*d*e^7*x^7) + (6*c^2*f*atan(((b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5)*(x^2*((36*c^6*e^8*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*f^2*(b^5*c^2*e^10 - 8*a*b^3*c^3*e^10 + 16*a^2*b*c^4*e^10))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + x*((72*c^6*d*e^7*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (72*b*c^4*f^2*(b^5*c^2*d*e^9 - 8*a*b^3*c^3*d*e^9 + 16*a^2*b*c^4*d*e^9))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (36*c^6*d^2*e^6*f^2)/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (36*b*c^4*f^2*(32*a^3*c^4*e^8 + 2*a*b^4*c^2*e^8 - 16*a^2*b^2*c^3*e^8 + b^5*c^2*d^2*e^8 - 8*a*b^3*c^3*d^2*e^8 + 16*a^2*b*c^4*d^2*e^8))/(a*e^2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))))/(72*c^6*e^6*f^2)))/(e*(4*a*c - b^2)^(5/2))

3.658 $\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=270

$$\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2b^4-15ab^2c+16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2}{4a^2\left(b^2-4ac\right)^2ef\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b(b^4-1}{}$$

```
[Out] 1/4*(b^2-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^4
)^2+1/4*(2*b^4-15*a*b^2*c+16*a^2*c^2+2*b*c*(-7*a*c+b^2)*(e*x+d)^2)/a^2/(-4*
a*c+b^2)^2/e/f/(a+b*(e*x+d)^2+c*(e*x+d)^4)+1/2*b*(30*a^2*c^2-10*a*b^2*c+b^4
)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(5/2)/e/f+
ln(e*x+d)/a^3/e/f-1/4*ln(a+b*(e*x+d)^2+c*(e*x+d)^4)/a^3/e/f
```

Rubi [A]
time = 0.33, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1156, 1128, 754, 836, 814, 648, 632, 212, 642}

$$-\frac{\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^3ef}+\frac{\log(d+ex)}{a^3ef}+\frac{16a^2c^2+2bc\left(b^2-7ac\right)(d+ex)^2-15ab^2c+2b^4}{4a^2ef\left(b^2-4ac\right)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^3ef\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef\left(b^2-4ac\right)\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f*(a + b*(d + e*x)^2 +
 c*(d + e*x)^4)^2) + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)
*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f*(a + b*(d + e*x)^2 + c*(d + e*x)^4
)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[
b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)*e*f) + Log[d + e*x]/(a^3*e*f) - L
og[a + b*(d + e*x)^2 + c*(d + e*x)^4]/(4*a^3*e*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx=\frac{\text{Subst}\left(\int \frac{1}{x(a+bx^2+cx^4)^3}\,dx,x,d+ex\right)}{ef}$$

$$=\frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^3}\,dx,x,(d+ex)^2\right)}{2ef}$$

$$=\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}-\text{Subst}$$

$$=\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2b^4-}{4a^2(b}$$

$$=\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2b^4-}{4a^2(b}$$

$$=\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2b^4-}{4a^2(b}$$

$$=\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2b^4-}{4a^2(b}$$

$$=\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2b^4-}{4a^2(b}$$

$$=\frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{2b^4-}{4a^2(b}$$

Mathematica [A]
time = 2.68, size = 394, normalized size = 1.46

$$\frac{\frac{a^2\left(-b^2+2ac-bc(d+ex)^2\right)}{(-b^2+4ac)(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{a\left(2b^4-15ab^2c+16a^2c^2+2b^3c(d+ex)^2-14abc^2(d+ex)^2\right)}{(b^2-4ac)^2(a+(d+ex)^2(b+c(d+ex)^2))}+4\log(d+ex)-\frac{\left(b^5-10ab^3c+30a^2bc^2+b^4\sqrt{b^2-4ac}-8ab^2c\sqrt{b^2-4ac}+16a^2c^2\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{5/2}}+\frac{\left(b^5-10ab^3c+30a^2bc^2-b^4\sqrt{b^2-4ac}+8ab^2c\sqrt{b^2-4ac}-16a^2c^2\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{(b^2-4ac)^{5/2}}}{4a^3ef}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]

[Out] ((a^2*(-b^2 + 2*a*c - b*c*(d + e*x)^2))/((-b^2 + 4*a*c)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*(d + e*x)^2 - 14*a*b*c^2*(d + e*x)^2))/((b^2 - 4*a*c)^2*(a + (d + e*x)^2*(b + c*(d + e*x)^2))) + 4*Log[d + e*x] - ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2) + ((b^5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3*e*f)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.28, size = 970, normalized size = 3.59

method	result
default	$\frac{\frac{\ln(ex+d)}{a^3 e} - \frac{\frac{c^2e^5\left(7ac-b^2\right)ab\,x^6}{32a^2c^2-16a\,b^2c+2b^4}+\frac{3\left(7ac-b^2\right)ab\,c^2d\,e^4x^5}{16a^2c^2-8a\,b^2c+b^4}-\frac{e^3ac\left(-210ab\,c^2d^2+30b^3c\,d^2+16a^2c^2-29a\,b^2c+4b^4\right)x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{cd\,e^2a\left(-70ab\,c^2d^2+10b^3c\,d^2\right.}{16a^2c^2-8a}}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)

[Out] 1/f*(ln(e*x+d)/a^3/e-1/a^3*((1/2*c^2*e^5*(7*a*c-b^2)*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3*(7*a*c-b^2)*a*b*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4*e^3*a*c*(-210*a*b*c^2*d^2+30*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-c*d*e^2*a*(-70*a*b*c^2*d^2+10*b^3*c*d^2+16*a^2*c^2-29*a*b^2*c+4*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/2*e*a*(105*a*b*c^3*d^4-15*b^3*c^2*d^4-48*a^2*c^3*d^2+87*a*b^2*c^2*d^2-12*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+d*a*(21*a*b*c^3*d^4-3*b^3*c^2*d^4-16*a^2*c^3*d^2+29*a*b^2*c^2*d^2-4*b^4*c*d^2+a^2*b*c^2+6*a*b^3*c-b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x-1/4/e*a*(-14*a*b*c^3*d^6+2*b^3*c^2*d^6+16*a^2*c^3*d^4-29*a*b^2*c^2*d^4+4*b^4*c*d^4-2*a^2*b*c^2*d^2-12*a*b^3*c*d^2+2*b^5*d^2+24*a^3*c^2-21*a^2*b^2*c+3*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+1/2/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((e^3*c*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^3+3*d*e^2*c*(16*a^2*c^2-8*a*b^2*c+b^4)*_R^2+e*(48*a^2*c^3*d^2-24*a*b^2*c^2*d^2+3*b^4*c*d^2+23*a^2*b*c^2-9*a*b^3*c+b^5)*_R+16*a^2*c^3*d^3-8*a*b^2*c^2*d^3+b^4*c*d^3+23*a^2*b*c^2*d-9*a*b^3*c*d+b^5*d)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxima"
)
```

```
[Out] 1/4*(2*(b^3*c^2 - 7*a*b*c^3)*d^6 + 12*(b^3*c^2*e^5 - 7*a*b*c^3*e^5)*d*x^5 +
 2*(b^3*c^2*e^6 - 7*a*b*c^3*e^6)*x^6 + 3*a*b^4 - 21*a^2*b^2*c + 24*a^3*c^2
+ (4*b^4*c - 29*a*b^2*c^2 + 16*a^2*c^3)*d^4 + (4*b^4*c*e^4 - 29*a*b^2*c^2*e
^4 + 16*a^2*c^3*e^4 + 30*(b^3*c^2*e^4 - 7*a*b*c^3*e^4)*d^2)*x^4 + 4*(10*(b^
3*c^2*e^3 - 7*a*b*c^3*e^3)*d^3 + (4*b^4*c*e^3 - 29*a*b^2*c^2*e^3 + 16*a^2*c
^3*e^3)*d)*x^3 + 2*(b^5 - 6*a*b^3*c - a^2*b*c^2)*d^2 + 2*(b^5*e^2 - 6*a*b^3
*c*e^2 - a^2*b*c^2*e^2 + 15*(b^3*c^2*e^2 - 7*a*b*c^3*e^2)*d^4 + 3*(4*b^4*c*
e^2 - 29*a*b^2*c^2*e^2 + 16*a^2*c^3*e^2)*d^2)*x^2 + 4*(3*(b^3*c^2*e - 7*a*b
*c^3*e)*d^5 + (4*b^4*c*e - 29*a*b^2*c^2*e + 16*a^2*c^3*e)*d^3 + (b^5*e - 6*
a*b^3*c*e - a^2*b*c^2*e)*d)*x)/(8*(a^2*b^4*c^2*e^8 - 8*a^3*b^2*c^3*e^8 + 16
*a^4*c^4*e^8)*d*f*x^7 + (a^2*b^4*c^2*e^9 - 8*a^3*b^2*c^3*e^9 + 16*a^4*c^4*e
^9)*f*x^8 + 2*(a^2*b^5*c*e^7 - 8*a^3*b^3*c^2*e^7 + 16*a^4*b*c^3*e^7 + 14*(a
^2*b^4*c^2*e^7 - 8*a^3*b^2*c^3*e^7 + 16*a^4*c^4*e^7)*d^2)*f*x^6 + 4*(14*(a^
2*b^4*c^2*e^6 - 8*a^3*b^2*c^3*e^6 + 16*a^4*c^4*e^6)*d^3 + 3*(a^2*b^5*c*e^6
- 8*a^3*b^3*c^2*e^6 + 16*a^4*b*c^3*e^6)*d)*f*x^5 + (a^2*b^6*e^5 - 6*a^3*b^4
*c*e^5 + 32*a^5*c^3*e^5 + 70*(a^2*b^4*c^2*e^5 - 8*a^3*b^2*c^3*e^5 + 16*a^4*
c^4*e^5)*d^4 + 30*(a^2*b^5*c*e^5 - 8*a^3*b^3*c^2*e^5 + 16*a^4*b*c^3*e^5)*d^
2)*f*x^4 + 4*(14*(a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 16*a^4*c^4*e^4)*d^5
 + 10*(a^2*b^5*c*e^4 - 8*a^3*b^3*c^2*e^4 + 16*a^4*b*c^3*e^4)*d^3 + (a^2*b^6
*e^4 - 6*a^3*b^4*c*e^4 + 32*a^5*c^3*e^4)*d)*f*x^3 + 2*(a^3*b^5*e^3 - 8*a^4*
b^3*c*e^3 + 16*a^5*b*c^2*e^3 + 14*(a^2*b^4*c^2*e^3 - 8*a^3*b^2*c^3*e^3 + 16
*a^4*c^4*e^3)*d^6 + 15*(a^2*b^5*c*e^3 - 8*a^3*b^3*c^2*e^3 + 16*a^4*b*c^3*e^
3)*d^4 + 3*(a^2*b^6*e^3 - 6*a^3*b^4*c*e^3 + 32*a^5*c^3*e^3)*d^2)*f*x^2 + 4*
(2*(a^2*b^4*c^2*e^2 - 8*a^3*b^2*c^3*e^2 + 16*a^4*c^4*e^2)*d^7 + 3*(a^2*b^5*
c*e^2 - 8*a^3*b^3*c^2*e^2 + 16*a^4*b*c^3*e^2)*d^5 + (a^2*b^6*e^2 - 6*a^3*b^
4*c*e^2 + 32*a^5*c^3*e^2)*d^3 + (a^3*b^5*e^2 - 8*a^4*b^3*c*e^2 + 16*a^5*b*c
^2*e^2)*d)*f*x + ((a^2*b^4*c^2*e - 8*a^3*b^2*c^3*e + 16*a^4*c^4*e)*d^8 + a^
4*b^4*e - 8*a^5*b^2*c*e + 16*a^6*c^2*e + 2*(a^2*b^5*c*e - 8*a^3*b^3*c^2*e +
 16*a^4*b*c^3*e)*d^6 + (a^2*b^6*e - 6*a^3*b^4*c*e + 32*a^5*c^3*e)*d^4 + 2*(
a^3*b^5*e - 8*a^4*b^3*c*e + 16*a^5*b*c^2*e)*d^2)*f) - integrate(((b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*d^3 + 3*(b^4*c*e^2 - 8*a*b^2*c^2*e^2 + 16*a^2*c^3*
e^2)*d*x^2 + (b^4*c*e^3 - 8*a*b^2*c^2*e^3 + 16*a^2*c^3*e^3)*x^3 + (b^5 - 9*
a*b^3*c + 23*a^2*b*c^2)*d + (b^5*e - 9*a*b^3*c*e + 23*a^2*b*c^2*e + 3*(b^4*
c*e - 8*a*b^2*c^2*e + 16*a^2*c^3*e)*d^2)*x)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*
d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/
((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f) + e^(-1)*log(x*e + d)/(a^3*f)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4868 vs. $2(263) = 526$.
time = 1.71, size = 9862, normalized size = 36.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="fricas"
)
```

```
[Out] [1/4*(3*a^2*b^6 - 33*a^3*b^4*c + 108*a^4*b^2*c^2 - 96*a^5*c^3 + 2*(a*b^5*c^
2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*x^6*e^6 + 12*(a*b^5*c^2 - 11*a^2*b^3*c^3
 + 28*a^3*b*c^4)*d*x^5*e^5 + 2*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*
d^6 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4 + 30*(a*b^
5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^2)*x^4*e^4 + (4*a*b^6*c - 45*a^2*b
^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^4 + 4*(10*(a*b^5*c^2 - 11*a^2*b^3*
c^3 + 28*a^3*b*c^4)*d^3 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 6
4*a^4*c^4)*d)*x^3*e^3 + 2*(a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*
c^3 + 15*(a*b^5*c^2 - 11*a^2*b^3*c^3 + 28*a^3*b*c^4)*d^4 + 3*(4*a*b^6*c - 4
5*a^2*b^4*c^2 + 132*a^3*b^2*c^3 - 64*a^4*c^4)*d^2)*x^2*e^2 + 2*(a*b^7 - 10*
a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d^2 + 4*(3*(a*b^5*c^2 - 11*a^2*b^
3*c^3 + 28*a^3*b*c^4)*d^5 + (4*a*b^6*c - 45*a^2*b^4*c^2 + 132*a^3*b^2*c^3 -
 64*a^4*c^4)*d^3 + (a*b^7 - 10*a^2*b^5*c + 23*a^3*b^3*c^2 + 4*a^4*b*c^3)*d)
*x*e + ((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8*e^8 + 8*(b^5*c^2 - 10*a
*b^3*c^3 + 30*a^2*b*c^4)*d*x^7*e^7 + (b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4
)*d^8 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3 + 14*(b^5*c^2 - 10*a*b^3*c
^3 + 30*a^2*b*c^4)*d^2)*x^6*e^6 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2
*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^6 + 4*(14*(b^5*c^2 - 10*a*b^3*c^
3 + 30*a^2*b*c^4)*d^3 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d)*x^5*e^
5 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3 + 70*(b^5*c^2 - 10*a*b
^3*c^3 + 30*a^2*b*c^4)*d^4 + 30*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^2
)*x^4*e^4 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^4 + 4*(14*(
b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*d^5 + 10*(b^6*c - 10*a*b^4*c^2 + 30*
a^2*b^2*c^3)*d^3 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d)*x^3
*e^3 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2 + 14*(b^5*c^2 - 10*a*b^3*c^
3 + 30*a^2*b*c^4)*d^6 + 15*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^4 + 3*
(b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^2)*x^2*e^2 + 2*(a*b^6 -
 10*a^2*b^4*c + 30*a^3*b^2*c^2)*d^2 + 4*(2*(b^5*c^2 - 10*a*b^3*c^3 + 30*a^2
*b*c^4)*d^7 + 3*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*c^3)*d^5 + (b^7 - 8*a*b^
5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*d^3 + (a*b^6 - 10*a^2*b^4*c + 30*a^3*b
^2*c^2)*d)*x*e)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*
c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x
*e + b^2 - 2*a*c + (2*c*x^2*e^2 + 4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c
))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(
2*c*d^3 + b*d)*x*e + a)) - ((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
```

```
^3*c^5)*x^8*e^8 + 8*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*
d*x^7*e^7 + (b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^
2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^
2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2
*c^4 - 64*a^3*c^5)*d^2)*x^6*e^6 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
- 64*a^3*b*c^4)*d^6 + 4*(14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5)*d^3 + 3*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*x
^5*e^5 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4
+ 70*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^4 + 30*(b^7*c
 - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2)*x^4*e^4 + (b^8 - 10*a
*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^4 + 4*(14*(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^5 + 10*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c
^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d)*x^3*e^3 + 2*(a*b^7 - 12*a^2*b^5*c + 4
8*a^3*b^3*c^2 - 64*a^4*b*c^3 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5)*d^6 + 15*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^
4)*d^4 + 3*(b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^
4)*d^2)*x^2*e^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*
d^2 + 4*(2*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^7 + 3*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^5 + (b^8 - 10*a*b^6
*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^3 + (a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d)*x*e)*log(c*x^4*e^4 + 4*c*d*x^3*e^3
 + c*d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a) + 4*(
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8*e^8 + 8*(b^6*c^2
 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^7*e^7 + (b^6*c^2 - 12*a*
b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*
b*c^4 + 14*(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2)*x^6*
e^6 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^6 + 4*(14*
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^3 + 3*(b^7*c - 12*
a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d)*x^5*e^5 + (b^8 - 10*a*b^6*c +
 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(263) = 526$.

time = 4.40, size = 1044, normalized size = 3.87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac")
```

```
[Out] -1/4*((a^3*b^7*c*f*e^3 - 14*a^4*b^5*c^2*f*e^3 + 70*a^5*b^3*c^3*f*e^3 - 120*
a^6*b*c^4*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*x^2*e^2 + 2*b*d*x*e + sqrt(b^2
 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e + b*d^2 + sqrt(b^2 - 4*a*c)*d
^2 + 2*a)) - (a^3*b^7*c*f*e^3 - 14*a^4*b^5*c^2*f*e^3 + 70*a^5*b^3*c^3*f*e^3
 - 120*a^6*b*c^4*f*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e +
sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 -
4*a*c)*d^2 - 2*a)))/(a^6*b^8*c*f^2*e^4 - 16*a^7*b^6*c^2*f^2*e^4 + 96*a^8*b^
4*c^3*f^2*e^4 - 256*a^9*b^2*c^4*f^2*e^4 + 256*a^10*c^5*f^2*e^4) - 1/4*e^(-1
)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4
 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^3*f) + e^(-1)*log(abs(x*e + d))/(
a^3*f) + 1/4*(2*a*b^3*c^2*d^6 - 14*a^2*b*c^3*d^6 + 4*a*b^4*c*d^4 - 29*a^2*b
^2*c^2*d^4 + 16*a^3*c^3*d^4 + 2*a*b^5*d^2 - 12*a^2*b^3*c*d^2 - 2*a^3*b*c^2*
d^2 + 2*(a*b^3*c^2*e^6 - 7*a^2*b*c^3*e^6)*x^6 + 3*a^2*b^4 - 21*a^3*b^2*c +
24*a^4*c^2 + 12*(a*b^3*c^2*d*e^5 - 7*a^2*b*c^3*d*e^5)*x^5 + (30*a*b^3*c^2*d
^2*e^4 - 210*a^2*b*c^3*d^2*e^4 + 4*a*b^4*c*e^4 - 29*a^2*b^2*c^2*e^4 + 16*a^
3*c^3*e^4)*x^4 + 4*(10*a*b^3*c^2*d^3*e^3 - 70*a^2*b*c^3*d^3*e^3 + 4*a*b^4*c
*d*e^3 - 29*a^2*b^2*c^2*d*e^3 + 16*a^3*c^3*d*e^3)*x^3 + 2*(15*a*b^3*c^2*d^4
*e^2 - 105*a^2*b*c^3*d^4*e^2 + 12*a*b^4*c*d^2*e^2 - 87*a^2*b^2*c^2*d^2*e^2
+ 48*a^3*c^3*d^2*e^2 + a*b^5*e^2 - 6*a^2*b^3*c*e^2 - a^3*b*c^2*e^2)*x^2 + 4
*(3*a*b^3*c^2*d^5*e - 21*a^2*b*c^3*d^5*e + 4*a*b^4*c*d^3*e - 29*a^2*b^2*c^2
*d^3*e + 16*a^3*c^3*d^3*e + a*b^5*d*e - 6*a^2*b^3*c*d*e - a^3*b*c^2*d*e)*x)
*e^(-1)/((c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e + c*d^4
 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a)^2*(b^2 - 4*a*c)^2*a^3*f)
```

Mupad [B]
time = 18.49, size = 2500, normalized size = 9.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] ((x^2*(b^5*e + 48*a^2*c^3*d^2*e + 15*b^3*c^2*d^4*e - 6*a*b^3*c*e - a^2*b*c^
2*e + 12*b^4*c*d^2*e - 105*a*b*c^3*d^4*e - 87*a*b^2*c^2*d^2*e))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x^4*(4*b^4*c*e^3 + 16*a^2*c^3*e^3 - 29*a*b
^2*c^2*e^3 + 30*b^3*c^2*d^2*e^3 - 210*a*b*c^3*d^2*e^3))/(4*(a^2*b^4 + 16*a^
4*c^2 - 8*a^3*b^2*c)) + (x^3*(16*a^2*c^3*d*e^2 + 10*b^3*c^2*d^3*e^2 + 4*b^4
*c*d*e^2 - 29*a*b^2*c^2*d*e^2 - 70*a*b*c^3*d^3*e^2))/(a^2*b^4 + 16*a^4*c^2
```

- 8*a^3*b^2*c) + (3*x^5*(b^3*c^2*d*e^4 - 7*a*b*c^3*d*e^4))/(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (x^6*(b^3*c^2*e^5 - 7*a*b*c^3*e^5))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)) + (x*(b^5*d + 4*b^4*c*d^3 + 16*a^2*c^3*d^3 + 3*b^3*c^2*d^5 - 29*a*b^2*c^2*d^3 - 6*a*b^3*c*d - a^2*b*c^2*d - 21*a*b*c^3*d^5))/(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c) + (3*a*b^4 + 24*a^3*c^2 + 2*b^5*d^2 - 21*a^2*b^2*c + 4*b^4*c*d^4 + 16*a^2*c^3*d^4 + 2*b^3*c^2*d^6 - 2*a^2*b*c^2*d^2 - 29*a*b^2*c^2*d^4 - 12*a*b^3*c*d^2 - 14*a*b*c^3*d^6)/(4*e*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))/(x^3*(56*c^2*d^5*e^3*f + 4*b^2*d*e^3*f + 40*b*c*d^3*e^3*f + 8*a*c*d*e^3*f) + x^2*(6*b^2*d^2*e^2*f + 28*c^2*d^6*e^2*f + 2*a*b*e^2*f + 12*a*c*d^2*e^2*f + 30*b*c*d^4*e^2*f) + x*(4*b^2*d^3*e*f + 8*c^2*d^7*e*f + 4*a*b*d*e*f + 8*a*c*d^3*e*f + 12*b*c*d^5*e*f) + x^4*(b^2*e^4*f + 70*c^2*d^4*e^4*f + 2*a*c*e^4*f + 30*b*c*d^2*e^4*f) + x^5*(56*c^2*d^3*e^5*f + 12*b*c*d*e^5*f) + a^2*f + x^6*(28*c^2*d^2*e^6*f + 2*b*c*e^6*f) + b^2*d^4*f + c^2*d^8*f + c^2*e^8*f*x^8 + 2*a*b*d^2*f + 2*a*c*d^4*f + 2*b*c*d^6*f + 8*c^2*d*e^7*f*x^7) - (log((((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*(((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*((2*b*c^2*e^16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2))/(a^2*f*(4*a*c - b^2)^2) + (b*c^2*e^16*(a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) + 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^3*f) + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*f*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*f*(4*a*c - b^2)^2)))/(4*a^3*e*f) + (b*c^3*e^15*(7*a*c - b^2)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2))/(a^4*f^2*(4*a*c - b^2)^4) - (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a*c - b^2)^4) - (2*b*c^4*d*e^16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a*c - b^2)^4)))/(4*a^3*e*f) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c^2*d^2))/(a^6*f^3*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^15*x*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6))*(((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) - 1)*(((a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) - 1)*((2*b*c^2*e^16*(2*b^5 + 46*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 18*a*b^3*c - 2*a*b^2*c^2*d^2))/(a^2*f*(4*a*c - b^2)^2) - (b*c^2*e^16*(a^3*e*f*(-(b^2*(b^4 + 30*a^2*c^2 - 10*a*b^2*c)^2)/(a^6*e^2*f^2*(4*a*c - b^2)^5))^(1/2) - 1)*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^3*f) + (2*b*c^3*e^18*x^2*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*f*(4*a*c - b^2)^2) + (4*b*c^3*d*e^17*x*(b^4 + 10*a^2*c^2 - 2*a*b^2*c))/(a^2*f*(4*a*c - b^2)^2)))/(4*a^3*e*f) - (b*c^3*e^15*(7*a*c - b^2)*(4*b^5 + 71*a^2*b*c^2 + 6*b^4*c*d^2 + 80*a^2*c^3*d^2 - 33*a*b^3*c - 47*a*b^2*c^2*d^2))/(a^4*f^2*(4*a*c - b^2)^4) + (b*c^4*e^17*x^2*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a*c - b^2)^4) + (2*b*c^4*d*e^16*x*(6*b^6 - 560*a^3*c^3 + 409*a^2*b^2*c^2 - 89*a*b^4*c))/(a^4*f^2*(4*a *c - b^2)^4)))/(4*a^3*e*f) - (b^3*c^5*e^16*x^2*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6) + (b^2*c^4*e^14*(7*a*c - b^2)^2*(b^4 + 16*a^2*c^2 + b^3*c*d^2 - 8*a*b^2*c - 7*a*b*c^2*d^2))/(a^6*f^3*(4*a*c - b^2)^6) - (2*b^3*c^5*d*e^15*x*(7*a*c - b^2)^3)/(a^6*f^3*(4*a*c - b^2)^6)))*(2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f))/(2*(4*a^3*b^10*e^2*f^2 - 4096*a^8*c^5*e^2*f^2 + 640*a^5*b^6*c^2*e^2*f^2 - 2560*a^6*b^4*c^3*e^2*f^2 + 5120*a^7*b^2*c^4*e^2*f^2 - 80*a^4*b^8*c*e^2*f^2)) + log(d + e*x)/(a^3*e*f) - (b*atan((x*((((((b*((2*(5120*a^10*b*c^9*d*e^17*f^2 + 2*a^4*b^13*c^3*d*e^17*f^2 - 36*a^5*b^11*c^4*d*e^17*f^2 + 276*a^6*b^9*c^5*d*e^17*f^2 - 1216*a^7*b^7*c^6*d*e^17*f^2 + 3456*a^8*b^5*c^7*d*e^17*f^2 - 6144*a^9*b^3*c^8*d*e^17*f^2))/(a^6*b^12*f^3 + 4096*a^12*c^6*f^3 - 24*a^7*b^10*c*f^3 + 240*a^8*b^8*c^2*f^3 - 1280*a^9*b^6*c^3*f^3 + 3840*a^10*b^4*c^4*f^3 - 6144*a^11*b^2*c^5*f^3) - ((2*b^10*e*f - 2048*a^5*c^5*e*f + 320*a^2*b^6*c^2*e*f - 1280*a^3*b^4*c^3*e*f + 2560*a^4*b^2*c^4*e*f - 40*a*b^8*c*e*f)*(163840*a^13*b*c^9*d*e^18*f^3 - 12*a^6*b^15*c^2*d*e^18*f^3 + 328*a^7*b^13*c^3*d*e^18*f^3 - 3840*a^8*b^11...

3.659 $\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=499

$$-\frac{3(5b^2-12ac)\,(b^2-5ac)}{8a^3\,(b^2-4ac)^2\,ef^2(d+ex)}+\frac{b^2-2ac+bc(d+ex)^2}{4a\,(b^2-4ac)\,ef^2(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{5b^4-35ab^2c}{8a^2\,(b^2-4ac)^2\,ef}$$

```
[Out] -3/8*(-12*a*c+5*b^2)*(-5*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/f^2/(e*x+d)+1/4*(b^2
-2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)
^4)^2+1/8*(5*b^4-35*a*b^2*c+36*a^2*c^2+b*c*(-32*a*c+5*b^2)*(e*x+d)^2)/a^2/(
-4*a*c+b^2)^2/e/f^2/(e*x+d)/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/16*arctan((e*x+d)
*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5
*a*c+b^2)+b*(124*a^2*c^2-47*a*b^2*c+5*b^4)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+
b^2)^2/e/f^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-3/16*arctan((e*x+d)*2^(1/
2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*((-12*a*c+5*b^2)*(-5*a*c+b
^2)+(-124*a^2*b*c^2+47*a*b^3*c-5*b^5)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^
2/e/f^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1156, 1135, 1291, 1295, 1180, 211}

$$-\frac{3(5b^2-12ac)\,(b^2-5ac)}{8a^3ef^2\,(b^2-4ac)^2\,(d+ex)}+\frac{36a^2c^2+bc(5b^2-32ac)\,(d+ex)^2-35ab^2c+5b^4}{8a^2ef^2\,(b^2-4ac)^2\,(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)}-\frac{3\sqrt{c}\left(\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}+(5b^2-12ac)\,(b^2-5ac)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^3ef^2\,(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left((5b^2-12ac)\,(b^2-5ac)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,(d+ex)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a^3ef^2\,(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef^2\,(b^2-4ac)\,(d+ex)\,(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*e*f^2*(d + e*x))
 + (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^2*(d + e*x)*(a +
b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*
(5*b^2 - 32*a*c)*(d + e*x)^2)/(8*a^2*(b^2 - 4*a*c)^2*e*f^2*(d + e*x)*(a + b
*(d + e*x)^2 + c*(d + e*x)^4)) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c)
 + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*
a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b
^2 - 5*a*c) - (5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*
(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
```

```
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{1}{(df+efx)^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{ef^2}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^2(d+ex)\left(a+b(d+ex)^2+c(d+ex)^4\right)$$

$$= -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2ef^2(d+ex)} + \frac{b^2-2}{4a\left(b^2-4ac\right)ef^2(d+e}$$

$$= -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2ef^2(d+ex)} + \frac{b^2-2}{4a\left(b^2-4ac\right)ef^2(d+e}$$

$$= -\frac{3(5b^2-12ac)\left(b^2-5ac\right)}{8a^3\left(b^2-4ac\right)^2ef^2(d+ex)} + \frac{b^2-2}{4a\left(b^2-4ac\right)ef^2(d+e}$$

Mathematica [A]
time = 6.15, size = 575, normalized size = 1.15

$$-\frac{1}{a^3ef^2(d+ex)}+\frac{b^3(d+ex)-3abc(d+ex)+b^2c(d+ex)^3-2ac^2(d+ex)^3}{4a^2\left(-b^2+4ac\right)ef^2\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{-7b^5(d+ex)+52ab^3c(d+ex)-84a^2bc^2(d+ex)-7b^4c(d+ex)^3+47ab^2c^2(d+ex)^3-52a^2c^3(d+ex)^3}{8a^3\left(-b^2+4ac\right)^2ef^2\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3\sqrt{c}\left(5b^5-47ab^3c+124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,ef^2}-\frac{3\sqrt{c}\left(-5b^5+47ab^3c-124a^2bc^2+5b^4\sqrt{b^2-4ac}-37ab^2c\sqrt{b^2-4ac}+60a^2c^2\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}(d+ex)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3\left(b^2-4ac\right)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}\,ef^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -(1/(a^3*e*f^2*(d + e*x))) + (b^3*(d + e*x) - 3*a*b*c*(d + e*x) + b^2*c*(d
+ e*x)^3 - 2*a*c^2*(d + e*x)^3)/(4*a^2*(-b^2 + 4*a*c)*e*f^2*(a + b*(d + e*x
)^2 + c*(d + e*x)^4)^2) + (-7*b^5*(d + e*x) + 52*a*b^3*c*(d + e*x) - 84*a^2
*b*c^2*(d + e*x) - 7*b^4*c*(d + e*x)^3 + 47*a*b^2*c^2*(d + e*x)^3 - 52*a^2*
c^3*(d + e*x)^3)/(8*a^3*(-b^2 + 4*a*c)^2*e*f^2*(a + b*(d + e*x)^2 + c*(d +
e*x)^4)) - (3*Sqrt[c]*(5*b^5 - 47*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2
- 4*a*c] - 37*a*b^2*c*Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*(d + e*x))/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^
3*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*e*f^2) - (3*Sqrt[c]*(-5*b
```

```
^5 + 47*a*b^3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt
[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*(d +
e*x))/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^(5/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]*e*f^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.29, size = 1201, normalized size = 2.41

method	result
default	$-\frac{1}{a^3 e(ex+d)}-\frac{\frac{\frac{c^2e^6\left(52a^2c^2-47a\,b^2c+7b^4\right)x^7}{128a^2c^2-64a\,b^2c+8b^4}+\frac{7c^2d\,e^5\left(52a^2c^2-47a\,b^2c+7b^4\right)x^6}{8\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(1092a^2c^3d^2-987a\,b^2c^2d^2+147b^4c\,d^2+136a^2b\,c^2-99a\,b^3c+14\right.}{128a^2c^2-64a\,b^2c+8b^4}}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^2*(-1/a^3/e/(e*x+d)-1/a^3*((1/8*c^2*e^6*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^7+7/8*c^2*d*e^5*(52*a^2*c^2-47*a*b^2*c+7*b^4)/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/8*(1092*a^2*c^3*d^2-987*a*b^2*c^2*d^2+147*b
^4*c*d^2+136*a^2*b*c^2-99*a*b^3*c+14*b^5)*e^4*c/(16*a^2*c^2-8*a*b^2*c+b^4)*
x^5+5/8*c*d*e^3*(364*a^2*c^3*d^2-329*a*b^2*c^2*d^2+49*b^4*c*d^2+136*a^2*b*c
^2-99*a*b^3*c+14*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+1/8*e^2*(1820*a^2*c^4*
d^4-1645*a*b^2*c^3*d^4+245*b^4*c^2*d^4+1360*a^2*b*c^3*d^2-990*a*b^3*c^2*d^2
+140*b^5*c*d^2+68*a^3*c^3+25*a^2*b^2*c^2-43*a*b^4*c+7*b^6)/(16*a^2*c^2-8*a*
b^2*c+b^4)*x^3+1/8*d*e*(1092*a^2*c^4*d^4-987*a*b^2*c^3*d^4+147*b^4*c^2*d^4+
1360*a^2*b*c^3*d^2-990*a*b^3*c^2*d^2+140*b^5*c*d^2+204*a^3*c^3+75*a^2*b^2*c
^2-129*a*b^4*c+21*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/8*(364*a^2*c^4*d^6-
329*a*b^2*c^3*d^6+49*b^4*c^2*d^6+680*a^2*b*c^3*d^4-495*a*b^3*c^2*d^4+70*b^5
*c*d^4+204*a^3*c^3*d^2+75*a^2*b^2*c^2*d^2-129*a*b^4*c*d^2+21*b^6*d^2+108*a^
3*b*c^2-66*a^2*b^3*c+9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/8*d/e*(52*a^2*
c^4*d^6-47*a*b^2*c^3*d^6+7*b^4*c^2*d^6+136*a^2*b*c^3*d^4-99*a*b^3*c^2*d^4+1
4*b^5*c*d^4+68*a^3*c^3*d^2+25*a^2*b^2*c^2*d^2-43*a*b^4*c*d^2+7*b^6*d^2+108*
a^3*b*c^2-66*a^2*b^3*c+9*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*
d*e^3*x^3+6*c*d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+
3/16/(16*a^2*c^2-8*a*b^2*c+b^4)/e*sum((c*e^2*(60*a^2*c^2-37*a*b^2*c+5*b^4)*
_R^2+2*c*d*e*(60*a^2*c^2-37*a*b^2*c+5*b^4)*_R+60*a^2*c^3*d^2-37*a*b^2*c^2*d
^2+5*b^4*c*d^2+92*a^2*b*c^2-42*a*b^3*c+5*b^5)/(2*_R^3*c*e^3+6*_R^2*c*d*e^2+
6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootOf(e^4*c*_Z^4+4*d*e^3*c*_Z
^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxim
a")

[Out] -1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^8 + 24*(5*b^4*c^2*e^7 - 3
7*a*b^2*c^3*e^7 + 60*a^2*c^4*e^7)*d*x^7 + 3*(5*b^4*c^2*e^8 - 37*a*b^2*c^3*e
^8 + 60*a^2*c^4*e^8)*x^8 + (30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^6 +
 (30*b^5*c*e^6 - 227*a*b^3*c^2*e^6 + 392*a^2*b*c^3*e^6 + 84*(5*b^4*c^2*e^6
- 37*a*b^2*c^3*e^6 + 60*a^2*c^4*e^6)*d^2)*x^6 + 8*a^2*b^4 - 64*a^3*b^2*c +
128*a^4*c^2 + 6*(28*(5*b^4*c^2*e^5 - 37*a*b^2*c^3*e^5 + 60*a^2*c^4*e^5)*d^3
 + (30*b^5*c*e^5 - 227*a*b^3*c^2*e^5 + 392*a^2*b*c^3*e^5)*d)*x^5 + (15*b^6
- 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3)*d^4 + (15*b^6*e^4 - 91*a*b^4*c
*e^4 + 25*a^2*b^2*c^2*e^4 + 324*a^3*c^3*e^4 + 210*(5*b^4*c^2*e^4 - 37*a*b^2
*c^3*e^4 + 60*a^2*c^4*e^4)*d^4 + 15*(30*b^5*c*e^4 - 227*a*b^3*c^2*e^4 + 392
*a^2*b*c^3*e^4)*d^2)*x^4 + 4*(42*(5*b^4*c^2*e^3 - 37*a*b^2*c^3*e^3 + 60*a^2
*c^4*e^3)*d^5 + 5*(30*b^5*c*e^3 - 227*a*b^3*c^2*e^3 + 392*a^2*b*c^3*e^3)*d^
3 + (15*b^6*e^3 - 91*a*b^4*c*e^3 + 25*a^2*b^2*c^2*e^3 + 324*a^3*c^3*e^3)*d)
*x^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d^2 + (84*(5*b^4*c^2*e^2
- 37*a*b^2*c^3*e^2 + 60*a^2*c^4*e^2)*d^6 + 25*a*b^5*e^2 - 194*a^2*b^3*c*e^2
 + 364*a^3*b*c^2*e^2 + 15*(30*b^5*c*e^2 - 227*a*b^3*c^2*e^2 + 392*a^2*b*c^3
*e^2)*d^4 + 6*(15*b^6*e^2 - 91*a*b^4*c*e^2 + 25*a^2*b^2*c^2*e^2 + 324*a^3*c
^3*e^2)*d^2)*x^2 + 2*(12*(5*b^4*c^2*e - 37*a*b^2*c^3*e + 60*a^2*c^4*e)*d^7
+ 3*(30*b^5*c*e - 227*a*b^3*c^2*e + 392*a^2*b*c^3*e)*d^5 + 2*(15*b^6*e - 91
*a*b^4*c*e + 25*a^2*b^2*c^2*e + 324*a^3*c^3*e)*d^3 + (25*a*b^5*e - 194*a^2*
b^3*c*e + 364*a^3*b*c^2*e)*d)*x)/(9*(a^3*b^4*c^2*e^9 - 8*a^4*b^2*c^3*e^9 +
16*a^5*c^4*e^9)*d*f^2*x^8 + (a^3*b^4*c^2*e^10 - 8*a^4*b^2*c^3*e^10 + 16*a^5
*c^4*e^10)*f^2*x^9 + 2*(a^3*b^5*c*e^8 - 8*a^4*b^3*c^2*e^8 + 16*a^5*b*c^3*e^
8 + 18*(a^3*b^4*c^2*e^8 - 8*a^4*b^2*c^3*e^8 + 16*a^5*c^4*e^8)*d^2)*f^2*x^7
+ 14*(6*(a^3*b^4*c^2*e^7 - 8*a^4*b^2*c^3*e^7 + 16*a^5*c^4*e^7)*d^3 + (a^3*b
^5*c*e^7 - 8*a^4*b^3*c^2*e^7 + 16*a^5*b*c^3*e^7)*d)*f^2*x^6 + (a^3*b^6*e^6
- 6*a^4*b^4*c*e^6 + 32*a^6*c^3*e^6 + 126*(a^3*b^4*c^2*e^6 - 8*a^4*b^2*c^3*e
^6 + 16*a^5*c^4*e^6)*d^4 + 42*(a^3*b^5*c*e^6 - 8*a^4*b^3*c^2*e^6 + 16*a^5*b
*c^3*e^6)*d^2)*f^2*x^5 + (126*(a^3*b^4*c^2*e^5 - 8*a^4*b^2*c^3*e^5 + 16*a^5
*c^4*e^5)*d^5 + 70*(a^3*b^5*c*e^5 - 8*a^4*b^3*c^2*e^5 + 16*a^5*b*c^3*e^5)*d
^3 + 5*(a^3*b^6*e^5 - 6*a^4*b^4*c*e^5 + 32*a^6*c^3*e^5)*d)*f^2*x^4 + 2*(a^4
*b^5*e^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 + 42*(a^3*b^4*c^2*e^4 - 8*a^4
*b^2*c^3*e^4 + 16*a^5*c^4*e^4)*d^6 + 35*(a^3*b^5*c*e^4 - 8*a^4*b^3*c^2*e^4
+ 16*a^5*b*c^3*e^4)*d^4 + 5*(a^3*b^6*e^4 - 6*a^4*b^4*c*e^4 + 32*a^6*c^3*e^4
)*d^2)*f^2*x^3 + 2*(18*(a^3*b^4*c^2*e^3 - 8*a^4*b^2*c^3*e^3 + 16*a^5*c^4*e^
3)*d^7 + 21*(a^3*b^5*c*e^3 - 8*a^4*b^3*c^2*e^3 + 16*a^5*b*c^3*e^3)*d^5 + 5*
(a^3*b^6*e^3 - 6*a^4*b^4*c*e^3 + 32*a^6*c^3*e^3)*d^3 + 3*(a^4*b^5*e^3 - 8*a
^5*b^3*c*e^3 + 16*a^6*b*c^2*e^3)*d)*f^2*x^2 + (a^5*b^4*e^2 - 8*a^6*b^2*c*e^
2 + 16*a^7*c^2*e^2 + 9*(a^3*b^4*c^2*e^2 - 8*a^4*b^2*c^3*e^2 + 16*a^5*c^4*e^
2)*d^8 + 14*(a^3*b^5*c*e^2 - 8*a^4*b^3*c^2*e^2 + 16*a^5*b*c^3*e^2)*d^6 + 5*
```

```
(a^3*b^6*e^2 - 6*a^4*b^4*c*e^2 + 32*a^6*c^3*e^2)*d^4 + 6*(a^4*b^5*e^2 - 8*a
^5*b^3*c*e^2 + 16*a^6*b*c^2*e^2)*d^2)*f^2*x + ((a^3*b^4*c^2*e - 8*a^4*b^2*c
^3*e + 16*a^5*c^4*e)*d^9 + 2*(a^3*b^5*c*e - 8*a^4*b^3*c^2*e + 16*a^5*b*c^3*
e)*d^7 + (a^3*b^6*e - 6*a^4*b^4*c*e + 32*a^6*c^3*e)*d^5 + 2*(a^4*b^5*e - 8*
a^5*b^3*c*e + 16*a^6*b*c^2*e)*d^3 + (a^5*b^4*e - 8*a^6*b^2*c*e + 16*a^7*c^2
*e)*d)*f^2) - 3/8*integrate((5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2 + (5*b^4*c -
 37*a*b^2*c^2 + 60*a^2*c^3)*d^2 + 2*(5*b^4*c*e - 37*a*b^2*c^2*e + 60*a^2*c^
3*e)*d*x + (5*b^4*c*e^2 - 37*a*b^2*c^2*e^2 + 60*a^2*c^3*e^2)*x^2)/(c*x^4*e^
4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2 + b*e^2)*x^2 + 2*(2*c*d^3*
e + b*d*e)*x + a), x)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 10408 vs. $2(459) = 918$.
time = 1.17, size = 10408, normalized size = 20.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="frica
s")
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8*e^8 + 48*(5*b^4*c^2 -
37*a*b^2*c^3 + 60*a^2*c^4)*d*x^7*e^7 + 6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2
*c^4)*d^8 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3 + 84*(5*b^4*c^2 - 3
7*a*b^2*c^3 + 60*a^2*c^4)*d^2)*x^6*e^6 + 2*(30*b^5*c - 227*a*b^3*c^2 + 392*
a^2*b*c^3)*d^6 + 12*(28*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^3 + (30*b
^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d)*x^5*e^5 + 16*a^2*b^4 - 128*a^3*b^2
*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3 +
210*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^4 + 15*(30*b^5*c - 227*a*b^3*
c^2 + 392*a^2*b*c^3)*d^2)*x^4*e^4 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2
 + 324*a^3*c^3)*d^4 + 8*(42*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^5 + 5
*(30*b^5*c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^3 + (15*b^6 - 91*a*b^4*c + 25
*a^2*b^2*c^2 + 324*a^3*c^3)*d)*x^3*e^3 + 2*(84*(5*b^4*c^2 - 37*a*b^2*c^3 +
60*a^2*c^4)*d^6 + 25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2 + 15*(30*b^5*c -
 227*a*b^3*c^2 + 392*a^2*b*c^3)*d^4 + 6*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c
^2 + 324*a^3*c^3)*d^2)*x^2*e^2 + 2*(25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^
2)*d^2 + 4*(12*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*d^7 + 3*(30*b^5*c -
227*a*b^3*c^2 + 392*a^2*b*c^3)*d^5 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^
2 + 324*a^3*c^3)*d^3 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*d)*x*e -
3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*f^2*x^9*e^10 + 9*(a
^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d*f^2*x^8*e^9 + 2*(a^3*b^5*c - 8*a
^4*b^3*c^2 + 16*a^5*b*c^3 + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d
^2)*f^2*x^7*e^8 + 14*(6*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^3 + (a
^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d)*f^2*x^6*e^7 + (a^3*b^6 - 6*a^4*
b^4*c + 32*a^6*c^3 + 126*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^4 + 4
```

2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^2)*f^2*x^5*e^6 + (126*(a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^5 + 70*(a^3*b^5*c - 8*a^4*b^3*c^2 +
16*a^5*b*c^3)*d^3 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d)*f^2*x^4*e^5 +
2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2 + 42*(a^3*b^4*c^2 - 8*a^4*b^2*c^3
+ 16*a^5*c^4)*d^6 + 35*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^4 + 5*(
a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^2)*f^2*x^3*e^4 + 2*(18*(a^3*b^4*c^2 -
8*a^4*b^2*c^3 + 16*a^5*c^4)*d^7 + 21*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b
*c^3)*d^5 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^3 + 3*(a^4*b^5 - 8*a^5
*b^3*c + 16*a^6*b*c^2)*d)*f^2*x^2*e^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
+ 9*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*d^8 + 14*(a^3*b^5*c - 8*a^4
*b^3*c^2 + 16*a^5*b*c^3)*d^6 + 5*(a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^4 +
6*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*d^2)*f^2*x*e^2 + ((a^3*b^4*c^2 -
8*a^4*b^2*c^3 + 16*a^5*c^4)*d^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c
^3)*d^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*d^5 + 2*(a^4*b^5 - 8*a^5*b^3
*c + 16*a^6*b*c^2)*d^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*d)*f^2*e)*sqr
t(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^
4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 -
640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*f^4*sqrt((625*b^12 -
12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^
4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a
^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*f^8)))*
e^(-2)/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 128
0*a^11*b^2*c^4 - 1024*a^12*c^5)*f^4))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*
c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 8100
00*a^5*c^9)*x*e - 27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6
- 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*d + 27/2*sqrt
(1/2)*((5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c
^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 32358
4*a^14*b^2*c^7 + 122880*a^15*c^8)*f^6*sqrt((625*b^12 - 12250*a*b^10*c + 947
25*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c
^5 + 50625*a^6*c^6)/((a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^
17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)*f^8))*e - (125*b^17 - 3775*
a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 -
4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*
a^8*b*c^8)*f^2*e)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a
^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c
+ 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*
f^4*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^
3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/((a^14*b^10 -
20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1
024*a^19*c^5)*f^8)))*e^(-2)/((a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 6
40*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*f^4))) + 3*sqrt(1/2)*(
(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*f^2*...

Sympy [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

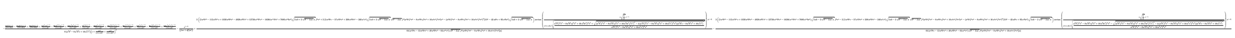
Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**2/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1658 vs. 2(459) = 918.
time = 4.01, size = 1658, normalized size = 3.32



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac"
)
```

```
[Out] -1/8*(7*b^4*c^2*e^(-1)/((f*x*e + d*f)*f) - 47*a*b^2*c^3*e^(-1)/((f*x*e + d*
f)*f) + 52*a^2*c^4*e^(-1)/((f*x*e + d*f)*f) + 14*b^5*c*f*e^(-1)/(f*x*e + d*
f)^3 - 99*a*b^3*c^2*f*e^(-1)/(f*x*e + d*f)^3 + 136*a^2*b*c^3*f*e^(-1)/(f*x*
e + d*f)^3 + 7*b^6*f^3*e^(-1)/(f*x*e + d*f)^5 - 43*a*b^4*c*f^3*e^(-1)/(f*x*
e + d*f)^5 + 25*a^2*b^2*c^2*f^3*e^(-1)/(f*x*e + d*f)^5 + 68*a^3*c^3*f^3*e^(
-1)/(f*x*e + d*f)^5 + 9*a*b^5*f^5*e^(-1)/(f*x*e + d*f)^7 - 66*a^2*b^3*c*f^5
*e^(-1)/(f*x*e + d*f)^7 + 108*a^3*b*c^2*f^5*e^(-1)/(f*x*e + d*f)^7)/((a^3*b
^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c + b*f^2/(f*x*e + d*f)^2 + a*f^4/(f*x*e +
d*f)^4)^2) - e^(-1)/((f*x*e + d*f)*a^3*f) + 3/64*((5*a^6*b^13 - 112*a^7*b^1
1*c + 1030*a^8*b^9*c^2 - 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11
*b^3*c^5 + 7680*a^12*b*c^6)*sqrt(2*a*b + 2*sqrt(b^2 - 4*a*c)*a)*f^8*e^4 + 2
*(5*a^4*b^6*c - 57*a^5*b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b
+ 2*sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*f^4*abs(a^3*b^4*f^4*e^2 - 8*a^4*
b^2*c*f^4*e^2 + 16*a^5*c^2*f^4*e^2)*e^2 - (a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^
4*e^2 + 16*a^5*c^2*f^4*e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*
b + 2*sqrt(b^2 - 4*a*c)*a))*arctan(2*sqrt(1/2)*e^(-1)/((f*x*e + d*f)*f*sqrt
((a^3*b^5*f^4*e^2 - 8*a^4*b^3*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2 + sqrt((a^3*
b^5*f^4*e^2 - 8*a^4*b^3*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2)^2 - 4*(a^4*b^4*f^
8*e^4 - 8*a^5*b^2*c*f^8*e^4 + 16*a^6*c^2*f^8*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^
2 + 16*a^5*c^3)))/(a^4*b^4*f^8*e^4 - 8*a^5*b^2*c*f^8*e^4 + 16*a^6*c^2*f^8*e
^4))))*e^(-3)/((a^7*b^6*c - 12*a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*
sqrt(b^2 - 4*a*c)*f^6*abs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*c^
2*f^4*e^2)*abs(a)) - 3/64*((5*a^6*b^13 - 112*a^7*b^11*c + 1030*a^8*b^9*c^2
- 4928*a^9*b^7*c^3 + 12736*a^10*b^5*c^4 - 16384*a^11*b^3*c^5 + 7680*a^12*b*
c^6)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a)*f^8*e^4 - 2*(5*a^4*b^6*c - 57*a^5*
b^4*c^2 + 208*a^6*b^2*c^3 - 240*a^7*c^4)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)*a
```

```
)*sqrt(b^2 - 4*a*c)*f^4*abs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*
c^2*f^4*e^2)*e^2 - (a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*c^2*f^4*
e^2)^2*(5*b^5 - 42*a*b^3*c + 92*a^2*b*c^2)*sqrt(2*a*b - 2*sqrt(b^2 - 4*a*c)
*a))*arctan(2*sqrt(1/2)*e^(-1)/((f*x*e + d*f)*f*sqrt((a^3*b^5*f^4*e^2 - 8*a
^4*b^3*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2 - sqrt((a^3*b^5*f^4*e^2 - 8*a^4*b^3
*c*f^4*e^2 + 16*a^5*b*c^2*f^4*e^2)^2 - 4*(a^4*b^4*f^8*e^4 - 8*a^5*b^2*c*f^8
*e^4 + 16*a^6*c^2*f^8*e^4)*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(a^4*
b^4*f^8*e^4 - 8*a^5*b^2*c*f^8*e^4 + 16*a^6*c^2*f^8*e^4))))*e^(-3)/((a^7*b^6
*c - 12*a^8*b^4*c^2 + 48*a^9*b^2*c^3 - 64*a^10*c^4)*sqrt(b^2 - 4*a*c)*f^6*a
bs(a^3*b^4*f^4*e^2 - 8*a^4*b^2*c*f^4*e^2 + 16*a^5*c^2*f^4*e^2)*abs(a))
```

Mupad [B]
time = 15.40, size = 2500, normalized size = 5.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^2*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] - atan(((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*
c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 61
26640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160
*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2)
 - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(
-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f
^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^1
2*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4
- 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^1
6*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(271790899200*a^20*c^
14*e^12*f^6 - 230400*a^9*b^22*c^3*e^12*f^6 + 9861120*a^10*b^20*c^4*e^12*f^6
 - 191038464*a^11*b^18*c^5*e^12*f^6 + 2207803392*a^12*b^16*c^6*e^12*f^6 - 1
6878108672*a^13*b^14*c^7*e^12*f^6 + 89374851072*a^14*b^12*c^8*e^12*f^6 - 33
3226967040*a^15*b^10*c^9*e^12*f^6 + 869815812096*a^16*b^8*c^10*e^12*f^6 - 1
543847804928*a^17*b^6*c^11*e^12*f^6 + 1747313491968*a^18*b^4*c^12*e^12*f^6
- 1101055131648*a^19*b^2*c^13*e^12*f^6) - (-(9*(25*b^21 - 25*b^6*(-(4*a*c -
 b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^
15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6
 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225
*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*
c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20
*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10
*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*
f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*
a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4
)))^(1/2)*((-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10
```

*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^10*c^5*e^2*f^4 + 860160*a^13*b^8*c^6*e^2*f^4 - 1966080*a^14*b^6*c^7*e^2*f^4 + 2949120*a^15*b^4*c^8*e^2*f^4 - 2621440*a^16*b^2*c^9*e^2*f^4 - 40*a^8*b^18*c*e^2*f^4)))^(1/2)*(x*(262144*a^15*b^23*c^2*e^14*f^10 - 11534336*a^16*b^21*c^3*e^14*f^10 + 230686720*a^17*b^19*c^4*e^14*f^10 - 2768240640*a^18*b^17*c^5*e^14*f^10 + 22145925120*a^19*b^15*c^6*e^14*f^10 - 124017180672*a^20*b^13*c^7*e^14*f^10 + 496068722688*a^21*b^11*c^8*e^14*f^10 - 1417339207680*a^22*b^9*c^9*e^14*f^10 + 2834678415360*a^23*b^7*c^10*e^14*f^10 - 3779571220480*a^24*b^5*c^11*e^14*f^10 + 3023656976384*a^25*b^3*c^12*e^14*f^10 - 1099511627776*a^26*b*c^13*e^14*f^10) - 1099511627776*a^26*b*c^13*d*e^13*f^10 + 262144*a^15*b^23*c^2*d*e^13*f^10 - 11534336*a^16*b^21*c^3*d*e^13*f^10 + 230686720*a^17*b^19*c^4*d*e^13*f^10 - 2768240640*a^18*b^17*c^5*d*e^13*f^10 + 22145925120*a^19*b^15*c^6*d*e^13*f^10 - 124017180672*a^20*b^13*c^7*d*e^13*f^10 + 496068722688*a^21*b^11*c^8*d*e^13*f^10 - 1417339207680*a^22*b^9*c^9*d*e^13*f^10 + 2834678415360*a^23*b^7*c^10*d*e^13*f^10 - 3779571220480*a^24*b^5*c^11*d*e^13*f^10 + 3023656976384*a^25*b^3*c^12*d*e^13*f^10) - 245760*a^12*b^23*c^2*e^12*f^8 + 10911744*a^13*b^21*c^3*e^12*f^8 - 220397568*a^14*b^19*c^4*e^12*f^8 + 2673082368*a^15*b^17*c^5*e^12*f^8 - 21630025728*a^16*b^15*c^6*e^12*f^8 + 122607894528*a^17*b^13*c^7*e^12*f^8 - 496773365760*a^18*b^11*c^8*e^12*f^8 + 1438679826432*a^19*b^9*c^9*e^12*f^8 - 2918430277632*a^20*b^7*c^10*e^12*f^8 + 3949222428672*a^21*b^5*c^11*e^12*f^8 - 3208340570112*a^22*b^3*c^12*e^12*f^8 + 1185410973696*a^23*b*c^13*e^12*f^8) + 271790899200*a^20*c^14*d*e^11*f^6 - 230400*a^9*b^22*c^3*d*e^11*f^6 + 9861120*a^10*b^20*c^4*d*e^11*f^6 - 191038464*a^11*b^18*c^5*d*e^11*f^6 + 2207803392*a^12*b^16*c^6*d*e^11*f^6 - 16878108672*a^13*b^14*c^7*d*e^11*f^6 + 89374851072*a^14*b^12*c^8*d*e^11*f^6 - 333226967040*a^15*b^10*c^9*d*e^11*f^6 + 869815812096*a^16*b^8*c^10*d*e^11*f^6 - 1543847804928*a^17*b^6*c^11*d*e^11*f^6 + 1747313491968*a^18*b^4*c^12*d*e^11*f^6 - 1101055131648*a^19*b^2*c^13*d*e^11*f^6)*1i + (-(9*(25*b^21 - 25*b^6*(-(4*a*c - b^2)^15)^(1/2) + 18923520*a^10*b*c^10 + 17794*a^2*b^17*c^2 - 188095*a^3*b^15*c^3 + 1299860*a^4*b^13*c^4 - 6126640*a^5*b^11*c^5 + 19905600*a^6*b^9*c^6 - 43904256*a^7*b^7*c^7 + 62684160*a^8*b^5*c^8 - 52039680*a^9*b^3*c^9 + 225*a^3*c^3*(-(4*a*c - b^2)^15)^(1/2) - 995*a*b^19*c - 694*a^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 245*a*b^4*c*(-(4*a*c - b^2)^15)^(1/2)))/(512*(a^7*b^20*e^2*f^4 + 1048576*a^17*c^10*e^2*f^4 + 720*a^9*b^16*c^2*e^2*f^4 - 7680*a^10*b^14*c^3*e^2*f^4 + 53760*a^11*b^12*c^4*e^2*f^4 - 258048*a^12*b^1...

3.660 $\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx$

Optimal. Leaf size=343

$$-\frac{3(b^2-5ac)\,(b^2-2ac)}{2a^3\,(b^2-4ac)^2\,ef^3(d+ex)^2}+\frac{b^2-2ac+bc(d+ex)^2}{4a\,(b^2-4ac)\,ef^3(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)^2}+\frac{3b^4-20ab}{4a^2\,(b^2-4ac)^2}$$

```
[Out] -3/2*(-5*a*c+b^2)*(-2*a*c+b^2)/a^3/(-4*a*c+b^2)^2/e/f^3/(e*x+d)^2+1/4*(b^2-
2*a*c+b*c*(e*x+d)^2)/a/(-4*a*c+b^2)/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d
)^4)^2+1/4*(3*b^4-20*a*b^2*c+20*a^2*c^2+3*b*c*(-6*a*c+b^2)*(e*x+d)^2)/a^2/(
-4*a*c+b^2)^2/e/f^3/(e*x+d)^2/(a+b*(e*x+d)^2+c*(e*x+d)^4)-3/2*(-20*a^3*c^3+
30*a^2*b^2*c^2-10*a*b^4*c+b^6)*arctanh((b+2*c*(e*x+d)^2)/(-4*a*c+b^2)^(1/2)
)/a^4/(-4*a*c+b^2)^(5/2)/e/f^3-3*b*ln(e*x+d)/a^4/e/f^3+3/4*b*ln(a+b*(e*x+d)
^2+c*(e*x+d)^4)/a^4/e/f^3
```

Rubi [A]
time = 0.38, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1156, 1128, 754, 836, 814, 648, 632, 212, 642}

$$\frac{3b\log\left(a+b(d+ex)^2+c(d+ex)^4\right)}{4a^4ef^3}-\frac{3b\log(d+ex)}{a^4ef^3}-\frac{3(b^2-5ac)\,(b^2-2ac)}{2a^3ef^3\,(b^2-4ac)^2\,(d+ex)^2}+\frac{20a^2c^2+3bc(b^2-6ac)\,(d+ex)^2-20ab^2c+3b^4}{4a^2ef^3\,(b^2-4ac)^2\,(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)}-\frac{3(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6)\tanh^{-1}\left(\frac{b+2c(d+ex)^2}{\sqrt{b^2-4ac}}\right)}{2a^4ef^3\,(b^2-4ac)^{5/2}}+\frac{-2ac+b^2+bc(d+ex)^2}{4aef^3\,(b^2-4ac)\,(d+ex)^2\,(a+b(d+ex)^2+c(d+ex)^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2)
+ (b^2 - 2*a*c + b*c*(d + e*x)^2)/(4*a*(b^2 - 4*a*c)*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)^2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b
*c*(b^2 - 6*a*c)*(d + e*x)^2)/(4*a^2*(b^2 - 4*a*c)^2*e*f^3*(d + e*x)^2*(a +
 b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 -
20*a^3*c^3)*ArcTanh[(b + 2*c*(d + e*x)^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 -
 4*a*c)^(5/2)*e*f^3) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3*b*Log[a + b*(d +
 e*x)^2 + c*(d + e*x)^4])/(4*a^4*e*f^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
```

```
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1156

```
Int[(u_)^(m_.)*((a_.) + (b_.)*(v_)^2 + (c_.)*(v_)^4)^(p_.), x_Symbol] :> Di
st[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^2 + c*x^(2*2))^p,
 x], x, v], x] /; FreeQ[{a, b, c, m, p}, x] && LinearPairQ[u, v, x]
```

Rubi steps

$$\int \frac{1}{(df+efx)^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx, x, d+ex\right)}{ef^3}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^3}\,dx, x, (d+ex)^2\right)}{2ef^3}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right.}$$

$$= \frac{b^2-2ac+bc(d+ex)^2}{4a\left(b^2-4ac\right)ef^3(d+ex)^2\left(a+b(d+ex)^2+c(d+ex)^4\right.}$$

$$= -\frac{3(b^2-5ac)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2ef^3(d+ex)^2} + \frac{b^2-}{4a\left(b^2-4ac\right)ef^3(d+}$$

$$= -\frac{3(b^2-5ac)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2ef^3(d+ex)^2} + \frac{b^2-}{4a\left(b^2-4ac\right)ef^3(d+}$$

$$= -\frac{3(b^2-5ac)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2ef^3(d+ex)^2} + \frac{b^2-}{4a\left(b^2-4ac\right)ef^3(d+}$$

$$= -\frac{3(b^2-5ac)\left(b^2-2ac\right)}{2a^3\left(b^2-4ac\right)^2ef^3(d+ex)^2} + \frac{b^2-}{4a\left(b^2-4ac\right)ef^3(d+}$$

Mathematica [A]
time = 6.10, size = 509, normalized size = 1.48

$$-\frac{1}{2a^3ef^3(d+ex)^2}+\frac{b^3-3abc+b^2c(d+ex)^2-2ac^2(d+ex)^2}{4a^2\left(-b^2+4ac\right)ef^3\left(a+b(d+ex)^2+c(d+ex)^4\right)^2}+\frac{-4b^5+29ab^3c-46a^2bc^2-4b^4c(d+ex)^2+26ab^2c^2(d+ex)^2-28a^2c^3(d+ex)^2}{4a^3\left(-b^2+4ac\right)^2ef^3\left(a+b(d+ex)^2+c(d+ex)^4\right)}-\frac{3b\log(d+ex)}{a^4ef^3}+\frac{3\left(b^6-10ab^4c+30a^2b^2c^2-20a^3c^3+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+16a^2bc^2\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{4a^4\left(b^2-4ac\right)^{5/2}ef^3}+\frac{3\left(-b^6+10ab^4c-30a^2b^2c^2+20a^3c^3+b^5\sqrt{b^2-4ac}-8ab^3c\sqrt{b^2-4ac}+16a^2bc^2\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2c(d+ex)^2\right)}{4a^4\left(b^2-4ac\right)^{5/2}ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x]
```

```
[Out] -1/2*1/(a^3*e*f^3*(d + e*x)^2) + (b^3 - 3*a*b*c + b^2*c*(d + e*x)^2 - 2*a*c
^2*(d + e*x)^2)/(4*a^2*(-b^2 + 4*a*c)*e*f^3*(a + b*(d + e*x)^2 + c*(d + e*x
)^4)^2) + (-4*b^5 + 29*a*b^3*c - 46*a^2*b*c^2 - 4*b^4*c*(d + e*x)^2 + 26*a*
b^2*c^2*(d + e*x)^2 - 28*a^2*c^3*(d + e*x)^2)/(4*a^3*(-b^2 + 4*a*c)^2*e*f^3
*(a + b*(d + e*x)^2 + c*(d + e*x)^4)) - (3*b*Log[d + e*x])/(a^4*e*f^3) + (3
*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] -
8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[
b^2 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3) + (3*(-b
^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a
*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2
 - 4*a*c] + 2*c*(d + e*x)^2])/(4*a^4*(b^2 - 4*a*c)^(5/2)*e*f^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.34, size = 1145, normalized size = 3.34

method	result
default	$-\frac{1}{2a^3e(ex+d)^2}-\frac{3b\ln(ex+d)}{a^4e}-\frac{\frac{c^2e^5\left(14a^2c^2-13a\,b^2c+2b^4\right)a\,x^6}{32a^2c^2-16a\,b^2c+2b^4}+\frac{3\left(14a^2c^2-13a\,b^2c+2b^4\right)a\,c^2d\,e^4x^5}{16a^2c^2-8a\,b^2c+b^4}+\frac{e^3ac\left(420a^2c^3d^2-390a\,b^2c^2d^2+60b^4c\,d^2+\right.}{64a^2c^2-32a\,b^2c+4}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^3*(-1/2/a^3/e/(e*x+d)^2-3*b*ln(e*x+d)/a^4/e-1/a^4*((1/2*c^2*e^5*(14*a^2
*c^2-13*a*b^2*c+2*b^4)*a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3*(14*a^2*c^2-13*a*
b^2*c+2*b^4)*a*c^2*d*e^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+1/4*e^3*a*c*(420*a^
2*c^3*d^2-390*a*b^2*c^2*d^2+60*b^4*c*d^2+74*a^2*b*c^2-55*a*b^3*c+8*b^5)/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^4+c*d*e^2*a*(140*a^2*c^3*d^2-130*a*b^2*c^2*d^2+20
*b^4*c*d^2+74*a^2*b*c^2-55*a*b^3*c+8*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3+1/
2*e*a*(210*a^2*c^4*d^4-195*a*b^2*c^3*d^4+30*b^4*c^2*d^4+222*a^2*b*c^3*d^2-1
65*a*b^3*c^2*d^2+24*b^5*c*d^2+18*a^3*c^3+7*a^2*b^2*c^2-12*a*b^4*c+2*b^6)/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^2+d*a*(42*a^2*c^4*d^4-39*a*b^2*c^3*d^4+6*b^4*c^2
*d^4+74*a^2*b*c^3*d^2-55*a*b^3*c^2*d^2+8*b^5*c*d^2+18*a^3*c^3+7*a^2*b^2*c^2
-12*a*b^4*c+2*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x+1/4/e*a*(28*a^2*c^4*d^6-26*
```

```
a*b^2*c^3*d^6+4*b^4*c^2*d^6+74*a^2*b*c^3*d^4-55*a*b^3*c^2*d^4+8*b^5*c*d^4+3
6*a^3*c^3*d^2+14*a^2*b^2*c^2*d^2-24*a*b^4*c*d^2+4*b^6*d^2+58*a^3*b*c^2-36*a
^2*b^3*c+5*a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*e^4*x^4+4*c*d*e^3*x^3+6*c*
d^2*e^2*x^2+4*c*d^3*e*x+b*e^2*x^2+c*d^4+2*b*d*e*x+b*d^2+a)^2+3/2/(16*a^2*c^
2-8*a*b^2*c+b^4)/e*sum((e^3*b*c*(-16*a^2*c^2+8*a*b^2*c-b^4)*_R^3+3*d*e^2*b*
c*(-16*a^2*c^2+8*a*b^2*c-b^4)*_R^2+e*(-48*a^2*b*c^3*d^2+24*a*b^3*c^2*d^2-3*
b^5*c*d^2+10*a^3*c^3-23*a^2*b^2*c^2+9*a*b^4*c-b^6)*_R-16*a^2*b*c^3*d^3+8*a*
b^3*c^2*d^3-b^5*c*d^3+10*a^3*c^3*d-23*a^2*b^2*c^2*d+9*a*b^4*c*d-b^6*d)/(2*_
R^3*c*e^3+6*_R^2*c*d*e^2+6*_R*c*d^2*e+2*c*d^3+_R*b*e+b*d)*ln(x-_R),_R=RootO
f(e^4*c*_Z^4+4*d*e^3*c*_Z^3+(6*c*d^2*e^2+b*e^2)*_Z^2+(4*c*d^3*e+2*b*d*e)*_Z
+d^4*c+d^2*b+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="maxim
a")
```

```
[Out] -1/4*(6*(b^4*c^2 - 7*a*b^2*c^3 + 10*a^2*c^4)*d^8 + 48*(b^4*c^2*e^7 - 7*a*b^
2*c^3*e^7 + 10*a^2*c^4*e^7)*d*x^7 + 6*(b^4*c^2*e^8 - 7*a*b^2*c^3*e^8 + 10*a
^2*c^4*e^8)*x^8 + 3*(4*b^5*c - 29*a*b^3*c^2 + 46*a^2*b*c^3)*d^6 + 3*(4*b^5*
c*e^6 - 29*a*b^3*c^2*e^6 + 46*a^2*b*c^3*e^6 + 56*(b^4*c^2*e^6 - 7*a*b^2*c^3
*e^6 + 10*a^2*c^4*e^6)*d^2)*x^6 + 2*a^2*b^4 - 16*a^3*b^2*c + 32*a^4*c^2 + 6
*(56*(b^4*c^2*e^5 - 7*a*b^2*c^3*e^5 + 10*a^2*c^4*e^5)*d^3 + 3*(4*b^5*c*e^5
- 29*a*b^3*c^2*e^5 + 46*a^2*b*c^3*e^5)*d)*x^5 + 2*(3*b^6 - 18*a*b^4*c + 7*a
^2*b^2*c^2 + 50*a^3*c^3)*d^4 + (6*b^6*e^4 - 36*a*b^4*c*e^4 + 14*a^2*b^2*c^2
*e^4 + 100*a^3*c^3*e^4 + 420*(b^4*c^2*e^4 - 7*a*b^2*c^3*e^4 + 10*a^2*c^4*e^
4)*d^4 + 45*(4*b^5*c*e^4 - 29*a*b^3*c^2*e^4 + 46*a^2*b*c^3*e^4)*d^2)*x^4 +
4*(84*(b^4*c^2*e^3 - 7*a*b^2*c^3*e^3 + 10*a^2*c^4*e^3)*d^5 + 15*(4*b^5*c*e^
3 - 29*a*b^3*c^2*e^3 + 46*a^2*b*c^3*e^3)*d^3 + 2*(3*b^6*e^3 - 18*a*b^4*c*e^
3 + 7*a^2*b^2*c^2*e^3 + 50*a^3*c^3*e^3)*d)*x^3 + (9*a*b^5 - 68*a^2*b^3*c +
122*a^3*b*c^2)*d^2 + (168*(b^4*c^2*e^2 - 7*a*b^2*c^3*e^2 + 10*a^2*c^4*e^2)*
d^6 + 9*a*b^5*e^2 - 68*a^2*b^3*c*e^2 + 122*a^3*b*c^2*e^2 + 45*(4*b^5*c*e^2
- 29*a*b^3*c^2*e^2 + 46*a^2*b*c^3*e^2)*d^4 + 12*(3*b^6*e^2 - 18*a*b^4*c*e^2
 + 7*a^2*b^2*c^2*e^2 + 50*a^3*c^3*e^2)*d^2)*x^2 + 2*(24*(b^4*c^2*e - 7*a*b^
2*c^3*e + 10*a^2*c^4*e)*d^7 + 9*(4*b^5*c*e - 29*a*b^3*c^2*e + 46*a^2*b*c^3*
e)*d^5 + 4*(3*b^6*e - 18*a*b^4*c*e + 7*a^2*b^2*c^2*e + 50*a^3*c^3*e)*d^3 +
(9*a*b^5*e - 68*a^2*b^3*c*e + 122*a^3*b*c^2*e)*d)*x)/(10*(a^3*b^4*c^2*e^10
- 8*a^4*b^2*c^3*e^10 + 16*a^5*c^4*e^10)*d*f^3*x^9 + (a^3*b^4*c^2*e^11 - 8*a
^4*b^2*c^3*e^11 + 16*a^5*c^4*e^11)*f^3*x^10 + (2*a^3*b^5*c*e^9 - 16*a^4*b^3
*c^2*e^9 + 32*a^5*b*c^3*e^9 + 45*(a^3*b^4*c^2*e^9 - 8*a^4*b^2*c^3*e^9 + 16*
a^5*c^4*e^9)*d^2)*f^3*x^8 + 8*(15*(a^3*b^4*c^2*e^8 - 8*a^4*b^2*c^3*e^8 + 16
```

```
*a^5*c^4*e^8)*d^3 + 2*(a^3*b^5*c*e^8 - 8*a^4*b^3*c^2*e^8 + 16*a^5*b*c^3*e^8
)*d)*f^3*x^7 + (a^3*b^6*e^7 - 6*a^4*b^4*c*e^7 + 32*a^6*c^3*e^7 + 210*(a^3*b
^4*c^2*e^7 - 8*a^4*b^2*c^3*e^7 + 16*a^5*c^4*e^7)*d^4 + 56*(a^3*b^5*c*e^7 -
8*a^4*b^3*c^2*e^7 + 16*a^5*b*c^3*e^7)*d^2)*f^3*x^6 + 2*(126*(a^3*b^4*c^2*e^
6 - 8*a^4*b^2*c^3*e^6 + 16*a^5*c^4*e^6)*d^5 + 56*(a^3*b^5*c*e^6 - 8*a^4*b^3
*c^2*e^6 + 16*a^5*b*c^3*e^6)*d^3 + 3*(a^3*b^6*e^6 - 6*a^4*b^4*c*e^6 + 32*a^
6*c^3*e^6)*d)*f^3*x^5 + (2*a^4*b^5*e^5 - 16*a^5*b^3*c*e^5 + 32*a^6*b*c^2*e^
5 + 210*(a^3*b^4*c^2*e^5 - 8*a^4*b^2*c^3*e^5 + 16*a^5*c^4*e^5)*d^6 + 140*(a
^3*b^5*c*e^5 - 8*a^4*b^3*c^2*e^5 + 16*a^5*b*c^3*e^5)*d^4 + 15*(a^3*b^6*e^5
- 6*a^4*b^4*c*e^5 + 32*a^6*c^3*e^5)*d^2)*f^3*x^4 + 4*(30*(a^3*b^4*c^2*e^4 -
 8*a^4*b^2*c^3*e^4 + 16*a^5*c^4*e^4)*d^7 + 28*(a^3*b^5*c*e^4 - 8*a^4*b^3*c^
2*e^4 + 16*a^5*b*c^3*e^4)*d^5 + 5*(a^3*b^6*e^4 - 6*a^4*b^4*c*e^4 + 32*a^6*c
^3*e^4)*d^3 + 2*(a^4*b^5*e^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4)*d)*f^3*x
^3 + (a^5*b^4*e^3 - 8*a^6*b^2*c*e^3 + 16*a^7*c^2*e^3 + 45*(a^3*b^4*c^2*e^3
- 8*a^4*b^2*c^3*e^3 + 16*a^5*c^4*e^3)*d^8 + 56*(a^3*b^5*c*e^3 - 8*a^4*b^3*c
^2*e^3 + 16*a^5*b*c^3*e^3)*d^6 + 15*(a^3*b^6*e^3 - 6*a^4*b^4*c*e^3 + 32*a^6
*c^3*e^3)*d^4 + 12*(a^4*b^5*e^3 - 8*a^5*b^3*c*e^3 + 16*a^6*b*c^2*e^3)*d^2)*
f^3*x^2 + 2*(5*(a^3*b^4*c^2*e^2 - 8*a^4*b^2*c^3*e^2 + 16*a^5*c^4*e^2)*d^9 +
 8*(a^3*b^5*c*e^2 - 8*a^4*b^3*c^2*e^2 + 16*a^5*b*c^3*e^2)*d^7 + 3*(a^3*b^6*
e^2 - 6*a^4*b^4*c*e^2 + 32*a^6*c^3*e^2)*d^5 + 4*(a^4*b^5*e^2 - 8*a^5*b^3*c*
e^2 + 16*a^6*b*c^2*e^2)*d^3 + (a^5*b^4*e^2 - 8*a^6*b^2*c*e^2 + 16*a^7*c^2*e
^2)*d)*f^3*x + ((a^3*b^4*c^2*e - 8*a^4*b^2*c^3*e + 16*a^5*c^4*e)*d^10 + 2*(
a^3*b^5*c*e - 8*a^4*b^3*c^2*e + 16*a^5*b*c^3*e)*d^8 + (a^3*b^6*e - 6*a^4*b^
4*c*e + 32*a^6*c^3*e)*d^6 + 2*(a^4*b^5*e - 8*a^5*b^3*c*e + 16*a^6*b*c^2*e)*
d^4 + (a^5*b^4*e - 8*a^6*b^2*c*e + 16*a^7*c^2*e)*d^2)*f^3) + 3*integrate(((
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d^3 + 3*(b^5*c*e^2 - 8*a*b^3*c^2*e^2 +
16*a^2*b*c^3*e^2)*d*x^2 + (b^5*c*e^3 - 8*a*b^3*c^2*e^3 + 16*a^2*b*c^3*e^3)*
x^3 + (b^6 - 9*a*b^4*c + 23*a^2*b^2*c^2 - 10*a^3*c^3)*d + (b^6*e - 9*a*b^4*
c*e + 23*a^2*b^2*c^2*e - 10*a^3*c^3*e + 3*(b^5*c*e - 8*a*b^3*c^2*e + 16*a^2
*b*c^3*e)*d^2)*x)/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*d^4 + b*d^2 + (6*c*d^2*e^2
 + b*e^2)*x^2 + 2*(2*c*d^3*e + b*d*e)*x + a), x)/((a^4*b^4 - 8*a^5*b^2*c +
16*a^6*c^2)*f^3) - 3*b*e^(-1)*log(x*e + d)/(a^4*f^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7510 vs. $2(336) = 672$.
time = 4.19, size = 15147, normalized size = 44.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="frica
s")
```

```
[Out] [-1/4*(6*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*x^8*e^8
 + 48*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d*x^7*e^7
```

+ 2*a^3*b^6 - 24*a^4*b^4*c + 96*a^5*b^2*c^2 - 128*a^6*c^3 + 6*(a*b^6*c^2 -
11*a^2*b^4*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^8 + 3*(4*a*b^7*c - 45*a^2*b
^5*c^2 + 162*a^3*b^3*c^3 - 184*a^4*b*c^4 + 56*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^2)*x^6*e^6 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2
+ 162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^6 + 6*(56*(a*b^6*c^2 - 11*a^2*b^4*c^3
+ 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^3 + 3*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^
3*b^3*c^3 - 184*a^4*b*c^4)*d)*x^5*e^5 + (6*a*b^8 - 60*a^2*b^6*c + 158*a^3*b
^4*c^2 + 44*a^4*b^2*c^3 - 400*a^5*c^4 + 420*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 3
8*a^3*b^2*c^4 - 40*a^4*c^5)*d^4 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*
b^3*c^3 - 184*a^4*b*c^4)*d^2)*x^4*e^4 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*
b^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^4 + 4*(84*(a*b^6*c^2 - 11*a^2*b^4
*c^3 + 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^5 + 15*(4*a*b^7*c - 45*a^2*b^5*c^2 +
162*a^3*b^3*c^3 - 184*a^4*b*c^4)*d^3 + 2*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b
^4*c^2 + 22*a^4*b^2*c^3 - 200*a^5*c^4)*d)*x^3*e^3 + (9*a^2*b^7 - 104*a^3*b^
5*c + 394*a^4*b^3*c^2 - 488*a^5*b*c^3 + 168*(a*b^6*c^2 - 11*a^2*b^4*c^3 + 3
8*a^3*b^2*c^4 - 40*a^4*c^5)*d^6 + 45*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3*
b^3*c^3 - 184*a^4*b*c^4)*d^4 + 12*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^2)*x^2*e^2 + (9*a^2*b^7 - 104*a^3*b^5*c +
 394*a^4*b^3*c^2 - 488*a^5*b*c^3)*d^2 + 2*(24*(a*b^6*c^2 - 11*a^2*b^4*c^3 +
 38*a^3*b^2*c^4 - 40*a^4*c^5)*d^7 + 9*(4*a*b^7*c - 45*a^2*b^5*c^2 + 162*a^3
*b^3*c^3 - 184*a^4*b*c^4)*d^5 + 4*(3*a*b^8 - 30*a^2*b^6*c + 79*a^3*b^4*c^2
+ 22*a^4*b^2*c^3 - 200*a^5*c^4)*d^3 + (9*a^2*b^7 - 104*a^3*b^5*c + 394*a^4*
b^3*c^2 - 488*a^5*b*c^3)*d)*x*e + 3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c
^4 - 20*a^3*c^5)*x^10*e^10 + 10*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 -
20*a^3*c^5)*d*x^9*e^9 + (b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c
^5)*d^10 + (2*b^7*c - 20*a*b^5*c^2 + 60*a^2*b^3*c^3 - 40*a^3*b*c^4 + 45*(b^
6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^2)*x^8*e^8 + 2*(b^7*c
 - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^8 + 8*(15*(b^6*c^2 - 10*
a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^3 + 2*(b^7*c - 10*a*b^5*c^2 + 30
*a^2*b^3*c^3 - 20*a^3*b*c^4)*d)*x^7*e^7 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2
 + 40*a^3*b^2*c^3 - 40*a^4*c^4 + 210*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c
^4 - 20*a^3*c^5)*d^4 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b
*c^4)*d^2)*x^6*e^6 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 4
0*a^4*c^4)*d^6 + 2*(126*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c
^5)*d^5 + 56*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^3 + 3
*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d)*x^5*e^
5 + (2*a*b^7 - 20*a^2*b^5*c + 60*a^3*b^3*c^2 - 40*a^4*b*c^3 + 210*(b^6*c^2
- 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^6 + 140*(b^7*c - 10*a*b^5*c
^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^4 + 15*(b^8 - 8*a*b^6*c + 10*a^2*b^4*
c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^2)*x^4*e^4 + 2*(a*b^7 - 10*a^2*b^5*c +
 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^4 + 4*(30*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^
2*b^2*c^4 - 20*a^3*c^5)*d^7 + 28*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 2
0*a^3*b*c^4)*d^5 + 5*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 4
0*a^4*c^4)*d^3 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d
)*x^3*e^3 + (45*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*d^8

```
+ a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3 + 56*(b^7*c - 10*a*b
^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*d^6 + 15*(b^8 - 8*a*b^6*c + 10*a^2*
b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*d^4 + 12*(a*b^7 - 10*a^2*b^5*c + 30*
a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2)*x^2*e^2 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4
*b^2*c^2 - 20*a^5*c^3)*d^2 + 2*(5*(b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4
- 20*a^3*c^5)*d^9 + 8*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4
)*d^7 + 3*(b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*
d^5 + 4*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^3 + (a^2*b
^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*d)*x*e)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^4*e^4 + 8*c^2*d*x^3*e^3 + 2*c^2*d^4 + 2*b*c*d^2 + 2*(6*c^2*d^2
 + b*c)*x^2*e^2 + 4*(2*c^2*d^3 + b*c*d)*x*e + b^2 - 2*a*c + (2*c*x^2*e^2 +
4*c*d*x*e + 2*c*d^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4*e^4 + 4*c*d*x^3*e^3 + c*
d^4 + (6*c*d^2 + b)*x^2*e^2 + b*d^2 + 2*(2*c*d^3 + b*d)*x*e + a)) - 3*((b^7
*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^10*e^10 + 10*(b^7*c^
2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d*x^9*e^9 + (b^7*c^2 - 12
*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*d^10 + (2*b^8*c - 24*a*b^6*c^2
+ 96*a^2*b^4*c^3 - 128*a^3*b^2*c^4 + 45*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^
3*c^4 - 64*a^3*b*c^5)*d^2)*x^8*e^8 + 2*(b^8*c - 12*a*b^6*c^2 + 48*a^2*b^4*c
^3 - 64*a^3*b^2*c^4)*d^8 + 8*(15*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 -
 64*a^3*b*c^5)*d^3 + 2*(b^8*c - 12*a*b^6*c^2 + ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)**3/(a+b*(e*x+d)**2+c*(e*x+d)**4)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. $2(336) = 672$.
time = 4.83, size = 1735, normalized size = 5.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*f*x+d*f)^3/(a+b*(e*x+d)^2+c*(e*x+d)^4)^3,x, algorithm="giac"
)
```

```
[Out] 3/4*((a^4*b^8*c*f^3*e^3 - 14*a^5*b^6*c^2*f^3*e^3 + 70*a^6*b^4*c^3*f^3*e^3 -
 140*a^7*b^2*c^4*f^3*e^3 + 80*a^8*c^5*f^3*e^3)*sqrt(b^2 - 4*a*c)*log(abs(b*
x^2*e^2 + 2*b*d*x*e + sqrt(b^2 - 4*a*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e
 + b*d^2 + sqrt(b^2 - 4*a*c)*d^2 + 2*a)) - (a^4*b^8*c*f^3*e^3 - 14*a^5*b^6*
c^2*f^3*e^3 + 70*a^6*b^4*c^3*f^3*e^3 - 140*a^7*b^2*c^4*f^3*e^3 + 80*a^8*c^5
```

```
*f^3*e^3)*sqrt(b^2 - 4*a*c)*log(abs(-b*x^2*e^2 - 2*b*d*x*e + sqrt(b^2 - 4*a
*c)*x^2*e^2 + 2*sqrt(b^2 - 4*a*c)*d*x*e - b*d^2 + sqrt(b^2 - 4*a*c)*d^2 - 2
*a)))/(a^8*b^8*c*f^6*e^4 - 16*a^9*b^6*c^2*f^6*e^4 + 96*a^10*b^4*c^3*f^6*e^4
 - 256*a^11*b^2*c^4*f^6*e^4 + 256*a^12*c^5*f^6*e^4) - 1/4*(6*b^4*c^2*x^8*e^
8 - 42*a*b^2*c^3*x^8*e^8 + 60*a^2*c^4*x^8*e^8 + 48*b^4*c^2*d*x^7*e^7 - 336*
a*b^2*c^3*d*x^7*e^7 + 480*a^2*c^4*d*x^7*e^7 + 168*b^4*c^2*d^2*x^6*e^6 - 117
6*a*b^2*c^3*d^2*x^6*e^6 + 1680*a^2*c^4*d^2*x^6*e^6 + 336*b^4*c^2*d^3*x^5*e^
5 - 2352*a*b^2*c^3*d^3*x^5*e^5 + 3360*a^2*c^4*d^3*x^5*e^5 + 420*b^4*c^2*d^4
*x^4*e^4 - 2940*a*b^2*c^3*d^4*x^4*e^4 + 4200*a^2*c^4*d^4*x^4*e^4 + 336*b^4*
c^2*d^5*x^3*e^3 - 2352*a*b^2*c^3*d^5*x^3*e^3 + 3360*a^2*c^4*d^5*x^3*e^3 + 1
68*b^4*c^2*d^6*x^2*e^2 - 1176*a*b^2*c^3*d^6*x^2*e^2 + 1680*a^2*c^4*d^6*x^2*
e^2 + 48*b^4*c^2*d^7*x*e - 336*a*b^2*c^3*d^7*x*e + 480*a^2*c^4*d^7*x*e + 6*
b^4*c^2*d^8 - 42*a*b^2*c^3*d^8 + 60*a^2*c^4*d^8 + 12*b^5*c*x^6*e^6 - 87*a*b
^3*c^2*x^6*e^6 + 138*a^2*b*c^3*x^6*e^6 + 72*b^5*c*d*x^5*e^5 - 522*a*b^3*c^2
*d*x^5*e^5 + 828*a^2*b*c^3*d*x^5*e^5 + 180*b^5*c*d^2*x^4*e^4 - 1305*a*b^3*c
^2*d^2*x^4*e^4 + 2070*a^2*b*c^3*d^2*x^4*e^4 + 240*b^5*c*d^3*x^3*e^3 - 1740*
a*b^3*c^2*d^3*x^3*e^3 + 2760*a^2*b*c^3*d^3*x^3*e^3 + 180*b^5*c*d^4*x^2*e^2
 - 1305*a*b^3*c^2*d^4*x^2*e^2 + 2070*a^2*b*c^3*d^4*x^2*e^2 + 72*b^5*c*d^5*x*
e - 522*a*b^3*c^2*d^5*x*e + 828*a^2*b*c^3*d^5*x*e + 12*b^5*c*d^6 - 87*a*b^3
*c^2*d^6 + 138*a^2*b*c^3*d^6 + 6*b^6*x^4*e^4 - 36*a*b^4*c*x^4*e^4 + 14*a^2*
b^2*c^2*x^4*e^4 + 100*a^3*c^3*x^4*e^4 + 24*b^6*d*x^3*e^3 - 144*a*b^4*c*d*x^
3*e^3 + 56*a^2*b^2*c^2*d*x^3*e^3 + 400*a^3*c^3*d*x^3*e^3 + 36*b^6*d^2*x^2*e
^2 - 216*a*b^4*c*d^2*x^2*e^2 + 84*a^2*b^2*c^2*d^2*x^2*e^2 + 600*a^3*c^3*d^2
*x^2*e^2 + 24*b^6*d^3*x*e - 144*a*b^4*c*d^3*x*e + 56*a^2*b^2*c^2*d^3*x*e +
400*a^3*c^3*d^3*x*e + 6*b^6*d^4 - 36*a*b^4*c*d^4 + 14*a^2*b^2*c^2*d^4 + 100
*a^3*c^3*d^4 + 9*a*b^5*x^2*e^2 - 68*a^2*b^3*c*x^2*e^2 + 122*a^3*b*c^2*x^2*e
^2 + 18*a*b^5*d*x*e - 136*a^2*b^3*c*d*x*e + 244*a^3*b*c^2*d*x*e + 9*a*b^5*d
^2 - 68*a^2*b^3*c*d^2 + 122*a^3*b*c^2*d^2 + 2*a^2*b^4 - 16*a^3*b^2*c + 32*a
^4*c^2)/((a^3*b^4*f^3*e - 8*a^4*b^2*c*f^3*e + 16*a^5*c^2*f^3*e)*(c*x^5*e^5
+ 5*c*d*x^4*e^4 + 10*c*d^2*x^3*e^3 + 10*c*d^3*x^2*e^2 + 5*c*d^4*x*e + c*d^5
 + b*x^3*e^3 + 3*b*d*x^2*e^2 + 3*b*d^2*x*e + b*d^3 + a*x*e + a*d)^2) + 3/4*
b*e^(-1)*log(abs(c*x^4*e^4 + 4*c*d*x^3*e^3 + 6*c*d^2*x^2*e^2 + 4*c*d^3*x*e
+ c*d^4 + b*x^2*e^2 + 2*b*d*x*e + b*d^2 + a))/(a^4*f^3) - 3*b*e^(-1)*log(ab
s(x*e + d))/(a^4*f^3)
```

Mupad [B]
time = 24.91, size = 2500, normalized size = 7.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*f + e*f*x)^3*(a + b*(d + e*x)^2 + c*(d + e*x)^4)^3),x)
```

```
[Out] (log(((27*c^5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c -
b^2)^6) - ((3*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b
```

^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^
2 - 7*a*b^2*c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^
4*c - 47*a*b^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*f^6*(4*a*c - b^2)^4) - ((3
*b - 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*
e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c
*d^2 + 46*a^2*b^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30
*a^2*b^2*c^3*d^2))/(a^3*f^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b - 3*a^4*e*f
^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c
 - b^2)^5))^(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*
e*x - 10*a*c*e^2*x^2 - 20*a*c*d*e*x))/(a^4*f^3) + (6*c^3*e^18*x^2*(b^6 + 10
0*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2) + (12*c^
3*d*e^17*x*(b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*
c - b^2)^2)))/(4*a^4*e*f^3) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4*c^4 + 479*
a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4) + (
18*b*c^4*d*e^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3
 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4)))/(4*a^4*e*f^3) + (27*c^4*e^14*(b
^4 + 10*a^2*c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3
*d^2 - 8*a*b^3*c - 7*a*b^2*c^2*d^2))/(a^9*f^9*(4*a*c - b^2)^6) + (54*c^5*d*
e^15*x*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6))*((27*c^
5*e^16*x^2*(b^4 + 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6) - ((
3*b + 3*a^4*e*f^3*(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8
*e^2*f^6*(4*a*c - b^2)^5))^(1/2))*((9*c^3*e^15*(b^4 + 10*a^2*c^2 - 7*a*b^2*
c)*(4*b^6 - 10*a^3*c^3 + 6*b^5*c*d^2 + 71*a^2*b^2*c^2 - 33*a*b^4*c - 47*a*b
^3*c^2*d^2 + 90*a^2*b*c^3*d^2))/(a^6*f^6*(4*a*c - b^2)^4) - ((3*b + 3*a^4*e
f^3(-(b^6 - 20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a
c - b^2)^5))^(1/2))((6*c^2*e^16*(2*b^7 - 20*a^3*b*c^3 + b^6*c*d^2 + 46*a^
2*b^3*c^2 + 100*a^3*c^4*d^2 - 18*a*b^5*c - 2*a*b^4*c^2*d^2 - 30*a^2*b^2*c^3
*d^2))/(a^3*f^3*(4*a*c - b^2)^2) + (b*c^2*e^16*(3*b + 3*a^4*e*f^3*(-(b^6 -
20*a^3*c^3 + 30*a^2*b^2*c^2 - 10*a*b^4*c)^2/(a^8*e^2*f^6*(4*a*c - b^2)^5))^
(1/2))*(a*b + 3*b^2*d^2 + 3*b^2*e^2*x^2 - 10*a*c*d^2 + 6*b^2*d*e*x - 10*a*c
*e^2*x^2 - 20*a*c*d*e*x))/(a^4*f^3) + (6*c^3*e^18*x^2*(b^6 + 100*a^3*c^3 -
30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2) + (12*c^3*d*e^17*x*(
b^6 + 100*a^3*c^3 - 30*a^2*b^2*c^2 - 2*a*b^4*c))/(a^3*f^3*(4*a*c - b^2)^2))
)/(4*a^4*e*f^3) + (9*b*c^4*e^17*x^2*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2
- 1100*a^3*b^2*c^3 - 89*a*b^6*c))/(a^6*f^6*(4*a*c - b^2)^4) + (18*b*c^4*d*e
^16*x*(6*b^8 + 900*a^4*c^4 + 479*a^2*b^4*c^2 - 1100*a^3*b^2*c^3 - 89*a*b^6*
c))/(a^6*f^6*(4*a*c - b^2)^4)))/(4*a^4*e*f^3) + (27*c^4*e^14*(b^4 + 10*a^2*
c^2 - 7*a*b^2*c)^2*(b^5 + 16*a^2*b*c^2 + b^4*c*d^2 + 10*a^2*c^3*d^2 - 8*a*b
^3*c - 7*a*b^2*c^2*d^2))/(a^9*f^9*(4*a*c - b^2)^6) + (54*c^5*d*e^15*x*(b^4
+ 10*a^2*c^2 - 7*a*b^2*c)^3)/(a^9*f^9*(4*a*c - b^2)^6)))*(6*b^11*e*f^3 - 12
0*a*b^9*c*e*f^3 - 6144*a^5*b*c^5*e*f^3 + 960*a^2*b^7*c^2*e*f^3 - 3840*a^3*b
^5*c^3*e*f^3 + 7680*a^4*b^3*c^4*e*f^3))/(2*(4*a^4*b^10*e^2*f^6 - 4096*a^9*c
^5*e^2*f^6 + 640*a^6*b^6*c^2*e^2*f^6 - 2560*a^7*b^4*c^3*e^2*f^6 + 5120*a^8*
b^2*c^4*e^2*f^6 - 80*a^5*b^8*c*e^2*f^6)) - ((x^4*(6*b^6*e^3 + 100*a^3*c^3*e
^3 + 180*b^5*c*d^2*e^3 + 14*a^2*b^2*c^2*e^3 + 4200*a^2*c^4*d^4*e^3 + 420*b^

4*c^2*d^4*e^3 - 36*a*b^4*c*e^3 - 1305*a*b^3*c^2*d^2*e^3 + 2070*a^2*b*c^3*d^2*e^3 - 2940*a*b^2*c^3*d^4*e^3))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^6*(4*b^5*c*e^5 - 29*a*b^3*c^2*e^5 + 46*a^2*b*c^3*e^5 + 560*a^2*c^4*d^2*e^5 + 56*b^4*c^2*d^2*e^5 - 392*a*b^2*c^3*d^2*e^5))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x*(12*b^6*d^3 + 36*b^5*c*d^5 + 200*a^3*c^3*d^3 + 240*a^2*c^4*d^7 + 24*b^4*c^2*d^7 + 9*a*b^5*d - 261*a*b^3*c^2*d^5 + 414*a^2*b*c^3*d^5 - 168*a*b^2*c^3*d^7 + 28*a^2*b^2*c^2*d^3 - 68*a^2*b^3*c*d + 122*a^3*b*c^2*d - 72*a*b^4*c*d^3))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^5*(560*a^2*c^4*d^3*e^4 + 56*b^4*c^2*d^3*e^4 + 12*b^5*c*d*e^4 - 87*a*b^3*c^2*d*e^4 + 138*a^2*b*c^3*d*e^4 - 392*a*b^2*c^3*d^3*e^4))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (3*x^8*(10*a^2*c^4*e^7 + b^4*c^2*e^7 - 7*a*b^2*c^3*e^7))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x^2*(36*b^6*d^2*e + 9*a*b^5*e + 600*a^3*c^3*d^2*e + 1680*a^2*c^4*d^6*e + 168*b^4*c^2*d^6*e - 68*a^2*b^3*c*e + 122*a^3*b*c^2*e + 180*b^5*c*d^4*e - 216*a*b^4*c*d^2*e - 1305*a*b^3*c^2*d^4*e + 2070*a^2*b*c^3*d^4*e - 1176*a*b^2*c^3*d^6*e + 84*a^2*b^2*c^2*d^2*e))/(4*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)) + (x^3*(6*b^6*d*e^2 + 100*a^3*c^3*d*e^2 + 60*b^5*c*d^3*e^2 + 840*a^2*c^4*d^5*e^2 + 84*b^4*c^2*d^5*e^2 - 36*a*b^4*c*d*e^2 + 14*a^2*b^2*c^2*d*e^2 - 435*a*b^3*c^2*d^3*e^2 + 690*a^2*b*c^3*d^3*e^2 - 588*a*b^2*c^3*d^5*e^2))/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (12*x^7*(10*a^2*c^4*d*e^6 + b^4*c^2*d*e^6 -...

3.661 $\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$

Optimal. Leaf size=340

$$-\frac{d(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}+\cdots$$

```
[Out] -d*(e*x+d)*AppellF1(1/3,1/2,1/2,4/3,-2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)),-
2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2
)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/e^2/(a+b*(e*x+d)^3
+c*(e*x+d)^6)^(1/2)+1/2*(e*x+d)^2*AppellF1(2/3,1/2,1/2,5/3,-2*c*(e*x+d)^3/(
b-(-4*a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*(e*x+d)
^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)/e^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)
```

Rubi [A]

time = 0.48, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1403, 1804, 1362, 440, 1399, 524}

$$\frac{(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{2e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^2\sqrt{a+b(d+ex)^3+c(d+ex)^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

```
[Out] -((d*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-
2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2
- 4*a*c])])/(e^2*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])) + ((d + e*x)^2*S
qrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)^
3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3)
/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(2*e
^2*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1403

```
Int[((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIntegrand[(x -
 Coefficient[v, x, 0])^m*(a + b*x^n + c*x^(2*n))^p, x], x], x, v], x] /; Fr
eeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[v, x] && IntegerQ[m] &&
NeQ[v, x]
```

Rule 1804

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \frac{\text{Subst}\left(\int \frac{-d+x}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^2}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{d}{\sqrt{a+bx^3+cx^6}}+\frac{x}{\sqrt{a+bx^3+cx^6}}\right)dx, x, d+ex\right)}{e^2}$$

$$= \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^2} - \frac{d\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^3+}}\right.}{e^2}$$

$$= \frac{\left(\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\right)\text{Subst}\left(\int \frac{}{\sqrt{1+}}\right.}{e^2\sqrt{a+b(d+ex)^3+c(}}$$

$$= -\frac{d(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2}\right.}{e^2\sqrt{a+b(d+ex)^3+c(d+ex}}$$

Mathematica [F]
time = 10.80, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
[Out] Integrate[x/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+b\,(ex+d)^3+c\,(ex+d)^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
[Out] int(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt((x*e + d)^6*c + (x*e + d)^3*b + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(c*x^6*e^6 + 6*c*d*x^5*e^5 + 15*c*d^2*x^4*e^4 + c*d^6 + (20*
c*d^3 + b)*x^3*e^3 + b*d^3 + 3*(5*c*d^4 + b*d)*x^2*e^2 + 3*(2*c*d^5 + b*d^2
)*x*e + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bd^3 + 3bd^2ex + 3bde^2x^2 + be^3x^3 + cd^6 + 6cd^5ex + 15cd^4e^2x^2 + 20cd^3e^3x^3 + 15cd^2e^4x^4 + 6cde^5x^5 + ce^6x^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**3 +
 c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*c*d
**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt((x*e + d)^6*c + (x*e + d)^3*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{a+b\,{\left(d+e\,x\right)}^3+c\,{\left(d+e\,x\right)}^6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2),x)
[Out] int(x/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2), x)
```

3.662 $\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$

Optimal. Leaf size=398

$$\frac{d^2(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-$$

```
[Out] 1/3*arctanh(1/2*(b+2*c*(e*x+d)^3)/c^(1/2)/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)
)/e^3/c^(1/2)+d^2*(e*x+d)*AppellF1(1/3,1/2,1/2,4/3,-2*c*(e*x+d)^3/(b-(-4*a*
c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*(e*x+d)^3/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/e^3
/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)-d*(e*x+d)^2*AppellF1(2/3,1/2,1/2,5/3,-2*
c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)),-2*c*(e*x+d)^3/(b+(-4*a*c+b^2)^(1/2)))*(
1+2*c*(e*x+d)^3/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*(e*x+d)^3/(b+(-4*a*c+b
^2)^(1/2)))^(1/2)/e^3/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 398, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1403, 1804, 1362, 440, 1399, 524, 1366, 635, 212}

$$\frac{d^2(d+ex)\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}-\frac{d(d+ex)^2\sqrt{\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2c(d+ex)^3}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}},-\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}\right)}{e^3\sqrt{a+b(d+ex)^3+c(d+ex)^6}}+\frac{\tanh^{-1}\left(\frac{b+2c(d+ex)^3}{2\sqrt{c}\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\right)}{3\sqrt{c}\,e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
```

```
[Out] (d^2*(d + e*x)*Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/3, 1/2, 1/2, 4/3, (-
2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2
- 4*a*c])])/(e^3*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) - (d*(d + e*x)^2*
Sqrt[1 + (2*c*(d + e*x)^3)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*(d + e*x)
^3)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[2/3, 1/2, 1/2, 5/3, (-2*c*(d + e*x)^3
)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*(d + e*x)^3)/(b + Sqrt[b^2 - 4*a*c])])/(e^
3*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6]) + ArcTanh[(b + 2*c*(d + e*x)^3)/
(2*Sqrt[c]*Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6])]/(3*Sqrt[c]*e^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1362

```
Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^
IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*c*(x^n/(b + Rt[b^2
- 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4*a*c, 2])))^FracPar
t[p])), Int[(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2*c*(x^n/(b - Sq
rt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] &
& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1403

```
Int[((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbo
l] :> Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIntegrand[(x -
```

```
 Coefficient[v, x, 0])^m*(a + b*x^n + c*x^(2*n))^p, x], x], x, v], x] /; Fr
eeQ[{a, b, c, n, p}, x] && EqQ[n2, 2*n] && LinearQ[v, x] && IntegerQ[m] &&
NeQ[v, x]
```

Rule 1804

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Mo
dule[{q = Expon[Pq, x], j, k}, Int[Sum[x^j*Sum[Coeff[Pq, x, j + k*n]*x^(k*n
), {k, 0, (q - j)/n + 1}]*(a + b*x^n + c*x^(2*n))^p, {j, 0, n - 1}], x]] /;
 FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && IGtQ[n, 0] &&  !PolyQ[Pq, x^n]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx = \frac{\text{Subst}\left(\int \frac{(-d+x)^2}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{d^2}{\sqrt{a+bx^3+cx^6}} - \frac{2dx}{\sqrt{a+bx^3+cx^6}} + \frac{x^2}{\sqrt{a+bx^3+cx^6}} \ldots\right.\right.}{e^3}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^3+cx^6}}\,dx, x, d+ex\right)}{e^3} - \frac{(2d)\text{Subst}\left(\int \frac{\ldots}{\sqrt{a+b\ldots}}\right.}{\ldots}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, (d+ex)^3\right)}{3e^3} - \frac{\left(2d\sqrt{1+\frac{2c(d+\ldots}{b-\sqrt{b^2\ldots}}}\right.}{\ldots}$$

$$= \frac{d^2(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\ldots\right.}{e^3\sqrt{a+b(d+ex)^3+c(d+ex\ldots}}$$

$$= \frac{d^2(d+ex)\sqrt{1+\frac{2c(d+ex)^3}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c(d+ex)^3}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{3};\frac{1}{2},\ldots\right.}{e^3\sqrt{a+b(d+ex)^3+c(d+ex\ldots}}$$

Mathematica [F]

time = 10.33, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+b(d+ex)^3+c(d+ex)^6}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6],x]
[Out] Integrate[x^2/Sqrt[a + b*(d + e*x)^3 + c*(d + e*x)^6], x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+b\left(ex+d\right)^3+c\left(ex+d\right)^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
[Out] int(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt((x*e + d)^6*c + (x*e + d)^3*b + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="fricas")
[Out] integral(x^2/sqrt(c*x^6*e^6 + 6*c*d*x^5*e^5 + 15*c*d^2*x^4*e^4 + c*d^6 + (2
0*c*d^3 + b)*x^3*e^3 + b*d^3 + 3*(5*c*d^4 + b*d)*x^2*e^2 + 3*(2*c*d^5 + b*d
^2)*x*e + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bd^3+3bd^2ex+3bde^2x^2+be^3x^3+cd^6+6cd^5ex+15cd^4e^2x^2+20cd^3e^3x^3+15cd^2e^4x^4+6cde^5x^5+ce^6x^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(e*x+d)**3+c*(e*x+d)**6)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*d**3 + 3*b*d**2*e*x + 3*b*d*e**2*x**2 + b*e**3*x**
3 + c*d**6 + 6*c*d**5*e*x + 15*c*d**4*e**2*x**2 + 20*c*d**3*e**3*x**3 + 15*
c*d**2*e**4*x**4 + 6*c*d*e**5*x**5 + c*e**6*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(e*x+d)^3+c*(e*x+d)^6)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt((x*e + d)^6*c + (x*e + d)^3*b + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a+b\,{\left(d+e\,x\right)}^3+c\,{\left(d+e\,x\right)}^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2),x)
```

```
[Out] int(x^2/(a + b*(d + e*x)^3 + c*(d + e*x)^6)^(1/2), x)
```

3.663 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)\,dx$

Optimal. Leaf size=34

$$\frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}$$

```
[Out] 1/21*(2+3*x)^7+1/42*(2+3*x)^14+1/63*(2+3*x)^21
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1404, 14}

$$\frac{1}{63}(3x+2)^{21}+\frac{1}{42}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 1404

```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n + c*x^(
2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && L
inearPairQ[u, v, x]
```

Rubi steps

$$\begin{aligned}\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)\,dx &= \frac{1}{3}\text{Subst}\left(\int x^6\left(1+x^7+x^{14}\right)\,dx,x,2+3x\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(x^6+x^{13}+x^{20}\right)\,dx,x,2+3x\right)\\ &= \frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 34, normalized size = 1.00

$$\frac{1}{21}(2+3x)^7+\frac{1}{42}(2+3x)^{14}+\frac{1}{63}(2+3x)^{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14),x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/42 + (2 + 3*x)^21/63
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(28) = 56.
time = 0.21, size = 105, normalized size = 3.09

method	result
gosper	$\frac{x(2324522934x^{20}+32543321076x^{19}+216955473840x^{18}+916034222880x^{17}+2748102668640x^{16}+6229032715584x^{15}+110738359388$
default	$1056832x+15808800x^2+149902032x^3+1010576952x^4+5149786572x^5+20588764518x^6+6$
norman	$1056832x+15808800x^2+149902032x^3+1010576952x^4+5149786572x^5+20588764518x^6+6$
risch	$1056832x+15808800x^2+149902032x^3+1010576952x^4+5149786572x^5+20588764518x^6+6$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x,method=_RETURNVERBOSE)
```

```
[Out] 1056832*x+15808800*x^2+149902032*x^3+1010576952*x^4+5149786572*x^5+20588764
518*x^6+66158154783*x^7+173635132896*x^8+376174427616*x^9+677082445416*x^10
+1015602174288*x^11+1269491970942*x^12+1318314865122*x^13+15819767221203/14
*x^14+790988281344*x^15+444930908256*x^16+196293047760*x^17+65431015920*x^1
8+15496819560*x^19+2324522934*x^20+1162261467/7*x^21
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(28) = 56.
time = 0.28, size = 104, normalized size = 3.06

$$\frac{1162261467}{7}x^{21}+2324522934x^{20}+15496819560x^{19}+65431015920x^{18}+196293047760x^{17}+444930908256x^{16}+790988281344x^{15}+\frac{15819767221203}{14}x^{14}+1318314865122x^{13}+1269491970942x^{12}+1015602174288x^{11}+677082445416x^{10}+376174427616x^9+173635132896x^8+66158154783x^7+20588764518x^6+5149786572x^5+1010576952x^4+149902032x^3+15808800x^2+1056832x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="maxima")
```

```
[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18 +
 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 15819767221203
/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11 + 6
77082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^7 +
```

20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 1580880
0*x^2 + 1056832*x

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(28) = 56$.
time = 0.36, size = 104, normalized size = 3.06

$$\frac{1162261467}{7}\,x^{21} + 2324522934\,x^{20} + 15496819560\,x^{19} + 65431015920\,x^{18} + 196293047760\,x^{17} + 444930908256\,x^{16} + 790988281344\,x^{15} + \frac{15819767221203}{14}\,x^{14} + 1318314865122\,x^{13} + 1269491970942\,x^{12} + 1015602174288\,x^{11} + 677082445416\,x^{10} + 376174427616\,x^{9} + 173635132896\,x^{8} + 66158154783\,x^{7} + 20588764518\,x^{6} + 5149786572\,x^{5} + 1010576952\,x^{4} + 149902032\,x^{3} + 15808800\,x^{2} + 1056832\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="fricas")
```

```
[Out] 1162261467/7*x^21 + 2324522934*x^20 + 15496819560*x^19 + 65431015920*x^18 +
 196293047760*x^17 + 444930908256*x^16 + 790988281344*x^15 + 15819767221203
/14*x^14 + 1318314865122*x^13 + 1269491970942*x^12 + 1015602174288*x^11 + 6
77082445416*x^10 + 376174427616*x^9 + 173635132896*x^8 + 66158154783*x^7 +
20588764518*x^6 + 5149786572*x^5 + 1010576952*x^4 + 149902032*x^3 + 1580880
0*x^2 + 1056832*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(24) = 48$.
time = 0.03, size = 107, normalized size = 3.15

$$\frac{1162261467x^{21}}{7} + 2324522934x^{20} + 15496819560x^{19} + 65431015920x^{18} + 196293047760x^{17} + 444930908256x^{16} + 790988281344x^{15} + \frac{15819767221203x^{14}}{14} + 1318314865122x^{13} + 1269491970942x^{12} + 1015602174288x^{11} + 677082445416x^{10} + 376174427616x^{9} + 173635132896x^{8} + 66158154783x^{7} + 20588764518x^{6} + 5149786572x^{5} + 1010576952x^{4} + 149902032x^{3} + 15808800x^{2} + 1056832x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14),x)
```

```
[Out] 1162261467*x**21/7 + 2324522934*x**20 + 15496819560*x**19 + 65431015920*x**
18 + 196293047760*x**17 + 444930908256*x**16 + 790988281344*x**15 + 1581976
7221203*x**14/14 + 1318314865122*x**13 + 1269491970942*x**12 + 101560217428
8*x**11 + 677082445416*x**10 + 376174427616*x**9 + 173635132896*x**8 + 6615
8154783*x**7 + 20588764518*x**6 + 5149786572*x**5 + 1010576952*x**4 + 14990
2032*x**3 + 15808800*x**2 + 1056832*x
```

Giac [A]
time = 3.13, size = 28, normalized size = 0.82

$$\frac{1}{63}\,(3\,x+2)^{21} + \frac{1}{42}\,(3\,x+2)^{14} + \frac{1}{21}\,(3\,x+2)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14),x, algorithm="giac")
```

```
[Out] 1/63*(3*x + 2)^21 + 1/42*(3*x + 2)^14 + 1/21*(3*x + 2)^7
```

Mupad [B]
time = 1.58, size = 29, normalized size = 0.85

$$\frac{(3x+2)^7 \left(3 (3x+2)^7+2 (3x+2)^{14}+6\right)}{126}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1),x)
[Out] ((3*x + 2)^7*(3*(3*x + 2)^7 + 2*(3*x + 2)^14 + 6))/126
```

3.664 $\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{1}{21}(2+3x)^7+\frac{1}{21}(2+3x)^{14}+\frac{1}{21}(2+3x)^{21}+\frac{1}{42}(2+3x)^{28}+\frac{1}{105}(2+3x)^{35}$$

```
[Out] 1/21*(2+3*x)^7+1/21*(2+3*x)^14+1/21*(2+3*x)^21+1/42*(2+3*x)^28+1/105*(2+3*x
)^35
```

Rubi [A]
time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1404, 1366, 625}

$$\frac{1}{105}(3x+2)^{35}+\frac{1}{42}(3x+2)^{28}+\frac{1}{21}(3x+2)^{21}+\frac{1}{21}(3x+2)^{14}+\frac{1}{21}(3x+2)^7$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] (2 + 3*x)^7/21 + (2 + 3*x)^14/21 + (2 + 3*x)^21/21 + (2 + 3*x)^28/42 + (2 +
 3*x)^35/105
```

Rule 625

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1366

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 1404

```
Int[(u_)^(m_.)*((a_.) + (c_.)*(v_)^(n2_.) + (b_.)*(v_)^(n_))^(p_.), x_Symbo
l] :> Dist[u^m/(Coefficient[v, x, 1]*v^m), Subst[Int[x^m*(a + b*x^n + c*x^(
2*n))^p, x], x, v], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && L
inearPairQ[u, v, x]
```

Rubi steps

$$\int (2+3x)^6 \left(1+(2+3x)^7+(2+3x)^{14}\right)^2 \, dx = \frac{1}{3}\text{Subst}\left(\int x^6 \left(1+x^7+x^{14}\right)^2 \, dx, x, 2+3x\right)$$
$$= \frac{1}{21}\text{Subst}\left(\int \left(1+x+x^2\right)^2 \, dx, x, (2+3x)^7\right)$$
$$= \frac{1}{21}\text{Subst}\left(\int \left(1+2x+3x^2+2x^3+x^4\right) \, dx, x, (2+3x)^7\right)$$
$$= \frac{1}{21}(2+3x)^7 + \frac{1}{21}(2+3x)^{14} + \frac{1}{21}(2+3x)^{21} + \frac{1}{42}(2+3x)^2$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 188 vs. 2(56) = 112.
time = 0.01, size = 188, normalized size = 3.36

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^6*(1 + (2 + 3*x)^7 + (2 + 3*x)^14)^2,x]
```

```
[Out] 17451466816*x + 443569828128*x^2 + 7299544818384*x^3 + 87406679578680*x^4 +
 (4057390785756924*x^5)/5 + 6077684727888102*x^6 + 37727143432895007*x^7 +
197897276851452864*x^8 + 889942562270387136*x^9 + (17344958593049772048*x^1
0)/5 + 11821487501620716192*x^11 + 35454069480572048124*x^12 + 940692639189
29616324*x^13 + 221699757548270194389*x^14 + 465517091041681015296*x^15 + 8
72775774067455498528*x^16 + 1463104032160519033200*x^17 + 21945771660147522
40080*x^18 + 2945285062308448290360*x^19 + 3534290697929473864098*x^20 + (2
6506949038858918036881*x^21)/7 + 3614565944605222108800*x^22 + 306451507651
2846852480*x^23 + 2298383223254096766840*x^24 + (7584660010542711771792*x^2
5)/5 + 875152864622814086340*x^26 + 437576396725285446564*x^27 + (262545832
6972530284475*x^28)/14 + 67899784121041365504*x^29 + (101849676181562048256
*x^30)/5 + 4928210137817518464*x^31 + 924039400840784712*x^32 + 12600537284
1925188*x^33 + 11118121133111046*x^34 + (16677181699666569*x^35)/35
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(46) = 92.
time = 0.24, size = 175, normalized size = 3.12

method	result
gosper	$\frac{x(333543633399333138x^{34}+7782684793177773220x^{33}+88203760989347631 60x^{32}+64682758058854929840x^{31}+344974709647226292}{}$
default	$17451466816x+924039400840784712x^{32}+4928210137817518464x^{31}+\frac{101849676181562048256}{5}x^{30}$
norman	$17451466816x+924039400840784712x^{32}+4928210137817518464x^{31}+\frac{101849676181562048256}{5}x^{30}$

risch	$17451466816x + 924039400840784712x^{32} + 4928210137817518464x^{31} + \frac{101849676181562048256}{5}x^{30} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 17451466816*x+924039400840784712*x^32+4928210137817518464*x^31+101849676181
562048256/5*x^30+16677181699666569/35*x^35+111181211331110 46*x^34+126005372
841925188*x^33+30645150765128468524 80*x^23+3614565944605222108800*x^22+8751
52864622814086340*x^26+7584660010542711771792/5*x^25+2298383223254096766840
*x^24+437576396725285446564*x^27+197897276851452864*x^8+889942562270387136*
x^9+37727143432895007*x^7+6077684727888102*x^6+4057390785756924/5*x^5+443569828
128*x^2+7299544818384*x^3+4057390785756924/5*x^5+17344958593049772048/5*x^1
0+35454069480572048124*x^12+94069263918929616324*x^13+221699757548270194389
*x^14+465517091041681015296*x^15+3534290697929473864098*x^20+29452850623084
48290360*x^19+2194577166014752240080*x^18+1463104032160519033200*x^17+87277
5774067455498528*x^16+26506949038858918036881/7*x^21+678997841210413655 04*x
^29+2625458326972530284475/14*x^28+1182148750162071 6192*x^11
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(46) = 92.
time = 0.27, size = 174, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="maxima")
```

```
[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^3
3 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 101849676181562048
256/5*x^30 + 67899784121041365504*x^29 + 2625458326972530284475/14*x^28 + 4
37576396725285446564*x^27 + 875152864622814086340*x^26 + 758466001054271177
1792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^23 + 3
614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290697929
473864098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*x^18
 + 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 46551709104168
1015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x^13 + 354
54069480572048124*x^12 + 11821487501620716192*x^11 + 17344958593049772048/5
*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 37727143432895007
*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 +
 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(46) = 92.
time = 0.40, size = 174, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="fricas")
```

```
[Out] 16677181699666569/35*x^35 + 11118121133111046*x^34 + 126005372841925188*x^3
3 + 924039400840784712*x^32 + 4928210137817518464*x^31 + 101849676181562048
256/5*x^30 + 67899784121041365504*x^29 + 2625458326972530284475/14*x^28 + 4
37576396725285446564*x^27 + 875152864622814086340*x^26 + 758466001054271177
1792/5*x^25 + 2298383223254096766840*x^24 + 3064515076512846852480*x^23 + 3
614565944605222108800*x^22 + 26506949038858918036881/7*x^21 + 3534290697929
473864098*x^20 + 2945285062308448290360*x^19 + 2194577166014752240080*x^18
+ 1463104032160519033200*x^17 + 872775774067455498528*x^16 + 46551709104168
1015296*x^15 + 221699757548270194389*x^14 + 94069263918929616324*x^13 + 354
54069480572048124*x^12 + 11821487501620716192*x^11 + 17344958593049772048/5
*x^10 + 889942562270387136*x^9 + 197897276851452864*x^8 + 37727143432895007
*x^7 + 6077684727888102*x^6 + 4057390785756924/5*x^5 + 87406679578680*x^4 +
 7299544818384*x^3 + 443569828128*x^2 + 17451466816*x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(41) = 82$.
time = 0.05, size = 187, normalized size = 3.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)**6*(1+(2+3*x)**7+(2+3*x)**14)**2,x)
```

```
[Out] 16677181699666569*x**35/35 + 11118121133111046*x**34 + 126005372841925188*x
**33 + 924039400840784712*x**32 + 4928210137817518464*x**31 + 1018496761815
62048256*x**30/5 + 67899784121041365504*x**29 + 2625458326972530284475*x**2
8/14 + 437576396725285446564*x**27 + 875152864622814086340*x**26 + 75846600
10542711771792*x**25/5 + 2298383223254096766840*x**24 + 3064515076512846852
480*x**23 + 3614565944605222108800*x**22 + 26506949038858918036881*x**21/7
+ 3534290697929473864098*x**20 + 2945285062308448290360*x**19 + 21945771660
14752240080*x**18 + 1463104032160519033200*x**17 + 872775774067455498528*x*
*16 + 465517091041681015296*x**15 + 221699757548270194389*x**14 + 940692639
18929616324*x**13 + 35454069480572048124*x**12 + 11821487501620716192*x**11
 + 17344958593049772048*x**10/5 + 889942562270387136*x**9 + 197897276851452
864*x**8 + 37727143432895007*x**7 + 6077684727888102*x**6 + 405739078575692
4*x**5/5 + 87406679578680*x**4 + 7299544818384*x**3 + 443569828128*x**2 + 1
7451466816*x
```

Giac [A]
time = 3.30, size = 46, normalized size = 0.82

$$\frac{1}{105}\left(3\,x+2\right)^{35}+\frac{1}{42}\left(3\,x+2\right)^{28}+\frac{1}{21}\left(3\,x+2\right)^{21}+\frac{1}{21}\left(3\,x+2\right)^{14}+\frac{1}{21}\left(3\,x+2\right)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)^6*(1+(2+3*x)^7+(2+3*x)^14)^2,x, algorithm="giac")
```

```
[Out] 1/105*(3*x + 2)^35 + 1/42*(3*x + 2)^28 + 1/21*(3*x + 2)^21 + 1/21*(3*x + 2)
^14 + 1/21*(3*x + 2)^7
```

Mupad [B]
time = 1.60, size = 46, normalized size = 0.82

$$\frac{(3\,x+2)^7}{21}+\frac{(3\,x+2)^{14}}{21}+\frac{(3\,x+2)^{21}}{21}+\frac{(3\,x+2)^{28}}{42}+\frac{(3\,x+2)^{35}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x + 2)^6*((3*x + 2)^7 + (3*x + 2)^14 + 1)^2,x)
```

```
[Out] (3*x + 2)^7/21 + (3*x + 2)^14/21 + (3*x + 2)^21/21 + (3*x + 2)^28/42 + (3*x
 + 2)^35/105
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

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        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

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#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).___name___ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
          return expnType(expn.operands()[0])   #expnType(expn.args[0])
       elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
          if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```